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Resurgence of Academic Responses

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at West Virginia University

in partial fulfillment of the requirements for the degree of

Masters of Science
in Psychology.

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ABSTRACT

Resurgence of Academic Responses

Catherine Stephens

Resurgence is the recurrence of a previously reinforced response after a more recently reinforced response is placed on extinction. Resurgence may explain the recurrence of socially appropriate behavior, including academic responding, but this had not yet been empirically demonstrated. The aim of this study was to determine if resurgence would occur when a participant solved quadratic equations using multiple methods. Each participant was taught two methods of solving quadratic equations across experimental phases, followed by a phase in which neither method resulted in the correct solution. In the first phase, only simple factoring was reinforced. In the second phase, only the AC method was reinforced. In the third phase, neither of these methods was reinforced (both were placed on extinction). Half of the participants attempted to use simple factoring to solve an equation in the third phase, but the extent to which this recurrence constituted resurgence was unclear. The lack of consistent intersubject replication indicates that an uncontrolled variable may be affecting the likelihood that a response will persist in an individual's repertoire. Identifying the variables that increase the persistence of a response may inform ways to promote maintenance of academic responses.

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Resurgence of Academic Responding

The consequences of a response determine how likely the response is to occur again. This relation between responses and their consequences is described as a contingency (Skinner, 1990). An example of a contingency in a classroom is providing a good grade if a student accurately completes homework. If the probability of accurately completing homework increases when followed by a good grade, the good grade is considered a reinforcer. Reinforcement is one example of the way the consequence of a response affects whether the response is likely to occur.

Another consequence of a response that affects the likelihood of a response occurring is extinction. Extinction refers to breaking the response-reinforcer dependency, such that responding no longer produces reinforcers. Because the response is no longer reinforced, it is less likely to occur again, and rates of responding decrease. However, other responses may increase (Lattal, St. Peter, & Escobar, 2013). One such increase in responding during extinction, termed *resurgence*, refers to an increase in a previously reinforced response when the reinforcement conditions for an alternative response worsen (Lattal, Cançado, Cook, Kincaid, Nighbor, & Oliver, 2017). Resurgence is typically studied using a three-phase procedure (Lattal et al., 2013). First, a target response is reinforced. Next, that response is no longer reinforced (extinction), and a distinct alternative is reinforced. Finally, the alternative response is also placed on extinction. When the alternative response is placed on extinction, an increase in rates of the target response is resurgence.

Resurgence is a robust phenomenon that has been observed across many settings, populations, and responses. Resurgence of arbitrary responses, such as pressing levers or clicking mouse buttons, has been studied in laboratory settings with non-humans (e.g., Lieving & Lattal, 2003) and humans (e.g., Marsteller & St. Peter, 2012, Experiment 1). Resurgence also occurs with socially significant responses, including aggression (e.g., Pritchard, Hoerger, & Mace, 2014), disruptive behavior (e.g., Marsteller & St. Peter, 2014), self-injurious behavior (e.g., Wacker et al., 2011), property destruction (e.g., Mace et al., 2010; Wacker et al., 2013), and inappropriate vocal behavior (e.g., Marsteller & St. Peter, 2012). As this list suggests, resurgence of socially significant behavior is often studied in terms of the undesirable recurrence of maladaptive behavior (e.g., Wacker et al., 2011). However, resurgence may be a desirable phenomenon when it is beneficial for responses to recur (Epstein, 1987; Kestner & Peterson, 2017), such as appropriate requests (Hoffman & Falcomata, 2014), caregiving responses (Bruzek, Thompson, & Peters, 2009), and play sequences (Reed & Clark, 2011).

Recent research has investigated ways to cause resurgence of appropriate behavior that competes with maladaptive behavior. For example, Hoffman and Falcomata (2014) taught multiple appropriate requests to children with autism spectrum disorders who engaged in challenging behavior. After teaching one appropriate request (e.g., handing over a card), the request was placed on extinction while a second appropriate request was taught (e.g., pushing a button). After the second request was taught, both requests were placed on extinction. The first appropriate response taught resurged before challenging behavior, demonstrating how resurgence of a desirable response might delay the reemergence of challenging behavior.

Another instance of resurgence of a desirable response was investigated by Bruzek et al. (2009), who targeted and reinforced an appropriate caregiving response (either vertical rocking, feeding, or playing) in a simulated caregiving situation. In this study, undergraduate students in a laboratory setting were asked to provide “care” for a baby doll. Experimenters controlled whether the baby doll cried remotely from an observation room. Every session started with the doll crying. When the targeted caregiving response occurred, the experimenters temporarily stopped the crying. After the targeted caregiving response occurred for five consecutive minutes, it was placed on extinction (i.e., the baby doll did not stop crying) and a second caregiving response was reinforced. When the second response occurred for five consecutive minutes, both responses were placed on extinction (the crying continued regardless of participant behavior). During extinction, the participants engaged primarily in both of the previously reinforced responses, and relatively few other responses. In this case, the resurgence of appropriate caregiving responses resulted in participants using other appropriate forms of caregiving when the current form was not working, rather than engaging in inappropriate caregiving responses (e.g., shaking the baby).

Resurgence of desirable responses can occur outside of the context of unwanted behavior. Reed and Clark (2010) investigated the resurgence of play sequences (e.g., putting together a Mr. Potato Head) for children diagnosed with autism. One play sequence was reinforced. Then a second play sequence was reinforced while the first play sequence was placed on extinction. Finally, both play sequences were placed on extinction. When the second play sequence was no longer reinforced, the children resumed the first play sequence. In the case of this study, resurgence did not delay an undesirable response; rather, resurgence demonstrated that a desirable response in an individual’s repertoire persisted when periods of extinction occurred (Kestner & Peterson, 2017). Such longevity may be important for academic responses. During academic instruction, teachers often teach their students a sequence of several skills. Once the students learn one skill, teachers move to the next skill. While teaching the next skill, the teachers may not continue to teach the first skill; that is, the reinforcement schedule for the first skill worsens as the second skill is reinforced. However, teachers do not want the first skill to disappear completely from students’ repertoires. Instead, the hope is that, when the first skill is needed in the students’ futures, it will recur. The recurrence of the first skill after a period of time when another skill was reinforced is akin to resurgence (Lattal & St. Peter Pipkin, 2009).

Although resurgence is a generally well-established phenomenon across species, reinforcers, and experimental arrangements, we know of no empirical demonstrations of resurgence of academic responding. However, the existing body of literature on resurgence suggests that the phenomenon may also apply to academic behavior (Lattal & St. Peter Pipkin, 2009). Further support for the possible resurgence of academic responding is provided by studies on the resurgence of arbitrary relations (Doughty, Cash, Finch, Holloway, & Wallington, 2010; Doughty, Kastner, & Bismark, 2011; Doughty, Leak, & Stoudemire, 2014). In these studies, undergraduate students were asked to make responses by clicking on one of several stimuli on the screen. During the target-reinforcement phase, clicks on one set of stimuli were reinforced. During the alternative-reinforcement phase, clicks on the original stimuli were no longer reinforced; clicks on a second, distinct, set of stimuli resulted in a reinforcer. During the third phase, when no reinforcers were available (extinction), participants resumed clicking on the first set of stimuli, demonstrating resurgence of responses similar to academic responding. However, to our knowledge, no investigations have evaluated resurgence of complex academic responding. The purpose of this study was to demonstrate such resurgence.

