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Using the Concrete-Representational-Abstract Sequence to Connect Manipulatives, Problem Solving Schemas, and Equations in Word Problems with Fractions

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ABSTRACT

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Students with learning disabilities or learning difficulties in mathematics often have difficulties solving word problems with fractions. These difficulties limit students' abilities to solve everyday math problems and develop the skills necessary for higher level mathematics. Prior research on problem solving indicates that direct instruction on problem schemas can improve problem solving performance. Previous research also suggests that instruction using the concrete-representational-abstract (CRA) sequence and instruction with virtual manipulatives can enhance understanding of mathematical concepts. However, a CRA sequence that incorporates virtual manipulatives has not been combined with schema-based instruction to help students solve word problems with fractions. The purpose of this study was to examine the effects of using an intervention that combined the CRA sequence with virtual manipulatives and schema-based instruction to improve the problem solving performance of students with learning disabilities or learning problems in mathematics on word problems with fractions. This sequence of instruction was combined with a mnemonic strategy called the LISTS strategy to help students remember the steps in the problem solving sequence. Using a single-case multiple baseline across participants design, the researcher provided an intervention to five students in the fifth grade that included instruction in three problem schemas for addition and subtraction (change, compare, and group). Results indicated that all students made some gains in performance on problems similar to those presented during the intervention, but the three students who were able to make connections between problem schemas and equations demonstrated significant gains in performance. The concrete models and virtual models used in the CRA sequence enhanced understanding of fraction word problems for some, but not all, students. Additionally, analysis of student performance on pre- and post-tests of problems with novel features indicated that students made only small gains in performance on fraction word problems that included difficult vocabulary, irrelevant information, or information that required different conceptualizations than those presented during the intervention.

Dedication

I dedicate this work to my family. To my wonderful husband Paul, I could not have done this without your support. Thank you for all that you gave up for me to reach my goals. To my daughters Kirsten, Rachael, and Sarah, thanks for being patient with me when I had to complete my work. This would not have possible without your love and understanding.

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Mom and Dad thank you for instilling in me the importance of lifelong learning and for providing me with the best education possible. I am grateful that I was fortunate enough to have such wonderful parents who always believed in me. Thank you to Mom and Pop who have always treated me as part of the Reneau family. Your help throughout this process has been invaluable.

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Introduction

Chapter one will describe concerns related to student performance on mathematics assessments with a specific focus on students with learning problems or disabilities in math. The importance of mathematical problem solving in these assessments and in national standards for mathematics instruction will be discussed. An overview of effective instructional strategies in problem solving for students with learning problems or disabilities will be presented with an analysis of the gaps in the research on problem solving instruction. A statement of purpose and research questions based on these gaps in the research will address concerns related to the lack of research on problem solving instruction with fractions. The final section of the chapter will include the possible limitations of the study and a glossary of key terms that will be an integral part of the proposed research.

Problem Context: Student Performance in Mathematics

Research suggests that strong student performance in mathematics can lead to individual success in the workplace and may impact the success of the United States in our global economy (Achieve, 2008). Yet recent national and international assessments indicate that students in the United States have difficulty with higher level reasoning skills in mathematics and score below students from other industrialized nations on assessments of these skills (TIMMS, 2007; NAEP, 2009). In the 2009 Program for International Assessment (PISA), students from the United States scored 25th out of 34 countries on problem solving and math literacy when compared to other industrialized countries. According to a report by the U.S. Department of Education, 24% of the fifteen year old students who took the assessment scored at the basic level, meaning that they were only able to make direct inferences from a single source and conduct literal interpretations of problem information. Furthermore, 23% of U.S. students scored below this basic level of achievement. These students were unable to apply mathematical concepts in problem solving contexts (Fleischman, Hopstock, Pelczar, Shelley, & Xie, 2010).

Results of assessments conducted in the United States indicate that students with disabilities perform lower than students without disabilities. The 2011 reports on the National Assessment of Educational Progress show that 45% of students with disabilities at the fourth grade level scored at the "below basic" level compared to 18% of students without disabilities. The achievement gap between students with disabilities and students without disabilities was even greater by eighth grade with 64% of students with disabilities scoring below basic and only 27% of students without disabilities scoring at this level (NAEP, 2011). Students at this "below basic" level did not possess understanding of grade level concepts and were not able to solve simple grade level word problems (National Center of Educational Statistics, 2011).

Students with low achievement in mathematics obtained scores that were slightly better than students with disabilities on national assessments, but they still performed below their average achieving peers (National Mathematics Advisory Panel, 2008). Furthermore, reports on the 2011 NAEP assessments state that low performing students in the fourth and eighth grades who scored at the 10^{th} percentile or lower did not show significant improvements from 2009 to 2011, while their higher performing peers did show significant gains. While almost one-fourth of the students who scored at the 10^{th} percentile were students with disabilities, over three-fourths of these students were not classified as students with disabilities (NCES, 2011). These low levels of achievement are problematic for students with low achievement or learning disabilities in mathematics. Problem solving skills are necessary for all individuals to function at school, home, and work. According to the National Mathematics Advisory Panel Report

(NMAP), higher-level mathematical skills have been correlated with greater access to college and to careers with greater incomes (NMAP, 2008). Future jobs will require a broader and more thoughtful understanding of quantitative concepts than current positions and mathematical problem solving processes will be necessary for many jobs in the 21st century (Xin, Jitendra, & Deatline-Buchman, 2005).

National standards and policies related to problem solving. The National Assessment of Educational Progress in mathematics is aligned with current national guidelines for teaching mathematics which include problem solving as a critical element of instruction across all topics in mathematics and across all grade levels (National Council of Teachers of Mathematics, 2000). Problem solving is so important that it was identified as one of the five major process standards in the National Council of Teachers of Mathematics (NCTM) *Principles and Standards* (2000), the guiding standards for mathematics instruction across the United States. The NCTM stated that "problem solving…is not only a goal of learning mathematics, but a major means of doing so" (NCTM, 2000, p. 52). This emphasis on problem solving was incorporated into the NCTM standards in response to poor student performance on national and international assessments in the 1980s and 1990s (Maccini et al., 2007). As a result of the standards, researchers began to focus on higher level problem solving instruction for all students. The passage of the Individuals with Disabilities Act (IDEA1997), which required that students with disabilities have access to the general education curriculum, and the No Child Left Behind Act (NCLB, 2002), which required that all students meet grade level expectations, led to additional research on problem solving instruction for students with learning problems because students were expected to master grade level content in mathematics (Maccini et al, 2007). Even with the additional emphasis on problem solving instruction for students with learning problems, many challenges still remain.

