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Unknown System Identification using LMS Algorithm

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Abstract

An adaptive filter is a digital filter that self adjusts its transfer function according to an optimizing algorithm which is most frequently Least Mean Square (LMS) algorithm. Due to the complexity of adaptive filtering most digital filters are FIR filter. There are numerous applications of adaptive filters like noise cancellations, echo cancellation, system modelling and identification, inverse system modelling, adaptive beam-forming etc. In this research article, adaptive LMS algorithm has been used for unknown system identification. The system identification is a category of adaptive filtering which find its numerous applications in diverse field like communication, image processing, speech processing etc.

Keywords: *FIR filter, adaptive filtering, LMS algorithm, system identification, MATLAB*

INTRODUCTION

A digital filter is basically a digital hardware performing some digital operation. The basic building blocks of digital filter are only three elements; namely adder, multiplier and delay network. The digital filters are superior over their analog counterparts because of various regions; and one important reason is digital filters are programmable. It means that when the design requirements changes, then keeping the same digital hardware we can redesign the filter merely by changing the program or in other words by changing the multiplier coefficients also called as filter weight or filter coefficients.

There are two types of digital filter, Finite Impulse Response (FIR) and Infinite Impulse Response (IIR). A FIR filter is a filter whose impulse response contains finite number of non-zero values. Similarly, an IIR filter is a filter whose impulse response has infinite number of non-zero values. FIR filter is most frequently implemented as non recursive filter while IIR filter as recursive filter. FIR filtering is basically linear convolution operation. For the same filter design specifications, the order of the digital FIR filter is significantly higher than IIR filters. Thus, the cost of FIR filter is always higher than IIR filter and at the same time the quality of FIR filter is always better than IIR filter. In pole-zero diagrams, FIR filter has pole at the center only and remaining zeros and thus the filter is constructed using zeros. Similarly, IIR filter has no pole at the center and the filter is basically designed using poles. An FIR filter has only numerator filter coefficients in their transfer function while IIR filter has both numerator and denominator filter coefficients in their transfer function. The complexity of adaptive filter is high and, therefore, here the choice is always FIR filter. The two basic requirements that must be fulfilled by any digital filter are 1) Stability and 2) Causality. A non causal filter would be non-realizable. An unstable filter would be of no use. FIR filter are guaranteed stable and for IIR filter the guarantee for stability of the filter cannot be given. Because of this issue also, FIR filters are superior to IIR filters. The additional desirable feature of digital filter is linear phase. For a linear phase filter, all frequency components of a digital signal are delayed by the same amount. An IIR filter is never a linear phase filter while FIR filter may be linear phase. This is one more reason because of which FIR filters are considered superior to IIR filters.

ADAPTIVE FILTER

One more way of classification of digital filters is fixed filter and adaptive filter. In fixed filter, the filter coefficients are calculated at design time, then the digital hardware is constructed and the filter coefficients do not change themselves. On the other hand an adaptive filter is a filter whose coefficients keep on changing continuously under the control of optimizing algorithm, most commonly LMS algorithm.

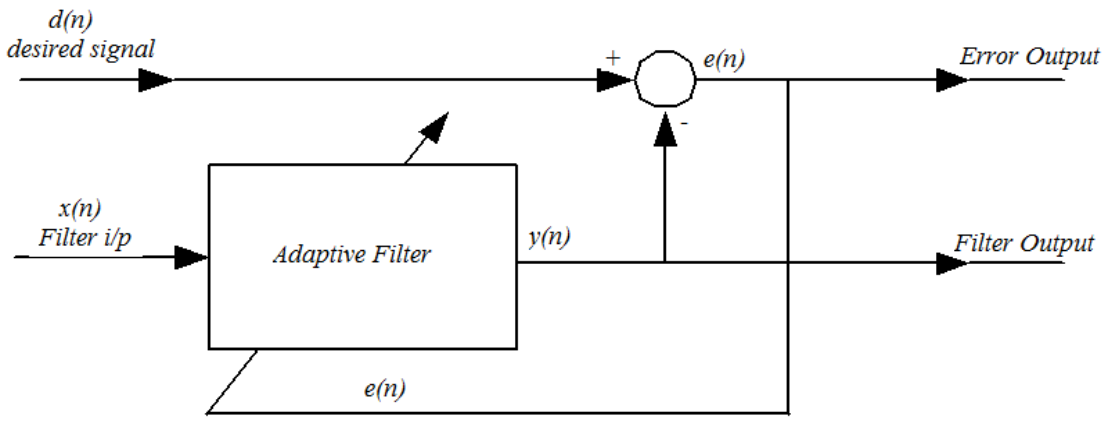

Fig. 1: *Adaptive Filter.*

It is the work of Wiener in 1942, and Kolmogorov in 1939, that derived optimal filter, which is popularly known as 'Wiener filter' [1]. But the Wiener filter is based on priori statistical information of the input signal. But in practice, though this information is available for stationary signals, it is not available for non stationary signals. So, in other words we can say that Wiener filter cannot be used for practical signals which are non stationary. But the significance of Wiener filter is that it gives optimal solution in mean square error (MSE) sense. If we plot the mean square error signal versus the adjustable parameters of the linear filter, then the minimum error point on the y-axis represents Wiener solution 'w_o'. However, for all non stationary signals, Kalman filters are used [2, 3]. Most commonly an adaptive filter is realized as transversal filter, also called as 'tapped delay line filter'. Figure 2 represents the transversal filter which is basic building block of any adaptive filter. The role of the adaptive filtering algorithm is to iteratively adjust filter coefficients w_0, w_1,.., w_{M-1} to achieve the desired goal.