Method

Participants and Setting

Participants were undergraduate students at West Virginia University who were enrolled in a psychology class that offered extra credit. Enrollment was not restricted based on race, gender, or age, except that the participant must have been at least 18 years old. Potential participants signed up for a 3.5-hr session using an online system provided by the university. Each participant signed up for an individual session; only the experimenter and participant were in the room during the session.

A total of 22 undergraduates completed the consent process for the experiment. Students asked to solve three problems at the beginning of the session. These pre-experimental probes are described in detail below. Students who solved any probes correctly were excluded to minimize the effects of extra-experimental instruction on responding. Eleven participants were dismissed from the study because they were able to solve quadratic equations during pre-experiment probes. Data from three additional participants were excluded because the participants did not complete at least six equations in the extinction phase; one withdrew from the study before its conclusion and two exceeded the 3.5-hr time limit before completing the extinction phase. Thus, data from a total of eight participants were included in the study. Table 1 shows the demographics of all participants in the study; there were no consistent differences between included (shown in bold) and excluded participants. The average age of the included participants was 20.6 years old (range, 18-26 years, *SD*=2.6). All were native English speakers. One participant was black/white, one was Asian/white, and the remaining six were white. The mode of the family-income bracket was $75-99K (*n*=4). Three participants reported having a learning disorder. On average, the participants had completed 3.1 years of college (range, 1-5 years, *SD*=1.6) and four semesters of school had passed since the participants last took a math class (range, 0-8 semesters, *SD*=3.16).

Participants earned extra credit in a Psychology course based on the amount of time that they spent participating, independently of their performance during the experiment. The amount of extra credit varied at the discretion of the instructor, but the Psychology Department specified that no more than 3% of a course grade could come from extra credit. In addition to extra credit, participants earned $1.25 for each equation they answered correctly during the three phases of the resurgence procedure and $5 for completing the session. Participants earned $20.31 on average (range, $16.25 to $23.50). Both money and credit were delivered at the end of the session. Participants who were not eligible for the study received 0.5 hr of extra credit and no money; the three participants who started but did not complete the study received earned $14.58 on average (range, $11.25 to $20.00), as well as extra credit for the time spent in the session, rounded up to the nearest half-hour.

Sessions were conducted in a 4.1-m by 3-m university laboratory with a table and two office chairs. Seventy-two worksheets, each with a different equation printed on it (24 for each of the three phases), were in two folders on the table. The sheets for the first phase were in one folder, and the sheets for the second and third phases were in a second folder. Printed scripts, instruction sheets, a 22-cm by 28-cm whiteboard, a white-board marker, data-collection sheets, and pens were also on the table. The participant used pens to solve the equations so that they could not erase their work, allowing for data to be collected on all strategies used. An Ipevo Ziggi-HD Plus document camera video recorded the participant's work as it occurred. In addition, a WEILIANTE full-HD digital camera recorded both the participant and instructor.

Response Measurement

Participants wrote out their work on standard letter-sized paper. Each equation appeared on a new piece of paper. Data were collected by two observers using the written work and document-camera recording to categorize the method(s) by which the participant attempted to solve the equation. In addition to the method used to solve the equation, the observers noted the time spent on each equation, and whether the answer was correct and reinforced. These data were recorded from videos of the session on paper data sheets and used to create cumulative records of participant performance (similar to the displays used by Bruzek et al., 2009). The experimenter also collected data on the methods used and the accuracy of each answer *in vivo* during the participant's session to make in-session decisions, but these data were not used in the analysis.

Three algebraic techniques were the focus of this study. The first was referred to as simple factoring, the second was the AC method, and the third was the quadratic formula. Although there are many additional methods that could be used to solve quadratic equations, these methods were selected because equations could be presented for which only some of these methods would work to find the solution. The selected set of methods allowed the experimenter to select equations that would place some methods, but not others, on extinction. Not all equations that could be solved using the AC method could be solved using simple factoring, and not all equations that could be solved using the quadratic formula could be solved using the AC method.

In reference to the equation $ax^2 + bx + c = 0$, *simple factoring* was identified when both statements were written referencing the addends of *b* and *b* was not split into multiple terms, or when statements were written referencing the factors of *c*. The *AC method* occurred when statements were written referencing the addends of *b*, when *b* was split into multiple terms (this could also be indicated by two lines drawn out from the *b* term), or when statements or equations were written referencing the factors of the product of a and *c*. The *quadratic formula* occurred when the quadratic formula ($\frac{-b\pm\sqrt{b^2-4ac}}{2a}$) was written (using variables or values to fill the equation). The quadratic formula was identified based on the presence of a square-root sign and a fraction. If the subject used a method at least three times that did not fit into the three methods defined above, it was categorized as “other method”.

Pre-Experimental Procedure

When the participant arrived, the experimenter reviewed the consent form, which included a general description of experimental procedures, compensation, and minimal risks. Participants who consented then completed a series of pre-experimental assessments. First, the participant filled out a brief demographics questionnaire. Next, before asking the participant to complete any equations, the experimenter demonstrated how the participant should show their work in a legible and logical manner and informed the participant of where to write answers on the worksheet.

Then, the experimenter asked the participant to solve and factor three equations. The equations were presented one at a time. An equation that could be solved using simple factoring was presented first, followed by one that could be solved using the AC method, and then by one that could not be solved using either of these methods. Participants were not told what method to use to solve each of these equations. Following each equation, the experimenter provided feedback about the extent to which the participant showed all algebraic steps after each equation. If participants incorrectly solved the equations, they were not told their responses were incorrect, but were just asked to solve the next equation until they either incorrectly completed all three or correctly solved one equation. Participants who correctly solved any equation were told immediately following the correct response that their solutions were correct (but received no payment for correct solutions) and were excluded from further participation.

Experimental Design

We used a three-phase resurgence procedure (target reinforcement, alternative reinforcement, and extinction) to demonstrate potential resurgence of academic responding. Additionally, the experimenter taught participants the algebraic techniques during two instruction phases (akin to the training phases used by Hoffman & Falcomata, 2014), which occurred immediately before target-reinforcement (for instruction on simple factoring) and alternative-reinforcement (for instruction on the AC method) phases.

Instruction Phases

There were five steps for solving quadratic equations using simple factoring and six steps for solving quadratic equations using the AC method. The specific steps for each method appear in Table 2. The experimenter modeled each step for the participant. After modeling, the experimenter asked the participant to demonstrate that step. If the participant was unable to demonstrate the step, the experimenter repeated the modeled example and asked the participant to try again.

After the participant demonstrated each of the steps, the experimenter presented equations for the participant to complete independently using all the steps. The participant must have correctly solved three of these equations without any aid from the experimenter to complete each instructional phase. The experimenter provided feedback about the accuracy of each solution immediately after the equation was completed. If the solution was correct, the experimenter praised the response but did not increment the count on the whiteboard (recall that participants were paid for correct responses only during the three resurgence phases). If the equation was solved incorrectly or solved correctly using a method other than that taught in the instruction, the experimenter demonstrated the method again. This process was repeated until the participant completed three equations correctly. A correct response was one where the participant used the method being taught and wrote the correct factored form and x values on the answer sheet. If the participant completed six equations without answering three correctly, the equations the participant answered incorrectly were presented again in a random order until three correct responses were given.

Resurgence Procedure

Each of the resurgence phases (target reinforcement [simple factoring], alternative reinforcement [AC method], and extinction [quadratic equation]) had 24 possible equations, which are listed in the Appendix. In reference to the equation $ax^2 + bx + c = 0$, the equations for each phase contained equal numbers of those where $a > 0$, $b > 0$, and $c > 0$; where $a > 0$, $b < 0$, and $c > 0$; and where $a > 0$, $b < 0$, and $c < 0$.

