2016

Determination of Cost-Effective Range in Surface Finish for Single Pass Turning

Mazin Fahad Alahmadi

Determination of Cost-Effective Range in Surface Finish for Single Pass Turning

Mazin Fahad Alahmadi

**Dissertation submitted
to the College of Engineering and Mineral Resources
at West Virginia University**

in partial fulfillment of the requirements for the degree of

**Doctor of Philosophy in
Industrial Engineering**

**B. Gopalakrishnan, Ph.D., Chair
Leily Farrokhvar, Ph.D.
Thorsten Wuest, Ph.D.
K. Sierros, Ph.D.
Powsiri Klinkhachorn, Ph.D.**

Department of Industrial and Management Systems Engineering

**Morgantown, West Virginia
2016**

Keywords: Machining, Turning, Geometric Programming, Sensitivity Analysis

Abstract

Surface finish is considered a critical characteristic for manufacturing components when manufacturers strive to produce components with high-quality characteristics predefined by design engineers. The objective of this research is to provide a cost-effective range in surface finish for single pass turning that enables the design engineers to explore a wider spectrum of alternative solutions without significantly affecting the functionality of the part. Apart from the one optimal solution, the proposed methodology, which is based on Geometric Programming, would provide a range of cutting conditions solutions that satisfy the economic and functional needs for the designer. This can be achieved by switching cost reduction focus from tooling to labor cost, particularly by adjusting variables values such as spindle speed and feed. An algorithm has been developed to find the new variables values. In addition, a sensitivity analysis model, based on metaheuristic techniques, will also be developed to further give a set of possible solutions that are practically preferable to the practitioners. In addition, the developed methodology can be applied to other engineering applications. The proposed methodology will provide a tool that enhances the design for manufacturability for companies to become more competitive.

Acknowledgement

I would like to express the deepest appreciation to my committee chair Professor Gopalakrishnan, who has the attitude and the substance of a genius as he continually and convincingly conveyed a spirit of adventure in regard to my research, and excitement in regard to teaching. Without his guidance and persistent help, this dissertation would not have been possible.

I also would like to thank my wife and parents who have been supporting me during my stay aboard. Without them, it would not have been possible to have the energy and excitement as to pursue my dreams.

I would like to thank my committee members, professor Farrokhvar, professor Klinkhachorn, professor Wuest, and professor Sierros whose comments and feedback made it possible to improve the value of the work done. Many thanks to them for their dedicated time and efforts.

Table of Contents

List of Tables

List of Figures

Nomenclature

b, c, and *e*	=	Exponents in the machine power constraint
C	=	Constant in Taylor's tool life equation
c_1	=	Coefficient of the first term (labor related) in the objective function
c_2	=	Coefficient of the second term (tool related) in the objective function
c_m	=	Coefficient in the power constraint
c_m'	=	Coefficient in the power constraint for the model in standard form
c_s	=	Coefficient in the surface finish constraint
c_s'	=	Coefficient in the surface finish constraint for the model in standard form
d	=	Depth of cut, inches
D	=	Diameter of the workpiece, inches
D_i	=	The dual variables each corresponding with the ith term in the primal model
f	=	Cutting feed, inches per revolution (*ipr)*
g, h, and *i*	=	Exponents in the surface finish constraint
HP_{max}	=	Maximum machine power available, *hp*
l	=	Length of the workpiece, inches
LC	=	Labor cost, $
L_r	=	Labor cost per minute, $/min
N	=	Spindle speed, revolutions per minute (*rpm*)
n, m, and *p*	=	Exponents in Taylor's tool life equation
Q	=	The dual objective function
SF_{max}	=	Surface finish required, micro-inches
T	=	Tool life, minutes
TC	=	Tool cost, $
T_c	=	Average time to change a tool, minutes
T_{cost}	=	Tool replacement cost, $
Ter	=	Number of terms in the primal model
T_m	=	Time of machining in turning process, minutes
v	=	Cutting Speed, surface feet per minute (*sfpm*)
Var	=	Number of variables in the primal model
μ_i	=	Percentage of cost reduction at iteration *i*
ψ_i	=	Percentage of surface finish increase at interation *i*
ρ_i	=	The ratio of percentage cost reduction to percentage surface finish increase at *i*

Chapter 1: Introduction

1.1 Machining and Metal Cutting

Machining is a manufacturing process which plays an important role in the production of a variety of different products. Machining processes can be differentiated by means of cutting, how the workpiece and cutting tool move with respect to one another. They can be grouped into two main categories, traditional and non-traditional machining processes.

Traditional machining processes include broaching, boring, drilling, facing, filing, grinding, honing, milling, planning, reaming, sawing, shaping, tapping, and turning. Non-traditional manufacturing processes include chemical, electrochemical, electro-discharge, electron beam, laser beam, plasma beam, ultrasonic, and water/abrasive jet machining.

In machining, one particularly important and widely used manufacturing process is metal cutting. The science and technology of metal cutting is of great interest for the aeronautics, aerospace, alternative energy, automotive, biomedical, molds and dies, and other industries. Metal cutting refers to a manufacturing process in which parts are shaped by the removal of unwanted material. Interest in this topic has increased over the last several decades, driven largely by rapid advances in automation and control, computer technology, and materials science [1]. Micro- and nano cutting also have grown in importance, as the manufacture of microscale components has become increasingly important to the development of new products in the modern industry [1].

Reducing the costs of direct manufacturing associated with machining operations is a primary objective of manufacturing plants. Increased use of special alloys with advanced properties, an increasing number of quality requirements for machined parts, cycle time reduction, and an increased use of high-speed and near-dry machining add additional considerations to cost reduction measures. In response, leading tool and machine manufacturers have developed new tool materials and coating, cutting insert and tool designs, tool holders, powerful precision machines, part fixtures, and advanced controllers [1]. These provide a wide spectrum of information about cutting processes and other aspects of machining and increase the efficiency of machining operations in the industry by increasing feed rates, reliability, tool life, and working speeds [1]. Different operations have different ways to reduce costs.

In turning operation, for example, one way to reduce cost is optimization of process parameters. Such optimization problems may be classified into two categories: single-pass and multi-pass. In single-pass operations, the total depth of the cut is removed in one pass. Many studies have considered these types of operations [2-4]. Most turning operations, however, require multiple passes; therefore, multipass operations have also been extensively investigated in the literature [5-9].

The exact definition of the optimization criterion for a particular machining process is critical. Most studies employed one of the following criteria: minimum production cost, minimum production time, a combination of minimum production cost and minimum production time, or maximum profit rate. The first three approaches have received the most attention in the literature. The fourth approach is not widely used, due to the lack of information and several uncertainties, such as variation of source material cost and product unit price during manufacturing.

Many different optimization methods have been applied to determine the optimal machining parameters in turning operations, including the Nelder–Mead simplex method

[2], genetic algorithms [5], simulated annealing [6], a combination of simulated annealing and Hooke–Jeeves pattern search [7], geometric programming [8], and particle swarm optimization [10].

Turning is one of the most widely used processes in manufacturing industries, and has been the preferred choice of most operations research groups for developing and analyzing machining economics models [11]. The following sections will outline the turning process and illustrate the importance of parameter optimization for machining economics.

1.2 The Turning Process

Turning is the removal of an unwanted section from the outer diameter of a rotating cylindrical workpiece, and is used to reduce the diameter to a specified size, and to produce a smooth finish on the part. The machining economics problems related to turning, milling etc. consist of determining the process parameters, usually the cutting speed, feed rate, and depth of cut, to optimize the objective functions subject to machining constraints.

Turning is the most used metal cutting process across the manufacturing industry [12]. The machine tool on which turning is accomplished is a lathe, with which the workpiece is held in a chuck and rotated. The turning tool is held rigidly in the tool post and moved at a constant rate along the axis of a bar, cutting away a layer of metal to form a cylinder or a surface of a more complex profile. **Figure 1.1** illustrates a turning operation in which the tool moves an axial distance f, the feed distance, in one revolution to reduce the bar radius by an amount d, called the depth of cut. **Figure 1.1** also shows the original diameter D of the workpiece being cut, and the angular spindle speed N at which the bar rotates.

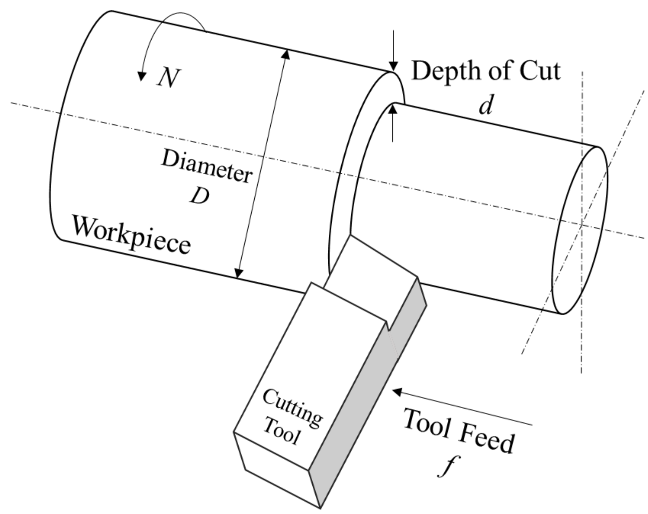

Figure 1.1: *The turning process*

The basic objective of turning process is to generate the desired shapes at minimum cost along with the required quality and delivery time. Achieving this objective is a challenging task, because of the variations in production requirements, tolerances, and materials used either in tools or workpieces. Main factors affecting the turning process can be grouped into the following [13]:

1. Geometries.
2. Cutting conditions.

Geometries are related to both the geometry of the part being produced and the geometry of the cutting tool. The geometry of the part is perhaps an essential factor that affects the design of the process as well as the economics of the machining. Many considerations are required as to the size of the component, complexity of the shape, dimensions, tolerances, interaction with other parts, surface finish requirements. The process engineer must decide whether minor changes in part geometry are appropriate to increase the ease of manufacturing. In the scope of this research, the part dimensions are to be incorporated in the sensitivity of the model. Also, tool geometry is an area of major significance and has many aspects. The primary objective here is optimizing the geometry of the cutting tool for the tool material used in the turning process. Separate studies are dedicated to the optimization of the cutting tools [14-16] and will be excluded in this paper.

Cutting conditions consider the selection of speed, feed, and depth of cut. This selection determines, to a large extent, the economic success of the operation. It is one of the tasks in process planning. Firstly, depth of cut is regularly predetermined by workpiece geometry and operation sequence and is to be considered constant in our model, which will be described in Chapter 3. It is regulated, however, by the available horsepower, machine tool, strength of the cutting tool, and so on. Therefore, the remaining variables to be optimized are feed and speed. Secondly, feed rate selection mainly depends on many factors including tool type and hardness, metal removal rate, and surface finish requirements. For example, high-speed steel (HSS) can tolerate higher feed rates as opposed to cemented carbides or ceramics tools. It is known that, generally, feed rate has the highest influence on surface finish followed by speed and depth of cut [17]. This result can be validated for the turning process, through sensitivity analysis. Finally, cutting speed plays a major role in tool life. It is required to achieve high metal removal rate as to speed up the process but yet maintains longer tool life. Thus, the ability to predict tool life is vitally important in tool management. Mathematical models were built to achieve a balance between these two contradictory factors. Most of these models are based on the well-known Taylor tool life equation, who machined approximately 30,000 tons of work material to establish tool life data. Taylor's tool life equation [18] was initially influenced by cutting speed:

$$vT^{n} = C \tag{1.1}$$

where v is the cutting speed, T is the tool life, C is a constant, and n is an exponent exhibits the sensitivity of the specific tool life to changes in speed. An expanded version of Taylor's equation [18] evolved with the changes in work material and tools which incorporate, in addition to speed, feed, and depth of cut:

$$vT^{n}f^{m}d^{p} = C \tag{1.2}$$

The exponents m and p are to be determined experimentally for each combination of the cutting conditions, although typical values for certain tools are readily available. In a general analysis provided by De Garmo et.al [19], the cutting speed has a higher influence on tool life compared to feed and depth of cut. It is found that 50% increase in speed, feed, and depth of cut results in a 90%, 60%, and 15% decrease in tool life correspondingly. A key player in this is the horsepower consumed. For example, if power is limited, depth of cut then feed should be maximized while holding the speed constant.

Determining the optimal process parameters is an essential part of planning machining processes since the process parameters have a significant effect on the cost, productivity, and quality of machining operations. Previous studies involving machining parameter optimization of turning operations concentrated on single-tool operations, where the process is performed by means of one cutting edge. It was shown that there exists an optimum speed in single pass turning operations, where the required cutting is achieved in one pass as oppose to several roughing cuts and a finishing cut. It was found that increasing the cutting speed would reduce the actual (traverse) cutting time and cost in machining a component, but that the production interruption time and cost due to tool failure and replacement would increase. A compromise between the two produces an optimum speed for which an overall minimum time or cost per component would need to be selected. Investigators have also realized that both speed and feed must be optimized according to the desired criterion while satisfying practical constraints, such as the machine tool available power, speeds, and feeds. Numerous mathematical optimization analyses and strategies for the selection of cutting speed and feed have been reported in the literature, with most depending on knowledge of predictive (empirical) equations for machining performance characteristics such as tool life, force, and power. In addition, optimization strategies incorporating practical constraints require significant knowledge of the machine tool specifications and capabilities. Thus, solving the more general problem of selecting optimal machining conditions requires more sophisticated techniques. The mathematical models are inherently nonlinear, constrained by the available speeds, feeds, horse-power, surface quality, tool life, and other factors. Complex relations occur between process parameters and the constraints in turning process, and in machining in general. Taking all such constraints into account, especially when they are non-linear, further complicates the matter.

1.3 Need for Research

Most of the recent attention in machining economics is focused on finding the optimal cutting parameters [20-32], as will be shown in section 2.1.2. Although operating with optimal cutting conditions plays an important role in reducing machining issues such as tool wear, other economic and managerial aspects are also important when selecting cutting conditions. Moreover, the manufacturing engineer is limited to the theoretical solution provided by optimization models. Sometimes a specific theoretical optimal solution is no longer practically applicable and, hence, more alternative solutions must be obtained using post-optimal analysis.

Depending on work and tool material and turning environment, cutting conditions affect surface finish significantly [33-35]. Also, One of the principles of design for manufacturability (DFM) is to specify “acceptable” surface finish for functionality [36]. Thus, assuming a range of surface finish requirement has been declared acceptable for the specification of the turned part, we are interested in determining what cutting conditions would provide the most reduction in cost within the defined surface finish range. It is also of interest that a set of possible solutions is provided according to a required reduction in unit cost by changing input data, such as operating costs, tools costs, and constraints boundaries. For example, what solutions are possible that would give a 1.5% required reduction in cost, in compromise to specific parameter changes. The user, in this case, pre-specifies what input parameters to be changed. Finally, an interesting result would be to find out what small changes in process parameters would provide the most significant change in production unit cost. For instance, the model suggests that small changes in design with a slight variation in the surface finish would give the most significant changes in unit cost. These changes, again, are defined by the user depending on the specific application. The developments of such scenarios would help the manufacturing engineer to obtain practical solutions easily.

1.4 Research Objectives

In this research, a mathematical model will be developed for a single-pass turning process with surface finish and power constraints. The model will provide alternatives to achieve certain economic goals. The variables to be optimized will include the speed and feed rate. The objectives can be stated as:

(1) Design and develop a model that incorporates cost-effective surface finish range constraints,

(2) Develop a solution procedure for model, and

(3) Conduct sensitivity analysis for key variables and parameters used in the model.

1.5 System Diagram

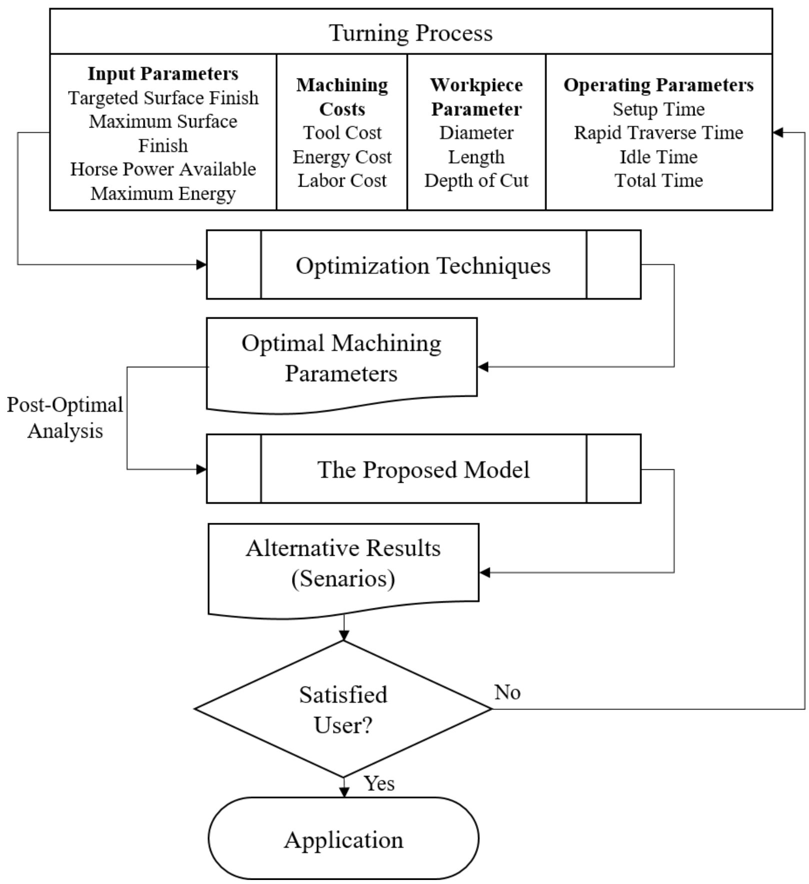

Figure 1.2: *System diagram*

Figure 1.2 provides a schematic view of the optimization process. The process begins with the selection of input parameters. These parameters can, of course, be changed after the first full cycle of the process to obtain different solutions. These parameters are altered arbitrarily at first. After a significant improvement to the optimization process has been realized, one can negotiate or improve the real system to obtain the recommended value of that parameter. Second, the model should provide the optimal solution for the inputted parameters. This optimal solution provides a starting point for sensitivity analysis. Third, comes the post-optimal model, in which more suitable solutions are obtained analytically based on some user criteria. These new solutions may violate one or more constraints, yet still be preferable to the user for economic reasons. Finally, if the user is satisfied with the recommended solutions, they can be applied; for example, allowing the surface finish to go off target. These solutions provide economically better results, regardless of the quality characteristic being violated.

1.6 Conclusion

One of the major goals of industrial production is to reduce manufacturing costs without tolerating the quality of the components. Consequently, sophisticated modern machine tools require an optimization procedure to determine optimal operating parameters such as cutting speed, feed rate, and depth of cut. The problem of selecting these optimal parameters in turning is a substantial problem and thus has been analyzed with varying degrees of generality by many investigators. Moreover, further analysis beyond just obtaining a theoretically optimal solution must be considered, even though a certain quality characteristic being tolerated.

Chapter 2: Literature Review

2.1 Literature Survey

The main objective in machining is to produce high-quality products while minimizing production costs. Cost consciousness with respect to the metal cutting process is an essential element in efficient manufacturing, and thus it is essential to analyze the metal cutting operations in the context of economic conditions. Due to the high capital cost and machining cost of CNC machines, there is a clear economic need to operate machines as efficiently as possible. The success or failure of a machining operation thus heavily depends on the selection of machining parameters such as cutting speed, feed, and depth of cut. A process planner selects the machining parameters based on experience and based on available handbooks, but these parameters do not necessarily yield optimal values or minimize production costs. Further, theoretically optimal solutions obtained by optimization tools are not always practically applicable.

2.1.1 The Early Literature

The single-pass turning operation has been thoroughly investigated, and several optimization techniques have been developed for it. The classical approach [37, 38], the probabilistic approach [4, 39], the adaptive approach [40], the Monte Carlo Simulation Technique [41] and others [42], [43] are some of the commonly used basic techniques. Mukherjee and Ray [44] classified the optimization of machining mainly into conventional and non-conventional techniques. **Figure 2.1** shows the structure of this classification.

Conventional methods of determining optimal machining parameters require the use of a large number of mathematical formulas that have been developed from experimental data.

However, these fail to account for the systematic and random errors associated with any set of experiments. Since optimization is a decision-making process, the results so obtained should serve to achieve the user's objectives. Some notable examples from the literature are given below.

Gilbert [38] previously used an analytical procedure for determining cutting speed in a way that minimizes machine cost for a single-pass turning operation. Armarego and Russel [45] later used the calculus method to solve this optimization problem, whereas Bhattacharyya et al. [46] and Brewer [47] used the Lagrange method. Ermer [48] illustrated a geometric programming technique that can be used to determine the optimum machining condition by considering cutting velocity and feed as variables, reducing cost as the objective, and surface finish and feed as constraints. Gopalakrishnan et al. [49] then improved upon this work by developing an analytical approach based on geometric programming. Iwata et al. [4] presented a dynamic programming model for the simultaneous determination of the optimal value of cutting speed, feed and depth of cut for an individual pass, and also determined the optimum number of passes. Rao and Hati [50] used computerized methods in the selection of optimized machining parameters for a job requiring multiple operations. Wang et al. [51] used a deterministic approach to maximize production rate.