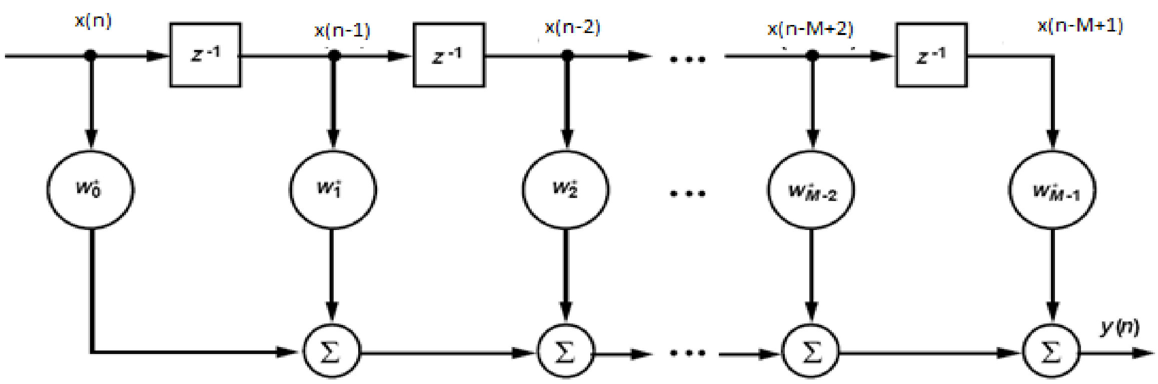

Fig. 2: *Transversal FIR Filter Structure.*

LITERATURE REVIEW

LMS algorithm invented by Widrow and Hoff has been extensively analyzed in the literature and a large number of results on its steady state misadjustment and its tracking performance are available. But this algorithm suffers from the disadvantages of slow rate of convergence, larger misadjustment error etc. [4]. In order to speed up, many frequency domain and block based algorithms were developed which take advantage of the FFT [5]. Standard LMS algorithm has been modified by S. G. Chen *et al.* to make it more computationally efficient. This new algorithms requires N/2 -1 less multiplication at the expense of N/2 +5 more additions S. C. Douglas indicates that use of non linear functions in LMS algorithm can yield a significant performance improvement in fast adaptation situations [6]. N. J. Bershad has developed a new theoretical model for predicting the behavior of the first and second moments of the LMS algorithm with a tapped delay line filtering structure [7]. S. Marcos has studied the tracking capability of a time varying system by an adaptive filter through the LMS algorithm [8]. R.H. Kwong has proposed a variable step-size LMS algorithm where the step size adjustment is controlled by the square of the prediction error [9]. The motivation is that a large prediction error will cause the step size to increase to provide faster tracking while a small prediction error will result in decrease in the step size to yield smaller misadjustment. Similar reporting is done by Li Yan and Wang Xinan in their paper describing that instead of conventional LMS algorithm; which has the fixed step size; VS-LMS algorithm can be used to improve convergence speed and low residual error level. Similarly, Thamer M. Jamel in his paper, “Combined Adjusted Step Size LMS Algorithm, and Active Tap Detection Technique for Adaptive Noise Cancellation” describes that there is improvement in performance if standard LMS algorithm is used along with the Adjustable step size LMS algorithm [10]. E. A. Hernandez has proposed a new variant of LMS algorithm known as Averaged, Over-determined, and Generalized LMS (AOGLMS) algorithm. This algorithm possesses a much lower computation burden than LMS. S. C. Chan and Y. Zhou have studied mean and mean square convergence behavior of the NLMS algorithm with Gaussian input and additive white Gaussian noise [11]. LMS algorithm has found wide applications for the stationary environment due to its implementation simplicity. However, its performance degrades substantially in the non-stationary or time varying environment. This degradation arises mainly because of the lag noise in addition to the gradient noise, which is measured in terms of dimensionless quantity called "misadjustment". To tackle with this problem, A. K. Kohli and D. K. Mehra has proposed a modified version of the two step LMS type algorithm and used random-walk model for developing the tracking theory of the LMS algorithm [12].

IMPROVED ADAPTIVE FILTER

There are many woks done in past to improve the conventional LMS algorithm and they are listed below.

1. The conventional LMS algorithm has fixed step size and thus rate of convergence is slow. Works have been done to improve it by using the Variable-Step Size LMS algorithm. Gear shifting is a popular approach by Widrow, which is based on using large step-size values when the filter weights are far from the optimal solution and small step size values when near the optimum solution.
2. Normalized LMS (NLMS) algorithm is not suitable for large memory applications. In these cases, earlier works suggest Wavelet Transform Domain LMS (WLMS). Rate of convergence became faster when the WLMS algorithm was used instead of NLMS algorithm.
3. Earlier works have been done on sub-band filter and it is shown that they have improved performance in terms of faster convergence speed and reduction of computational complexity due to shorter adaptive filters in the sub-bands.
4. Robust NLMS Concentrated section algorithm have been developed earlier based on the empirical evidence that only a small number of filter taps are needed to generate the replica of the PSTN impulse response, the remaining taps contribute only to error. Thus, the concentrated section allows independent processing of the filter coefficients using an increased adaptation gain within the section while maintaining uniform stability across the entire filter.
5. Modified VSS-LMS Algorithm: This algorithm is a modification of the already described Variable Step Size Least Mean Square (VSS-LMS) algorithm. It provides fast convergence at early stages of adaptation while ensuring small final misadjustment. The performance of the algorithm is

not affected by existing uncorrelated noise disturbances. Simulation results comparing this algorithm with the current variable step size algorithm clearly indicates its superior performance. For stationary environments, this algorithm performs better compared to VSS-LMS algorithm and for non stationary environments its performance is almost similar to the existing LMS algorithm.