Before starting the target-reinforcement phase, the experimenter told the participant to solve and factor the equations, where to write their answers, that they would earn $1.25 for each correct answer, and that the number of correct answers would be written on the whiteboard (this number was intended to function as an immediate reinforcer, similar to a token in a token economy). The same instructions were repeated before starting the alternative-reinforcement phase. In both of these phases, after giving these instructions, the experimenter set the first sheet in front of the participant. If participants asked for help at any point, the experimenter told them to do their best, but did not give any additional instruction. No instructions were read between the alternative-reinforcement and extinction phases.

Equations from each set were presented in a random order, one at a time. When the participant handed the completed worksheet to the experimenter, the experimenter checked to ensure that answers appeared in the appropriate areas on the page. If the participant wrote answers and did not hand the answer sheet to the experimenter within 10 s, the experimenter reminded the participant to pass the sheet over if they were done. If the participant tried to hand the answer sheet to the experimenter without writing either the factored form or solution in the appropriate spaces, the experimenter returned the sheet to the participant, and prompted the participant to complete those sections. Once the participant handed the experimenter a completed worksheet, the experimenter checked the equation and told the participant whether the answer was correct. If the equation was completed correctly (the specified method was used and the correct factored form and x values were written), the experimenter increased the count of correct responses on a small whiteboard by one. If the participant completed the equation incorrectly, the experimenter told the participant that the answer was incorrect and did not change the number on the whiteboard. The experimenter then set a sheet with the next equation in front of the participant and repeated the procedure.

During the target-reinforcement phase, the experimenter presented equations that were most easily solved using simple factoring and reinforced the use of simple factoring to obtain the correct answer. During the alternative-reinforcement phase, the experimenter presented equations that could not be solved using simple factoring (using simple factoring was on extinction). Instead, participants must have reached the correct solution using an alternative response (the AC method) to receive money. For the target- and alternative-reinforcement phases, this procedure was repeated until the participant answered at least six equations and three consecutive equations correctly, or until 24 total equations were completed in the phase.

During the extinction phase, equations could only be solved using the quadratic formula (simple factoring and the AC method were on extinction). Use of the quadratic formula to correctly solve these equations would have resulted in the experimenter providing feedback that the response was correct and increasing the count on the whiteboard; however, this never occurred. There were no stimulus changes or instructions that signaled the start of the extinction phase, and the equations presented generally looked like those from the AC phase (when the greatest common factor of the three terms was factored out, the coefficient a was greater than one). The extinction phase continued until the participant completed the same number of equations as the longer of the target- or alternative-reinforcement phases.

Post-Experimental Procedure

When the participant finished the extinction phase, the experimenter explained the purpose of the study, described why the last set of equations was difficult for them to solve, and demonstrated how to solve quadratic equations using the quadratic formula. After this explanation, participants completed a survey about their previous math experience using the simple factoring, AC method, or quadratic formula. They also were asked to rank how confident they were that they could use each of these methods before and after the session, and for a narrative description of how they tried to solve the last equations. After completing the survey, the participant signed a receipt and received the money earned in cash. After the session, the experimenter also granted the participant extra credit using the department's online system.

Interobserver Agreement. The experimenter trained observers by demonstrating each method of solving quadratic equations and how those equations were scored. The observers were given a key containing the correct factored form and solutions for all equations presented in the study. The solutions for the equations were calculated by hand by the experimenter and checked using Wolfram Alpha (an online computational knowledge engine). The observers then used videos and paper products from pilot sessions to independently score equations. To be a reliable data collector, the observer must have correctly scored three consecutive examples of the three methods targeted in this study to factor and solve quadratic equations.

After training, observers used the video and written products produced by the participant to obtain primary and interobserver-agreement data for each session. Interobserver agreement was calculated for three aspects of the equation: 1) the algebraic technique(s) used, 2) the accuracy of the response, 3) the duration of the response. Interobserver agreement for the algebraic techniques and accuracy of the response were each scored as an agreement if the observers classified the response the same way (simple factoring, AC method, quadratic equation, or other) and as a disagreement if the values differed. Interobserver agreement for the duration of the equation was scored as an agreement if the durations were within two seconds of each other and a disagreement if they differed by more than two seconds. For each measure, interobserver agreement scores were calculated by summing the number of agreements, dividing the total agreements by the number of equations (agreements plus disagreements), and converting the quotient to a percentage. This calculation was done for each participant; the percentages were then averaged across participants for each phase of the experiment. The average phase-specific interobserver agreement for each measure is shown in Table 3. Across all phases and participants, interobserver agreement was 95.8% (range 89-100%) for the algebraic techniques used, 100% for the accuracy of the response, and 95% (range 91-100%) for the duration of the equation.

Procedural fidelity. The experimenter trained the observer to collect procedural-fidelity data by describing how to complete a phase-specific fidelity checklist and by having the observer collect data on mock video-recorded sessions. Training continued until the observer scored the fidelity of two mock sessions with at least 90% accuracy, as compared to a key written by the experimenter.

The trained observer used the video recording and paper products from each session to score procedural fidelity using the same phase-specific checklist as in training. Global fidelity scores were calculated by counting the number of items scored as correct and dividing this number by the number of items scored as either correct or incorrect. Average fidelity was 99.3% (range 96.2-100%). Component fidelity scores were also calculated for six components of the procedure (two components in the instruction phases and four components in the resurgence phases). Component scores were calculated by dividing the number of times each component was implemented correctly by the number of times the component occurred. Procedural fidelity was 100% for all components except giving feedback and reinforcing the response. The experimenter failed to give feedback a total of three times and failed to reinforce a correct response twice across all sessions.

Results

There was no recurrence of simple factoring for four of the eight participants. Figure 1 shows results for these four participants. The y-axis shows the cumulative number of times each algebraic method was observed during each phase; the x-axis shows the session time at which that equation was completed. Unfilled shapes denote answers that were not reinforced with money, and filled shapes denote answers that were reinforced with money. Different symbols denote the arithmetic method used for that equation. Each method is plotted on its own line. Each solid vertical line indicates a phase change.

For all four participants whose data are shown in Figure 1, only the response eligible for reinforcement was observed in the first two experimental phases (labeled "S^R Simple" and "S^R AC" in the graphs). These participants continued to use the AC method (the alternative response) throughout the extinction phase. In addition to the AC method, P20 used an "other method" of solving quadratic equations during the fifth equation in the extinction phase; P20 had also used this method during all three pre-experimental probe equations. This method approximated the quadratic formula, but consisted of moving the *c* term of the equation to the other side of the equation and finding the square root of at least one side of the equation.

Figure 2 shows data for the four participants for whom simple factoring recurred in the extinction phase. The graphs are arranged identically to Figure 1. All four participants used simple factoring exclusively during the target-reinforcement phase (labeled "S^R Simple" on the graphs). Unlike participants whose data appear in Figure 1, P13 (upper-right graph) and P22 (lower-right graph) continued to use simple factoring initially during the alternative-reinforcement phase, when only use of the AC method was reinforced. By the end of this phase, however, all participants consistently used the AC method to solve the equations. During the extinction phase, use of the AC method persisted for all participants, but simple factoring occurred at least once. The use of simple factoring tended to occur early in the phase (within the first three equations for all but P13) and was highly transient, persisting for only one or two equations. In all cases, simple factoring was used only after the alternative response (AC method) had already been tried for that equation. P3 also attempted to use the quadratic equation to solve the third equation in the phase, demonstrating potential extinction-induced variability in responding. The use of the quadratic equation occurred after both the AC method (alternative response) and simple factoring (target response) had occurred, in that order. The sequence in which these responses occurred is consistent with a regression phenomenon sometimes observed in resurgence experiments (Epstein, 1983, Hoffman & Falcomata, 2014; Lieving, Hagopian, Long, & O'Conner, 2004; Reed & Morgan, 2006).

