Optimal Pollution Tax within Labor-Managed and Profit-Maximizing Cournot Oligopolies

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Abstract

Optimal pollution tax rate is derived for labor-managed and profit-maximizing Cournot oligopolies with polluting firms and without product differentiation. If all firms are identical, the optimal pollution tax rate is higher than the marginal value of the environmental damage for labor-managed cournot oligopoly and lower than the marginal value of the environmental damage for profit-maximizing Cournot oligopoly.

1. Introduction

A labor-managed firm is defined to be a firm which maximizes its surplus per unit labor. Labor-managed firms were prevalent in the former Yugoslavia, and many Japanese firms are considered to be labor-managed (see Komiya (1988)). The labor-managed firm under perfect competition has been known to behave perversely if the price of the product it is producing changes (see Ward (1958)), that is, its output decreases in the event of an increase in the product price. Labor-managed Cournot oligop-

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※ This paper is dedicated to the late Professor Moriyuki Yoshioka. I heard of his sudden death while I was at Shiraz University in Iran. I was in Sydney for a few months some years ago at the same time while he was on sabbatical leave. After my return to Japan, he sent me from there one of the papers cited here, which was indispensable in writing this paper. I am very grateful to him for this kindness and for his long friendship with me.

oly has been first formulated by Hill and Waterson (1983), and its behavior has been analyzed in relation to profit-maximizing Cournot oligopoly by them, Neary (1984) and Okuguchi (1993), among others. In all of these works, firms have been implicitly assumed to produce products without emitting harmful effluents generally known as pollution which damages the environment. One of governmental policies to mitigate the environmental damage is to levy pollution tax on firms' effluents. The optimal pollution tax rate which maximizes the total social welfare has been investigated for perfectly competitive profit-maximizing firms (see Barnett (1980), and Baumol and Oates (1980)). Ebert (1992), Okuguchi and Yamazaki (1994) have derived the optimal pollution tax rate within profit-maximizing Cournot oligopoly, assuming that firms' harmful effluents are proportional to their outputs. Simpson (1995) has conducted a similar analysis for profit-maximizing Cournot duopoly using Shephard's duality lemma on the relationship between a firm's cost function and its factor demand. Okuguchi (2003) has extended Simpson's analysis to profit-maximizing Cournot oligopoly where the firm's emission is not necessarily proportional to its output.

To the best of my knowledge, no one has analyzed the effects of pollution tax and the optimal pollution taxation for labor-managed firms and labor-managed Cournot oligopoly. In this paper I will analyze the optimal pollution tax rate within labor-managed Cournot oligopoly and within profit-maximizing Cournot oligopoly. In Section 2, I will formulate labor-managed Cournot oligopoly without product differentiation and with polluting firms, and prove that given the pollution tax rate, there exists a unique Cournot equilibrium. On the basis of this existence result, I will then derive the optimal pollution tax rate that maximizes the net total social welfare. In Section 3, I will prove the existence of a unique Cournot equilibrium for a given pollution tax rate for profit-maximizing Cournot oligopoly without product differentiation and derive the optimal pollution tax rate maximizing the total social welfare, without using Shephard's duality lemma as in Okuguchi (2003). As I have already mentioned, a labor-managed firm, of which objective is maximization of the surplus per unit of labor, behaves differently from a profit-maximizing one in the event of a change in the product prtce. In this paper I will show that the optimal pollution tax rate for labor-managed Cournot oligopoly and that for profit-maximizing Cournot oligopoly have asymmetric properties in relation to the marginal value of the environmental damage. I will simplify my analysis in Sections 2 and 3 by reducing the existence problem for the Cournot equilibrium to a fixed-point problem for a function involving only one variable, that is, industry output. Section 4 concludes.