Similarly, a variety of other algorithms may be implemented and their performance may be compared with LMS and NLMS algorithm. So, in order to overcome these limitations of the conventional LMS algorithm, the improved LMS algorithm will be proposed for various signal processing applications.

LMS ALGORITHM

Consider following notations:

$w(n)$: Filter coefficient at n^{th} iteration.

$w(n+1)$: Updated filter coefficient at next iteration.

μ : Step size parameter.

$x(n)$: Input signal to filter.

$y(n)$: Output of the filter.

$e^2(n)$: Squared error signal known as cost function.

$d(n)$: Desired response.

w_{opt} : optimal filter weight.

R : Autocorrelation matrix of x (n).

P : Cross correlation matrix between x (n) and d (n).

$\left(-\frac{\partial e^2(n)}{\partial w(n)}\right)$: Negative gradient of the mean square error.

The popular optimization equation studied in statistical signal processing known as Weiner-Hopf equation is given by, $w_{opt} = R^{-1}P$. This equation gives the optimal filter weight, but the computation of optimal solution involves matrix inversion and thus it is highly computation intensive. Therefore, this method cannot be used for real time applications. However, this problem can be solved by another approach where instead of directly calculating the optimal solution by Weiner-Hopf method, an iterative approach is used. In this iterative method, the algorithm starts by assuming small initial weights, zero in most cases, and by finding the gradient of the cost function, the weights are updated iteratively at each step. That is, if the gradient is positive, it implies the error is increasing positively, which indicates to reduce the weights. In the same way if the gradient is negative, it indicates to increase the weights. So, the basic weight update equation is given as;

$W_{n+1} = W_n - \mu \nabla \varepsilon[n]$ where ε represents the mean-square error. The negative sign indicates that, we need to change the weights in a direction opposite to that of the gradient slope. LMS algorithm follows stochastic gradient descent method in that the filter is only adapted based on the error at the current time. The LMS algorithm utilizes the gradient vector of the filter tap weights to converge to the optimal wiener solution. The LMS algorithm iteratively solves the Wiener-Hopf equation and finds the filter coefficients. The LMS algorithm is based on the steepest descendent method from numerical optimization where the cost function is the squared error signal, i. e. $\varepsilon(n) = e^2(n)$. LMS algorithm stands as the benchmark against which all other adaptive filtering algorithms are judged. The LMS algorithm utilizes fewer computational resources and memory than the RLS algorithms. The implementation of the LMS algorithms is less complicated than the RLS algorithms. These advantages have made LMS algorithm the first choice.

The celebrated LMS algorithm was invented in 1959 by Stanford University professor Bernard Widrow and his first doctoral research scholar, Ted Hoff through their studies of pattern recognition. This algorithm is the most popular adaptive filtering algorithm because it gives simple and robust design. This algorithm uses the optimization concept of 'steepest descent', i.e., moving towards the negative gradient on the error surface to get the minimum. Iteration by iteration LMS algorithm goes closer to the minima and it can reach minima after few iterations and. But the major drawback of LMS algorithm is that LMS algorithm can search only local minima but not the global minima.

However, by simultaneously starting the search at multiple points, this drawback of LMS algorithm can be overcome. In stationary environment, the LMS filter can converge to optimal Wiener filter, however, in non stationary environments, the filter is expected to track time variations of the signal and vary its filter coefficients accordingly. Thus, LMS algorithm tries to modify the filter coefficients in order to make cost function minimum.

Now, the filter coefficient weight vectors and input vector are represented as,

$$w(n) = [w_0, w_1, w_3, ...w_{M-1}] \qquad (1)$$

$$x(n) = [x_n, x_{n-1}, x_{n-N}] \qquad (2)$$

The output of the transversal filter, which is also an estimate of the desired signal, is given as,

$$y(n) = \sum_{k=0}^{M-1} w_k^{\ *} x(n-k) \qquad (3)$$

The error signal is given as,

$$e(n) = d(n) - y(n) \qquad (4)$$

As per theory of LMS algorithm, the next iteration cost function is given as,

$$w(n+1) = w(n) - \mu \nabla e^2(n) \qquad (5)$$

$$\frac{\partial e^2(n)}{\partial w_i} = 2e(n)\frac{\partial e(n)}{\partial w_i} \xrightarrow{e(n)=d(n)-y(n)} -2e(n)\frac{\partial y(n)}{\partial w_i}$$

We know, $y = \sum_{i=0}^{N-1} w(n)\, x(n-i)$ (6)

Therefore,

$$\frac{\partial e^2(n)}{\partial w_i} = -2e(n)\, x(n-i)$$

$$\nabla e^2(n) = -2e(n)\, x(n) \qquad (7)$$

Thus, tap weight update equation of standard LMS algorithm is,

$$w(n+1) = w(n) + 2\mu\, x(n)\, e(n) \qquad (8)$$

This can also be written as,

$$\begin{pmatrix} update\ value \\ of\ tap\ weight \\ vector \end{pmatrix} = \begin{pmatrix} Old\ value \\ of\ tap-weight \\ vector \end{pmatrix} + 2\begin{pmatrix} learning \\ rate \\ parameter \end{pmatrix}\begin{pmatrix} tap- \\ input \\ vector \end{pmatrix}\begin{pmatrix} error \\ signal \end{pmatrix}$$

This algorithm defines cost function in terms of mean squared error (MSE). The adaptive process involves the use of the cost function to feed an algorithm which determines how to modify the filter coefficients to minimize the cost of the next iteration. Now, the foremost important thing in filter design is to see that the designed filter is stable. In following section, this issue is discussed.