Each participant's responses on the math-experience survey are shown in Table 4. All participants reported having learned the simple-factoring and quadratic-formula methods previously. Six of the eight participants also reported learning the AC method previously. In addition, all participants indicated that they were more confident that they could correctly use the simple-factoring and AC methods at the end of the session than they were at the beginning, suggesting that the participants learned about these methods during the session.

Discussion

The purpose of this study was to determine if the initially taught academic response would recur when an alternative academic response was placed on extinction. Although such desirable resurgence has been speculated (Lattal & St. Peter Pipkin, 2009), it had not yet been empirically demonstrated. Recurrence of the initially taught academic response occurred for only half of the participants. Additional research is needed to determine if extinction may lead to the reemergence of previously taught academic skills. However, when the recurrence did occur, it was highly transient and combined with other responses, suggesting that the degree to which this responding should be classified as resurgence is questionable.

Previous studies have identified resurgence in at least two ways (Lattal et al., 2017), two of which may be particularly relevant to the current results. First, resurgence may be identified if the target response occurred more often during the extinction phase than during the alternative-reinforcement phase. Simple factoring (the target response) occurred during the extinction phase for four participants, but also occurred at least once during the alternative-reinforcement phase for two of those participants (P13 and P22). Because the target response occurred at least as many times in the alternative-reinforcement phase as it did in the extinction phase for these participants, it may not be appropriate to call the occurrence of the target response during the extinction phase resurgence.

Second, resurgence may be identified by comparing the rate of the target response to the rate of unreinforced (control) responses that occur during the extinction phase. Because resurgence is specifically the reemergence of a *previously reinforced* response, such responses should recur more frequently than those of a *never-reinforced* response. By this logic, a second approach to identifying resurgence is by comparing frequency of the target response to frequency of responses that were never reinforced during the session (hereafter, *variable* responses) during extinction. An increase in variable responses when reinforcement of a response is discontinued is called extinction-induced variability (Lattal et al., 2013). In this study, the variable responses observed included the use of the quadratic formula in the extinction phase for P3 and the use of the "other" method in the extinction phase for P20. In both cases, these responses occurred only once during the extinction phase, though both variable responses were also seen during the probe phase at the beginning of the session. Variable responses were likely due to extinction-induced variability rather than a recent reinforcement history, particularly given that P20 had not completed a mathematics course in the previous six semesters. For P3, the variable response occurred as often as simple factoring (the target response) during extinction. For P20, the variable response occurred more often during the extinction phase than did simple factoring. Thus, the recurrence of simple factoring may have been extinction-induced variability rather than resurgence.

Most studies investigating resurgence have used a free-operant procedure (i.e., participants could respond many times in a row at any time in the session). However, the current study used a trial-based procedure (i.e., participants could respond only once using each method per equation). Although trial-based procedures are atypical, Doughty et al. (2010, 2011, 2014) used trial-based procedures to investigate resurgence as a function of reinforcement history using a matching-to-sample task. Participants responded to the trial by clicking on an item with a mouse. Participants could respond rapidly to each trial, allowing each participant to experience hundreds of trials during the experiment. The trial-based procedures in the current study differed from those used by Doughty et al. in at least two ways that may have complicated the identification of resurgence. First, participants in the present study completed only 18 to 30 total trials because each trial could last several minutes. The lengthy response chains in the current study permitted only a few exposures to the contingencies in each phase. The few exposures to the contingencies may affect resurgence (Doughty et al., 2010). Second, participants in the current study were limited to using each method once per trial, where a trial began when an equation was presented to a participant and ended when the participant wrote solutions to the equation. Thus, the maximum number of times that simple factoring could have occurred during the final phase was limited to six times for the majority of participants. The limited opportunities for simple factoring to resurge complicates data analysis.

Resurgence is generally considered a robust phenomenon with wide generality across species and experimental arrangements and is just one of several ways that previously reinforced responses can recur when the environment changes. However, simple factoring recurred for only four of the eight participants in this study. It is not clear what differed for the four participants that used simple factoring compared to the four that did not. One possibility is that some participants had extra-experimental histories that allowed features of the equations to signal which algebraic method was more likely to work. When using simple factoring, the first term of the equation (x^2) has a coefficient of one; a coefficient greater than one is obtained when the AC method can be used to solve an equation. Although participants were not taught this discrimination, it is possible that they learned it previously or identified the pattern during the course of the study. Like the equations in the alternative-reinforcement (AC-method) phase, the equations presented in the extinction phase involved a coefficient greater than one, potentially making it less likely that participants tried simple factoring. Unfortunately, no assessments of the possible discrimination were included in the current study. Future research should evaluate the extent to which participants discriminate between equation types at the end of the study, and should manipulate features of the equations to determine effects of the equation structure on recurrence of previously taught academic responses.

The current study examined solving quadratic equations as a form of academic responding. One reason quadratic equations were selected to use in this study was because participants can learn these techniques in one session. Each method was taught based on the use of a series of steps. Although this approach was experimentally advantageous, it does not mimic typical teaching procedures. Bransford, Sherwood, Vye, and Rieser (1986) noted that participants were more likely to use academic skills again in the future when they had previously learned to organize the material and understand its significance. Thus, the specific teaching procedures are likely to impact the extent to which academic responses recur when solving new problems. Future studies could explicitly evaluate the role of various specific teaching strategies on desirable recurrence of academic responses.

Although the role of resurgence in the recurrence of academic responding remains unclear, this study demonstrated that academic responses recur during extinction for some participants. These findings add to the literature on extinction-induced responding by providing an example of when recurrence of responses during extinction may be a desirable phenomenon to increase the longevity of an academic response.

References

Bransford, J., Sherwood, R., Vye, N., & Rieser, J. (1986). Teaching thinking and problem solving: Research foundations. *American Psychologist*, *41*, 1078-1089. doi:10.1037/0003-066X.41.10.1078

Bruzek, J. L., Thompson, R. H., & Peters, L. (2009). Resurgence of infant caregiving responses. *Journal of the Experimental Analysis of Behavior*, *92*, 327-343. doi:10.1901/jeab.2009-92-327

Doughty, A. H., Cash, J. D., Finch, E. A., Holloway, C., & Wallington, L. K. (2010). Effects of training history on resurgence in humans. *Behavioural Processes*, *83*, 340-343. doi:10.1016/j.beproc.2009.12.001

Doughty, A. H., Kastner, R. M., & Bismark, B. D. (2011). Resurgence of derived stimulus relations: Replication and extensions. *Behavioural Processes*, *86*, 152-155. doi:10.1016/j.beproc.2010.08.006

Doughty, A. H., Leake, L. W., & Stoudemire, M. L. (2014). Failure to observe untested derived stimulus relations in extinction: Implications for understanding stimulus-equivalence formation. *Journal of the Experimental Analysis of Behavior*, *102*, 311-326. doi:10.1002/jeab.111

Epstein, R. (1983). Resurgence of previously reinforced behavior during extinction. *Behaviour Analysis Letters*, *3*, 391-397.

