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Being Mathematical Relations: Dynamic Gestures Support Mathematical Reasoning

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Abstract: In mathematics classrooms, body-based actions, including gestures, offer an important way for students to *become* mathematical ideas as they engage in mathematical practices. In particular, a type of gesture that we call a *dynamic depictive gesture* allows learners to model and represent fluid transformations of mathematical objects with their bodies. In this paper, we report on two empirical studies – one in which dynamic gestures were observed, and one where these gestures were directed. We conclude that dynamic gestures are a key element in successful justification and proof activities in mathematics.

Introduction

According to the embodied cognition perspective, body-based behaviors associated with intellectual performance are not merely epiphenomenal, but are constitutive of the mental process (Wilson, 2002). One way learning is enhanced is by *grounding* abstract and unfamiliar ideas through situated action and body states (e.g., Barsalou, 2008). Some accounts proffer reciprocity between the action system and cognition such that cognitive states and goals can lead directly (and unconsciously) to actions and actions can induce cognitive states (Nathan et al., under review); in this way, the very boundaries between thinking and acting become blurred. Such embodied perspectives are particularly salient for the domain of mathematics. In effect, one way of *knowing* a mathematical relationship is by *being* the relationship. In particular, learners can enact and therefore *become* mathematical relations is by using *gestures*, an important type of body-based action.

Recent empirical findings lend support to this view. Abrahamson Trninic, and Gutierrez (2012) explored how enactment of the covariation of two constant rates helped foster proportional reasoning. Gerofsky (2011) describes students' accounts of how the ways in which they "become" the Cartesian graphs affects their understanding of the mathematical relations represented. Petrick and Martin (2012) discuss how having high school students physically enact rather than observe geometric relations improved learning gains on conceptual assessment items. As learners engage in the situated practice of mathematical reasoning, body-based actions are an important element of how they become competent members of a community of practice. The ICLS theme, *Learning and Becoming in Practice*, thus connects to our work. Our studies were inspired by observations of teachers and students using gestures to represent dynamic mathematical ideas (Walkington et al., in press), and our research underscores the importance of body-based actions as a way of becoming in mathematical practice.

Here we expand on current research by focusing on a subset of gestures and the utility of these gestures for supporting students' reasoning abilities. Specifically, we explore the distinction between *static* depictive gestures, which display an unmoving, unchanging mathematical object in bodily form, and *dynamic* depictive gestures, which display a mathematical object being transformed using the affordances of the body. We report on two empirical studies designed to explore the nature of mathematical reasoning in the form of proof practices and how these practices are influenced by action. Study 1 uses an observational approach to examine how gestures that occur spontaneously relate to one's justifying and proving. Study 2 uses an experimental paradigm to investigate whether directed gestures can improve proof practices. We explore the implications of these findings for theories of learning and instruction, with a focus on the enactment of mathematical relations.

Theoretical Framework

Embodied Cognition

Theories of embodied cognition posit that cognition is deeply rooted in action and perception (Wilson, 2002). This perspective rejects the view that cognition involves algorithmic processes that use amodal symbol systems, and identifies the body itself as a crucial element in cognition. This implies that mental representations of objects are experiential, perception-based, and multimodal. In mathematics, embodied theories stand in stark contrast to a view of mathematics as an amodal, transcendental, objective feature of the universe. Instead, embodied theories view mathematics as constructed of body-based experiences of human beings with the world (Lakoff & Nunez, 2000). For example, understanding of number is spatial and tied to bodily orientations (Dehaene, Bossini, & Giraux, 1993), and children approach arithmetic problems using modeling approaches in which they manipulate objects or count with their fingers (Carpenter & Moser, 1984). When learning fractions, actions coupled with interpretations serve as developmental precursors to general mathematical procedures, which can later be enacted mentally (Martin & Schwartz, 2005). Even when working with algebraic equations, students perceive symbols and equations as having concrete, spatial and perceptual qualities (Landy, Brooks, & Smout, 2012). Thus, we posit that all mathematical cognition is embodied. In this work, we study a particular type of body-based action that provides evidence for the embodiment of cognitive processes – gesture.