Students with learning disabilities or with learning problems in math continue to have difficulty understanding problem components, identifying effective strategies, or applying strategies when attempting to solve word problems (Hegarty, Mayer, & Monk, 1995; Jordan & Montani, 1997; Gurganus, 2007). Additionally, these students often have weak reading comprehension skills that limit their understanding of written mathematical problems (Fuchs & Fuchs, 2002).

Problem solving with fractions. Difficulty with problem solving is compounded by lack of procedural and conceptual knowledge related to specific content areas. One of the most difficult content areas for students in math is fractions. Authors of the NMAP report suggest that almost half of middle and high school students have difficulty with basic fraction concepts (Misquitta, 2011). Visual models of fractions that are typically used in the elementary grades, such as pizzas and pies, only provide a limited representation and often hinder student understanding when they move to more complex problems (Wu, 2008). Word problems with fractions pose even more difficulty for students with learning disabilities due to deficits in working memory (Hecht, Close, & Santisi, 2003). Additionally, students with learning problems in math often use ineffective strategies to solve fraction problems (Grobecker, 1999). According to the National Mathematics Advisory Panel this difficulty with fractions can be an “obstacle to further progress in mathematics” (U.S. Department of Education, 2008, p. 28). Because of these concerns, the U.S. Department of Educational Sciences issued the following recommendations in its report on evidence-based practices for students who struggle with mathematics: (1) Focus on rational numbers in grades four through eight; (2) Emphasize problem solving instruction based on common underlying structures; and (3) Include materials that provide visual representations of mathematical ideas to help students develop understandings of mathematical concepts (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). Recommendations were made for students with learning disabilities and students with low achievement because students identified with LD or as low achievers are often combined in the research and similar instructional strategies have been found to be effective for both groups (Misquitta, 2011).

Problem Solving Instruction for Students with Learning Problems in Math

Recent research on mathematics instruction for students with disabilities or learning problems in mathematics reflects this focus on using visual representations and problem solving in math instruction. In a review of studies on interventions for secondary students with learning disabilities or learning problems in mathematics, eleven studies conducted from 1995-2006 included some assessment of problem solving performance and seven studies focused explicitly on instruction and assessment of problem solving (Maccini et al., 2007). Furthermore, an analysis of the results of these studies and studies conducted with students in the elementary grades suggests that cognitive interventions including instruction using a concrete-representational-abstract sequence, mnemonic strategy instruction, and schema-based instruction led to significant gains in student achievement (Maccini et al., 2007; Powell, 2011).

The CRA sequence with virtual manipulatives and mnemonic strategies. When using a concrete-representational-abstract sequence, researchers help students develop an understanding of concepts by linking instruction with physical manipulatives to representational drawings of those manipulatives and abstract equations of the problems represented by those drawings. Research indicates that this type of instruction has led to improved performance on fraction equivalence concepts (Butler, Miller, Crehan, Babbitt, & Pierce, 2003), area and perimeter problems (Cass, Cates, Smith, & Jackson, 2003), problem solving with integers (Maccini & Ruhl, 2000), and algebra equations (Witzel, Mercer, & Miller, 2003). Research suggests that student performance with the CRA sequence could be enhanced by incorporating virtual manipulatives into the sequence. Virtual manipulatives are computer models of physical manipulatives. Unlike representational drawings, virtual manipulatives can be moved or turned to simulate student experiences with physical manipulatives (Moyer-Packenham, 2010). Instruction combining physical and virtual manipulatives can lead to improved student performance on computation and problem solving tasks that can be effectively modeled with both types of manipulatives (Gire, Carmichael, Chini, Rouinfar, & Rebello, 2010; Terry, 1996).

Research also indicates that mnemonic strategy instruction can lead to improvements in procedural skills such as solving fraction problems with unlike denominators (Test & Ellis, 2005). Mnemonic strategies have been combined with instruction in the CRA sequence to enhance understanding of addition and subtraction word problems (Mancl, 2011), integers, and algebraic equations (Maccini & Hughes, 2000; Gagnon & Maccini, 2001). When using mnemonic strategy instruction in these studies, researchers made a cue word for students by creating a word from the first letter of each problem solving step. This word was used to help students remember each step of the problem solving process.

Schema-based instruction and schema-broadening instruction. Another instructional approach that has led to gains in problem solving achievement for students with learning difficulties is schema-based instruction. In this type of instruction students were asked to identify a problem type based on conceptual understanding of the problem structure and then use a diagram to represent the problem (Xin et al., 2005). This use of schema diagrams provided students with a graphic representation to help them identify critical elements in word problems and organize this information so they could visualize the problem. Identifying specific schema types also helped students determine the operations necessary to solve different types of problems (Bender, 2009).

Initial research on schema-based instruction focused on individual instruction or small group instruction in pull-out special education settings with elementary and middle school students (Jitendra, Hoff, & Beck, 1999; Jitendra, DiPipi, & Jones, 2002). More recent studies have compared schema-based instruction to general strategy instruction in both small group and inclusive settings at the elementary and middle school levels (Xin et al., 2005; Jitendra & Star, 2011). An analysis of the addition and subtraction word problems from the examples, teacher scripts, and article manuscripts used in these studies shows that most research interventions in schema-based instruction focused on problems with whole numbers operations (Jitendra et al., 1999; Fuchs et al., 2010; Powell, Fuchs, & Fuchs, 2010; Griffin & Jitendra, 2008; Fuchs Seethaler, Powell, Fuchs, Hamlett, Fletcher, 2008a; Xin, Wiles, & Lin, 2008). Problems with fractions were only noted in several studies that included a schema called a "half" schema (Fuchs et al., 2006; Fuchs, Fuchs, Craddock, Hollenbeck, Hamlett, & Schatschneider, 2008b). While these studies did incorporate some problems where students had to determine half price or a half of an amount, no other fractions were included and instruction only addressed splitting objects or sets into halves.

Recent research also has focused on explicit instruction in transfer skills and connecting schematic diagrams to algebraic equations. This type of instruction, called schema-broadening instruction, includes instruction in the specific word problem schemas, but it also includes explicit instruction in transferring understanding of problem schemas to problems with novel features (i.e. – irrelevant information, unfamiliar vocabulary). Research on schema-broadening instruction has been conducted with elementary students in one-on-one, small group, and general classroom settings (Fuchs et al., 2008b; Fuchs et al., 2010). Recent interventions using schema-broadening instruction helped students generalize the information in word problem schemas to algebraic equations (Fuchs et al, 2008a; Fuchs et al., 2010). Similar to schema-based instruction, schema-broadening instruction primarily included problems that involved whole number operations (Fuchs et al., 2008b; Fuchs et al, 2010).

