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# Robust Cross-dock Location Model Accounting for Demand Uncertainty

## **Stephanie Spangler**

Thesis submitted to the Benjamin M. Statler College of Engineering and Mineral Resources at West Virginia University

in partial fulfillment of the requirements for the degree of

Master of Science in Civil Engineering

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**Department of Civil and Environmental Engineering** 

Morgantown, West Virginia 2013

Keywords: Cross-docks, Robust optimization, *P*-median facility location model Copyright 2013 Stephanie Spangler

## ABSTRACT

# Robust Cross-dock Location Model Accounting for Demand Uncertainty

### **Stephanie Spangler**

The objective of this thesis was to develop optimization models to locate cross-docks in supply chain networks. Cross-docks are a type of intermediate facility which aid in the consolidation of shipments, in which the goods spend little or no time in storage. Instead, the goods are quickly and efficiently moved from the inbound trucks to the outbound docks. Two deterministic facility location models were developed. One followed the *p*-median facility problem type, where only p facilities were opened in order to minimize total network costs. In the second model, as many cross-docks as necessary were opened and facility location costs were considered while minimizing total network costs. In order to account for uncertainty in demands, a robust optimization model was created based on the initial deterministic one. Robust counterparts were developed for each equation that contained the demand term. The robust model allowed for the creation of a network with the ability to handle variations in demand due to factors such as inclement weather, seasonal variations, and fuel prices. Numerical analysis was performed extensively on both the deterministic and robust models, following the *p*-median facility problem type, using three networks and parameters coherent with industry standards. The results showed that accounting for uncertainty in demands had a real effect on the facilities which were opened and total network costs. While the deterministic network was less expensive, it was unable to handle increases in demand due to uncertainty, whereas the robust network had no capacity shortages in any scenario. Simple demand inflation, along with the use of a robust model for baseline comparison, also proved to be a legitimate strategy to account for uncertainties in demand among small freight carriers.

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Stephanie Spangler

West Virginia University December 2013

# TABLE OF CONTENTS

CHAPTER 1. INTRODUCTION
1.1 Motivation1
1.2 Contribution
CHAPTER 2. LITERATURE REVIEW
2.1 Freight Operations and Pricing7
2.2 Cross-dock Operation Modeling10
2.3 Facility Location Decision Problem16
2.4 Uncertainty in Cross-dock Location Models
2.5 Literature Summary
CHAPTER 3. DETERMINISTIC LOCATING MODEL FOR CROSS-DOCK NETWORK DESIGN22
3.1 Introduction
3.2 Problem Definition and Formulation
CHAPTER 4. ROBUST LOCATING MODEL FOR CROSS-DOCK NETWORK DESIGN WITH DEMAND UNCERTAINTY
4.1 Robust Optimization
4.2 Robust Optimization Problem Formulation
CHAPTER 5. NUMERICAL RESULTS
5.1 Description of the Network
5.2 Demand Uncertainty
5.2.1 Effect on Opened Cross-docks
5.2.2 Effect on Total Cost
5.2.3 Relative Cost
5.3 Deterministic and Robust Performance Comparison46
5.3.1 Comparison with Average Demands for Deterministic Case
5.3.2 Comparison with Inflated Demands for the Deterministic Case
CHAPTER 6. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH
6.1 Summary61
6.2 Conclusions
6.3 Directions for Future Research
REFERENCES
APPENDICES
Appendix A69
Appendix B
Appendix C 104

## LIST OF TABLES

Table 1: Robust Counterpart for Constraint Equations Affected with Demand Uncertainty	. 31
Table 2: Description of Network 1	. 34
Table 3: Description of Network 2	. 35
Table 4: Description of Network 3	. 35
Table 5: Cross-docks Opened for Network 1 with Low Uncertainty	. 38
Table 6: Cross-docks Opened for Network 2 with Medium Uncertainty	. 39
Table 7: Cross-docks Opened for Network 3 with High Uncertainty	. 40
Table 8: Opened Cross-docks for Each Network and Uncertainty Scenario	. 47
Table 9: Initial Total Cost for Each Network and Uncertainty Scenario	. 47
Table 10: Capacity Shortage and Total Cost for Network 1	. 49
Table 11: Capacity Shortage and Total Cost for Network 2	. 50
Table 12: Capacity Shortage and Total Cost for Network 3	. 51
Table 13: Total Cost after Capacity Shortage Penalties for All Three Networks	. 53
Table 14: Average Capacity Shortages (In Pallets) and Total Costs for All Three Networks	. 54
Table 15: Average Total Cost after Capacity Shortage Penalties for All Three Networks	. 55
Table 16: Capacity Shortages (In Pallets) for Demand Inflation of the Deterministic vs. the	
Robust Approach	. 57
Table 17: Demand Inflation Cost of the Deterministic vs. the Robust Approach	. 58
Table 18: Internode Mileage for Network 1 from Origin (i) to Destination (j)	. 69
Table 19: Internode Mileage for Network 1 from Origin (i) to Cross-dock (k)	. 70
Table 20: Internode Mileage for Network 1 from Cross-dock (k) to Destination (j)	. 71
Table 21: Internode Mileage for Network 2 from Origin (i) to Destination (j)	. 72

Table 22: Internode Mileage for Network 2 from Origin (i) to Cross-dock (k)    73
Table 23: Internode Mileage for Network 2 from Cross-dock (k) to Destination (j)
Table 24: Internode Mileage for Network 3 from Origin (i) to Destination (j)
Table 25: Internode Mileage for Network 3 from Origin (i) to Destination (j)
Table 26: Internode Mileage for Network 3 from Origin (i) to Cross-dock (k)
Table 27: Internode Mileage for Network 3 from Origin (i) to Cross-dock (k)
Table 28: Internode Mileage for Network 3 from Cross-dock (k) to Destination (j)
Table 29: Internode Mileage for Network 3 from Cross-dock (k) to Destination (j) 80
Table 30: Average Demands (in pallets) for Network 1
Table 31: Average Demands (in pallets) for Network 2
Table 32: Average Demands (in pallets) for Network 3
Table 33: Average Demands (in pallets) for Network 3
Table 34: Average Demands (in pallets) for Network 3
Table 35: Average Demands (in pallets) for Network 3
Table 36: Average Demands (in pallets) for Network 3
Table 37: Average Demands for Network 3
Table 38: Cross-docks Opened for Network 1 with Medium Uncertainty    104
Table 39: Cross-docks Opened for Network 1 with High Uncertainty    105
Table 40: Cross-docks Opened for Network 2 with Low Uncertainty    106
Table 41: Cross-docks Opened for Network 2 with High Uncertainty    107
Table 42: Cross-docks Opened for Network 3 for Low Uncertainty
Table 43: Cross-docks Opened for Network 3 with Medium Uncertainty

## LIST OF FIGURES

Figure 1: MAUD Cost Structure (Chan et al. 2002)	9
Figure 2: Typical Flow in a Cross-Dock System (Liao et al. 2012)	12
Figure 3: Value of Cross-Dock (Faint 2011)	
Figure 4: Some Typical Cross-Dock Shapes (Bartholdi and Gue 2004)	14
Figure 5: Facility Location Decision Problem (Sung and Song 2003)	17
Figure 6: Total Cost for Network 1 with Low Uncertainty	
Figure 7: Total Cost for Network 2 with Medium Uncertainty	
Figure 8: Total Cost for Network 3 with High Uncertainty	
Figure 9: Relative Cost for Network 1 with Low Uncertainty	
Figure 10: Relative Cost for Network 2 with Medium Uncertainty	
Figure 11: Relative Cost for Network 3 with High Uncertainty	
Figure 12: Total Cost for Network 1 with Medium Uncertainty	110
Figure 13: Total Cost for Network 1 with High Uncertainty	111
Figure 14: Total Cost for Network 2 with Low Uncertainty	112
Figure 15: Total Cost for Network 2 with High Uncertainty	113
Figure 16: Total Cost for Network 3 with Low Uncertainty	114
Figure 17: Total Cost for Network 3 with Medium Uncertainty	115
Figure 18: Relative Cost for Network 1 with Medium Uncertainty	116
Figure 19: Relative Cost for Network 1 with High Uncertainty	117
Figure 20: Relative Cost for Network 2 with Low Uncertainty	118
Figure 21: Relative Cost for Network 2 with High Uncertainty	119
Figure 22: Relative Cost for Network 3 with Low Uncertainty	120

Figure 23: R	elative Cost for	Network 3 with	Medium	Uncertainty	,	121

#### **CHAPTER 1. INTRODUCTION**

#### **1.1 Motivation**

The trucking industry plays a vital role in the United States economy and society. Trucking allows for the transport of goods across the country from inter-city to coastline-tocoastline trips; thus allowing for product specialization in the United States. With the construction of the Eisenhower Interstate System in the 1950's, the trucking industry flourished and became a very popular form of freight transport (USDOT- FHWA 2013). Since a vast number of companies rely on trucking today as their main source of shipping, it is important to make this aspect of business as cost-effective as possible.

From the trucking industry perspective, freight agents have to make both long term strategic decisions as well as short term operational decisions. Freight agent could correspond to the owner of a small trucking company, the supply chain manager of a large business enterprise like FedEx, Caterpillar, or Walmart, or a Third Party Logistics (3PL) company. 3PL companies provide different types of freight services – actual transportation through trucks, recommendations on supply chain management, etc. – depending on their size and scope. One of the main long-term strategic decisions made by freight agents is locating warehouses or facilities for storage and transhipment. Short term operational decisions involve routing decisions associated with trucks – when to pick up, when to deliver, what route to follow, how often deliveries occur, etc. Operational decisions normally change on a daily, weekly, or monthly basis depending on the scope of operations. While the specific details and the costs of operational decisions are not considered while making strategic location decisions, freight agents often use estimates of routing costs. Costs are approximated because it is usually difficult to determine accurate details of routing and the associated costs when making facility location

decisions for ten to thirty years in the future. The importance of strategic long term and short term operational decision making has increased in recent times. With the rise of fuel prices and all of the regulations placed on trucking companies, including noise and exhaust emissions standards, inspection, repair and maintenance regulations as well as driver regulations, it becomes vital for these companies to work and run efficiently to reduce their environmental impact. This thesis was primarily concerned with developing mathematical models which aid strategic long term facility decision making while accounting for uncertainty in demand. In particular, this research focused on a particular type of facility called cross-docks.

Cross-docking (CD) refers to the method of transporting freight or goods from the manufacturer to a cross-dock facility and then to the customer. At the cross-dock, the goods are unloaded, sorted and then loaded onto another truck to be delivered to the customer. The idea of a cross-dock is different than that of a typical warehouse in that goods spend little or no idle time at the cross-dock facility, while warehouses store goods. Once the goods are delivered at the facility, in most cases, they are immediately taken to their destination truck. Intensive planning and expert logistics must be in place to manage the incoming goods and efficiently allocate them to their destination trucks with little or no wait time. Companies looking to implement cross-docking in their shipping system must first choose the optimal location for their cross-dock facility. Cross-dock facility location is very important because it improves shipping procedures, reduces costs, and also makes freight transport more efficient which reduces the environmental impact (Murray 2013).

The use of cross-docking has many advantages in the shipping field. Unlike traditional warehouses, goods flow through the facility in a timely manner and the goods or products are never stored. Since the products are never stored, there is no wasted time and labor in locating

the items when they are needed for delivery. Cross-docking saves companies from the costly warehouse procedures of storing and picking. With the removal of storing, delivery to the customer can be expedited since there is no wait time on locating and picking goods. Cross-docking also leads to the benefits of a smaller facility, less equipment, smaller labor force, and a decreased risk of items being damaged or becoming outdated. The use of cross-docking is important in the transportation industry because it reduces the burden of transportation costs on companies (Galbreth et al. 2008). Cross-docking reduces labor costs to stock and move goods in the warehouse, reduces delivery times to customers, and reduces the amount of space needed in the intermediate facilities.

Cross-docking also provides opportunities for consolidation. Shipments arriving at a crossdock may be consolidated into a single truck if they are to be delivered to a single destination. Consolidation creates many benefits for companies including the following: discounts on less than truck load (LTL) shipments, fuel savings, driver wage savings, less empty waste space on trucks, and reduction of environmental impacts. Through consolidation of LTL into truck load (TL) shipments, companies can see a reduction in freight transportation costs by 20-35%, according to Chris Kane, vice president of sales and marketing for a third-party logistics provider based in Scranton, PA (O'Reilly 2009). While consolidation can lead to major cost savings for companies, it also serves as a mechanism to help "green" the freight industry. Ülkü (2012) talks about Green Supply Chain Management (GSCM) which can be used to help reduce freight related pollution, such as emissions and noise pollution. Srivastava (2007) defines GSCM as "integrating environmental thinking into supply chain management, including product design, material sourcing and selection, manufacturing processes, delivery of the final product to the consumers as well as end-of-life management of the product after its useful life." Shipment

consolidation, cross-docking, and vehicle routing optimize delivery of the final product to consumers and reduce wasted trips and truck space; therefore, these strategies can be seen as excellent ways to help "green" the supply chain.

As stated above, shipment consolidation has the ability to generate cost savings for freight companies. However, determining discounts for companies taking part in LTL consolidation can be quite complicated and time-consuming. Carriers can implement a few different types of cost structures and discounting strategies in order to reward shipment consolidation. The modified all-unit discount (MAUD) cost structure is just one of these methods. Chan et al. (2002) define the MAUD cost structure as a piecewise function in which a small shipment incurs a fixed cost and as the number of units being shipped increases, the transportation cost per unit decreases. These costs per unit are based on predefined ranges of shipment size (in units). This type of cost function allows companies to transport LTL shipments at a lower cost than if they were to use a full truckload, because of the consolidation opportunities. When companies have opportunities for consolidation, they are able to send small shipments via an LTL carrier and only use TL carriers for near truck-load size shipments. With many small shipments consolidated to one truck rather than many large trucks with wasted space, transportation cost reductions are incurred through truck maintenance, fuel, and driver wage savings. Note that the MAUD cost structure is very detailed and is suitable when making optimal operational plans. This thesis focused on long term strategic location decisions. Therefore, a simple discounted cost structure was used rather than the MAUD cost structure.

One major issue in making long term strategic location decisions is accounting for uncertainty in parameters. When freight companies are looking at a time frame of five to ten years, there will be significant sources of uncertainty in terms of the demand for commodities,

transportation costs, etc. The future is going to become more and more unpredictable from the perspective of freight managers due to changes occurring in the freight industry such as globalization, new industry practices like electronic commerce, and volatility in fuel prices. Lessard (2013) provides a detailed review of the various uncertainties and risks in global supply chain management. Past research has shown that freight decisions which do not account for uncertainty in parameters can lead to poor decisions (Unnikrishnan et al. 2009). In this thesis, the focus was on the impact of demand uncertainty on location decisions. The goal of this thesis was to develop a mathematical model which helps immunize the location decisions against the impact of demand uncertainty.

#### **1.2 Contribution**

There have been many research papers written concerning the usage of vehicle routing and cross-docking in freight networks. These papers typically involve linear programming formulations used to model particular aspects of the cross-docking or vehicle routing system which serve to minimize total transportation costs. For example, Wen et al. (2009) developed a mathematical formulation to model vehicle routing with cross-docking which placed time constraints on shipment pick-up. Lee et al. (2006) also developed a vehicle routing schedule for cross-docking which placed constraints on vehicle departure time and arrival time. Some just studied cross-docking without vehicle routing such as Chen et al. (2006) who developed a multiple cross-docks model with inventory and time windows. Gümüş and Bookbinder (2004) looked at cross-docking and its impact on location-distribution systems in which their decision variables decide which of a group of potential cross-docks should be opened under certain network circumstances. Sung and Song (2003) also developed a model to determine cross-dock location while adding vehicle allocation, truck load capacities, and service time limits. Galbreth

et al. (2008) investigated the usage of cross-docking and its potential value for supply chain management in which a modified all unit discount (MAUD) function was used to model LTL costs.

While these formulations were all beneficial to the industry in some way, they only modeled certain aspects of the vehicle routing and cross-docking systems. Most of them ignored the impact of uncertainty in cross-docking networks; whether it was in the form of demand uncertainty, transportation cost uncertainty, or capacity uncertainty. The contributions of this thesis are given below:

- Present a cross-docking optimization model which includes a location decision variable following the *p*-median problem, handling cost at the cross-docks, vehicle and cross-dock capacity constraints, multiple commodities, and a discount parameter for consolidated freight transport.
- (ii) Develop a cross-docking optimization model which accounts for the impact of demand uncertainty through the utilization of robust optimization techniques.
- (iii) Use real world freight networks, demands, and commodities from a Third Party Logistics Company in these models.
- (iv) Study the value of accounting for demand uncertainty by comparing deterministic and robust total costs and capacity shortages to each other.
- (v) Evaluate the potential of demand inflation strategies to combat demand uncertainty when compared to robust optimization strategies.

#### **CHAPTER 2. LITERATURE REVIEW**

The literature review of this thesis provides an overview of four pertinent literature areas: freight operations and pricing, cross-dock operation modeling, facility location decision problems, and uncertainty in cross-dock location models.

#### 2.1 Freight Operations and Pricing

Since the trucking industry plays such a vital role in the operation and survival of nearly all transportation and distribution networks, it is very important to be familiar with freight classification and pricing when working with cross-dock systems. In the trucking industry, shipments or freight are generally classified as truckload (TL) or as less than truckload (LTL). TL refers to a shipment which fills the entire truck, thus making a full truck load. LTL refers to relatively small freight which does not fill a standard truck, such that it is less than a truckload. LTL shippers often look for consolidation opportunities or use smaller trucks in order to reduce wasted space and cut costs, whereas TL shippers do not have the opportunity for consolidation due to their already full trucks (Carr 2009).

TL and LTL shippers have very different operating systems. TL service providers typically transport single large shipments to one customer or retailer. It is beneficial for them to use the largest truck allowed in order to move as much freight as possible with one trip. TL shippers also try to consult with customers in order to plan shipments for back hauls so that dead loads may be avoided (Carr 2009). An LTL carrier operates in an opposite manner compared to TL, such that the LTL carrier's goal is to carry many small shipments to many different places (Carr 2009). An LTL network operates with intermediate facilities such as hubs or in this case, cross-docks. These intermediate facilities allow the carrier to consolidate small shipments from many different local customers onto one truckload. The individual shipments act as a TL when

they are transported over a long distance to another hub. At the hub close to the multiple destinations, shipments are usually separated onto smaller trucks and sent to their individual destinations. According to Carr (2009), "LTL networks like Yellow Freight or FedEx may consist of more than one hundred transshipment terminals" or hubs. LTL shipments fall in the weight range of 100 to 20,000 lbs., whereas TL shipments weigh more than 20,000 lbs., up to truck weight allowance (Carr 2009).

Because TL and LTL shipping are so different from each other, they have different pricing mechanisms. TL shipping is rather simple in that the rate is based on a dollar amount per mile. The rate may change due to factors such as "geography, accessory services, and delivery deadline" (Carr 2009). LTL is much more complicated to price than TL shipping. LTL pricing is determined by the National Motor Freight Association who establishes rates based on freight class and other very important parameters. The rates range in value from 50 to 500% in which 100% is the base rate. The rates are based on a 100 lb. shipment, so that they are defined as cwt (hundred weight). For example, a 400 lb. shipment with a rate of 78 cwt would cost \$312 to ship. When the size of shipments approaches TL capacity, they are often discounted by the trucking company (Carr 2009). In an effort to reduce shipping costs, companies consolidate small shipments with the hope of many benefits such as decreased fuel consumption, reduced pollution, and less wasted space.

Different pricing scenarios may be used in order to determine rates for consolidated shipping. One prominent method is the modified all-unit discount (MAUD) cost function. According to Chan et al. (2002), if a customer orders Q units of a commodity, the transportation cost is determined by the following piecewise function.

$$G(Q) = \begin{cases} 0 & if Q = 0, \\ c & if 0 < Q < M_1, \\ \alpha_1 Q & if M_1 \le Q < M_2, \\ \alpha_i Q & if M_i \le Q < M_{i+1}, \\ \alpha_n Q & if M_n \le Q, \end{cases}$$

Where  $\alpha_1 > \alpha_2 > \ldots \ge 0$  and  $\alpha_1 M_1 = c$ .

In the function, Q is the shipment quantity or units,  $M_i$  is a cutoff quantity, c is a fixed cost for shipping a small quantity, and  $\alpha_i$  is the discounted cost greater than or equal to zero. If a shipment size (Q) falls into the range between  $M_1$  and  $M_2$ , then the shipment cost is equal to the size of the shipment multiplied by the discounted cost,  $\alpha_1$ . Figure 1 below shows how the quantity ranges and discounts are defined.

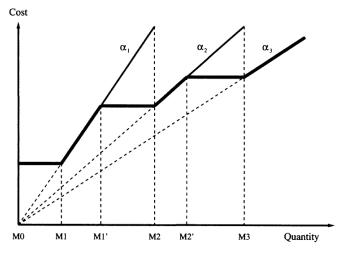


Figure 1: MAUD Cost Structure (Chan et al. 2002)

In some cases, customers may over declare in order to receive a greater discount (Chan et al. 2002). The horizontal lines on the figure above are the areas where customers benefit from over declaring (Galbreth et al. 2008). Because of consolidation, this type of pricing allows customers to ship at lower cost than if they were forced to ship TL.

A quite simpler solution to LTL pricing is the method of using a discount factor on any portions of transport where consolidation takes place. The discount factor pricing method was

used in this cross-docking model, such that shipments from the cross-dock nodes to the destination nodes had a discount factor ranging from 0 to 1 applied to them. The focus of this thesis was on long term strategic location decisions. For making such decisions, in general, an approximate estimate of the routing costs is needed. When planning for a time frame of five to ten years in the future, it is difficult to come up with accurate estimates of the quantity cutoff points and the prices for each load range. A discount factor of 0.8 was used in this study, which was based on industry standards, as discussed with Aerostream Logistics. This meant that consolidated shipments cost only 80% of the TL shipping costs, which were charged on a cost per mile basis.

#### 2.2 Cross-dock Operation Modeling

Cross-docking is a warehousing strategy in which goods are sent from an origin or supplier to a cross-dock and then to the destination or customer with little or no wait time at the intermediate facility. Many different models have been developed in transportation research in an attempt to capture the system of cross-docking and all of its many facets. These models aim to realistically generate a formulation or algorithm which represents real world transportation and warehousing networks in order to optimize the flow of goods and cut costs. The simplest models of networks using cross-docks are generally the easiest to solve, but models with more realistic constraints usually give a better representation of the actual network. Therefore, researchers must determine which constraints, such as vehicle capacity, warehouse capacity, arrival time window, departure time window, etc., should be integrated into models in order to make them more realistic and what constraints should not be included in order for the models to be able to be solved by computer programs or heuristics. This section synthesizes the various mathematical models used to optimize the different aspects of cross-dock operations.

According to Apte and Viswanathan (2000) a cross-docking warehouse functions in much the same way as a traditional mixed warehouse, in which truckload shipments enter the facility, are broken up and then consolidated to other trucks to create a variety of product shipments which are then sent direct to the customer. In a traditional warehouse, goods are sent to the warehouse from the supplier and then are stored until they are requested by customer. Once requested, the products are picked from the warehouse and shipped to the customer. According to Van Belle et al. (2012) product storage and order picking are usually the two most expensive activities. This makes cross-docking a realistic cost-saving approach as it eliminates the need to store and pick goods. Cross-docking offers many benefits to the transportation industry including network cost savings and reduced shipping times. However, not all products are right for this type of transport; Richardson (1999) says that goods which have a large, foreseeable demand and rather short delivery times are some of the best contenders for crossdocking. Walmart has realized the benefits of cross-docking in its supply chain network which is vital to the company's logistics management (Sung and Song 2003). In fact, as stated by Stalk et al. (1992), Walmart was able to decrease its "costs of sales by 2% to 3% compared with the industry average" by moving a larger percentage of goods through its own warehouses than its competitors, who were moving more goods through a third party warehouse.

The use of cross-docks inherently allows for the removal of the costly warehouse operations of storing and picking. Cross-docks operate in three main steps: 1) products arrive at the facility where they are scanned into the system, 2) products are sorted based on their destination (destination information is generally included in the bar code of the package), 3) products are transferred to the correct shipping dock and sent to their destination (Liao et al.

2012). The three step process flows continuously unlike a traditional warehouse. The figure below shows a typical flow in a cross-dock facility as described above.

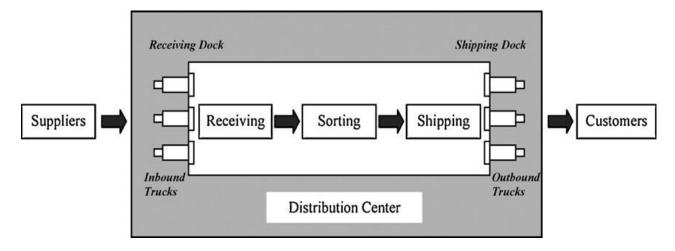


Figure 2: Typical Flow in a Cross-Dock System (Liao et al. 2012)

Cross-docks enable consolidation at centralized or optimal locations between the suppliers and customers. This means that LTL shipments which create inefficiencies due to wasted space can be consolidated into TL shipments which make use of the entire truck. The figure below shows a schematic of a network before and after cross-docking is implemented.