2. Labor-managed Cournot Oligopoly

Let there be n firms aiming to maximize the surplus per unit labor. The firms are assumed to form expectations on all its rivals' outputs à la Cournot. I use the following notation.

l_i : firm i's labor

x_i : firm i's output

$l_i = h_i(x_i)$, firm i's inverse production functions, *i. e.* factor demand

$e_i = g_i(x_i)$, firm i's emission rate of harmful effluent

s_i : firm i's surplus per unit labor

k_i : firm i's fixed cost

p :product price

$p = f(\Sigma x_i)$, inverse demand function

w : competitive wage rate

t :pollution tax rate per unit effluent

$X \equiv \Sigma x_j$, industry output

$L \equiv \Sigma l_j$, industry factor demand

$E \equiv \Sigma e_j$, total emission

$D \equiv D(E)$, value of the environmental damage

By definition, the surplus per unit labor for firm i is

$$s_i \equiv \frac{x_i f(\Sigma x_j) - wh_i(x_i) - tg_i(x_i) - k_i}{h_i(x_i)} \tag{1}$$

The functions f, h_i and g_i are assumed to have the following general properties.

Assumption 1 : $f' < 0, \quad h_i' > 0, \quad h_i'' \geqq 0, \quad g_i' > 0, \quad g_i'' \geqq 0,$

$$i = 1, 2, \cdots, n. \tag{2}$$

Inequality $h_i'' \geqq 0$ implies labor's non-increasing marginal product. If all firms behave under Cournot expectations on their rivals' outputs and if, in addition, the maximum is interior, firm i's first order condition for profit maximization is given by

$$\frac{\partial s_i}{\partial x_i} = \frac{A}{h_i^2(x_i)} = 0,$$

where $A = \{f(\Sigma x_j) + x_i f'(\Sigma x_j) - wh_i'(x_i) - tg_i'(x_i)\}h_i(x_i)$

$$- \{x_i f(\Sigma x_i) - wh_i(x_i) - tg_i(x_i) - k_i\}h_i'(x_i)$$

$$i = 1, 2, \cdots, n. \tag{3}$$

Let the numerator in (3) be v_i. Then the second order condition is simplified in light of $v_i = 0$ as

$$\frac{\partial^2 s_i}{\partial x_i^2} = \frac{\dfrac{\partial x_i}{\partial v_i}}{h_i^2(x_i)} < 0, \qquad \mathrm{i} = 1, 2\cdots, n, \tag{4}$$

where

$$\frac{\partial v_i}{\partial x_i} = \left(2f' + x_i f'' - wh_i'' - tg_i''\right)h_i \ - kh_i' \\ - \left(x_i f - wh_i - th_i - k_i\right)h_i'', \qquad i = 1, 2, \cdots, n. \tag{5}$$

I now introduce the following assumption.

Assumption 2 : $f' + x_i f'' < 0, \qquad i = 1, 2, \cdots, n.$

This assumption has been widely used in the literature on profit-maximizing Cournot oligopoly. It is satisfied if the inverse demand function is linear, and if it is concave. It holds also if the degree of its convexity is not too large. If a labor-managed firm is to be viable, surplus per unit labor has to be nonnegative, *i. e.* $s_i \geq 0$. The second order condition $\frac{\partial v_i}{\partial x_i} < 0$ holds under Assumption 2 in view of $h_i'' \geqq 0$, $g_i'' \geqq 0$ and $s_i \geqq 0$.
Using the definition $X \equiv \Sigma x_j$, I rewrite the first order condition (3) as

$$v_i(x_i, X, t) = \{f(X) + x_i f'(X) - w_i h_i'(x_i) - tg_i'(x_i)\}h_i(x_i) \\ - \{x_i f(X) - wh_i(x_i) - tg_i(x_i) - h_i\}h_i'(x_i) \\ = 0, \qquad i = 1, 2, \cdots, n, \tag{6}$$

which is an implicit function among x_i, X and t. Simple calculations yield

$$\frac{\partial v_i}{\partial x_i} = (h_i + x_i h_i')f' - (wh_i'' + tg_i'') - (x_i f - wh_i - tg_i - k_i)h_i'' < 0 \qquad i = 1, 2, \cdots, n. \tag{7.1}$$

$$\frac{\partial v_i}{\partial t} = x_i f'' h_i + x_i f' \left(\frac{h_i}{x_i} - h_i'\right) \gtreqless 0, \qquad i = 1, 2, \cdots, n. \tag{7.2}$$