Gesture

Gestures are an form of action (Goldin-Meadow & Beilock, 2010) that has been theorized to emerge from embodied perceptual and motor simulations that underlie mental imagery and language processing (Hostetter & Alibali, 2008). Gestures can guide attention and communicate spatial, relational, and embodied concepts (Alibali, Nathan, & Fujimori, 2011). Gestures can also serve to link ideas and representations, with gestural catchments (i.e., repeated iconic gestures, see McNeill & Duncan, 2000) creating structural mappings between different entities to show relatedness (Alibali et al., 2011; Nathan, 2008). Recent research has begun to explore how performing gestures can influence the gesturer's thought processes (Goldin-Meadow & Beilock, 2010; Goldin-Meadow, Cook, & Mitchell, 2009). For instance, requiring students to to represent ideas through gesture supports long-term retention of concepts (Cook, Mitchell, & Goldin-Meadow, 2008), and directing students to gesture can instigate the creation of novel ideas (Goldin-Meadow et al., 2009).

One important type of gesture is *depictive* or *iconic* gestures (McNeill, 1992). Here, speakers directly represent objects or ideas with their bodies – e.g., they may form two crossing line segments with their hands, or use their fingers to connect three sides of a triangle. Our research on gesture during mathematical problem solving, as well as research of others (e.g., Goksun et al., 2013), suggests an important distinction between two types of depictive gestures. In *static depictive gestures*, problem-solvers represent an object (like a triangle or line segment), but do not attempt to directly act upon that object. The gesture shows a static representation of a single object that is not interacting with other objects. In *dynamic depictive gestures*, problem-solvers first represent an object, and then engage in fluid transformations of that object using the affordances of their body. For example, a problem-solver might "collapse" a triangle formed with their hands into two line segments on top of each other, or create a rectangle with their hands that "grows" as they move their hands outwards. From an embodied cognition perspective, physical action both results from and initiates cognitive states; thus, performing dynamic gestures with the body might both be a by-product of reasoning processes *and* also give rise to novel ideas. In this paper, we explore the idea of a dynamic gesture, and show how these gestures are important in two studies of students' reasoning when engaging in justification and proof activities in geometry.

Justification and Proof

Justification and proof are challenging practices for students to master as they reach secondary mathematics classes, particularly high school geometry, which more heavily emphasizes this type of complex mathematical thinking (Healy & Hoyles, 2000). Research has shown that students often test examples rather than engaging in general justification (Knuth et al., 2002), and rely on description and perception rather than formal mathematical reasoning (Jones, 2000). Proofs that are mathematically valid have three key characteristics: (1) they are general and show that the argument is true for all possible cases; (2) they involve operational thought with a progression through sub-goals that correctly anticipate the results of mathematical transformations; (3) they involve logical inference in which conclusions are drawn from valid premises (Harel & Sowder, 2005). Harel and Sowder (2005) distinguish such valid proofs from other types of proof by naming them *transformational proofs*. One tool that supports understanding of mathematical proofs with action is Dynamic Geometry Systems. These systems allow students to engage in action-based manipulations of objects on a screen, in order to support students' understanding and exploration of mathematical conjectures (Christou et al., 2004; Marrades & Gutierrez, 2000). Although these technology systems are powerful, we argue that dynamic depictive gestures can provide some of the same affordances, while also remaining highly portable, flexible, and personalized.

Research Purpose

Here we report two studies of learners' engaging in geometric proofs. As we observed learners' proof activities, we discovered that there was a particular class of gestures – dynamic depictive gestures – that seemed important in valid reasoning processes. Thus our overarching research purpose was to explore these dynamic gestures in the context of mathematical conjectures. We examined the dynamic gestures that learners spontaneously produce (Study 1) and we specifically directed learners to use dynamic gestures and tested the effects (Study 2). In Study 1, we identify and describe the characteristics of dynamic gestures and their association with different types of reasoning practices. In Study 2, we seek to tease apart an important distinction regarding whether dynamic gestures are simply the natural result or by-product of valid mathematical reasoning, or whether they also function to spur novel insights by allowing learners to experience geometric ideas in body-based form. Pen and paper, critical resources typically used by students in mathematics class, were removed in both studies to encourage participants to use their bodies.

Study 1

Our research questions in Study 1 were: (1) What are the characteristics of the gestures that problem-solvers spontaneously produce when justifying a set of geometry conjectures? (2) How are different types of gestures related to production of valid proofs? and (3) How does freedom of gesture production (standing without writing materials vs. seated with writing materials) influence (a) the types of gestures participants produce, and (b) the nature of their mathematical reasoning and proof practices?