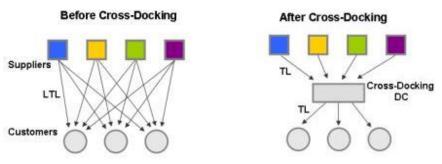


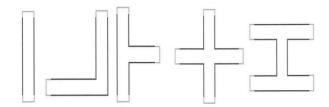
Figure 3: Value of Cross-Dock (Faint 2011)

Figure 3 shows that before cross-docking, an LTL shipment must be sent from each supplier to each destination, totaling 12 LTL shipments. After cross-docking, a single TL shipment is sent from each one of the suppliers to the cross-dock, and a single TL shipment is

sent to each customer from the cross-dock, totaling seven shipments. The use of the cross-dock in this sample network allows for fewer more cost-effective and efficient shipments.

Schaffer (1998) makes a very logical point in stating that certain activities, such as shipping, receiving, storing, and picking, can be made more efficient to cut down on costs, but the most significant cost reductions would come from removing the operation altogether. When implementing cross-docking, there are many needs that must be met in order to make the warehousing strategy successful. Schaffer (1998) outlines requirements for successful cross-docking which fall into six key groupings including the following: "1) partnering with other members of the distribution chain, 2) absolute confidence in the quality and availability of product, 3) communications between supply chain members, 4) communications and control within the cross docking operation, 5) personnel, equipment and facilities, and 5) tactical management". Most failures within companies attempting to introduce cross-docking occur when proper planning and intensive logistics are not in place to manage the flow of goods (Schaffer 1998).

Another vital aspect of cross-docking is the actual layout of the cross-dock facility. According to Bartholdi and Gue (2004), most of the cross-docks in the United States were operated by LTL carriers due the fact that cross-docking was still a rather new operation. They observed a total of six different shapes for cross-dock layouts. Most cross-docks fell into the Ishape, while L-, U-, T-, H-, and E-shapes have also been found. The wide variety in facility shape can be attributed to many different causes including the following: companies leasing an existing warehouse, constraints on facility size, constraints on land availability, lot restrictions, cost constraints, and poor designs, among other reasons. Figure 4 shows some, but not all, typical cross-dock shapes found in practice (Bartholdi and Gue 2004).



**Figure 4:** Some Typical Cross-Dock Shapes (Bartholdi and Gue 2004) Bartholdi and Gue (2004) measured the performance of different cross-dock facility shapes and found that the I-shaped cross-dock was the most efficient for small facilities (less than 150 doors), the T-shaped was best for medium sized cross-docks (between 150 -200 doors) and the X-shape was the most efficient for large warehouses (more than 200 doors). Vis and Roodbergen (2011) outlined three general facility design stages used to minimize handling and waiting times in cross-docks. The first step involved defining the general layout of the facility also called the "block layout", most importantly the location of the loading docks and the temporary product storage space. The second stage was to determine exactly how the loading docks and doors and storage space would be designed. And finally, the last step was to determine procedures that would allow the cross-dock to run efficiently at each individual block and as a whole (Vis and Roodbergen 2011).

Sung and Song (2003) state that a transportation network which has cross-docking centers integrated into its system should cogitate certain constraints and parameters which arise from consolidation. Some of the issues they note involve schedules of vehicles taking part in consolidation, time restrictions on entry, exit and the transfer of goods, determining optimal facility location, and allocating vehicles. Therefore, Sung and Song (2003) develop a model to optimize cross-dock facility location and allocate vehicles to their service network while meeting certain time constraints and staying within vehicle capacity. Lee et al. (2006) model vehicle routing with cross-docking and constraints on vehicle departure and arrival times in order to

determine an optimal route and schedule for each vehicle and arrival time at a cross-dock center to minimize total transportation costs for the network. Chen et al. (2006) considered a network of cross-docks, rather than just a single cross-dock, in order to better model larger service networks. In the study, they modeled delivery and pickup time windows, cross-dock capacity constraints and handling costs at the cross-dock facilities in order to minimize transportation cost based on known supplies and demands for the network. Gümüş and Bookbinder (2004) developed several cross-docking models in order to compare the effect of different numbers of manufacturers, cross-docks, and retailers. The purpose of their individual models was to determine the number of cross-docks which were opened from a set of potential cross-docks, the number of trucks used to transport goods through the cross-docks, and the consolidation details for each different scenario. Galbreth et al. (2008) created a cross-docking model in which LTL costs were determined using the MAUD cost function, in order to study three main research questions. The three questions Galbreth et al. (2008) were concerned with were the value of cross-docking when demand variability is low, when average demands are close to TL capacity, and when the holding cost at customer locations is high. A mathematical formulation for a cross-docking network with vehicle routing and time constraints for pick-up and delivery as well as consolidation decisions at the cross-dock was developed by Wen et al. (2009).

The extent to which models accurately replicate real world situations defines the complexity of solving them. In general, mathematical formulations developed for cross-docking are binary integer linear programs. Traditional solvers like GAMS and CPLEX can solve binary integer linear programs for small to medium sized formulations, but are unable to handle very large problems. Because of this, other methods have been developed to solve or simplify large problems. Many heuristic search based algorithms can be found in research to help solve large,

complex programs. Heuristic methods provide near optimal solutions in a computationally efficient manner. According to Chen et al. (2006), using tabu search heuristics generated better solutions, more quickly to their model than CPLEX. Liao et al. (2010) developed a new tabu search algorithm to solve their vehicle routing problem with cross-docking that they said outperformed an existing tabu search algorithm with a shorter solver time. Vahdani and Zandieh (2010) used five meta-heuristic algorithms including a genetic algorithm, tabu search, simulated annealing, electromagnetism-like algorithm, and variable neighborhood search algorithm to solve a cross-docking schedule problem in order to minimize operation time.

The next section reviews relevant literature on strategic location decisions with respect to cross-docks and other general facilities.

#### **2.3 Facility Location Decision Problem**

Another very important aspect of optimizing a cross-docking network is the facility location decision problem. The facility location decision problem involves determining which of a group of potential cross-docks or other intermediate facilities should be opened and which customers should be served from those facilities in order to maximize efficiency and minimize total cost. Figure 5 shows a typical basic layout to help better explain the facility location decision problem.

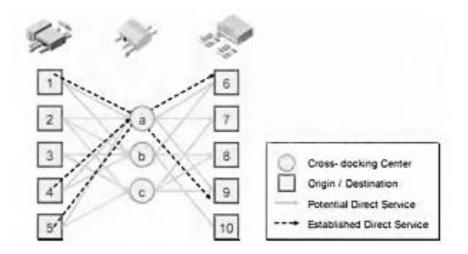


Figure 5: Facility Location Decision Problem (Sung and Song 2003)

In Figure 5, the squares labeled one through ten represent origins or destinations, the circles represent cross-docking centers, the solid lines represent potential direct service, and the dotted lines represent the established direct service. In this problem, three potential cross-dock locations were considered which are represented by a, b, and c. Through optimization techniques, cross-dock location a was chosen to be opened since it generated the lowest transportation cost (Sung and Song 2003).

The location decision for intermediate facilities in a transportation network is a very important aspect in the modeling and optimization of cross-docking networks. In fact, according to Daskin et al. (2005), "location decisions may be the most critical and most difficult of the decisions needed to realize an efficient supply chain". Deciding where to locate cross-docks is a very difficult task because once a facility is built it cannot be easily changed or moved due to changes in demand, transportation costs or availability, and element prices (Daskin et al. 2005). Unlike inventory and transportation plans which can be changed on relatively short notice, facility location is usually permanent and moving cross-docks is very costly. Ko (2005) states that "facilities location and the distribution process are two key components of a distribution system". Because the facility location and the distribution process are interrelated, they should be used together when making decisions about either of these two elements (Ko 2005). Clearly, cross-dock location decision should be included in models developed to optimize transportation networks because of the important role it plays in the overall distribution system.

Sung and Song (2003) developed a model to locate cross-dock facilities and allocate vehicles for a supply chain network in order to minimize costs. The model was optimized using both a proposed tabu search heuristic algorithm and the valid-inequality separation algorithm. Gümüş and Bookbinder (2004) solved a cross-docking model using four varieties of distributor,

cross-dock, customer, and product combinations in order to determine which cross-docks should be opened from a set of feasible locations, how many trucks should be used for direct shipment and shipment through the cross-docks, and how consolidation should be executed.

Daskin et al. (2005) discuss a few facility location decision models including the fixed charge facility location problem, integrated location/routing models, integrated location/inventory models, planning under uncertainty, and location models with facility failures. The literature on facility location models can be primarily classified into *p*-median facility location models and transportation and facility location cost minimizing models. The model developed in this research is most similar to the *p*-median facility problem. According to Alp et al. (2003) "the goal of the model is to select the locations of p facilities to serve n demand points so as to minimize the total travel." In the *p*-median problem, there is no cost incurred for opening a certain cross-dock facility. It is assumed that the cross-docks are already built and all that needs to be determined is which ones to open and use. The number of facilities to be opened is set to a certain number, p. The facility location decision problem is included in order to realistically model transportation network scenarios for companies wishing to begin cross-docking or optimize their current system. The goal of the transportation and facility location cost minimizing models are to establish any number of facilities so that the total cost of transporting goods, establishing facilities, and operating facilities is minimized. The literature can be further classified into uncapacitated and capacitated facility location problems. In the uncapacitated variant, no hard capacities are established at the facilities. The facilities can handle any number of commodities. In the capacitated variant, the facilities have a hard capacity which depends on the type of facility, number of workers, quantity of machinery available to process the goods, etc. The amount of goods handled by the facility has to be less than the capacity. Owen and Daskin

(1998), Melo et al. (2009), and Klose and Drexl (2005) provide a detailed review of the various mathematical models used for network facility location.

This research focused on the cross-dock facility location which is marginally more complicated than a simple facility location. The simple facility location models are from the perspective of a warehouse supplying goods to consumers or collecting goods from suppliers. The cross-docks are established from the perspective of consolidation. In addition, this research considered the capacity of trucks and cross-docks. Another important contribution of this work was the consideration of demand uncertainty in a cross-dock facility location context.

#### **2.4 Uncertainty in Cross-dock Location Models**

Researchers have used two main mathematical paradigms to capture the impact of uncertainty in optimization models – stochastic programming and robust optimization based strategies. Stochastic programming models assume that the uncertainty of a parameter or input is captured by a pre-specified probability distribution (Birge and Louveaux 1997; Kall and Wallace 1994). In this thesis, a stochastic programming approach would have implied that the probability distribution of the demand was known. In many situations, it is difficult to predict the future probability distribution of demand. Several researchers have recognized this shortcoming of stochastic programming and are of the view that while stochastic programming is a theoretically rigorous way of capturing uncertainty, it is often difficult to obtain accurate estimates of the probability distribution. They assumed that the uncertain parameters can vary in a pre-specified interval or range. Ranges of the uncertain parameters are easier to obtain than the probability distribution. In fact, freight agents with significant experience in the field should be able to come up with reasonable estimates on the ranges in which the demand should lie. This stream of work is collectively known as robust optimization (Ben-Tal and Nemirovski 1998, 1999, 2000; Bertsimas and Sim 2003, 2004). Accounting for uncertainty significantly increases the complexity of both the formulations and the solution algorithms as compared to deterministic models. However, past work in relevant areas such as transportation network analysis and supply chain management have shown that not accounting for uncertainty can lead to incorrect estimates of system performance, as well as poor strategic and operational decisions (Waller and Ziliaskopoulos 2001; Waller et al. 2001; Gardner et al. 2008, 2009; Unnikrishnan 2008; Duthie et al. 2009; Unnikrishnan and Waller 2009; Unnikrishnan et al. 2009; Unnikrishnan and Figliozzi 2011).

Snyder (2006) provides a detailed review of literature on the impact of various forms of uncertainty on facility location models. However, the work on capturing the impact of uncertainty in the context of cross-dock facility location is limited. Soanpet (2012) adopted a stochastic programming approach to model capacity uncertainty in cross-dock facility location models. In this thesis, a robust optimization approach was adopted to capture the impact of demand uncertainty in cross-dock facility location models. The robust optimization approach was chosen as it was believed that the inputs to the model could be more easily estimated by practitioners.

#### 2.5 Literature Summary

A review of the literature shows that various models have been developed to locate crossdocks and route trucks to optimize freight networks through minimizing total transportation costs. In this study, a cross-dock facility location formulation was developed which followed the *p*-median facility problem, as explained above. The major contributions come from the use of real world data and the modeling of demand uncertainties. In most studies, demands and networks are randomly generated, but in this study a real freight network was used with real origin-destination demands for dry, refrigerated, and frozen goods. Also, in the other cross-

docking models, demands were assumed to be known, or deterministic. But, in this study, the demands were assumed to be uncertain and were modeled through the utilization of robust optimization. In reality, exact demands are not generally known, but are subject to outside forces, such as seasons, fuel prices, weather, economic stability, etc. Robust optimization accounts for these variations in demand in order to better model real world situations. The performance and total cost of the deterministic and the robust models were also compared to each other. The next chapter provides the deterministic cross-dock facility location formulation.

# CHAPTER 3. DETERMINISTIC LOCATING MODEL FOR CROSS-DOCK NETWORK DESIGN

#### **3.1 Introduction**

This section describes the basic deterministic cross-dock network design formulation. This deterministic formulation has been used by Soanpet (2012). The deterministic formulation was used as a basis for generating the robust formulation accounting for demand uncertainty in the next chapter.

#### **3.2 Problem Definition and Formulation**

Let *N* denote the set of nodes and *A* the set of arcs. Let *O*, *D*, *K* represent the set of origin, destination, and potential cross-dock location nodes, respectively. Let *L* represent the set of commodities and  $Z_+$  the set of positive integers. Let *i*, *j*, *k* represent indexes for nodes and  $l \in L$ represent indexes for the commodities.

The inputs to the model are described next. Let  $q_{ij}^l$  represent the demand in pallets of commodity  $l \in L$  which needs to be transported from origin node  $i \in O$  to destination node  $j \in D$ . Let U and W represent the capacities of the truck and the cross-dock, respectively. Let  $c_{ij}^l$ represent the unit truckload cost, in dollars per mile, for transporting commodity  $l \in L$ . Let  $\gamma$ represent the discount factor in transporting goods from cross-docks to destinations. Let  $s_{ij}$ denote the distance, in miles, between node  $i \in N$  and node  $j \in N$ . Let  $h_k$  denote the unit cost of handling a pallet in a cross-dock at location  $k \in K$ .

The mathematical formulation is provided from the perspective of a Third Party Logistics (3PL) company. The goal of the 3PL firm is to choose P facilities out of established cross-docks with the objective of minimizing the total transportation and facility handling costs subject to constraints on routing and capacity.

The decision variables for the model are described next. The decision variables can be categorized into those corresponding to flow and those corresponding to location. The decision variable  $x_{ijk}^{l}$  takes the value 1 if commodity  $l \in L$  is transported from origin node  $i \in O$  to destination node  $j \in D$  through cross-dock location  $k \in K$  and 0 otherwise. The decision variable  $v_{ij}^{l}$  takes the value 1 if commodity  $l \in L$  is transported from origin node  $i \in O$  to destination node  $j \in D$  directly without using cross-docks. The decision variable  $y_{ij}^{l}$  denotes the number of trucks transporting commodity  $l \in L$  from origin node node  $i \in O$  to destination node  $j \in D$ . The decision variable  $z_k$  takes the value 1 if a cross-dock facility is established at location  $k \in K$  and 0 otherwise. The mathematical programming formulation which corresponds to the objective of the 3PL company and flow constraints is provided next.

$$Min \qquad \sum_{i \in O} \sum_{k \in K} \sum_{l \in L} c_{ik}^{l} s_{ik} y_{ik}^{l} + \gamma \sum_{k \in K} \sum_{j \in D} \sum_{l \in L} c_{kj}^{l} s_{kj} y_{kj}^{l} + \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} c_{ij}^{l} s_{ij} y_{ij}^{l} \qquad (1)$$
$$+ \sum_{k \in K} h_{k} \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} q_{ij}^{l} X_{ijk}^{l}$$

Subject to:

$$\sum_{k \in K} z_k = P \tag{2}$$

$$\sum_{k \in K} x_{ijk}^{l} + v_{ij}^{l} = 1 \qquad \forall i \in O, j \in D, l \in L \quad (3)$$
$$\sum_{j \in D} q_{ij}^{l} x_{ijk}^{l} \leq U y_{ik}^{l} \qquad \forall i \in O, k \in K, l \in L \quad (4)$$

$$\sum_{i \in O} q_{ij}^{l} x_{ijk}^{l} \leq U y_{kj}^{l} \qquad \forall j \in D, k \in K, l \in L \quad (5)$$

$$q_{ij}^{l} v_{ij}^{l} \leq U y_{ij}^{l} \qquad \forall i \in O, j \in D, l \in L \quad (6)$$

$$x_{ijk}^{l} \leq z_{k} \qquad \forall i \in O, j \in D, k \in K, l \in L \quad (7)$$

$$\sum_{i \in O} \sum_{j \in D} \sum_{l \in L} q_{ij}^{l} x_{ijk}^{l} \leq W z_{k} \qquad \forall k \in K \quad (8)$$

$$z_{k} \in \{0,1\} \qquad \forall k \in K \quad (9)$$

$$x_{ijk}^{l} \in \{0,1\} \qquad \forall i \in O, j \in D, l \in L, k \in K \quad (10)$$

$$v_{ij}^{l} \in \{0,1\} \qquad \forall i \in O, j \in D, l \in L \quad (11)$$

 $y_{ij}^l \in Z_+ \qquad \qquad \forall i \in N, j \in N, l \in L \quad (12)$ 

Equation (1) corresponds to the objective function which has four terms. The first term of equation (1) corresponds to the total routing costs from origins to cross-docks; the second term denotes the total routing cost from cross-docks to destinations; the third term represents the total routing cost of all goods which are transported directly from origins to destinations without using cross-docks. The final and fourth term denotes handling costs of all goods at the cross-docks.

Equations (2)-(12) denote the constraints of the optimization formulation. Equation (2) constrains the total number of cross-docks to be opened to be equal to P. Equation (3) ensures that all demands are transported to their destinations either through the cross-docks or directly

without using a cross-dock. Equation (4), (5), and (6) correspond to truck capacity constraints. Equation (4) ensures that the volume of goods being transported between origin node  $i \in O$  and cross-dock  $k \in K$  for a specific commodity  $l \in L$  is less than the capacity of the trucks operating between those two nodes. Similarly equation (5) ensures the volume of goods being transported between cross-dock  $k \in K$  and destination node  $j \in I$  for specific commodity  $l \in L$  is less than the truck capacity on that route. Equation (6) enforces the truck capacity constraint on the direct route between origin node  $i \in O$  and destination node  $j \in D$ . Constraint (7) ensures that goods are transported through a cross-dock only if it is opened. If a cross-dock at location  $k \in K$  is not opened, then  $z_k = 0$  which makes  $Wz_k = 0$ . Since  $\sum_{i \in O} \sum_{j \in D} \sum_{l \in L} q_{ij}^l x_{ijk}^l \leq Wz_k$ , this will ensure  $\sum_{i \in O} \sum_{j \in D} \sum_{l \in L} q_{ij}^l x_{ijk}^l = 0$  which implies if the cross-dock is not selected, it will not be used to handle goods. Equation (8) ensures that the total volume of goods being transported through a cross-dock is less than the capacity of the cross-dock. Equation (9) defines the location decision variable to be a binary variable. Equations (10) and (11) define the routing variables to be binary variables. Equation (12) enforces integrality constraints on the number of trucks on each route.

The above formulation can be easily modified to consider the case where the objective of the 3PL company is to minimize both the transportation and the facility location costs. In this case let  $f_k$  denote the cost of establishing the facility at location  $k \in K$ . The objective function can be modified as follows.

$$Min \qquad \sum_{i \in O} \sum_{k \in K} \sum_{l \in L} c_{ik}^{l} s_{ik} y_{ik}^{l} + \gamma \sum_{k \in K} \sum_{j \in D} \sum_{l \in L} c_{kj}^{l} s_{kj} y_{kj}^{l} + \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} c_{ij}^{l} s_{ij} y_{ij}^{l} \qquad (13)$$
$$+ \sum_{k \in K} h_{k} \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} q_{ij}^{l} X_{ijk}^{l} + \sum_{k \in K} f_{k} z_{k}$$

The constraints of this formulation correspond to equations (3) to (12). The only modification in the constraint is the removal of equation (2). In this case the number of facilities to be opened is not constrained to be equal to P.

# CHAPTER 4. ROBUST LOCATING MODEL FOR CROSS-DOCK NETWORK DESIGN WITH DEMAND UNCERTAINTY

#### **4.1 Robust Optimization**

In general, there are two major streams of research for dealing with the uncertainty in data, namely: robust optimization and stochastic programming. While stochastic optimization relies on the distribution of the uncertain parameters, robust optimization does not make any assumptions regarding the distributions of the uncertain parameters. Instead, it seeks to minimize the worst case realization of the uncertain parameters with respect to a predefined uncertainty set. Therefore, considering the limited information on specifications of the distribution function for the uncertain parameter, robust optimization framework is a desired approach for dealing with the uncertainty in models. Specifically, Wagner et al. (2009), Baron et al. (2011), and Gülpinar et al. (2013) have adopted a robust optimization framework for solving facility location problems.

#### **4.2 Robust Optimization Problem Formulation**

In this section, the robust optimization framework was adopted to deal with the uncertainty in demand. More specifically, it is assumed that the demand is uncertain and limited information regarding its distribution function is available. Mathematically it is also assumed that the demand is described by mean  $\bar{q}_{ij}^l$  and an associated uncertainty term  $\Delta_{ij}^l$ . The random uncertainty term  $\Delta_{ij}$  is a representation of variation in demand which mathematically can be expressed as the combination of M independent sources of uncertainty. Each source has an associated weight of  $b_{ij}^{lm}$ . Therefore, uncertain demand  $\tilde{q}_{ij}^l$  can be expressed as the following equation. Note that Chen et al. (2007) and Wagner et al. (2009) have adopted a similar approach for modeling input parameter uncertainty. Specifically, Wagner et al. (2009) discussed how the

following uncertainty form can capture the long term and short term demand uncertainties in facility location problems.

$$\tilde{q}_{ij}^l = \bar{q}_{ij}^l + \sum_{m=1}^M b_{ij}^{lm} \tilde{r}_m \tag{14}$$

In the above equation  $\tilde{r}_m$  is a random variable which can be described based on three following assumptions:

$$\mathbf{i})E(\tilde{r}_m)=0$$

ii)
$$|\tilde{r}_m| \leq 1$$

iii)  $\tilde{r}_m$  are all independent.