STABILITY OF LMS ALGORITHM

The LMS algorithm is convergent in mean square only if the step size parameters satisfy the following relationship,

$$0 < \mu < \frac{2}{\lambda_{\max}} \qquad (9)$$

Where, $\lambda_{\max}$ = largest Eigen value of the correlation matrix of the input data

PERFORMANCE PARAMETERS OF ADAPTIVE FILTERS

Various performance evaluation parameters for adaptive filter exist. A designed adaptive filter should fit with respect to these parameters for a particular application.

Rate of Convergence

This indicates the number of iteration a filter has to perform to decrease the error equal to steady state error. Real time applications require high rate of convergence.

Misadjustment

This is a measure of the steady state error performance of the filter. It indicates how much higher is the residual error of the filter from the theoretical minimum error given by optimal Weiner solution.

Computational Requirements

This indicates how much addition, multiplication, delay and memory is required to implement the filter. The computational complexity of a filter affects the speed of operation as well as cost of the filter.

Stability

Stability is the basic requirement of any digital filter. FIR filters are inherently stable whereas IIR filters may become unstable. A filter is said to be stable if the mean-squared error converges to a finite value.

Applications of Adaptive Filters

LMS algorithm and its variants are used in number of signal processing applications. Few of these applications are described below.

Line Echo Cancellation

Echo cancellation is a burning area of research now days and attracting various researchers in this field. Echoes are delayed or distorted versions of a sound or signal which have been reflected back to the source. They become distinct and disruptive when their round trip delay is longer than a few tens of milliseconds. In telecommunications, echoes are categorized as network echoes and acoustic echoes. There is always a need for improved echo cancellers to cancel both the network and acoustic echo. The basic principle of echo cancellation is to eliminate the echo by subtracting from it a synthesized replica.

Network Echo

Network echoes appear in telephone calls over the public switched telephone network (PSTN). The link connecting the two users is comprised of a two-wire line to connect both phones to their respective local central office and two separate unidirectional lines that make a four-wire inter-office link, as shown in Figure 3. The hybrid transformer is the device that connects the two wire circuit to the four-wire circuit. Ideally, the hybrid would transfer all energy from the incoming signal on the four-wire circuit to the two-wire circuit. However, due to imperfect impedance matching, some of the energy is reflected back to its source on the four-wire branch as an echo. Thus, hybrid or network echoes in the PSTN arise from hybrid devices.

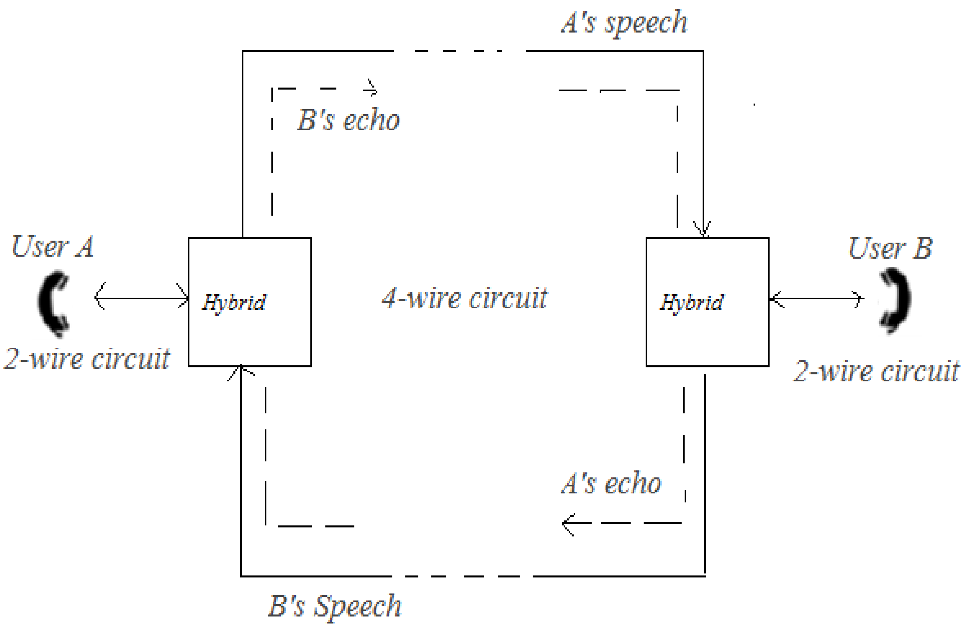

Fig. 3: *Network Echo in Telephone Network.*

Acoustic Echo

Acoustic echo cancellation is a common occurrence in today's telecommunication systems. The signal interference caused by acoustic echo is distracting to users and causes a reduction in the quality of the communication. Acoustic echoes occur in a loudspeaker-enclosure microphone (LEM) system. In the LEM system, there exists an electro-acoustic coupling between the loudspeaker and the microphone, resulting in the microphone picking up signals from the loudspeaker as well as signal reflections off surrounding objects and boundaries as illustrated in Figure 4. Acoustic echoes occur in applications such as teleconferencing and hands-free telephony.