Research gaps. While results of research in the CRA sequence, mnemonic strategy instruction, and schema-based instruction indicate that all of these strategies can improve the mathematical performance of students with learning disabilities or who struggle with math, no research could be found that combines instruction in problem schemas with the CRA sequence. Furthermore, in a review of major educational databases such as Education Research Complete and ProQuest Dissertations and Theses, no studies could be found that use concrete and virtual manipulatives within the full CRA sequence. Additionally, research related to schema-based and schema-broadening instruction has not focused on word problems with fractions. This gap in the research is problematic because students with learning problems in mathematics often do not have a strong conceptual understanding of fractions and are unable to visualize problems with fractions. Schema-based instruction that does not include concrete and pictorial representations through the CRA sequence may not be sufficient to develop student understanding of how to represent fractional parts and solve word problems with fractions.

Rationale for this Study

Research indicates that students with learning disabilities or learning problems in mathematics often have more difficulty representing problems correctly, identifying relevant information when solving word problems (Jitendra et al., 2002) or using effective strategies (Gurganus, 2007) than students with higher achievement in math. These limitations make problem solving extremely difficult for students with learning problems in math. In a review of literature on fraction instruction for students with learning disabilities or who struggle with math, Misquitta (2011) suggests that problems with working memory, whole number models, and confusion from representations used in early elementary school make fraction problems even more difficult for these students.

Problem solving skills with fractions are important for daily activities and help students develop understandings of more abstract mathematical concepts (Misquitta, 2011). Yet, little research has been conducted on effective instructional strategies with fractions for students with learning problems in mathematics. A search of the literature between 1990 and 2008 resulted in only 10 empirical studies that focused on current instructional practices for teaching fractions to students with learning problems, with only three of these studies focusing on problem solving with fractions. None of the studies that focused on problem solving with fractions included instruction with the CRA sequence, mnemonic strategies, or schema-based instruction. Because of this lack of research and concerns related to the performance of students with learning problems on problem solving tasks and fractions, this study focused on an instructional routine that could be used to increase achievement in problem solving with fractions for students with learning problems in mathematics. This instructional routine combined multiple evidence-based practices including the CRA sequence with virtual manipulatives, schema-based instruction, and mnemonic strategies to help students develop a conceptual understanding of fraction concepts within the context of specific problem schemas for addition and subtraction word problems. A mnemonic strategy was provided to help students remember the steps for solving the problems when using this routine. This combination of effective instructional strategies was necessary to try to address the complex reading, memory, and processing challenges encountered by students with learning problems when they solve word problems with fractions.

Statement of Purpose

The purpose of this study was to examine the effects of using a concrete-representational-abstract (CRA) sequence that included explicit connections between concrete manipulatives, virtual manipulatives, representational problem solving schemas, and abstract equations on the problem solving performance of students with learning disabilities or students with learning problems in mathematics. This sequence of instruction was combined with a mnemonic strategy called the LISTS strategy, to help students remember the steps in the problem solving sequence. Using a single-case multiple baseline across participants design, the researcher provided an intervention to five students in the fifth grade that included instruction in three problem schemas for addition and subtraction (change, compare, and group). The intervention also connected concrete manipulatives, virtual manipulatives, schemas, and equations to help students solve word problem with fractions.

Research Questions

Listed below are the research questions for this study:

1. When using the concrete-representational-abstract (CRA) sequence, can students connect the concrete manipulatives and virtual fraction manipulatives to the representational change, compare, and group schemas?
2. When using the CRA sequence can students connect the representational change, compare, and group schemas to the abstract equations?
3. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem-solving schemas and equations improve student performance on problems similar to the problems used during the intervention?

4. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem-solving schemas and equations improve student performance on problems that require generalization from the models provided during the intervention?

Limitations

This study was conducted with only five fifth grade students with learning disabilities or with learning problems in math, so the generalizability of the results to students with other disabilities or in different grade levels was limited. Additionally, this study was conducted in a small rural town in the eastern United States, so the results may not be generalizable to students in cities or in other locations. There were several limitations related to the content of the study as well. First, only group, change, and compare problems were addressed in this study, so the results may not be applicable to other types of problems. Second, this study focused on using the CRA sequence with fraction problems. The results may not be applicable to other content areas because students may not need the CRA sequence to visualize problems with whole numbers. Finally, the virtual manipulatives from Conceptua Fractions included a limited number of fractions that can be modeled with the manipulatives. Students were not able to complete problems with manipulatives for numbers greater than thirty.

Glossary

Algebraic Reasoning. The ability to use problem solving, representation, and quantitative reasoning skills to understand the language of algebra, generalize patterns in arithmetic, and use algebra as a tool for modeling these patterns. Students who demonstrate algebraic reasoning are able to reverse mathematical processes and build abstract rules from mathematical patterns (Kriegler, n.d.; Driscoll, 1999).

Cognitive Strategy. A mental routine such as a mnemonic strategy, self-instruction, or graduated instructional sequence that is used to help students understand and remember information (Maccini et al., 2007; Dole, Nokes, & Dritts, 2009).

Concrete-Representational-Abstract Sequence. An instructional series that helps students connect concrete objects to pictures of objects and equations to develop an understanding of mathematical concepts and processes. The sequence includes an introduction to mathematical concepts through concrete or physical manipulatives. The concrete manipulatives are then linked to a representation or picture of the manipulatives. In the final abstract stage, the pictorial representation is linked to an abstract equation that shows the mathematical problem. This sequence is also referred to as a graduated instructional sequence or a concrete-semiconcrete-abstract sequence (Maccini et al, 2007).

Enhanced Anchored Instruction (EAI). Use of video-based problems and hands on projects to improve student performance on problem solving tasks. Problems are based in authentic contexts and require students to identify relevant information and solve several smaller problems to determine an overall solution (Bottge, Rueda, Grant, Stephens, & Laroque, 2010).

Explicit Instruction. An instructional sequence that incorporates direct instruction through advanced organizers that identify the objective and rational for the lesson, teacher modeling of skills, guided student practice, and independent student practice. (Strickland & Maccini, 2010).