Wang and Liu [52] used geometric programming principle to develop a solution method capable of deriving the interval unit production cost with interval parameters. A pair of two-level machining problems is formulated to calculate the upper and lower bounds of the unit production cost. The results indicated that the cost interval contains more information relevant to the decision-making process. Agapiou [2] has investigated the optimization problem for a multi-stage machining system. This work proposed the Nelder-Mead simplex (NMS) method for optimization. The author used the idle time to the full extent at all machining stations, with the intention of improving tool life and thereby achieving the desired cost reduction. Later, the author developed a combined objective of cost and time using the weighted coefficient method. Lambert and Walvekar [53] also developed a dynamic programming model for the multipass turning operation based on the constraints of force, cutting power, and surface finish. Geometric programming has been used to determine the values of machining variables with the objective of minimizing production costs in two-pass turning examples only. Subsequently, Yellowley and Gun [54] have shown that the optimal subdivision of depth of cut for both turning and milling operations may be determined without knowledge of the relevant tool life equation. A calculation of machining parameters in a turning operation using machining theory was carried out by Meng et al. [55], with an objective criterion of minimizing cost. Prased et al. [56] used a combination of geometric and linear programming techniques to solve the multipass turning optimization problem as part of a PC-based generative CAPP system. Multipass turning optimization with the optimal subdivision of depth of cut was developed by Gupta et al. [57]. Tan and Crease [58] implemented linear programming with branch and bound to explore the optimization of machining parameters in multipass operations. Also, the goal programming method has been used by many researchers [59] to solve similar problems. Similar problems have also been solved using several other methods, namely, dynamic programming [60], mathematical programming [61], and sequential quadratic programming [62]

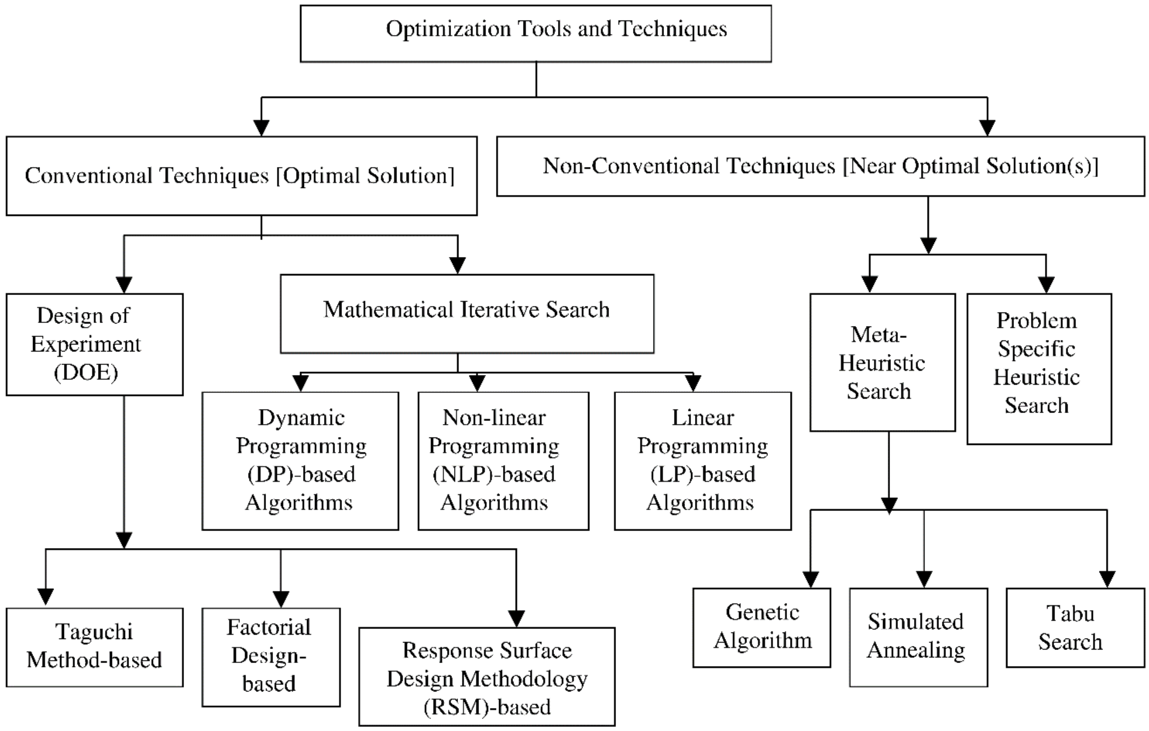

Figure 2.1: *Optimization tools/techniques used for machining economics problems* [44]

Non-conventional, meta-heuristic, search-based techniques, which are sufficiently general and extensively used by modern researchers are based on genetic algorithm (GA), tabu search (TS), and simulated annealing (SA). Some examples of these techniques are given below.

Chen and Tsai [6] developed an optimization model for a continuous profile using the SA approach. This model simultaneously considers straight turning, taper turning and circular turning. Bhaskara, Reddy, et al. [63] used the GA to select the optimal depth of cut which minimizes production cost in multipass turning operations. Onwubolu and Kumalo [9] implemented GA to determine the optimal values of cutting variables in multipass machining operations, but they did not consider the depth of cut constraint. Wong and Hamouda [64] presented GA and a fuzzy expert system for use in developing a design for metal cutting data selection. In addition to traditional parameters, they also used tool material, tool shape, cutting fluid, and characteristics of the machine tool as major independent variables. Saravanan et al. [65] used GA and SA separately and compared the values for turning cylindrical stock into a continuous finished profile. The machining variables were determined by minimizing the unit production cost, and subject to a number of practical constraints. Vijayakumar et al. [66] developed a model based on the ant colony algorithm for a multipass turning operation, which remains one of the non-traditional optimization techniques that researchers believe could give optimal global solutions. Cus and Balic [67] used GA to reduce the production cost and time by implementing a new methodology for continuous improvement of the cutting condition with GA. Ping et al. [68] used the particle swarm optimization (PSO) technique to find the optimal choice for machining parameters. The constriction factor, velocity constraint, and population size were all found to impact the performance of PSO significantly. Increasing the population size can improve the solution quality, but may also increase the required computational time. Sardinas et al. [69] also used GA for a multi-objective optimization problem. The two conflicting objectives are to increase tool life and decrease operation time. Ruy Mesquita [70] used the Hook–Jeeves search method for finding the optimum operating parameters. Chen et al. [71] developed an optimization model for machining a continuous profile from bar stock using an SA approach. A direct search procedure was used by Arsecularatne et al. [72] to determine the optimum cutting parameters for right- and left-hand turning, boring, facing, and threading. Most researchers in the area of machining have used various techniques to find the optimal machining parameters for single- and multi-pass turning operations. Saravanan et al. [73] attempted to utilize various non-traditional techniques to optimize the machining parameters in turning operations. Natarajan et al. [74, 75] suggested using the PSO method and GA to predict tool life and optimize cutting parameters. The literature supports the fact that the optimal selection of cutting speed, feed rate, depth of cut, and the number of passes is important in machining operations because of their significant influence on machining quality and machining economics.

Other techniques based on empirical input-output and in-process optimization have also been reported in the literature. One example of such a technique was reported by Zuperl and Cus [76], who used a neural network to optimize cutting conditions, with an objective function of increasing productivity and reducing cost.

2.1.2 The Most Recent Literature

In the next section, the most recent models and optimization solutions will be reviewed for single pass turning and multiple-pass turning operations. A summary table will be presented at the end showing what method has been used for the specific study.

Devaki et al. have used RSM to optimize the process parameters for a straight turning [23]. Surface roughness was considered as a quality measure and material removal rate (MRR) as a productivity measure. The process parameters considered were spindle speed, feed, depth of cut, and type of coolant. Design of experiments (DOE) was used to set up and conduct the experiment and analysis of variance (ANOVA) was used to check the adequacy of the linear order model suggested. It was found that feed and depth of cut have less effect on MRR while speed has a more significant impact on MRR. Moreover, feed has more significant effect on surface roughness. Also, cutting speed has more significant effect on tool life. The optimal setting of machining parameters was also reported after using RSM.

Çolak employed a hybrid model based on genetic algorithm and used multi-regression to determine the optimum cutting parameters for multi-objective single-pass turning [22]. The performance measures considered were surface roughness, MRR, and cutting power. Taguchi designs were utilized to conduct the experiment. It was mainly concluded that tool life is remarkably different in each cooling condition specified, namely conventional and high-pressure cooling. Also, it was observed that cutting conditions did not change significantly under the cooling conditions specified.

Raja et al. implemented a metaheuristic algorithm called firefly algorithm (FA) to select the optimum process parameters while minimizing production time and production cost [29]. The researchers implemented the method and compared its performance with well-known algorithms like particle swarm optimization (PSO), genetic algorithm, and Nelder-Mead Simplex (NMS), and three other methods. Out of the seven methods investigated, FA was ranked 5th among the other methods, whereas PSO was ranked 1st.

Senthilkumaar et al. have coupled GA with ANN for the optimization of Single-pass finish turning [30]. Data were collected from experiments conducted based on design of experiments. Process parameters were cutting speed, feed, and depth of cut and the responses were flank wear and surface roughness. It was concluded that all main factors and their interactions are not statistically significant to predict the surface roughness whereas they are statistically significant to predict flank wear. Confirmation experiments were conducted for the optimal machining parameters, and the results agreed with the model prediction.

Jain et al. have used the conventional Taguchi method to optimize the MRR for single pass turning [27]. Three levels were specified for each process parameter, namely speed, feed, and depth of cut. It was found that spindle speed and feed rate are the only significant factors affecting MRR for the identified experiment.

Carmita has utilized RSM and developed regression models to optimize a multi-objective rough turning [21]. Energy consumption and surface roughness were minimized, while the material removal rate of the process was maximized. It was found that feed rate and depth of cut were the most significant factors for minimizing the total specific energy consumed, and for minimizing the surface roughness, feed rate was the most significant factor. It was also found that the optimal turning parameters suggested by the proposed optimization model can reduce the energy consumption by around 14%, and the surface roughness by around 360%.

Durairaj and Gowri have applied GA to optimize the process parameter of micro-turning Inconel 600 alloy with titanium carbide coated tool [24]. Full factorial experiments were conducted, and a non-linear regression model was developed. The objectives considered were conflicting; surface roughness and tool wear. The optimal settings were reported for the specified case. It was concluded that best surface finish could be obtained with low cutting speed, low feed rate and low depth of cut.

Yildiz has applied a relatively new optimization algorithm called teaching–learning-based optimization (TLBO) and coupled it with Taguchi's method [31]. TLBO is a teaching–learning inspired process algorithm to solve nonlinear optimization problems proposed by Rao et al. [77], which is a population-based method. TLBO algorithm imitates the influence of a teacher on the output of learners in class. Yildiz has considered multiple passes with the objective of minimizing production cost. It was concluded that the proposed model performed quite well and that it provided better solutions compared to other approaches.

Aryanfar and Solimanpur have used GA to simultaneously optimize the multi-pass roughing and single-pass finishing parameters [20]. In addition to traditional process parameters, the number of roughing cuts is also considered in multi-pass turning optimization. It was concluded that the proposed GA model overcomes other conventional and non-conventional methods proposed in the literature.

Jabri et al. have also considered GA model to minimize the cutting cost while maximizing tool life [26]. The model was built to consider multi-pass turning. It was mainly concluded that cutting cost could be minimized by selecting large values for cutting speed and feed, but small values should be selected for both speed and feed to maximize tool life. The results obtained from the GA model were plotted in a Pareto frontier graph to help in the decision-making process. Similar model and methodology were also followed by Ganesan and Mohankumar [25]. Pareto frontier graphs were plotted for unit production time and cost, tool wear and unit cost, and tool wear and unit production cost. The optimal values were reported for the case provided.

Lu et al. have used a hybrid genetic algorithm and sequential quadratic programming technique to minimize the production cost [28]. They, however, added a second phase to the problem where the optimal cutting sequence is found using dynamic programming. The cutting sequence for multi-pass turning "has not gained much attention in many previous studies," they stated. It was shown that the sequence of the cuts does affect the optimization process.

Yildiz has employed a hybrid optimization approach based on artificial bee colony algorithm and Taguchi method to optimize multi-pass turning [32]. As Yildiz mentioned, artificial bee colony algorithm is an optimization algorithm which is based on the intelligent foraging behavior of honey bee swarm. The proposed method was tested and compared to previous work. The results showed that the proposed method is highly competitive to previously published methods for multi-pass turning.

Table 2.1: *Summary table of recent academic models and solutions in turning.*

Researchers	Year	Passes	Objective	Method/Model Used
Devaki et al.	2015	Single	Multiple	Response Surface Methodology
Çolak	2014	Single	Multiple	Hybrid genetic algorithm
Raja et al.	2012	Single	Multiple	Firefly algorithm
Senth. et al.	2012	Single	Multiple	Genetic algorithm and artificial neural network
Jain et al.	2015	Single	Single	Taguchi
Carmita	2015	Single	Multiple	Response Surface Methodology
Durairaj et al.	2013	Single	Multiple	Genetic algorithm
Yildiz	2013	Multiple	Single	Teaching–learning–based optimization and Taguchi
Aryanfar et al.	2012	Multiple	Single	Genetic algorithm
Jabri et al.	2013	Multiple	Multiple	Genetic algorithm
Ganesan et al.	2013	Multiple	Multiple	Genetic algorithm
Lu et al.	2013	Single	Multiple	Genetic algorithm and sequential quadratic programming
Yildiz	2013	Multiple	Single	Hybrid artificial bee colony algorithm and Taguchi

We can see from Table 2.1 that recently non-conventional methods and hybrid models have relatively gained popularity over other optimization methods. This is because researchers have recently been considering more complex aspects of the optimization process for turning. For example, almost none of the reviewed most recent literature has considered the very basic problem, a single pass with one objective. Researchers have considered, multiple passes, cutting sequence, MRR, energy consumed, among other variables and objectives. In such situations, non-conventional optimization methods surpass other methods [78]. Most of the early researchers, however, have used traditional optimization techniques for solving machining problems. These techniques, again, are not efficient when the practical search space is large [79]. Numerous constraints and the number of passes complicate the machining optimization problem. Traditional techniques such as geometric programming, dynamic programming, and branch and bound techniques have difficulty solving such problems and are inclined to obtain only locally optimal solutions. Despite these drawbacks, this research attempts to use traditional optimization techniques, specifically geometric programming, to further develop the post-optimality analysis. Major reasons include:

(1) The core idea of the research was based on the dual of a geometric programming model.
(2) The ability of traditional methods to give exact solutions.
(3) The ease of obtaining analytical solutions for relatively small size problems.
(4) The use of analytical solutions in post-optimality analysis instead of problem resolution.

2.2 Geometric Programming

It is now four decades since the initial development of geometric programming (GP) by Duffin, Peterson, and Zener [80]. It has proved to be valuable for a variety of disciplines, engineering design, transportation, management science, planning, and reliability, among other. The text of Beightler and Phillips [81] gives a broad selection of applications.

Geometric programming can be defined as a methodology for solving nonlinear algebraic optimization problems. It can be seen as a subset of nonlinear programming. It can also be seen as a broader method than nonlinear programming as it has been shown by Beightler and Phillips that nonlinear programs may be transformed to geometric programs using simple algebra.

Geometric programming has many powerful properties that make it useful. The first, and possibly the most useful, the property that a program can be transformed into an equivalent program, called the dual, which has linear constraints. This transformation makes it much easier to solve the problem. The second property is that in a special case the solution to the dual program is independent of the coefficients used in the problem. This special case is when we have zero degrees of difficulty, which is the number of terms minus the number of variables minus one, as defined by Duffin et al. This will be further illustrated in this report. Finally, many engineering design problems, which has cost and constraints that are power functions of the variables, can be modeled using geometric programming.

Many algorithms were developed for GP in the first two decades after its inception, surveys of which may be found in Dembo [82], Sarma et al. [83] and Rijckaert and Martens [84]. While modern development has slowed, recently several new techniques based on interior point methods have been presented by, to name only a few, Bricker and Yang [85]; Kortanek and No [86]; and Kortanek, Xu, and Ye [87]. These solution techniques may be categorized as either primal-based algorithms that directly solve the nonlinear primal problem, or dual-based algorithms that solve the equivalent linearly constrained dual. While the dual is intuitively more attractive due to its relative structural simplicity, it also presents serious computational problems. These are particularly problematic when slack primal constraints are present at the optimum. Also, difficulties arise when the objective function becomes non-differentiable when some dual variables become zero at an optimal solution. Also, recovering the primal variables values in these cases requires a more complicated sub-problem, called the subsidiary problem. These issues have caused some researchers to abandon the linear structure and address the primal directly. However, in our research, we will follow the dual-based procedures for the benefits explained.

2.3 Conclusion

Several researchers have investigated the optimization of machining economics models. The machining economics models can be divided into conventional and non-conventional models based on the nature of the solution procedure. Various structure parameters may be included in these models, including optimization criteria, unconstrained or constrained models, deterministic or probabilistic tool life models, and solution techniques.

One must also note the importance of the number of cutting variables and the form of the tool life equation in model formulation. Various machining economics models have been developed based on the optimization criteria, tool life equations, cutting variables, and constraints. In general, the criteria of optimization can be categorized into:

(1) maximizing the unit profit of machining,

(2) minimizing the unit cost of machining,

(3) minimizing the unit time of production or maximizing the production rate, or

(4) maximizing the material removal rate of machining.

These four categories are interrelated, and the problem of selecting machining variables can be formulated in accordance with any of these objective criteria according to a user's unique practical concerns.

Chapter 3: The Model

3.1 Model Assumptions

Here are some assumptions for our model:

1. Single pass turning is considered.
2. The following input parameters are known:
    a. Labor cost
    b. Tool cost
    c. Exponent constants.
3. Machining cost includes only actual cutting, while idle time cost and rapid traverse are excluded.
4. The depth of cut is constant and set to 0.2 in.
5. No constraints on cutting speed and feed rate.

3.2 Primal and Dual Problems

The formulation of the problem starts with the development of the objective function, which includes, in our case, labor cost and machining cost. These cost terms are dependent on machining time. Machining time, T_m, can be calculated as,

$$T_m = \frac{l}{fN} \tag{3.1}$$

where,
l: The length of the workpiece.
f: The feed rate.
N: The rotational speed (spindle speed).

The value of rotational speed in terms of speed and workpiece diameter is expressed as,

$$N = \frac{12v}{\pi D} \tag{3.2}$$

where v is the speed in feet/minute and D is the diameter of the workpiece in inches. Now we can substitute equation (3.2) in equation (3.1) to obtain the cutting/machining time equation that can be used to derive other cost terms:

$$T_m = \frac{\pi D l}{12 f v} \tag{3.3}$$

The first term in our objective function is the labor cost (*LC*), which is the labor cost per unit, denoted as L_r, multiplied by machining time. Hence, *LC* can be expressed as:

$$LC = L_r \times \left(\frac{\pi D l}{12 f v} \right) \tag{3.4}$$

This equation (3.4) can be reduced to:

$$LC = c_1 f^{-1} v^{-1} \tag{3.5}$$

where c_1 is a constant giving by,

$$c_1 = \frac{\pi D l}{12} L_r \tag{3.6}$$

The second term is the tool cost (*TC*), which include labor cost to replace the tools and the actual tool cost.

$$TC = \frac{T_m}{T} L_r T_c + \frac{T_m}{T} T_{cost} \tag{3.7}$$

Where T_c is the average time in minutes to change a tool and T_{cost} is the tool replacement cost. These costs are dependent on the number of times tools fail. The expanded Taylor's tool life equation used early by Ermer [88] can be used to derive tool cost:

$$v T^n f^m d^p = C \tag{3.8}$$

where T is the tool life in minutes, d is the depth of cut, and n, m, p, and C are constants related to the material being used. Hence, T can be expressed as:

$$T = \frac{C^{1/n}}{v^{1/n} f^{m/n} d^{p/n}} \tag{3.9}$$

Thus, the average number of times the tools fail can be given by dividing machining time by tool life; that is equation (3.3) by (3.9):

$$\frac{T_m}{T} = \frac{\pi D l C^{-1/n} d^{p/n}}{12} v^{\frac{1}{n}-1} f^{\frac{m}{n}-1} \tag{3.10}$$

It can be shown that equation (3.7) can be reduced to:

$$TC = c_2 v^{\frac{1}{n}-1} f^{\frac{m}{n}-1} \tag{3.11}$$

where c_2 is a constant expressed by,

$$c_2 = \left(\frac{\pi D l C^{-1/n} d^{p/n}}{12} \right) \left(L_r T_c + T_{cost} \right) \tag{3.12}$$

Therefore, the objective function can be stated as:

$$\underset{(v,f)}{\text{Minimize}} \ C_u = c_1 v^{-1} f^{-1} + c_2 v^{\frac{1}{n}-1} f^{\frac{m}{n}-1} \tag{3.13}$$

The constraints are as used in some literature [48, 49]. The first constraint is the power constraint:

$$c_m v^b f^c d^e \leq HP_{max} \tag{3.14}$$

The second constraint is the surface finish required and can be expressed as,

$$c_s v^g f^h d^i \leq SF_{max} \tag{3.15}$$

Therefore, the primal problem can be put together in standard geometric programming form as:

$$\underset{(v,f)}{\text{Minimize}}\ C_u = c_1 v^{-1} f^{-1} + c_2 v^{\frac{1}{n}-1} f^{\frac{m}{n}-1}$$

Power Constraint: $$c_m^{'} v^{b} f^{c} \le 1 \tag{3.16}$$

Surface Finish Constraint: $$c_s^{'} v^{g} f^{h} \le 1 \tag{3.17}$$

$$v, f \ge 0$$

where,

$$c_m^{'} = \frac{c_m d^{e}}{HP_{max}} \text{ and } c_s^{'} = \frac{c_s d^{i}}{SF_{max}} \tag{3.18}$$

The dual geometric programming formulation is obtained as the following:

$$\underset{(D_i)}{\text{Maximize}}\ Q = \left(\frac{c_1}{D_1}\right)^{D_1} \left(\frac{c_2}{D_2}\right)^{D_2} \left(c_m^{'}\right)^{D_3} \left(c_s^{'}\right)^{D_4} \tag{3.19}$$

Subject to the normality condition,

$$D_1 + D_2 = 1 \tag{3.20}$$

and the orthogonality conditions,

$$-D_1 + \left(\frac{1}{n} - 1\right) D_2 + bD_3 + gD_4 = 0 \tag{3.21}$$

$$-D_1 + \left(\frac{m}{n} - 1\right) D_2 + cD_3 + hD_4 = 0 \tag{3.22}$$

$$D_1, D_2, D_3, D_4 \geq 0$$

Computationally, the *dual problem* is easier to solve than the *primal problem*, which maximizes $Q^*(D_i)$ subject to a linear normality equation and the orthogonality conditions (one for each variable). The degree of difficulty of the dual problem can be calculated as defined by Duffin as,

$$Ter - Var - 1 \tag{3.23}$$

where *Ter* is the number of terms in the primal problem or the number of dual variables, and *Var* is the number of variables in the primal problem, or the number of orthogonality constraints in the dual problem. It can be seen that if the degree of difficulty is zero, then the solution to the system is unique. Also, if it is equal to one, then the system can be rendered in terms of one dual variable and the optimal solution can easily be found. The larger the degree of difficulty the more complex the problem becomes. It turns out for our case that the degree of difficulty is equal to two.