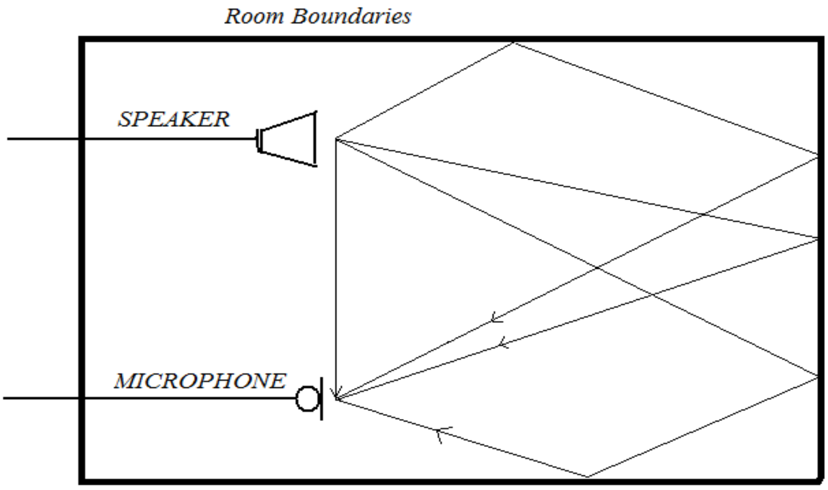

Fig. 4: *Acoustic Echo in Telephone Network.*

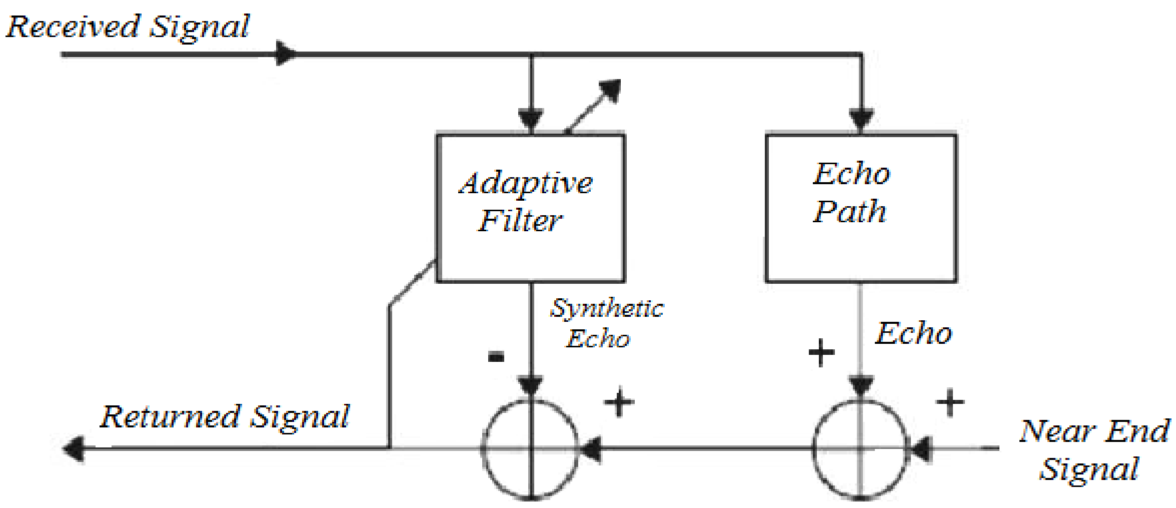

Fig. 5: *Network Echo Cancellation.*

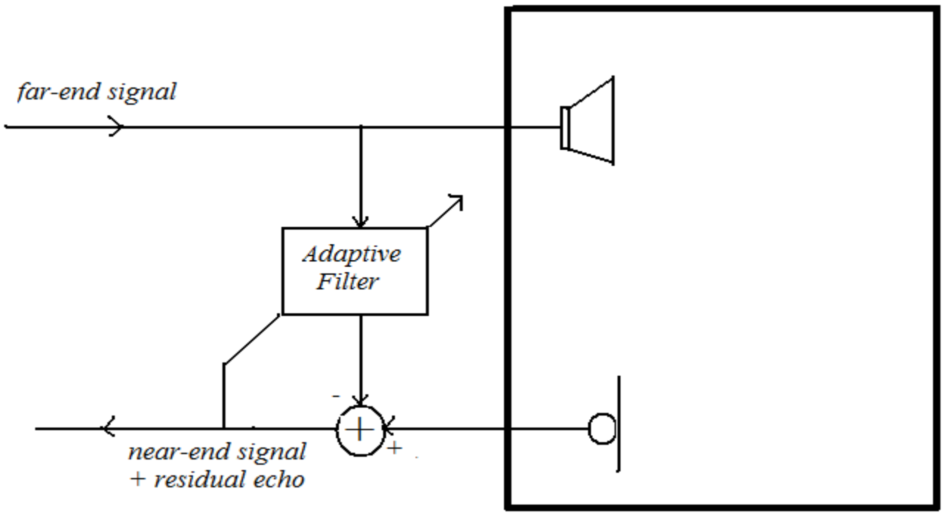

Fig. 6: *Acoustic Echo Cancellation.*

Echo cancellers are amongst the most widely used digital signal processing devices in the world because each telephone call requires a pair of echo cancellers. Basically, a transversal filter, which is adaptively modelling the echo path impulse responses, generates an estimate of the echo; with this an echo estimate is created at the right time to cancel the actual echo. The common problems faced by echo cancellation are the convergence rate and the misadjustment. Convergence time is the time taken to reach an acceptable level of steady state residual echo which depends upon number of iterations done for minimum stable MSE error.