Method

Students were solicited to participate in a problem solving experiment on the campus of a selective private university in the South. Fifteen students (9 female, average age = 20.7 years) were asked to provide justifications for 7 geometric conjectures that were mathematically true or false (Table 1). Eleven of the participants had taken Calculus I or higher. Two conditions were alternated among participants. In both conditions, participants were asked to read aloud the conjectures and generate concurrent verbal reports (i.e., think alouds) while being videoed. Participants in the control condition were seated facing the conjectures displayed on screen in front of them and were given a paper and pen. The interviewer sat off to the side but facing the participants and gave only scripted prompts. Participants in the treatment condition were asked to stand within a defined area facing the screen. No paper or pen was provided. The interviewer stood in the same place facing the students and gave the same scripted prompts. Conjectures were presented in random order.

Table 1: Geometric conjectures given to participants in Study 1

If you double the length and width of a rectangle, then the area is also doubled. *(False)*	Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements. *(False)*
The area of a parallelogram is the same as the area of a rectangle with the same length and height. *(True)*	The sum of the lengths of two sides of a triangle is always greater than the length of the third side. *(True)*
The diagonals of a rectangle are always congruent (i.e., they have the same length). *(True)*	The segment that joins the midpoints of two sides of any triangle, called the *midsegment*, is parallel to the third side. *(True)*
All four-sided figures have angles that add up to 360 degrees. *(True)*	

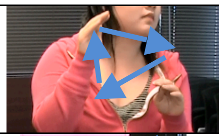
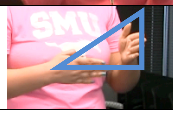
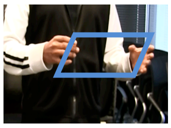
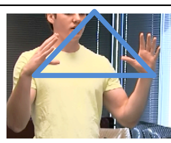
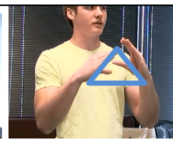
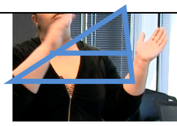
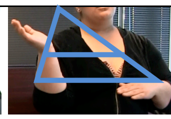
Participants' speech and gestures were examined from video. Justifications were analyzed to determine if a participant judged a conjecture to be true or false (*T/F Judgment*). Proofs were analyzed as to whether participants constructed a valid, transformational proof of the conjecture (*Proof*). Gestures were coded into 4 categories: (1) The participant made only static gestures that represented a stationary mathematical object (*Static*), (2) the participant made at least one dynamic gesture that involved a movement-based transformation of a mathematical object (*Dynamic*), (3) the participant drew on their paper and potentially used pointing gestures indicating positions on the paper (*Drawing*), or (4) the participant made no gestures or drawing actions (*None*). The Drawing code could co-occur with Static or Dynamic – for example, the participant may have begun their justification by producing a drawing and gesturing at it, but then abandoned that drawing to engage in standalone Static or Dynamic gestures that were not related to their drawing. Analyses were conducted based on 15 participants generating 7 justifications each (15 × 7 = 105). One justification was missing due to a video malfunction, and another due to a participants' refusal to give a response, for a final count of 103 justifications.

Results

1. Characteristics of Gestures

We noted several different types of depictive gestures that occurred as students provided justifications for the conjectures, which are illustrated in Table 2 below. We coded whether the referent object of the gesture (e.g., the triangle, rectangle, line segment, etc. the gesturer is modeling) was static (i.e., non-moving) or dynamic (i.e., moving). We thus call a gesture that displays a static object to be a static gesture, and a gesture that displays a dynamic object to be a dynamic gesture. This distinction is illustrated with the Static-Trace category in Table 2 – although a trace gesture involves continuous dynamic movement on the part of the gesturer as they outline an object, the *object being depicted itself* is static and non-moving, and thus this was classified as a static gesture. Among the seven conjectures, all had instances of participants using static gestures and drawing, while six of the seven had instances of participants using dynamic gestures. Of the 15 participants, 11 used dynamic gestures at some point while justifying a conjecture, and 14 used static gestures.

