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O.O. Tkachenko

DETERMINATION OF ANALYTICAL CALCULATION ERROR OF MAGNETIC FIELD OF HIGH-VOLTAGE CABLE LINES WITH TWO-POINT BONDED CABLE SHIELDS CAUSED BY NON-UNIFORM CURRENT DISTRIBUTION IN THE SHIELDS

This paper deals with the determination of analytical calculation error of magnetic field of high-voltage cable lines in two-point bonded cable shields caused by non-uniformity of the current distribution in the shields. The relative error is determined by comparing numerical calculation of magnetic field obtained in the COMSOL Multiphysics software with the analytical one. It is shown that the maximum value of relative error does not exceed 8 %. The obtained error values are verified by testing the numerical calculation and confirmed by results of experiment. In paper proves the correctness of the analytical calculation of magnetic field of cable lines at the points of its normalization, which is carried out without taking into account the non-uniform current distribution in the cable shields. References 13, tables 2, figures 4.

Key words: **cable line, cable shield, bonded shields, magnetic field, calculation error.**

В работе определена относительная погрешность аналитического расчета индукции магнитного поля трехфазной кабельной линии, обусловленная неравномерностью плотности тока в экранах одножильных кабелей. Погрешность получена путем сравнения численного расчета в программной среде COMSOL Multiphysics с аналитическим методом расчета. Показано, что максимальное значение погрешности не превышает 8 %. Полученные значения погрешности верифицированы путем тестирования численного расчета и подтверждены результатами эксперимента. Обоснована корректность аналитического расчета магнитного поля кабельных линий в точках его нормирования при двухстороннем замыкании экранов кабелей, выполняемого без учета неравномерности плотности тока в экранах кабелей. Библ. 13, рис. 4, табл. 2.

Ключевые слова: **кабельная линия, экран кабеля, двухстороннее заземление, магнитное поле, погрешность расчета.**

Introduction. The problem of calculating the RMS value of the flux density of the magnetic field (MF) of high-voltage cable lines consisting of single-core cables [1, 2] arises when they are designed and is necessary to limit the MF along the cable lines to the maximum permissible level. In Ukraine, this level is regulated by normative documents [3, 4] and for residential development is 10 μT over the cable lines, at an altitude of 0.5 m from the surface of the earth, and 0.5 μT in nearby residential areas.

A certain complexity is the calculation of the flux density of the MF of cable lines with two-point bonded cable shields [4], when longitudinal currents are induced in them [5, 6]. The known engineering methods for calculating MF of cable lines [1, 4, 5], for such cases, are based on numerical methods. When using these techniques, the results of the calculation are presented in the form of tables with a limited number of discrete values of the calculated quantities, which causes difficulties in their practical application in the design of the cable lines.

In [7], an analytical technique is proposed for calculating the MF of cable lines with the two-point bonded cable shields, free from the above disadvantages, and allowing calculation at any cable line parameters. However, this technique does not take into account the influence of the proximity effect on the MF of cable line [8, 9], which related to non-uniformity of the densities of the longitudinal currents in the cable shields, the description of which is analytically difficult. In this connection, the problem arises of determining the error in the analytical calculation of the MF of cable lines.

The goal of this paper is the determination of the relative error of the analytical calculation of the magnetic flux density of three-phase high-voltage cable line at the points of its normalization caused by non-uniform current distribution in the cable shields.

Investigation technique. The required relative error ε of the analytical calculation of the flux density of the MF of cable line is defined as

$$\varepsilon = \left| 1 - \frac{B}{B_{num}} \right| \cdot 100\% , \qquad (1)$$

where B is analytical value of the magnetic flux density, without taking into account the non-uniform current distribution in the cable shields; B_{num} is the exact RMS value of the magnetic flux density, which it is determined with taking into account the non-uniformity current distribution.

To determine B_{num}, a numerical calculation is used that has a verified relative error, which is substantially less than the error permissible in engineering calculations.

Calculation of the magnetic flux density is carried out for real high-voltage cable line [1, 4] at the point of normalization P for trefoil and flat arrangement (Fig. 1). The value of h varies from 0.5 m to 2 m by step 0.5 m, taking into account [3, 4]. In this case, the distance between the cables axes d varies from 0.1 m to 0.3 m by step 0.1 m. The diameter of the cable shields D is 55 mm and 70 mm, the cross-section of shields S is 100 mm^2, 200 mm^2 and 300 mm^2.