Also, it is assumed that the uncertain random variables can be described by an ellipsoidal uncertainty set  $U = {\tilde{r}_m, ||r|| \le \Omega}$  as described by Bertsimas and Sim (2004). The goal of the robust cross-dock network design problem is to determine the locations of cross-docks under worst-case outcomes of the uncertain variables belonging to the uncertainty set. In the above set,  $\Omega$  corresponds to the budget of uncertainty, which adjusts the desired level of robustness. The size of the uncertainty set is a reflection of the uncertainty protection needed at cross-dock locations. Since the uncertain demand is affecting the constraints, the robust counterpart for the constraints which are affected by the uncertain demand must be provided. For example, considering demand based on equation (14), the robust counterpart for constraint (2) can be obtained according to the following procedure:

$$\max_{\|r\| \le \Omega} \left\{ \sum_{j \in D} \bar{q}_{ij}^l x_{ijk}^l + \sum_{j \in D} \sum_{m=1}^M b_{ij}^{lm} \tilde{r}_m x_{ijk}^l \right\} \le U y_{ik}^l$$
(15)

The goal of equation (15) is to maximize the worst case realization of demand based on ellipsoidal uncertainty set of  $||r|| \leq \Omega$ .

$$\sum_{j\in D} \bar{q}_{ij}^l x_{ijk}^l + \max_{\|r\| \le \Omega} \sum_{j\in D} \sum_{m=1}^M b_{ij}^{lm} \tilde{r}_m x_{ijk}^l \le U y_{ik}^l$$
(16)

Consider  $Z_2$  as the following maximization problem:

$$Z_{2} = \max_{\|r\| \le \Omega} \sum_{j \in D} \sum_{m=1}^{M} b_{ij}^{lm} \tilde{r}_{m} x_{ijk}^{l}$$
(17)

In order to solve the above maximization problem, the Lagrangian relaxation method was adopted in which the constraint  $||r|| \leq \Omega$  is relaxed assuming the Lagrangian multiplier  $\alpha$ . The derivation below is based on Gülpinar et al. (2013). Therefore, the Lagrangian function can be written as below:

$$L(r, \alpha) = \min_{r_m} \max_{\alpha} \sum_{j \in D} \sum_{m=1}^{M} -b_{ij}^{lm} r_m x_{ijk}^l + \alpha(||r|| - \Omega)$$
(18)

The solution of the Lagrangian function (18) can be achieved by first order optimality condition as below:

$$\frac{\partial L(r,\alpha)}{\partial r_m} = \sum_{j \in D} -b_{ij}^{lm} x_{ijk}^l + \alpha \frac{r_m}{\|r\|} = 0$$
<sup>(19)</sup>

Therefore the optimal value for  $r_m$  is obtained by equation (20):

$$r_m = \frac{\|r\|}{\alpha} \sum_{j \in D} b_{ij}^{lm} x_{ijk}^l \qquad \forall m \in M$$
<sup>(20)</sup>

Considering the complementary condition  $\alpha(||r|| - \Omega) = 0$  and assuming  $\alpha \neq 0$ , ||r|| can be set equal to  $\Omega$ . Therefore

$$r_m = \frac{\alpha}{\alpha} \sum_{j \in D} b_{ij}^{lm} x_{ijk}^l \qquad \forall m \in M$$
<sup>(21)</sup>

Given the expression for  $r_m$ , ||r|| can be derived as below:

$$\|r\| = \frac{\Omega}{\alpha} \sqrt{\sum_{m=1}^{M} \left(\sum_{j \in D} b_{ij}^{lm} x_{ijk}^{l}\right)^2}$$
(22)

Similarly since  $||r|| = \Omega$  the optimal value for Lagrangian multiplier  $\alpha$  can be achieved through the following equation:

$$\alpha = \sqrt{\sum_{m=1}^{M} \left(\sum_{j \in D} b_{ij}^{lm} x_{ijk}^{l}\right)^2}$$
(23)

Substituting optimal value of  $\alpha$  into equation (21) will provide us with the optimal expression for  $r_m$  as below:

$$r_m = \Omega \frac{\sum_{j \in D} b_{ij}^{lm} x_{ijk}^l}{\sqrt{\sum_{m=1}^{M} \left(\sum_{j \in D} b_{ij}^{lm} x_{ijk}^l\right)^2}} \qquad \forall m \in M$$
(24)

Finally the optimal value for  $\mathrm{Z}_2$  is updated through the following expression

$$Z_{2} = \Omega \frac{\sum_{m} (\sum_{j \in D} b_{ij}^{lm} x_{ijk}^{l})^{2}}{\sqrt{\sum_{m} (\sum_{j \in D} b_{ij}^{lm} x_{ijk}^{l})^{2}}} = \Omega \sqrt{\sum_{m=1}^{M} \left(\sum_{j \in D} b_{ij}^{lm} x_{ijk}^{l}\right)^{2}}$$
(25)

And the robust counterpart for constraint (2) can be written as below:

$$\sum_{j \in D} \bar{q}_{ij}^l x_{ijk}^l + \Omega \sqrt{\sum_{m=1}^M \left(\sum_{j \in D} b_{ij}^{lm} x_{ijk}^l\right)^2} \le U y_{ik}^l$$
(26)

Following the same strategy the robust counterpart for constraint equations which are affected with demand uncertainty (equations: (3), (4) and (6)) can be presented as according to the following table:

Base Equation	Robust Counterpart
$\sum_{i \in O} q_{ij}^l x_{ijk}^l \le U y_{kj}^l$	$\sum_{i \in O} \bar{q}_{ij}^{l} x_{ijk}^{l} + \Omega \sqrt{\sum_{m=1}^{M} \left(\sum_{i \in O} b_{ij}^{lm} x_{ijk}^{l}\right)^{2}} \le U y_{kj}^{l}$
$q_{ij}^l v_{ij}^l \leq U y_{ij}^l$	$\bar{q}_{ij}^{l}v_{ij}^{l} + \Omega \sqrt{\sum_{m=1}^{M} (b_{ij}^{lm}v_{ij}^{l})^{2}} \leq Uy_{ij}^{l}$
$\sum_{i \in O} q_{ij}^l x_{ijk}^l \le U y_{kj}^l$	$\sum_{i \in O} \sum_{j \in D} \sum_{l \in L} \bar{q}_{ij}^l x_{ijk}^l + \Omega \sqrt{\sum_m^M \left(\sum_{i \in O} \sum_{j \in D} \sum_{l \in L} b_{ij}^{lm} x_{ijk}^l\right)^2} \le W z_k$

Table 1: Robust Counterpart for Constraint Equations Affected with Demand Uncertainty

Also, the robust counterpart for the objective function can be formulated as the following equation:

$$\operatorname{Min} \qquad \sum_{i \in O} \sum_{k \in K} \sum_{l \in L} c_{ik}^{l} s_{ik} y_{ik}^{l} + \gamma \sum_{k \in K} \sum_{j \in D} \sum_{l \in L} c_{kj}^{l} s_{kj} y_{kj}^{l} + \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} c_{ij}^{l} s_{ij} y_{ij}^{l}$$

$$+ \sum_{k \in K} h_{k} \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} q_{ij}^{l} X_{ijk}^{l} + \Omega \sum_{k \in K} h_{k} \sqrt{\sum_{m}^{M} \left(\sum_{i \in O} \sum_{j \in D} \sum_{l \in L} b_{ij}^{lm} x_{ijk}^{l}\right)^{2}}$$

$$(27)$$

Therefore the robust modeling for the p-median cross docks network design is presented as below:

$$\text{Min} \qquad \sum_{i \in O} \sum_{k \in K} \sum_{l \in L} c_{ik}^{l} s_{ik} y_{ik}^{l} + \gamma \sum_{k \in K} \sum_{j \in D} \sum_{l \in L} c_{kj}^{l} s_{kj} y_{kj}^{l} + \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} c_{ij}^{l} s_{ij} y_{ij}^{l}$$
$$+ \sum_{k \in K} h_{k} \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} \overline{q}_{ij}^{l} X_{ijk}^{l} + \Omega \sum_{k \in K} h_{k} \sqrt{\sum_{m} \left( \sum_{i \in O} \sum_{j \in D} \sum_{l \in L} b_{ij}^{lm} x_{ijk}^{l} \right)^{2} }$$

Subject to:

$$\sum_{k \in K} z_k = P$$

$$\sum_{k \in K} x_{ijk}^l + v_{ij}^l = 1$$

$$\forall i \in 0, j \in D, l \in L$$
(29)

(30)

(31)

 $\forall j \in D, k \in K, l \in L$ 

$$\sum_{j \in D} \overline{q}_{ij}^{l} x_{ijk}^{l} + \Omega \sqrt{\sum_{m=1}^{M} \left(\sum_{j \in D} b_{ij}^{lm} x_{ijk}^{l}\right)^{2}} \le U y_{ik}^{l} \qquad \forall i \in O, k \in K, l \in L$$

$$\sum_{i \in O} \bar{q}_{ij}^l x_{ijk}^l + \Omega \sqrt{\sum_{m=1}^M \left(\sum_{i \in O} b_{ij}^{lm} x_{ijk}^l\right)^2} \le U y_{kj}^l$$

$$\bar{q}_{ij}^{l}v_{ij}^{l} + \Omega \sqrt{\sum_{m=1}^{M} (b_{ij}^{lm}v_{ij}^{l})^{2}} \leq Uy_{ij}^{l} \qquad \forall i \in 0, j \in D, l \in L \quad (32)$$

$$x_{ijk}^{l} \leq z_{k} \qquad \forall i \in 0, j \in D, k \in K, l \in L \quad (33)$$

$$\sum_{i\in O}\sum_{j\in D}\sum_{l\in L}\bar{q}_{ij}^{l}x_{ijk}^{l} + \Omega \sqrt{\sum_{m}^{M} \left(\sum_{i\in O}\sum_{j\in D}\sum_{l\in L}b_{ij}^{lm}x_{ijk}^{l}\right)^{2}} \le Wz_{k} \qquad \forall k\in K$$
(34)

$$z_k \in \{0,1\} \qquad \qquad \forall k \in K \tag{35}$$

$x_{ijk}^l \in \{0,1\}$	$\forall i \in O, j \in D, l \in L, k \in K$	(36)
$v_{ij}^l \in \{0,1\}$	$\forall i \in O, j \in D, l \in L$	(37)

$$y_{ij}^{l} \in Z_{+} \qquad \forall i \in N, j \in N, l \in L \quad (38)$$

The above formulation is the robust model for p-median cross-dock network design where demand is assumed to be uncertain. Compared with the deterministic formulation it is observed that the equations including the uncertain parameters have been replaced by their robust counterpart. Also, it is worth mentioning that the p-median assumption in the above formulation can simply be relaxed by dropping equation (28) which guarantees location of pfacilities. However, in this study, the p-median variant of problem is just considered where it is assumed that all of the cross-docks are already located and built and all that must be decided is which ones to open and use.

The above formulation is a nonlinear binary integer program. General nonlinear binary integer programs are difficult problems to solve. However, the above nonlinear binary integer programs belong to a category of models called Second Order Conic Integer Programs (Atamtürk et al. 2012). Recently solvers like CPLEX have developed efficient algorithms to solve Second Order Conic Integer Programs. In this thesis, the transformation using the Lagrangian enables the exploitation of advances made by CPLEX to solve reasonably sized problems efficiently.

#### **CHAPTER 5. NUMERICAL RESULTS**

The purpose of this chapter is to demonstrate the importance of cross-docking networks and the effects of accounting for demand uncertainties in real world freight situations through the utilization of robust optimization. The next section describes the real world freight networks, datasets, and parameters used in this study. The experimental runs for the robust and deterministic networks are then defined along with the significant results. The formulation was programmed in GAMS software and solved using the CPLEX 12.0 solver.

#### **5.1 Description of the Network**

Three networks were used for the analysis of robust optimization and comparison with deterministic demand. Network 1 consisted of five origin nodes, five destination nodes, and five potential cross-dock nodes. Network 2 consisted of ten origin nodes, ten destination nodes, and ten potential cross-dock nodes. Network 3 consisted of 20 origin nodes, 20 destination nodes, and 20 potential cross-dock nodes, descriptions of the three networks are in Tables 2, 3, and 4, respectively. The distance, in miles, between each node,  $s_{ij}$ , was found using an online mapping website. The internode mileage for Network 1 can be found in Tables 18, 19 and 20 in Appendix A. The internode mileage for Network 2 can be found in Tables 21, 22, and 23 in Appendix A.

	Network 1	
Origins	Destinations	Potential Cross-docks
Montgomery, AL	Westborough, MA	Lexington, KY
Atlanta, GA	Kirkwood, NY	Charlotte, NC
Unadilla, GA	Columbus, OH	Knoxville, TN
Haines City, FL	Fairborn, OH	Charlottesville, VA
Hattiesburg, MS	Eighty Four, PA	Charleston, WV

**Table 2:** Description of Network 1

	Network 2	
Origins	Destinations	Potential Cross-docks
Everett, MA	Alsip, IL	Indianapolis, IN
Franklin, MA	Danville, IL	Fort Wayne, IN
Baltimore, MD	Des Plaines, IL	Cumberland, MD
Elizabeth, NJ	Hanover Park, IL	Canton, OH
Newark, NJ	Monroe, MI	Cincinnati, OH
Sayreville, NJ	Troy, MI	Columbus, OH
Delhi, NY	Kansas City, MO	Altoona, PA
Waterford, NY	Springfield, MO	Pittsburgh, PA
Hanover, PA	St. Louis, MO	Parkersburg, WV
Hatfield, PA	Milwaukee, WI	Wheeling, WV

**Table 3:** Description of Network 2

# **Table 4:** Description of Network 3

	Network 3	
Origins	Destinations	Potential Cross-docks
New Haven, CT	Alsip, IL	Fairfield, IL
Everett, MA	Chicago, IL	Bloomington, IN
Franklin, MA	Danville, IL	Fort Wayne, IN
Baltimore, MD	Des Plaines, IL	Hebron, IN
Belcamp, MD	Hanover Park, IL	Indianapolis, IN
Elizabeth, NJ	Taylorville, IL	Lexington, KY
Kearny, NJ	Mason City, IA	Summit, KY
Newark, NJ	Louisville, KY	Cumberland, MD
Sayreville, NJ	Dearborn, MI	Canton, OH
Delhi, NY	Monroe, MI	Cincinnati, OH
Rochester, NY	Troy, MI	Columbus, OH
Waterford, NY	Rogers, MN	Lima, OH
Williamson, NY	St. Paul, MN	Altoona, PA
Blandon, PA	Kansas City, MO	Mansfield, PA
Hanover, PA	Springfield, MO	Oil City, PA
Hatfield, PA	St. Louis, MO	Pittsburgh, PA
Alexandria, VA	Arlington, TN	Beckley, WV
Lyndhurst, VA	Elkhorn, WI	Buckhannon, WV
Newport News, VA	Milwaukee, WI	Parkersburg, WV
Richmond, VA	Oak Creek, WI	Wheeling, WV

The system demands,  $q_{ij}^l$ , were defined as the number of pallets which need to be transported from origin node i to destination node j. The demands consisted of three types of commodities, l: dry goods (D), refrigerated goods (R), and frozen goods (F). All of the networks along with their origin-destination demands and commodity type were extracted from real world data provided by a Third Party Logistics Company, Aerostream Logistics. Average demands for Networks 1 and 2 are in Tables 30 and 31, respectively, in Appendix A. Average demands for Network 3 are in Tables 32 through 37 in Appendix A as well. The demands for Network 3 had to be split into six tables, because of the immense size of Network 3.

Each of the commodities had a different shipping cost per mile,  $c_{ij}^l$ , which were as follows: \$1.40 for dry, \$1.60 for refrigerated, and \$1.80 for frozen. The truck capacity, U, was set to be equal to 28 pallets. The discount factor,  $\gamma$ , was set to be 0.8. The handling cost at each cross-dock,  $h_k$ , was set to be equal to \$3.00 per pallet. The number of opened cross-docks, P, was chosen to be four for all of the networks. The values for the above parameters were based on industry standards as discussed with Aerostream Logistics.

The cross-dock capacity, W, in Network 1 was set to 175 pallets, Network 2 was set to 150 pallets, and Network 3 was set to 250 pallets. The cross-dock capacities in each network were determined based on total average demands for that specific network. The number of uncertain parameters, m, was set to be equal to three to account for inclement weather, seasonal variations, and fuel prices. The weights associated with each random variable,  $b_{ij}^{lm}$ , were varied for low, medium, and high uncertainty. Such that for low uncertainty, the demand could be increased anywhere from zero to 30%; for medium uncertainty, the demand could be increased anywhere from zero to 60%; and for high uncertainty, the demand could be increased anywhere from zero to 90%. The bound on overall uncertainty, omega, ranged from zero to three. When

omega was set equal to zero, this was defined as the deterministic case, because no variance from the mean was allowed. Therefore, the demand was equal to the average demand.

## **5.2 Demand Uncertainty**

In this section, the impact of accounting for uncertainty in demand was studied by varying the bound on overall uncertainty. It is important to remember that omega = 0.0 represents the deterministic case, or known demand.

#### 5.2.1 Effect on Opened Cross-docks

The goal of varying the bound on the overall uncertainty, or omega, was to see how the decision variables were affected when uncertainty in demand was accounted for and when it was not accounted for. All three networks were studied at three levels of uncertainty (low, medium, and high), while varying omega from zero to three at 0.1 increments. The resulting opened cross-docks were as follows (Tables 5, 6, and 7).

Omega	CD's Opened	Omega	CD's Opened	Omega	<b>CD's Opened</b>	Omega	CD's Opened
	Charlotte		Lexington		Lexington		Lexington
0.0	Knoxville	0.8 Charlotte Knoxville	Charlotte	1.6	Charlotte	2.4	Charlotte
0.0	Charlottesville	0.0	Knoxville	1.0	Knoxville	2.4	Knoxville
	Charleston		Charlottesville		Charleston		Charleston
	Lexington		Lexington		Lexington		Lexington
0.1	Charlotte	0.9	Charlotte	1.7	Charlotte	2.5	Charlotte
0.1	Knoxville	0.7	Knoxville	1./	Knoxville	2.5	Knoxville
	Charleston		Charleston		Charleston		Charlottesville
	Lexington		Charlotte		Lexington		Lexington
0.2	Charlotte	1.0	Knoxville	1.8	Charlotte	2.6	Charlotte
0.2	Knoxville	1.0	Charlottesville	1.0	Knoxville	2.0	Knoxville
	Charleston		Charleston		Charlottesville		Charlottesville
	Lexington		Lexington		Lexington		Lexington
0.3	Charlotte	1.1 Knoxv	Charlotte	1.9	Charlotte	2.7	Charlotte
0.5	Knoxville		Knoxville		Knoxville	2.1	Knoxville
	Charlottesville		Charleston		Charleston		Charleston
	Charlotte		Lexington		Lexington		Lexington
0.4	Knoxville	1.2	Charlotte	2.0	Charlotte	2.8	Charlotte
0.4	Charlottesville	1.2	Knoxville	2.0	Knoxville		Knoxville
	Charleston		Charleston		Charlottesville		Charleston
	Lexington		Lexington		Lexington		Lexington
0.5	Charlotte	1.3	Charlotte	2.1	Charlotte	2.9	Charlotte
0.5	Knoxville	1.5	Knoxville	2.1	Knoxville	2.9	Knoxville
	Charlottesville		Charlottesville		Charlottesville		Charleston
	Lexington		Lexington		Lexington		Lexington
0.6	Charlotte	1.4	Charlotte	2.2	Charlotte	3.0	Charlotte
0.0	Knoxville	1.7	Knoxville	2.2	Knoxville	5.0	Knoxville
	Charleston		Charleston		Charleston		Charlottesville
	Lexington		Charlotte		Lexington		
0.7	Charlotte	1.5	Knoxville	2.3	Charlotte		
0.7	Knoxville	1.5	Charlottesville	2.3	Knoxville		
	Charleston		Charleston		Charleston		

 Table 5: Cross-docks Opened for Network 1 with Low Uncertainty

Omega	CD's Opened	Omega	CD's Opened	Omega	CD's Opened	Omega	<b>CD's Opened</b>
	Cumberland		Cumberland		Cumberland		Columbus
0.0	Canton	0.8	Canton	1.6	Canton	2.4	Altoona
0.0	Altoona	0.8	Altoona	1.0	Altoona	2.4	Pittsburgh
	Pittsburgh		Pittsburgh		Pittsburgh		Wheeling
	Fort_Wayne		Fort_Wayne		Canton		Fort_Wayne
0.1	Altoona	0.9	Columbus	1.7	Columbus	2.5	Cumberland
0.1	Pittsburgh	0.9	Altoona	1.7	Altoona	2.5	Altoona
	Wheeling		Pittsburgh		Wheeling		Pittsburgh
	Canton		Indianapolis		Fort_Wayne		Cumberland
0.2	Altoona	1.0	Cumberland	1.8	Cumberland	2.6	Columbus
0.2	Pittsburgh	1.0	Altoona	1.0	Altoona	2.0	Altoona
	Wheeling		Pittsburgh		Pittsburgh		Pittsburgh
	Columbus		Fort_Wayne		Fort_Wayne		Indianapolis
0.3	Altoona	1.1	Cumberland	1.9	Cumberland	2.7	Columbus
0.5	Pittsburgh		Canton		Altoona		Altoona
	Wheeling		Pittsburgh		Pittsburgh		Wheeling
	Fort_Wayne		Indianapolis		Cumberland		Cumberland
0.4	Altoona	1.2	Cumberland	2.0	Columbus	2.8	Columbus
0.4	Pittsburgh	1.2	Columbus	2.0	Altoona		Altoona
	Wheeling		Pittsburgh		Pittsburgh		Pittsburgh
	Cumberland		Canton		Fort_Wayne		Cumberland
0.5	Columbus	1.3	Altoona	2.1	Cumberland	2.9	Canton
0.5	Altoona	1.5	Pittsburgh	2.1	Altoona	2.9	Altoona
	Pittsburgh		Wheeling		Pittsburgh		Pittsburgh
	Cumberland		Cumberland		Cumberland		Fort_Wayne
0.6	Canton	1.4	Canton	2.2	Columbus	3.0	Cumberland
0.0	Altoona	1.4	Pittsburgh	2.2	Altoona	5.0	Pittsburgh
	Pittsburgh		Wheeling		Pittsburgh		Wheeling
	Cumberland		Fort_Wayne		Cumberland		
0.7	Canton	1.5	Columbus	2.3	Canton		
0.7	Altoona	1.3	Altoona	2.3	Altoona		
	Pittsburgh		Pittsburgh		Wheeling		

**Table 6:** Cross-docks Opened for Network 2 with Medium Uncertainty

Omega	<b>CD's Opened</b>	Omega	<b>CD's Opened</b>	Omega	CD's Opened	Omega	<b>CD's Opened</b>
	Fort_Wayne		Cumberland		Fort_Wayne		Cumberland
0.0	Cumberland	0.8	Lima	1.6	Cumberland	2.4	Lima
0.0	Altoona	0.0	Oil_City	1.0	Altoona	2.4	Altoona
	Mansfield		Pittsburgh Fort Wayne		Pittsburgh		Pittsburgh
	Hebron		Fort_Wayne		Fort_Wayne		Fort_Wayne
0.1	Cumberland	0.9	Cumberland	1.7	Cumberland	2.5	Columbus
0.1	Altoona	0.7	Altoona	1.7	Columbus	2.5	Altoona
	Oil_City		Oil_City		Altoona		Pittsburgh
	Cumberland		Fort_Wayne		Fort_Wayne		Fort_Wayne
0.2	Canton	1.0	Cumberland	1.8	Cumberland	2.6	Columbus
0.2	Altoona	1.0	Columbus	1.0	Altoona	2.0	Altoona
	Oil_City		Wheeling		Pittsburgh		Oil_City
	Cumberland		Cumberland		Hebron		Hebron
0.3	Lima	1 1	Columbus	1.9	Cumberland	2.7	Columbus
0.5	Altoona	1.1	1.1 Oil_City	1.9	Columbus	2.1	Altoona
	Mansfield		Pittsburgh		Altoona		Pittsburgh
	Canton		Cumberland		Cumberland		Hebron
0.4	Altoona	1.2	Canton	2.0	Lima	2.8	Cumberland
0.4	Pittsburgh	1.2	Mansfield	2.0	Altoona	2.0	Columbus
	Wheeling		Pittsburgh		Wheeling		Altoona
	Cumberland		Cumberland		Hebron		Fort_Wayne
0.5	Lima	1.3	Lima	2.1	Cumberland	2.9	Indianapolis
0.5	Altoona	1.5	Altoona	2.1	Altoona	2.9	Altoona
	Pittsburgh		Pittsburgh		Oil_City		Oil_City
	Fort_Wayne		Hebron		Cumberland		Fort_Wayne
0.6	Hebron	1.4	Altoona	2.2	Lima	3.0	Altoona
0.0	Cumberland	1.4	Oil_City 2.2	2.2	Altoona	5.0	Pittsburgh
	Altoona		Pittsburgh		Pittsburgh		Wheeling
	Fort_Wayne		Fort_Wayne		Fort_Wayne		
0.7	Cumberland	1.5	Cumberland	2.3	Cumberland		
0.7	Oil_City	1.3	Altoona	2.3	Pittsburgh		
	Pittsburgh		Oil_City		Wheeling		

 Table 7: Cross-docks Opened for Network 3 with High Uncertainty

As can be seen from Tables 5, 6, and 7, the cross-docks which were opened varied with omega for all three networks and uncertainty levels. This means that accounting for uncertainty in demand had a distinct effect on the outputs of a system. If a particular network had uncertain demand, but it was assumed to be deterministic, and the average value of demand was used, the wrong cross-docks would be opened. In some cases, this could mean that the cross-dock capacity may be exceeded or that the total transportation cost would be much greater than expected. While deterministic models may be easier to solve and lead to less costly solutions, they may not be practical in cases where demand or other inputs are uncertain. Ignoring such uncertainty can lead to serious design flaws, such as too little warehouse capacity, too few trucks and even unexpected transportation costs. Tables containing the results for medium and high uncertainty for Network 1, low and high uncertainty for Network 2, and low and medium uncertainty for Network 3 are included in Appendix C; the results for those scenarios were consistent with the ones shown in Tables 5, 6, and 7.

#### **5.2.2 Effect on Total Cost**

In this section, the effects of demand uncertainty on total system costs are presented. Once again, all three networks were studied at three levels of uncertainty (low, medium, and high), while varying omega from zero to three at 0.1 increments. The resulting total system costs are shown on Figures 6, 7, and 8.