3.3 Computer Model

3.3.1 Obtaining the Optimal Solution

The initial mathematical models developed in Section 3.2 is modeled using Microsoft Excel. For simplicity, input parameters will be adapted and modified in our preliminary analysis from the example proposed early by Ermer and Kromodihardjo [88]. They are as follow,

$b = 0.91$	$p = 0.35$	$C_S = 204.62 \times 10^6$	$T_C = 0.5\,\text{min}.$
$c = 0.78$	$g = -1.52$	$C = 80$	$d = 0.2in.$
$e = 0.75$	$h = 1.004$	$HP_{max} = 2hp$	$D = 6in.$
$n = 0.25$	$i = 0.25$	$SF_{max} = 50\mu in.$	$l = 8in.$
$m = 0.29$	$C_m = 2.394$	$T_{cost} = \$0.5/edge$	$L_r = \$0.1/\text{min}.$

the *primal problem* can be reinstated as,

$$\begin{array}{l} \text{Minimize } C_u = c_1 v^{-1} f^{-1} + c_2 v^{3} f^{0.16} \\ \text{Subject to:} \\ \qquad c_m^{'} \quad v^{0.91} f^{0.78} \leq 1 \quad (HP) \\ \qquad c_s^{'} \quad v^{-1.52} f^{1.004} \leq 1 \quad (SF) \\ \qquad\qquad v, \quad f \geq 0 \end{array} \tag{3.24}$$

whereas the *dual problem* is:

$$\begin{array}{l} Max\ Q = \left(\dfrac{c_1}{D_1}\right)^{D_1} \left(\dfrac{c_2}{D_2}\right)^{D_2} \left(c_m^{'}\right)^{D_3} \left(c_s^{'}\right)^{D_4} \\ \text{Subject To:} \\ \begin{array}{rcrcrcrcl} & D_1 & + & D_2 & & & & = & 1 \\ - & D_1 & + & 3\ D_2 & + & 0.91\ D_3 & - & 1.52\ D_4 = & 0 \\ - & D_1 & + & 0.16\ D_2 & + & 0.78\ D_3 & + & 1.004 D_4 = & 0 \\ & D_1, & & D_2, & & D_3, & \text{and} & D_4 \geq & 0 \end{array} \end{array} \tag{3.25}$$

with the constants being:

$$c_1 = 1.256 \qquad c_2 = 1.77188 \times 10^{-8} \qquad c_m^{'} = 0.35799 \qquad c_s^{'} = 2736752.82 \tag{3.26}$$

Note that D_1 and D_2 from *dual problem* correspond to the first and second term in the *primal problem* respectively while D_3 and D_4 corresponds to the first and second constraint terms respectively. It can be shown that all the dual variables in system (3.25) can be expressed in terms of only one variable, say D_1. The resulted substitutions give the following reduced system:

$$
\begin{aligned}
&Max\ \ Q\ =\left(\frac{c_1}{D_1}\right)^{D_1}\left(\frac{c_2}{D_2}\right)^{D_2}\left(c_m^{'}\right)^{D_3}\left(c_s^{'}\right)^{D_4}\\
&D_2 = 1 - D_1 \ge 0\\
&D_3 = -1.5507 + 2.753 D_1 \ge 0\\
&D_4 = 1.0453 - 0.9834 D_1 \ge 0\\
&0 \le D_1 \le 1
\end{aligned}
\tag{3.27}
$$

The objective function can now be plotted dependent only on D_1 (See **Figure 3.1**). The resulted curve is concave and differential calculus can be used to obtain the maximum value.

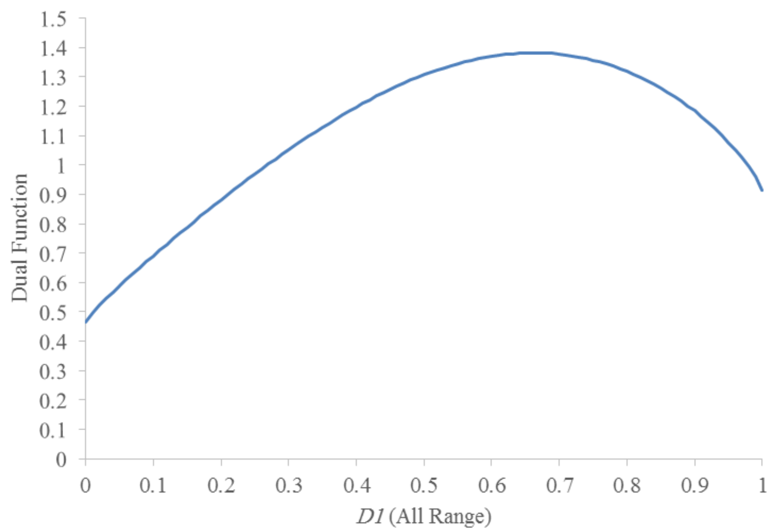

Figure 3.1: *The dual objective function in terms of only one dual variable*

This plot shows the behavior of the dual objective function over all possible values for D_1. Note that not all values are feasible. Any value of D_1, however, in the given range gives values for the other dual variables which satisfy the normality and orthogonality conditions. The problem is to select the optimal weight of D_1 among these infinite possibilities. It can be shown, however, from the set of inequalities in system (3.27) that the feasible region lies where D_1 is in the interval [0.5632, 1]. Within this range, the solutions obtained will satisfy the normality and orthogonality condition for the dual system. Differential calculus can be used to show that the optimal solution occurs at $D_1 = 0.662$ with an optimal dual objective of \$1.38. The generalized solution procedure will be specified later.

3.3.2 Preliminary Analysis

In this section, the principle idea of this research is illustrated. First, let us examine the traditional way to perform a post-optimal analysis or construct a cost analysis curve. In the traditional post-optimal analysis the number one on the right-hand side for the constraint of interest, in a standard geometric primal problem, is changed slightly to see the effect on the dual function [89]. A perturbed primal problem is formed by replacing the number one by a positive parameter, say u_s. The perturbed problem becomes:

$$\begin{array}{llllll} \text{Minimize } Z = c_1 v^{-1} f^{-1} + c_2 v^{3} f^{0.16} & & & & & \\ \text{Subject to:} & & & & & \\ c'_m & v^{0.91} & f^{0.78} & \leq & 1 & (HP) \\ c'_s & v^{-1.52} & f^{1.004} & \leq & u_s & (SF) \\ & v, & f & \geq & 0 & u_s > 0 \end{array} \tag{3.28}$$

Table 3.1 summarizes the different cases when varying this parameter:

Table 3.1: *Effect of right-hand-side change on cost*

Case	Range	Interpretation	Effect
A	$u_s = 1$	The problem reduces back to the original primal problem	None
B	$u_s > 1$	The surface finish constraint is loosened	The surface finish constraint has been loosened by $100(u_s - 1)\%$ or SF_{max} is increased by the same percent change.
C	$u_s < 1$	The surface finish constraint is tightened	The surface finish constraint has been tightened by $100(u_s - 1)\%$ or SF_{max} is decreased by the same percent change.

Recall that for our model $c_s^{'} = c_s d^{0.25} / SF_{max}$. If the surface finish constraint is loosened by $100(u_s - 1)\%$, we can equivalently say that the maximum desired surface finish is increased by $100(u_s - 1)\%$, keeping all other parameters constant. Also, note that the optimal solution for the perturbed problem is not necessarily a feasible solution for the original primal problem. An optimal cost/surface finish curve can now be obtained (**Figure 3.2**).

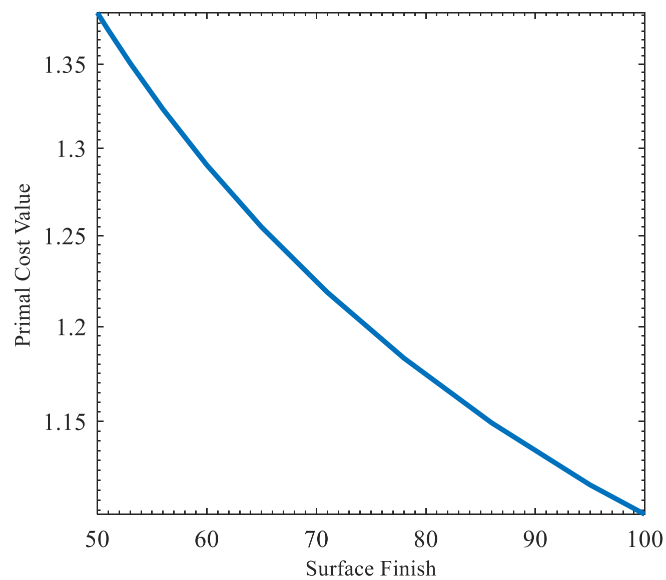

Figure 3.2: *Cost versus surface finish curve*

In this plots, the optimal cost curve is plotted. At each iteration (gradual increments in surface finish) a new optimization is performed. Consequently, a new optimal D_1 value is obtained with different cost value. Also, an optimal solution at iteration *i* is not necessarily a feasible solution for the base model. In fact, it would be most likely considered an infeasible solution for the original problem, because the surface finish constraint would be violated. This analysis is performed on the *primal problem*. One can perform traditional post-optimal analysis on the *dual problem* by gradually varying surface finish, for example, and keeping the dual variable D_1 at the optimal value, resulting in exactly similar results as if conducted on a *primal problem*.

Our approach for post-optimal analysis uses the *dual problem*. The basic principle is that we deviate D_1 from the optimal value, resulting in a new spectrum of solutions. Therefore, it is unnecessary that the value of D_1 at iteration *i* would correspond to an optimal value to that iteration. In other words, we are arbitrary assigning more weight to one term in the objective function over the other. This could practically mean that investing more in labor is economically preferred over investing in tools within certain limitations, although some specific quality characteristics being not met.

Let us define the following measure to help assess the relation between cost and the quality characteristic; the surface finish in our case.

$$\rho_i = \frac{\mu_i}{\psi_i} \quad i \in \mathbb{R}^+ \tag{3.29}$$

where,

$$\mu_i = \frac{Q_o - Q_i}{Q_o} \tag{3.30}$$

$$\psi_i = \frac{SF_i - SF_o}{SF_o} \tag{3.31}$$

ρ_i is the measure of the optimization process, where it computes the ratio of the percent decrease in cost to the percent increase in surface finish at iteration *i* from the base solution. Thus, we need to find out the point at which this measure reaches the maximum value during the analysis. μ_i is the percentage of cost reduction at iteration i. Similarly, ψ_i is the percentage of surface finish increment at iteration i. Note that the dual variable D_1 gradually increases, at a constant rate, from iteration to the other.

Looking at **Figure 3.3**, we note that ρ is initially equal to zero for the base model. Then it gradually increases to a maximum point where the magnitude of ρ reaches its maximum as we deviate from the dual optimal value $D_1 = 0.662$. The more we deviate from optimality, the more the surface finish becomes over tight, more violated. We can see that, in this case, one can increase surface finish from 50 microinches to approximately 67 microinches (a 35.14% increment), to obtain the most significant reduction in cost, 3.89% decrease. This occurs at approximately $D_1 = 0.79$. Unlike the traditional relation charts like in **Figure 3.2**, obtained by the primal problem, this type of analysis provide information on the significance of each increment of the surface finish.

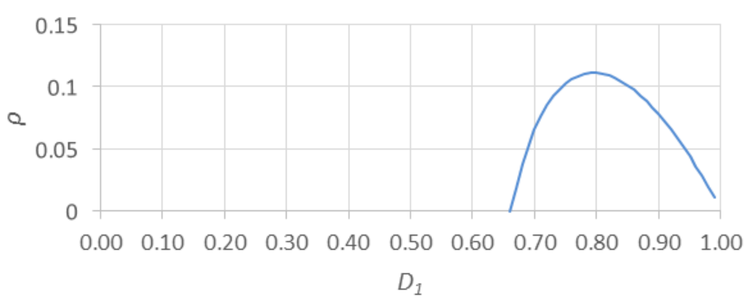

Figure 3.3: *The relation curve with the new measure ρ*

3.4 Future Analytical Model

The computer model exposed an opportunity for cost reduction in the cutting process. Although this reduction seems small for a unit production, it can be vital in annual savings.

Hence, it is worth exploring the opportunity and find new ways to save money. It is essential to building a mathematical base for this type of analysis in the future. One should investigate the following:

(1) What other measures would be appropriate for post-optimal analysis for geometric programming?

(2) How can one obtain an analytical solution to the problem?

(3) How can the dual variable D_1 be varied so that the specified measure be optimized?

(4) What other constraints are appropriate and impose significant contribution to the cost?

3.5 Conclusion

The machining economics problem is a popular problem that still has the potential for improvements. Although obtaining the optimal solution is a relatively well-addressed problem, the post-optimal analysis is still an open space for research. The computer model and the preliminary analysis illustrated show potential for improvement. More investigation and future analytical solution shall be conducted.

Chapter 4: The Solution Approach

4.1 Introduction

In this chapter, an analytical procedure will be developed and presented to solve the problem. In addition, some of the relations that are necessary for the solution approach will be presented.

4.2 Derivation of Solution Cases

In this section, the problem will be decomposed into two smaller cases assuming the surface finish constraint is always tight at the optimal solution, meaning the constraint holds with equality at optimality.

Consider the *primal system* consisting of equations (3.13), (3.16), and (3.17) along with the non-negativity conditions. Also consider the *dual system* of equations (3.19), (3.20), (3.21), and (3.22) with the non-negativity conditions. The dual system is easier to work with since the set of constraints are linear. Assuming the surface finish required, equation 3.17, is always attained at the optimal solution, we should have two and only two cases. The first case is true when the power constraint, equation 3.16, is loose at optimality, while the second case holds when the power constraint is tight at an optimal solution. We shall derive the solutions to each case in the following lines:

4.2.1 Case A: Loose Power Constraint

The following derivation for **Case A** has been reported by professor Gopalakrishnan and Al-Khayyal [49] but will be reported again for the sake of completeness. According to geometric programming duality theory and complementary slackness, when a specific primal constraint is loose at optimality, the corresponding dual variable will be equal to zero. This can be represented mathematically for our case as:

$$D_3(c_m v^b f^c d^e - HP_{max}) = 0 \tag{4.1}$$

Having $D_3 = 0$ will leave us with the following system of dual constraints:

$$\begin{aligned} &D_1 + D_2 = 1 \\ &-D_1 + (\frac{1}{n} - 1)D_2 + gD_4 = 0 \\ &-D_1 + (\frac{m}{n} - 1)D_2 + hD_4 = 0 \end{aligned}$$

This system has a unique solution and can be easily found as:

$$D_1 = \frac{n(g-n)}{h-gm} + 1 \tag{4.2}$$

$$D_2 = \frac{n(n-g)}{h-gm} \tag{4.3}$$

$$D_3 = 0 \tag{4.4}$$

$$D_4 = \frac{m-1}{gm-h} \tag{4.5}$$

Hence, we can note that the values of optimal dual variables values are independent of the primal coefficients in case we have loose horsepower but tight surface finish.

4.2.2 Case B: Tight Power Constraint

The case when we have tight power constraint in addition to the surface finish is a more difficult case, where we can render the dual constraint in terms of one dual variable, say D_1, and then substitute in the dual objective function:

$$0 \le D_1 \le 1 \tag{4.6}$$

$$D_2 = 1 - D_1 \tag{4.7}$$

$$D_3 = \frac{c_{31} + c_{32} D_1}{c_d} \tag{4.8}$$

$$D_4 = \frac{c_{41} + c_{42} D_1}{c_d} \tag{4.9}$$

where,

$$c_{31} = h(n-1) + g(m-n) \tag{4.10}$$

$$c_{32} = h - gm \tag{4.11}$$

$$c_{41} = c(1-n) - b(m-n) \tag{4.12}$$

$$c_{42} = bm - c \tag{4.13}$$

$$c_d = n(bh - cg) \tag{4.14}$$

Now we can substitute (4.7), (4.8), and (4.9) into (3.19) to get an unconstrained dual system:

$$\underset{(D_1)}{\text{Maximize}}\ Q = \left(\frac{c_1}{D_1}\right)^{D_1} \left(\frac{c_2}{1-D_1}\right)^{1-D_1} \left(c_m^{'}\right)^{(c_{31}+c_{32}D_1)/c_d} \left(c_s^{'}\right)^{(c_{41}+c_{42}D_1)/c_d} \tag{4.15}$$

Differential calculus can be used to obtain the maximum of (4.15). Before we do so, let us take the natural logarithm of the function to make the differentiation process easier. Taking the natural logarithm to (4.15) yields:

$$\begin{aligned} Z = \ln Q = D_1 \ln\left(\frac{c_1}{D_1}\right) + (1-D_1)\ln\left(\frac{c_2}{1-D_1}\right) \\ + \left(\frac{c_{31}+c_{32}D_1}{c_d}\right)\ln\left(c_m^{'}\right) + \left(\frac{c_{41}+c_{42}D_1}{c_d}\right)\left(c_s^{'}\right) \end{aligned} \tag{4.16}$$

Equation (4.16) can be expanded and rearranged as:

$$\begin{aligned} Z = \ln Q = D_1 \ln c_1 - D_1 \ln D_1 + (1-D_1)\ln c_2 - (1-D_1)\ln(1-D_1) \\ + \left(\frac{c_{31}}{c_d}\right)\ln\left(c_m^{'}\right) + \left(\frac{c_{41}}{c_d}\right)\ln\left(c_s^{'}\right) \\ + \left[\left(\frac{c_{32}}{c_d}\right)\ln\left(c_m^{'}\right)\right]D_1 + \left[\left(\frac{c_{42}}{c_d}\right)\ln\left(c_s^{'}\right)\right]D_1 \end{aligned} \tag{4.17}$$

Equation (4.17) can be differentiated with respect to D_1 and get:

$$Z' = \ln(1-D_1) - \ln D_1 + \ln c_1 - \ln c_2 + \frac{c_{31}}{c_d}\ln c_m^{'} + \frac{c_{41}}{c_d}\ln c_s^{'} \tag{4.18}$$

For simplicity define the following new constants:

$$c_x = \ln c_1 - \ln c_2 \tag{4.19}$$

$$c_y = \frac{c_{31}}{c_d}\ln c_m^{'} + \frac{c_{41}}{c_d}\ln c_s^{'} \tag{4.20}$$

Equation (4.18) becomes:

$$Z' = \ln(1 - D_1) - \ln D_1 + c_x + c_y \qquad (4.21)$$

Now we can equate equation (4.21) to zero and solve for D_1 to obtain a closed-form solution that gives the maximum value for the objective function. With simple algebra we get:

$$D_1 = \frac{1}{1 + e^{-(c_x + c_y)}} \qquad (4.22)$$

4.3 Recovering Primal Solution

We know from geometric programming theory that the solutions obtained for the dual variables present in the normality constraint, equation (3.20), represent a fraction. This fraction represents a proportion of total cost the corresponding term, in the primal objective function, holds. Mathematically, we can state the following:

$$c_1 v^{-1} f^{-1} = D_1 Q \qquad (4.23)$$

$$c_2 v^{\frac{1}{n}-1} f^{\frac{m}{n}-1} = D_2 Q \qquad (4.24)$$

For simplicity, the exponents in (4.24) will be renamed as:

$$c_2 v^{a_3} f^{a_4} = D_2 Q \qquad (4.25)$$

Thus, from (4.23) we can easily get:

$$v = \frac{c_1 f^{-1}}{D_1 Q} \qquad (4.26)$$

Now we can substitute (4.26) into (4.25) and solve for f to get:

$$f = \left(\frac{D_1^{a_3} D_2 Q^{a_3+1}}{c_1^{a_3} c_2} \right)^{\frac{1}{a_4 - a_3}} \tag{4.27}$$

The expressions in (4.26) and (4.27) can be reduced to simplify calculations for the next section. Let $w_1 = c_1 / (D_1 Q)$ and $w_2 = (D_2 Q) / c_2$. Then we have:

$$v = w_1 f^{-1} \tag{4.28}$$

$$f = w_1^{\frac{-a_3}{a_4 - a_3}} w_2^{\frac{1}{a_4 - a_3}} \tag{4.29}$$

Now we can substitute (4.29) into (4.28) to get:

$$v = w_1^{\frac{a_3}{a_4 - a_3}+1} w_2^{\frac{-1}{a_4 - a_3}} \tag{4.30}$$

If you let $a = 1/(a_4 - a_3)$ and substitute into (4.29) and (4.30), we finally get:

$$f = w_1^{-a \cdot a_3} w_2^{a} \tag{4.31}$$

$$v = w_1^{a \cdot a_3 + 1} w_2^{-a} \tag{4.32}$$

The expressions in (4.31) and (4.32) will also be used in the primal-dual representation of the problem next section.