Noise Cancellation of Speech Signals

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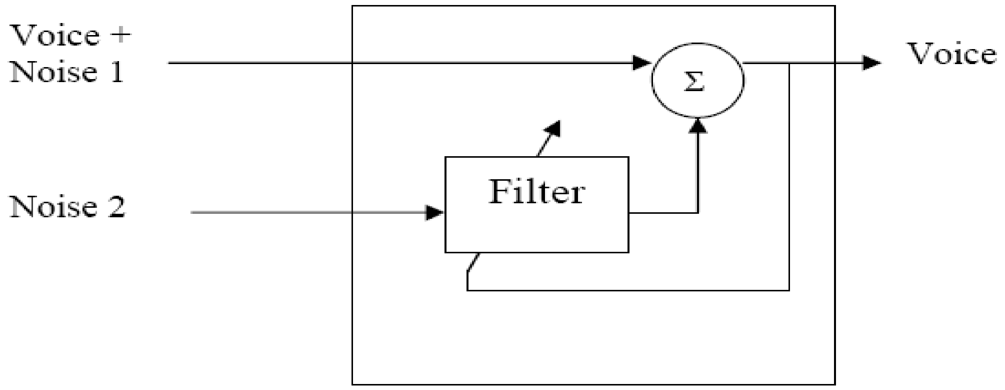

Fig. 7: *Noise Cancellation Model.*

Adaptive filtering can be extremely useful in cases where a speech signal is submerged in a very noisy environment with many periodic components lying in the same bandwidth as that of speech. The adaptive noise canceller for speech signals needs two inputs. The main input is containing the voice that is corrupted by noise. The other input (noise reference input) contains noise related in some way to that of the main input (background noise). The system filters the noise reference signal to make it more similar to that of the main input and that filtered version is subtracted from the main input. Ideally, it removes the noise and leaves intact the speech. In practical systems noise is not completely removed but its level is reduced considerably.

Audio Noise Cancellation using Two Microphones

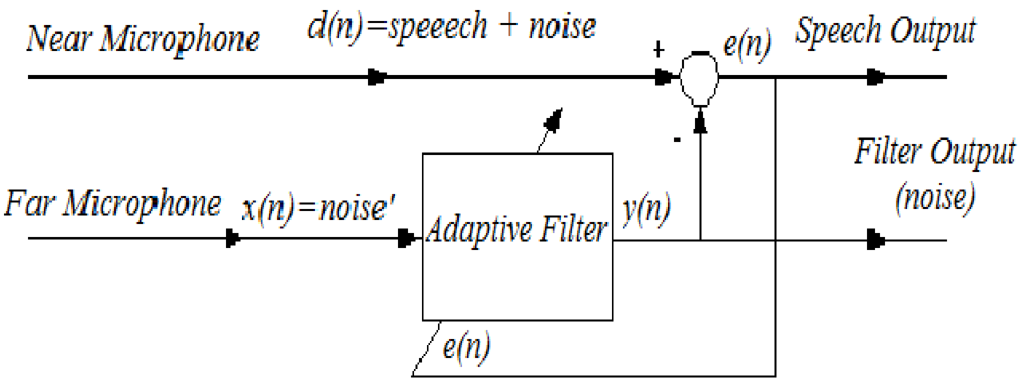

Fig. 8: *Audio Noise Cancellation using Two Microphone.*

Unknown System Identification

Used to provide a linear model of an unknown plant

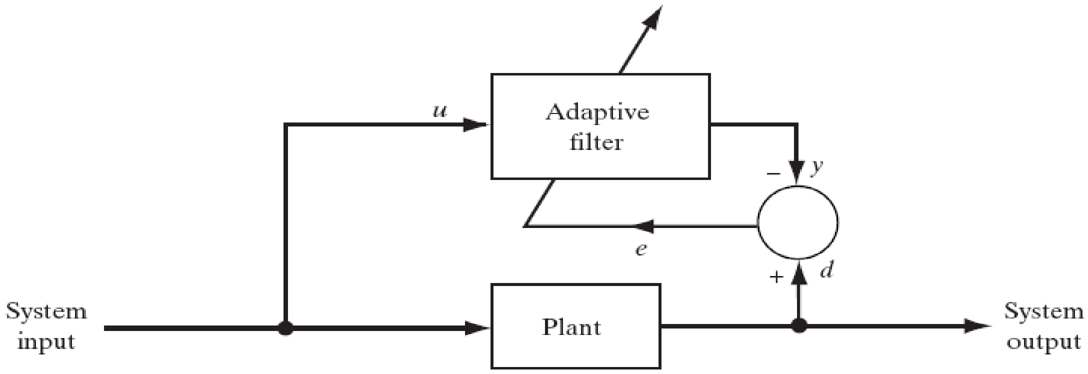

Fig. 9: *Unknown System Identification.*

Equalization

Equalizers are used to provide an inverse model of unknown plant.