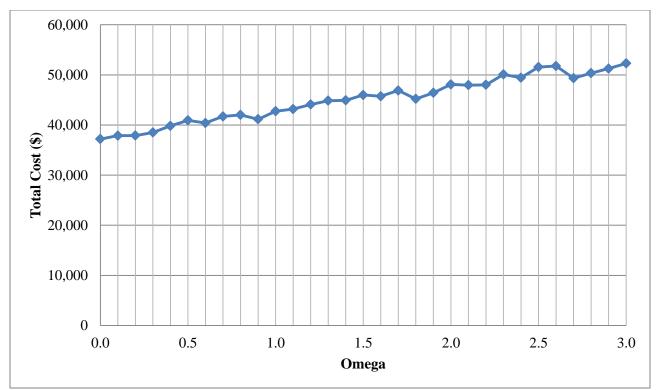


Figure 6: Total Cost for Network 1 with Low Uncertainty

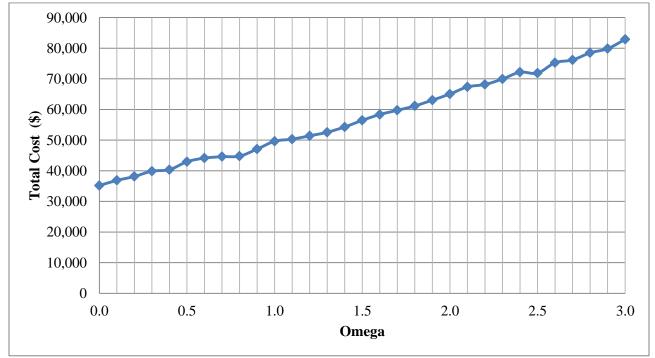


Figure 7: Total Cost for Network 2 with Medium Uncertainty

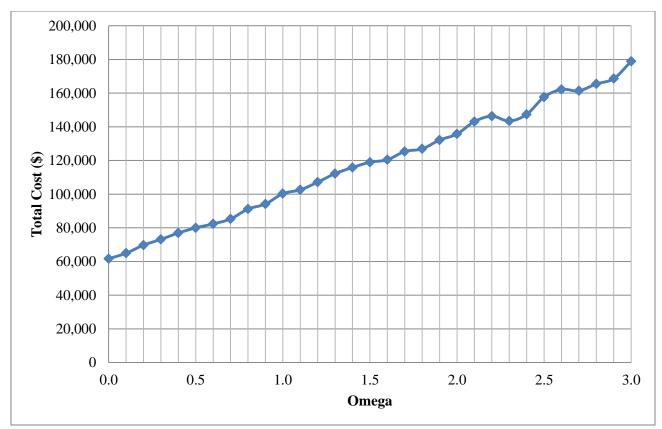


Figure 8: Total Cost for Network 3 with High Uncertainty

As expected, the total cost of the system tended to increase as omega increased. This occurred because the increase in omega allowed for higher demands, higher demands meant more trucks, and more trucks meant higher transportation prices. Network 3 had the highest total cost, because it had the largest overall demand and also had high uncertainty. High uncertainty meant that the factors that cause uncertainty, inclement weather, seasonal variations, and fuel prices, had a larger effect on the increases in demand than in the case of low and medium uncertainty. Networks 1 and 2 had very similar total demands and therefore, their cost at omega=0 was similar. However, because Network 2 had medium uncertainty its total cost increased much more rapidly than the total cost of Network 1 with low uncertainty. Graphs containing the results for medium and high uncertainty for Network 1, low and high uncertainty

for Network 2, and low and medium uncertainty for Network 3 are included in Appendix C; the results for those scenarios were consistent with the ones shown in Figures 6, 7, and 8.

## 5.2.3 Relative Cost

In order to better compare the deterministic and robust solutions for the cross-dock facility location problem, the relative costs were calculated. The relative cost was simply the total cost of a network for a certain scenario (evaluated at mean demands) divided by the base total cost of the network. In this case, the deterministic model solution was used, where omega is set equal to zero, as the base total cost. It was then expected that the relative cost for the deterministic case would be equal to one. Once again, all three networks were studied at three levels of uncertainty (low, medium, and high), while varying omega from zero to three at 0.1 increments. The resulting relative costs are shown in the following graphs.

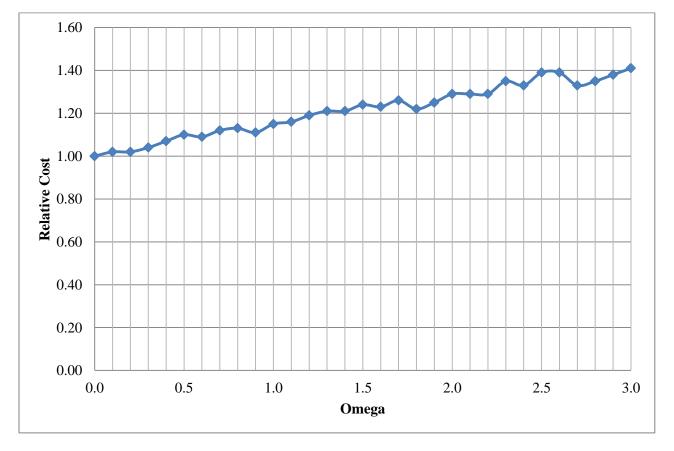


Figure 9: Relative Cost for Network 1 with Low Uncertainty

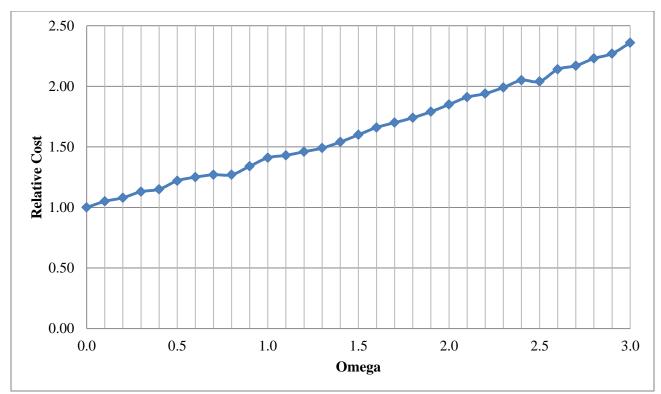


Figure 10: Relative Cost for Network 2 with Medium Uncertainty

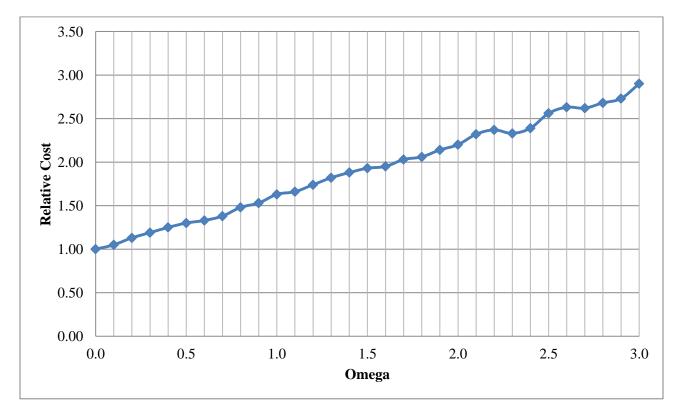


Figure 11: Relative Cost for Network 3 with High Uncertainty

As expected, the relative cost increased as omega increased. The relative cost was equal to one for the deterministic solution for all three networks, because omega equal to zero was used as the base scenario. In Figure 9, the relative cost when omega was equal to three was 1.41. This means that for network 1, under low uncertainty, accounting for total demand uncertainty with a budget of three, the total cost was 41% more than the deterministic case at mean demand levels. This shows that there was a significant difference in the deterministic and robust cases. While the deterministic solution may look more appealing, because it costs less, the robust solution is more resilient and can handle variations in demand. If assuming an average demand and modeling using a deterministic model, even slight increases in demand or slight uncertainties can cause the network to be overloaded or exceed capacity at the nodes or on the trucks. The above graphs show how much more a robust network, which is able to handle uncertainty, will cost as compared to the deterministic case. Graphs containing the results for medium and high uncertainty for Network 1, low and high uncertainty for Network 2, and low and medium uncertainty for Network 3 are included in Appendix C; the results for those scenarios were consistent with the ones shown in Figures 9, 10, and 11.

## **5.3 Deterministic and Robust Performance Comparison**

In order to further compare the performance of the deterministic and robust cross-docking models, their ability to handle demand uncertainties was studied. The first comparison used the average demands for the deterministic case and the second used inflated demands for the deterministic case.

#### 5.3.1 Comparison with Average Demands for Deterministic Case

This comparison was achieved by first solving each of the cross-dock location decision and routing problems for the deterministic case (omega = 0) and a robust case (omega = 3, high uncertainty) for each of the three networks. The resulting opened cross-docks and total system costs are given below.

Cross-dock's Opened						
Network 1         Network 2         Network 3						
Deterministic	Robust	Robust Deterministic Robust			Robust	
Charlotte	Lexington	Columbus	Fort_Wayne	Fort_Wayne	Hebron	
Knoxville	Charlotte	Altoona	Cumberland	Cumberland	Cumberland	
Charlottesville	Knoxville	Pittsburgh	Altoona	Canton	Oil_City	
Charleston	Charlottesville	Wheeling	Pittsburgh	Altoona	Pittsburgh	

**Table 8:** Opened Cross-docks for Each Network and Uncertainty Scenario

**Table 9:** Initial Total Cost for Each Network and Uncertainty Scenario

Initial Total Cost (\$)						
Netwo	Network 1 Network 2 Network 3					
Deterministic	Robust	Deterministic	Robust	Deterministic	Robust	
37,197	116,013	35,992	104,878	62,676	175,307	

As can be seen in the above tables, the opened cross-docks were different in each network for the deterministic and robust cases; this was consistent with the results in the previous section. The total costs were also much higher for the robust cases due to the increases in demands.

After the four cross-docks were established for each uncertainty level and network, and the numbers of trucks that were scheduled to transport commodities for each node pair were determined, the ability of each network to handle uncertainty was tested. This was done by creating 30 scenarios in which the demands were subject to uncertainty. The random demand values were calculated as follows:

Random demand = average demand + 
$$\sum_{m}$$
 uniform(0,0.9) (average demand)

This means that the demand values were increased by a value equal to the average demand multiplied by a random value between 0 and 0.9 with a uniform distribution, summed over *m* uncertain parameters. In all cases, m was set equal to three as previously discussed and the random value between 0 and 0.9 meant high uncertainty, such that the demand could be increased by up to 90%.

The main premise behind this study was that a network was established with four opened cross-docks chosen from a set of potential locations, and a set number of trucks traveling between different origins and destinations and cross-dock locations was determined. For the deterministic case, these outputs were determined based on average demands. For the robust case, these outputs were determined based on uncertain demands, with high uncertainty and a budget of uncertainty, omega, equal to three. The next set of experiments tested the resilience of the deterministic solution and the robust solution to demand uncertainty. It was assumed that the robust network would be better able to handle the variations in demand as compared to the deterministic case.

The ability of a network to handle uncertainty was based on the total capacity shortage in the network. The total shortage was calculated as the sum of truck capacity shortage plus the sum of cross-dock capacity shortage. The truck capacity shortage was equal to the truck capacity minus the total demand needing shipped by the truck. The cross-dock capacity shortage was equal to the cross-dock capacity minus the total demand needing to be sorted and consolidated at the cross-dock. The following three tables show the capacity shortages and total costs for the deterministic and robust case in each network, with 30 demand scenarios.

Demand	Total Capacity Shortage (In Pallets)		Total Co	st (\$)
Scenario	Deterministic	Robust	Deterministic	Robust
1	1,327	0	38,851	115,796
2	1,068	0	38,612	115,853
3	1,400	0	38,869	115,815
4	1,712	0	39,116	115,910
5	1,161	0	38,812	115,779
6	940	0	38,485	115,688
7	982	0	38,497	115,673
8	1,311	0	38,681	115,688
9	1,133	0	38,804	115,685
10	1,014	0	38,541	115,656
11	1,159	0	38,603	115,687
12	1,134	0	38,785	115,911
13	889	0	38,417	115,718
14	986	0	38,681	115,696
15	886	0	38,633	115,722
16	1,344	0	38,839	115,785
17	1,371	0	38,874	115,729
18	1,240	0	38,689	115,692
19	1,232	0	38,964	115,824
20	1,241	0	38,770	115,859
21	952	0	38,547	115,687
22	1,031	0	38,699	115,853
23	1,300	0	38,747	115,728
24	826	0	38,609	115,737
25	1,022	0	38,559	115,680
26	1,256	0	38,618	115,771
27	1,283	0	39,000	115,843
28	1,336	0	38,846	115,837
29	1,006	0	38,614	115,699
30	1,009	0	38,613	115,810

**Table 10:** Capacity Shortage and Total Cost for Network 1<br/>( $\Omega$ =3 for Robust Case)

Demand	Total Capacity Short	age (In Pallets)	Total Cost (\$)		
Scenario	Deterministic	Robust	Deterministic	Robust	
1	1,202	0	37,795	104,892	
2	1,498	0	38,080	104,987	
3	1,368	0	37,969	104,897	
4	1,472	0	38,057	104,911	
5	1,503	0	38,081	104,989	
6	1,450	0	38,033	105,000	
7	1,269	0	37,909	104,839	
8	1,263	0	37,888	104,847	
9	1,424	0	37,984	104,959	
10	1,408	0	38,002	104,919	
11	1,129	0	37,711	104,933	
12	1,094	0	37,681	104,956	
13	1,295	0	37,869	104,881	
14	1,380	0	37,958	104,879	
15	1,169	0	37,761	104,802	
16	1,068	0	37,651	104,879	
17	1,411	0	38,000	104,965	
18	1,130	0	37,705	104,921	
19	1,500	0	38,139	104,917	
20	1,166	0	37,780	104,971	
21	1,240	0	37,834	104,863	
22	1,239	0	37,844	104,856	
23	1,140	0	37,728	104,855	
24	1,520	0	38,124	105,009	
25	1,261	0	37,838	104,875	
26	1,064	0	37,632	104,701	
27	1,053	0	37,647	104,851	
28	1,242	0	37,805	104,899	
29	1,353	0	37,946	104,885	
30	1,271	0	37,888	104,841	

**Table 11:** Capacity Shortage and Total Cost for Network 2<br/>( $\Omega$ =3 for Robust Case)

Demand	Total Capacity Short	age (In Pallets)	Total Cost (\$)			
Scenario	Deterministic	Robust	Deterministic	Robust		
1	2,231	0	66,101	175,670		
2	2,109	0	66,016	175,701		
3	2,433	0	66,314	175,663		
4	2,071	0	65,958	175,573		
5	2,195	0	66,070	175,604		
6	2,336	0	66,211	175,792		
7	2,308	0	66,205	175,613		
8	2,193	0	66,050	175,609		
9	2,276	0	66,151	175,573		
10	2,264	0	66,160	175,787		
11	2,140	0	66,038	175,594		
12	2,184	0	66,037	175,616		
13	2,427	0	66,281	175,924		
14	2,095	0	65,966	175,612		
15	2,349	0	66,245	175,792		
16	2,236	0	66,144	175,849		
17	2,469	0	66,372	175,550		
18	2,114	0	65,995	175,908		
19	2,435	0	66,354	175,768		
20	2,491	0	66,369	175,657		
21	2,274	0	66,175	175,789		
22	2,128	0	66,009	175,637		
23	2,024	0	65,955	175,723		
24	1,954	0	65,853	175,539		
25	1,850	0	65,755	175,619		
26	2,199	0	66,089	175,633		
27	1,953	0	65,845	175,552		
28	2,023	0	65,914	175,470		
29	2,434	0	66,331	175,762		
30	2,313	0	66,197	175,776		

**Table 12:** Capacity Shortage and Total Cost for Network 3<br/>( $\Omega$ =3 for Robust Case)

In all three networks, there were large capacity shortages for every scenario in the deterministic model, but in every scenario for each network, the robust model was able to transport all of the pallets with no capacity shortages. It is obvious that the robust models were better equipped to handle demand uncertainty, than the deterministic models. The total costs were still much lower for the deterministic cases than the robust cases. However, in most real world situations, trucking companies would be largely penalized for being unable to move demand; they would either lose clients, be forced to buy or lease more trucks to ship directly, open more cross-docks or find other carriers to ship their excess demands. All of these solutions would be extremely costly and detrimental to the trucking company.

In order to better compare the real world costs for the deterministic and robust models, a penalty charge was added for each pallet that could not be transported. The tables below show the total costs adjusted for penalized capacity shortages, based on a penalty cost of \$100 per pallet. The penalty value was an estimate based on an assumed monthly leasing cost of about \$2,000 per truck. If a truck can hold 28 pallets, the cost per pallet would be about \$71.00 plus the price of fuel. The penalty value could be changed based on the trucking company or the alternative solution to ship excess capacity. If a trucking company loses a client because of its inability to ship the excess demand, the cost would likely be much greater than \$100 per pallet. The total cost after penalty was calculated by:

*Total Cost* (\$) *after Penalty* = *total cost* + (\$100 \* *capacity shortage*)

	Total Cost (\$) After Penalty										
Demand	Networl	x 1	Networ	k 2	Networ	k 3					
Scenario	Deterministic	Robust	Deterministic	Robust	Deterministic	Robust					
1	171,502	115,796	157,984	104,892	289,142	175,670					
2			187,877	104,987	276,842	175,701					
3	178,863	115,815	174,738	104,897	309,586	175,663					
4	210,306	115,910	185,189	104,911	273,031	175,573					
5	154,893	115,779	188,321	104,989	285,568	175,604					
6	132,480	115,688	182,967	105,000	299,753	175,792					
7	136,656	115,673	164,760	104,839	296,928	175,613					
8	169,755	115,688	164,171	104,847	285,303	175,609					
9	152,012	115,685	180,310	104,959	293,723	175,573					
10	139,881	115,656	178,785	104,919	292,514	175,787					
11	154,459	115,687	150,602	104,933	279,993	175,594					
12	152,147	115,911	147,043	104,956	284,375	175,616					
13	127,240	115,718	167,272	104,881	308,945	175,924					
14	137,273	115,696	175,941	104,879	275,380	175,612					
15	127,166	115,722	154,610	104,802	301,076	175,792					
16	173,185	115,785	144,420	104,879	289,729	175,849					
17	175,895	115,729	179,002	104,965	313,200	175,550					
18	162,602	115,692	150,613	104,921	277,322	175,908					
19	162,097	115,824	188,092	104,917	309,834	175,768					
20	162,817	115,859	154,354	104,971	315,428	175,657					
21	133,728	115,687	161,806	104,863	293,568	175,789					
22	141,719	115,853	161,665	104,856	278,727	175,637					
23	168,687	115,728	151,711	104,855	268,301	175,723					
24	121,164	115,737	190,079	105,009	261,241	175,539					
25	140,753	115,680	163,929	104,875	250,738	175,619					
26	164,149	115,771	143,990	104,701	285,975	175,633					
27	167,296	115,843	142,939	104,851	261,145	175,552					
28	172,405	115,837	162,002	104,899	268,150	175,470					
29	139,118	115,699	173,152	104,885	309,653	175,762					
30	139,487	115,810	164,930	104,841	297,413	175,776					

**Table 13:** Total Cost after Capacity Shortage Penalties for All Three Networks<br/>( $\Omega$ =3 for Robust Case)

In the above table, the total costs for the robust models are now less than the total costs for the deterministic models. Because, there were no capacity shortages for the robust models, their adjusted total costs did not change; they incurred no penalties. Penalizing for capacity

shortages made the robust models look much more appealing than the deterministic. As stated above, the penalty value was simply an estimate. It would be better determined on a case by case basis for trucking companies to estimate the penalties or loss in business they would incur for not accounting for uncertainties in demand.

The analysis was repeated to test different values of omega, but instead of listing values for every demand scenario, the averages were computed. The average capacity shortages and total costs for the three networks across the thirty demand scenarios for the deterministic and three robust optimization strategies (omega = 1, 2, and 3) are shown below in Table 14. The results confirm the fact that the deterministic solution suffered from capacity shortages in all the scenarios whereas this was not an issue in the robust framework.

Network			N1		N2			N3			
	rtainty		Omega			Omega			Omega		
Bu	dget	1	2	3	1	2	3	1	2	3	
Capacity Shortage	Deterministic	1,045	1,045	1,045	1,338	1,338	1,338	2,102	2,102	2,102	
Capac	Robust	0	0	0	0	0	0	0	0	0	
Total Cost (\$)	Deterministic	49,881	49,881	49,881	29,420	29,420	29,420	65,997	65,997	65,997	
	Robust	62,252	99,856	110,081	57,855	73,539	95,347	103,544	141,826	179,350	

Table 14: Average Capacity Shortages (In Pallets) and Total Costs for All Three Networks

The average total cost of 30 simulated demand scenarios for the three networks considered in this research is reported in Table 15, with the appropriate capacity shortage penalty added. The cost of the deterministic setting, assuming the penalty cost on capacity shortage, was higher than the robust approach. The total costs for all of the robust scenarios are now lower than the deterministic. These results were consistent with the results in Table 13.

Network		N1			N2			N3		
Uncert	ainty		Omega		Omega			Omega		
Budg	get	1	2	3	1	2	3	1	2	3
Total Cost (\$)	Deterministic	154,381	154,381	154,381	163,220	163,220	163,220	276,197	276,197	276,197
Tot	Robust	62,252	99,856	110,081	57,855	73,539	95,347	103,544	141,826	179,350

 Table 15: Average Total Cost after Capacity Shortage Penalties for All Three Networks

#### 5.3.2 Comparison with Inflated Demands for the Deterministic Case

One specific way to deal with uncertainty in demand is to plan for an inflated demand. For example, in a number of civil engineering applications the uncertainty in loads is accounted for by inflating the load using a safety factor. The goal of this set of experiments was to determine if a similar safety inflation factor could be applied to the demand. The advantage of this method is that freight companies need not do complicated robust optimization or stochastic optimization based planning. However, before embarking on such a method it was crucial to compare the performance of the deterministic model with an inflated demand to the robust model. First the deterministic demand was inflated according to five levels: 10%, 20%, 30%, 40%, and 50%. The optimal strategy was obtained for the five inflated demand levels. Then the robust optimization strategy was determined for high uncertainty for three uncertainty budget levels. Then the performances of the inflated demand deterministic strategy and the robust strategy were compared by simulating 30 random demand values. The capacity shortages as well as the average costs (considering capacity shortage penalties) are shown in Table 16 and Table 17 on the next two pages.

Network			N1	Rot	ust Appro	N2			N3		
Una	Uncertainty Budget		Omega			Omega			Omega		
Uncertainty Budget		1	2	3	1	2	3	1	2	3	
	10%	Deterministic Model	504.3	504.3	504.3	821.5	821.5	821.5	1,401.3	1,401.3	1,401.3
		Robust Model	0	0	0	0	0	0	0	0	0
	20%	Deterministic Model	12.36	12.36	12.36	0	0	0	94.3	94.3	94.3
		Robust Model	0	0	0	0	0	0	0	0	0
Demand Inflation Level	30%	Deterministic Model	0	0	0	0	0	0	0	0	0
Demand		Robust Model	0	0	0	0	0	0	0	0	0
	40%	Deterministic Model	0	0	0	0	0	0	0	0	0
		Robust Model	0	0	0	0	0	0	0	0	0
	50%	Deterministic Robust D Model Model	0	0	0	0	0	0	0	0	0
	4,	Robust Model	0	0	0	0	0	0	0	0	0

 Table 16: Capacity Shortages (In Pallets) for Demand Inflation of the Deterministic vs. the Robust Approach

	Network			N1			N2		N3			
Uno	Uncertainty Budget		Omega			Omega			Omega			
Uncertainty Budget		1	2	3	1	2	3	1	2	3		
	10%	Deterministic Cost	94,636	94,636	94,636	122,632	122,632	122,632	143,103	143,103	143,103	
		Robust Cost	62,608	88,844	116,352	67,007	78,702	101,281	110,174	151,751	182,466	
	20%	Deterministic Cost	58,524	58,524	58,524	78,619	78,619	78,619	94,680	94,680	94,680	
		Robust Cost	62,608	88,844	116,352	67,007	78,702	101,281	110,174	151,751	182,466	
Demand Inflation Level	30%	Deterministic Cost	63,145	63,145	63,145	58,255	58,255	58,255	108,406	108,406	108,406	
Demand I		Robust Cost	62,608	88,844	116,352	67,007	78,702	101,281	110,174	151,751	182,466	
	40%	Deterministic Cost	73,640	73,640	73,640	67,847	67,847	67,847	129,597.	129,597	129,597	
		Robust Cost	62,608	88,844	116,352	67,007	78,702	101,281	110,174	151,751	182,466	
	50%	Deterministic Cost	87,839	87,839	87,839	71,262	71,841	72,126	142,191	142,191	142,191	
	4)	Robust Cost	62,608	88,844	116,352	67,007	78,702	101,281	110,174	151,751	182,466	

 Table 17: Demand Inflation Cost of the Deterministic vs. the Robust Approach (Accounting for Capacity Shortage Penalties)

It can be seen in Table 16 that the robust model had no capacity shortages for any of the networks or omega values. The robust model was able to handle all of the demands in this case just as it was in the comparison in Section 5.3.1. The deterministic models had shortages when the demands were only increased by 10% in all three networks, and shortages in Networks 1 and 3 when the demands were increased by 20%. When the demands were increased by 30, 40, and 50% there were no capacity shortages for the deterministic cases. This observation shows that inflating demands by a reasonable percentage can allow the model to handle uncertainty.

In order to better compare the demand inflation strategy to the robust strategy, the total costs were also considered. Table 17 shows the total costs for all scenarios. Penalties have already been applied to the costs in Table 17 in cases where capacity shortages occurred. The penalty was \$100 per pallet, which was the same as in Section 5.3.1. When demands were inflated by 10% and 20% for the deterministic case, shortages still occurred and thus a penalty was added to the average total cost for every pallet not shipped. In this test of demand inflation, increasing the demands by 10% or 20% would be undesirable because not only were there capacity shortages in these cases, but the total costs were increased because of the inflation. In other words, the total costs were increasing even without the penalties and the networks were still unable to handle uncertainty in demand.

