O.V. Golik

# STATISTICAL PROCEDURES FOR TWO-SIDED LIMIT OF A CONTROLLED PARAMETER IN THE PROCESS OF PRODUCTION OF CABLE AND WIRE PRODUCTS

*Purpose. To consider issues of statistical control in the process of mass production of cable and wire products on the example of enameled wire. To analyze the results of direct control of the diameter of the wire in two-layer polyimide insulation in a continuous technological cycle. To submit to the control map of maximum probability of the exit diameter outside a specific range. To analyze the conditions under which maximum sensitivity of process control. Methodology. Study of the sensitivity of the control map of maximum probability of the exit option for regulatory of limit in the field deviations of the centered parameter close to zero. The existence of stable trends in the change of a controlled parameter can reduce the sensitivity of punishment to instability of the process. Results. To achieve maximum sensitivity of control of the technological frontier should be selected on the basis of the achieved level of the average value of the parameter and its statistical scattering. Process boundaries must be changed in accordance with the achieved level of the average value of the parameter and its statistical scattering. Such a change may serve as a quantitative indicator of trends in the increase or decrease in the reliability of the technological system. Originality. In particular the tasks of current control using engineering tolerances for controlled parameter are impractical. Control on $P_{max}$ should be directed to the exception of manufacturing, the parameters of which extend beyond the technical tolerances. Practical value. The exception is the manufacture of bulk cable products, the parameters of which extend beyond the technical tolerances.* References 5, figures 1.
*Key words:* **enameled wire, double polyimide insulation, control card, maximum probability of the parameter exit for the regulatory limit.**

*Рассмотрены вопросы статистического контроля в процессе производства массовой кабельно-проводниковой продукции на примере эмальпроводов. Получены результаты непосредственного контроля диаметра эмальпровода с полиимидной двухслойной изоляцией в непрерывном технологическом цикле. Представлена контрольная карта максимальной вероятности выхода диаметра за пределы определенного диапазона.* Библ. 5, рис. 1.
*Ключевые слова:* **эмальпровод, двойная полиимидная изоляция, контрольная карта, максимальная вероятность выхода параметра за нормативный предел.**

**Introduction.** In the process of quality control of insulation of enameled wires it is convenient to use the so-called control card. Control card is a special form in which the statistical indicators for measured indicator in the chronological sequence of manufacturing [1] are included. On the form control frontiers which limit the range of permissible values of statistical indicators are presented. If during the inspection results are outside the regulatory frontier, it is perceived as information on the rejection of the process from normal. Using control cards the main is a method used to determine the control limits.

In the production the following types of cards are used most often: a card of arithmetic, a card of the standard deviation, a card of number of defects per unit of product. The most effective is the use of these control cards in the complex, from the input control stage to the output control. Here, it is necessary to select such a card type and parameters of the card that are informative in all stages of control. For example, this is the application for card building of the mathematical apparatus of interval statistical models.

**Analysis of publications.** In [2] tasks of the theoretical basis of application of methods of interval statistical models to the lower values of the mean interval are solved, in fact, in one phrase: «to take everything with a minus.» For solving applied mathematical problems of process control that is not enough, since at the bilateral frontiers upper and lower deviations of values of the controlled parameter may appear in any order.

For all bounded signs $f$ belonging to the class $\mathfrak{I}_{00} : \mathfrak{I}_{00} = \left\{ f : \sup |f(x)| < \infty \right\}$ interval averages $\underline{M} f$ ; $\overline{M} f$ exist. Axioms of interval models of averages adopted [2] as main uniquely associate lower and upper averages by changes of sign of the controlled parameter. For all upper bounded signs:

$$\mathfrak{I}_0 = \left\{ f : \sup f(x) < \infty \right\}; \overline{M} f < \infty.$$

According to the axiom of the change of sign [2] for all upper bounded: $\underline{M}(-f) = -\overline{M} f$, from here $\underline{M}(-f)$ is determined on $-\mathfrak{I}_0$ .

That is the change of sign of the parameters of the class $\mathfrak{I}_0$ results in the class where there are lower averages $\underline{M} f$ and at their intersection there are those and other that are the interval averages.

Such unambiguous linking the upper and lower averages by change of sign can, in principle, contrary to the physical meaning of some parameters in real control

problems. However, it may be convenient for the mathematical description and simultaneously be physically adequate if in a particular problem to use *centered* feature set.

**The goal of the paper** is investigation of maximum sensitivity of control at bilateral limitation of the controlled parameter in the production of power cables and wires.

**Results of investigations.** Using of centered parameters set becomes fundamental for the real control problem in which the measured value is only positive, and technological limitation bilateral.