Table 2: Types of depictive gestures observed

Referent Object is…	Gesture Type	Description	Image
Static	*Trace*	Participant traces over the outline of a stationary line or a shape in the air, similar to drawing on a page. *(On right, participant traces triangle in the air)*	
Static	*Represent*	Participant uses hands or fingers to physically formulate a complete or semi-complete object. *(On right, participant forms a right triangle)*	
Dynamic	*Rotate/ Reflect/ Translate*	Participant "picks up" an object represented with their hands/fingers, and then slides, rotates or reflects to change its orientation or position. *(On right, participant makes a parallelogram with hands, and then slides the two consecutive angles together to show they equal 180°)*	 
Dynamic	*Dilate*	Participant begins by representing a static object with hands, and then moves hands outwards and inwards to show the object growing or shrinking. *(On right, participant makes the triangle he forms larger then smaller)*	 
Dynamic	*Test Interactivity*	Participant modifies one element of an object to predict impact on the rest of the object. *(On right, participant shifts apex of triangle to see what happens to midsegment)*	 

2. Relationship between Gestures and Valid Proofs

We hypothesized that the dynamic gestures in particular were an important way to engage with mathematical relationships during justification practices and to formulate transformational proofs. Indeed, this is the premise behind dynamic geometry systems like Geometer's Sketchpad and Cabri Geometry. Although this was a small data set, we looked at trends in the relationship between gesture types and valid proofs to see if this idea was worthy of further investigation. Results are shown in Table 3. Dynamic gestures were associated with the highest accuracy on both true/false judgments and production of transformational proof. Making no gestures or drawings was associated with the lowest performance. Static gestures and drawing actions fell in between.

Table 3: Associations between gesture codes and average accuracy, for each geometric conjecture ($N = 103$)

Gesture Performed	% Correct on T/F Judgment	% Formulating Valid Proof
Dynamic	90.9%	63.6%
Static	74.3%	34.3%
Drawing	84.4%	27.3%
None	57.7%	11.5%

Table 4: Associations between condition, gestures, and average accuracy, for each conjecture ($N = 103$)

Condition	Accuracy on T/F	Accuracy on Proof	% of justifications involving dynamic gesture(s)	% of justifications involving only static gestures
Pen	76.4%	20.0%	12.7%	30.9%
No Pen	75.5%	52.1%	30.6%	36.7%

3. Relationship between Condition (Pen/No Pen) and Valid Proofs

Given that drawing gestures were associated with lower accuracy on formulating valid proofs than depictive gestures (static and dynamic), we also investigated how students' accuracy and tendency to gesture varied by whether or not they were given a pen and paper. Results are shown in Table 4. Participants in the "No Pen" condition had higher accuracy when formulating proofs, and were more likely to make dynamic gestures.

Discussion

This study suggests that dynamic gestures are an important component of formulating and communicating valid proofs in geometry. Dynamic gestures may be promoted when learners are denied tools of pen and paper. These traditional tools may in some cases be less productive for mathematical reasoning than simply encouraging students to use their bodies. This study also suggests some characteristics of dynamic gestures used in geometry, and illustrates different types of dynamic gestures used across a variety of conjectures. Dynamic gestures appear to be an important part of justification and proof activities, so their use should be encouraged by providing students with greater freedom to gesture. In Study 2, we investigate the potential of dynamic gestures further in an experimental paradigm, by explicitly directing participants to perform dynamic gestures prior to a proof task.

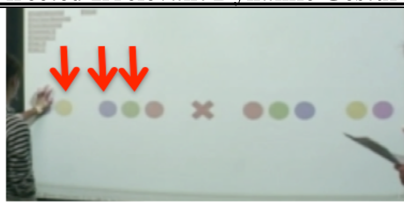
Study 2

Our research questions in Study 2 were: (1) Does explicitly directing participants to perform dynamic depictive gestures influence their accuracy when justifying a geometry conjecture? (2) Is there a significant association between learners generating their *own* dynamic gestures during the process of proof and justification, and their accuracy when justifying a geometry conjecture?