After studying Tables 16 and 17, it was clear that inflating demands by 30% would be the "best strategy". There were no capacity shortages in these cases and the average total costs were either less than the robust or comparable. Inflating demands by 40 and 50% began to increase the total costs by a rather large amount, which would be undesirable to freight carriers.

The idea behind testing demand inflation against the robust strategy was not to negate the usefulness of robust optimization. Instead it was to test the effects of demand inflation and how

this method compared to the robust model. Based on the results, the demand inflation or "factor of safety" strategy could be a very useful tool for small freight carriers who do not have the means or knowledge to use robust optimization. Telling a carrier to inflate demands by 30% is much simpler than giving them a robust model to use. This simple strategy could allow even small carriers to account for some demand uncertainty. However, the robust model is still vital such that it gives a basis to which the results of inflated demands can be compared. The robust optimization is also a powerful tool for larger freight companies who have the ability to understand and use this method. Unlike demand inflation, there is a sound analytical process behind the robust approach.

# CHAPTER 6. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH 6.1 Summary

Innovations in technology, population booms, fuels prices, traffic congestion, tighter trucking restrictions, and the demand for overnight and next-day delivery have led trucking companies to seek ways to reduce costs and increase efficiency. The desire to reduce wasted space on trucks led to the implementation of LTL consolidation and better planning for return shipments to eliminate empty backhauling. The idea of shipment consolidation created a need for intermediate facilities where trucks could consolidate their loads with other loads from different origins. Cross-dock warehouses filled that need. The use of cross-docks allowed for the efficient consolidation of shipments at intermediate points. Because goods spend little or no time in the actual facility, storage costs are eliminated as well as costs for picking. In order to utilize cross-docks, companies must first decide where to locate and open these facilities.

Therefore, a formulation was presented in order to choose which cross-docks should be opened from a set of potential nodes with the goal of minimizing total transportation costs. The first formulation assumed deterministic, or known, demands and followed the *p*-median facility problem. Three separate networks were studied, each of a different size. All networks and demands were extracted from real world data from a Third Party Logistics Company. The purpose of the deterministic formulation was to determine which cross-docks would be opened for each network and the total network costs.

The deterministic model then served as the baseline or control for the robust optimization. In the robust formulation, demand uncertainty was accounted for in each of the three networks at three levels of uncertainty – low, medium, and high. The robust formulation

was created as a variation of the deterministic formulation by adding uncertainty terms to the appropriate constraints and the objective function.

#### **6.2** Conclusions

The effects of demand uncertainty and the comparison between the performance of the deterministic and robust scenarios were tested on three realistic freight networks with parameters that followed industry standards. First, the deterministic and robust networks were compared to see simply how their results differed. In other words, the goal was to determine if accounting for uncertainty in demands changed which cross-docks would be opened and the overall total costs. In order to compare the costliness of the robust solutions, the relative costs were determined for various levels of omega. While the opened cross-docks, total costs, and relative costs provided important information regarding the differing results between deterministic and robust optimization, a better way to test the performance of each scenario was needed. This was accomplished in section 5.3, where the ability of each "optimized" network to handle uncertainty was studied.

Numerical analysis revealed that varying values of omega and the level of uncertainty for each network resulted in the opening of different cross-docks. This is an important result because it showed that accounting for uncertainty in demand did affect the results of the optimization. In other words, if demands are assumed to be known but in reality the demands actually vary, the system may have too little capacity or much greater total cost than anticipated. As expected, the total costs rose with the increase in omega and demand uncertainty. This occurred because larger omega values and higher uncertainty allowed the demands to increase by greater amounts. Larger demands resulted in higher transportation and handling costs. The relative cost showed the comparison between the total cost of the deterministic scenario (omega = 0) and all of the

robust scenarios. Because the total costs of the robust scenarios were greater than the deterministic case, all relative costs were greater than one, as expected. Of course, because omega set equal to zero was used as the baseline, that relative cost was one.

Simply looking at the total costs and relative costs could be misleading, because it makes the deterministic case look most appealing. In situations where demands are certain, the deterministic case is the best solution as it provides the lowest total cost. However, in networks where uncertainty exists, the robust optimization may be the best choice. Comparing the ability of the deterministic case (with average demands) and the robust case to handle uncertainty showed that the robust case was in fact more resilient when demands vary. In all networks, demand scenarios, and budgets of uncertainty in Section 5.3.1, the deterministic case came up short, such that it did not have enough capacity on either the trucks or in the cross-docks to handle increases in demand due to uncertainty. The robust case had no capacity shortages in any of the networks, demand scenarios, or budgets of uncertainty; it was able to handle and transport all random demands. In real world situations if a trucking company were unable to transport all of their demands, they would face some type of penalty. In this study, a penalty of \$100 per pallet was estimated. When the penalties were added to the total costs, the robust case ended up being less expensive than the deterministic case for every network, demand scenario, and budget of uncertainty.

In the analysis (Section 5.3.2) where demands were inflated by a percentage for the deterministic model and compared to the robust optimization, there were some valuable results. While small inflations still led to capacity shortages, and large inflations led to very high costs, a 30% demand inflation had no capacity shortages and total costs less than or comparable to the robust case.

This analysis demonstrated the importance of accounting for uncertainty in demands. While assuming deterministic demands may result in more attractive total costs, accounting for uncertainties with a robust formulation can save money and reputation in the long run. In some situations for small carriers, simple demand inflation can be an extremely effective planning method. However, the robust model is still needed as a baseline to determine what percent of inflation will yield the best results. When inputs, such as demand, are uncertain, robust optimization is a powerful tool to develop a network that is able to handle increases in demand without shortages or unexpected costs.

#### **6.3 Directions for Future Research**

The work in this thesis could be expanded in a few ways. The first would be to model other parameters that may be uncertain, such as capacity or travel times, in order to see how they affect the cross-dock locations and total costs. This formulation could also be tested on even larger networks with much higher demands to see how the robust solution is able to handle greater uncertainty.

The discounts created due to consolidation could be modeled in other ways, such as using the MAUD function or some type of discount sharing technique. Another way to expand upon this work would be to discuss the penalty cost with real freight companies to get a more accurate or realistic value. This would allow for better comparison of the deterministic and robust cases. Finally, adding more constraints, such as delivery and pick-up times, would make the model more realistic, but also greatly increase the complexity.

64

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### APPENDICES

### Appendix A

Table 18: Internode Mileage for Network 1 f	from Origin (i) to Destination (j)
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		Destination								
		Westborough	Kirkwood	Columbus	Fairborn	Eighty Four				
	Montgomery	1233	1047	660	616	819				
	Atlanta	1082	897	570	524	674				
Origin	Unadilla	1128	976	694	645	753				
	Haines City	1310	1176	1025	982	982				
	Hattiesburg	1379	1197	799	755	960				

		Cross-dock									
		Lexington	Charlotte	Knoxville	Charlottesville	Charleston					
	Montgomery	494	405	343	671	661					
	Atlanta	380	245	214	511	502					
Origin	Unadilla	501	333	334	603	596					
	Haines City	830	562	664	847	825					
	Hattiesburg	635	622	489	855	798					

### **Table 19:** Internode Mileage for Network 1 from Origin (i) to Cross-dock (k)

		Destination									
		Westborough	Kirkwood	Columbus	Fairborn	Eighty Four					
	Lexington	891	696	191	148	351					
	Charlotte	808	646	426	454	422					
Cross-dock	Knoxville	892	709	357	314	467					
	Charlottesville	527	413	410	437	293					
	Charleston	717	533	162	189	202					

 Table 20: Internode Mileage for Network 1 from Cross-dock (k) to Destination (j)

		Destination													
_		Alsip	Danville	Des Plaines	Hanover Park	Monroe	Troy	Kansas City	Springfield	St. Louis	Milwaukee				
	Everett	982	1036	1005	1018	776	713	1439	1411	1196	1079				
	Franklin	965	1020	988	1002	759	696	1423	1394	1179	1062				
	Baltimore	696	667	718	733	491	549	1060	1036	821	797				
	Elizabeth	778	800	800	814	575	622	1203	1159	953	880				
Origin	Newark	775	788	798	811	572	627	1181	1167	941	871				
Origin	Sayreville	799	787	822	819	580	651	1180	1161	941	895				
	Delhi	769	829	792	806	565	525	1222	1202	983	866				
	Waterford	818	878	843	856	612	687	1271	1247	1064	930				
	Hanover	676	651	699	694	455	528	1054	1015	808	773				
	Hatfield	748	734	770	783	544	592	1127	1103	902	849				

**Table 21:** Internode Mileage for Network 2 from Origin (i) to Destination (j)

						Cross-o	dock				
		Fort Wayne	Indianapolis	Cumberland	Canton	Cincinnati	Columbus	Altoona	Pittsburgh	Parkersburg	Wheeling
	Everett	847	940	525	663	858	755	483	576	706	628
	Franklin	830	912	508	646	841	737	466	559	689	611
	Baltimore	560	579	139	366	508	404	181	244	319	278
	Elizabeth	643	701	293	454	630	526	274	363	478	400
Origin	Newark	636	699	291	447	628	524	267	360	476	398
Ongin	Sayreville	660	699	289	470	628	524	279	354	471	392
	Delhi	634	740	347	461	671	564	281	374	528	434
	Waterford	682	789	439	523	722	615	366	470	620	530
	Hanover	538	565	119	317	488	384	133	199	299	237
	Hatfield	610	646	237	420	575	471	233	307	418	345

 Table 22: Internode Mileage for Network 2 from Origin (i) to Cross-dock (k)

						]	Destination	1			
				Des	Hanover			Kansas			
		Alsip	Danville	Plaines	Park	Monroe	Troy	City	Springfield	St. Louis	Milwaukee
	Fort Wayne	155	180	179	191	127	185	608	584	368	253
	Indianapolis	174	89.3	206	210	251	309	482	458	242	280
	Cumberland	556	529	579	592	350	408	922	898	683	652
	Canton	372	389	396	420	177	231	782	758	543	469
Cross-	Cincinnati	287	202	319	323	227	285	589	565	350	392
dock	Columbus	318	264	342	384	163	227	657	633	418	416
	Altoona	548	547	567	584	342	400	940	915	700	645
	Pittsburgh	456	448	475	492	250	308	841	817	602	553
	Parkersburg	458	375	453	492	279	337	768	743	528	526
	Wheeling	445	391	505	510	272	330	784	759	544	543

 Table 23: Internode Mileage for Network 2 from Cross-dock (k) to Destination (j)

							Destination				
		Alsip	Chicago	Danville	Des Plaines	Hanover Park	Taylorville	Mason City	Louisville	Dearborn	Monroe
	New Haven	858	864	906	882	895	989	1231	817	683	652
	Everett	982	987	1036	1005	1018	1141	1355	973	719	776
	Franklin	965	971	1020	988	1002	1124	1338	956	702	759
	Baltimore	696	704	667	718	733	779	1070	610	522	491
	Belcamp	724	729	694	748	760	806	1094	638	549	518
	Elizabeth	778	787	800	800	814	902	1154	730	603	575
	Kearny	776	785	805	798	813	902	1149	731	602	571
	Newark	775	780	788	798	811	899	1147	728	599	572
	Sayreville	799	789	787	822	819	900	1157	728	609	580
Origin	Delhi	769	775	829	792	806	951	1142	773	532	565
Ongin	Rochester	597	602	657	620	633	777	970	601	333	391
	Waterford	818	825	878	843	856	1001	1192	823	552	612
	Williamson	624	629	684	648	660	806	997	628	361	418
	Blandon	719	724	697	743	755	809	1092	668	544	513
	Hanover	676	654	651	699	694	738	1022	590	474	455
	Hatfield	748	756	734	770	783	861	1118	674	573	544
	Alexandria	699	704	669	723	735	781	1072	613	524	493
	Lyndhurst	706	714	623	738	742	753	1057	470	566	535
	Newport News	855	860	784	879	892	915	1218	631	680	649
	Richmond	793	798	714	817	829	847	1150	563	618	587

# **Table 24:** Internode Mileage for Network 3 from Origin (i) to Destination (j)(Destinations 1 through 10 are shown)

						De	stination				
		Troy	Rogers	St. Paul	Kansas City	Springfield	St. Louis	Arlington	Elkhorn	Milwaukee	Oak Creek
	New Haven	710	1294	1264	1270	1246	1031	1150	956	954	946
	Everett	713	1417	1388	1439	1411	1196	1288	1080	1079	1070
	Franklin	696	1401	1371	1423	1394	1179	1272	1063	1062	1053
	Baltimore	549	1132	1102	1060	1036	821	888	794	797	783
	Belcamp	576	1159	1130	1087	1063	848	916	821	820	812
	Elizabeth	622	1217	1194	1203	1159	953	1062	879	880	869
	Kearny	620	1212	1192	1208	1159	944	1064	874	878	864
	Newark	627	1210	1180	1181	1167	941	1061	872	871	862
	Sayreville	651	1219	1189	1180	1161	941	1049	881	895	872
Origin	Delhi	525	1205	1175	1222	1202	983	1131	867	866	857
Oligin	Rochester	328	1032	1002	1050	1025	810	958	699	692	684
	Waterford	687	1255	1236	1271	1247	1064	1181	917	930	907
	Williamson	355	1064	1029	1077	1053	837	985	721	719	724
	Blandon	571	1207	1124	1090	1066	850	964	816	814	852
	Hanover	528	1084	1054	1054	1015	808	883	746	773	737
	Hatfield	592	1190	1164	1127	1103	902	1008	844	849	836
	Alexandria	551	1134	1104	1085	1038	823	855	796	794	787
	Lyndhurst	593	1144	1115	974	943	728	706	806	805	796
	Newport News	707	1290	1260	1135	1104	889	867	952	950	943
	Richmond	645	1228	1199	1067	1037	821	799	891	889	881

# **Table 25:** Internode Mileage for Network 3 from Origin (i) to Destination (j)(Destinations 11 through 20 are shown)

						Cross-c	lock				
		Fairfield	Bloomington	Fort Wayne	Hebron	Indianapolis	Lexington	Summit	Cumberland	Canton	Cincinnati
	New Haven	972	838	720	842	788	784	680	385	530	718
	Everett	1128	990	847	966	940	923	818	525	663	858
	Franklin	1111	973	830	949	912	906	801	508	646	841
	Baltimore	765	628	560	680	579	538	434	139	366	508
	Belcamp	792	655	586	707	606	566	461	166	396	535
	Elizabeth	885	751	643	765	701	697	592	293	454	630
	Kearny	886	752	638	760	702	698	593	298	448	631
	Newark	883	749	636	758	699	695	590	291	447	628
	Sayreville	884	749	660	767	699	691	586	289	470	628
Origin	Delhi	928	790	634	753	740	756	640	347	461	671
Ongin	Rochester	756	618	461	580	568	584	520	341	302	501
	Waterford	979	840	682	803	789	807	734	439	523	722
	Williamson	783	645	489	607	595	611	548	332	329	528
	Blandon	792	658	581	702	608	599	494	199	391	537
	Hanover	745	588	538	632	565	518	414	119	317	488
	Hatfield	830	695	610	732	646	637	532	237	420	575
	Alexandria	767	630	561	682	581	541	436	141	371	510
	Lyndhurst	624	570	543	667	534	398	293	165	415	427
	Newport News	786	732	704	828	695	559	454	297	527	588
	Richmond	718	664	636	760	628	491	387	235	466	521

# Table 26: Internode Mileage for Network 3 from Origin (i) to Cross-dock (k) (Cross-docks 1 through 10 are shown)

						(	Cross-dock				
		Columbus	Lima	Altoona	Mansfield	Oil City	Pittsburgh	Beckley	Buckhannon	Parkersburg	Wheeling
	New Haven	614	663	351	267	422	450	605	523	565	488
	Everett	755	796	483	395	555	576	743	661	706	628
	Franklin	737	779	466	378	538	559	727	644	689	611
	Baltimore	404	502	181	222	291	244	343	277	319	278
	Belcamp	431	529	207	222	292	275	371	304	347	305
	Elizabeth	526	586	274	235	346	363	517	435	478	400
	Kearny	527	581	269	230	341	363	519	436	479	401
	Newark	524	579	267	228	339	360	516	433	476	398
	Sayreville	524	589	279	238	348	354	504	429	471	392
Origin	Delhi	564	596	281	157	342	374	568	483	528	434
Ongin	Rochester	394	424	277	131	205	284	519	417	422	336
	Waterford	615	646	366	232	428	470	659	577	620	530
	Williamson	421	451	268	122	232	312	547	444	449	363
	Blandon	433	524	189	151	261	270	420	337	380	307
	Hanover	384	454	133	183	252	199	339	257	299	237
	Hatfield	471	553	233	202	312	307	466	375	418	345
	Alexandria	406	504	182	270	315	250	310	224	322	280
	Lyndhurst	384	471	224	359	357	292	161	147	299	289
	Newport News	546	633	338	425	471	406	322	309	461	436
	Richmond	478	565	276	363	409	344	254	241	393	374

## **Table 27:** Internode Mileage for Network 3 from Origin (i) to Cross-dock (k)(Cross-docks 11 through 20 are shown)

						Ι	Destination				
					Des	Hanover		Mason			
		Alsip	Chicago	Danville	Plaines	Park	Taylorville	City	Louisville	Dearborn	Monroe
	Fairfield	256	271	174	288	292	121	538	157	483	453
	Bloomington	223	232	136	256	259	184	571	105	331	301
	Fort Wayne	155	162	180	179	191	313	528	236	158	127
	Hebron	47.4	56	110	80.1	83.5	196	420	250	261	251
	Indianapolis	174	183	89.3	206	210	201	523	114	282	251
	Lexington	365	373	281	398	401	364	715	80.6	341	310
	Summit	424	432	340	457	460	470	774	187	322	291
	Cumberland	556	561	529	579	592	641	929	472	381	350
	Canton	372	389	389	396	420	501	756	333	208	177
Cross-	Cincinnati	287	295	202	319	323	314	637	99.4	258	227
dock	Columbus	318	356	264	342	384	376	699	206	199	163
	Lima	222	228	262	247	258	374	594	224	133	103
	Altoona	548	553	547	567	584	658	921	487	373	342
	Mansfield	651	656	693	674	687	805	1023	637	415	445
	Oil City	457	462	495	481	493	607	830	439	282	251
	Pittsburgh	456	461	448	475	492	560	829	388	281	250
	Beckley	542	550	459	574	578	589	893	306	416	386
	Buckhannon	566	593	483	612	602	595	917	359	413	382
	Parkersburg	458	464	375	453	492	485	807	286	326	279
	Wheeling	445	482	391	505	510	502	825	331	289	272

## **Table 28:** Internode Mileage for Network 3 from Cross-dock (k) to Destination (j)(Destinations 1 through 10 are shown)

						De	stination				
		Troy	Rogers	St. Paul	Kansas City	Springfield	St. Louis	Arlington	Elkhorn	Milwaukee	Oak Creek
	Fairfield	511	669	639	367	335	120	289	360	361	353
	Bloomington	359	662	632	464	440	225	391	324	322	314
	Fort Wayne	185	592	562	608	584	368	593	254	253	244
	Hebron	280	486	456	511	511	296	513	148	146	138
	Indianapolis	309	613	583	482	458	242	472	275	280	265
	Lexington	368	803	774	586	554	339	398	466	464	456
	Summit	349	863	833	691	660	445	511	525	523	515
	Cumberland	408	991	961	922	898	683	797	653	652	644
	Canton	231	819	789	782	758	543	691	481	469	471
Cross-	Cincinnati	285	725	695	589	565	350	457	387	392	378
dock	Columbus	227	786	756	657	633	418	563	448	416	438
	Lima	161	659	629	655	631	416	581	321	319	311
	Altoona	400	983	954	940	915	700	865	645	645	636
	Mansfield	410	1086	1056	1086	1062	846	994	748	746	738
	Oil City	309	892	863	888	864	649	796	555	553	545
	Pittsburgh	308	891	861	841	817	602	746	553	553	544
	Beckley	444	980	951	810	779	564	617	642	641	633
	Buckhannon	440	1023	994	863	833	617	684	685	684	676
	Parkersburg	337	894	865	768	743	528	620	557	526	547
	Wheeling	330	912	882	784	759	544	688	574	543	565

## **Table 29:** Internode Mileage for Network 3 from Cross-dock (k) to Destination (j)(Destinations 11 through 20 are shown)

				D	estination (j)	1	
		Commodity Type	Westborough	Kirkwood	Columbus	Fairborn	Eighty Four
	Montgomery		9.91	0	0	0	0
	Atlanta		1.95	22.56	15	24.97	6.23
	Unadilla	Dry	0	0	0	0	0
	Haines City		0	14.34	0	0	0
	Hattiesburg		0	30	0	30	0
	Montgomery		0	0	237.73	0	0
	Atlanta		1.95	0	0	0	0
Origin (i)	Unadilla	Refrigerated	0	0	0	0	0
	Haines City		12.03	0	0	0	0
	Hattiesburg		0	0	0	0	0
	Montgomery		0	23.67	0	0	0
	Atlanta		1.95	22.56	15	24.97	0
	Unadilla	Frozen	0	27.2	51.47	0	28.26
	Haines City		12.03	0	2	0	0
	Hattiesburg		0	0	0	0	0

### Table 30: Average Demands (in pallets) for Network 1

			-	Destination (j)										
		Commodity			Des	Hanover	Destr	()	Kansas		St.			
		Туре	Alsip	Danville	Plaines	Park	Monroe	Troy	City	Springfield	Louis	Milwaukee		
	Everett	~ ~	0	0.71	0	0	0	0	1	0	0	0		
	Franklin		0	0	0	0	0	0	0	0	0	0		
	Baltimore		9.68	0	9.62	0	0	0	0	0	0	9.62		
	Elizabeth		2	7.02	2.72	0	0	7.01	4.77	0	2.87	3.03		
	Newark	Dry	0	0	0	0	0	0	0	0	0	0		
	Sayreville	Diy	0	0	0	25.36	23.72	0	0	27.36	0	0		
	Delhi		0	0	0	0	0	0	0	0	0	0		
	Waterford		0	0	28.4	0	0	21.43	0	0	27.74	27.83		
	Hanover		0	5.8	0	14.41	37.42	0	15.92	1	0	0		
	Hatfield		47.63	0	12.44	24.54	27.75	4.83	0	0	6.52	17.28		
	Everett		0	0	0	0	0	0	0	0	0	0		
	Franklin		0	17.7	0	0	0	0	15.65	0	0	0		
	Baltimore		0	0	0	0	0	0	0	0	0	0		
	Elizabeth		0	7.02	0	0	0	0	4.77	0	0	0		
Origin	Newark	Refrigerated	0	0	0	0	0	0	0	0	0	0		
(i)	Sayreville	Kenngerateu	0	0	0	0	0	0	0	0	0	0		
	Delhi		0	0	0	0	3.13	0	0	3	0	0		
	Waterford		0	0	0	0	0	0	0	0	0	0		
	Hanover		0	0	0	0	0	0	0	0	0	0		
	Hatfield		0	0	0	0	0	0	0	0	0	0		
	Everett		0	0	0	0	0	0	0	0	0	0		
	Franklin		0	0	0	0	0	0	0	0	0	0		
	Baltimore		0	0	0	0	0	0	0	0	0	0		
	Elizabeth		0	0	0	2	8.67	0	0	0	0	0		
	Newark	Frozen	0	0	0	2	12	0	0	14.83	0	0		
	Sayreville	1102011	0	0	0	0	0	0	0	0	0	0		
	Delhi		0	0	0	0	0	0	0	0	0	0		
	Waterford		0	0	0	0	0	0	0	0	0	0		
	Hanover	-	0	0	0	0	0	0	0	0	0	0		
	Hatfield		0	0	0	0	0	0	0	0	0	0		

 Table 31: Average Demands (in pallets) for Network 2

							De	stination (j)				
		Commodity Type	Alsip	Chicago	Danville	Des Plaines	Hanover Park	Taylorville	Mason City	Louisville	Dearborn	Monroe
	New Haven		0	0	0	0	0	0	0	0	0	0
	Everett		0	0	0.71	0	0	0	0	0	0	0
	Franklin		0	0	0	0	0	0	0	0	0	0
	Baltimore		9.68	26.98	0	9.62	0	0	0	0	0	0
	Belcamp		0	0	0	0	0	0	2.78	0	0	0
	Elizabeth		2	4.66	7.02	2.72	0	0	0	0	5.13	0
	Kearny		9.8	19.49	2	11.35	0	0	0	0	0.95	0
	Newark		0	0	0	0	0	0	0	0	0	0
	Sayreville		0	0	0	0	25.36	0	0	0	0	23.72
Origin	Delhi	Dry	0	0	0	0	0	0	0	0	0	0
(i)	Rochester	Diy	0	0	0	0	0	0.18	0	0	0	0
	Waterford		0	0	0	28.4	0	0	0	0	28.72	0
	Williamson		0	0	0	0	0	0	0	0	0	0
	Blandon		0	0	0	0	0	0	0	0	0	0
	Hanover		0	0	5.8	0	14.41	0	0	0	0	37.42
	Hatfield		47.63	16.11	0	12.44	24.54	6.71	6.44	0	5.78	27.75
	Alexandria		0	0	0	0	0	0	0	0	0	0
	Lyndhurst		0	0	0	0	0	0	0	0	0	0
	Newport News		0	0	0	0	0	0	0	0	0	0
	Richmond		0	0	21.98	0	0	0	0	0	0	0