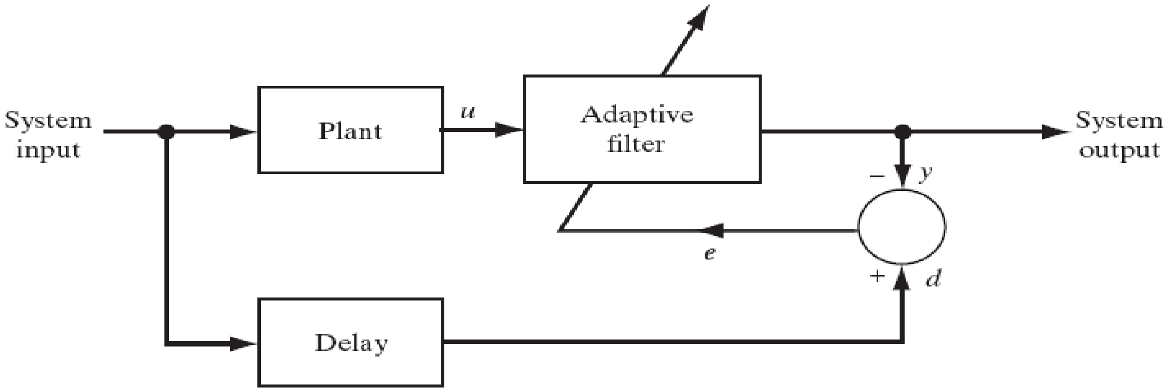

Fig. 10: *Inverse Channel Modeling.*

Linear Predictive Coding

This is used to provide a prediction of the present value of a random signal.

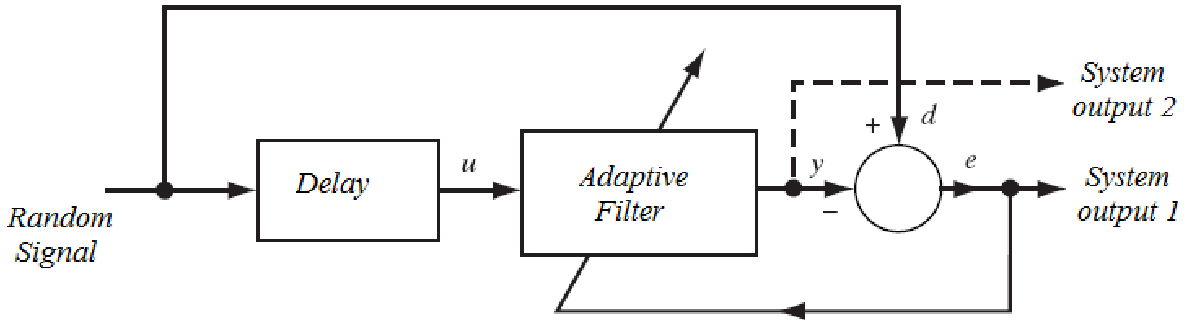

Fig. 11: *Linear Predictive Coding.*

SOFTWARE PROGRAM

The MATLAB program for implementation of LMS adaptive filter for system identification is given as follows. In this program, unknown system has 5 numbers of coefficients.

```
% MATLAB Program for Unknown System Identification
clc, clear all, close all;
%channel system order
sysorder = 5 ;
% Number of system points
N=2100;
inp = randn (N, 1);
n = randn (N, 1);
[b, a] = butter (3, 0.25);
Gz = tf (b, a,-1);
h= [0.0875;
   0.2567;
   0.3500;
   0.2190;
   0.0875];
y = lsim(Gz,inp);
%add some noise
n = n * std(y)/ (10*std (n));
d = y + n;
total length=size (d, 1);
%Take 60 points for training
N=50;
%begin of algorithm
w = zeros (sysorder, 1);
for n = sysorder: N
   u = inp (n:-1: n-sysorder+1);
   y (n) = w' * u;
   e (n) = d (n) - y (n);
% Start with big mu for speeding the convergence then slow down to reach the correct weights
   if n < 20
      mu=0.30;
```

```
    else
       mu=0.15;
    end
    w = w + mu * u * e (n);
end
%check of results
for n = N+1: total length
    u = inp (n:-1: n-sysorder+1);
    y (n) = w' * u;
    e (n) = d (n) - y (n);
end
hold on
plot (d)
Plot(y,'g');
title ('System output');
xlabel ('Samples')
ylabel ('Actual and estimated output')
figure
semilogy ((abs (e)));
title ('Error signal');
xlabel ('Samples')
ylabel ('Error amplitude')
figure
plot (h, 'k+')
hold on
plot (w, 'r*')
legend ('Actual weights', 'Estimated weights')
title ('System Identification by comparing actual and estimated weights');
axis ([0 7 0.01 0.50])
```

RESULTS AND DISCUSSION

Figure 12 indicates actual and estimated output for every input signal on sample by sample basis. This indicates that initially there is larger difference in actual and estimated output, but as time progresses, estimated output goes closer and closer to the actual output. Similarly, Figure 13 indicates estimation error. Figure 14 indicates the actual system coefficients versus estimated system coefficients.