LISTS Strategy. The LISTS strategy is a mnemonic strategy that helps students remember the steps of the instructional routine that combines the CRA sequence and schema-based instruction. The LISTS strategy includes the following steps: 1.) Locate key terms; 2.) Identify the problem type and model; 3) Show the model with concrete or virtual manipulatives; 4.) Tie the model to the diagram; and 5.) Select the correct equation and solve for the unknown amount.

Learning Disabilities in Mathematics. To qualify as a student with a specific learning disability in math for this study, a student must meet the following criteria: 1.) an average to above average score on the WISC-IV (Wechsler, 2003); 2.) below an 85 on the Broad Math section of the Woodcock-Johnson III Tests of Achievement (Woodcock, 2001) or other similar standardized achievement test; and 3.) a severe discrepancy between intelligence and achievement in math. Students with learning disabilities must be receiving services in math in self-contained special education or inclusive general education settings.

Learning Problems in Mathematics. To qualify as a student with learning problems in mathematics for this study, the student must meet the following criteria: 1.) score partial mastery or novice on the state assessment, Westest II, and below the mastery level in problem-solving on benchmark assessments; 2.) perform at least 2 years below grade level on curriculum-based measures in Number Operations and Algebra when using measures from easyCBM (University of Oregon, 2010); and 3.) score below the 16th percentile (one standard deviation below the mean) on the Applications subtests on Foundations of Problem Solving and Applied Problem Solving of the Key Math-3 Diagnostic Assessment (2007).

Mnemonics. An instructional strategy used to help students remember information by linking the information to keywords, peg words, or acronyms (U.S. Office of Special Education Programs, n.d.). In mathematical problem solving instruction, individual letters are often combined to create a word. Each letter from the word represents a step in the problem solving process.

Schemas. The underlying structures of different types of mathematical problems. The following list includes definitions of the three schemas for addition and subtraction word problems that will be included in this study:

(1) Group schema: A schema that includes problems with "two distinct groups or parts combine to form a new group" (Griffin & Jitendra, 2008, p. 188)

(2) Change schema: A schema that includes problems that have an "increase or decrease of an initial quantity to result in a new quantity" (Griffin & Jitendra, 2008, p. 188)

(3) Compare schema: problems include comparisons of two different groups where the relationship stays the same (Griffin & Jitendra, 2008).

Schema-Based Instruction. A type of problem solving instruction where students are explicitly taught the underlying structures of different types of mathematical word problems and are given specific guidelines on how to solve each type of problem. This type of instruction typically includes diagrams to help students organize their work (Powell, 2011).

Schema-Broadening Instruction. A type of problem solving instruction where students are explicitly taught the underlying structures of different types of mathematical word problems and how to apply their understandings of the different types of problems to new problems that contain novel features. These novel features may include information presented in charts or graphs, irrelevant information, difficult vocabulary, or irrelevant information (Powell, 2011). While students are given diagrams to help organize work, instruction focuses on organizing information through mathematical equations (Powell, 2011; Fuchs et al., 2010)

Schematic Diagrams. A graphic organizer that provides a visual representation of the structure of each schema and provides visual cues to help students solve problems from each schema (Xin et al., 2005).

Virtual Manipulatives. Dynamic visual representations of concrete or physical manipulatives that can be flipped, turned, or moved with the computer mouse (Moyer, Bolyard, & Spikell, 2002).

Chapter Two

Review of Literature

This literature review will begin with an overview of the difficulties that students who struggle with math or who have learning disabilities encounter when solving story problems, and the strategies that these learners typically employ when solving these types of word problems. Next, an overview of the literature on effective instructional strategies for students with learning difficulties in math will be discussed. This synthesis of the literature will include specific research on the CRA sequence; the use of mnemonics and cognitive strategies within a CRA sequence; and the use of virtual manipulatives alone or combined with concrete manipulatives in the CRA sequence. The review will also include an analysis of the literature on teaching word problems with fractions using manipulatives and on teaching fraction problem solving using the entire CRA sequence. Finally, a detailed review of research on schema-based instruction, schema-broadening instruction, and schema-based instruction combined with cognitive strategy instruction will highlight effective instructional routines that have been used in the implementation of schema instruction.

Listed below is an outline of the topics that will be covered in this review:

1. Challenges with Word Problems
2. Direct Instruction
3. Instruction using the Concrete-Representational-Abstract Sequence (CRA)
4. Instruction with Virtual Manipulatives
5. Problem Solving Instruction with Fractions
6. Fractions and the CRA Sequence

7. Schema-Based and Schema-Broadening Instruction
    a. Addition and Subtraction Schemas
    b. Schematic Diagrams and Cue Cards
    c. Schema Instruction and Algebraic Reasoning
    d. Schema Instruction and Cognitive Strategies
8. Summary

Challenges with Word Problems

Solving word problems in mathematics requires a complex combination of procedural skills and conceptual understanding. Research indicates that students must be able to understand the relevant information and semantic structure of problems to solve them effectively (Jonassen, 2003; Lucangeli, Tressoldi, & Cendron, 1998). Furthermore, effective problem solvers are able to create good visual representations of problem information and use these representations to determine steps toward a solution (Van Garderen & Montague, 2003; Lucangeli et al., 1998). Students who have difficulty in mathematics often struggle with word problems due to procedural deficits in working memory, lack of conceptual knowledge (Geary, 2004), and difficulty representing the underlying structure of problems (Van Garderen & Montague, 2003). Additionally, difficulties in reading can further influence performance on word problems. In studies where the performance of students with math difficulties only is compared to students who have both math and reading difficulties, students with difficulties in both math and reading performed significantly lower on problem solving tasks than students with math difficulties only (Hanich, Jordan, Kaplan, & Dick, 2001; Fuchs & Fuchs, 2002; Jordan & Montani, 1997). The authors suggest that difficulties with interpreting the linguistic information in word problems or in conceptualizing problem situations could contribute to this lower performance.