## **Table 32:** Average Demands (in pallets) for Network 3(Dry Commodity, Destinations 1 through 10)

	s (in puncts) for retwork s	
(Dry Commodity, Destin	nations 11 through 20)	

Table 33: Average Demands (in pallets) for Network 3	
(Dry Commodity, Destinations 11 through 20)	

							Desti	nation (j)				
		Commodity Type	Troy	Rogers	St. Paul	Kansas City	Springfield	St. Louis	Arlington	Elkhorn	Milwaukee	Oak Creek
	New Haven		0	0	0	0	0	0	0	0	0	0
	Everett		0	0	0	1	0	0	0	0	0	0
	Franklin		0	0	0	0	0	0	0	0	0	0
	Baltimore		0	0	0	0	0	0	0	0	9.62	3.75
	Belcamp		0	0	0	0	0	0	4.83	0	0	0
	Elizabeth		7.01	0	5.02	4.77	0	2.87	0	0	3.03	0
	Kearny		5.6	0	6.66	1	0	0	0	0	13.9	0
	Newark		0	0	0	0	0	0	0	0	0	0
	Sayreville		0	0	0	0	27.36	0	0	0	0	0
Origin	Delhi	Dry	0	0	0	0	0	0	0	0	0	0
(i)	Rochester	Dry	0	0	0	0	0	0	0	0.23	0	0
	Waterford		21.43	0	28.25	0	0	27.74	0	0	27.83	0
	Williamson		0	8	0	0	0	0	0	0	0	7.13
	Blandon		0	0	0	0	0	0	0	0	0	0
	Hanover		0	0	0	15.92	1	0	16	0	0	0
	Hatfield		4.83	18.8	3.98	0	0	6.52	28	10.62	17.28	0
	Alexandria		0	0	0	0	0	0	0	0	0	0
	Lyndhurst		0	0	0	0	0	0	0	0	0	0
	Newport News		0	0	0	0	0	0	0	0	0	0
	Richmond		0	0	0	0	0	0	0	0	0	0

<b>Table 34:</b> Average Demands (in pallets) for Network 3	
(Refrigerated Commodity, Destinations 1 through 10)	

							Dest	ination (j)				
		Commodity				Des	Hanover		Mason			
		Туре	Alsip	Chicago	Danville	Plaines	Park	Taylorville	City	Louisville	Dearborn	Monroe
	New Haven		0	0	0	0	0	0	0	0	0	0
	Everett		0	0	0	0	0	0	0	0	0	0
	Franklin		0	0	17.7	0	0	0	0	0	0	0
	Baltimore		0	0	0	0	0	0	0	0	0	0
	Belcamp		0	0	0	0	0	0	0	0	0	0
	Elizabeth		0	0	7.02	0	0	0	0	0	0	0
	Kearny		0	0	0	0	0	0	0	0	0	0
	Newark		0	0	0	0	0	0	0	0	0	0
	Sayreville		0	0	0	0	0	0	0	0	0	0
Origin	Delhi	Refrigerated	0	0	0	0	0	0	0	0	0	3.13
(i)	Rochester	Kenngeraleu	0	0	0	0	0	0	0	0	0	0
	Waterford		0	0	0	0	0	0	0	0	0	0
	Williamson		0	0	0	0	0	0	0	0	0	0
	Blandon		0	0	0	0	0	0	0	0	0	0
	Hanover		0	0	0	0	0	0	0	0	0	0
	Hatfield		0	0	0	0	0	0	0	0	0	0
	Alexandria		0	0	0	0	0	0	0	0	0	0
	Lyndhurst		0	0	0	0	0	0	0	0	0	0
	Newport News		0	0	0	0	0	0	0	0	0	0
	Richmond		0	0	0	0	0	0	0	0	0	0

**Table 35:** Average Demands (in pallets) for Network 3(Refrigerated Commodity, Destinations 11 through 20)

						-	Desti	nation (j)				
		Commodity Type	Troy	Rogers	St. Paul	Kansas City	Springfield	St. Louis	Arlington	Elkhorn	Milwaukee	Oak Creek
	New Haven		0	0	0	0	0	0	0	0	0	0
	Everett		0	0	0	0	0	0	0	0	0	0
	Franklin		0	0	0	15.65	0	0	0	0	0	0
	Baltimore		0	0	0	0	0	0	0	0	0	0
	Belcamp		0	0	0	0	0	0	0	0	0	0
	Elizabeth		0	0	0	4.77	0	0	0	0	0	0
	Kearny		0	0	0	0	0	0	0	0	0	0
	Newark		0	0	0	0	0	0	0	0	0	0
	Sayreville		0	0	0	0	0	0	0	0	0	0
Origin	Delhi	Refrigerated	0	0	0	0	3	0	0	0	0	0
(i)	Rochester	Reffigerated	0	0	0	0	0	0	0	0	0	0
	Waterford		0	0	0	0	0	0	0	0	0	0
	Williamson		0	0	0	0	0	0	0	0	0	0
	Blandon		0	0	0	0	0	0	0	0	0	0
	Hanover		0	0	0	0	0	0	0	0	0	0
	Hatfield		0	0	0	0	0	0	0	0	0	0
	Alexandria		0	0	0	0	0	0	0	0	0	0
	Lyndhurst		0	0	0	0	0	0	0	0	0	0
	Newport News		0	0	0	0	0	0	0	0	0	0
	Richmond		0	0	0	0	0	0	0	0	0	0

							Dest	ination (j)				
		Commodity Type	Alsip	Chicago	Danville	Des Plaines	Hanover Park	Taylorville	Mason City	Louisville	Dearborn	Monroe
	New Haven		0	0	1.3	0	0	0	0	0	0	0
	Everett		0	0	0	0	0	0	0	0	0	0
	Franklin		0	0	0	0	0	0	0	0	0	0
	Baltimore		0	0	0	0	0	0	0	0	0	0
	Belcamp		0	0	0	0	0	0	0	0	0	0
	Elizabeth		0	0	0	0	2	0	0	0	0	8.67
	Kearny		0	0	0	0	0	0	0	0	0	0
	Newark		0	0	0	0	2	0	0	0	0	12
	Sayreville		0	0	0	0	0	0	0	0	0	0
Origin	Delhi	Frozen	0	0	0	0	0	0	0	0	0	0
(i)	Rochester	Tiozen	0	0	0	0	0	0	0	0	0	0
	Waterford		0	0	0	0	0	0	0	0	0	0
	Williamson		0	0	0	0	0	0	0	0	0	0
	Blandon		0	0	0	0	0	0	0	2.5	0	0
	Hanover		0	0	0	0	0	0	0	0	0	0
	Hatfield		0	0	0	0	0	0	0	0	0	0
	Alexandria		0	0	0	0	0	0	0	0	0	0
	Lyndhurst		0	0	0	0	0	0	0	0	0	0
	Newport News		0	0	0	0	3	0	0	0	0	0
	Richmond		0	0	0	0	0	0	0	0	0	0

## **Table 36:** Average Demands (in pallets) for Network 3(Frozen Commodity, Destinations 1 through 10)

# **Table 37:** Average Demands for Network 3(Frozen Commodity, Destinations 11 through 20)

							Dest	ination (j)				
		Commodity Type	Troy	Rogers	St. Paul	Kansas City	Springfield	St. Louis	Arlington	Elkhorn	Milwaukee	Oak Creek
	New Haven		0	0	0	0	0	0	0	0	0	0
	Everett		0	0	0	0	0	0	0	0	0	0
	Franklin		0	0	0	0	0	0	0	0	0	0
	Baltimore		0	0	0	0	0	0	0	0	0	0
	Belcamp		0	0	0	0	0	0	0	0	0	0
	Elizabeth		0	0	0	0	0	0	0	0	0	0
	Kearny		0	0	0	0	0	0	0	0	0	0
	Newark		0	0	0	0	14.83	0	0	0	0	0
	Sayreville		0	0	0	0	0	0	0	0	0	0
Origin	Delhi	Frozen	0	0	0	0	0	0	0	0	0	0
(i)	Rochester	Tiozen	0	0	0	0	0	0	0	0	0	0
	Waterford		0	0	0	0	0	0	0	0	0	0
	Williamson		0	0	0	0	0	0	0	0	0	0
	Blandon		0	2	0	0	0	0	0	0	0	2.17
	Hanover		0	0	0	0	0	0	0	0	0	0
	Hatfield		0	0	0	0	0	0	0	0	0	0
	Alexandria		0	0	0	1.11	0	0	0	0	0	0
	Lyndhurst		0	0	0	0	0	0	0	0	0	26.24
	Newport News		0	0	0	0	0	0	0	0	0	0
	Richmond		0	0	0	0	0	0	0	0	0	0

#### Appendix B

GAMS Code for Network 1 Varying Omega from 0 to 3 (Corresponds to the results in Sections 5.2.1, 5.2.2, 5.2.3, and Appendix C for N1)

\$offlisting \$offsymxref offsymlist

file demandUncert1 / demandUncert1.csv / ;

\*Network 1 (50, 5D, 5K)

set i 'origin'/Montgomery, Atlanta, Unadilla, Haines\_City, Hattiesburg/; set j 'destination' /Westborough, Kirkwood, Columbus, Fairborn, Eighty\_Four/; set k 'cross-docks'/Lexington, Charlotte, Knoxville, Charlottesville, Charleston/; set l 'commodity'/D, R, F/;

Table s(i,j) 'distance from origin node i to destination node j'

	Westborough	Kirkwood	Columbus	Fairborn	Eighty_Four
Montgomery	1233	1047	660	616	819
Atlanta	1082	897	570	524	674
Unadilla	1128	976	694	645	753
Haines_City	1310	1176	1025	982	982
Hattiesburg	1379	1197	799	755	960;
display i,j,s;					

Table s1(i,k) 'distance from origin node i to cross-dock node k'

]	Lexington	Charlotte	Knoxville	Charlottesville	Charleston
Montgomer	y 494	405	343	671	661
Atlanta	380	245	214	511	502
Unadilla	501	333	334	603	596
Haines_City	y 830	562	664	847	825
Hattiesburg	635	622	489	855	798;
display i,j,s	1;				

Table s2(k,j) 'distance from cross-dock node k to destination node j'

We	estborough	Kirkwood	Columbus	Fairborn	Eighty_Four
Lexington	891	696	191	148	351
Charlotte	808	646	426	454	422
Knoxville	892	709	357	314	467
Charlottesvill	e 527	413	410	437	293
Charleston	717	533	162	189	202;
display i,k,s2					

parameter c(l)'unit truckload cost for transporting commodity l' /D=1.40, R=1.60, F=1.80/; display l,c;

	Westborough	Kirkwood	Columbus	Fairborn	Eighty_Four
D.Montgomery	9.91	0	0	0	0
<b>R.Montgomery</b>	0	0	237.73	0	0
F.Montgomery	0	23.67	0	0	0
D.Atlanta	1.95	22.56	15	24.97	6.23
R.Atlanta	1.95	0	0	0	0
F.Atlanta	1.95	22.56	15	24.97	0
D.Unadilla	0	0	0	0	0
R.Unadilla	0	0	0	0	0
F.Unadilla	0	27.2	51.47	0	28.26
D.Haines_City	0	14.34	0	0	0
R.Haines_City	12.03	0	0	0	0
F.Haines_City	12.03	0	2	0	0
D.Hattiesburg	0	30	0	30	0
<b>R</b> .Hattiesburg	0	0	0	0	0
F.Hattiesburg	0	0	0	0	0;

Table q(l,i,j)'amount of pallets of commodity l which needs to go from node i to node j'

parameter gamma 'discount factor';

gamma = 0.8;

display l,i,j,q;

parameter h(k)'unit cost of handling a pallet in a cross-dock at location k' /Lexington=3,Charlotte=3,Knoxville=3,Charlottesville=3,Charleston=3/;

parameter u 'truck capacity';

u = 28;

```
parameter p 'number of cross-docks';
```

p =4;

```
parameter w 'cross-dock capacity';
```

w =175;

parameter omega 'bound on overall uncertainty';

set m 'number of uncertain parameters' /1\*3/;
parameter b(i,j,l,m)'weights associated with m random variables';
b(i,j,l,m)=uniform(0,0.3)\*q(l,i,j);
\*Variation in code for medium uncertainty [b(i,j,l,m)=uniform(0,0.6)\*q(l,i,j);]
\*Variation in code for high uncertainty [b(i,j,l,m)=uniform(0,0.9)\*q(l,i,j);]
variables
O 'objective value'
;
positive variables
t2(j,l,k)
t1(i,l,k)
t3(k)
t4(i,j,l)
t5(k);
Binary variables

```
x(i,j,l,k) 'commodity l transported from node i to node j through cross-dock k'
v(i,j,l) 'commodity l transported from node i to node j without using cross-dock k'
z(k) 'cross-dock established at location k'
integer variables
y(i,j,l) 'number of trucks transporting commodity l from node i to node j'
y1(i,k,l) 'number of trucks transporting commodity l from node i to node k'
y2(k,j,l) 'number of trucks transporting commodity l from node k to node j'
equations
total_cost 'objective value'
number_crossdocks 'total number of cross-docks opened is equal to P'
routing(i,j,l) 'demand must be routed through a cross-dock or direct'
truck_capacity1(i,k,l) 'capacity constraint on trucks from origin to cross-dock'
truck_capacity2(j,k,l) 'capacity constraint on trucks from cross-dock to destination'
truck_capacity3(i,j,l) 'capacity constraint on trucks from origin to destination'
crossdock opened(i,j,k,l) 'cross-dock k can only be used if the cross-dock at k is opened'
crossdock_capacity(k) 'capacity constraint on cross-docks'
inequality7(i,l,k) 'helps to solve truck_capacity1'
inequality8(j,l,k) 'helps to solve truck_capacity2'
inequality9(k) 'helps to solve crossdock_capacity'
inequality10(i,j,l) 'helps to solve truck capacity3'
inequality11(k) 'helps to solve objective'
total cost..
sum((i,k,l),c(l)*s1(i,k)*y1(i,k,l))+(gamma*sum((k,j,l),c(l)*s2(k,j)*y2(k,j,l)))+sum((i,j,l),c(l)*s(i,j,l))
y(i,j,l) + sum(k, omega*h(k)*t5(k)) + sum((k), h(k)*sum((i,j,l),x(i,j,l,k)*q(l,i,j))) = e=O;
number_crossdocks.. sum((k), z(k))=e=p;
routing(i,j,l).. sum((k), x(i,j,l,k))+ v(i,j,l)=e=1;
truck_capacity1(i,k,l).. sum((j),q(l,i,j)*x(i,j,l,k))+omega*t1(i,l,k)=l=u*y1(i,k,l);
truck_capacity2(j,k,l).. sum((i),q(l,i,j)*x(i,j,l,k))+omega*t2(j,l,k)=l=u*y2(k,j,l);
truck_capacity3(i,j,l)..q(l,i,j)*v(i,j,l)+omega*t4(i,j,l)=l=u*y(i,j,l);
crossdock_opened(i,j,k,l)..x(i,j,l,k)=l=z(k);
crossdock\_capacity(k)..sum((i,j,l),q(l,i,j)*x(i,j,l,k))+omega*t3(k)=l=w*z(k);
inequality7(i,l,k).. (sum(m,sqr(sum(j,b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t1(i,l,k));
inequality8(j,l,k).. (sum(m,sqr(sum(i,b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t2(j,l,k));
```

```
inequality9(k)..(sum(m,sqr(sum((l,i,j),b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t3(k)); inequality10(i,j,l).. sum(m,sqr(b(i,j,l,m)*v(i,j,l)))=l=sqr(t4(i,j,l)); inequality11(k) .. sum(m,sum((i,j,l),sqr(x(i,j,l,k)*b(i,j,l,m))))=l=sqr(t5(k));
```

```
model test/all/;
option miqcp=cplex;
set level
level/1*31/;
 demandUncert1.ap = 0;
omega=0;
solve test minimizing O using MIQCP;
parameter base_cost;
base_cost=o.l;
parameter relative_cost;
loop(level,
     put demandUncert1;
     Put "Omega, Date, Time, TotalCost, RunningTime, SolverStatus, ModelStatus, Relative
Cost" / ;
     demandUncert1.ap = 1;
     put omega "," system.date "," system.time "," ;
solve test minimizing O using MIQCP;
relative_cost=o.l/base_cost;
      put o.l:0:3 "," test.Resusd:0:3 "," test.solvestat:0:0 "," test.modelstat:0:0 ","relative_cost / ;
put / "k, z" / ;
     loop((k)$(z.l(k)>0), put k.tl:0:0 "," z.l(k):0:0 /);
display z.l, y1.l,y2.l, y.l,x.l,v.l,o.l,omega;
omega=omega+0.1;);
```

#### GAMS Code for Network 1 Using Random Demands to Test Deterministic and Robust Model Performance with Uncertainty with Average Demands (Corresponds to the results in Section 5.3.1 for N1)

\$offlisting
\$offsymxref offsymlist
execseed = 1e8\*(frac(jnow));
file demandUncert1 / demandUncert1.csv /;

\*Network 1 (50, 5D, 5K)

set i 'origin'/Montgomery, Atlanta, Unadilla, Haines\_City, Hattiesburg/; set j 'destination' /Westborough, Kirkwood, Columbus, Fairborn, Eighty\_Four/; set k 'cross-docks'/Lexington, Charlotte, Knoxville, Charlottesville, Charleston/; set l 'commodity'/D, R, F/;

Table s(i,j) 'distance from origin node i to destination node j'									
۲	Westborough	Kirkwood	Columbus	Fairborn	Eighty_Four				
Montgomer	y 1233	1047	660	616	819				
Atlanta	1082	897	570	524	674				
Unadilla	1128	976	694	645	753				
Haines_City	y 1310	1176	1025	982	982				
Hattiesburg	1379	1197	799	755	960;				
display i,j,s	;								

Table s1(i,k) 'distance from origin node i to cross-dock node k'

]	Lexington	Charlotte	Knoxville	Charlottesville	Charleston
Montgomery	494	405	343	671	661
Atlanta	380	245	214	511	502
Unadilla	501	333	334	603	596
Haines_City	830	562	664	847	825
Hattiesburg	635	622	489	855	798;
display i,j,s1	;				

Table s2(k,j) 'distance from cross-dock node k to destination node j'

,	Westborough	Kirkwood	Columbus	Fairborn	Eighty_Four
Lexington	891	696	191	148	351
Charlotte	808	646	426	454	422
Knoxville	892	709	357	314	467
Charlottesv	ille 527	413	410	437	293
Charleston	717	533	162	189	202;
display i,k,s	\$2;				

parameter c(l)'unit truckload cost for transporting commodity l' /D=1.40, R=1.60, F=1.80/; display l,c;

West	tborough	Kirkwood	Columbus	Fairborn	Eighty_	_Four
D.Montgomery	9.91	0		0	0	0
R.Montgomery	0	0		237.73	0	0
F.Montgomery	0	23	3.67	0	0	0
D.Atlanta	1.95	22	2.56	15	24.97	6.23
R.Atlanta	1.95	0		0	0	0
F.Atlanta	1.95	22	2.56	15	24.97	0
D.Unadilla	0	0		0	0	0
R.Unadilla	0	0		0	0	0
F.Unadilla	0	27	7.2	51.47	0	28.26
D.Haines_City	0	14	4.34	0	0	0
R.Haines_City	12.0	3 0		0	0	0
F.Haines_City	12.0	3 0		2	0	0
D.Hattiesburg	0	30	C	0	30	0
R.Hattiesburg	0	0		0	0	0
F.Hattiesburg	0	0		0	0	0;

Table q(l,i,j)'amount of pallets of commodity l which needs to go from node i to node j' Westborough Kirkwood Columbus Eairborn Eighty Four

parameter gamma 'discount factor';

gamma = 0.8;

display l,i,j,q;

parameter h(k)'unit cost of handling a pallet in a cross-dock at location k' /Lexington=3,Charlotte=3,Knoxville=3,Charlottesville=3,Charleston=3/;

parameter u 'truck capacity';

u = 28;

```
parameter p 'number of cross-docks';
```

```
p =4;
```

```
parameter w 'cross-dock capacity';
```

w =175;

parameter omega 'bound on overall uncertainty';

```
set m 'number of uncertain parameters' /1*3/;

parameter b(i,j,l,m)'weights associated with m random variables';

b(i,j,l,m)=uniform(0,0.9)*q(l,i,j);

variables

O 'objective value'

;

positive variables

t2(j,l,k)

t1(i,l,k)

t3(k)

t4(i,j,l)

t5(k);

Binary variables

x(i,j,l,k) 'commodity l transported from node i to node j through cross-dock k'

v(i,j,l) 'commodity l transported from node i to node j without using cross-dock k'
```

z(k) 'cross-dock established at location k'

;

integer variables

y(i,j,l) 'number of trucks transporting commodity l from node i to node j'

y1(i,k,l) 'number of trucks transporting commodity l from node i to node k'

 $y_{2(k,j,l)}$  'number of trucks transporting commodity l from node k to node j'

, equations

total\_cost 'objective value'

number\_crossdocks 'total number of cross-docks opened is equal to P' routing(i,j,l) 'demand must be routed through a cross-dock or direct' truck\_capacity1(i,k,l) 'capacity constraint on trucks from origin to cross-dock' truck\_capacity2(j,k,l) 'capacity constraint on trucks from origin to destination' truck\_capacity3(i,j,l) 'capacity constraint on trucks from origin to destination' crossdock\_opened(i,j,k,l) 'cross-dock k can only be used if the cross-dock at k is opened' crossdock\_capacity(k) 'capacity constraint on cross-docks' inequality7(i,l,k) 'helps to solve truck\_capacity1' inequality8(j,l,k) 'helps to solve truck\_capacity2' inequality9(k) 'helps to solve truck\_capacity3' inequality10(i,j,l) 'helps to solve truck\_capacity3' inequality11(k) 'helps to solve objective'

;

total\_cost..

sum((i,k,l),c(l)\*s1(i,k)\*y1(i,k,l))+(gamma\*sum((k,j,l),c(l)\*s2(k,j)\*y2(k,j,l)))+sum((i,j,l),c(l)\*s(i,j))\*y(i,j,l))+sum(k, omega\*h(k)\*t5(k))+sum((k), h(k)\*sum((i,j,l),x(i,j,l,k)\*q(l,i,j)))=e=O;

number\_crossdocks.. sum((k), z(k))=e=p;

routing(i,j,l).. sum((k), x(i,j,l,k))+ v(i,j,l)=e=1;

 $truck\_capacity1(i,k,l).. sum((j),q(l,i,j)*x(i,j,l,k))+omega*t1(i,l,k)=l=u*y1(i,k,l);$ 

 $truck\_capacity2(j,k,l).. sum((i),q(l,i,j)*x(i,j,l,k))+omega*t2(j,l,k)=l=u*y2(k,j,l);$ 

truck\_capacity3(i,j,l)..q(l,i,j)\*v(i,j,l)+omega\*t4(i,j,l)=l=u\*y(i,j,l);

crossdock\_opened(i,j,k,l)..x(i,j,l,k)=l=z(k);

 $crossdock\_capacity(k)..sum((i,j,l),q(l,i,j)*x(i,j,l,k))+omega*t3(k)=l=w*z(k);$ 

```
inequality7(i,l,k).. (sum(m,sqr(sum(j,b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t1(i,l,k)); inequality8(j,l,k).. (sum(m,sqr(sum(i,b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t2(j,l,k)); inequality9(k)..(sum(m,sqr(sum((l,i,j),b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t3(k)); inequality10(i,j,l).. sum(m,sqr(b(i,j,l,m)*v(i,j,l)))=l=sqr(t4(i,j,l)); inequality11(k) .. sum(m,sum((i,j,l),sqr(x(i,j,l,k)*b(i,j,l,m))))=l=sqr(t5(k));
```

model test/all/;

```
option miqcp=cplex;
set level
level/1*31/;
    demandUncert1.ap = 0;
omega=0;
parameter rand demand(l,i,j);
parameter add_dir_truck(i,j,l);
rand demand(1,i,j)=0;
parameter induce demand;
induce_demand=0;
rand_demand(l,i,j)=0;
parameter base demand(l,i,j);
base_demand(l,i,j)= q(l,i,j);
             parameter simulated cost;
             simulated_cost=0;
             put demandUncert1;
             Put "Date, Time, TotalCost, RunningTime, SolverStatus, ModelStatus, Relative Cost" /;
             demandUncert1.ap = 1;
             put system.date "," system.time "," ;
             solve test minimizing O using MIQCP;
             put o.1:0:3 "," test.Resusd:0:3 "," test.solvestat:0:0 "," test.modelstat:0:0 /;
             put / "k, z" / ;
             loop((k)$(z.l(k)>0), put k.tl:0:0 "," z.l(k):0:0 /);
             display z.l, y1.l,y2.l, y.l,x.l,v.l,o.l,omega;
             parameter base location(k), base assignment,
base_inf(k), base_y1(i,k,l), base_y2(k,j,l), base_y(i,j,l), base_v(i,j,l), base
base_inf1(i,k,l),base_inf2(k,j,l),base_inf3(i,j,l),base_cost;
             base location(k)=z.l(k);
             base_assignment(i,j,l,k)=x.l(i,j,l,k);
             base y_1(i,k,l)=y_1.l(i,k,l);
             base_y2(k,j,l) = y2.l(k,j,l);
             base_y(i,j,l) = y.l(i,j,l);
             base_v(i,j,l) = v.l(i,j,l);
omega=3;
Put "Omega, Date, Time, TotalCost, RunningTime, SolverStatus, ModelStatus "/;
             demandUncert1.ap = 1;
             put omega "," system.date "," system.time "," ;
solve test minimizing O using MIQCP;
             put o.1:0:3 "," test.Resusd:0:3 "," test.solvestat:0:0 "," test.modelstat:0:0 /;
 put / "k, z" / ;
             loop((k)(z.l(k)>0), put k.tl:0:0 ", "z.l(k):0:0 /);
```

parameter robust\_location, robust\_assignment, robust\_inf(k), sum\_robust\_inf, sum\_base\_inf, robust\_y1(i,k,l),robust\_y2(k,j,l),robust\_y(i,j,l),robust\_v(i,j,l),robust\_inf1(i,k,l),robust\_inf2(k,j,l),r obust\_inf3(i,j,l),robust\_cost;