$$\frac{\partial v_i}{\partial t} = hg_i \left(\frac{h_i'}{h_i} - \frac{g_i'}{g_i}\right)$$
$$\gtreqless 0 \text{ according as } \alpha_i \equiv \frac{x_i h_i'}{h_i} \gtreqless \frac{xg_i'}{g_i} \equiv \beta_i, \qquad i = 1, 2, \cdots, n, \tag{7.3}$$

where α_i and β_i are elasticities of h_i and g_i with respect to x_i, respectively. Note that $h_i'' \geqq 0$ implies that $\frac{h_i}{x_i} - h_i' \leqq 0$. Hence, in view of this and $f' < 0$, I have $\frac{\partial v_i}{\partial X} > 0$ if $f'' \geqq 0$ (convex or linear inverse demand function). Otherwise, the sign of $\frac{\partial v_i}{\partial X}$ is indeterminate.

Now solve (6) with respect to x_i to get

$$x_i \equiv \varphi^i (X, t), \qquad i = 1, 2, \cdots, n. \tag{8}$$

where

$$\frac{\partial \varphi^i}{\partial X} = \frac{\frac{\partial v_i}{\partial X}}{\frac{\partial v_i}{\partial x_i}}, \qquad i = 1, 2, \cdots, n. \tag{9.1}$$

$$\frac{\partial \varphi^i}{\partial t} = \frac{\frac{\partial v_i}{\partial t}}{\frac{\partial v_i}{\partial x_i}}, \qquad i = 1, 2, \cdots, n. \tag{9.2}$$

The signs of the above partial derivatives are indeterminate in light of (7.2) and (7.3). Given an arbitrary value of t, the Cournot equilibrium industry output is identifiable as a fixed-point of one variable function $\varphi(X,t)$, *i. e.* the solution of one variable equation,

$$X \equiv \Sigma\varphi^i(X,t) \equiv \varphi(X,t), \tag{10}$$

where the partial derivatives of φ are in general indeterminate. I introduce the following assumption.

Assumption 3 : $\varphi^i(0,t) > 0, \qquad i = 1,2,\cdots,n.$

It is clear that under this assumption (10) has a unique solution if $\frac{\partial\varphi}{\partial X} < 0$. It has also a unique solution if $0 \leqq \frac{\partial\varphi}{\partial X} < 1$. Inequality $\frac{\partial\varphi}{\partial X} < 0$ is true if f is sufficiently concave to ensure $\frac{\partial v_i}{\partial X} < 0$ for all i. If f is convex, I get $\frac{\partial v_i}{\partial x} \geqq 0$ and $\frac{\partial\varphi}{\partial x} \geqq 0$. Hence, in this case (10) has a unique solution if $\frac{\partial\varphi}{\partial X} < 1$. Let the unique solution of (10) be

$$X \equiv X(t), \tag{11}$$

where

$$\frac{dX}{dt} = \frac{\frac{\partial\varphi}{\partial t}}{1 - \frac{\partial\varphi}{\partial X}}. \tag{12}$$

If $\frac{\partial\varphi}{\partial X} < 0$ or $\frac{\partial\varphi}{\partial X} < 1$, the denominator of the above expression is positive. However, the sign of $\frac{\partial\varphi}{\partial t} < 0$ is indeterminate. If $\alpha_i > \beta_i$ for all i, $\frac{\partial\varphi_i}{\partial t} > 0$, consequently $\frac{\partial\varphi}{\partial t} > 0$. On the other hand, $\frac{\partial\varphi}{\partial t} < 0$ if $\alpha_i < \beta_i$ for all i.

The case where $\frac{\partial \varphi}{\partial X} < 0$ and $\frac{\partial \varphi}{\partial t} > 0$ hold simultaneously is shown in the figure.