Numerical calculation of B_{num}. To calculate B_{num}, the software package *COMSOL Multiphysics* [10] is used, the relative error of which is verified in [5] when solving a similar problem and does not exceed 1 %. The computational model, in contrast to [5], is performed in accordance with [11] and allows one to find the flux distribution of the MF of a three-phase cable line with allowance for the non-uniform current distribution in shields with a smaller error.

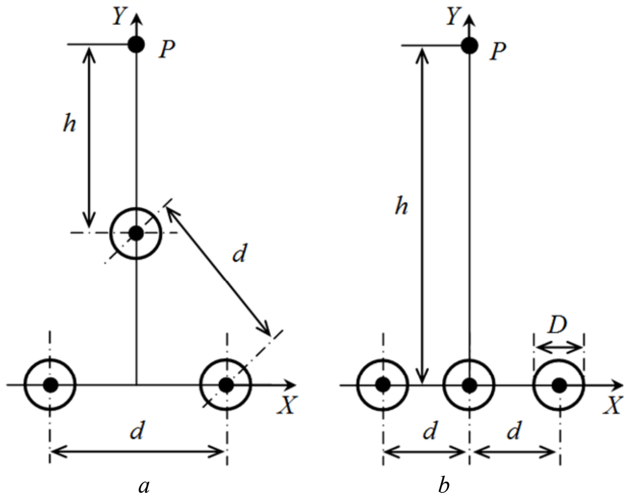

Fig. 1

The following assumptions are made in the calculation:

1) Cables are infinitely long and laid parallel to each other;

2) The cable line operates in the steady-state mode, the RMS values of currents in the cores have a frequency of 50 Hz, are equal in magnitude and their phases are shifted from each other by 120°.

In this case, the MF of the cable line is a plane-parallel, which allows us to solve the problem in a two-dimensional formulation (Fig. 2).

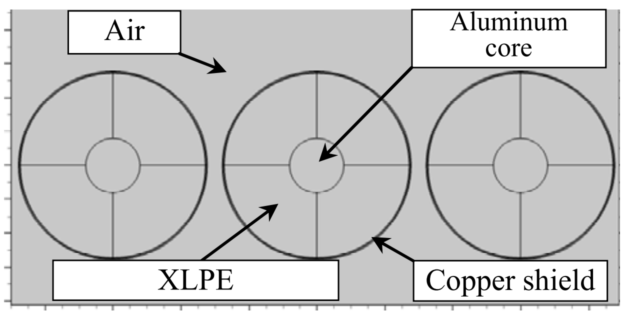

Fig. 2

The theoretical basis for describing the accepted design model of MF is the equation of the law of the total current in the quasi-stationary approximation [8, 12]. Taking into account the connection between the vector potential and the magnetic field strength and taking into account that the vector potential has only one non-zero component, this equation takes the form:

$$\frac{\partial^2 \dot{A}_z}{\partial x^2} + \frac{\partial^2 \dot{A}_z}{\partial y^2} - j\mu_0 \omega \sigma \dot{A}_z = 0 ,$$

where $\dot{A}_z$ is the complex amplitude of the component of the vector potential of the electromagnetic field along the *Z* axis directed parallel to the cable line; *j* is the imaginary unit; $\mu_0=4\pi\cdot 10^{-7}$ H/m is the magnetic constant; $\omega=2\pi\cdot 50$ s^{-1} is the cyclic current frequency; σ is the conductivity of the medium for which the equation is written ($\sigma_{Al} = 3.8\cdot 10^7$ S/m, $\sigma_{Cu} = 5.0\cdot 10^7$ S/m, $\sigma_{air} = 0$ S/m).

The numerical simulation is carried out using the *«Magnetic Fields»* interface, which is included in the *«AC/DC Module»* [10] of the software package. When constructing a two-dimensional model in the Cartesian coordinate system, the option *«Space Dimension»* was set to *«2D»*.

The calculation area is a circle with a diameter *L*=6 m. Inside it there are the cable line and airspace. To reduce the size of the calculated area on the periphery of the circle, the layer *«Infinite element domains»* of thickness *L*/3 is located.