Use of ineffective strategies may also contribute to lower performance on word problems. In a study on the word problem solving performance of 38 college students, Hegarty, Mayer, and Monk (1995) found that effective problem solvers used a meaningful model approach when solving problems while ineffective problem solvers used a direct translation approach. When using a meaningful model approach students changed problems into mental models with concrete representations. Students who used a direct translation approach focused on numbers in the problems and the key terms (i.e. – more, less). The students who correctly solved problems with meaningful models answered more problems correctly and could remember the essential meanings of the problems more accurately than those students who used the less successful translation strategies (Hegarty et al., 1995). Jordan and Montani (1997) suggest that younger students with difficulties in math exhibit deficits similar to those students who used the direct translation approach. In a study that compared the performance of 24 third graders with math difficulties to 24 third graders without math difficulties, the authors found that students with math difficulties often could not effectively solve problems or develop "back-up" strategies when they could not solve the problems initially. When these students could not solve problems, they would typically refer to known ineffective procedures that focused on the key terms in the problems (Jordan & Montani, 1997). Additionally, Rosenzweig, Krawec, and Montague (2011) found that students with learning disabilities in mathematics were significantly more likely to discuss processes or events that did not help them solve more difficult word problems than low achieving or average achieving students. The authors concluded that these students did not have or could not apply effective strategies to these problem solving tasks.

Direct Instruction

Because of concerns related to student difficulties with problem solving and mandates for grade level assessments for students with disabilities in the No Child Left Behind Act of 2001(NCLB, 2002), more researchers have focused on identifying effective instructional strategies in mathematics. Multiple studies have supported the use of explicit, direct instruction in problem procedures and mathematical concepts for students with difficulties or learning disabilities (Gersten et al., 2008, Witzel, Mercer, & Miller; 2003; Strickland & Maccini, 2010; Kroesbergen, Van Luit, & Mass, 2004; Xin, Jitendra, and Deatline-Buchman, 2005). In a study that compared explicit instruction and constructivist instruction, Kroesbergen et al. (2004) found that elementary students with math difficulties who worked together to construct their own understanding of mathematical problems did not perform as well on problem solving assessments as students who received explicit, direct instruction. Using explicit teaching strategies was also supported in a meta-analysis of literature on mathematics instruction for students who struggle or have disabilities in math. Gersten et al. (2008) reviewed 11 studies on explicit instruction and found that explicit instruction was highly effective with a mean effect size of 1.22. Based on these results, Gersten et al. (2008) argues that "explicit instruction should play a key role in mathematics instruction for students with LD" (p. 1). After conducting a review of research on teaching algebra to secondary students, Strickland and Maccini (2010) concurred with this analysis and recommended using an explicit instructional sequence that includes teacher modeling through a think aloud process, guided practice with teacher prompts, and independent practice using the teacher modeled strategies.

Instruction using the Concrete-Representational-Abstract Sequence

Research indicates that using this type of explicit instruction in a specific instructional sequence called the concrete-representational-abstract sequence (CRA) can improve understanding of mathematical concepts and lead to gains in achievement for students with learning disabilities. When using this sequence, students manipulate concrete objects to show a mathematical problem, draw a picture of the manipulatives in the problem, and then tie that picture to the abstract numerals that could be used to solve the problems (Witzel, 2005; Maccini & Hughes, 2000; Allsopp et al., 2007). This approach has been proven effective for students with math difficulties in a variety of areas. According to Cass, Cates, Smith and Jackson (2003), using concrete geoboards to model geometric figures through an instructional sequence of modeling, guided practice and independent practice led to significant improvements in student performance on word problems involving perimeter and area. Witzel, Mercer, and Miller (2003) found that sixth and seventh grade students with learning difficulties in mathematics instructed using the CRA sequence performed significantly better on algebra transformation equations than students receiving traditional instruction on equations.

Students with learning disabilities often need additional support to learn steps for problem solving and apply those steps to novel problems when using the CRA sequence. Instruction in mnemonic strategies can help students choose and implement effective problem solving strategies (Montague, Enders, & Dietz, 2011; Maccini & Hughes, 2000; Witzel, Riccomini, & Schneider, 2008). These mnemonic strategies can help students remember the steps of a problem solving process or prompt students to self-instruct or self-monitor their own work (Maccini et al., 2007). When these strategies are combined with the CRA sequence, student performance can be enhanced. For example, several researchers have found that using a STAR strategy with the CRA sequence can improve performance of students with learning disabilities on word problems with integers and algebraic equations (Maccini & Hughes, 2000; Gagnon & Maccini, 2001). When using the STAR strategy students were taught to search the word problem to determine known and unknown facts; translate the problem to concrete, representational, and then abstract forms; answer the problem, and review the solution to see if it makes sense. Researchers found that students who could implement the strategy with accurate models could accurately solve abstract problems with integers (Maccini & Hughes, 2000; Maccini & Ruhl, 2000). According to Maccini & Hughes (2000) the combined CRA sequence and STAR strategy provided scaffolding specific to algebra and helped cue students on how to represent and solve problems. This scaffolding was critical for students with learning disabilities that have difficulty accessing and applying information to problem-solving situations.

Scheuermann et al. (2009) combined the explicit sequencing and instruction using the CRA sequence with a routine that included self-monitoring to help students model and solve word problems that included one variable equations. Students used the CRA sequence to model and solve problems for each skill. Students were also expected to guide the teacher in the modeling process, explain the process to a partner, and provide self-instruction through private dialogue for each skill and at the concrete, representational, and abstract levels. The researchers found that student performance on similar and novel algebra word problems improved significantly after instruction on the routine. The researchers suggested that this type of routine could help students with disabilities have greater access to grade level content (Scheuermann et al., 2009).

Instruction with Virtual Manipulatives

Even though the use of concrete manipulatives has been supported, teachers of middle school students are less likely to use them than their elementary peers. Challenges with organizing materials, monitoring performance, and concerns with student behavior limit the use of concrete manipulatives (Butler et al., 2003). Additionally, as students move into middle school, the increased complexity of problems make the use of concrete manipulatives more difficult (Xin, 2008). One solution is to provide instruction with virtual manipulatives. According to Moyer et al. (2002), virtual manipulatives are web-based visual representations can be moved or changed by teachers or students to develop understandings of mathematical concepts. Suh (2005) found that students who used virtual manipulatives performed better on addition of fraction tasks than students who used physical manipulatives. Some authors suggest that virtual manipulatives can be used in place of concrete manipulatives when implementing the CRA sequence because they help students see mathematical relationships and explicitly connect pictorial and abstract representations (Suh, Moyer, & Heo, 2005). In a study that compared the performance of eighth-graders on solving problems with polyominoes, Yuan, Lee, and Wang (2010) found that eighth grade students performed equally well when using physical or virtual manipulatives.