Epstein, R. (1987). The spontaneous interconnection of four repertoires of behavior in a pigeon (Columba livia). *Journal of Comparative Psychology, 101*, 197-201. doi:10.1037/0735-7036.101.2.197

Hoffman, K., & Falcomata, T. S. (2014). An evaluation of resurgence of appropriate communication in individuals with autism who exhibit severe problem behavior. *Journal of Applied Behavior Analysis, 47*, 651-656. doi:10.1002/jaba.144

Kestner, K. M., & Peterson, S. M. (2017). A review of resurgence literature with human participants. *Behavior Analysis: Research and Practice, 17*, 1-17. doi:10.1037/bar0000039

Lattal, K. A., Cançado, C. X., Cook, J. E., Kincaid, S. L., Nighbor, T. D., & Oliver, A. C. (2017). On defining resurgence. *Behavioural Processes*, *141*, 85-91. doi:10.1016/j.beproc.2017.04.018

Lattal, K. A., & St. Peter Pipkin, C.C. (2009). Resurgence of previously reinforced responding: Research and application. *The Behavior Analyst Today*, *10*, 254-266. doi:10.1037/h0100669

Lattal, K. A., St. Peter, C.C., & Escobar, R. (2013). Operant extinction: Elimination and generation of behavior. In G. Madden (Ed.), *Handbook of Behavior Analysis* (2nd ed., pp. 77-107). Washington, D.C.: American Psychological Association.

Lieving, G. A., & Lattal, K. A. (2003). Recency, repeatability, and reinforcer retrenchment: An experimental analysis of resurgence. *Journal of the Experimental Analysis of Behavior*, *80*(2), 217–233. doi:10.1901/jeab.2003.80-217

Lieving, G. A., Hagopian, L. P., Long, E. S., & O'Connor, J. (2004). Response–class hierarchies and resurgence of severe problem behavior. *The Psychological Record*, *54*, 621–634. doi:10.1007/BF03395495

Mace, F. C., McComas, J. J., Mauro, B. C., Progar, P. R., Taylor, B., Ervin, R., & Zangrillo, A. N. (2010). Differential reinforcement of alternative behavior increases resistance to extinction: Clinical demonstration, animal modeling, and clinical test of one solution.

Journal of the Experimental Analysis of Behavior, 93, 349-67. doi:10.1901/jeab.2010.93-349

Marsteller, T. M., & St. Peter, C. C. (2012). Resurgence during treatment challenges. *Revista Mexicana de Análisis de la Conducta, 38,* 7–23.

Marsteller, T. M., & St. Peter, C. C. (2014). Effects of fixed-time reinforcement schedules on resurgence of problem behavior. *Journal of Applied Behavior Analysis*, *47,* 455-469. doi:10.1002/jaba.134

Pritchard, D., Hoerger, M., & Mace, F. C. (2014). Treatment relapse and behavioral momentum theory. *Journal of Applied Behavior Analysis*, *47*, 814-833. doi:10.1002/jaba.163

Reed, P., & Clark, C. (2011). Impact of intervening learning on resurgence in humans with Autism Spectrum Disorders. *Learning & Behavior*, *39*, 163–170. doi:10.3758/s13420-010-0014-2

Reed, P., & Morgan, T. A. (2006). Resurgence of response sequences during extinction in rats shows a primacy effect. *Journal of the Experimental Analysis of Behavior, 86,* 307-315. doi:10.1901/jeab.2006.20-05

Wacker, D. P., Harding, J. W., Berg, W. K., Lee, J. F., Schieltz, K. M., Padilla, Y. C., … Shahan, T. A. (2011). An evaluation of persistence of treatment effects during long-term treatment of destructive behavior. *Journal of the Experimental Analysis of Behavior, 96,* 261–282. doi:10.1901/jeab.2011.96-261

Wacker, D. P., Harding, W., Morgan, T. A., Berg, W. K., Schieltz, K. M., Lee, J. F., & Padilla, Y. C. (2013). An evaluation of resurgence during functional communication training. *The Psychological Record*, *63,* 3–20. doi:10.11133/j.tpr.2013.63.1.001

Table 1
Participant Demographics

ID #	Age	Gender	Years in College	Native English Speaker?	Ethnicity	Annual Family Income ($)	Learning Disability?	# of Semesters Since Last Math Class
1	**18**	**Female**	**1**	**Yes**	**White**	**75K-100K**	**No**	**1**
2	**21**	**Male**	**4**	**Yes**	**Black/White**	**10K-25K**	**No**	**3**
3	**19**	**Female**	**2**	**Yes**	**White**	**>100K**	**Yes**	**0**
4	19	Female	2	Yes	White	25K-50K	No	4
5	20	Female	3	Yes	White	75K-100K	Yes	2
6	19	Male	1	Yes	White	50K-75K	Yes	0
7	18	Female	1	Yes	White	50K-75K	No	2
8	19	Female	2	Yes	White		No	2
9	19	Female	2	Yes	White	75K-100K	No	3
10	22	Female	6	Yes	White	50K-75K	No	0
11	19	Male	2	Yes	White	>100K	Yes	2
12	**18**	**Female**	**1**	**Yes**	**White**	**75K -100K**	**Yes**	**1**
13	**20**	**Male**	**3**	**Yes**	**White**	**75K -100K**	**Yes**	**7**
14	18	Female	1	Yes	White	>100K	No	0
15	19	Female	2	No	White	75K-100K		0
16	20	Female	3	Yes	White		No	2
17	20	Female	3	Yes	White	>100K	No	6
18	21	Male	4	Yes	Hispanic	75K-100K		5
19	**22**	**Female**	**5**	**Yes**	**Asian/White**	**>100K**	**No**	**8**
20	**21**	**Male**	**4**	**Yes**	**White**	**75K -100K**	**No**	**7**
21	20	Male	3	Yes	White	>100K	No	7
22	**26**	**Female**	**5**	**Yes**	**White**	**>100K**	**No**	**5**

Note. Participant numbers whose data were included in the study are in bold.

Table 2

The Steps Taught for Each Method

Step	Simple-Factoring Instruction	AC-Method Instruction
1	Look for a common factor	Look for a common factor
2	Factor out the common factor	Factor out the common factor
3	Figure out the factors	Split up the “b” term
4	Split up the factors	Factor out each half
5	Solve for x	Combine the halves
6	n/a	Solve for x

Table 3

Phase-Specific Interobserver Agreement

Phase	Duration % Agreement (range)	Correct Answer % Agreement (range)	Method Used % Agreement (range)
Probe	95.8 (66.7-100)	100	91.7 (33.3-100)
Simple-Factoring Instruction	96.9 (75-100)	100	100
Simple-Factoring Reinforced	95.8 (83.3-100)	100	100
AC-Method Instruction	96.9 (75-100)	100	100
AC-Method Reinforced	98.1 (91.7-100)	100	93.3 (66.7-100)
Extinction	95.8 (83.3-100)	100	96.7 (83.3-100)

Table 4
Math-Experience Survey

Participant #	Did they learn simple factoring before?	Did they learn AC method before?	Did they learn the quadratic formula before?	How confident they were that they could correctly use each method? (1: Not confident- 5: Very confident)					
				Simple Factoring		AC Method		Quadratic Equation	
				Pre	Post	Pre	Post	Pre	Post
1	Yes	No	Yes	4	5	1	5	4	5
2	Yes	No	Yes	2	4	1	5	2	3
3	Yes	Yes	Yes	4	5	3	5	2	4
12	Yes	Yes	Yes	2	5	1	4	1	2
13	Yes	Yes	Yes	1	4	1	3	1	1
19	Yes	Yes	Yes	1	4	1	4	1	1
20	Yes	Yes	Yes	2	4	2	5	2	3
22	Yes	Yes	Yes	3	5	2	5	1	1