The calculation model of each of the three cables consists of an aluminum core and a copper shield (Fig. 2). Since the electrical conductivities of the external medium and cross-linked polyethylene (XLPE) are negligible compared to the conductivity of the shield, their effect on the current density distribution in the shields is not taken into account.

The current in each cable core is set using the *«Single-Turn Coil»* function in the *«Magnetic Fields»* menu. For the option *«Coil excitation»*, the variant *«Current»* is selected, for which:

$$\dot{I}_k^c = I_0 e^{j\phi_k} ,$$

where I_0 is the current amplitude in cores; $\phi_k = \{-2\pi/3, 0, 2\pi/3\}$ is the current phase in the cable core; $k = \overline{1,3}$ is the cable number.

The boundary conditions are following:

$$\begin{cases} \dot{A}_z^i = \dot{A}_z , \\ \dfrac{\partial \dot{A}_z^i}{\partial n} = \dfrac{\partial \dot{A}_z}{\partial n} , \end{cases}$$

where n is the unit vector of the normal to the boundary; the superscript *i* indicates the conductive medium.

In the area of the shields, the mesh of the *«Mapped»* type is used (Fig. 3,*b*). Along the thickness, the shield is divided into 20 elements, and along the perimeter – into 200. In the other regions, the mesh of the *«Free Triangular»* type is used (Fig. 3,*a*). The mesh density is *«Extremely fine»*, the minimum element size is 1/40 of the diameter of the cable core.

The result of the calculation is the distribution of $\dot{A}_z$. Taking into account that the complex amplitudes of the magnetic flux density are equal to $\dot{B}_x = \partial \dot{A}_z / \partial y$ and $\dot{B}_y = -\partial \dot{A}_z / \partial x$, the expression for the RMS value of the magnetic flux density of the cable line takes the form:

$$B_{num} = \frac{1}{\sqrt{2}} \sqrt{\left|\frac{\partial \dot{A}_z}{\partial y}\right|^2 + \left|\frac{\partial \dot{A}_z}{\partial x}\right|^2} .$$

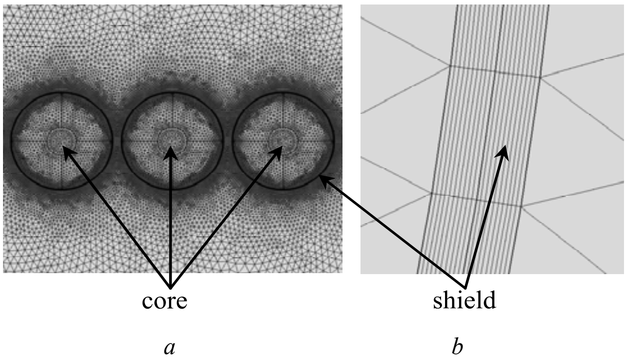

Fig. 3

It is worth noting that the proposed calculation model is applicable for an arbitrary method of cables arrangement.

The verification of the numerical calculation was performed by comparison with the solutions obtained by doubling the size of the calculated area and using a finest grid. In this case, the error of calculation does not exceed 0.5 %. The results of the calculation also coincide with the experimental data [5].

Analytical calculation of *B*. The analytical method for calculating the MF of cable line is proposed in [7]. When obtaining the calculated ratios, the same assumptions were used in it as for the numerical calculation considered above, but the distribution of the current density in the shields of each cable was assumed to be uniform.

As the calculated one we use the following relation obtained from [7] by transition from the complex amplitude of the magnetic flux density to its RMS value:

$$B=\frac{1}{\sqrt{2}}\sqrt{\left|\frac{\mu_0}{2\pi}\sum_{k=1}^{3}\left(\dot I_k^c+\dot I_k^{sh}\right)\left(\frac{-(y-y_k)}{(x-x_k)^2+(y-y_k)^2}\right)\right|^2+\left|\frac{\mu_0}{2\pi}\sum_{k=1}^{3}\left(\dot I_k^c+\dot I_k^{sh}\right)\left(\frac{x-x_k}{(x-x_k)^2+(y-y_k)^2}\right)\right|^2}, \quad (2)$$

where $\dot I_k^c$ and $\dot I_k^{sh}$ are the complex amplitudes of currents in the core and shield of the k-th cable, respectively; (x, y) are the coordinates of the point P, in which MF is calculated (see Fig. 1); (x_k, y_k) are the coordinates of the k-th cable axis; $k=\overline{1,3}$ is the cable number.

