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Numerical Evaluation and Monte Carlo Simulation for Performance Analysis of 4G LTE Networks

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Abstract

Two main and fundamental key performance indicators which have been used to evaluate the performance of the wireless communication systems are the Bit Error Rate (BER), and the data throughput or its related channel capacity bound. For a system using one antenna at the transmitter and one antenna at the receiver and in non fading AWGN channel, the evaluation of the BER for most of the known modulation schemes is well known. Using more antennas at the transmitter and at the receiver and also in more realistic fading channel models, such as Rayleigh, Rician and Nakagami, the evaluation becomes more complex and necessitates the use and the development of advanced mathematical tools. In this context, the theory of BER evaluation had experienced an important evolution during the past decade. Initially, the evaluation of the BER performance was based on a classical approach where the Gaussian Q-function (also known as Gaussian Probability Integral) was used. Instead of using the alternative representation of the Gaussian Q-function, the Marcum Q-function was used to derive an approximate expression of BER analysis of Alamouti-MRC scheme with imperfect channel state information in Rician fading channel. In LTE, the BER is mainly evaluated by simulation and to the best of our knowledge, the BER analysis is rarely treated in the literature. One of our goal is to develop in this thesis some closed form expressions of the BER for the main MIMO schemes as used in LTE. To study the performance of LTE systems, a MATLAB based downlink physical layer simulator for Link Level Simulation (LLS) has been developed. A System Level Simulation of the Simulator is also available. The main goal of this proposed research work is to provide accurate BER analysis for different modulation schemes and their comprehensive comparison adopted in LTE networks

Keywords: *AWGN channel, BER, link level simulation, LTE, monte carlo simulation*

INTRODUCTION

In a MIMO system with Nr receive antennas and Nt transmit antennas, the relation between the received and the transmitted signals on OFDM subcarrier frequency k (k ∈ 1, ...,N), at sampling instant time n is given by where yk,n ∈ CNr×1 is the received output vector, Hk,n ∈ CNr×Nt represents the channel matrix on subcarrier k at instant time n, xk,n ∈ CNt×1 is the transmit symbol vector and nk,n ∼ CN(0, σ2n .I) is a white, complex valued Gaussian noise vector with variance σ2nand I is an Nr × Nr identity matrix. Assuming perfect channel estimation, the channel matrix and noise variance are considered to be known at the receiver. A linear equalizer filter given by a matrix Fk,n ∈ CNr×Nr is applied on the received symbol vector yk,n to determine the post-equalization symbol vector rk,n as follows [1–7].

$$r_{k,n} = F_{k,n} y_{k,n} = F_{k,n} H_{k,n} X_{k,n} + F_{k,n} n_{k,n} \quad (1)$$

The Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) design criterion is typically used for the linear receiver and the input signal vector is normalized to unit power [8–11]. In MIMO-OFDM systems, the key factor of link error prediction and performances is the signal to noise ratio (SNR) which represents the measurement for the channel quality information. In this study, the SNR is defined as follows [4]:

$$\gamma_{k,n} = \frac{\| H_{k,n} X_{k,n} \|2F}{Nt\sigma 2n} \quad (2)$$

where xk,n is the transmitted symbol vector, k.k2 F is the squared Frobenius norm of a matrix.

Numerical Analysis for 2*1 SFBC OFDM Scheme

In LTE, the transmit diversity techniques are defined only for 2 and 4 transmit antennas and one data stream. When two eNodeB antennas are available for transmit diversity operation, the Space Frequency Block Code (SFBC) is used [8]. SFBC is based on the well known Space Time Block Code (STBC), derived by Alamouti for two transmit antennas [1]. STBC is employed with the UMTS and it operates on pairs of adjacent symbols in the time domain. Since the signal in LTE is a two dimensional signal (in time and frequency domains) and the number of available OFDM symbols in a subframe is not always an even number, the direct application of STBC is not straightforward. In LTE, for SFBC transmission, the symbols are transmitted from two eNodeB antenna ports on each pair of adjacent subcarriers as follows [8]:

$$\begin{bmatrix} y(0)(1) & y(0)(2) \\ y(1)(1) & y(1)(2) \end{bmatrix} = \begin{bmatrix} x1 & x2 \\ -x2* & x1* \end{bmatrix} \quad (3)$$