4.4 Primal-Dual Representation

It is important in this research to develop equations that represent what we call tightness of the primal constraints at a specific solution, which will help particularly in the development of the solution. Basically, the expressions developed in equations (4.31) and (4.32) will be substituted into the primal constraints and ultimately obtain a representation of each primal constraint in terms of only one dual variable. The procedure starts by considering the first primal constraint in (3.16). Assume we change that constraint to the following:

$$\alpha = c_m^{'} v^{b} f^{c} \tag{4.33}$$

Where α represent a nonnegative real number that signifies how much of the specific constraint is attained at a given solution. For example, if $\alpha = 0.65$ then the horsepower constraint is loose and only 65% percent of the maximum power available has been used. Likewise, if, for instance, $\alpha = 1.30$ then we say that the horsepower constraint is overtight, meaning the solution requires 30% extra power to perform the recommended cutting conditions.

Let us now substitute (4.31) and (4.32) into (4.33).

$$\begin{aligned}
\alpha &= c_m^{'} v^{b} f^{c} \\
&= c_m^{'} (w_1^{a \cdot a_3 + 1} w_2^{-a})^{b} (w_1^{-a \cdot a_3} w_2^{a})^{c} \\
&= c_m^{'} (w_1^{b \cdot a \cdot a_3 + 1} w_2^{-ba})(w_1^{-c \cdot a \cdot a_3} w_2^{c \cdot a}) \\
&= c_m^{'} (w_1^{b \cdot a \cdot a_3 + 1 - c \cdot a \cdot a_3} w_2^{-b \cdot a + c \cdot a}) \\
&= c_m^{'} (w_1^{a \cdot a_3 \cdot (b - c) + b} w_2^{a \cdot (c - b)})
\end{aligned}$$

Now for simplicity, let:

$$\begin{aligned}
c_{\alpha_1} &= a a_3 (b - c) + b = \frac{b(m - n) + c(n - 1)}{m - 1} \\
c_{\alpha_2} &= a(c - b) = \frac{n(b - c)}{m - 1}
\end{aligned} \tag{4.34}$$

Hence, we have:

$$\alpha = c_m^{'} (w_1^{c_{\alpha 1}} w_2^{c_{\alpha 2}}) \tag{4.35}$$

Similarly, we can develop an expression for the second primal constraint, the surface finish constraint in (3.17):

$$\beta = c_s^{'} (w_1^{c_{\beta 1}} w_2^{c_{\beta 2}}) \tag{4.36}$$

Where,

$$\begin{aligned} c_{\beta_1} &= aa_3(g-h)+g = \frac{g(m-n)+h(n-1)}{m-1} \\ c_{\beta_2} &= a(h-g) = \frac{n(g-h)}{m-1} \end{aligned} \tag{4.37}$$

Now we would like to refine (4.35) and (4.36), and have both of them depend only on the dual variables. Starting with (4.35),

$$\begin{aligned}
\alpha &= c_m^{'} \left[\left(\frac{c_1}{D_1 Q} \right)^{c_{\alpha 1}} \left(\frac{D_2 Q}{c_2} \right)^{c_{\alpha 2}} \right] \\
&= c_m^{'} \left(\frac{c_1}{D_1} \right)^{c_{\alpha 1}} \left(\frac{D_2}{c_2} \right)^{c_{\alpha 2}} \left[\left(\frac{c_1}{D_1} \right)^{D_1} \left(\frac{c_2}{D_2} \right)^{D_2} \left(c_m^{'} \right)^{D_3} \left(c_s^{'} \right)^{D_4} \right]^{c_{\alpha 2} - c_{\alpha 1}} \qquad \text{from (3.19)} \\
&= \left(\frac{c_1}{D_1} \right)^{(c_{\alpha 2} - c_{\alpha 1})D_1 + c_{\alpha 1}} \left(\frac{c_2}{D_2} \right)^{(c_{\alpha 2} - c_{\alpha 1})D_2 - c_{\alpha 2}} \left(c_m^{'} \right)^{1+(c_{\alpha 2} - c_{\alpha 1})D_3} \left(c_s^{'} \right)^{(c_{\alpha 2} - c_{\alpha 1})D_4} \\
&= \left(\frac{c_1}{D_1} \right)^{(c_{\alpha 2} D_1 + c_{\alpha 1} \mathrm{D}_2)} \left(\frac{c_2}{D_2} \right)^{-(c_{\alpha 2} D_1 + c_{\alpha 1} \mathrm{D}_2)} \left(c_m^{'} \right)^{1+(c_{\alpha 2} - c_{\alpha 1})D_3} \left(c_s^{'} \right)^{(c_{\alpha 2} - c_{\alpha 1})D_4} \\
&= \left(\frac{c_1 D_2}{c_2 D_1} \right)^{(c_{\alpha 2} D_1 + c_{\alpha 1} \mathrm{D}_2)} \left(c_m^{'} \right)^{1+(c_{\alpha 2} - c_{\alpha 1})D_3} \left(c_s^{'} \right)^{(c_{\alpha 2} - c_{\alpha 1})D_4}
\end{aligned}$$

Let $c_\alpha = c_{\alpha 2} - c_{\alpha 1}$, and we get:

$$\alpha = \left(\frac{c_1 D_2}{c_2 D_1} \right)^{c_\alpha D_1 + c_{\alpha_1}} \left(c_m^{'} \right)^{1 + c_\alpha D_3} \left(c_s^{'} \right)^{c_\alpha D_4} \tag{4.38}$$

Similarly, we obtain similar expression to β as,

$$\beta = \left(\frac{c_1 D_2}{c_2 D_1} \right)^{c_\beta D_1 + c_{\beta_1}} \left(c_m^{'} \right)^{c_\beta D_3} \left(c_s^{'} \right)^{1 + c_\beta D_4} \tag{4.39}$$

Where $c_\beta = c_{\beta_2} - c_{\beta_1}$.

Note that, by observation, the roots of the natural logarithm of (4.39), when rendered in terms of only D_1, will represent the two cases developed earlier in **Section 4.2**.

4.5 Solution Approach

In this section, the solution cases developed in **Section 4.2** as well as the primal-dual expressions developed in **Section 4.4**, will be utilized to help obtain the required solution. Recall that what is required is to find an analytical method to optimize the measure shown in Equation (3.29), which represent a ratio of the change in cost to change in surface finish in an interval starts from optimal dual values for the corresponding case. Note that Equation (3.31), by construction, is equivalent to the following:

$$\Psi_i = \beta_i - 1 \tag{4.40}$$

This is true because β, developed in Equation (4.39), is a factor of the attained surface finish for a specific solution. Hence, the percent change of this factor is equivalent to the percent change of surface finish attained. We can restate Equation (3.29) as:

$$\text{Maximize } \rho(D_1, D_2, D_3, D_4) = \frac{1 - cQ_i(D_1, D_2, D_3, D_4)}{\beta_i - 1} \qquad i \in \mathbb{R}^+ \tag{4.41}$$

Where,

c is the inverse of the cost function at the base model or iteration.

$Q_i(D_j)$ is the cost at iteration i at the specified dual values D_1, D_2, D_3, and D_4.

β_i is the factor of the surface finish attained at iteration i.

When running the optimization, the solution will provide dual values that give the maximum reduction in cost while keeping surface finish increase at lower values. Note that, by observation, this works only when we have an initial base model coming from **Case 2**, developed in **Section 4.2.2**, where the dual variable D_1 value is greater than that of **Case 1**. This happens intuitively for the following reasons:

1. Changing the dual variables change speed and feed in primal solution. In **Case 1**, the solution can be obtained with fairly enough power. Hence, no need to modify speed and feed.
2. In **Case 2**, when we have tight power, speed and feed need to be adjusted accordingly, which help reduce cost, but increase surface finish.

The optimization function (4.41) can be represented as:

$$\text{Minimize } \rho(D_1, D_2, D_3, D_4) = \frac{c\left(\frac{c_1}{D_1}\right)^{D_1}\left(\frac{c_2}{D_2}\right)^{D_2}(c_m^{'})^{D_3}(c_s^{'})^{D_4} - 1}{\left(\frac{c_1 D_2}{c_2 D_1}\right)^{c\beta D_1 + c\beta_1}(c_m^{'})^{c\beta D_3}(c_s^{'})^{1 + c\beta D_4} - 1} \tag{4.42}$$

Now, the problem can be rendered in terms of one dual variable as:

$$\underset{D_1 \in (D_1^*,1)}{\text{Minimize}}\ \rho(D_1) = \frac{c\left(\dfrac{c_1}{D_1}\right)^{D_1}\left[\dfrac{c_2}{(1-D_1)}\right]^{(1-D_1)}(c_m^{'})^{(c_{31}+c_{32}D_1)/c_d}(c_s^{'})^{(c_{41}+c_{42}D_1)/c_d}-1}{\left[\dfrac{c_1(1-D_1)}{c_2 D_1}\right]^{c_\beta D_1 + c_{\beta 1}}(c_m^{'})^{c_\beta\left[(c_{31}+c_{32}D_1)/c_d\right]}(c_s^{'})^{1+c_\beta\left[(c_{41}+c_{42}D_1)/c_d\right]}-1} \tag{4.43}$$

Where D_1^* is the base model value for the dual variable D_1 obtained from one of the cases developed in Section 4.2. In fact, D_1 should be evaluated for both cases and the bigger D_1 is considered the optimal solution.

Practically, the problem has been reduced initially from multidimensional constrained problem to multidimensional unconstrained problem. Then, from to multidimensional unconstrained problem to one-dimensional unconstrained. Thus, the search methods or approximation methods available to optimize one-dimensional problems can be utilized for the problem at hand, the mathematical optimization model in (4.43). However, a new method, called *finite evaluations*, was developed to improve the convergence rate and computational efficiency, since we know by observation that the function $\rho(D_1)$ is a convex function. The following section describes the *finite evaluations algorithm*.

4.5.1 The Finite Evaluations Algorithm

Usually, in practice, the function being minimized has multiple local extremum points. For convex functions, however, like $\rho(D_1)$, if local extremum point does exist, then it is also a global extremum point. Hence, the *finite evaluations algorithm* was developed based on the behavior of the function $\rho(D_1)$ to find the extremum point in an iterative manner.

Unlike search methods and approximation methods, the *finite evaluations algorithm* is only applicable to convex functions. **Figure 4.1** shows the pseudocode for the algorithm. The algorithm starts by setting the initial value of the dual variable D_1^*. The uncertainty region is the interval from D_1^* to 1. Then the user specifies a required tolerance which will be compared to the magnitude of the uncertainty region after each iteration, or alternatively, the user can provide a maximum number of iterations. Each iteration the uncertainty interval is cut in the middle as it is shown in **Figure 4.2**. The midpoint is set to be either the new upper bound or lower bound for the next iteration depending on the evolution of ρ function at this midpoint plus and minus some constant called the *finite evolution constant.* The algorithm will always exclude the region that does not contain the optimal point and obtain smaller uncertainty intervals.

Pseudocode - *Finite Evaluations Algorithm*

Use solution cases to get D_1^*
Set *finite difference constant* ϵ, *tolerance* τ or *maximum number of iterations* I_{max}
Set uncertainty region $\Omega_o = [D_1^*, 1] = [\Omega_L, \Omega_U]$
While $i = 0$ to I_{max} or $|\Omega_U - \Omega_L| > \tau$
    Set $D_1^i = (\Omega_U + \Omega_L)/2$
    Calculate $\rho(D_1^i + \epsilon)$ and $\rho(D_1^i - \epsilon)$
    If $\rho(D_1^i + \epsilon) > \rho(D_1^i - \epsilon)$ **then**
        Set $\Omega_L = D_1^i$, **Otherwise Set** $\Omega_U = D_1^i$
    $i = i + 1$
    Set $D_1^{**} = D_1^i$
End

Figure 4.1: *Pseudocode for finite evaluations algorithm.*

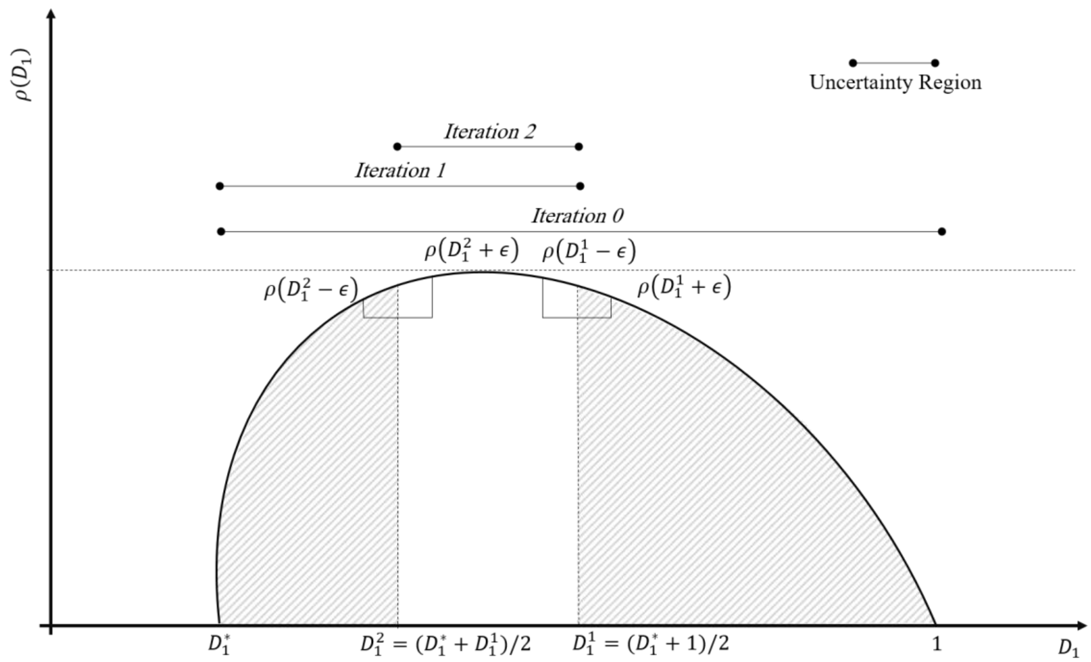

Figure 4.2: *Illustration of the finite evaluations algorithm.*

4.6 Conclusion

It was shown in this chapter that the problem could be formulated and solved mathematically. The solution is an iterative procedure that was coded as an algorithm that showed better convergence rate especially for the problem at hand than some of the available tools available such as golden search algorithm and parabolic interpolation algorithm. Validation of the mathematical part will be presented next chapter.

Chapter 5: Model Validation and Sensitivity Analysis

5.1 Introduction

In this chapter, the model formulation developed in the previous chapter will be validated mathematically. This is done to ensure the solutions provided by the model are at least theoretically applicable since conducting real experimentations is economically inapplicable at this time. Also, applying part of the model in a previously published case to validate the model is presented in this chapter although the rest of the model, or the theory behind the model, is novel. Later in the chapter, a sensitivity analysis model will be presented to illustrate further the model's ability to provide alternative solutions.

5.2 Model Validation

Consider the relations developed in (4.38) and (4.39). Taking the natural logarithm for both equations yields:

$$\ln\alpha = \left(c_\alpha D_1 + c_{\alpha_1}\right)\ln\left(\frac{c_1 D_2}{c_2 D_1}\right) + \left(1 + c_\alpha D_3\right)\ln\left(c_m^{'}\right) + \left(c_\alpha D_4\right)\ln\left(c_s^{'}\right) \tag{5.1}$$

$$\ln\beta = \left(c_\beta D_1 + c_{\beta_1}\right)\ln\left(\frac{c_1 D_2}{c_2 D_1}\right) + \left(c_\beta D_3\right)\ln\left(c_m^{'}\right) + \left(1 + c_\beta D_4\right)\ln\left(c_s^{'}\right) \tag{5.2}$$

From (5.1), we can get:

$$\ln\left(\frac{c_1 D_2}{c_2 D_1}\right) = \frac{\ln\alpha - \left[\left(1 + c_\alpha D_3\right)\ln\left(c_m^{'}\right) + \left(c_\alpha D_4\right)\ln\left(c_s^{'}\right)\right]}{\left(c_\alpha D_1 + c_{\alpha_1}\right)} \tag{5.3}$$

Similarly, from (5.2) we can get:

$$\ln\left(\frac{c_1 D_2}{c_2 D_1}\right) = \frac{\ln\beta - \left[\left(c_\beta D_3\right)\ln\left(c_m^{'}\right) + \left(1 + c_\beta D_4\right)\ln\left(c_s^{'}\right)\right]}{\left(c_\beta D_1 + c_{\beta_1}\right)} \tag{5.4}$$

Now we can equate both quantities in (5.3) and (5.4), and rearrange terms to get:

$$\frac{c_\alpha D_1 + c_{\alpha_1}}{c_\beta D_1 + c_{\beta_1}} = \frac{\ln\alpha - \left[\left(c_\alpha D_4\right)\ln\left(c_s^{'}\right) + \left(1 + c_\alpha D_3\right)\ln\left(c_m^{'}\right)\right]}{\ln\beta - \left[\left(c_\beta D_3\right)\ln\left(c_m^{'}\right) + \left(1 + c_\beta D_4\right)\ln\left(c_s^{'}\right)\right]} \tag{5.5}$$

The equation (5.5) is given the name the *master relation*. Whenever this master relation holds true, the solution provided is theoretically applicable, but not necessarily optimal. However, for the case when we have loose horsepower and tight surface finish constraints at optimally, equation (5.5) reduces to:

$$\frac{c_\alpha D_1 + c_{\alpha_1}}{c_\beta D_1 + c_{\beta_1}} = \frac{\ln\alpha - \left[\left(c_\alpha D_4\right)\ln\left(c_s^{'}\right) + \ln\left(c_m^{'}\right)\right]}{-\left[\left(1 + c_\beta D_4\right)\ln\left(c_s^{'}\right)\right]} \tag{5.6}$$

Since at this specific solution $D_3 = 0$ and $\beta = 1$.

Likewise, when we have both constraints tight at optimal solution equation (5.5) becomes:

$$\frac{c_\alpha D_1 + c_{\alpha_1}}{c_\beta D_1 + c_{\beta_1}} = \frac{\left(c_\alpha D_4\right)\ln\left(c_s^{'}\right) + \left(1 + c_\alpha D_3\right)\ln\left(c_m^{'}\right)}{\left(c_\beta D_3\right)\ln\left(c_m^{'}\right) + \left(1 + c_\beta D_4\right)\ln\left(c_s^{'}\right)} \tag{5.7}$$

Since $\alpha = 1$ and $\beta = 1$ at this specific solution.

The master relation represents a relation between the obtained proportion of the cost terms, the "tightness" of the constraints, and the other dual variables. These components interact according to the master relation, and any invalid input would make the two sides of the equation not equal.

In order to use the master relation, the dual variables values should be known. Also, one can obtain α and β if the dual values are known. A numerical example will be presented in section 5.5.

Figure 5.1 summarizes the equations and constants used for the master relation equation.

The preceding equations developed in this section are to verify that a given solution is mathematically valid. Next, the accuracy of the developed solution **Case II** is to be examined by comparing the results to certain examples present in the literature.

5.2.1 Example 1

The first example is taken from [90]. The following are the input values:

$b = 1$	$p = 0$	$C_s = 2.2\times10^4$	$T_c = 4\min$
$c = 0.83$	$g = -1.52$	$C = 348$	$d = 3mm$
$e = 0$	$h = 1$	$HP_{max} = 5.5KW$	$D = 80mm$
$n = 0.3$	$i = 0$	$SF_{max} = 2\mu m$	$l = 300mm$
$m = 0.42$	$C_m = 10.6\times10^{-2}$	$T_{cost} = \$0.19$	$L_r = \$0.07$

The solution was obtained using **Case II** and the results were comparable with 2.03% less speed, 4.17% smaller feed rate, and 1.63% more cost. Results are shown in **Table 5.1**.

5.2.2 Example 2

The second example is also a case where we have both tight constraints at optimality, and it is found in [48]. Note that in this example, the handling cost considered in the reference paper were excluded. Input parameters are as follow:

$$
\begin{array}{llll}
b = 0.91 & p = 0 & C_s = 1.36\times10^8 & T_c = 0.5\min \\
c = 0.78 & g = -1.52 & C = 140 & d = 0.2in \\
e = 1 & h = 1.004 & HP_{max} = 2hp & D = 6in \\
n = 0.25 & i = 0 & SF_{max} = 100\mu in & l = 8in \\
m = 0.29 & C_m = 3.58 & T_{cost} = \$0.5 & L_r = \$0.07
\end{array}
$$

The results obtained from the developed closed-form equation (Case II) also matches what was reported in [48] and shown in **Table 5.1**.

5.2.3 Example 3

The last example is extracted from [88]. Handling cost is also excluded as in the previous example. Also, this example has an additional constraint on feed rate, which is redundant in this case as it was loose at optimality. Input parameters are as follow:

$$
\begin{array}{llll}
b = 0.91 & p = 0.35 & C_s = 204.62\times10^6 & T_c = 0.5\min \\
c = 0.78 & g = -1.52 & C = 80 & d = 0.2in \\
e = 0.75 & h = 1.004 & HP_{max} = 1.5hp & D = 6in \\
n = 0.25 & i = 0.25 & SF_{max} = 50\mu in & l = 8in \\
m = 0.29 & C_m = 2.394 & T_{cost} = 0.5 & L_r = 0.1
\end{array}
$$

The results show that the developed equation provides accurate results as can be seen in **Table 5.1**.