 $\label{eq:constraint} \begin{array}{l} \mbox{robust\_location(k)=z.l(k);} \\ \mbox{robust\_assignment(i,j,l,k)=x.l(i,j,l,k);} \\ \mbox{robust\_y1(i,k,l)=y1.l(i,k,l);} \\ \mbox{robust\_y2(k,j,l)=y2.l(k,j,l);} \\ \mbox{robust\_y(i,j,l)=y.l(i,j,l);} \\ \mbox{robust\_v(i,j,l)=v.l(i,j,l);} \\ \end{array}$ 

loop(level,

rand\_demand(l,i,j)=0; rand\_demand(l,i,j)=sum(m,uniform(0,0.9)\*base\_demand(l,i,j))+base\_demand(l,i,j); q(l,i,j)=rand\_demand(l,i,j);

 $robust\_inf(k)=w*robust\_location(k)-sum((i,j,l),q(l,i,j)* robust\_assignment(i,j,l,k));$   $base\_inf(k)=w*base\_location(k)-sum((i,j,l),q(l,i,j)* base\_assignment(i,j,l,k));$   $robust\_inf1(i,k,l)=u*robust\_y1(i,k,l)-sum((j),q(l,i,j)*robust\_assignment(i,j,l,k));$   $base\_inf1(i,k,l)=u*base\_y1(i,k,l)-sum((i),q(l,i,j)*robust\_assignment(i,j,l,k));$   $robust\_inf2(k,j,l)=u*robust\_y2(k,j,l)-sum((i),q(l,i,j)*robust\_assignment(i,j,l,k));$   $base\_inf2(k,j,l)=u*base\_y2(k,j,l)-sum((i),q(l,i,j)*base\_assignment(i,j,l,k));$   $robust\_inf3(i,j,l)=u*robust\_y(i,j,l)-q(l,i,j)*robust\_v(i,j,l);$  $base\_inf3(i,j,l)=u*base\_y(i,j,l)-q(l,i,j)*base\_v(i,j,l);$ 

```
\label{eq:cost_cost_sum} \begin{split} & \text{robust\_cost=sum}((i,k,l),c(l)*s1(i,k)*\text{robust\_y1}(i,k,l)) + (gamma*sum((k,j,l),c(l)*s2(k,j)*\text{robust\_y2}(k,j,l))) + sum((i,j,l),c(l)*s(i,j)*\text{robust\_y}(i,j,l)) + sum((k), \\ & h(k)*sum((i,j,l),\text{robust\_assignment}(i,j,l,k)*q(l,i,j))); \end{split}
```

```
base_cost=sum((i,k,l),c(l)*s1(i,k)*base_y1(i,k,l))+(gamma*sum((k,j,l),c(l)*s2(k,j)*base_y2(k,j,l
)))+sum((i,j,l),c(l)*s(i,j)*base_y(i,j,l))+sum((k),
h(k)*sum((i,j,l),base_assignment(i,j,l,k)*q(l,i,j)));
sum_robust_inf=sum(k, robust_inf(k))+sum((i,k,l),
robust_inf1(i,k,l))+sum((k,j,l),robust_inf2(k,j,l))+sum((i,j,l),robust_inf3(i,j,l));
sum_base_inf=sum(k, base_inf(k))+sum((i,k,l),
base_inf1(i,k,l))+sum((k,j,l),base_inf2(k,j,l))+sum((i,j,l),base_inf3(i,j,l));
Put "Deterministic ,Base Cost, Robust, Robust Cost"/;
put sum_base_inf "," base_cost "," sum_robust_inf "," robust_cost /;
```

);

#### GAMS Code for Network 1 Using Random Demands to Test Deterministic and Robust Model Performance with Uncertainty with Inflated Demands (Corresponds to the results in Section 5.3.2 for N1)

\$offlisting

\$offsymxref offsymlist execseed = 1e8\*(frac(jnow)); file demandUncert1 / demandUncert1.csv / ; \*Network 1 (5O, 5D, 5K) set i 'origin'/Montgomery, Atlanta, Unadilla, Haines\_City, Hattiesburg/; set j 'destination' /Westborough, Kirkwood, Columbus, Fairborn, Eighty\_Four/; set k 'cross-docks'/Lexington, Charlotte, Knoxville, Charlottesville, Charleston/; set l 'commodity'/D, R, F/;

Table s(i,j) 'distance from origin node i to destination node j'									
Westbo	rough	Kirkwood	Columbus	Fairborn	Eighty_Four				
Montgomery	1233	1047	660	616	819				
Atlanta	1082	897	570	524	674				
Unadilla	1128	976	694	645	753				
Haines_City	1310	1176	1025	982	982				
Hattiesburg	1379	1197	799	755	960;				
display i,j,s;									

Table s1(i,k) 'distance from origin node i to cross-dock node k'

Lexi	ngton	Charlotte	Knoxville	Charlottesville	Charleston
Montgomery	494	405	343	671	661
Atlanta	380	245	214	511	502
Unadilla	501	333	334	603	596
Haines_City	830	562	664	847	825
Hattiesburg	635	622	489	855	798;
display i,j,s1;					

Table s2(k,j) 'distance from cross-dock node k to destination node j'

V	Westborough	Kirkwood	Columbus	Fairborn	Eighty_Four
Lexington	891	696	191	148	351
Charlotte	808	646	426	454	422
Knoxville	892	709	357	314	467
Charlottesv	ille 527	413	410	437	293
Charleston	717	533	162	189	202;
display i,k,	s2;				

parameter c(l)'unit truckload cost for transporting commodity l' /D=1.40, R=1.60, F=1.80/; display l,c;

Table q(l,i,j)'amount of pallets of commodity l which needs to go from node i to node j'

West	borough	Kirkwood	Columbus	Fairborn	Eighty	_Four
D.Montgomery	9.91	0		0	0	0
R.Montgomery	0	0		237.73	0	0
F.Montgomery	0	23	3.67	0	0	0
D.Atlanta	1.95	22	2.56	15	24.97	6.23
R.Atlanta	1.95	0		0	0	0
F.Atlanta	1.95	22	2.56	15	24.97	0
D.Unadilla	0	0		0	0	0
R.Unadilla	0	0		0	0	0
F.Unadilla	0	27	7.2	51.47	0	28.26
D.Haines_City	0	14	1.34	0	0	0
R.Haines_City	12.03	3 0		0	0	0
F.Haines_City	12.03	3 0		2	0	0
D.Hattiesburg	0	30	)	0	30	0
<b>R</b> .Hattiesburg	0	0		0	0	0
F.Hattiesburg	0	0		0	0	0;

parameter gamma 'discount factor';

gamma = 0.8;

display l,i,j,q;

\*parameter f(k) 'fixed cost of establishing a cross-dock at location k'

```
/1=500,2=300,3=450,4=350,5=600,6=650,7=450,8=300,9=400,10=550/;
```

parameter h(k)'unit cost of handling a pallet in a cross-dock at location k' /Fort\_Wayne=3,

Indianapolis=3, Cumberland=3, Canton=3, Cincinnati=3, Columbus=3, Altoona=3,

Pittsburgh=3, Parkersburg=3, Wheeling=3/;

parameter u 'truck capacity';

u = 28;

parameter p 'number of cross-docks';

p =4;

parameter w 'cross-dock capacity';

w =150;

parameter omega 'bound on overall uncertainty';

```
set m 'number of uncertain parameters' /1*3/;
parameter b(i,j,l,m)'weights associated with m random variables';
b(i,j,l,m)=uniform(0,0.9)*q(l,i,j);
variables
O 'objective value'
;
positive variables
t2(j,l,k)
t1(i,l,k)
t3(k)
t4(i,j,l)
t5(k)
;
```

Binary variables

x(i,j,l,k) 'commodity l transported from node i to node j through cross-dock k' v(i,j,l) 'commodity l transported from node i to node j without using cross-dock k' z(k) 'cross-dock established at location k'

;

integer variables

y(i,j,l) 'number of trucks transporting commodity l from node i to node j' y1(i,k,l) 'number of trucks transporting commodity l from node i to node k' y2(k,j,l) 'number of trucks transporting commodity l from node k to node j'

;

equations

total\_cost 'objective value'

number\_crossdocks 'total number of cross-docks opened is equal to P' routing(i,j,l) 'demand must be routed through a cross-dock or direct' truck\_capacity1(i,k,l) 'capacity constraint on trucks from origin to cross-dock' truck\_capacity2(j,k,l) 'capacity constraint on trucks from origin to destination' truck\_capacity3(i,j,l) 'capacity constraint on trucks from origin to destination' crossdock\_opened(i,j,k,l) 'cross-dock k can only be used if the cross-dock at k is opened' crossdock\_capacity(k) 'capacity constraint on cross-docks' inequality7(i,l,k) 'helps to solve truck\_capacity1' inequality8(j,l,k) 'helps to solve truck\_capacity2' inequality9(k) 'helps to solve truck\_capacity3' inequality10(i,j,l) 'helps to solve truck\_capacity3' inequality11(k) 'helps to solve objective'

total\_cost..

sum((i,k,l),c(l)\*s1(i,k)\*y1(i,k,l))+(gamma\*sum((k,j,l),c(l)\*s2(k,j)\*y2(k,j,l)))+sum((i,j,l),c(l)\*s(i,j))\*y(i,j,l))+sum(k, omega\*h(k)\*t5(k))+sum((k), h(k)\*sum((i,j,l),x(i,j,l,k)\*q(l,i,j)))=e=O;

```
number_crossdocks.. sum((k), z(k))=e=p;
```

routing(i,j,l).. sum((k), x(i,j,l,k)) + v(i,j,l) = e = 1;

 $truck\_capacity1(i,k,l).. sum((j),q(l,i,j)*x(i,j,l,k))+omega*t1(i,l,k)=l=u*y1(i,k,l);$ 

truck\_capacity2(j,k,l).. sum((i),q(l,i,j)\*x(i,j,l,k))+omega\*t2(j,l,k)=l=u\*y2(k,j,l);

 $truck\_capacity3(i,j,l)..q(l,i,j)*v(i,j,l)+omega*t4(i,j,l)=l=u*y(i,j,l);$ 

crossdock\_opened(i,j,k,l)..x(i,j,l,k)=l=z(k);

 $crossdock\_capacity(k)..sum((i,j,l),q(l,i,j)*x(i,j,l,k))+omega*t3(k)=l=w*z(k);$ 

```
\label{eq:constraint} \begin{split} & \text{inequality7}(i,l,k)..~(sum(m,sqr(sum(j,b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t1(i,l,k))~;\\ & \text{inequality8}(j,l,k)..~(sum(m,sqr(sum(i,b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t2(j,l,k));\\ & \text{inequality9}(k)..(sum(m,sqr(sum((l,i,j),b(i,j,l,m)*x(i,j,l,k)))))=l=sqr(t3(k))~;\\ & \text{inequality10}(i,j,l)..~sum(m,sqr(b(i,j,l,m)*v(i,j,l)))=l=sqr(t4(i,j,l))~;\\ \end{split}
```

```
inequality11(k) .. sum(m,sum((i,j,l),sqr(x(i,j,l,k)*b(i,j,l,m))))=l=sqr(t5(k));
set iii/1*31/;
model test/all/;
option miqcp=cplex;
alias(iii, level);
    demandUncert3.ap = 0;
parameter rand_demand(l,i,j,iii);
parameter add_dir_truck(i,j,l);
rand_demand(l,i,j,iii)=0;
parameter induce demand;
induce_demand=0;
rand demand(l,i,j,iii)=0;
parameter base_demand(l,i,j);
base demand(1,i,j) = q(1,i,j);
parameter bbb;
 parameter base_location(k), base_assignment,
base_inf(k), base_y1(i,k,l), base_y2(k,j,l), base_y(i,j,l), base_v(i,j,l), base
base_inf1(i,k,l),base_inf2(k,j,l),base_inf3(i,j,l),base_cost;
  parameter robust location, robust assignment, robust inf(k), sum robust inf, sum base inf,
robust_y1(i,k,l),robust_y2(k,j,l),robust_y(i,j,l),robust_v(i,j,l),robust_inf1(i,k,l),robust_inf2(k,j,l),r
obust_inf3(i,j,l),robust_cost;
bbb=0:
        parameter simulated_cost;
set uncertain/1*5/;
rand_demand(l,i,j,iii) = sum(m,uniform(0,0.9)*base_demand(l,i,j)) + base_demand(l,i,j);
loop(uncertain,
       bbb=bbb+0.1;
q(l,i,j)=sum(m,bbb*base\_demand(l,i,j))+base\_demand(l,i,j);
  omega=0;
            simulated_cost=0;
            put demandUncert3:
            Put "bbb,Date, Time, TotalCost, RunningTime, SolverStatus, ModelStatus, Relative Cost"
/;
            demandUncert3.ap = 1;
             put Omega","bbb","system.date "," system.time "," ;
            solve test minimizing O using MIQCP;
             put o.l:0:3 "," test.Resusd:0:3 "," test.solvestat:0:0 "," test.modelstat:0:0 /;
             put / "k, z" / ;
            loop((k)$(z.l(k)>0), put k.tl:0:0 "," z.l(k):0:0 /);
            display z.l, y1.l,y2.l, y.l,x.l,v.l,o.l,omega;
```

base\_location(k)=z.l(k); base\_assignment(i,j,l,k)=x.l(i,j,l,k); base\_y1(i,k,l)=y1.l(i,k,l); base\_y2(k,j,l)= y2.l(k,j,l); base\_y(i,j,l)= y.l(i,j,l); base\_v(i,j,l)= v.l(i,j,l);

loop(level,

\* rand\_demand(l,i,j,iii)=0;

```
q(l,i,j)=rand_demand(l,i,j,level);
```

```
base_inf(k)=w*base_location(k)-sum((i,j,l),q(l,i,j)* base_assignment(i,j,l,k));
```

base\_inf1(i,k,l)=u\*base\_y1(i,k,l)- sum((j),q(l,i,j)\*base\_assignment(i,j,l,k));

base\_inf2(k,j,l)= u\*base\_y2(k,j,l)-sum((i),q(l,i,j)\*base\_assignment(i,j,l,k));

base\_inf3(i,j,l)= u\*base\_y(i,j,l)-q(l,i,j)\*base\_v(i,j,l);

```
base\_cost=sum((i,k,l),c(l)*s1(i,k)*base\_y1(i,k,l))+(gamma*sum((k,j,l),c(l)*s2(k,j)*base\_y2(k,j,l)))+sum((i,j,l),c(l)*s(i,j)*base\_y(i,j,l))+sum((k), h(k)*sum((i,j,l),base\_assignment(i,j,l,k)*q(l,i,j)));
```

```
sum_base_inf=sum(k, base_inf(k))+sum((i,k,l),
base_inf1(i,k,l)+sum((k,j,l),base_inf2(k,j,l))+sum((i,j,l),base_inf3(i,j,l));
     Put "Deterministic ,Base Cost"/;
    put sum_base_inf "," base_cost /;
);
);
set ppp/1*3/;
omega=0;
loop(ppp,
omega=omega+1;
q(l,i,j) = base\_demand(l,i,j);
Put "Omega, bbb, Date, Time, TotalCost, RunningTime, SolverStatus, ModelStatus "/;
     demandUncert3.ap = 1;
     put omega ","bbb"," system.date "," system.time "," ;
option optcr=0.15;
solve test minimizing O using MIQCP;
     put o.1:0:3 "," test.Resusd:0:3 "," test.solvestat:0:0 "," test.modelstat:0:0 /;
put / "k, z" / ;
```

```
loop((k)$(z.l(k)>0), put k.tl:0:0 "," z.l(k):0:0 /);
```

```
\label{eq:construction} robust\_location(k)=z.l(k); \\ robust\_assignment(i,j,l,k)=x.l(i,j,l,k); \\ robust\_y1(i,k,l)=y1.l(i,k,l) ; \\ robust\_y2(k,j,l)=y2.l(k,j,l); \\ robust\_y(i,j,l)=y.l(i,j,l) ; \\ robust\_v(i,j,l)=v.l(i,j,l) ; \\ \end{cases}
```

loop(level,

q(l,i,j)=rand\_demand(l,i,j,level);

```
robust_inf(k)=w*robust_location(k)-sum((i,j,l),q(l,i,j)* robust_assignment(i,j,l,k));
```

robust\_inf1(i,k,l)= u\*robust\_y1(i,k,l)- sum((j),q(l,i,j)\*robust\_assignment(i,j,l,k));

robust\_inf2(k,j,l)=u\*robust\_y2(k,j,l)-sum((i),q(l,i,j)\*robust\_assignment(i,j,l,k));

robust\_inf3(i,j,l)= u\*robust\_y(i,j,l)-q(l,i,j)\*robust\_v(i,j,l);

```
\label{eq:cost_cost_sum} \begin{split} & \text{robust\_cost}=sum((i,k,l),c(l)*s1(i,k)*\text{robust\_y1}(i,k,l)) + (gamma*sum((k,j,l),c(l)*s2(k,j)*\text{robust\_y2}(k,j,l))) + sum((i,j,l),c(l)*s(i,j)*\text{robust\_y}(i,j,l)) + sum((k), \\ & h(k)*sum((i,j,l),\text{robust\_assignment}(i,j,l,k)*q(l,i,j))); \end{split}
```

```
sum_robust_inf=sum(k, robust_inf(k))+sum((i,k,l),
robust_inf1(i,k,l))+sum((k,j,l),robust_inf2(k,j,l))+sum((i,j,l),robust_inf3(i,j,l));
```

```
Put " Robust, Robust Cost"/;
    put sum_robust_inf "," robust_cost /;
););
display rand_demand;
```

## Appendix C

Table 38: Cross-docks O	pened for Network 1	with Medium Uncerta	ainty

Omega	<b>CD's Opened</b>						
	Charlotte		Lexington		Lexington		Lexington
0.0	Knoxville	0.8	Charlotte	1.6	Charlotte	2.4	Charlotte
	Charlottesville	0.8	Knoxville	1.0	Knoxville		Charlottesville
	Charleston		Charleston		Charleston		Charleston
	Lexington		Lexington		Lexington		Lexington
0.1	Charlotte	0.9	Charlotte	1.7	Charlotte	2.5	Charlotte
0.1	Knoxville	0.9	Knoxville	1.7	Knoxville	2.5	Knoxville
	Charleston		Charleston		Charleston		Charlottesville
	Charlotte		Lexington		Charlotte		Lexington
0.2	Knoxville	1.0	Charlotte	1.8	Knoxville	2.6	Charlotte
0.2	Charlottesville	1.0	Knoxville	1.0	Charlottesville		Knoxville
	Charleston		Charleston		Charleston		Charlottesville
	Lexington	1.1	Lexington	1.9	Lexington	2.7	Lexington
0.3	Charlotte		Charlotte		Charlotte		Charlotte
0.5	Knoxville		Knoxville		Knoxville		Knoxville
	Charleston		Charleston		Charleston		Charleston
	Lexington		Lexington	2.0	Lexington	2.8	Lexington
0.4	Charlotte	1.2	Charlotte		Charlotte		Charlotte
0.4	Knoxville		Knoxville		Knoxville		Knoxville
	Charlottesville		Charleston		Charleston		Charlottesville
	Lexington		Lexington		Lexington		Lexington
0.5	Charlotte	1.3	Charlotte	2.1	Charlotte	2.9	Charlotte
0.5	Knoxville	1.5	Knoxville	2.1	Knoxville	2.7	Knoxville
	Charleston		Charlottesville		Charleston		Charleston
	Lexington		Lexington		Lexington		Charlotte
0.6	Charlotte	1.4	Charlotte	2.2	Charlotte	3.0	Knoxville
0.0	Knoxville	1.7	Knoxville	2.2	Knoxville	5.0	Charlottesville
	Charleston		Charleston		Charleston		Charleston
	Lexington		Lexington		Lexington		
0.7	Charlotte	1.5	Charlotte	2.3	Charlotte		
0.7	Knoxville		Knoxville		Knoxville		
	Charleston		Charleston		Charlottesville		

Omega	CD's Opened						
	Charlotte		Lexington		Charlotte		Lexington
0.0	Knoxville	0.8	Charlotte	1.6	Knoxville	2.4	Charlotte
	Charlottesville	0.0	Knoxville	1.0	Charlottesville		Knoxville
	Charleston		Charleston		Charleston		Charleston
	Lexington		Lexington		Lexington		Lexington
0.1	Charlotte	0.9	Charlotte	1.7	Charlotte	2.5	Charlotte
0.1	Knoxville	0.7	Knoxville	1.7	Charlottesville	2.5	Knoxville
	Charlottesville		Charleston		Charleston		Charlottesville
	Lexington		Lexington		Lexington		Lexington
0.2	Charlotte	1.0	Charlotte	1.8	Charlotte	2.6	Charlotte
0.2	Knoxville	1.0	Knoxville		Knoxville	2.0	Knoxville
	Charlottesville		Charlottesville		Charlottesville		Charlottesville
	Lexington		Charlotte	1.9	Lexington	2.7	Lexington
0.3	Charlotte	1.1	Knoxville		Charlotte		Charlotte
0.5	Knoxville		Charlottesville		Knoxville		Knoxville
	Charlottesville		Charleston		Charlottesville		Charlottesville
	Lexington		Lexington	2.0	Lexington	2.8	Lexington
0.4	Charlotte	1.2	Charlotte		Charlotte		Charlotte
0.1	Knoxville	1.2	Knoxville		Knoxville		Knoxville
	Charleston		Charleston		Charlottesville		Charleston
	Lexington		Lexington		Lexington	2.9	Lexington
0.5	Charlotte	1.3	Charlotte	2.1	Charlotte		Charlotte
0.5	Knoxville	1.5	Knoxville	2.1	Knoxville		Knoxville
	Charleston		Charleston		Charleston		Charleston
	Lexington		Lexington		Lexington		Lexington
0.6	Charlotte	1.4	Charlotte	2.2	Charlotte	3.0	Charlotte
0.0	Knoxville	1.1	Knoxville	2.2	Knoxville		Knoxville
	Charlottesville		Charleston		Charlottesville		Charlottesville
	Lexington		Lexington		Lexington		
0.7	Charlotte	1.5	Charlotte	2.3	Charlotte		
0.7	Knoxville		Knoxville		Knoxville		
	Charleston		Charlottesville		Charleston		

 Table 39: Cross-docks Opened for Network 1 with High Uncertainty

Omega	CD's Opened	Omega	CD's Opened	Omega	CD's Opened	Omega	CD's Opened
	Fort_Wayne		Cumberland		Cumberland		Cumberland
0.0	Cincinnati	0.8	Canton	1.6	Altoona	2.4	Cincinnati
0.0	Altoona	0.0	Altoona		Pittsburgh		Altoona
	Pittsburgh		Pittsburgh		Wheeling		Pittsburgh
	Indianapolis		Cumberland		Canton		Indianapolis
0.1	Altoona	0.9	Columbus	1.7	Columbus	2.5	Columbus
0.1	Pittsburgh	0.7	Altoona	1.7	Altoona	2.5	Altoona
	Wheeling		Pittsburgh		Wheeling		Pittsburgh
	Fort_Wayne		Cumberland		Fort_Wayne		Fort_Wayne
0.2	Cumberland	1.0	Canton	1.8	Cumberland	2.6	Cumberland
0.2	Altoona	1.0	Altoona		Altoona	2.0	Canton
	Pittsburgh		Pittsburgh		Pittsburgh		Altoona
	Fort_Wayne		Fort_Wayne		Indianapolis		Cumberland
0.3	Canton	1.1	Cumberland	1.9	Altoona	2.7	Columbus
0.5	Altoona		Altoona		Pittsburgh		Altoona
	Pittsburgh		Pittsburgh		Wheeling		Pittsburgh
	Cumberland		Fort_Wayne	2.0	Fort_Wayne	2.8	Cumberland
0.4	Canton	1.2	Cumberland		Indianapolis		Altoona
0.4	Altoona	1.2	Altoona		Canton		Pittsburgh
	Pittsburgh		Pittsburgh		Altoona		Wheeling
	Cumberland		Fort_Wayne		Columbus		Cumberland
0.5	Columbus	1.3	Cumberland	2.1	Altoona	2.9	Canton
0.5	Altoona	1.5	Altoona	2.1	Pittsburgh	2.9	Columbus
	Wheeling		Pittsburgh		Wheeling		Pittsburgh
	Cumberland		Cumberland		Indianapolis		Canton
0.6	Columbus	1.4	Altoona	2.2	Canton	3.0	Columbus
0.0	Altoona	1.4	Pittsburgh	2.2	Altoona	5.0	Altoona
	Pittsburgh		Wheeling		Pittsburgh		Pittsburgh
	Fort_Wayne		Cumberland		Cumberland		
0.7	Cumberland	1.5	Canton	2.3	Columbus		
0.7	Altoona	1.5	Altoona	2.3	Altoona		
	Pittsburgh		Pittsburgh		Pittsburgh		