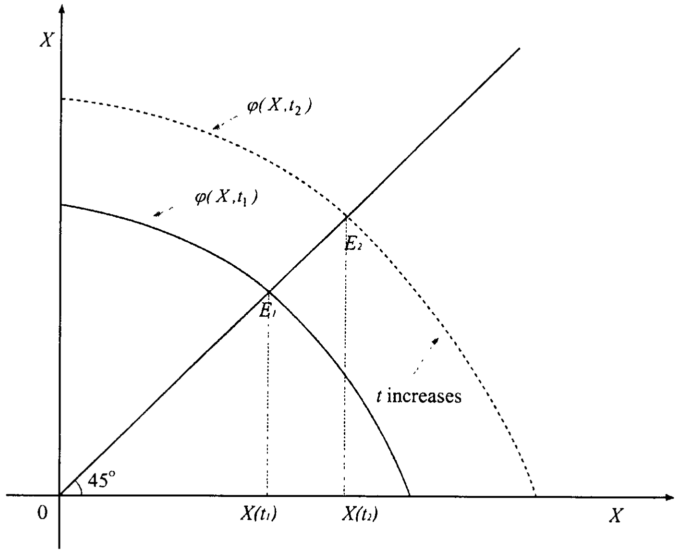

Figure: Equilibrium industry output as an increasing function of t, where $t_1 < t_2$.

I am now in a position to determine the optimal pollution tax rate for labor-managed Cournot oligopoly. The total social welfare W, which is to be maximized with respect to the pollution tax rate, is defined as the sum of the firms' profits, the consumers' surplus and the governmental revenue from pollution tax minus the value of the environmental damaged caused by firms' effluents. Thus

$$W = \Sigma \pi_j + \int_0^X f(\gamma) d\gamma - Xf(X) + tE - D(E)$$

$$= \int_0^X f(\gamma) d\gamma - w\Sigma h_j(x_j) - \Sigma k_j - D\left(\Sigma g_j(x_j)\right), \tag{13}$$

where X and $x_j s$ in the above expression are evaluated at the equilibrium and are functions of t. Differentiating W with respect to t, I get

$$\frac{dW}{dt} = f(x)\frac{dx}{dt} - w\Sigma h_j'\frac{dx_j}{dt} - D'\Sigma g_j'\frac{dx_j}{dt}$$

$$= \Sigma(f - wh_j')\frac{dx_j}{dt} - D'\Sigma g_j'\frac{dx_j}{dt}$$

$$= (t - D')\Sigma g_j'\frac{dx_j}{dt} + \Sigma\left(s_j\, h_j' - x_j f'\right)\frac{dx_j}{dt}, \tag{14}$$

where I have made use of the first order condition (3) as well as the definition of s_i . The optimal pollution tax rate must satisfy $\frac{dW}{dt} = 0$. Hence

$$t = D' + \frac{\Sigma\left(s_i\, h_j' - x_j f'\right)\frac{dx_j}{dt}}{\Sigma g_j'\frac{dx_j}{dt}}. \tag{15}$$

Note that the sign of the expression $s_j\, h_j' - x_j f'$ is positive for all j. However, I can not say anything definite regarding the sign of $\frac{dx_j}{dt}$, consequently, that of the second term on the right hand side of (15). Hence, I get $t \gtreqless D'$in general.

In order to get a clearer result, I consider a symmetric case in which all firms are assumed to be identical, *i*, *e*.

$$l_i = h_i\,(x_i) \equiv h(x_i)$$

$$e_i = g_i\,(x_i) \equiv g(x_i)$$

for all i. In this case, $x_i \equiv x$ for all i and

$$\frac{dX}{dt} = n\frac{dx}{dt} \gtreqless \text{ according as } \alpha_i \equiv \alpha \gtreqless \beta \equiv \beta_i \text{ for all } i.$$

Hence, regardless whether $\alpha > \beta$ or $\alpha < \beta$, the second term in (15) is unambiguously positive, leading to

$$t > D'. \tag{16}$$

To wit, if all firms are identical, the optimal pollution tax rate is set at a level higher than the marginal value of the environmental damage. As a corollary of this result, I can assert that the optimal pollution tax rate for labor-managed monopoly is higher than the marginal value of the environmental damage.