The values of the currents in (2) are determined as follows. For the case of cable lines in the flat arrangement (Fig. 1,*b*), the currents in the cores form a system of direct sequence, and the currents in the shields are determined by the following relationships [7, 13]:

$$\dot I_1^{sh}=-\dot I_1^c\cdot\frac{Q\cdot\ln 2\Delta\cdot\ln\frac{\Delta^3}{2}+\sqrt{3}\ln 2-j\ln 4\Delta^3}{Q\cdot\ln 2\Delta\cdot\ln\frac{\Delta^3}{2}-\frac{3}{Q}-2j\ln 2\Delta^3},$$

$$\dot I_2^{sh}=-\dot I_2^c\cdot\frac{jQ\ln\frac{\Delta^3}{2}}{3+jQ\ln\frac{\Delta^3}{2}}, \quad (3)$$

$$\dot I_3^{sh}=-\dot I_3^c\cdot\frac{Q\cdot\ln 2\Delta\cdot\ln\frac{\Delta^3}{2}-\sqrt{3}\ln 2-j\ln 4\Delta^3}{Q\cdot\ln 2\Delta\cdot\ln\frac{\Delta^3}{2}-\frac{3}{Q}-2j\ln 2\Delta^3},$$

where $Q=\frac{\mu_0\omega}{2\pi R}$ and $\Delta=\frac{d}{r}$ are the dimensionless parameters of the cable line; R is the cable length unit resistance, Ω/m; d is the inter-phase distance (distance between axes of neighbor cables), m; r is the shield radius, m.

In the case of trefoil arrangement of the cable line (Fig. 1,*a*), the calculation dependence for currents in shields has a more compact form [7, 13]:

$$\dot I_k^{sh}=-\dot I_k^c\cdot\frac{jQ\ln\Delta}{1+jQ\ln\Delta}. \quad (4)$$

Consequently, B is analytically calculated using expression (2) with (3) or (4), for cables with a trefoil or flat arrangement, respectively.

The analysis of analytical calculation error. Fig. 4 shows the results of the calculation of the RMS values of B (analytical calculation) and B_{num} (numerical calculation) using the proposed methods for the 1000A current. The calculation of the magnetic flux density is performed as a function of the distance h to the observation point P (see Fig. 1) for the case of maximum non-uniformity of the current distribution in cable shields. At the same time, the shield diameter is maximal D=70 mm [4], the shield cross section is S =300 mm^2 and the cables are laid close d=0.1 m. As can be seen on Fig. 4, the scatter of the results of the analytical and numerical calculations is negligible.

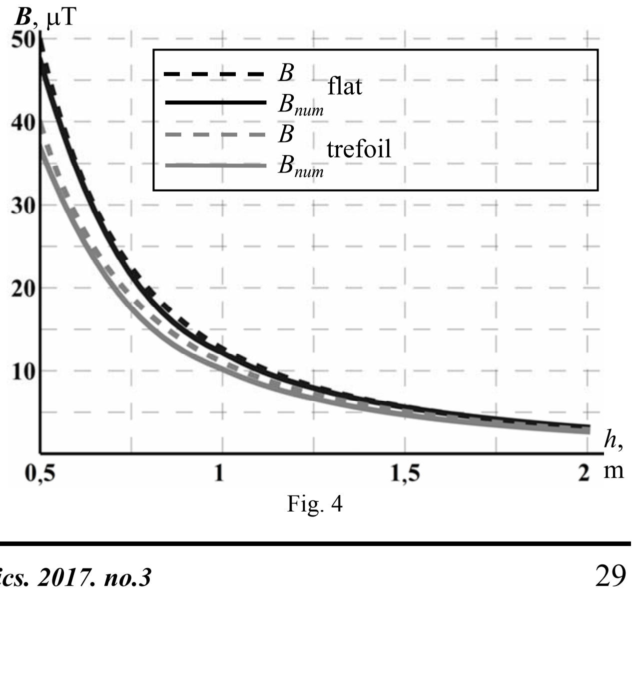

Fig. 4

To quantify the error of the analytical calculation, Table 1, 2 show the results of calculating the value of ε (1) for real high-voltage cable lines [4].