where y(p)(k) denotes the symbols transmitted on the kth subcarrier from antenna port p. An important characteristic of such codes is that the transmitted signal streams are orthogonal and a simple linear receiver is required for optimal performance. Since OFDM converts the multipath channel into N frequency-flat fading channel, we first derive the BER expressions over flat Rayleigh fading channels, given by Pb (E). Then, the overall average BER over N subcarriers, in each case can be calculated from

$$BER = \frac{1}{N}\sum_{K=1}^{N} p_{b,}\, k\,(E) \quad (4)$$

where the index k (subcarrier index) is ignored for the sake of brevity. In addition, the impact of cyclic prefix in OFDM is assumed to be negligible. For the 2× 1 SFBC MIMO scheme, the probability density function of the SNR for each subcarrier is given by a chi-square distribution function as follows [8]:

$$f(\gamma) = \frac{2}{\gamma 2}\, \gamma e - \frac{2}{\gamma} \quad (5)$$

where $^{-}\gamma$ is the average SNR per symbol given by $^{-}\gamma$ = Es/N0. To derive the BER, we follow the unified approach to the performance analysis of digital communication systems over generalized fading channel. To this end, we first derive the expression of Moment Generating Function (MGF) of the derived probability density function of the instantaneous SNR as [6]:

$$M\gamma(s) = \int_0^{\infty} e^{-s\gamma} f(\gamma)\, d\gamma \quad (6)$$

Inserting (5) into (6) and solving the integral, the MGF yields:

$$M\gamma(s) = \frac{4}{\gamma^2 \left(s + \frac{2}{\gamma}\right)2} \quad (7)$$

The average BER expression for M-QAM modulation scheme can be obtained from [6] as:

$$Pb(E) \cong B \sum_{i=1}^{\frac{\sqrt{M}}{2}} 1/\pi \int_0^{\pi/2} M\gamma(Ai,\theta)\, d\theta \quad (8)$$

Where Ai, _ = $\sum_{i=1}^{\frac{\sqrt{M}}{2}} 1/\pi$ and B is defined by

$$B = 4\left(\frac{\sqrt{M-1}}{\sqrt{M}}\right)\left(\frac{1}{log2M}\right) \quad (9)$$

Then, using the MGF expression in (7), the associated MGF becomes:

$$My(Ai,\theta) = \frac{4}{\gamma 2\, \frac{(2i-1)2}{2\sin 2\theta}} + \frac{2}{\gamma} \quad (10)$$

The BER expression is then obtained by substituting (10) into (8). After some manipulations, we obtain:

$$Pb(E) \cong B \sum_{i=1}^{\sqrt{M/2}} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\frac{sin2\theta}{sin2\theta+Ci})2 \, d\theta$$

(11)

Equation (11) represents the average BER performance as a function of ¯γ = Es/N0 and it can be evaluated by numerical evaluation of the integral in (11) for M-QAM modulation schemes. Alternatively, the integral in Equation (11) can be solved analytically using [6], which give us a closed-form expression for the average BER of M-QAM modulation as follows:

$$Pb(E) \cong B \sum_{i=1}^{\sqrt{M/2}} I2(\frac{\pi}{2}, Ci) \quad (12)$$

where the closed-form expression for I2(., .) can be obtained from [6] as follows:

$$In(\emptyset, D) = \frac{1}{\pi} \int_0^{\emptyset} \left(\frac{sin2\theta}{sin2\theta+D}\right) n \, d\theta,$$

$$= \frac{\emptyset}{\pi} - \frac{\beta}{\pi} \left\{ \left(\frac{\pi}{2} + tan-1\,\alpha\right) \sum_{q=0}^{n-1} \left(\frac{2q}{q}\right) \frac{1}{(4(1+D))q} + \sin(tan-1\alpha) \sum_{q=1}^{n-1} \sum_{p=1}^{9} \frac{Tpq}{(1+D)q} [\cos(tan-1\alpha)] 2(q-p)+1 \right.$$

(13)

Analysis 4*2 FSTD OFDM Scheme

In LTE, the frequency space code, designed for 4 transmit antennas is defined as follows:

$$\begin{bmatrix} y(0)(1) & y(0)(2) & y(0)(3) & y(0)(4) \\ y(1)(1) & y(1)(2) & y(1)(3) & y(0)(4) \\ y(2)(1) & y(2)(2) & y(2)(3) & y(2)(4) \end{bmatrix} = \begin{bmatrix} x1 & x2 & 0 & 0 \\ 0 & 0 & x3 & x4 \\ x5 & x6 & 0 & 0 \end{bmatrix}$$