3. Profit-Maximizing Cournot Oligopoly

In this section I will derive the optimal pollution tax rate for profit-maximizing Cournot oligopoly, without using Shephard's duality lemma as in Okuguchi (2003). I will stick to the notaton in Section 2. However, I will use the production function,

$$x_i = f_i(l_i), \quad f_i' > 0, \quad f_i'' \leq 0, \qquad i = 1, 2, \cdots, n, \tag{17}$$

instead of the factor demand function $l_i = h_i(x_i)$. Moreover, I will denote the inverse demand function as $p \equiv p(X)$ instead of $p = f(X)$ as in Section 2 to avoid possible confusion of f with the producton function. Therefore, firm i's profit π_i is given as

$$\begin{aligned} \pi_i &\equiv p(X)x_i - wl_i - tg_i(f_i(l_i)) \\ &= p(\Sigma f_j(l_j))f_i(l_i) - wl_i - tg_i(f_i(l_i)), \\ & \qquad\qquad i = 1, 2, \cdots, n. \end{aligned} \tag{18}$$

where I have assumed away fixed cost. The first and second order conditions for profit maximization for firm i's profit under the Cournot behavioristic assumption yield (19) and (20), respectively.

$$\frac{\partial \pi_i}{\partial l_i} = p'(\Sigma f_j(l_j)) f_i(l_i) f_i'(l_i) + p(\Sigma f_j(l_j)) f_i'(l_i)$$
$$- w - tg_i'(f_i(l_i)) f_i'(l_i) = 0, \qquad i = 1, 2, \cdots, n. \qquad (19)$$

$$\frac{\partial^2 \pi_i}{\partial l_i^2} = (p' + f_i \ p_j'') f_i^2 + (p + f_i \ p') f_i'' + p' f_i''^2$$
$$- t(g_i'' f_i'^2 + g_i' f_i'') < 0, \qquad i = 1, 2, \cdots, n, \qquad (20)$$

where in (19), I have assumed away corner maximum. As it stands, it is not clear if (20) holds. I therefore introduce the following assumption to resolve this indeterminacy.

Assumption 4 : $p + x_i p' > 0, \quad p' + x_i p'' < 0, \quad g_i'' f_i'^2 + g_i' f_i'' > 0,$

$$i = 1, 2, \cdots, n. \qquad (21)$$

The first inequality implies that firm i's marginal revenue with respect to increase in its output is positive. The second one asserts that the marginal revenue is decreasing with respect to increase in its output. The third one holds $f_i'' = 0$. I rewrite the first order condition (19) as

$$H_i(l_i, X, t) \equiv p'(X) f'(l_i) f_i(l_i) + p(X) f_i'(l_i) - w$$
$$- tg_i'(f_i(l_i)) f_i'(l_i) = 0, \quad i = 1, 2, \cdots, n. \qquad (22)$$

Solving (22) with respect to li, I get

$$l_i \equiv \psi^i(X, t), \qquad i = 1, 2, \cdots, n, \qquad (23)$$

where

$$\psi^i{}'_X \equiv \frac{\partial \psi^i}{\partial X} = - \frac{(p' + x_i \ p'') f_i'}{p'(f_i'' f_i + f_i'^2) + p f_i'' - t(g_i'' f_i'^2 + g_i' f_i'')} < 0,$$
$$i = 1, 2, \cdots, n. \qquad (24.1)$$

$$\psi^{i}{}_{X} \equiv \frac{\partial \psi^{i}}{\partial t} = -\frac{g_i'' f_i'^2 + g_i' f_i''}{p'(f_i'' f_i + f_i'^2) + p f_i'' - t(g_i'' f_i'^2 + g_i' f_i'')} < 0,$$

$$i = 1, 2, \cdots, n. \qquad (24.2)$$

The Cournot equilibrium industry output is indentical to the fixed point of a single variable function $\psi(x, t)$, *i. e.* it is the solution of

$$X = \Sigma f_j(\psi^{j}(X, t)) \equiv \psi(X, t), \qquad (25)$$

where

$$\psi_X \equiv \frac{\partial \psi}{\partial X} = \Sigma f_j' \psi_X^j < 0. \qquad (26.1)$$

$$\psi_t \equiv \frac{\partial \psi}{\partial t} = \Sigma f_j' \psi_t^j < 0. \qquad (26.2)$$

Introduce the following assumption.