The function $g(x)$ majorizing the set of primary characteristics $f(x)$ belongs to a semilinear shell with non-negative coefficients $c^+_i$ and an arbitrary free term $c$: $g(x) = c + \sum c^+_i g_i(x)$; $g_i(x) \geq f_i(x)$. Centering of the majorizing function $g(x)$ allows to find the best approximation of the primary function $f(x)$ [2]:

$$g^\circ_i(x) = g_i(x) - \overline{M} g_i ; \quad \overline{M} g^{\circ i} = 0;$$
$$\overline{M} = \inf \left\{ [c + c^+_i g_i(x)] : c + c^+_i g_i(x) \geq f(x) - \sum c^+_i g(x) \right\} = (1)$$
$$= \inf \sup \left[ f(x) - \sum c^+_i g^0_i(x) \right].$$

In addition, centering of the majorizing function $g(x)$ permits to use in real control the deviation of the measured parameter from its primary average, i.e. instead of $x$ to use the $\Delta x$ that allows to remove this contradiction between the considered positive random value of the measured characteristic at the control with two-way restriction, with one side, and the of the change of sign [2] for all low limited parameters, from the other side: $\underline{M}(-f) = -\overline{M}(f)$, from here $\underline{M}(-f)$ is defined only on $-\mathfrak{I}_0$.

For example, for controlling the primary indication of the dielectric loss tangent in [3] as a controlled parameter they use the deflection of the measured characteristic $Y$ from its primary average $M^*[Y]$ and use the majorizing function as a parabola with three parameters:

$$g(Y - M[Y]) = C + C_{2(+)}((Y - M[Y]) - C_1)^2, \quad (2)$$

which majorizes the indicator sign (the relative number of the primary characteristic values that have fallen into the set interval) $\alpha_1 \dots \alpha_2$, and if the upper limit $\alpha_2$ has no technical meaning as in the problem of control of the dielectric loss tangent), then:

$$A\{\alpha_1 \leq (Y - M[Y]) \leq \alpha_2\} \leq C + C_{2(+)}$$
$$\alpha_1 - C_1)^2 \geq 1. \quad (3)$$

That $M(Y - M^*[Y]) = 0$, for the minimal majorizing function $\inf \left\{ C + C^{(+)2}(\alpha_1 - C_1)^2 \right\} = 1$ determines the parabola's parameters:

$$C = 0, C_1 = -M_{\max}((Y - M[Y]))/\alpha_1,$$
$$C_{2(+)} = (\alpha_1 - C_1)^{-2}. \quad (4)$$

Relations (3), (4) allowed the use of an assessment of the relative maximum number of primary attribute values that exceed the upper permissible limit $\alpha_1$ to organize control at the top technological limitation [3]:

$$A_{\max}\{\Delta Y \geq \alpha_1\} = (1 + \alpha_1^2/M_{\max}[(\Delta Y)^2])^{-1}, \quad (5)$$

where the indicator sign $A_{\max}$ is the relative maximum average number of the primary characteristic values that exceed $\alpha_1$, i.e. the maximum probability of controlled parameter $\Delta Y$ exit for the upper limit $\alpha_1$: $P_{\max}\{\Delta Y \geq \alpha_1\}$.

For bilateral limitation in accordance with the axiom of the change of sign [2] of interval models medium, lower and upper averaged are connected by controlled characteristic's sign change. For example, at the control of the enamel wire diameter $D$ the maximum probability $P_{\max}$ of the controlled parameter $\Delta D$ exit outside the range frontiers $\overline{E} \dots \underline{E}$ is defined as the sum of the corresponding probabilities of the parameter exit outside of one-sided limits. Here, the probability of the controlled parameter $\Delta D$ exit outside the lower bound is taken with minus:

$$P_{\max i} = \overline{P_{\max i}} - \underline{P_{\max i}}; \quad (6)$$

$$\overline{P_{\max i}} = \frac{[\sup(\Delta D_{i \cdot 2} \Delta D_{i \cdot 2 - 1})]^2}{[\sup(\Delta D_{i \cdot 2} \Delta D_{i \cdot 2 - 1})]^2 + (\overline{E} - \frac{1}{2}\sum_{i \cdot 2 - 1}^{i \cdot 2} D)^2}; \quad (7)$$

$$\underline{P_{\max i}} = \frac{[\inf(\Delta D_{i \cdot 2} \Delta D_{i \cdot 2 - 1})]^2}{[\inf(\Delta D_{i \cdot 2} \Delta D_{i \cdot 2 - 1})]^2 + (\underline{E} - \frac{1}{2}\sum_{i \cdot 2 - 1}^{i \cdot 2} D)^2}. \quad (8)$$

where $D$ is the wire's diameter; $\overline{E}$ is the upper technological frontier of the diameter; $\underline{E}$ is the lower technological frontier of the diameter; $\Delta D_{i \cdot 2}$ is the difference between the current diameter of the sample No. $i$ and average diameter value determined during the process cycle: $\Delta D_{i2} = D_{i2} - \frac{1}{i*2}\sum_1^{i2} D$.