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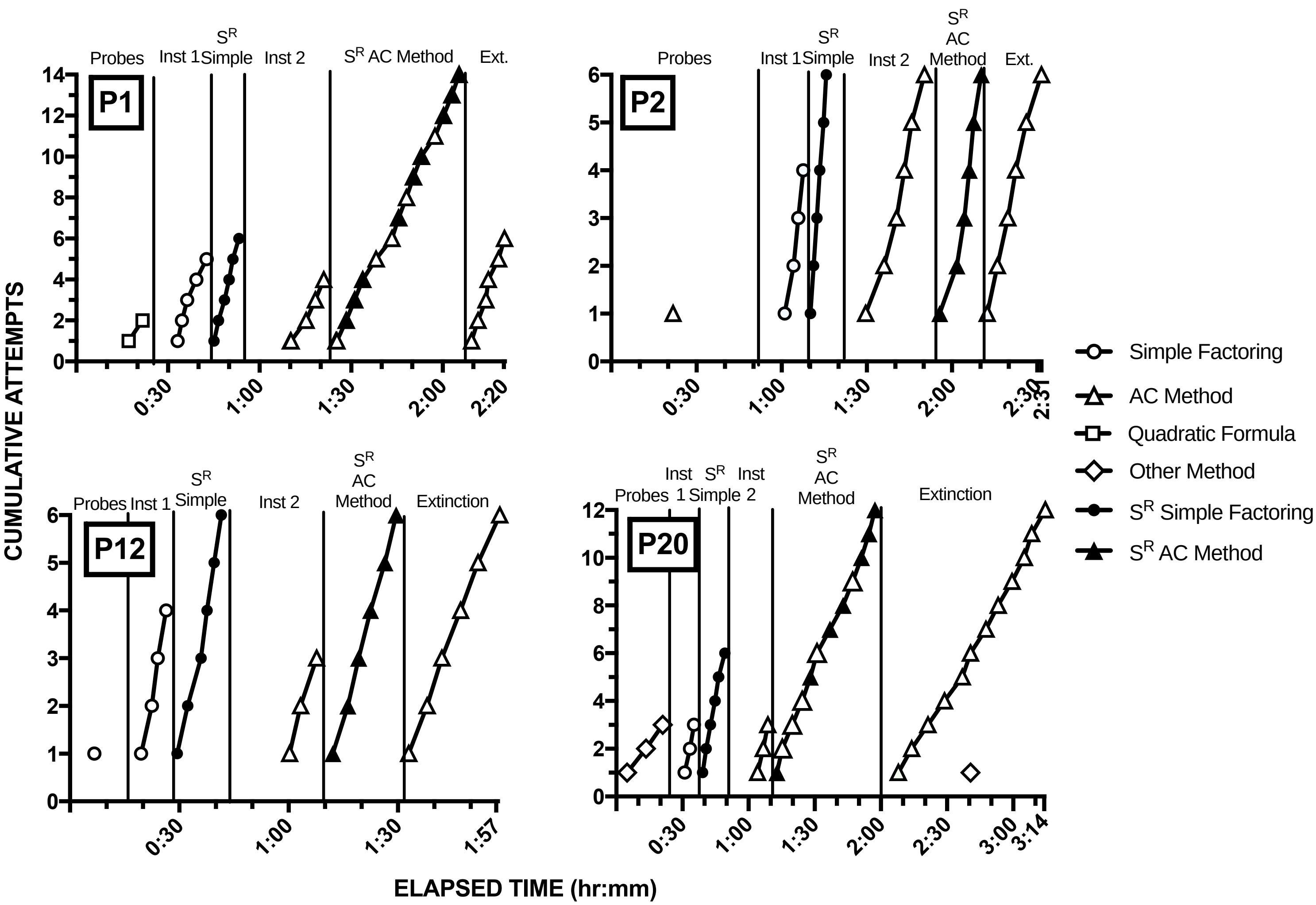

Figure 1. The cumulative number of times each algebraic technique was used to attempt to solve the equations. Each algebraic technique is graphed on its own line. This figure shows the four participants for whom simple factoring was not seen during extinction. Filled shapes indicate that the response was reinforced.

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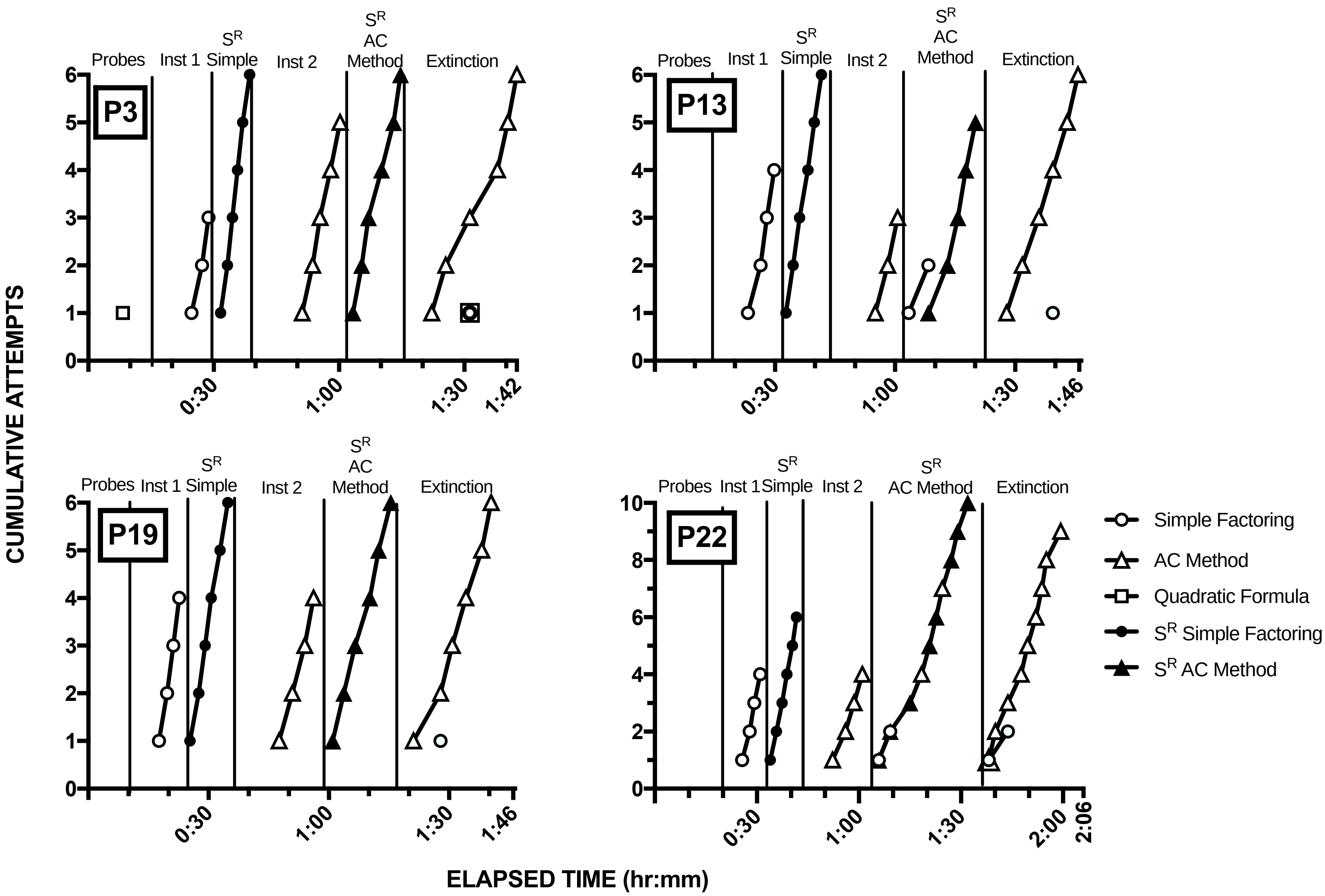

Figure 2. The cumulative number of times each algebraic technique was used to attempt to solve the equations. Each algebraic technique is graphed on its own line. This figure shows the four participants for whom simple factoring was seen during extinction. The use of simple factoring on an equation is indicated by the presence of an open circle during the extinction phase. Filled shapes indicate that the response was reinforced.