Other researchers suggest that a combination of both physical manipulatives and virtual manipulatives may provide more effective instruction on concepts that can be clearly shown and manipulated with concrete objects. Researchers at the university level in science compared a combined instructional approach of using physical and virtual manipulatives to help students understand concepts related to pulleys. In one condition the students used physical manipulatives first and then used virtual manipulatives. In the other condition students used virtual manipulatives and then physical manipulatives. The researchers found that for concepts that could be modeled more concretely related to effort force, the students performed better when working with physical manipulatives before virtual manipulatives, but when working with concepts that were more difficult to model, the students performed better when working with virtual manipulatives before concrete manipulatives. The authors concluded a combination of physical and virtual manipulatives can be effective, but the order of presentation depends on the concepts that are taught. They also suggested that using the concrete, real pulleys provided an important kinesthetic experience that helped even college level students understand more difficult concepts (Gire, Carmichael, Chini, Rouinfar, & Rebello, 2010).

Research by Suh and Moyer (2007) comparing student use of concrete Hands-On equation model in algebra to a virtual balance scale manipulative extends this research in science to math. When providing instruction to third grade students, the authors found that both groups made significant gains, but that each group made gains in different areas. The virtual manipulative balance scale helped students develop a better understanding of equality while the Hands-On Equation model led to more significant improvements in mental math and using invented methods. The authors concluded that "different manipulative models, both in the physical and virtual environments, may have unique features that encourage relational thinking and promote algebraic reasoning" (Suh & Moyer, 2007, p. 171).

Problem Solving Instruction with Fractions

Studies on problem solving with fractions have incorporated concrete manipulatives, virtual manipulatives, and the CRA sequence. Bottge, Rueda, Serlin, Ya-Hui, & Jung Min (2007) used an instructional strategy called Enhanced Anchored Instruction (EAI) to help students improve their problem solving abilities in several content areas including fractions. In EAI, students watched short video clips of real world scenarios with embedded math problems related to the NCTM curriculum. The students worked with their peers in the general education classroom to solve problems in the video and then applied the information they learned to a hands-on project. In one of these projects titled, "A Fraction of the Cost", students focused on student understanding and application of fraction concepts. The students were supported through virtual manipulatives such as interactive tape measures and models of ramps that they could access as needed. Students applied the information from these virtual manipulatives to a concrete real world project of building a skateboard ramp. In a seven month study that included 13 students with learning disabilities in inclusive classrooms, the researchers found that the performance of students with learning disabilities remained below students without learning disabilities, but that students with learning disabilities learned at the same pace as their peers without disabilities. In a related mixed-methods study, researchers implemented a similar sequence of instruction using EAI with students with disabilities in a special education setting (Bottge, Rueda, LaRoque, Serlin, & Kwon, 2007). Data from the problem solving test results, teacher journals, and classroom observations indicated that problem solving skills improved and student motivation to work on problem solving tasks increased significantly. Researchers reported, however, that the scores of the students were not as high as the scores of students with learning disabilities in the general education environment from the earlier study. The special educators' lack of mathematical content and pedagogical knowledge and lower initial levels of student performance in math were two of the possible reasons given for these results (Bottge et al., 2007).

Because students lacked prerequisite skills and content knowledge, the researchers designed a new study which included direct instruction to develop understanding of fraction concepts and computation skills. The researchers implemented instruction using the same three EAI videos, problems, and informal supports that had been used in previous studies with students in collaborative classes. A second collaborative group received teacher-directed instruction that combined work with manipulatives and explicit computer-assisted instruction on fraction concepts. The second group then only completed two of the EAI videos and problems. The researchers found that the computation skills of the second group were higher than the group that only received problem solving instruction with informal support. Additionally, the problem solving skills of both groups improved, but were not significantly different from each other. The authors concluded that direct instruction in procedural knowledge combined with EAI could enhance student performance in computational skills without impeding problem solving performance (Bottge, Rueda, Grant, Stephens, & Laroque, 2010).

CRA Sequence and Fraction Instruction

Direct instruction using the CRA sequence can also lead to gains in conceptual understanding and problem solving performance on problems with fractions for students with learning disabilities. When comparing explicit instruction using the CRA sequence to instruction with the representational and abstract (RA) components of the sequence, Butler, Miller, Crehan, Babbitt, and Pierce (2003) found that all students improved in fraction equivalence tasks, but that the CRA group performed better on all measures. A significant finding in the study was that both the CRA group and the RA group performed better than a comparison group of eighth graders in general education on solving word problems with fractions. After examining student papers, the researchers noted that students in the CRA group and RA group drew representations that helped them solve the word problems. These results are supported by Hecht, Close, and Santisi (2003) who found that students with strong conceptual understandings of fractions did well on problem solving tasks. The authors argue that students must be able to form accurate mental models of fraction word problems to solve these problems.

Schema-Based Instruction and Schema-Broadening Instruction

Research suggests that implementation of the CRA sequence may be enhanced by having students classify word problems based on common characteristics such as comparisons between two quantities or changes in the amount of an initial quantity. Schema-based instruction, an approach that uses these problem structures, has led to gains in problem solving achievement for students with learning difficulties (Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Xin et. al., 2005; Jitendra, Hoff, & Beck, 1999). In this type of instruction students are asked to identify a problem type based on conceptual understanding of the problem structure and then use a schematic diagram or graphic organizer of the schema to represent the problem (Xin et al., 2005). According to a meta-analysis of research on effective instructional strategies for students who struggle with mathematics, this focus on underlying problem structures, or schemas, has strong support from the research and leads to significant improvements in problem solving performance (Gersten et al., 2009).

Many studies support the use of schema-based instruction for students with learning disabilities. Initial studies indicated that explicit schema-based instruction for middle school students in a pullout setting improved problem-solving performance on addition, subtraction, multiplication, and division word problems (Jitendra et al., 1999; Jitendra, Dipipi, & Jones, 2002) and was more effective than general strategy-based instruction (Xin et al., 2005). Recent studies suggested that explicit schema-based instruction was more effective than general strategy instruction for students with disabilities in collaborative elementary classrooms as well (Griffin & Jitendra, 2008). Xin (2008) found that schema-based instruction combined with algebraic expressions of mathematical relationships led to improvements in problem solving performance and algebraic understanding of multiplicative compare and equal group problems middle school students. In a follow-up study, students were able to accurately apply these strategies to different types of equal group and multiplicative compare problems (Xin & Zhang, 2009). Research related to schema-based instruction has moved toward helping students with disabilities have access to the general education curriculum. Initial research was conducted on the instructional level of middle school students using third grade addition and subtraction word problems in pullout settings (Jitendra et al., 1999), but more recent research has indicated that schema-based instruction improves student performance on grade level standards in both collaborative and pullout settings (Xin et al., 2005; Griffin & Jitendra, 2008).