Method

Participants were 80 undergraduates (44 female, average age = 19.5 years) enrolled in a Psychology course at a large Midwestern university. Sixty of the participants had taken Calculus I or higher, and their average ACT/SAT math percentile was 87. Participants provided a justification for the triangle conjecture (*"For any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side."*) while being video-recorded. They key idea was that if the two sides were shorter they would not be able to reach the endpoints of the remaining side to close the triangle. Of the 80 participants, 40 were first explicitly directed to perform relevant dynamic gestures related to the triangle conjecture (Table 5), and were directed to use their bodies to form changing versions of the referent object (a triangle). In effect, they formed possible and impossible triangles with their arms or hands, with one side of the triangle dynamically "growing" until a triangle could no longer be formed. They were not told these gestures were related to the conjecture. The other 40 were directed to enact irrelevant gestures (Table 5). Of the 40 participants in the relevant condition, only 4 reported that they saw a connection between the directed gestures and the conjecture, and the results were similar with or without those participants. As Study 1 suggested dynamic gestures were facilitated in absence of pen and paper, no participants were allowed to use these tools. Participants were asked to think aloud.

Table 5: Dynamic gestures that participants were directed to perform

Directed Relevant Dynamic Gestures	**Directed Irrelevant Dynamic Gestures**
	
Participants touched concentric pairs of circles with outstretched arms, with the last pair being too far apart to touch. (A second version involved hands instead of arms, not shown here.) In pilot work, we found participants spontaneously using gestures similar to these.	Participants were asked to walk back and forth in front of concentric pairs of circles, touching one circle at a time. (A second version involved smaller circles that were closer together, not shown here.)

Analyzing video records, we coded each participant's judgment of the conjecture as true or false (*T/F Judgment*), and coded whether they generated a valid transformational proof (*Proof*). A kappa reliability of 0.82 was obtained by 3 coders on a list of proof categories adapted from Harel and Sowder (2005) that also included the T/F judgment. We then coded the gestures participants spontaneously made while justifying the conjecture. Gestures were coded into 3 categories: (1) The participant made no gestures (*None*), (2) The participant made only static gestures that represented a stationary triangle or triangle part (*Static*), and (3) The participant made at least one dynamic gesture that involved a movement-based transformation of a triangle (*Dynamic*). This coding represented only self-initiated gestures that participants produced while justifying the conjecture, not gestures that participants were directed to perform; inter-coder reliability was kappa = 0.78. We fit logistic regression models predicting a correct T/F Judgment or Proof (coded as 0/1). Predictor 1 was whether the participant had been directed to perform relevant dynamic gestures prior to being given the conjecture (Condition-Experimental) or whether they had been directed to perform irrelevant gestures (Condition-Control). Predictor 2 was whether the gestures participants spontaneously produced during justification were Not Dynamic or Dynamic. We controlled for math achievement using self-report ACT/SAT math percentile as a covariate.

Results

1. Effects of Dynamic Gestures that are Explicitly Directed

We first examined whether the relevant dynamic gestures we directed participants to perform prior to being given the triangle conjecture affected their subsequent accuracy when justifying the conjecture. Although participants who were directed to perform relevant gestures constructed a valid proof of the conjecture more often than participants who were directed to perform irrelevant gestures (50% vs. 40%), this difference did not reach significance (z = 0.095, p = 0.924). Likewise, participants who were directed to perform relevant gestures correctly judged the conjecture was true more often than participants who were directed to perform irrelevant gestures (92.5 vs. 82.5% of cases), but this difference also did not reach significance (z = 1.317, p = 0.188).

At the end of the session, all participants who had performed directed relevant gestures (n = 40, first column of Table 5) were informed of the relevance of these gestures to the triangle conjecture. During this debriefing these participants were then given an opportunity to provide a second justification for the triangle conjecture based on this information. The 40 participants were more likely to give a correct proof on this second attempt (z = 2.190, p = 0.0285), with their chances of obtaining a correct proof increasing from 50% to 70%. Directed relevant gestures may only be effective when the learner is explicitly made aware of their relevance.

2. The Impact of Dynamic Gestures produced during Justification and Proof Activities

Although all participants were explicitly directed to perform some gestures prior to being shown the triangle conjecture (Table 5), we were also interested in the gestures that participants spontaneously produced as they engaged in the proof and justification activities. Figure 1 gives an example of a dynamic gesture sequence that was spontaneously used to prove the triangle conjecture. The participant gives a specific example of an equilateral triangle and then generalizes to all triangles. She moves her thumbs apart, representing the bottom of the triangle as she says that "it wouldn't connect," and then moves her thumbs together as she says "make 'em a triangle." Finally, she collapses the triangle into a line as she says, "they would be like flattened out." This participant produces gestures where she is dynamically modifying one aspect of the triangle (the side lengths) using her body, and then using gestures to see what would happen to the rest of the triangle as a result.