Table 1

Values of ε, % for cable line of trefoil arrangement

h, m	*d*, m	Shield diameter *D*, mm					
		55			70		
		Shield section *S*, mm^2					
		100	200	300	100	200	300
0.5	0.1	**1.0**	**2.7**	**4.3**	**1.6**	**4.8**	**7.9**
	0.2	**0.2**	**0.4**	**0.6**	**0.4**	**1.0**	**1.5**
	0.3	**0.1**	**0.1**	**0.1**	**0.2**	**0.3**	**0.5**
1.0	0.1	**1.0**	**2.8**	**4.3**	**1.6**	**4.8**	**7.9**
	0.2	**0.2**	**0.4**	**0.6**	**0.4**	**1.0**	**1.5**
	0.3	**0.1**	**0.1**	**0.1**	**0.2**	**0.3**	**0.5**
1.5	0.1	**1.0**	**2.7**	**4.3**	**1.6**	**4.8**	**7.9**
	0.2	**0.2**	**0.4**	**0.6**	**0.4**	**1.0**	**1.5**
	0.3	**0.1**	**0.1**	**0.1**	**0.2**	**0.3**	**0.5**
2.0	0.1	**1.0**	**2.7**	**4.3**	**1.6**	**4.8**	**7.9**
	0.2	**0.2**	**0.4**	**0.6**	**0.4**	**1.0**	**1.5**
	0.3	**0.1**	**0.1**	**0.1**	**0.2**	**0.3**	**0.5**

Table 2

Values of ε, % for cable line of flat arrangement

h, m	*d*, m	Shield diameter *D*, mm					
		55			70		
		Shield section *S*, mm^2					
		100	200	300	100	200	300
0.5	0.1	**1.5**	**2.5**	**3.0**	**2.0**	**3.8**	**4.8**
	0.2	**1.1**	**1.5**	**1.6**	**1.2**	**1.8**	**2.1**
	0.3	**1.1**	**1.2**	**1.3**	**1.1**	**1.4**	**1.6**
1.0	0.1	**0.6**	**1.3**	**1.5**	**1.1**	**2.6**	**3.2**
	0.2	**0.9**	**1.2**	**1.3**	**1.0**	**1.5**	**1.7**
	0.3	**1.0**	**1.1**	**1.2**	**1.0**	**1.4**	**1.5**
1.5	0.1	**0.8**	**0.7**	**0.9**	**0.2**	**0.7**	**0.8**
	0.2	**0.5**	**0.6**	**0.6**	**0.6**	**0.9**	**1.0**
	0.3	**0.8**	**0.9**	**0.9**	**0.8**	**1.1**	**1.2**
2.0	0.1	**2.1**	**2.6**	**3.2**	**2.0**	**1.8**	**2.3**
	0.2	**0.1**	**0.1**	**0.1**	**0.1**	**0.2**	**0.2**
	0.3	**0.5**	**0.6**	**0.6**	**0.6**	**0.8**	**0.8**

As follows from Table 1, 2, the maximum error of the analytical calculation of MF is 7.9 % for cable line of trefoil and 4.8 % of flat arrangement, and occurs at the minimum value of *d*(0.1 m), maximum values of *D*(70 mm) and *S*(300 mm^2). This case corresponds to dense cable routing, which is often unacceptable, since it can cause a limitation of the current carrying capacity of the cable. With increasing *d* to 0.2 m, the value of ε does not exceed 2.5 %.

The obtained results of the calculation with a spread of not more than 5 % agree with the results of the experimental investigations of the MF of cable line carried out in [5].

Thus, the relative error ε of the analytical calculation of the MF of cable line does not exceed 8 %, which confirms the correctness of the assumption made in it about the uniformity of the current distribution in each of the cable shields in the engineering calculation of the MF of cable line.

Conclusions.

1. The maximum value of the error of the analytical calculation of magnetic field does not exceed 7.9 % for cable line of trefoil and 4.8 % of flat arrangement, and occurs with the maximum diameter and cross section of the cable shields (70 mm and 300 mm^2), and the minimum distance between the cable cores axes (0.1 m).
2. The obtained error values are verified by testing the numerical calculation performed in the *COMSOL Multiphysics* software (the error of less than 0.5 %), and comparing the results with the experiment.
3. The presented analysis confirms the correctness of the analytical calculation of the flux density of magnetic field of cable lines at the points of its normalization, which is carried out without taking into account the non-uniformity of current distribution in the cable shields.