Table 5.1: *Comparing results from the two cases developed to literature examples*

	Solution from Reference			Our Solution		
Example Reference	*v*	*f*	C_u ($/unit)	*v*	*f*	C_u ($/unit)
[90]	178 (*m/min*)	0.24 (*mm/rev*)	11.64	174.39 (*m/min*)	0.23 (*mm/rev*)	11.83
[48]	311 (*sfpm*)	$0.46\text{x}10^{-2}$ (*ipr*)	1.11	310.71 (*sfpm*)	$0.46\text{x}10^{-2}$ (*ipr*)	1.10
[88]	350 (*sfpm*)	$0.28\text{x}10^{-2}$ (*ipr*)	1.60	351.12 (*sfpm*)	$27.68\text{x}10^{-4}$ (*ipr*)	1.59

Hence, **Table 5.1** shows that the developed closed-form solution in **Case II** in section 4.2.2 provides valid solutions that are comparable to literature.

5.3 Sensitivity Analysis

A sensitivity analysis for the developed model should tell us something about the importance of the parameters in equation (4.43). A new model was built for sensitivity that allows the user to add uncertainty in the input parameters. The solution of the sensitivity model is based on evolutionary techniques, namely the genetic algorithm. Basic features of the sensitivity model include:

1. Providing ranked parameters as results according to their impact on the objective function.
2. Providing alternative solutions, based on the input from the user or a targeted cost, that can be sorted based on the preferences of the user too.
3. Allowing the user to include or exclude the parameters to be changed.
4. Allowing the user to input the number of alternative solutions required.
5. Because the genetic algorithm may provide inaccurate solutions, the resulted solutions are compared to a tolerance provided by the user.

The following constraints, which give bounds for uncertain variables, were added to the problem in (4.43):

$$c_{mL} \leq c_m \leq c_{mU} \tag{5.8}$$

$$c_{sL} \leq c_s \leq c_{sU} \tag{5.9}$$

$$T_{cL} \leq T_c \leq T_{cU} \tag{5.10}$$

The Master Relation:

$$\frac{c_\alpha D_1 + c_{\alpha_1}}{c_\beta D_1 + c_{\beta_1}} = \frac{\ln\alpha - (c_\alpha D_3 + 1)\ln c_m^{'} - c_\alpha D_4 \ln c_s^{'}}{\ln\beta - (c_\beta D_4 + 1)\ln c_s^{'} - c_\beta D_3 \ln c_m^{'}}$$

$D_1 \in (0,1)$, $\quad D_2 = 1 - D_1$

$D_3 = (c_{31} + c_{32}D_1)/c_d$, $\quad D_4 = (c_{41} + c_{42}D_1)/c_d$

$c_x = \ln c_1 - \ln c_2$, $\quad c_y = (c_{32}/c_d)\ln c_m^{'} + (c_{42}/c_d)\ln c_s^{'}$, $\quad c_m^{'} = \dfrac{c_m d^e}{HP_{max}}$ and $c_s^{'} = \dfrac{c_s d^i}{SF_{max}}$.

$$\alpha = \left(\frac{c_1 D_2}{c_2 D_1}\right)^{c_\alpha D_1 + c_{\alpha_1}} (c_m^{'})^{c_\alpha D_3 + 1} (c_s^{'})^{c_\alpha D_4}, \qquad \beta = \left(\frac{c_1 D_2}{c_2 D_1}\right)^{c_\beta D_1 + c_{\beta_1}} (c_m^{'})^{c_\beta D_3} (c_s^{'})^{c_\beta D_4 + 1}$$

Other constants:

$c_{31} = h(n-1) + g(m-n)$ $\quad c_{32} = h - gm$ $\quad c_{\alpha_1} = \dfrac{c_{41}}{m-1}$ $\quad c_{\beta_1} = \dfrac{c_{31}}{m-1}$ $\quad c_\beta = c_{\beta_2} - c_{\beta_1}$

$c_{41} = c(1-n) + b(n-m)$ $\quad c_{42} = bm - c$ $\quad c_{\alpha_2} = \dfrac{n(b-c)}{m-1}$ $\quad c_{\beta_2} = \dfrac{n(g-h)}{m-1}$ $\quad c_\alpha = c_{\alpha_2} - c_{\alpha_1}$

$c_d = n(bh - cg)$.

Figure 5.1: *Summary of the mathematical validation equations*

$$T_{\mathrm{cost}L} \le T_{\mathrm{cost}} \le T_{\mathrm{cost}U} \tag{5.11}$$

$$L_{rL} \le L_r \le L_{rU} \tag{5.12}$$

$$C_L \le C \le C_U \tag{5.13}$$

$$d_L \le d \le d_U \tag{5.14}$$

$$l_L \le l \le l_U \tag{5.15}$$

$$D_L \le D \le D_U \tag{5.16}$$

$$HP_{\max L} \le HP_{\max} \le HP_{\max U} \tag{5.17}$$

$$SF_{\max} \le SF_{\max} + SF_{allowance} \tag{5.18}$$

The addition of bounds for parameters increases the complexity of the problem to a limit where analytical solutions are no longer easy to obtain. Hence, evolutionary techniques, namely genetic algorithm, will be used to provide solutions for the sensitivity model. Microsoft Excel Solver ®, which is an add-in readily available with Microsoft Excel ® that is considered a general-purpose optimization modeling system, will be used to solve the sensitivity analysis model. Detailed solution procedure for the sensitivity analysis is out of the scope of this research. **Figure 5.2** shows a summary of the equations involved in the sensitivity analysis model.

In the sensitivity analysis model, the user starts by identifying what parameters will be involved as bounded parameters and provide those bounds. Setting up bounds is crucial for the sensitivity analysis procedure.

Maximize $\rho(D_1,D_2,D_3,D_4)=\dfrac{1-c\left(\frac{c_1}{D_1}\right)^{D_1}\left(\frac{c_2}{D_2}\right)^{D_2}(c_m^{'})^{D_3}(c_s^{'})^{D_4}}{\left(\frac{c_1D_2}{c_2D_1}\right)^{c\beta D_1+c\beta_1}(c_m^{'})^{c\beta D_3}(c_s^{'})^{1+c\beta D_4}-1}$, $\quad D_1\in(D_1^*,1)$

Subject to: $c_{mL}\le c_m\le c_{mU}$, $c_{sL}\le c_s\le c_{sU}$, $T_{cL}\le T_c\le T_{cU}$, $T_{\mathrm{cost}L}\le T_{\mathrm{cost}}\le T_{\mathrm{cost}U}$,
$L_{rL}\le L_r\le L_{rU}$

$C_L\le C\le C_U$, $d_L\le d\le d_U$, $l_L\le l\le l_U$, $D_L\le D\le D_U$, $SF_{\max}\le SF_{\max}+SF_{allowance}$

Where, $D_1^*=1/[1+e^{-(c_x+c_y)}]$, $D_2=1-D_1^*$, $D_3=(c_{31}+c_{32}D_1^*)/c_d$, $D_4=(c_{41}+c_{42}D_1^*)/c_d$

$c_1=\dfrac{\pi Dl}{12}L_r$, $\quad c_2=\left(\dfrac{\pi DlC^{-1/n}d^{p/n}}{12}\right)(L_rT_c+T_{cost})$, $\quad c=\left[\left(\dfrac{c_1}{D_1^*}\right)^{D_1^*}\left(\dfrac{c_2}{D_2^*}\right)^{D_2^*}(c_m^{'})^{D_3^*}(c_s^{'})^{D_4^*}\right]^{-1}$,

$c_x=\ln c_1-\ln c_2$, $\quad c_y=(c_{32}/c_d)\ln c_m^{'}+(c_{42}/c_d)\ln c_s^{'}$, $c_m^{'}=\dfrac{c_m d^e}{HP_{max}}$ and $c_s^{'}=\dfrac{c_s d^i}{SF_{max}}$.

Other constants:

$c_{31}=h(n-1)+g(m-n)$ $\quad c_{32}=h-gm$ $\quad c_{\beta_1}=\dfrac{c_{31}}{m-1}$

$c_{41}=c(1-n)+b(n-m)$ $\quad c_{42}=bm-c$ $\quad c_{\beta_2}=\dfrac{n(g-h)}{m-1}$

$c_d=n(bh-cg)$.

Figure 5.2: *The sensitivity analysis model*

The numerical example, in section 5.5, will show important issues that must be considered when defining parameters bounds.

After input parameters bounds are defined, the user then would provide the number of required alternative solutions and the allowance for the surface finish, which is the allowed increment for the surface finish from the required value. There are two modes for the sensitivity analysis model:

1. ***Ranges mode***: In this mode, the solutions will be obtained with a maximum reduction in cost that could be achieved while considering the bound constraints (5.8) to (5.18).
2. ***Target mode***: Solutions will be provided according to the required cost and also constraints bounds (5.8) to (5.18) are considered.

In the case of target mode, the user should provide cost tolerance such that the solutions provided by the algorithm can be identified as within target or out of the target, since the genetic algorithm sometimes provide inaccurate results because of the population initialized.

The algorithm initializes a population that adhere to the specified constraints for the parameters and apply the steps in the genetic algorithm as well as the solution methodology developed in the previous chapter and come up with solutions. **Figure 5.3** shows an illustration of the how the sensitivity analysis model finds four alternative solutions. Each solution gives different values for primal variables and parameters. The user after that can sort the provided solutions according to his or her preferences. For example, if the user has asked for 200 alternative solutions, then these solutions can be sorted showing solutions that have faster speeds first. Multiple sorting preferences can also be utilized.

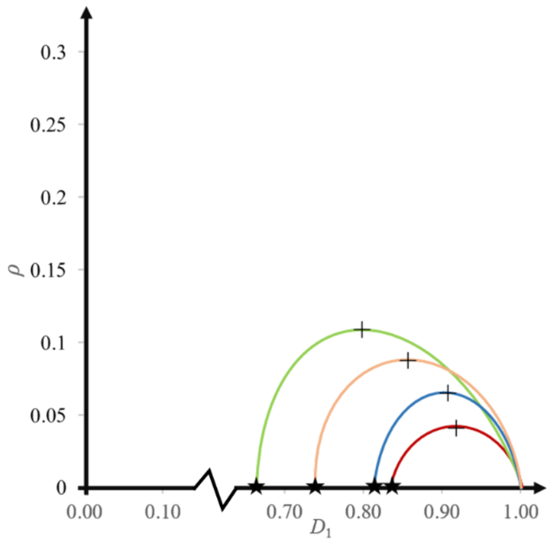

Figure 5.3: *Illustrative example of the sensitivity analysis for four required solutions*

5.4 Computer Model Development

The computer model was built in Microsoft Excel® since it is widely available. It has the following key features:

1. User-friendly interface to get input and show output, programmed with Visual Basic.
2. Calculate optimal solutions based on analytical geometric programming.
3. Calculate alternative solutions based the method and the solution approach developed in Section 4.5.
4. Show the relation between the constraints "tightness" and cost in a two-dimensional plot.
5. Allow to include or exclude parameters to be bounded.
6. Show the impact of the parameters on the objective function and present it as a tornado chart.
7. Allow the user to specify the number of alternative solutions required.
8. Perform sensitivity analysis with two modes, ranges and target solutions.
9. Show the results tabulated and allow to sort alternative based on the preferences of the user.
10. Compare alternative solutions graphically.

Figure 5.4 shows the main user interface for the computer model.

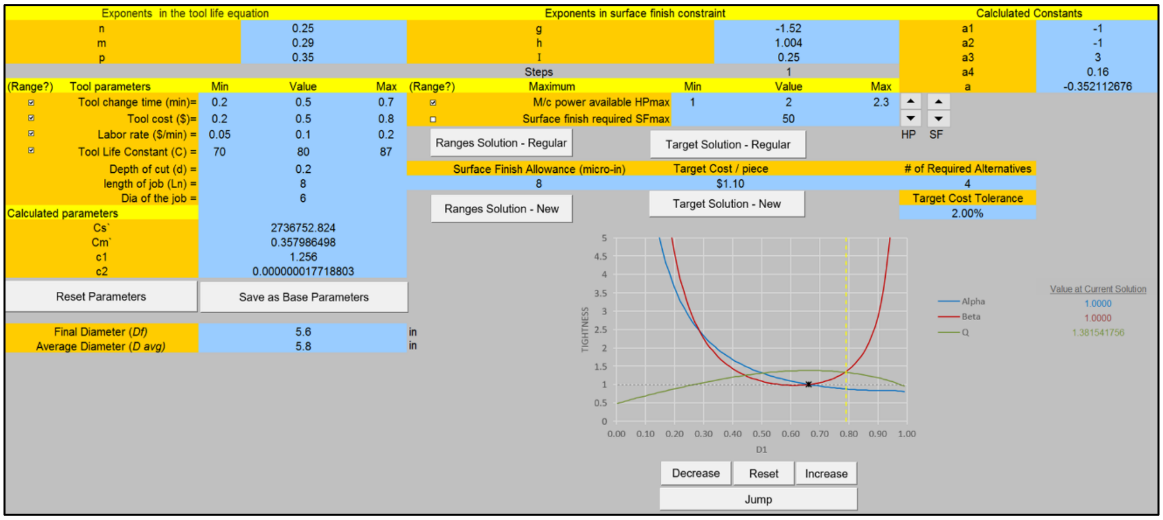

Figure 5.4: *The main user interface for the developed computer model.*

5.5 Numerical Example

In this section, a numerical example will be presented to illustrate the developed method and perform sensitivity analysis. The example initiated in chapter 3 will be used again, which was extracted and altered from [88]. The process input is as follow:

$$
\begin{array}{llll}
b = 0.91 & p = 0.35 & C_s = 204.62 \times 10^6 & T_c = 0.5\,\text{min}. \\
c = 0.78 & g = -1.52 & C = 80 & d = 0.2in. \\
e = 0.75 & h = 1.004 & HP_{max} = 2hp & D = 6in. \\
n = 0.25 & i = 0.25 & SF_{max} = 50\mu in. & l = 8in. \\
m = 0.29 & C_m = 2.394 & T_{cost} = \$0.5/edge & L_r = \$0.1/\text{min}.
\end{array}
$$

The optimal solution using geometric programming analytically is:

$D_1 = 0.6620$, $D_2 = 0.3380$, $D_3 = 0.2718$, $D_4 = 0.3943$ and $\alpha = \beta = 1$.

This solution is mathematically valid since if we use equation (5.7) we get -1.45 for both sides in the equation. The dual variable values were obtained from **Case 2** formulas developed in section 4.2.2, where the value of the dual variable D_1 happens to bigger than that of **Case 1**, a value of 0.5633. This suggests that our developed method can be applied. The horsepower used for this pass is exactly 2 *hp*, and the surface finish attained is 50 microinches. Also, the solution indicates that 66.20% of the total cost is due to labor cost 33.80% of the cost is due tooling cost. Primal solution can be recovered using equations from section 4.3 with v = 402.91 *sfpm* and f = 34.08x10^{-4} *ipr*. The primal-dual representation can be obtained using the relations in (4.38) and (4.39), and it is shown in Figure 5.5.

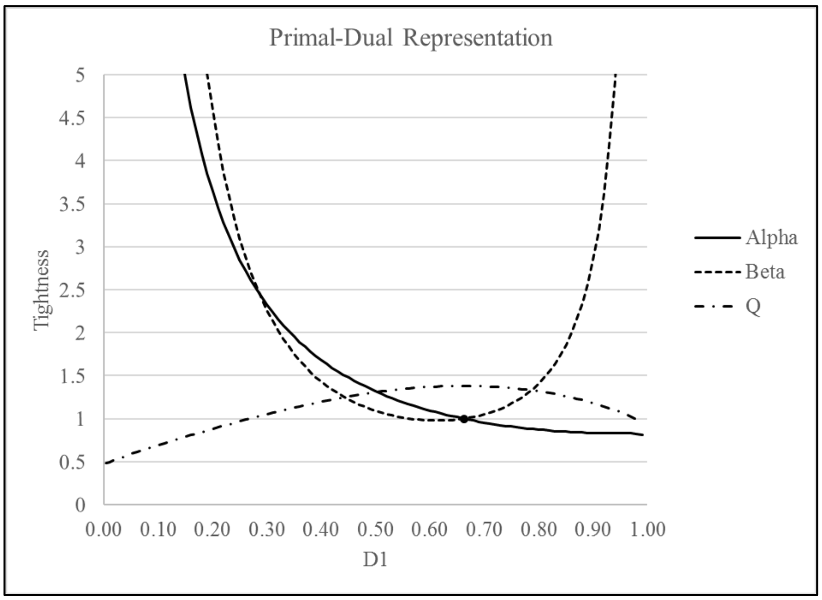

Figure 5.5: *Primal-Dual representation for the numerical example.*

Now, if we suppose that we can allow surface finish to go off the required 50 μin to reduce the cost, then we can apply the algorithm developed in section 4.5.1 to obtain the following dual values:

D_1 = 0.7947, D_2 = 0.2053, D_3 = 0.6371, D_4 = 0.2638, α = 0.8756 β = 1.378.

We may validate this solution using equation (5.5) and get -0.414 for both sides. With this new solution, the cutting speed v was reduced by 16.70% to 335.62 *sfpm* and feed was increased by 4.38% to 35.58×10^{-4} *ipr*. Labor proportion of cost was increased to 79.47% while machining proportion was reduced to 20.53%. Only 1.75 *hp* was used for the power while surface finish produced was 68.91 micro-inches. Finally, the cost was reduced from $1.38/piece to $1.32/piece.

Now, we can start the sensitivity analysis procedure by examining the main effect of input parameters on the cost and surface finish. The computer model provides main effect plots as in **Figure 5.6** and **Figure 5.7**. Note that the “non-smooth” points in the plots represent the points where the solution changes from a loose power constraint case to a tight power constraint. Let us examine, for example, the tool life constant main effect on cost in details, **Figure 5.8**. Note that we have added its effect on the cost using regular solution methods in the literature [49]. We can see the offset in the cost function occurs when the power constraint becomes loose, at a tool life constant of 73. This happens particularly because, in the developed method, we let the surface finish increases, which provides some reduction in the cost function. Similarly, **Figure 5.9** shows a comparison of the effect of labor rate on cost. It is evident that using the developed method, the larger the labor rate, the more saving on cost occurs since more portion of labor cost is assigned to the solution provided. The computer model also provides a tornado chart to help decide what parameters to include in the sensitivity procedure, as in **Figure 5.10**. It also sorts the parameter according to their impact, the highest first. For example, a 10% increase in tool life constant from the base value of 80 would provide 14.5% increase in cost.