Fig. 1 shows results of the direct control of enamel wire diameter with polyimide insulation in continuous process cycle and presents the control card of the maximum probability of the diameter exit outside the range frontiers determined in accordance with (6) – (8).

A comparison of Fig. 1,*a* and Fig. 1,*b* shows the information content of the technological control of the maximum probability $P_{\max}$ of the parameter exit outside the predetermined bilateral range:

1) the control card represents the processing stability period during which $P_{max}$ does not exceed by the absolute value the level of 0.25 (dotted line) which analytically from (9) reflects increase in sensitivity of control with increasing deviation from the average;

2) the control card reflects a stable tendency to reduce the control parameter values that will prevent $P_{max}$ exit outside the level of 0.25 which made it possible to determine the cause of the trend of the decreasing in the $D$ – increase the conductor drawing during the technological cycle.

Here we have used an analytical study of the derivatives of (4) to assess the control frontiers in the $P_{max}$ control card with unilateral constraint [3]. Control by $P_{max}$ at the bilateral limitation posed the question about the need to study the sensitivity of the $P_{max}$ control card in the centered parameter deviations area close to zero. In this area, the presence of the stable trend in the change of the controlled parameter may reduce the sensitivity of the card to the instability of the technological process.
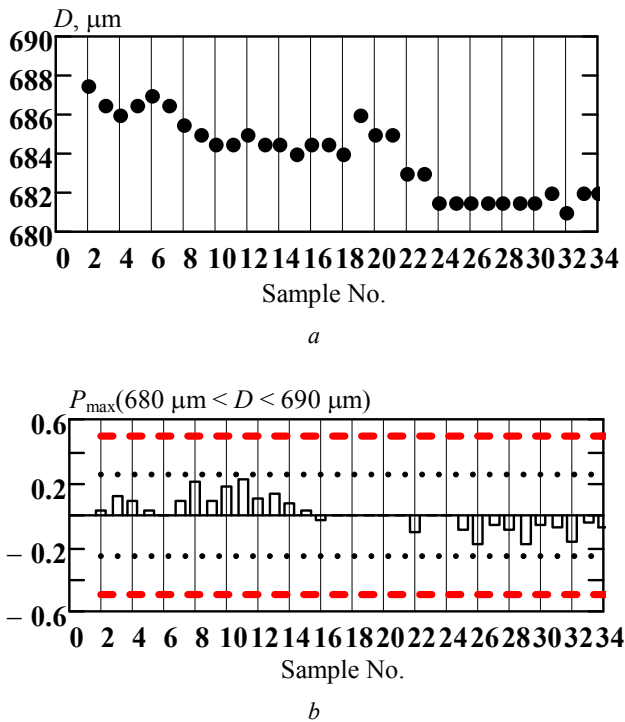


*a*



*b*

Fig. 1. Control of diameter $D$ of enameled wires in the continuous technological process cycle by the maximum probability $P_{max}$ of the parameter exit outside the predetermined frontiers of the given bilateral range: *a* – results of measurements of $D$; *b* – control card of the probability of the parameter exit outside of the range's frontiers (680 … 690) μm

Because the value $[(\varDelta D_{i \cdot 2} \varDelta D_{i \cdot 2-1})]^2$ is the square of the largest current change of the diameter, this value generally is the maximum estimate of the variance of the controlled parameter in the current sample $\underline{S}^2$. Accordingly, the change of sign for reversing for the probability of exit outside the lower limit, value

$\inf[(\varDelta D_{i \cdot 2} \varDelta D_{i \cdot 2-1})]$ is a square of the largest current change of diameter towards the lower limit of the specified range. That is, this value is the maximum variance estimation of the controlled parameter in the current sample at its changing toward the lower limit $\underline{S}^2$. We denote the current deviations of the average value of the parameter in the sample from the upper to lower bounds as $\underline{\alpha} \dots \overline{\alpha}$.