(14)

where y(p)(k) denotes the symbols transmitted on the kth subcarrier from antenna port p. For the 4×2 FSTD MIMO scheme, we can show that the instantaneous SNR of the system, for k th subcarrier, is equivalent to that for a 2 × 2 STBC MIMO system. Therefore, the probability density function of the SNR is given by a chi-square distribution function as follows [8]:

$$f(\gamma) = \frac{8}{3\gamma 4} \gamma 3 e^{-\frac{2}{\gamma}} \quad (15)$$

In this case, the MGF expression can be obtained by substituting (15) into (6), which Yields

$$M\gamma(s) = \frac{16}{\gamma^4} \left(s + \frac{2}{\gamma}\right) 4 \quad (16)$$

Similarly to the SFBC case discussed in previous Section, inserting (16) into (8), the average BER expression with M-QAM modulation for FTSD can be written as

$$Pb(E) \cong B \sum_{i=1}^{\sqrt{\frac{M}{2}}} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{sin2\theta}{sin2\theta+Ci}\right) 4 \, d\theta$$

(17)

where ci = 3(2i−1)2 2(M−1) ¯
2 , an integral which can be calculated numerically. Alternatively, the integral in (17) can be solved analytically, which yields a closed-form expression for the average BER of M-QAM modulation as follows:

$$Pb(E) \cong B \sum_{i=1}^{\sqrt{\frac{M}{2}}} I^4 \left(\frac{\pi}{2}, Ci\right) \quad (18)$$

where the closed-form expression for I4(., .) can be obtained from (17). Finally, for the sake of comparison, we can express the average BER for the SISO case, that has been derived for Rayleigh fading channels and M-QAM signals by [6]:

$$Pb(E) \cong \frac{B}{2} \sum_{i=1}^{\sqrt{\frac{M}{2}}} \left(1 - \sqrt{\frac{1.5(2i-1)2\gamma \, log2M}{M-1+1.5(2i-1)2\,\gamma \, log2\,M}}\right)$$

(3.20)

where B is defined in Equation (9). This concludes our BER analysis of 2 × 1 SFBC and 4 × 2 FSTD schemes in LTE. To verify our analysis, the numerical results of our analysis as well as the results of Monte-Carlo simulation.

Monte Carlo Simulation and validity of Numerical Method

Monte-Carlo simulation results, using the Link Level LTE Simulator, are also provided to prove the accuracy of the analysis [12, 13]. The common settings for Monte-Carlo simulations are summarized in Table 3.1. The average BER performance as a function of γ = Es/N0 for SISO, 2 × 1 SFBC and 4 × 2 FSTD MIMO schemes and different modulation modes have been considered. The results obtained for QPSK modulation appear in Figure 3.1. Figure 3.2 and Figure 3.3 show the results for 16-QAM modulation and 64-QAM modulation respectively.

In each Figure, the BER of SISO scheme as well as the BER of s SFBC and FSTD diversity schemes are shown. It can be seen that the average BER of QPSK, 16-QAM and 64-QAM schemes at high SNRs decrease by factors γ1, γ2, and γ4, for SISO, 2 × 1, and 4 × 2 cases, respectively. Thus, the corresponding diversity order (slope of the curves) is 1, 2 and 4 for SISO, 2 × 1 SFBC and 4×2 FSTD schemes, respectively. Since as stated earlier for the 4×2 FSTD scheme, for each time slot/frequency-slot 2 out of 4 transmit antennas are in use, therefore the diversity order must be 2×2 = 4. In fact, the corresponding average BER curve for 4×2 FSTD is somehow like the classical 2 × 2 STBC system, when the channel is not time-varying. In all cases it is clear that the BER performance improves as the number of transmit or receive antennas increases, as expected.