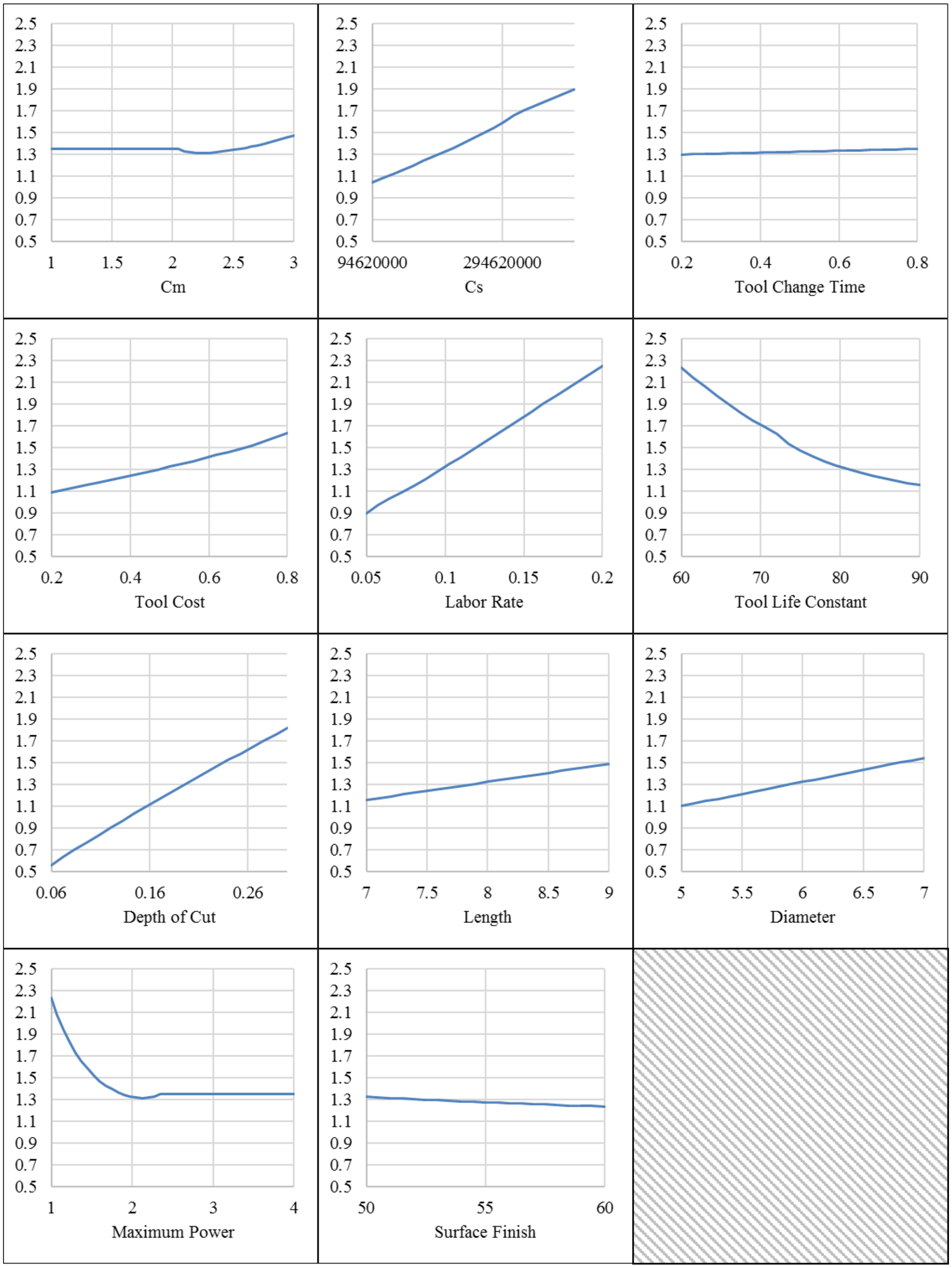

Figure 5.6: *Main Effect plots for the cost function*

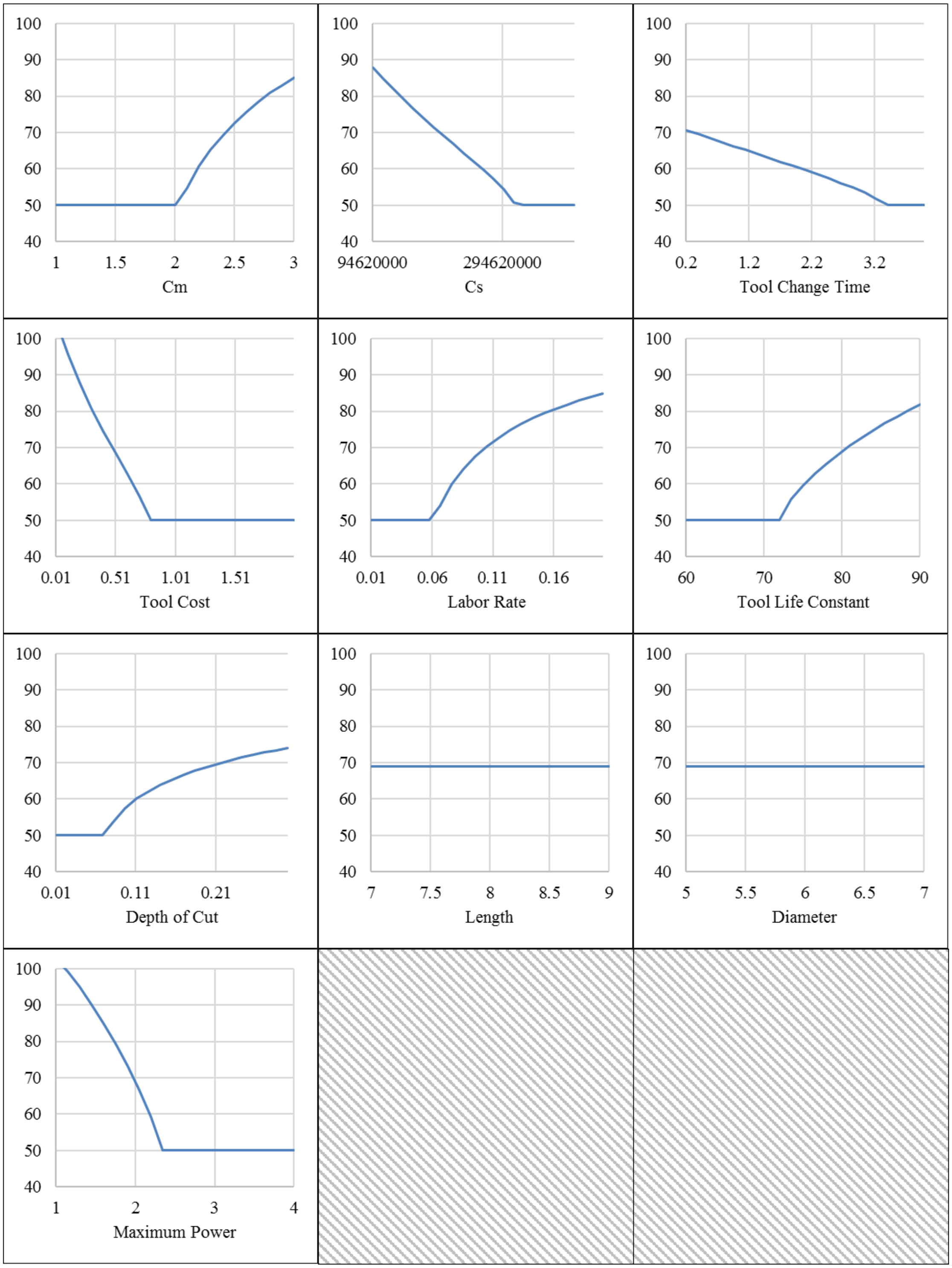

Figure 5.7: *Main effect plots for surface finish*

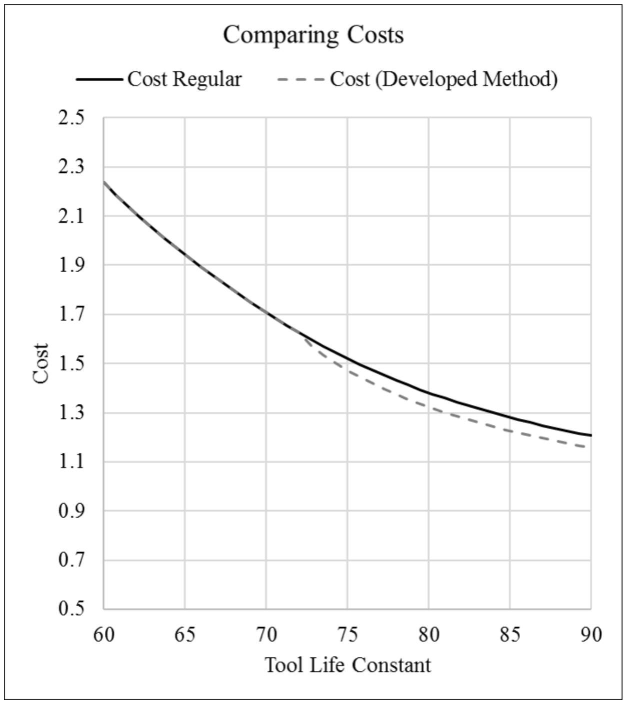

Figure 5.8: *Comparing the effect of the tool life constant on costs*

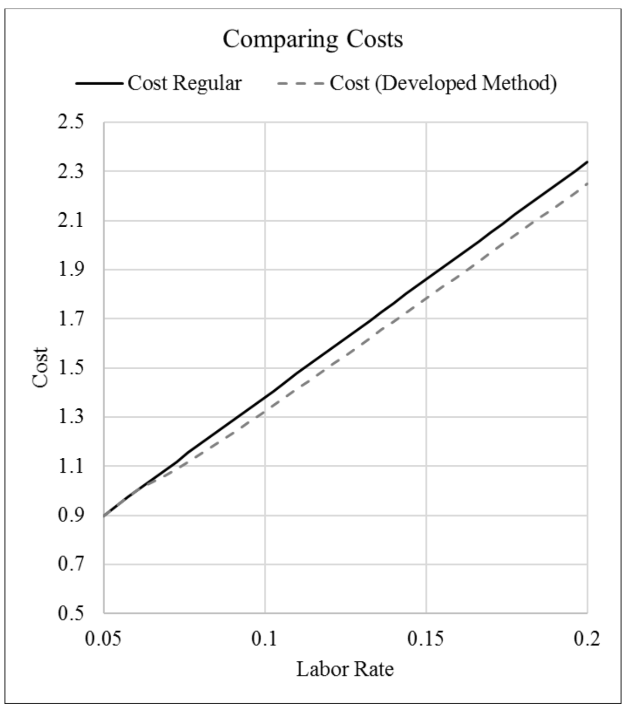

Figure 5.9: *Comparing the effect of labor rate on costs*

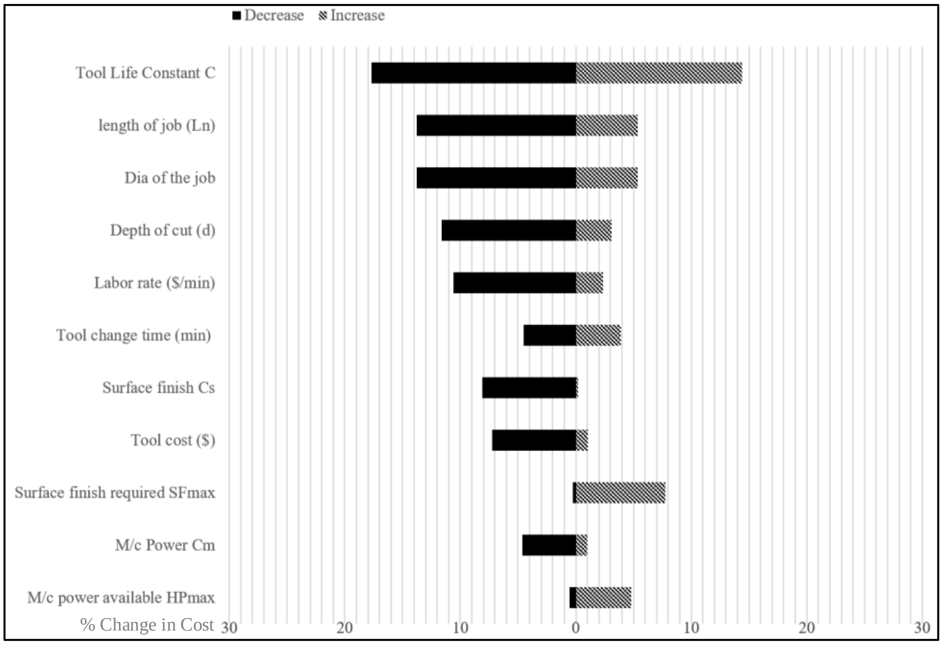

Figure 5.10: *A Tornado chart – Effect of 10% change in input parameters on cost*

Also, the computer model provides a bar chart which shows the effect of changes in input parameters to surface finish, as in **Figure 5.11**.

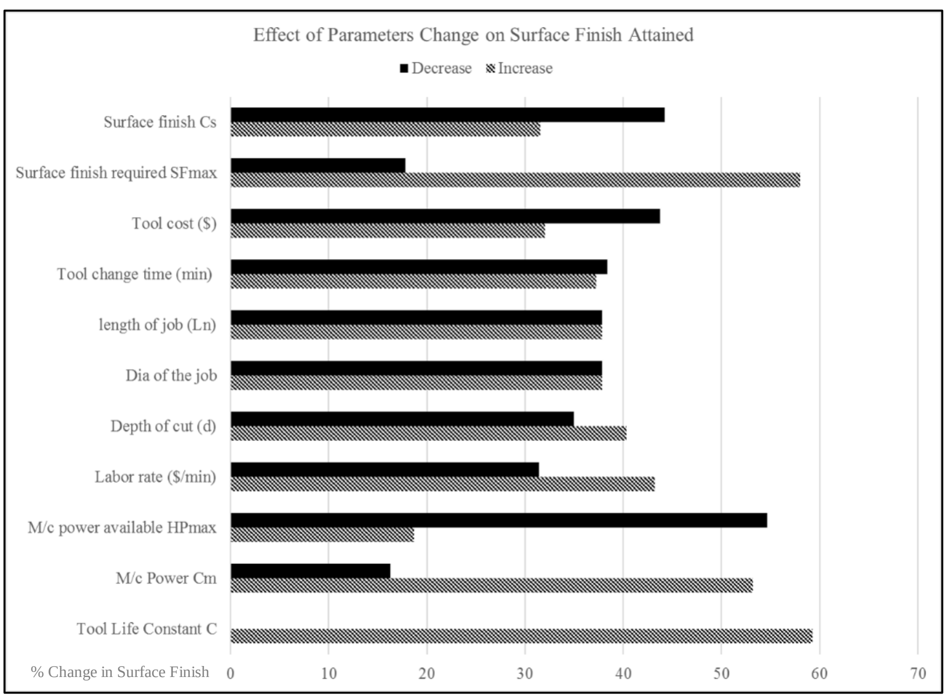

Figure 5.11: *A Bar chart – Effect of 10% change in input parameters on surface finish*

The impact on surface finish was plotted as a bar chart instead of a tornado chart because an increase or a decrease in input parameters may cause only an increase in surface finish. Also, it is important to note that the bar chart shows the impact of the individual input parameter on surface finish. In **Figure 5.11** we note that tool life constant has no effect on surface finish if it has been decreased by 10%, because the power constraint becomes loose at this point, making the appropriate solution unique as in **Case 1**. **Figure 5.12** shows the main effect of tool life constant on surface finish with some important points markers. We see that the lower bound as well as a 10% decrease from the base value of 80 would results in a solution with loose power constraint, with no change in attained surface finish. One could change these points to obtain the better results if tool life constant is the only parameter to be changed in the model. However, if two or more parameters are changed simultaneously, then the interaction becomes significant.

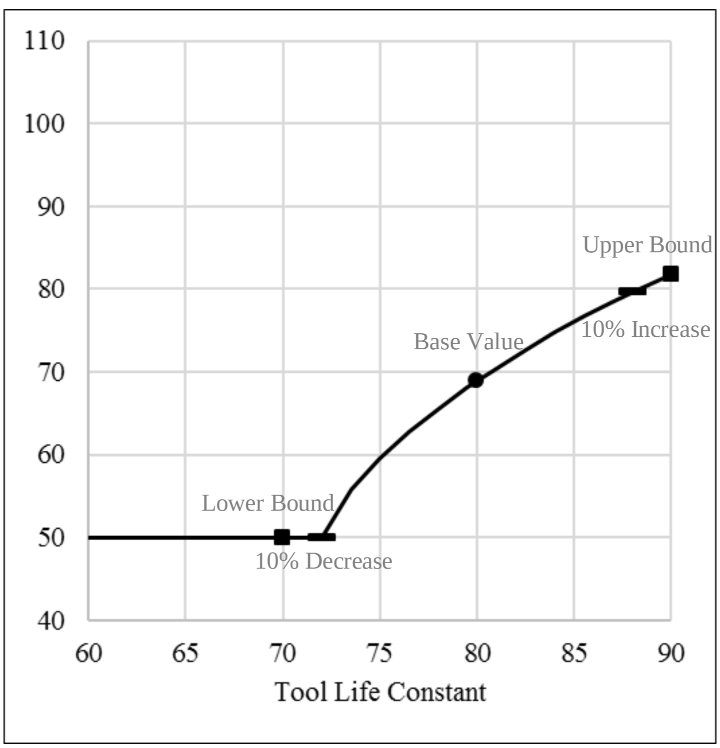

Figure 5.12: *Tool life constant bounds values*

For example, **Figure 5.13** shows the effect of changing tool life constant from 80 to 70 as well as its lower bound from 70 to 60. This change would make the solution becomes loose for horsepower value more than 2 *hp*.

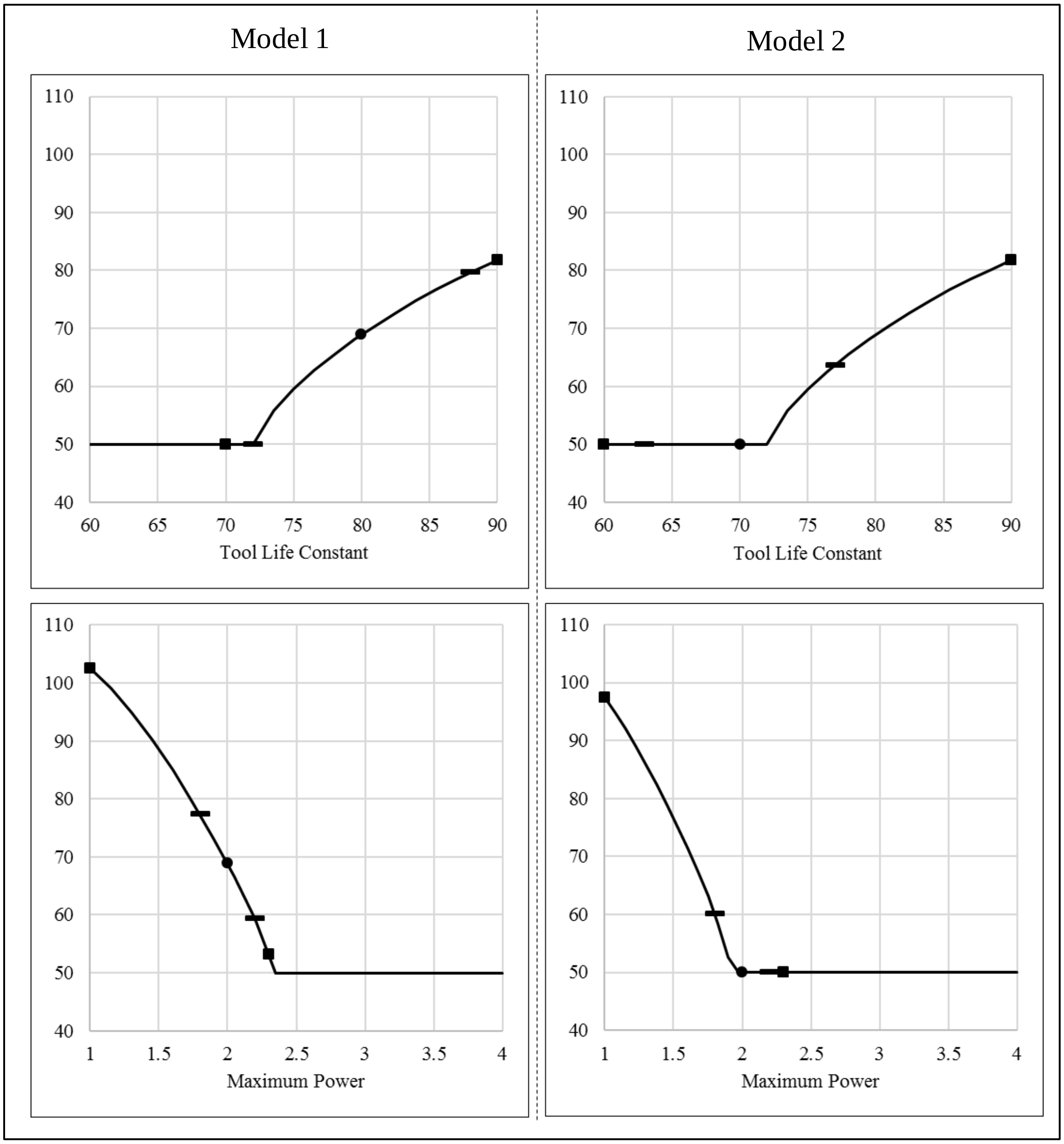

Figure 5.13: *Illustrating the effect of changing one input parameter on another*

Now, after examining the effect plots and the impact plots, suppose now that we would like to obtain 20 different alternative solutions while allowing the surface finish to go up to 8 micro-inches from the required 50 micro-inches with the following parameters bounds:

$$0.2 \le T_c \le 0.7$$

$$0.2 \le T_{\text{cost}} \le 0.8$$

$$0.05 \le L_r \le 0.2$$

$$70 \le C \le 90$$

$$1 \le HP_{\max} \le 2.3$$

$$SF_{\max} \le SF_{\max} + SF_{allowance} = 58$$

The model would suggest 20 alternatives as shown in **Table 5.2**.

Now suppose that we would not like to go the extremes of the bounds and obtain alternatives that adhere to a targeted cost of $1/piece with 2% tolerance. Then we can run the sensitivity analysis model with target mode and get 20 alternative solutions as shown in

Table 5.3. We note that the suggested alternatives are now more relaxed.