Then we can represent (2) as a function of four variables:

$$\overline{\alpha} = \overline{E} - \frac{1}{2}\sum_{i \cdot 2-1}^{i \cdot 2} D; \quad \underline{\alpha} = \underline{E} - \sum_{i \cdot 2-1}^{i \cdot 2} D, \tag{9}$$

$$P_{max} = \frac{\overline{S}^2}{\overline{S}^2 + \overline{\alpha}^2} - \frac{\underline{S}^2}{\underline{S}^2 + \underline{\alpha}^2}. \tag{10}$$

$P_{max}$ sensitivity to change in the controlled parameter values is a total differential (10). In the case of mutual independence of variables, the following expression permits to analyze theoretically $P_{max}$ sensitivity to bilateral limitation:

$$dPM = \frac{\partial P_{max}}{\partial \overline{S}} + \frac{\partial P_{max}}{\partial \overline{\alpha}} - \frac{\partial P_{max}}{\partial \underline{S}} - \frac{\partial P_{max}}{\partial \underline{\alpha}} =$$

$$= \frac{2 \cdot \overline{S} \cdot \overline{\alpha}^2 d\overline{S}}{(\overline{S}^2 + \overline{\alpha}^2)^2} - \frac{2 \cdot \overline{S}^2 \cdot \overline{\alpha} \, d\overline{\alpha}}{(\overline{S}^2 + \overline{\alpha}^2)^2} - \frac{2 \cdot \underline{S} \cdot \underline{\alpha}^2 d\underline{S}}{(\underline{S}^2 + \underline{\alpha}^2)^2} + \tag{11}$$

$$+ \frac{2 \cdot \underline{S}^2 \cdot \underline{\alpha} \, d\underline{\alpha}}{(\underline{S}^2 + \underline{\alpha}^2)^2}.$$

Separately for upper and lower limitations:

$$\overline{\partial P} = \frac{\partial P_{max}}{\partial \overline{S}} + \frac{\partial P_{max}}{\partial \overline{\alpha}} = \frac{2 \cdot \overline{S} \cdot \overline{\alpha}^2 d\overline{S}}{(\overline{S}^2 + \overline{\alpha}^2)^2} - \frac{2 \cdot \overline{S}^2 \cdot \overline{\alpha} \, d\overline{\alpha}}{(\overline{S}^2 + \overline{\alpha}^2)^2}, \tag{12}$$

$$\underline{\partial P} = \frac{\partial P_{max}}{\partial \underline{S}} + \frac{\partial P_{max}}{\partial \underline{\alpha}} = \frac{2 \cdot \underline{S} \cdot \underline{\alpha}^2 d\underline{S}}{(\underline{S}^2 + \underline{\alpha}^2)^2} + \frac{2 \cdot \underline{S}^2 \cdot \underline{\alpha} \, d\underline{\alpha}}{(\underline{S}^2 + \underline{\alpha}^2)^2}. \tag{13}$$

**Conclusions.**

Maximum sensitivity of the control card $P_{max}$ occurs at well-defined ratios of variables within the established boundaries. Outside these limits control by $P_{max}$ is not effective. Therefore, in the specific problems of monitoring of cable and wire products utilization of technical tolerances for the controlled parameter is inappropriate. Control by $P_{max}$ should be directed to the exclusion of the manufacture of products (such as enameled wire) the parameters of which are beyond the technical tolerances.

Therefore, to achieve the maximum sensitivity of control technological boundaries, first, must be chosen based on the achieved level of average value of the parameter and its statistical scattering.

Second, technological boundaries need to be changed in accordance with the achieved level of the average value of the parameter and its statistical

dispersion. Such a change can be a quantitative indicator of the trend of the increase or decrease the reliability of the technological system.

### REFERENCES

*1.* Shchebeniuk L.A., Golik O.V. *Matematychni osnovy nadiynosti izolyatsiyi elektroobladnannya* [Mathematical foundations of the reliability of electrical insulation]. Kharkiv, NTU «KhPI» Publ., 2003. 102 p. (Ukr).

*2.* Kuznetsov V.P. *Interval'nye statisticheskie modeli* [Interval statistical models]. Moscow, Radio i sviaz' Publ., 1991. 352 p. (Rus).

*3.* Karpushenko V.P., Shchebeniuk L.A., Antonets Y.O., Naumenko O.A. Control card – instrument of quality of cable products. *Power cables of low and medium voltage. Design, technology, quality.* Kharkiv, Region-Inform Publ., 2000. pp. 270-289. (Ukr).

*4.* Shchebeniuk L.A., Golik O.V. A method of constructing control charts the probability of an exit sign beyond the regulatory limit. *Bulletin of Kharkiv State Polytechnic University*, 2000, no.127, pp. 35-38. (Rus).

*5.* Golik O.V. *Metod operativnogo kontrolia parametrov dvukhsloinoi poliimidnoi izoliatsii emal'provoda v protsesse proizvodstva. Diss. cand. techn. nauk* [Method of controlling parameters of a two-layer polyimide insulation enameled wire in the production process. Cand. tech. sci. diss.]. Kharkov, 2009, 168 p. (Rus).

*O.V. Golik, Candidate of Technical Science, Associate Professor,*
National Technical University «Kharkiv Polytechnic Institute», 21, Kyrpychova Str., Kharkiv, 61002, Ukraine.
phone +380 57 7076010, e-mail: unona928@gmail.com