REFERENCES

1. SOU-N EE 20.179:2008. *Rozrakhunok elektrychnoho i mahnitnoho poliv linii elektroperedavannia. Metodyka* [Calculation of the electric and magnetic fields of power line. Method]. Kyiv, Minenergovugillya of Ukraine Publ., 2016, 34 p. (Ukr).

2. Rozov V.Yu., Reutskiy S.Yu., Piliugina O.Yu. The method of calculation of the magnetic field of three-phase power lines. *Tekhnichna Elektrodynamika*, 2014, no.5, pp. 11-13. (Rus).

3. *Pravila ulashtuvannya electroustanovok* [Electrical installation regulations]. 5th ed. Kharkiv, Minenergovugillya of Ukraine, 2014. 793 p. (Ukr).

4. SOU-N MEV 40.1-37471933-49:2011.2. *Proektuvannia kabelnykh linii napruhoiu do 330 kV. Nastanova* [Design of cable lines with voltage up to 330 kV. Guidance]. Kyiv, Minenergovugillya of Ukraine Publ., 2017, 139 p. (Ukr).

5. Rozov V.Yu., Kvytsynskyi A.A., Dobrodeyev P.N., Grinchenko V.S., Erisov A.V. and Tkachenko O.O. Study of the magnetic field of three phase lines of single core power cables with two-end bonding of their shields. *Electrical engineering & electromechanics*, 2015, no.4, pp. 56-61. (Rus). **doi: 10.20998/2074-272X.2015.4.11.**

6. *Kovrigin L.A.* The longitudinal currents in the screens of the single-core cables. *Kabel-news*, 2009, no.3, pp. 56-58. (Rus).

7. Rozov V.Yu., Tkachenko O.O., Erisov A.V. and Grinchenko V.S. Analytical calculation of magnetic field of three-phase cable lines with two-point bonded shields. *Tekhnichna Elektrodynamika*, 2017, no.2, pp. 13-18 (Rus).

8. Demirchyan K., Neiman L., Korovkin N. and Chechurin V. *Teoreticheskie osnovy elektrotekhniki: V 3 t.*[Theoretical Basis of Electrical Engineering: in 3 vols.]. Saint Petersburg: Piter, vol.3, 2003, 377 p. (Rus).

9. Podoltsev A., Kucheryavaya I. *Multifizicheskoe modelirovanie v elektrotehnike. Monografiya* [Multi-physical modeling in electrical engineering. Monograph]. Kyiv: Inst. of Electrodynamics of NAS of Ukraine, 2015, 305 p. (Rus).
10. https://www.comsol.com/models/acdc-module
11. Grinchenko V.S., Tkachenko O.O. and Chunikhin K.V. Calculation of shield currents in three-phase cable lines with a trefoil arrangement of phases. *Anotatsiї dopovidei 24 Mizhn. nauk.-prakt. konf. «Informatsiini tekhnologiї: nauka, tekhnika, tekhnologiia, osvita, zdorov'ia»* [Abstracts of 24th Int. Sci.-Pract. Conf. «Information technology: science, engineering, technology, education and health»]. Kharkiv, Ukraine, 2016, 18-20 May, 324 p. (Rus).
12. del-Pino-López J.C., Cruz-Romero P., Serrano-Iribarnegaray L. and Martínez-Román J. Magnetic field shielding optimization in underground power cable duct banks. *Electric Power Systems Research*, 2014, vol. 114, pp. 21-27. **doi: 10.1016/j.epsr.2014.04.001.**
13. Grinchenko V.S., Tkachenko O.O. and Grinchenko N.V. Improving calculation accuracy of currents in cable shields at double-sided grounding of three-phase cable line. *Electrical engineering & electromechanics*, 2017, no.2, pp. 39-42. **doi: 10.20998/2074-272X.2017.2.06.**

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O.O. Tkachenko, Postgraduate Student,
State Institution «Institute of Technical Problems
of Magnetism of the NAS of Ukraine»,
19, Industrialna Str., Kharkiv, 61106, Ukraine,
phone +380 572 992162,
e-mail: oleksandr.tk7@gmail.com

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