The user can then sort the alternative solutions based on his or her preferences. For example, the following solution, number 10, is best suited when higher production rate is preferred:

#	Q	D_1	D_2	v	f	HP	SF	T_c	T_{cost}	L_r	C
10	1.00	0.62	0.38	404.17	3.53E-03	2.06	51.47	0.60	0.42	0.07	81.12

Table 5.2: *Results of the numerical example – ranges mode.*

#	Q	D_1	D_2	v	f	HP	SF	T_c	T_{cost}	L_r	C
1	0.64	0.70	0.30	382.67	3.63E-03	2.00	57.55	0.58	0.21	0.05	78.16
2	0.72	0.71	0.29	363.26	3.38E-03	1.81	57.99	0.35	0.52	0.05	89.68
3	0.65	0.71	0.29	379.35	3.61E-03	1.98	57.97	0.22	0.20	0.05	75.65
4	0.70	0.70	0.30	369.33	3.44E-03	1.86	57.68	0.47	0.47	0.05	89.44
5	0.66	0.70	0.30	380.13	3.59E-03	1.97	57.48	0.32	0.23	0.05	77.84
6	0.65	0.71	0.29	379.58	3.60E-03	1.98	57.85	0.32	0.20	0.05	76.30
7	0.64	0.71	0.29	382.64	3.64E-03	2.01	57.84	0.58	0.31	0.05	86.06
8	0.66	0.71	0.29	378.06	3.58E-03	1.96	57.87	0.24	0.34	0.05	85.51
9	0.66	0.71	0.29	379.27	3.59E-03	1.97	57.78	0.23	0.31	0.05	83.43
10	0.72	0.71	0.29	363.80	3.38E-03	1.81	57.93	0.26	0.52	0.05	89.74
11	0.71	0.71	0.29	367.43	3.43E-03	1.85	57.94	0.37	0.33	0.05	81.99
12	0.70	0.71	0.29	368.09	3.44E-03	1.86	57.97	0.40	0.49	0.05	89.90
13	0.68	0.70	0.30	375.16	3.51E-03	1.92	57.34	0.33	0.45	0.05	89.50
14	0.68	0.71	0.29	372.31	3.50E-03	1.90	57.96	0.47	0.20	0.05	74.54
15	0.69	0.71	0.29	371.00	3.49E-03	1.89	57.96	0.30	0.45	0.05	89.21
16	0.73	0.71	0.29	362.30	3.36E-03	1.80	57.94	0.45	0.25	0.05	75.13
17	0.64	0.71	0.29	383.44	3.66E-03	2.02	57.85	0.55	0.37	0.05	89.86
18	0.70	0.70	0.30	371.59	3.46E-03	1.88	57.37	0.20	0.33	0.05	81.53
19	0.65	0.70	0.30	384.02	3.61E-03	2.00	57.04	0.38	0.24	0.05	79.80
20	0.66	0.70	0.30	380.57	3.58E-03	1.97	57.24	0.31	0.25	0.05	79.59

Table 5.3: *Results of the numerical example – target cost of $1/piece*

#	Q	D_1	D_2	v	f	HP	SF	T_c	T_{cost}	L_r	C
1	1.00	0.56	0.44	373.71	3.04E-03	1.71	50.00	0.51	0.49	0.05	75.28
2	1.00	0.68	0.32	359.02	3.15E-03	1.69	55.06	0.45	0.45	0.06	77.34
3	1.00	0.56	0.44	387.04	3.21E-03	1.84	50.00	0.43	0.57	0.06	80.07
4	1.00	0.70	0.30	344.57	3.06E-03	1.60	57.00	0.61	0.62	0.06	82.76
5	1.00	0.56	0.44	400.92	3.38E-03	1.98	50.00	0.30	0.41	0.06	76.12
6	1.00	0.56	0.44	399.58	3.37E-03	1.97	50.00	0.20	0.48	0.06	78.48
7	1.00	0.56	0.44	378.27	3.10E-03	1.75	50.00	0.25	0.41	0.05	72.33
8	0.99	0.69	0.31	350.21	3.11E-03	1.64	56.41	0.40	0.36	0.06	73.07
9	1.01	0.56	0.44	402.95	3.41E-03	2.00	50.00	0.55	0.31	0.06	72.23
10	1.00	0.62	0.38	404.17	3.53E-03	2.06	51.47	0.60	0.42	0.07	81.12
11	1.08*	0.56	0.44	362.08	2.90E-03	1.60	50.00	0.30	0.60	0.05	75.19
12	1.19*	0.56	0.44	409.54	3.49E-03	2.07	50.00	0.39	0.35	0.08	71.89
13	1.00	0.66	0.34	338.99	2.83E-03	1.48	53.94	0.51	0.73	0.05	82.07
14	1.00	0.56	0.44	401.65	3.39E-03	1.99	50.00	0.33	0.67	0.06	85.80
15	1.05*	0.70	0.30	347.21	3.11E-03	1.63	57.16	0.34	0.44	0.06	75.38
16	1.02*	0.56	0.44	378.77	3.10E-03	1.76	50.00	0.34	0.62	0.05	79.94
17	1.00	0.65	0.35	355.39	3.01E-03	1.62	53.33	0.51	0.60	0.06	80.79
18	1.00	0.56	0.44	371.00	3.01E-03	1.68	50.00	0.40	0.73	0.05	82.26
19	1.19*	0.56	0.44	360.81	2.88E-03	1.59	50.00	0.69	0.56	0.06	72.64
20	0.99	0.56	0.44	383.34	3.16E-03	1.80	50.00	0.34	0.60	0.05	80.50

*Solutions that are not within the required tolerance of 2%

Each solution has unique characteristics, and it is difficult to develop an algorithm that selects the best of these alternatives because that would depend on the preferences of users.

Based on the obtained results and the impact of the parameters the user can go back and forth and alter the input values until a satisfactory solution is obtained, as shown early in **Figure 1.2**.

5.6 Conclusion

Because the developed methodology is theoretically novel, a mathematical validation for the solutions obtained was developed. Moreover, some examples from the literature were tested in order to validate the accuracy of the developed equations. In addition, a sensitivity analysis model was developed to illustrate the power of the methodology further. An illustrative numerical example was also presented that showed how setting up bounds for parameters is examined and how alternative solutions can be obtained and selected to best suit user preferences.

Chapter 6: Discussion, Limitations, Conclusion, and Recommendations

6.1 Introduction

In this chapter, discussion on the importance of the developed methodology to the industry will be presented. Also, a framework of how the results obtained from the model should be communicated between different entities in a firm. In addition, other applications of the presented method will be stated. Finally, a summary of the research contribution, concluding remarks, and future work recommendation will be addressed.

6.2 Discussion

6.2.1 The Importance of the Developed Methodology

The developed methodology is connected to design for manufacturability (DMF) in some aspects. DFM is defined by Poli as a philosophy and mind-set in which manufacturing input is used at the earliest stages of design in order to design parts and products that can be produced more easily and more economically [91]. These aspects are the base ground for the importance of the developed method to manufacturing. These aspects are:

Aspect 1: Shifting cost reduction focus toward labor cost.

The developed method shifts the cost to include a higher percentage of labor cost, and consequently lower tooling cost. A cost analysis is recommended to be conducted to determine the degree of the automation best suited to the machining conditions.

Aspect 2: Making tools more available.

Tooling cost percentage becomes relatively minor, mostly because tool failures are minimized. Hence, tools become more available. Consequently, average production uptime will increase. In addition, more budget becomes readily available for more advanced tools.

Aspect 3: Reducing total cost while maintaining quality.

In this context, the quality of the part is defined as the fitness of the manufactured part for its purpose. Thus, assuming the manufacturer defines ranges for surface finish requirement, then the model provides alternatives cutting conditions that lead to economically better cost.

Each of the preceding aspects addresses an issue that eventually influences the design of the part and helps improve the manufacturing process.

The following table, Table 6.1, summarizes the advantages and disadvantages of applying the developed methodology and tools:

Table 6.1: *Advantages and disadvantages of the developed methodology and tools*

Advantages	Disadvantages
The method reduces cost	Some constraints must be tolerated
The method reduces tools failures	Production rate decreases because speed decreases
The method provides alternative solutions	Needs user analysis and input
The algorithm solves more efficient than search methods	Only applicable to this type of nonlinear functions
The sensitivity model systematically helps the design engineer	It is based on evolutionary techniques and takes time to provide results
The method is theoretically applicable	Has limited applications

6.2.2 Communicating Results

When certain firm implements the suggested model, different entities or stakeholders are involved in the decision-making process of establishing or changing the design of the process and part specifications. These decisions are affected based on the functional requirements of the part to be produced and the constraints that should be satisfied. The decision process is an iterative process which starts with an initial design and ends with a final design that includes part and process specifications such as dimensions, surface finish, cutting conditions. Figure 6.1 shows a framework that involves relevant stakeholders, internal and external, that might be affected by the decision after implementing our model.

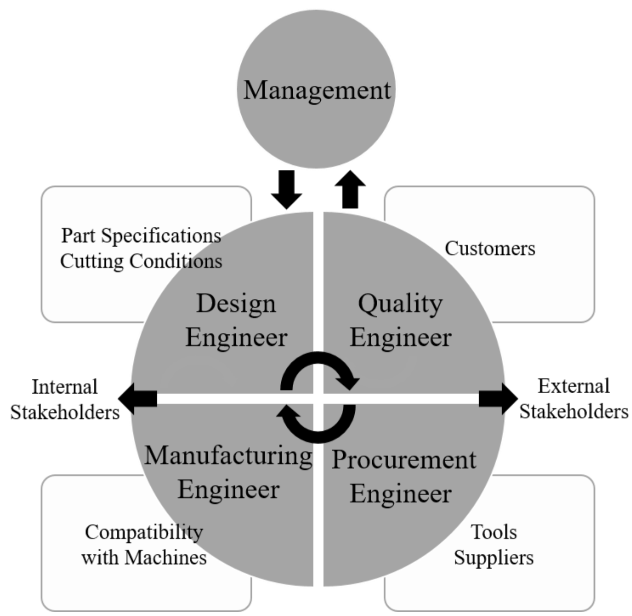

Figure 6.1: *A framework to communicate results between stakeholders*

The process starts by request from the management to reduce cost. Typically, a team of engineers is established to conduct the project. This team may involve professionals such as design engineer, manufacturing engineer, quality engineer, procurement engineer, and quality engineer. The role of each member of the team is to examine the design of the part and process based on relevant specialty. On the one hand, the design engineer uses the developed model and obtain an initial part and process design based on financial requirements from the management. He or she establishes the part knowledge necessary to carry out manufacturing. The manufacturing engineer assists the design engineer by providing inputs regarding the capability of the current machines that perform the turning process. He or she should review the design to make sure the part can be produced in manufacturing. Also, the manufacturing engineer is responsible for checking if the new cutting conditions satisfy the production rate required. On the other hand, the quality engineer examines the effect of the change in process parameters and cutting condition on the quality of the part, which ultimately leads to customer satisfaction. In addition, changes to cutting conditions may affect the procurement of tools as fewer failures should occur. Thus, procurement engineer role is vital so that further communication with external stakeholders such as tool suppliers are considered. The design process continues back and forth between the different stakeholders until a final design is accepted.

6.2.3 Other Applications

The book of professor Creese [92], named “geometric programming for design and cost optimization,” contains many engineering applications and case studies for geometric programming such as journal bearing design, liquefied petroleum gas (LPG) cylinders, the open cargo shipping box, and more. Most of the applications discussed are different in the structure of the model. For example, the journal bearing design problem has an objective function that has three cost terms, which will lead to three dual variables in the normality equation for the dual model. Such applications need further investigation, and perhaps more development, for the presented method to be applied on. Applications that has similar structure can be used as other examples where the developed methodology can be helpful. For example, the case of LPG Cylinders, which deals with the design of propane gas cylinders, can be used to demonstrate another application of our method. The problem is to design a tank by deep drawing with minimum drawing force such that the tank would have a minimum volume and the height to diameter ratio is less than one. Hence, the problem can be modeled as geometric programming model with the objective:

$$\text{Minimize } Z = T_1 hd + T_2 d^2 \tag{6.1}$$

Where h and d are the internal height and diameter of the cylinder respectively. T_1 and T_2 are constants. The constraints can be stated in standard geometric programming form as:

$$T_3 h^{-1} d^{-2} \leq 1 \tag{6.2}$$

$$hd^{-1} \leq 1 \tag{6.3}$$

Where,

$$T_1 = \pi PYC / F \tag{6.4}$$

$$T_2 = (2C - E)\pi PY / (2F) \tag{6.5}$$

$$T_3 = \frac{4}{\pi} V_{min} \tag{6.6}$$

Z = drawing force, P = internal gas pressure, Y = material yield strength, F = hoop stress, C = constant = 1.04, and E = constant = 0.65.

Note that the relation in (6.2) will be always considered tight at optimality since minimum volume is required. The *dual problem* can be formulated as:

$$\text{Maximize } G = \left(\frac{T_1}{D_1}\right)^{D_1} \left(\frac{T_2}{D_2}\right)^{D_2} (T_3)^{D_3} \tag{6.7}$$

With the normality condition:

$$D_1 + D_2 = 1 \tag{6.8}$$

Also, the orthogonality conditions:

$$D_1 - D_3 + D_4 = 0 \tag{6.9}$$

$$D_1 + 2D_2 - 2D_3 - D_4 = 0 \tag{6.10}$$

$$D_1, D_2, D_3, D_4 \geq 0$$

It can be shown that the general solution for the case when the two constraints are tight is:

$$D_1 = \frac{T_1}{T_1 + T_2} \tag{6.11}$$

$$D_2 = 1 - D_1 \tag{6.12}$$

$$D_3 = {}^2\!/_3 \tag{6.13}$$

$$D_4 = {}^2\!/_3 - D_1 \tag{6.14}$$

The other case, when the ratio constraint is loose, is not relevant to our demonstration.

The primal solutions can be recovered according to the following relations:

$$d = \sqrt{\frac{D_2 G}{T_2}} \tag{6.15}$$

$$h = \frac{D_1 G}{T_1 d} \tag{6.16}$$

Now we use the values provided for the constants from the evaluation questions following the case in professor Creese's book [92],

P = 0.2535 kg/mm^2
F = 32.33 kg/mm^2
Y = 25 kg/mm^2
C = 1.04
E = 0.65.
V_{min} =1.75x10^7 mm^3

Then we get $T_1 = 0.64$, $T_2 = 0.44$, and $T_3 = 22281692$. Using (6.11), the optimal D_1 value is 0.59 and for D_2 it is 0.41 with an optimal cost of $85579.82. The required minimum volume of 1.75x10^7 mm^3 can be attained by designing the tank with height and diameter of 281.39 *mm* each. Now suppose the cost need to be reduced further, but we do not want to violate the minimum required volume much.

Looking at Figure 6.2, which shows the primal-dual representation plot, we see that reducing the value of the optimal D_1 will violate the volume requirement, alpha, and would make the ratio constraint loose, beta. The function ρ, in this case, is defined as the ratio of the change in cost to change in volume. By applying the developed algorithm in Section 4.5.1, and changing initial uncertainty region to [0, D_1^*], we can obtain a new value for D_1 of 0.43. This new solution has 5.21% less cost, but a volume of 13,861,670.97 mm^3, which is 20.79% less than the required. Sensitivity analysis can also be applied to achieve workable alternative solutions if the new volume is not applicable.

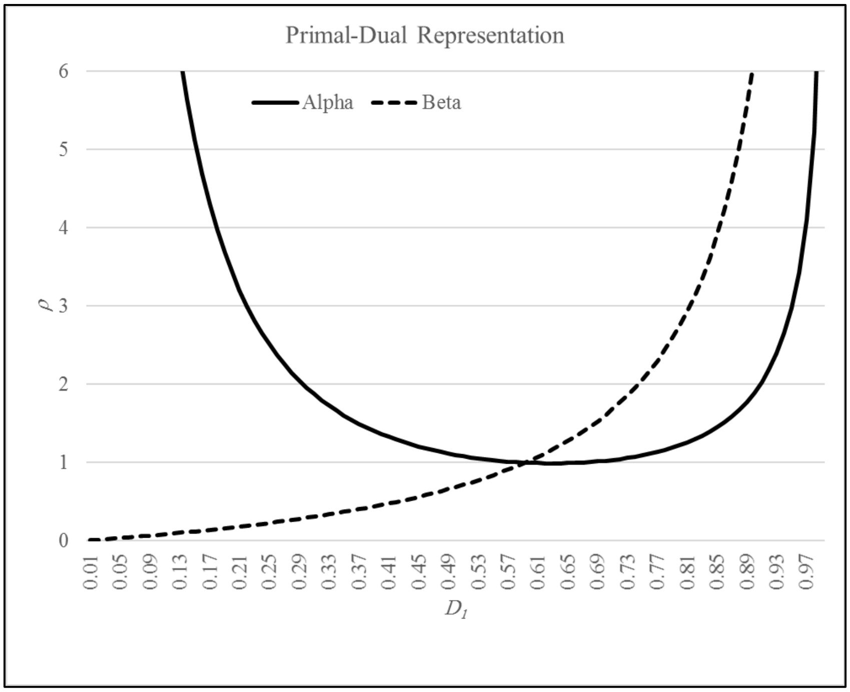

Figure 6.2: *Primal-dual representation for the LPG cylinders example*

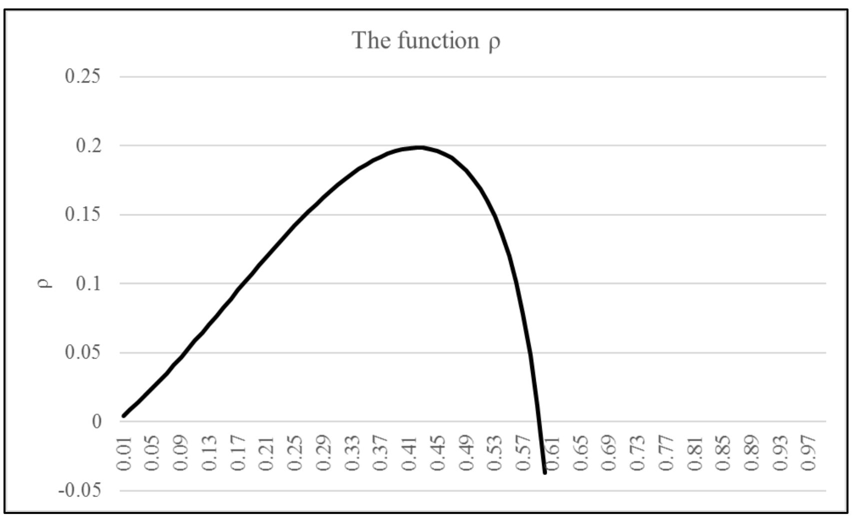

Figure 6.3: *The function ρ for the LPG cylinders example*

6.3 Research Limitations

In this section, the limitations of the research will be discussed. These shortcomings restrict the applicability of the developed methodology to a certain extent. They are as follow:

1. **The restriction to single-pass turning**: The single-pass restriction considered in this research may prevent additional cost reduction. Multi-pass turning was shown to be more efficient than single-pass turning in some cases as shown early by Ermer and Kromodihardjo [88], and then many studies used multiple passes in their optimization models [6, 9, 20, 26, 58, 93]. Going back to the numerical example presented in the previous chapter, we can see that depth of cut significantly affects cost, see Figure 6.4 (A). The smaller the depth of cut the less the cost. Moreover, changing depth of cut will affect the diameter of the workpiece after each pass, which is also a significant parameter to cost. Our analysis shows that, in certain cases, same surface finish value can be attained with some range of depth of cut when using the method developed and adjusting the required surface finish for the base model, Figure 6.4 (B). Each corresponding solution will have specific speed and feed that is different from the other solutions.

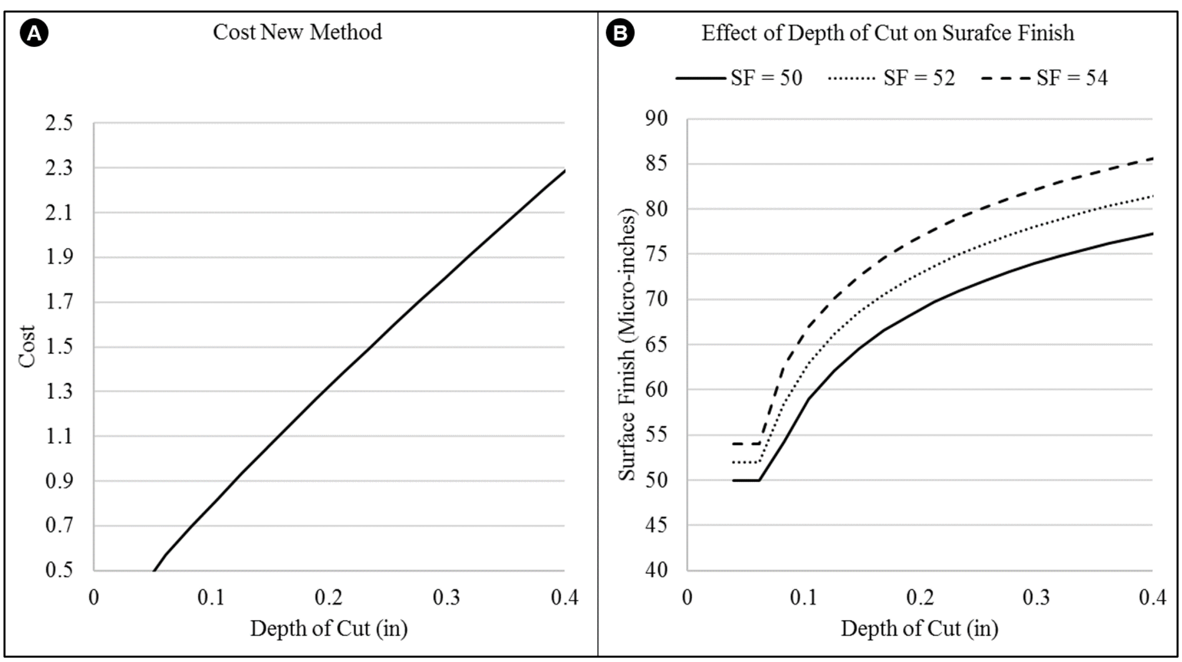

Figure 6.4: The effect of changing depth of cut on cost and surface finish

This type of analysis should be performed to find the least depth of cut required to perform either a roughing cut or finish cut in a multi-pass turning optimization. However, it is out of the scope of the project and is recommended to be performed in future research work.

2. **The developed model is not considered robust**: Most of the equations developed in this research are particularly applicable to problems with exactly similar model structure. In other words, the primal model should have two variables with two posynomial terms in the objective function as well as one in each of two constraints. This structure is important so that we would have a maximum degree of difficulty of one, where we can render the problem in terms of one dual variable and apply the algorithm. In addition, if three terms of costs are presents in the primal objective function then we would have three dual variables present in the normality constraint of the dual problem, which is considered a more complicated problem. Similarly, increasing the number of constraints would increase the number of dual variables and, hence, increase complexity.
3. **The exclusion of exponents in the analysis:** Sensitivity to the changes in exponents values were not included. Although the user can change the base values for the exponents when using different work material, the model provides no support as far as the individual effect of these values on either cost or surface finish. The reason for not including exponents in the analysis is that the practitioner prefixes these values beforehand for the work material used for the turning operation. Liu has developed a solution methodology for geometric programs that has exponents as intervals [94]. This methodology can be incorporated into our

model but that would increase the complexity of the problem, and evolutionary techniques would be the best tools to be used in such cases.