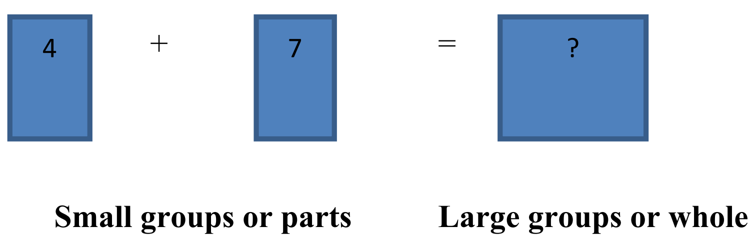
Schema-broadening instruction is a variation of schema-based instruction that focuses on helping students transfer their knowledge of existing schemas to similar problems with new features (Fuchs, Seethaler, Powell, Fuchs, Hamlett, & Fletcher, 2008). Research on schema-broadening instruction indicates that adding transfer features (i.e., irrelevant information, unfamiliar vocabulary or different formats) to schema-based instruction leads to increased problem-solving achievement for students with disabilities (Fuchs, Fuchs, Prentice, Hamlett, Finelli, & Courey, 2004; Powell, 2011). While schema-based instruction uses diagrams to help students model problems, schema-broadening instruction incorporates mathematical equations to help students model problems (Powell, 2011). Research indicates that relating common schemas to equations helps students develop beginning algebraic reasoning skills (Fuchs et al., 2008; Fuchs, Zumet, Schumacher, Powell, Seethaler, Hamlett, & Fuchs, 2010).

Addition and subtraction schemas. Schema-based instruction and schema-broadening instruction focus on three general types of addition and subtraction schemas. The first type, referred to in the literature as the group (Griffin & Jitendra, 2008), the total (Fuchs et al., 2008) or the part-part whole schema (Xin, Wiles, & Lin, 2008) refers to problems that include two parts that are combined to make a whole (Griffin & Jitendra, 2009). An example of a problem from this type of schema would be, "Farmer Joe has 88 animals on his farm. He only has horses and goats. There are 49 horses on the farm. How many goats are on the farm? (Griffin & Jitendra, 2009, p. 189). The second type of problem is referred to the change schema (Fuchs et al., 2008; Xin et al., 2008; Griffin & Jitendra). Problems are classified in the change schema if they include an initial quantity and then an action that causes an increase or decrease in quantity (Griffin & Jitendra, 2009). An example of a problem from the change schema would be, " Johnny had 21 pencils in his desk. Then he found another 16 pencils in the closet. How many pencils did Johnny have now? (Fuchs et al., 2008, p. 164). The final schema is referred to in the literature as the compare schema (Griffin & Jitendra, 2009), the difference schema (Fuchs et al., 2008), or the additive compare schema (Xin et al., 2008). When solving problems in this schema, students compare two sets to determine the relationship between two items (Griffin & Jitendra, 2009). The following is an example: "Craig saw a pine tree in the forest. Later, he saw a maple tree that was 9 feet tall. The maple tree was 5 feet shorter than the pine tree. How tall is the pine tree?" (Griffin & Jitendra, 2009, p. 189).

Table 2.1
Types of schemas and names of schema by each research group

Description of Schema Problem Types	**Sample Problem**	**Schema Names** *Griffin & Jitendra (2008); Jitendra et al.(1999)*	**Schema Names** *Fuchs et al. (2008); Fuchs et al. (2010)*	**Schema Names** *Xin et al., 2008*
Two separate groups joined to make a whole	Tim had 4 dogs. John had 7 cats. How many animals did they have altogether?	Group	Total	Part-Part-Whole (Combine subtype)
Increase or decrease in an initial quantity	Sarah had a bowl of 20 m & ms. She ate 7. How many were left?	Change	Change	Part-Part-Whole (Change subtype)
Comparison of two distinct sets	Casey's foot was 9 inches long. Bob's foot was 13 inches long. How much longer was Bob's foot?	Compare	Difference	Additive Compare

Schematic diagrams and cue cards. When providing instruction in the three types of addition and subtraction word problem schemas, all researchers used some combination of diagrams and cue cards to help students organize their work and solve the word problems. These schematic diagrams and cue cards provide a visual map for students to use to organize information for each schema (Powell, 2011). When using these diagrams or cue cards, students were instructed to place the correct values in the boxes or circles. These diagrams showed the connections of the values through arrows, plus, minus and equal signs. This use of schema diagrams and cues provided students with graphic representations to help them identify critical elements in each schema and organize this information so they can visualize the problem. For example, when providing instruction on the schema where two separate groups are joined to make a whole (i.e. – group, total, or part-part-whole/combine problems), Griffin and Jitendra (2008) used the following schematic diagram:



(Griffin & Jitendra, 2008, p. 189)

Figure 2.1. **Schematic Diagram for Group Schema**

Students were instructed to fill in the parts or whole based on the information in the problem and to place a question mark in the box for the information that was not known. If students were given the sample problem, “Tim had 4 dogs. John had 7 cats. How many animals did they have altogether?”, they would be instructed to identify the problem schema, determine whether the problem included information on the small parts or the whole, fill in the appropriate information in the boxes, and put a question mark in the box of the unknown information.

Both Fuchs et al. (2008) and Xin et al. (2008) used diagrams, but also included cue cards with phrases to help students identify the appropriate problem schemas and correctly organize information in the problem. Fuchs et al. (2008) provided a series of prompts with a diagram to help students solve problems. When asked to solve total problems students were given the following clues and diagram:

1. How many for part 1? (P1) $\boxed{}$
2. How many for part 2? (P2) $\boxed{}$
3. What is the total? (T) $\boxed{}$
4. Write the number sentence.

$$\underset{P1}{\boxed{}} \quad \underset{+}{\boxed{}} \quad \underset{P2}{\boxed{}} \quad \underset{=}{\boxed{}} \quad \underset{T}{\boxed{}}$$

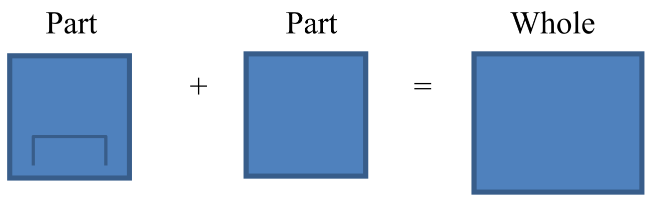
(Fuchs et al., 2008, p. 163; Powell, Fuchs, & Fuchs, 2010, p. 26)

***Figure 2.2*. Schematic Diagram for Total Schema**

When students were given a problem, they followed the same steps for identifying the schema and placing information in the correct positions, but they were instructed to put an x in the box to symbolize the unknown information (Fuchs et al., 2008). Xin et al. (2008) followed a similar pattern, but included more information to help students recognize the components of each schema. For example, when asking students to identify problems from the part-part-whole or combine schema, the authors gave students a cue card that included the definition of the schema, the schematic diagram, and specific story grammar questions to help students understand the schema.