More specifically, if we consider Figure 3.1, a BER of 10−5 for SISO scheme is achieved with an SNR of 48 dB. For the 2×1 SFBC scheme, the same level of BER (i.e., 10−5) is achieved in SNR = 27 dB, which represents a 21 dB improvement compared to the SISO scheme. The 4 × 2 FSTD scheme performs even more efficiently since an improvement of 32 dB is observed at the same BER level of 10−5, as the required SNR is found to be 16 dB. Hence, the SNR gain of the 4 × 2 FSTD scheme compared to the 2 × 1 SFBC scheme is about 11dB (27 dB - 16 dB). Analogous observations can be made for the 16-QAM modulation order (Figure 3.2). For this modulation scheme, a BER of 10−4 for SISO scheme is obtained with an SNR of almost 43 dB. For the 2×1 SFBC scheme only an SNR of 29 Db is needed to reach the 10−4 level of BER, which represents an improvement in SNR of 14 dB. The SNR improvement gain of the 4×2 FSTD with respect to SISO scheme is almost 23 dB as the required SNR to achieve 10^{-4} with 4×2 FSTD scheme is found to be 20 dB. The SNR gain of 4 × 2 FSTD with respect to 2 × 1 SFBC is only 9 dB (29 dB - 20 dB). For 64 QAM modulations (Figure 1), the 2x1 SFBC scheme shows a SNR improvement of almost 14 dB compared to the SISO scheme at 10^{-4} BER level.

Table 3.1 *Simulation Settings*

Parameter	Setting
Transmission Schemes	SISO; 2 × 1 SFBC; 4 × 2 FSTD
Bandwidth	5 MHz
Simulation length	5000 subframes
Channel Type	Flat Rayleigh
Channel knowledge	Perfect
CQI	6 (QPSK); 9 (16-QAM) and 16 (64-QAM)

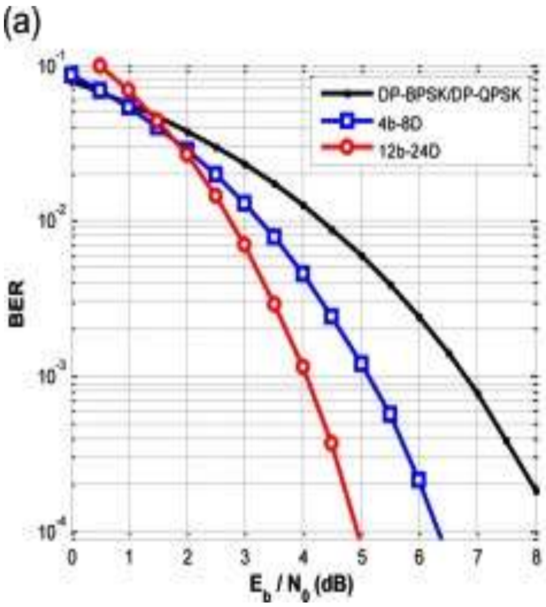

Fig. 1: *Numerical Evaluation and Monte-Carlo Simulations of the Average BER for QPSK Modulation.*

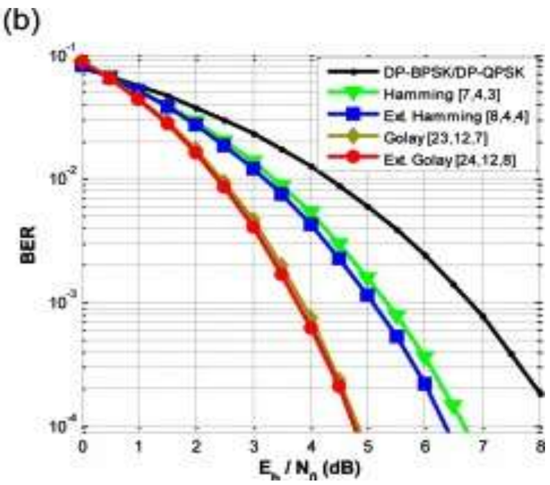

Fig. 2: *Numerical Evaluation and Monte-Carlo Simulations of the Average BER for 16- QAM Modulation.*

The 4×2 FSTD scheme however has a 22 dB SNR gain compared to SISO scheme at the same BER level (10−4). In addition to the previous observations, the comparison of the results with respect to modulation order is as expected. In fact, at 10−4 BER level, the required SNRs for SISO scheme are found to be 38 dB; 43 dB and 48 dB for QPSK; 16-QAM and 64-QAM, respectively.

This means that to reach the 10^{-4} BER level the 64-QAM requires 10 dB higher SNR compared to QPSK modulation, which is in compliance with the theory of performance of high order modulations. Finally, it can be observed from Figure 1, Figure 2 and Figure 3.3 that numerical evaluation results obtained from BER formulas match closely to the BER results obtained from Monte-Carlo simulations. This verifies the accuracy of our mathematical analysis.

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