 Table 40: Cross-docks Opened for Network 2 with Low Uncertainty

Omega	CD's Opened	Omega	CD's Opened	Omega	CD's Opened	Omega	<b>CD's Opened</b>
	Fort_Wayne		Cumberland		Cumberland		Cumberland
0.0	Altoona	0.8	Canton	1.6	Altoona	2.4	Columbus
	Pittsburgh	0.0	Columbus	1.0	Pittsburgh	2.7	Altoona
	Wheeling		Altoona		Wheeling		Pittsburgh
	Cumberland		Cumberland		Cumberland		Cumberland
0.1	Canton	0.9	Columbus	1.7	Columbus	2.5	Columbus
0.1	Altoona	0.7	Altoona	1.7	Altoona	2.5	Altoona
	Pittsburgh		Pittsburgh		Pittsburgh		Wheeling
	Cumberland		Canton		Indianapolis		Fort_Wayne
0.2	Altoona	1.0	Altoona	1.8	Cumberland	2.6	Cumberland
0.2	Pittsburgh	1.0	Pittsburgh	1.0	Altoona	2.0	Altoona
	Wheeling		Wheeling		Pittsburgh		Pittsburgh
	Cumberland	1.1	Cumberland		Cumberland		Columbus
0.3	Canton		Canton	1.9	Columbus	2.7	Altoona
0.5	Altoona		Columbus		Altoona		Pittsburgh
	Pittsburgh		Pittsburgh		Pittsburgh		Wheeling
	Fort_Wayne		Fort_Wayne	2.0	Cumberland	2.8	Canton
0.4	Cumberland	1.2	Columbus		Columbus		Cincinnati
0.4	Altoona		Altoona		Altoona		Altoona
	Wheeling		Wheeling		Pittsburgh		Wheeling
	Cumberland		Indianapolis		Fort_Wayne		Cumberland
0.5	Canton	1.3	Cumberland	2.1	Cumberland	2.9	Columbus
0.5	Altoona	1.5	Altoona	2.1	Altoona		Altoona
	Pittsburgh		Pittsburgh		Pittsburgh		Wheeling
	Canton		Cumberland		Indianapolis		Fort_Wayne
0.6	Altoona	1.4	Columbus	2.2	Cumberland	3.0	Canton
0.0	Pittsburgh	1.7	Altoona	2.2	Altoona	5.0	Pittsburgh
	Wheeling		Wheeling		Pittsburgh		Wheeling
	Cumberland		Fort_Wayne		Cumberland		
0.7	Altoona	1.5	Altoona	2.3	Altoona		
0.7	Pittsburgh	1.5	Pittsburgh		Pittsburgh		
	Wheeling		Wheeling		Parkersburg		

 Table 41: Cross-docks Opened for Network 2 with High Uncertainty

Omega	<b>CD's Opened</b>	Omega	CD's Opened	Omega	CD's Opened	Omega	<b>CD's Opened</b>
	Fort_Wayne		Cumberland		Fort_Wayne		Cumberland
0.0	Cumberland	0.8	Altoona	1.6	Cumberland	2.4	Lima
	Altoona	0.8	Oil_City	1.0	Altoona		Altoona
	Wheeling		Pittsburgh		Oil_City		Pittsburgh
	Fort_Wayne		Cumberland		Cumberland		Fort_Wayne
0.1	Cumberland	0.9	Lima	1.7	Columbus	2.5	Altoona
0.1	Mansfield	0.7	Altoona	1.7	Oil_City	2.5	Oil_City
	Pittsburgh		Mansfield		Pittsburgh		Pittsburgh
	Hebron		Altoona		Fort_Wayne		Columbus
0.2	Cumberland	1.0	Mansfield	1.8	Cumberland	2.6	Altoona
0.2	Altoona	1.0	Oil_City	1.0	Altoona	2.0	Mansfield
	Oil_City		Pittsburgh		Oil_City		Oil_City
	Canton		Cumberland		Fort_Wayne	2.7	Cumberland
0.3	Lima	1.1	Columbus	1.9	Cumberland		Altoona
0.5	Altoona		Lima		Altoona		Oil_City
	Pittsburgh		Oil_City		Pittsburgh		Wheeling
	Hebron		Cumberland	2.0	Fort_Wayne	2.8	Indianapolis
0.4	Canton	1.2	Canton		Cumberland		Cumberland
0.4	Altoona		Altoona		Canton		Altoona
	Oil_City		Mansfield		Columbus		Pittsburgh
	Fort_Wayne		Fort_Wayne		Hebron		Fort_Wayne
0.5	Altoona	1.3	Altoona	2.1	Cumberland	2.9	Cumberland
0.5	Oil_City	1.5	Mansfield	2.1	Columbus		Oil_City
	Pittsburgh		Pittsburgh		Altoona		Wheeling
	Cumberland		Fort_Wayne		Fort_Wayne		Bloomington
0.6	Canton	1.4	Cumberland	2.2	Cumberland	3.0	Cumberland
0.0	Altoona	1	Altoona	2.2	Altoona		Altoona
	Oil_City		Oil_City		Oil_City		Pittsburgh
	Cumberland		Fort_Wayne		Fort_Wayne		
0.7	Lima	1.5	Cumberland	2.3	Cumberland		
0.7	Altoona	1.5	Canton		Altoona		
	Mansfield		Altoona		Oil_City	]	

 Table 42: Cross-docks Opened for Network 3 for Low Uncertainty

Omega	CD's Opened	Omega	CD's Opened	Omega	CD's Opened	Omega	<b>CD's Opened</b>
	Cumberland		Fort_Wayne		Cumberland		Hebron
0.0	Lima	0.8	Cumberland	1.6	Oil_City	2.4	Cumberland
	Altoona	0.0	Altoona	1.0	Pittsburgh		Columbus
	Mansfield		Oil_City		Wheeling		Altoona
	Cumberland		Fort_Wayne		Fort_Wayne		Fort_Wayne
0.1	Lima	0.9	Cumberland	1.7	Mansfield	2.5	Mansfield
0.1	Altoona	0.7	Oil_City	1.7	Oil_City	2.5	Pittsburgh
	Wheeling		Wheeling		Pittsburgh		Wheeling
	Hebron		Fort_Wayne		Cumberland		Hebron
0.2	Cumberland	1.0	Cumberland	1.8	Lima	2.6	Cumberland
0.2	Altoona	1.0	Mansfield	1.0	Altoona	2.0	Altoona
	Oil_City		Pittsburgh		Mansfield		Oil_City
	Cumberland	1.1	Fort_Wayne	1.9	Cumberland		Cumberland
0.3	Altoona		Cumberland		Lima	2.7	Lima
0.5	Oil_City		Altoona		Oil_City		Altoona
	Pittsburgh		Pittsburgh		Pittsburgh		Pittsburgh
	Cumberland		Cumberland	2.0	Fort_Wayne	2.8	Fort_Wayne
0.4	Altoona	1.2	Lima		Cumberland		Cumberland
0.1	Oil_City		Oil_City		Altoona		Altoona
	Pittsburgh		Pittsburgh Oil_City	Oil_City		Oil_City	
	Fort_Wayne		Fort_Wayne		Canton	2.9	Fort_Wayne
0.5	Canton	1.3	Columbus	2.1	Altoona		Cumberland
0.5	Altoona	1.5	Altoona	2.1	Oil_City		Altoona
	Pittsburgh		Pittsburgh		Pittsburgh		Mansfield
	Cumberland		Fort_Wayne		Cumberland		Cumberland
0.6	Canton	1.4	Cumberland	2.2	Canton	3.0	Canton
0.0	Altoona	1	Altoona	2.2	Altoona	5.0	Altoona
	Mansfield		Oil_City		Pittsburgh		Pittsburgh
	Fort_Wayne		Fort_Wayne		Fort_Wayne		
0.7	Cumberland	1.5	Cumberland	2.3	Cumberland		
0.7	Altoona	1.5	Oil_City	2.3	Altoona		
	Oil_City		Pittsburgh		Oil_City	]	

 Table 43: Cross-docks Opened for Network 3 with Medium Uncertainty

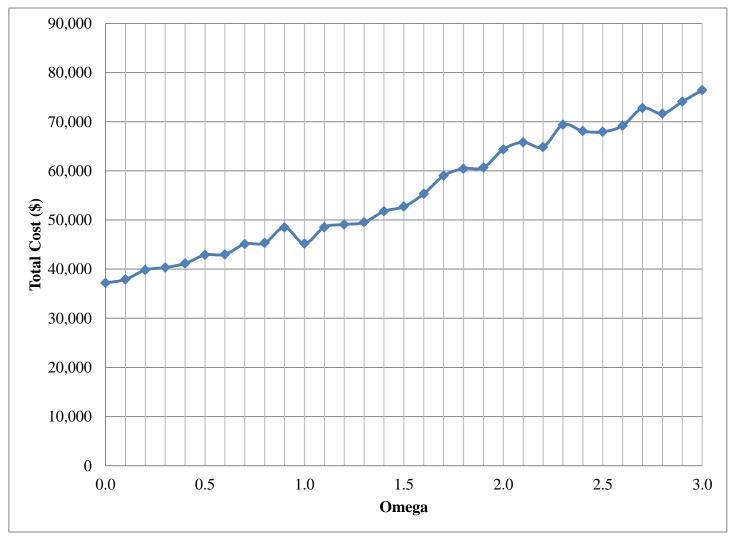


Figure 12: Total Cost for Network 1 with Medium Uncertainty

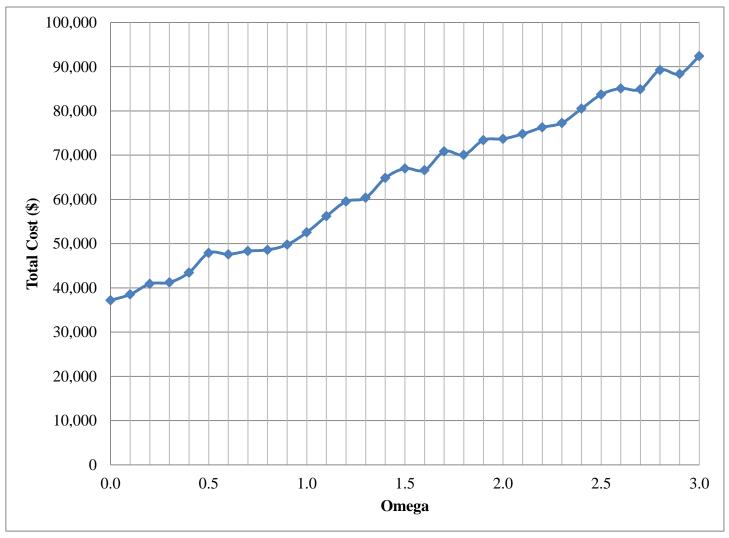


Figure 13: Total Cost for Network 1 with High Uncertainty

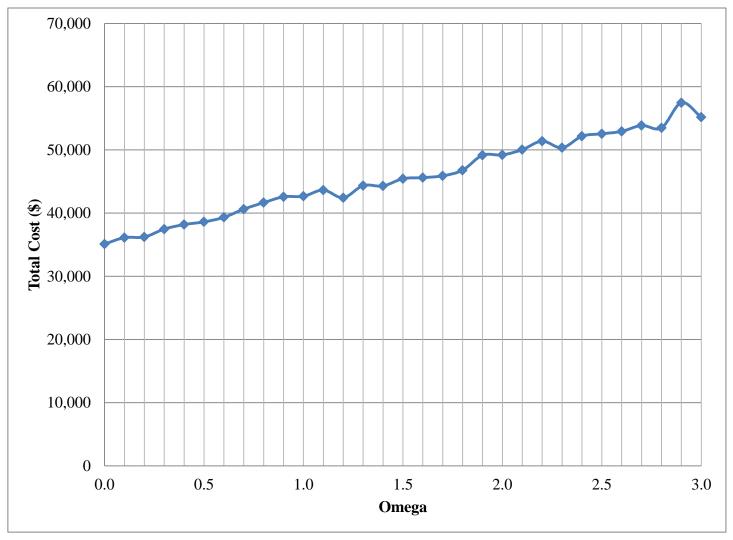


Figure 14: Total Cost for Network 2 with Low Uncertainty

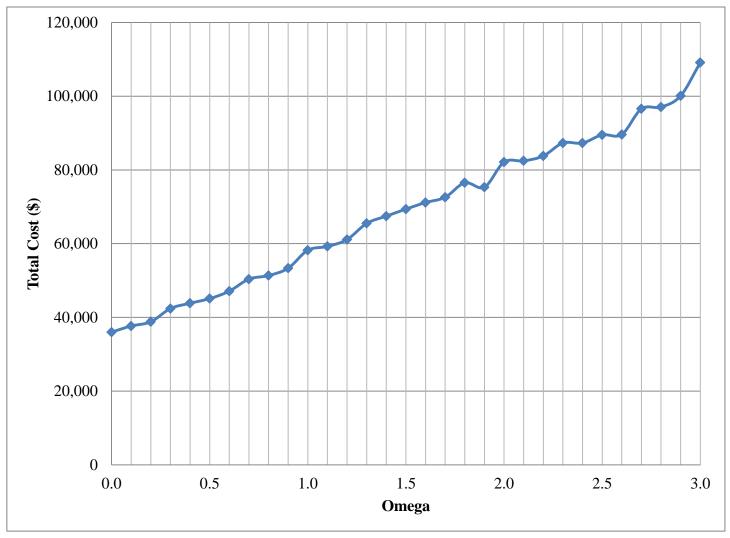


Figure 15: Total Cost for Network 2 with High Uncertainty

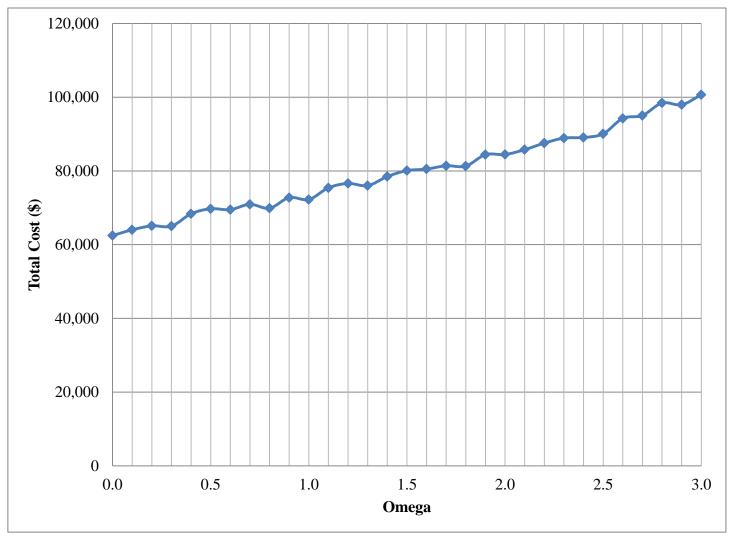


Figure 16: Total Cost for Network 3 with Low Uncertainty

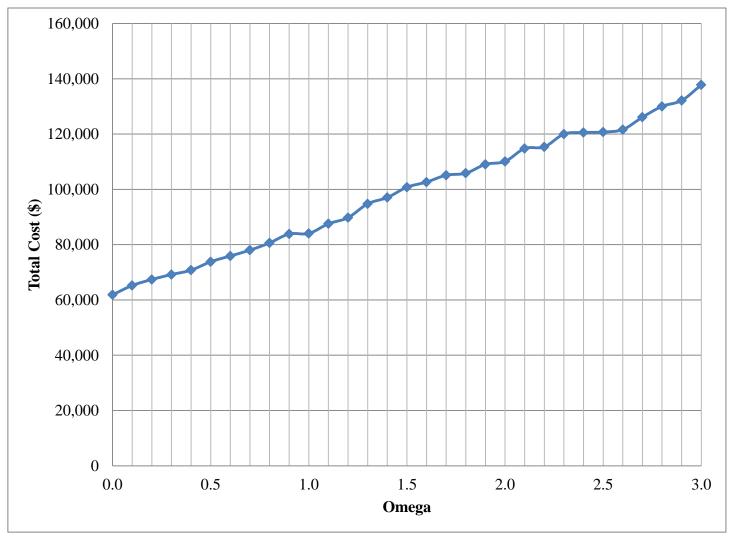


Figure 17: Total Cost for Network 3 with Medium Uncertainty

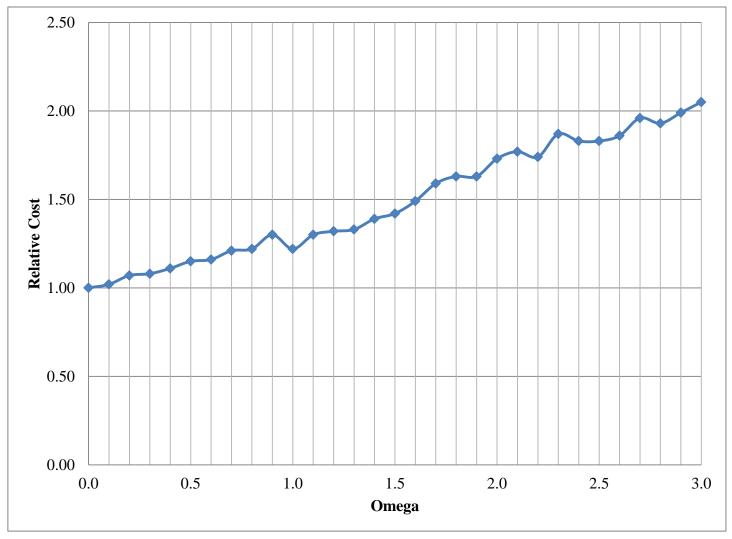


Figure 18: Relative Cost for Network 1 with Medium Uncertainty

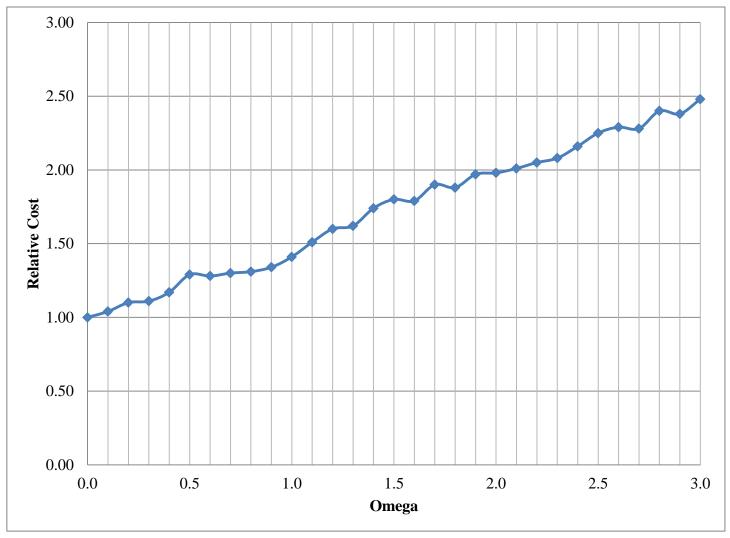


Figure 19: Relative Cost for Network 1 with High Uncertainty

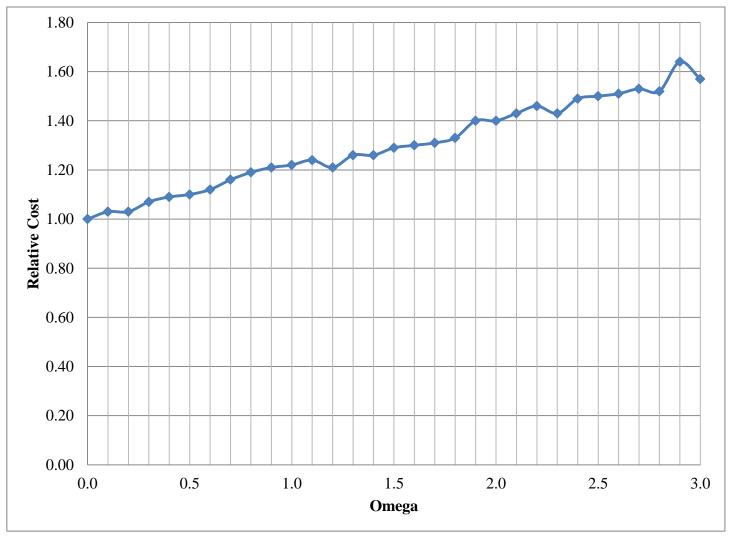


Figure 20: Relative Cost for Network 2 with Low Uncertainty

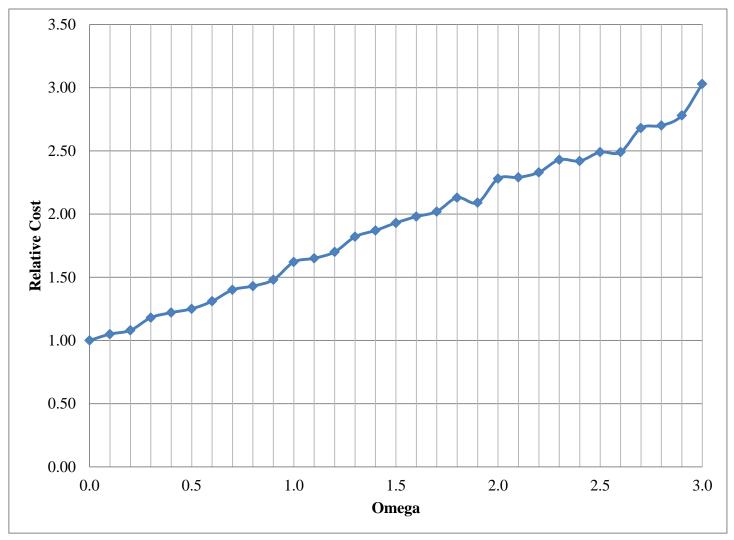


Figure 21: Relative Cost for Network 2 with High Uncertainty

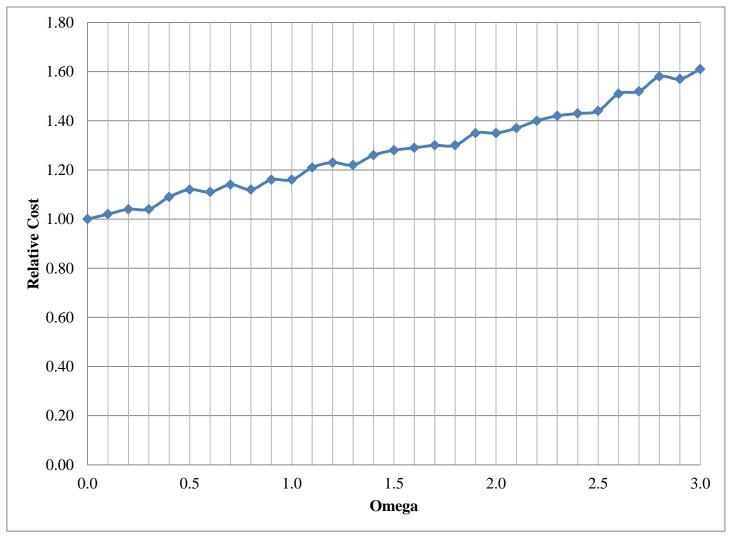


Figure 22: Relative Cost for Network 3 with Low Uncertainty

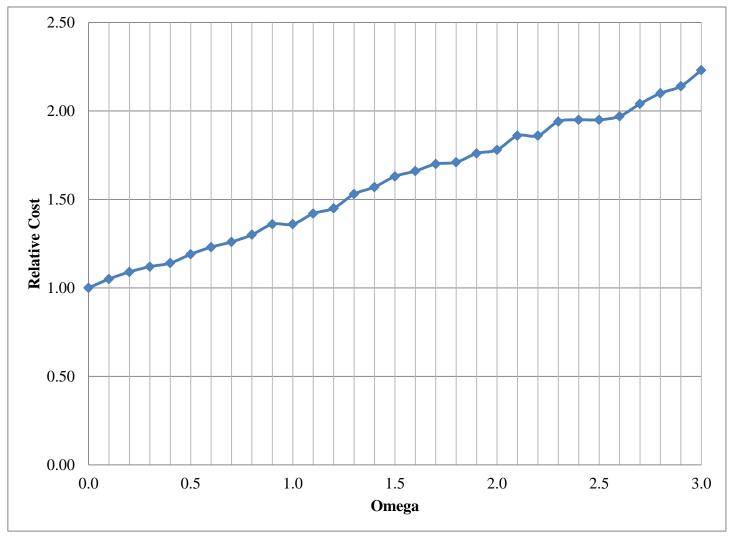


Figure 23: Relative Cost for Network 3 with Medium Uncertainty