Figure 1. Participant uses dynamic representation gestures to justify triangle conjecture

Many participants in both conditions made their own spontaneous dynamic gestures, and we were interested in whether these gestures were associated with correct T/F judgments and proofs. Participants who produced dynamic gestures were more likely to correctly judge the conjecture to be true than participants who did not produce dynamic gestures (100% vs. 75%). This trend was similar across conditions (experimental/control). However, this difference did not reach significance in the models due to the error term associated with the ceiling effect. Participants who spontaneously produced dynamic gestures were also more likley to provide a correct justification than participants who did not spontaneously produce dynamic gestures (57.5% vs. 32.5%). This difference was significant in the regression model (z = 2.90, p = 0.00375, d = 1.0). However, being directed to perform relevant dynamic gestures prior to being given the conjecture did not influence whether participants actually used dynamic gestures to justify the conjecture. In both condictions, exactly 12 of the 40 participants (30% of participants) made spontaneous dynamic gestures.

Discussion

Study 2 highlights the importance of learners' generating their own dynamic gestures as they spontaneously engage in justification and proof activities. These spontaneous gestures may simply be outward evidence of a stronger understanding of the geometric relations, or they may also indicate that giving dynamic imagery body-based form can support and enhance mathematical reasoning. In other words, spontaneous dynamic gestures may simply be a by-product that is often seen with advanced reasoning, or alternatively students may actually learn from "being" the geometric shape as they generate spontaneous gestures. Given episodes like Figure 1, in which a participant seems to be actively attending to and experimenting with geometric ideas using her body, we believe the latter is a strong possibility. However, Study 2 also cautions that directing participants to perform the physical action of dynamic gestures may not, in isolation, be useful to formulating valid proofs. If dynamic gestures are directed, the learner may need to pay explicit attention to and reflect on the gestures for them to be useful. Asking the learning to "become" a triangle may be of limited usefulness to proof generation if the learner believes they are simply making meaningless, non-mathematical motions. Thus, giving students prompts that relate directed dynamic gestures to the task at hand is important to facilitating valid proof generation.

General Discussion and Conclusions

The current studies support the idea that the dynamic gestures that learners spontaneously produce allow learners to utilize the affordances of their body to ground their understanding of mathematical relationships. Thus, the findings suggest that complex reasoning in a challenging area of mathematics can be fostered by recruiting body-based resources. Study 1 showed that participants who spontaneously used dynamic gestures demonstrated superior mathematical reasoning. The findings suggest that when people enact the key mathematical relations of a task in dynamic, body-based form, they are better able to accurately assess the validity of mathematical conjectures and are more likely to generate valid mathematical proofs to warrant their judgments. Study 2 reinforced the important relation between dynamic gestures and valid proofs, but also suggested that those who were directed to enact relevant relationships through dynamic gestures were more likely to construct valid proofs, provided the purpose of the directed gestures was made explicit.

Although further study is needed to clarify these findings, the studies corroborate the view that reasoning through enactment is associated with conceptual development in an area of study that students find quite challenging. In a recent study, Goksun et al. (2013) found that the gestures of adults with high spatial reasoning abilities were more likely to contain dynamic information about mental rotation. Similarly, Ehrlich et al. (2006) found that children who produce dynamic gestures performed better on a mental rotation task. Our findings further suggest that behaviors such as writing may inhibit dynamic gesture production and impair mathematical reasoning. In similar fashion, Martinez (2012) found that participants who had to type their responses to a science test made fewer relevant inferences than participants who instead spoke their responses and were consequently free to gesture. Both that study and the current one raise questions about whether assessment practices that impinge on people's ability to freely produce gestures may have the unintended consequence of impairing their abilities to generate inferences. Dynamic gestures offer a unique way for learners to "become" a mathematical idea as they engage in learning mathematical practices. Our work seeks to make explicit the importance of identifying, analyzing, and facilitating the dynamic gestures in the classroom.

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