4. **On the practical validation of the model:** One primary reason for creating mathematical models of complex systems is that the true relationships that govern the real system are often virtually impossible to know precisely. Other reasons would be time and cost constraints. Although the mathematical part was verified as described in section 5.2, the practical validity of the obtained results is essential to put the developed methodology into practice. In other words, theoretically, the method is applicable, but practically, further experimentation is needed. Applications may include basic turning operations where limited horsepower is available or more reduction in cost per piece is required given that the required surface finish can be tolerated.
5. **The sensitivity analysis is user-driven:** The sensitivity analysis is user-driven, and the developed system is not considered a smart system. In our model, the user decides which parameters to be included in the model or excluded. Also, he or she define the bounds for parameters. Moreover, the time to stop the analysis and declare that the best solution has been achieved is also decided by the user. Hence, the process requires user knowledge and input in order to succeed. Improvements to the model can be made to include computer interpretation that is fed to the user.

6.4 Research Contribution

The contribution of this research work lies in the following:

1. The development of a methodology that provides alternative solutions for geometric programming applications when optimal solutions are not applicable.

2. The development of an algorithm that solves convex functions more efficient in most cases than search methods particularly for this problem.
3. The development of a computer model that can be used to apply the developed methodology.
4. The development of a sensitivity analysis model that is incorporated into the developed model to provide further solutions.

6.5 Conclusion

It is evident that current literature prefers non-traditional optimization techniques to solve optimizations models particularly because of the increasing complexity of today's applications. Nevertheless, traditional techniques such as geometric programming should not be abandoned since theoretical developments lie mainly on such techniques. The methodology developed in this research was based on geometric programming where alternative solutions can be obtained by moving away from the optimal solution. These types of solutions are only applicable when the optimal solution is practically inapplicable, or a further reduction in cost is strictly required. Working within the framework suggested, a team of professionals is essential to conclude alternative solutions that work best for the management's financial requirements. The design engineer can use the developed methodology that is supported by an algorithm, which works better than search methods available in most cases, and comes up with the initial design. Sensitivity analysis is also essential to support the iterative procedure to obtain a final design ultimately.

Finally, it should be mentioned that the results obtained in this dissertation are theoretical and solely depend on the validity of the model developed. Hence, the applications of the developed methodology are limited at present to be taken and directly used by the industry.

6.6 Future Research Work Recommendations

Future work recommendations are listed as the following:

- It has been noted that a special pattern is present when developing the equations in this research case. Hence, it might be appropriate to examine the development for higher dimensions, where we have more than two terms in the objective function as well as more than two constraints.
- The solutions approach may be improved by looking at the following points:
  - Apply Nelder-Mead method as a solution approach for the problem to be handled in higher dimensions instead of rendering the equations in terms of one dual variable.
  - Apply the methods of roots finding to equation (4.39) as a solution procedure for the main problem, because the roots of the natural logarithm of that equation represent the two solution cases.
  - Use available methods to convert the initial geometric programming primal model to the convex problem and use interior point method as a solution approach.
- Utilize process planning knowledge to study if additional manufacturing process can be applied to composite the lost quality of surface finish.
- Utilize multiple pass turning environment as discussed in the limitations of this work.
- The sensitivity analysis model needs to be studied in much more details. For example, it is possible to set some termination rules for the sensitivity analysis model to get solutions faster.

References

1. Davim, J.P., *Metal Cutting: Research Advances*. 2010: Nova Science Publishers, Incorporated.
2. Agapiou, J., *The optimization of machining operations based on a combined criterion, part 1: the use of combined objectives in single-pass operations.* Journal of Manufacturing Science and Engineering, 1992. **114**(4): p. 500-507.
3. Ermer, D. and Morris, M., *A treatment of errors of estimation in determining optimum machining conditions.* International Journal of Machine Tool Design and Research, 1969. **9**(4): p. 357-362.
4. Iwata, K., et al., *A probabilistic approach to the determination of the optimum cutting conditions.* Journal of Manufacturing Science and Engineering, 1972. **94**(4): p. 1099-1107.
5. Wang, X., et al., *Performance-based optimal selection of cutting conditions and cutting tools in multipass turning operations using genetic algorithms.* International Journal of Production Research, 2002. **40**(9): p. 2053-2065.
6. Chen, M.-C. and Tsai, D.-M., *A simulated annealing approach for optimization of multi-pass turning operations.* International Journal of Production Research, 1996. **34**(10): p. 2803-2825.
7. Su, C.-T. and Chen, M.-C., *Computer-aided optimization of multi-pass turning operations for continuous forms on CNC lathes.* IIE transactions, 1999. **31**(7): p. 583-596.
8. Sönmez, A.İ., et al., *Dynamic optimization of multipass milling operations via geometric programming.* International Journal of Machine Tools and Manufacture, 1999. **39**(2): p. 297-320.
9. Onwubolu, G. and Kumalo, T., *Optimization of multipass turning operations with genetic algorithms.* International Journal of Production Research, 2001. **39**(16): p. 3727-3745.
10. Tandon, V., El-Mounayri, H. and Kishawy, H., *NC end milling optimization using evolutionary computation.* International Journal of Machine Tools and Manufacture, 2002. **42**(5): p. 595-605.

11. Singh, K., Bhangu, G.S. and Gill, S.S., *Optimization of Surface Roughness In CNC Turning of Aluminum Using ANOVA Technique.* Advancements in Engineering and Technology, 2015: p. 583.

12. Dave, H., Patel, L. and Raval, H., *Effect of machining conditions on MRR and surface roughness during CNC Turning of different Materials Using TiN Coated Cutting Tools–A Taguchi approach.* International Journal of Industrial Engineering Computations, 2012. **3**(5): p. 925-930.

13. King, R., *Handbook of High-Speed Machining Technology*. 2013: Springer US.

14. Çelik, Y.H., Kilickap, E. and Güney, M., *Investigation of cutting parameters affecting on tool wear and surface roughness in dry turning of Ti-6Al-4V using CVD and PVD coated tools.* Journal of the Brazilian Society of Mechanical Sciences and Engineering, 2016: p. 1-9.

15. Lawal, S. and Ugheoke, B., *Effect of HSS and Tungsten Carbide Tools on Surface Roughness of Aluminium Alloy during Turning Operation.* American Journal of Mechanical Engineering, 2016. **4**(2): p. 60-64.

16. Siddhpura, A. and Paurobally, R., *A review of flank wear prediction methods for tool condition monitoring in a turning process.* The International Journal of Advanced Manufacturing Technology, 2013. **65**(1-4): p. 371-393.

17. Abdullah, A., Chia, L. and Samad, Z., *The effect of feed rate and cutting speed to surface roughness.* Asian Journal of Scientific Research, 2010. **3**(4): p. 278-287.

18. Davim, J.P., *Machining: Fundamentals and Recent Advances*. 2008: Springer London.

19. De Garmo, E.P., Black, J.T. and Kohser, R.A., *DeGarmo's materials and processes in manufacturing*. 2011: John Wiley & Sons.

20. Aryanfar, A. and Solimanpur, M. *Optimization of multi-pass turning operations using genetic algorithms*. in *Proceedings of the 2012 International Conference on Industrial Engineering and Operations Management. Istanbul, Turkey*. 2012.

21. Camposeco-Negrete, C., *Optimization of cutting parameters using Response Surface Method for minimizing energy consumption and maximizing cutting quality in turning of AISI 6061 T6 aluminum.* Journal of cleaner production, 2015. **91**: p. 109-117.

22. Çolak, O., *Optimization of machining performance in high-pressure assisted turning of Ti6Al4V alloy.* Strojniški vestnik-Journal of Mechanical Engineering, 2014. **60**(10): p. 675-681.
23. Devaki, K., Babu, K.S. and Reddy, K.H., *Mathematical Modeling and Optimization of Turning Process Parameters Using Response Surface Methodology.* International Journal of Applied Science and Engineering 13 (1), 2015: p. 55-68.
24. Durairaj, M. and Gowri, S., *Parametric optimization for improved tool life and surface finish in micro turning using genetic algorithm.* Procedia Engineering, 2013. **64**: p. 878-887.
25. Ganesan, H. and Mohankumar, G., *Optimization of machining techniques in CNC turning centre using genetic algorithm.* Arabian Journal for Science and Engineering, 2013. **38**(6): p. 1529-1538.
26. Jabri, A., El Barkany, A. and El Khalfi, A., *Multi-objective optimization using genetic algorithms of multi-pass turning process.* Engineering, 2013. **5**(07): p. 601.
27. Jain, H., et al., *Optimisation and evaluation of machining parameters for turning operation of Inconel-625.* Materials Today: Proceedings, 2015. **2**(4): p. 2306-2313.
28. Lu, K., et al., *Optimization of sequential subdivision of depth of cut in turning operations using dynamic programming.* The International Journal of Advanced Manufacturing Technology, 2013. **68**(5-8): p. 1733-1744.
29. Raja, S.B., et al., *Optimization of constrained machining parameters in turning operation using firefly algorithm.* Journal of Applied Sciences, 2012. **12**(10): p. 1038.
30. Senthilkumaar, J., Selvarani, P. and Arunachalam, R., *Intelligent optimization and selection of machining parameters in finish turning and facing of Inconel 718.* The International Journal of Advanced Manufacturing Technology, 2012. **58**(9-12): p. 885-894.
31. Yildiz, A.R., *Optimization of multi-pass turning operations using hybrid teaching learning-based approach.* The International Journal of Advanced Manufacturing Technology, 2013. **66**(9-12): p. 1319-1326.
32. Yildiz, A.R., *Optimization of cutting parameters in multi-pass turning using artificial bee colony-based approach.* Information Sciences, 2013. **220**: p. 399-407.

33. Asiltürk, I. and Akkuş, H., *Determining the effect of cutting parameters on surface roughness in hard turning using the Taguchi method.* Measurement, 2011. **44**(9): p. 1697-1704.

34. Rao, C., Rao, D.N. and Srihari, P., *Influence of cutting parameters on cutting force and surface finish in turning operation.* Procedia Engineering, 2013. **64**: p. 1405-1415.

35. Singh, H., Khanna, R. and Garg, M., *Effect of cutting parameters on MRR and surface roughness in turning EN-8.* Current Trends in Engineering Research, 2011. **1**(1).

36. Verma, D.S. and Dikroos, F.W.B., *Development of Quality Circles in an Organisation (A Case Study in the Areas of Design for Manufacturing and Design for Assembly In Machine Shop of Tool Room, Indore).* PARIPEX-Indian Journal of Research, 2016. **4**(6).

37. Armarego, E. and Brown, R.H., *The machining of metals.* Prentice-Hall Inc, Englewood Cliffs, N. J., 1969, 437 P, 1969.

38. Gilbert, W., *Economics of machining.* Machining theory and practice, 1950: p. 465-485.

39. Hati, S. and Rao, S., *Determination of optimum machining conditions—Deterministic and probabilistic approaches.* Journal of Manufacturing Science and Engineering, 1976. **98**(1): p. 354-359.

40. Ermer, D.S., *A Bayesian model of machining economics for optimization by adaptive control.* Journal of Manufacturing Science and Engineering, 1970. **92**(3): p. 628-632.

41. Groover, M., *Monte Carlo simulation of the machining economics problem.* Journal of Manufacturing Science and Engineering, 1975. **97**(3): p. 931-938.

42. Ermer, D. and Shah, B., *Analytical Sensitivity Studies of the Optimum Machining Conditions for Milling, Drilling, Reaming, and Tapping.* Journal of Manufacturing Science and Engineering, 1973. **95**(1): p. 312-316.

43. Friedman, M. and Tipnis, V., *Cutting Rate-Tool Life Characteristic Functions for Material Removal Processes—Part 1: Theory.* Journal of Manufacturing Science and Engineering, 1976. **98**(2): p. 481-486.

44. Mukherjee, I. and Ray, P.K., *A review of optimization techniques in metal cutting processes.* Computers & Industrial Engineering, 2006. **50**(1): p. 15-34.

45. Armarego, E. and Russell, J., *Maximum profit rate as a criterion for the selection of machining conditions.* International Journal of Machine Tool Design and Research, 1966. **6**(1): p. 15-23.

46. Bhattacharyya, A., Faria-Gonzalez, R. and Ham, I., *Regression analysis for predicting surface finish and its application in the determination of optimum machining conditions.* Journal of Manufacturing Science and Engineering, 1970. **92**(3): p. 711-714.

47. Brewer, R., *Parameter selection problem in machining.* Ann. CIRP, 1966. **14**(11): p. 55-61.

48. Ermer, D., *Optimization of the constrained machining economics problem by geometric programming.* Journal of Manufacturing Science and Engineering, 1971. **93**(4): p. 1067-1072.

49. Gopalakrishnan, B. and Al-Khayyal, F., *Machine parameter selection for turning with constraints: an analytical approach based on geometric programming.* The International Journal of Production Research, 1991. **29**(9): p. 1897-1908.

50. Rao, S. and Hati, S., *Computerized selection of optimum machining conditions for a job requiring multiple operations.* Journal of Manufacturing Science and Engineering, 1978. **100**(3): p. 356-362.

51. Wang, J., et al., *Optimization of cutting conditions for single pass turning operations using a deterministic approach.* International Journal of Machine Tools and Manufacture, 2002. **42**(9): p. 1023-1033.

52. Wang, R.-T. and Liu, S.-T., *An economic machining process model with interval parameters.* The International Journal of Advanced Manufacturing Technology, 2007. **33**(9-10): p. 900-910.

53. Lambert, B. and Walvekar, A., *The application of geometric programming to machining variable selection.* Int J Prod Res, 1970. **8**: p. 123-133.

54. Yellowley, I. and Gunn, E., *The optimal subdivision of cut in multi-pass machining operations.* The International Journal of Production Research, 1989. **27**(9): p. 1573-1588.

55. Meng, Q., Arsecularatne, J. and Mathew, P., *Calculation of optimum cutting conditions for turning operations using a machining theory.* International Journal of Machine Tools and Manufacture, 2000. **40**(12): p. 1709-1733.

56. Prasad, A., Rao, P. and Rao, U., *Optimal selection of process parameters for turning operations in a CAPP system.* International Journal of Production Research, 1997. **35**(6): p. 1495-1522.

57. Gupta, R., Batra, J. and Lal, G., *Determination of optimal subdivision of depth of cut in multipass turning with constraints.* International Journal of Production Research, 1995. **33**(9): p. 2555-2565.

58. Tan, F. and Creese, R., *A generalized multi-pass machining model for machining parameter selection in turning.* The International Journal of Production Research, 1995. **33**(5): p. 1467-1487.

59. Satyanarayana, B., Rao, P. and Tewari, N. *Application of non-linear goal programming technique in metal cutting*. in *Proceedings of 12th AMTDR conference, IIT Delhi, India*. 1986.

60. Iwata, K. and Moriwaki, T., *An application of acoustic emission measurement to in-process sensing of tool wear.* Annals of the CIRP, 1977. **26**(1): p. 21-26.

61. Goranskii, G.K., *Calculation of cutting conditions by means of electronic computers*. 1967: National Lending Library for Science and Technology.

62. Chua, M., et al., *Optimization of cutting conditions for multi-pass turning operations using sequential quadratic programming.* Journal of Materials Processing Technology, 1991. **28**(1): p. 253-262.

63. Reddy, S.B., Shunmugam, M. and Narendran, T., *Optimal sub-division of the depth of cut to achieve minimum production cost in multi-pass turning using a genetic algorithm.* Journal of Materials Processing Technology, 1998. **79**(1): p. 101-108.

64. Wong, S. and Hamouda, A., *Development of genetic algorithm-based fuzzy rules design for metal cutting data selection.* Robotics and Computer-Integrated Manufacturing, 2002. **18**(1): p. 1-12.

65. Asokan, P., Saravanan, R. and Vijayakumar, K., *Machining parameters optimisation for turning cylindrical stock into a continuous finished profile using*

genetic algorithm (GA) and simulated annealing (SA). The International Journal of Advanced Manufacturing Technology, 2003. **21**(1): p. 1-9.

66. Vijayakumar, K., et al., *Optimization of multi-pass turning operations using ant colony system.* International Journal of Machine Tools and Manufacture, 2003. **43**(15): p. 1633-1639.

67. Cus, F. and Balic, J., *Optimization of cutting process by GA approach.* Robotics and Computer-Integrated Manufacturing, 2003. **19**(1): p. 113-121.

68. Li-Ping, Z., Huan-Jun, Y. and Shang-Xu, H., *Optimal choice of parameters for particle swarm optimization.* Journal of Zhejiang University Science A, 2005. **6**(6): p. 528-534.

69. Sardinas, R.Q., Santana, M.R. and Brindis, E.A., *Genetic algorithm-based multi-objective optimization of cutting parameters in turning processes.* Engineering Applications of Artificial Intelligence, 2006. **19**(2): p. 127-133.

70. Mesquita, R., Krasteva, E. and Doytchinov, S., *Computer-aided selection of optimum machining parameters in multipass turning.* The International Journal of Advanced Manufacturing Technology, 1995. **10**(1): p. 19-26.

71. Chen, M.-C. and Su, C.-T., *Optimization of machining conditions for turning cylindrical stocks into continuous finished profiles.* International Journal of Production Research, 1998. **36**(8): p. 2115-2130.

72. Arsecularatne, J.A., Hinduja, S. and Barrow, G., *Optimum cutting conditions for turned components.* Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 1992. **206**(1): p. 15-31.

73. Saravanan, R., et al., *Optimization of cutting conditions during continuous finished profile machining using non-traditional techniques.* The International Journal of Advanced Manufacturing Technology, 2005. **26**(1-2): p. 30-40.

74. Natarajan, U., Periasamy, V. and Saravanan, R., *Application of particle swarm optimisation in artificial neural network for the prediction of tool life.* The International Journal of Advanced Manufacturing Technology, 2007. **31**(9-10): p. 871-876.

75. Prasad, C.F., Jayabal, S. and Natarajan, U., *Optimization of tool wear in turning using genetic algorithm.* Indian Journal of Engineering and Materials Sciences, 2007. **14**(6): p. 403.

76. Zuperl, U. and Cus, F., *Optimization of cutting conditions during cutting by using neural networks.* Robotics and Computer-Integrated Manufacturing, 2003. **19**(1): p. 189-199.

77. Rao, R.V., Savsani, V.J. and Vakharia, D., *Teaching–learning-based optimization: an optimization method for continuous non-linear large scale problems.* Information Sciences, 2012. **183**(1): p. 1-15.

78. Belloufi, A., Assas, M. and Rezgui, I., *Optimization of turning operations by using a hybrid genetic algorithm with sequential quadratic programming.* Journal of applied research and technology, 2013. **11**(1): p. 88-94.

79. Saravanan, R., Asokan, P. and Sachithanandam, M., *Comparative analysis of conventional and non-conventional optimisation techniques for CNC turning process.* The International Journal of Advanced Manufacturing Technology, 2001. **17**(7): p. 471-476.

80. Duffin, R.J., Peterson, E.L. and Zener, C., *Geometric programming: theory and application*. 1967: Wiley New York.

81. Phillips, D.T. and Beightler, C.S., *Geometric programming: A technical state-of-the-art survey.* AIIE Transactions, 1973. **5**(2): p. 97-112.

82. Dembo, R.S., *A set of geometric programming test problems and their solutions.* Mathematical Programming, 1976. **10**(1): p. 192-213.

83. Sarma, P., et al., *A comparison of computational strategies for geometric programs.* Journal of Optimization Theory and Applications, 1978. **26**(2): p. 185-203.

84. Rijckaert, M. and Martens, X., *Comparison of generalized geometric programming algorithms.* Journal of Optimization Theory and Applications, 1978. **26**(2): p. 205-242.

85. Yang, H.-H. and Bricker, D.L., *Investigation of path-following algorithms for signomial geometric programming problems.* European Journal of Operational Research, 1997. **103**(1): p. 230-241.

86. Kortanek, K. and No, H., *A second order affine scaling algorithm for the geometric programming dual with logarithmic barrier.* Optimization, 1992. **23**(4): p. 303-322.

87. Kortanek, K.O., Xu, X. and Ye, Y., *An infeasible interior-point algorithm for solving primal and dual geometric programs.* Mathematical Programming, 1997. **76**(1): p. 155-181.

88. Ermer, D. and Kromodihardjo, S., *Optimization of multipass turning with constraints.* Journal of Manufacturing Science and Engineering, 1981. **103**(4): p. 462-468.

89. Boyd, S., et al., *A tutorial on geometric programming.* Optimization and engineering, 2007. **8**(1): p. 67-127.

90. PETROPOULOS, P.G., *Optimal selection of machining rate variables by geometric programming.* THE INTERNATIONAL JOURNAL OF PRODUCTION RESEARCH, 1973. **11**(4): p. 305-314.

91. Poli, C., *Design for manufacturing : a structured approach*. 2001, Boston: Butterworth-Heinemann.

92. Creese, R.C., *Geometric programming for design and cost optimization (with illustrative case study problems and solutions).* Synthesis Lectures On Engineering, 2010. **5**(1): p. 1-140.

93. Chandrasekaran, M., Muralidhar, M. and Dixit, U., *Online optimization of multipass machining based on cloud computing*. The International Journal of Advanced Manufacturing Technology, 2013. **65**(1-4): p. 239-250.

94. Liu, S.-T., *Posynomial geometric programming with interval exponents and coefficients.* European Journal of Operational Research, 2008. **186**(1): p. 17-27.