Assumption 5: $\psi^j(0, t) > 0, \qquad j = 1, 2, \cdots, n.$

As ψ is strictly decreasing in x and $\psi(0, t) > 0$ under this assumption, given t, (25) has a unique solution, which is strictly decreasing in t in light of (26.2) and

$$\frac{dX}{dx} = \frac{\psi_t}{1 - \psi_X} < 0. \qquad (27)$$

On the other hand, the individual firm's output response to a change in t is ambiguous as

$$\frac{dx_i}{dt} = f_i' \frac{dl_i}{dt}, \qquad (28)$$

$$\frac{dl_i}{dt} = \psi^{i}{}_{X} \frac{dX}{dt} + \psi_t^i \gtreqless 0. \qquad (29)$$

The total social welfare is defined by

$$
\begin{aligned}
W &= \Sigma\pi_j + \int_0^X p(\tau)d\tau - p(X)X \\
&\quad + t\Sigma g_j(f_j(l_j)) - D(\Sigma g_j(f_j(l_j)) \\
&= \int_o^X p(\tau)d\tau - wL - D(\Sigma g_i(f_j(l_j)).
\end{aligned} \tag{30}
$$

Noting that W is a function of t in the equilibrium and differentiating it with respect to t, I have

$$
\begin{aligned}
\frac{dW}{dt} &= p(X)\frac{dX}{dt} - w\frac{dL}{dt} - D'\Sigma g_j' f_j' \frac{dl_j}{dt} \\
&= \Sigma\{p(X)f_j' - w - D'\Sigma g_j' f_j'\}\frac{dl_j}{dt} \\
&= \Sigma\left(-p' f_j' f_j + t g_j' f_j' - D' g_j' f_j'\right)\frac{dl_j}{dt},
\end{aligned} \tag{31}
$$

where I have taken into account the first order condition (19) to derive the last equality in (31). Hence, the optimal tax rate which maximizes W is

$$
t = D' + \frac{\Sigma p' f_j' x_j \dfrac{dl_j}{dt}}{\Sigma g_i' f_j' \dfrac{dl_j}{dt}}. \tag{32}
$$

The sign of the second term on the right hand side of (32) is in determinate in general due to the indeterminacy of the sign of $\frac{dl_j}{dt}$. Hence, the optimal pollution tax rate may be higher, lower than or equal to the marginal value of the environmental damage. If all firms are symmetric, *i. e.* $f_i \equiv f$ and $g_i \equiv g$ for all i, I have $l_i = l$, $x_i \equiv x$. Hence,

$$
\frac{dx}{dt} = \frac{1}{n}\frac{dX}{dt} < 0,
$$

therefore,

$$\frac{dl}{dt} = \frac{\frac{dx}{dt}}{f'} < 0,$$

which leads to

$$t < D'. \tag{33}$$

This inequality is entirely opposite to (16) for labor-managed Cournot oligopoly. Note that (33) is true also for profit-maximizing monopoly. Note further that (32) shows that if the product market is perfectly competitive, the optimal pollution tax rate is equal to the marginal value of the environmental damage.

4. Conclusion

There has been no research on the pollution taxation on labor-managed competitive firm as well as on labor-managed Cournot oligopoly. In Section 2, I have formulated labor-managed Cournot oligopoly without product differentiation and with polluting firms which are subject to the pollution taxation, and derived the optimal pollution tax rate which maximizes the total social welfare. In general case with asymmetric firms, the optimal pollution tax rate may be higher, lower than or equal to the marginal value of the environmental damage. If all firms are symmetric in the sense that their labor demand functions and emission functions are identical, the optimal pollution tax rate is higher than the marginal value of the environmental damage. In section 3, I have derived the optimal pollution tax rate for profit-maximizing Cournot oligopoly without product differentiation and with polluting firms. If firms are asymmetric, the optimal pollution tax rate may be higher, lower than or equal to the marginal value of the environmental damage, as in the case of asymmetric labor-managed Cournot oligopoly. However, if all firms are identical, the optimal pollution tax rate is lower than the marginal value of the environmental damage. The asymmetric properties of the optimal pollution tax rate for labor-managed and profit-maximizing Cournot oligopolies may be comparable to the asymmetric responses to product price change of competitive labor-managed and profit-maximizing firms.

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