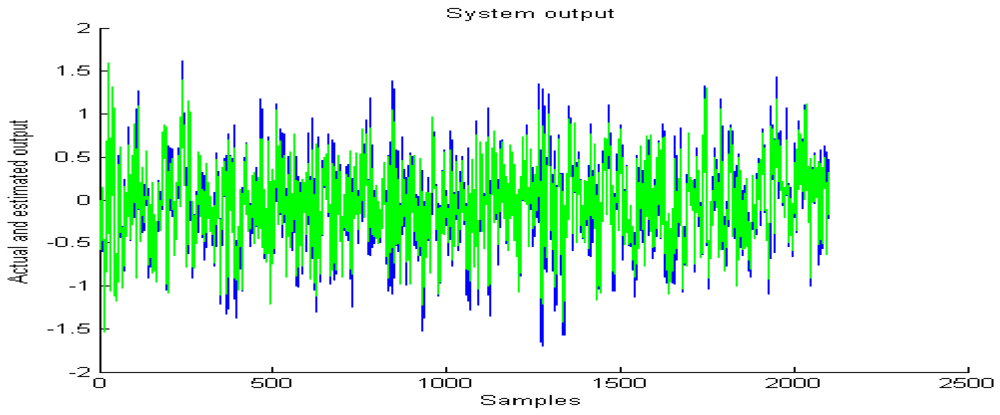

***Fig. 12:** System Output.*

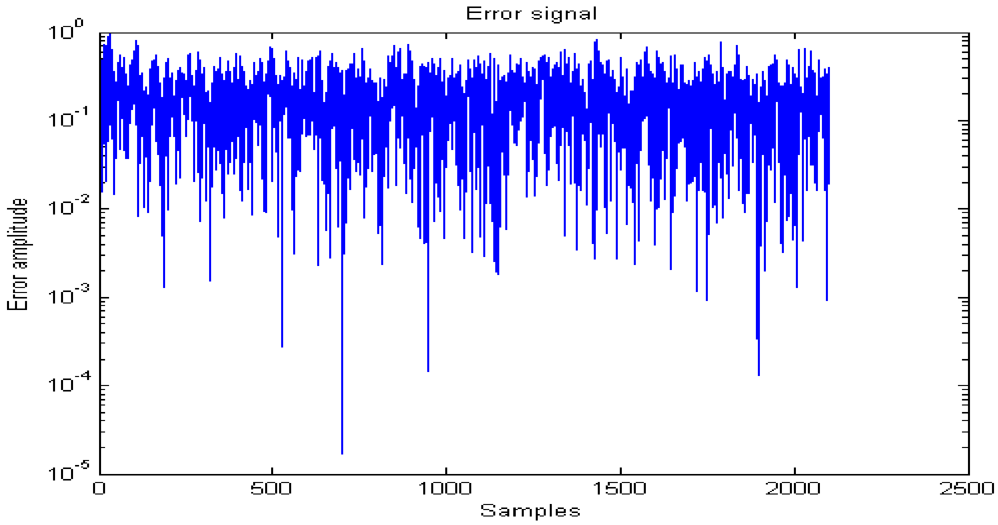

Fig. 13: *Error Signal.*

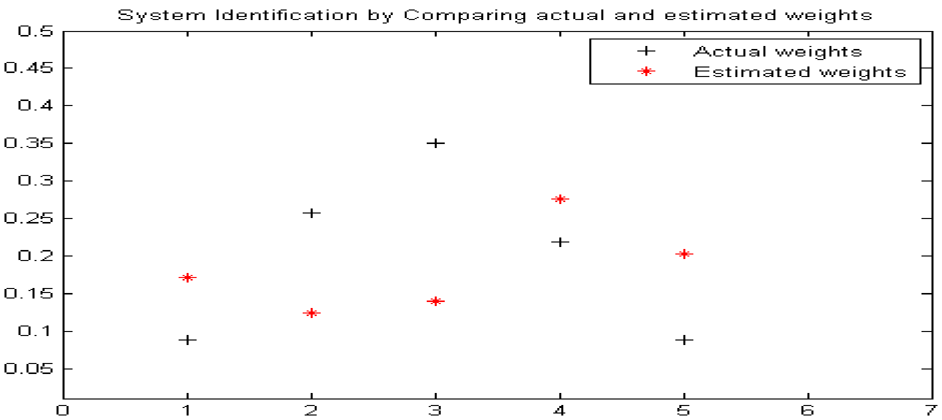

Fig. 14: *Actual and Estimated System.*

CONCLUSION

The experimental result clearly indicates that the estimated weight of the unknown system is matching with actual weight. Thus, the adaptive filter has successfully identified the unknown system. In the MATLAB, two different step size parameters are fixed in order to speed up the convergence of the filter. This system identification uses standard LMS algorithm. System identification finds a wide range of applications in various fields such as communication, navigation, radar, image and speech processing and many more. Adaptive filtering constitutes one of the core technologies in digital signal processing and finds numerous application areas in engineering education as well as industry. Adaptive filtering techniques are used in wide range of applications including echo cancellation, adaptive noise cancellation, system modeling, channel equalization and adaptive beam-forming and many more.

FUTURE IMPROVEMENTS

In this research article, the results of MATLAB implementation of adaptive filtering are presented. But further studies can be carried out for hardware implementation using suitable platform like Xilinx FPGA or DSP processors. Similarly, optimization of this system identification problem for different values of step size (μ), filter order etc. can be carried out in future. At the same time, the performance of this system for other variants of LMS algorithm can be observed and compared with standard LMS result.

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