Appendix

Equations and Solutions

Pre-experimental Screening Equations

Best Method	Equation	Factored form	Solutions
Simple Factoring	$7x^2 + 35x + 28 = 0$	$7(x + 1)(x + 4) = 0$	$x = -1; x = -4$
AC-Method	$18x^2 + 48x - 18 = 0$	$6(x + 3)(3x - 1) = 0$	$x = -3; x = 1/3$
Quadratic Formula	$7x^2 - 28x + 10 = 0$	$\frac{7}{18}(x - 2)^2 - 1 = 0$	$x = 2 \pm 3\sqrt{\frac{2}{7}}$

Simple-Factoring Instruction Equations

ID	Equation	Factored form	Solutions
1	$8x^2 - 48x + 40 = 0$	$8(x - 1)(x - 5) = 0$	$x = 1; x = 5$
2	$12x^2 + 24x - 36 = 0$	$12(x + 3)(x - 1) = 0$	$x = -3; x = 1$
3	$9x^2 + 9x - 18 = 0$	$9(x + 2)(x - 1) = 0$	$x = -2; x = 1$
4	$6x^2 - 6x - 36 = 0$	$6(x + 2)(x - 3) = 0$	$x = -2; x = 3$
5	$7x^2 - 35x + 42 = 0$	$7(x - 2)(x - 3) = 0$	$x = 2; x = 3$
6	$8x^2 + 32x + 24 = 0$	$8(x + 3)(x + 1) = 0$	$x = -3; x = -1$

Simple-Factoring Equations

ID	Equation	Factored form	Solutions
1	$2x^2 + 14x + 24 = 0$	$2(x + 3)(x + 4) = 0$	$x = -3;\ x = -4$
2	$2x^2 - 14x + 24 = 0$	$2(x - 3)(x - 4) = 0$	$x = 3;\ x = 4$
3	$2x^2 - 2x - 24 = 0$	$2(x + 3)(x - 4) = 0$	$x = -3;\ x = 4$
4	$3x^2 + 18x + 24 = 0$	$3(x + 2)(x + 4) = 0$	$x = -2; x = -4$
5	$3x^2 - 18x + 24 = 0$	$3(x - 2)(x - 4) = 0$	$x = 2; x = 4$

6	$3x^2+6x-24=0$	$3(x-2)(x+4)=0$	$x=2;\ x=-4$
7	$4x^2+20x+24=0$	$4(x+2)(x+3)=0$	$x=-2;\ x=-3$
8	$4x^2-20x+24=0$	$4(x-2)(x-3)=0$	$x=2;\ x=3$
9	$4x^2+4x-24=0$	$4(x-2)(x+3)=0$	$x=2;\ x=-3$
10	$5x^2+30x+40=0$	$5(x+2)(x+4)=0$	$x=-2;\ x=-4$
11	$5x^2+10x-40=0$	$5(x-2)(x+4)=0$	$x=2;\ x=-4$
12	$5x^2-30x+40=0$	$5(x-2)(x-4)=0$	$x=2;\ x=4$
13	$6x^2+18x+12=0$	$6(x+1)(x+2)=0$	$x=-1;\ x=-2$
14	$6x^2+6x-12=0$	$6(x-1)(x+2)=0$	$x=1;\ x=-2$
15	$6x^2-18x+12=0$	$6(x-1)(x-2)=0$	$x=1;\ x=2$
16	$7x^2+28x+21=0$	$7(x+1)(x+3)=0$	$x=-1;\ x=-3$
17	$7x^2+14x-21=0$	$7(x-1)(x+3)=0$	$x=1;\ x=-3$
18	$7x^2-28x+21=0$	$7(x-1)(x-3)=0$	$x=1;\ x=3$
19	$8x^2+40x+32=0$	$8(x+1)(x+4)=0$	$x=-1;\ x=-4$
20	$8x^2+24x-32=0$	$8(x-1)(x+4)=0$	$x=1;\ x=-4$
21	$8x^2-40x+32=0$	$8(x-1)(x-4)=0$	$x=1;\ x=4$
22	$9x^2+27x+18=0$	$9(x+1)(x+2)=0$	$x=-1;\ x=-2$
23	$9x^2-27x+18=0$	$9(x-1)(x-2)=0$	$x=1;\ x=2$
24	$9x^2-9x-18=0$	$9(x+1)(x-2)=0$	$x=-1;\ x=2$

AC-Method Instruction Equations

ID	Equation	Factored form	Solutions
1	$12x^2-52x+16=0$	$4(x-4)(3x-1)=0$	$x=4; x=1/3$
2	$12x^2+6x-18=0$	$6(x-1)(2x+3)=0$	$x=1; x=-3/2$
3	$12x^2+3x-9=0$	$3(x+1)(4x-3)=0$	$x=-1; x=3/4$

4	$14x^2 + 10x - 4 = 0$	$2(7x - 2)(x + 1) = 0$	$x = -1; x = 2/7$
5	$14x^2 - 21x - 14 = 0$	$7(x - 2)(2x + 1) = 0$	$x = 2; x = -1/2$
6	$14x^2 - 22x - 12 = 0$	$2(7x + 3)(x - 2) = 0$	$x = 2; x = -3/7$

AC-Method Equations

ID	Equation	Factored form	Solutions
1	$4x^2 + 2x - 12 = 0$	$2(x + 2)(2x - 3) = 0$	$x = -2;\ x = 3/2$
2	$4x^2 + 14x + 12 = 0$	$2(x + 2)(2x + 3) = 0$	$x = -2;\ x = -3/2$
3	$4x^2 - 14x + 12 = 0$	$2(x - 2)(2x - 3) = 0$	$x = 2;\ x = 3/2$
4	$6x^2 + 9x - 6 = 0$	$3(x + 2)(2x - 1) = 0$	$x = -2; x = 1/2$
5	$6x^2 + 15x + 6 = 0$	$3(x + 2)(2x + 1) = 0$	$x = -2; x = -1/2$
6	$6x^2 - 15x + 6 = 0$	$3(x - 2)(2x - 1) = 0$	$x = 2; x = 1/2$
7	$8x^2 + 28x + 24 = 0$	$4(x + 2)(2x + 3) = 0$	$x = -2;\ x = -3/2$
8	$8x^2 - 28x + 24 = 0$	$4(x - 2)(2x - 3) = 0$	$x = 2;\ x = 3/2$
9	$8x^2 - 4x - 24 = 0$	$4(x - 2)(2x + 3) = 0$	$x = 2;\ x = -3/2$
10	$15x^2 + 50x + 40 = 0$	$5(x + 2)(3x + 4) = 0$	$x = -2;\ x = -4/3$
11	$15x^2 - 10x - 40 = 0$	$5(x - 2)(3x + 4) = 0$	$x = 2;\ x = -4/3$
12	$15x^2 - 50x + 40 = 0$	$5(x - 2)(3x - 4) = 0$	$x = 2;\ x = 4/3$
13	$18x^2 + 30x + 12 = 0$	$6(x + 1)(3x + 2) = 0$	$x = -1;\ x = -2/3$
14	$18x^2 - 6x - 12 = 0$	$6(x - 1)(3x + 2) = 0$	$x = 1;\ x = -2/3$
15	$18x^2 - 30x + 12 = 0$	$6(x - 1)(3x - 2) = 0$	$x = 1;\ x = 2/3$
16	$21x^2 + 49x + 28 = 0$	$7(x + 1)(3x + 4) = 0$	$x = -1;\ x = -4/3$
17	$21x^2 + 7x - 28 = 0$	$7(x - 1)(3x + 4) = 0$	$x = 1;\ x = -4/3$
18	$21x^2 - 49x + 28 = 0$	$7(x - 1)(3x - 4) = 0$	$x = 1;\ x = 4/3$
19	$16x^2 + 40x + 24 = 0$	$8(x + 1)(2x + 3) = 0$	$x = -1;\ x = -3/2$

20	$16x^2 + 8x - 24 = 0$	$8(x-1)(2x+3) = 0$	$x = 1;\ x = -3/2$
21	$16x^2 - 40x + 24 = 0$	$8(x-1)(2x-3) = 0$	$x = 1;\ x = 3/2$
22	$18x^2 - 27x - 18 = 0$	$9(x-2)(2x+1) = 0$	$x = 2;\ x = -1/2$
23	$18x^2 + 45x + 18 = 0$	$9(x+2)(2x+1) = 0$	$x = -2;\ x = -1/2$
24	$18x^2 - 45x + 18 = 0$	$9(x-2)(2x-1) = 0$	$x = 2;\ x = 1/2$

Quadratic-Formula Equations

ID	Equation	Intermediate Step	Solutions
1	$4x^2 + 14x - 4 = 0$	$\frac{16}{65}\left(x + \frac{7}{4}\right)^2 - 1 = 0$	$x = -\frac{7}{4} \pm \frac{\sqrt{65}}{4}$
2	$4x^2 + 14x + 4 = 0$	$\frac{16}{33}\left(x + \frac{7}{4}\right)^2 - 1 = 0$	$x = -\frac{7}{4} \pm \frac{\sqrt{33}}{4}$
3	$4x^2 - 14x - 4 = 0$	$\frac{16}{65}\left(x - \frac{7}{4}\right)^2 - 1 = 0$	$x = \frac{7}{4} \pm \frac{\sqrt{65}}{4}$
4	$9x^2 - 9x - 6 = 0$	$\frac{12}{11}\left(x - \frac{1}{2}\right)^2 - 1 = 0$	$x = \frac{1}{2} \pm \frac{\sqrt{\frac{11}{3}}}{2}$
5	$9x^2 + 9x - 6 = 0$	$\frac{12}{11}\left(x + \frac{1}{2}\right)^2 - 1 = 0$	$x = -\frac{1}{2} \pm \frac{\sqrt{\frac{11}{3}}}{2}$
6	$9x^2 + 16x + 6 = 0$	$\frac{81}{10}\left(x + \frac{8}{9}\right)^2 - 1 = 0$	$x = -\frac{8}{9} \pm \frac{\sqrt{10}}{9}$
7	$6x^2 + 20x + 12 = 0$	$\frac{9}{7}\left(x + \frac{5}{3}\right)^2 - 1 = 0$	$x = -\frac{5}{3} \pm \frac{\sqrt{7}}{3}$
8	$6x^2 + 20x - 12 = 0$	$\frac{9}{43}\left(x + \frac{5}{3}\right)^2 - 1 = 0$	$x = -\frac{5}{3} \pm \frac{\sqrt{43}}{3}$
9	$6x^2 - 20x - 12 = 0$	$\frac{9}{43}\left(x - \frac{5}{3}\right)^2 - 1 = 0$	$x = \frac{5}{3} \pm \frac{\sqrt{43}}{3}$
10	$15x^2 + 30x + 10 = 0$	$3(x+1)^2 - 1 = 0$	$x = -1 \pm \frac{1}{\sqrt{3}}$

11	$15x^2 - 30x + 10 = 0$	$3(x-1)^2 - 1 = 0$	$x = 1 \pm \frac{1}{\sqrt{3}}$
12	$15x^2 - 30x - 10 = 0$	$\frac{3}{5}(x-1)^2 - 1 = 0$	$x = 1 \pm \sqrt{\frac{5}{3}}$
13	$6x^2 + 18x + 9 = 0$	$\frac{4}{3}\left(x+\frac{3}{2}\right)^2 - 1 = 0$	$x = \frac{-3 \pm \sqrt{3}}{2}$
14	$6x^2 - 18x + 9 = 0$	$\frac{4}{3}\left(x-\frac{3}{2}\right)^2 - 1 = 0$	$x = \frac{3 \pm \sqrt{3}}{2}$
15	$6x^2 - 18x - 9 = 0$	$\frac{4}{15}\left(x-\frac{3}{2}\right)^2 - 1 = 0$	$x = \frac{3 \pm \sqrt{15}}{2}$
16	$14x^2 + 28x + 7 = 0$	$2(x+1)^2 - 1 = 0$	$x = -1 \pm \frac{1}{\sqrt{2}}$
17	$14x^2 - 28x + 7 = 0$	$2(x-1)^2 - 1 = 0$	$x = 1 \pm \frac{1}{\sqrt{2}}$
18	$14x^2 - 28x - 7 = 0$	$\frac{2}{3}(x-1)^2 - 1 = 0$	$x = 1 \pm \sqrt{\frac{3}{2}}$
19	$8x^2 + 36x + 32 = 0$	$\frac{16}{17}\left(x+\frac{9}{4}\right)^2 - 1 = 0$	$x = -\frac{9}{4} \pm \frac{\sqrt{17}}{4}$
20	$8x^2 - 36x + 32 = 0$	$\frac{16}{17}\left(x-\frac{9}{4}\right)^2 - 1 = 0$	$x = \frac{9}{4} \pm \frac{\sqrt{17}}{4}$
21	$8x^2 - 36x - 32 = 0$	$\frac{16}{145}\left(x-\frac{9}{4}\right)^2 - 1 = 0$	$x = \frac{9}{4} \pm \frac{\sqrt{145}}{4}$
22	$12x^2 + 27x + 9 = 0$	$\frac{64}{33}\left(x+\frac{9}{8}\right)^2 - 1 = 0$	$x = -\frac{9}{8} \pm \frac{\sqrt{33}}{8}$
23	$12x^2 - 27x + 9 = 0$	$\frac{64}{33}\left(x-\frac{9}{8}\right)^2 - 1 = 0$	$x = \frac{9}{8} \pm \frac{\sqrt{33}}{8}$
24	$12x^2 - 27x - 9 = 0$	$\frac{64}{129}\left(x-\frac{9}{8}\right)^2 - 1 = 0$	$x = \frac{9}{8} \pm \frac{\sqrt{129}}{8}$