Part-Part Whole (PPW)

A PPW problem describes multiple parts that make up the whole



Which sentence or question tells about the "whole" or "combined" amount? Write quantity in the big box on one side of the equation by itself.

Which sentence or question tells about one of the parts that makes up the whole? Write that quantity in the first small box on the other side of the equation.

Which sentence or question tells about the other part that makes up the whole? Write that quantity in the 2nd small box (next to the first small box).

(Xin et al., 2008, p. 171)

Figure 2.3. **Cue Card for Part-Part Whole (Combine) Schema**

Schema instruction and algebraic reasoning. Several researchers have included algebraic equations with schematic diagrams or cue cards to help elementary and middle school students generalize problems from specific schemas to specific abstract equations that represent those schemas (Fuchs et al., 2010; Xin, 2008). According to these researchers, algebraic reasoning is important for students with disabilities because it provides the foundation for understanding abstract concepts used in higher levels of mathematics. To make the connection between the information in the problem, the schematic diagram, and the equation, students receiving this type of instruction would circle the numerical values for the parts and total amounts found in word problems in the total schema. The students would then label each value as Part One (P1), Part Two (P2), or Total (T). The numbers would be placed in the correct sequence in the equation P1 + P2 = T and the equation would be solved (Fuchs et al., 2008). Xin (2008) uses a similar approach when connecting schema to algebraic equations at the middle school level by combining the schematic diagrams with algebraic equations for word problems with multiplication and division (Xin, 2008; Xin & Zhang, 2009). Xin (2008) argues that this approach is more effective with students with learning difficulties because the connection between the schematic diagram and the equation is explicit.

Schema instruction and cognitive strategies. Similar to researchers on the CRA sequence, individuals investigating the effects of schema-based and schema-broadening instruction used specific cognitive strategies to help students remember the steps to solve problems from different schemas. Fuchs et al. (2008) used a simple mnemonic devise called RUN to remind students to read the problem, underline the question, and name the problem type. The mnemonic by Griffin and Jitendra (2008) called FOPS encouraged students to find the problem type, organize the information in the diagram, plan a way to solve the problem, and then solve the problem. A slightly more complex mnemonic devise called DOTS was implemented by Xin et. al (2008) and Xin and Zhang (2009) to have students detect the type of problem, organize information using the correct diagram, transform the diagram into an equation, solve the equation, and check the response. The authors of this mnemonic included a specific statement on connecting the diagram to an equation to help students make the connection between the visual schema and the overarching algebraic concept for the schema.

Summary

Students with math difficulties face many challenges when solving word problems. Deficits in working memory, conceptual knowledge, and reading comprehension can all create challenges when solving word problems (Geary, 2004; Van Garderen & Montague, 2003; Fuchs & Fuchs, 2002). Explicit instruction using the concrete-representational-abstract sequence can help students develop an understanding of math concepts incorporated into specific word problems (Gersten et al., 2008). Instruction using the CRA sequence can be enhanced by connecting virtual manipulatives and concrete manipulatives (Suh and Moyer, 2007). Additionally, the use of cognitive strategies can help students remember general problem solving steps when using these instructional routines (Maccini et al., 2007).

When conducting a review of the research, only one research study was found that combines the CRA sequence, schema-based instruction, and cognitive strategies. In his dissertation, Mancl (2011) developed an instructional routine that included the CRA sequence and a cognitive READER strategy. However, the schemas included in the study were not related to the work of Griffin and Jitendra (2005), Fuchs et al. (2008), or Xin et al. (2008). The schematic diagrams in the study helped students make connections between the concrete, representational, and abstract levels of the CRA sequence, but they did not address underlying problem schemas. Therefore, this study will focus on an instructional routine that combines the CRA sequence with schemas that will help students develop an understanding on specific problem structures. A cognitive strategy will be included to support students as they work through the steps of the problem. Instruction will specifically focus on addition and subtraction problems with fractions because fraction concepts are difficult for students with learning difficulties in mathematics.

Chapter Three

Method

Introduction

This chapter will begin with a review of the purpose and research questions for the study. Next, the process that was used to select participants and the setting of the study will be described. In the following section, a description of the instructional intervention that combined specific schema-based problems and the CRA sequence will be provided. The chapter concludes with an explanation of the data collection and analysis procedures that were used to answer each research question.

Research Questions

The purpose of this study was to examine the effects of using a concrete-representational-abstract (CRA) sequence on the problem solving performance of students who struggle with mathematics or have been identified with learning disabilities in mathematics. The problem solving instruction in this study included explicit connections between concrete manipulatives, virtual manipulatives, representational problem solving schemas, and abstract equations. The following questions guided this study:

1. When using the concrete-representational-abstract (CRA) sequence, can students connect the concrete manipulatives and virtual fraction manipulatives to the representational change, compare, and group schemas?
2. When using the CRA sequence can students connect the representational change, compare, and group schemas to the abstract equations?

3. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems similar to the problems used during the intervention?
4. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems that require generalization from the models provided during the intervention?

Design

A single-case multiple-baseline across participants design was used to evaluate the effects of the intervention on student performance on fraction word problems in this study. According to Kazdin (2011), a multiple-baseline design is appropriate when evaluating changes in a specific skill and when interventions can be implemented with one student or one group at a time. When implementing this design, data were gathered on the baseline performance of each student. After baseline performance was stable for all students, the intervention was started with one student or group while the other students continued to receive instruction under baseline conditions (Kazdin, 2011). Using a multiple-baseline across participants design in this study was appropriate because a stable baseline of performance on fraction word problems was established for each student. The intervention that combined the CRA sequence and schema-based instruction was implemented with the first two students while the other students only received classroom instruction in mathematics. The intervention did not begin with the second two students until changes in performance on fraction word problems were documented for the first students. The intervention began with the final student after changes in performance were documented with the second two students. This process showed whether performance on each type of word problem could be attributed to the instructional routine that combined the CRA sequence and schema-based instruction. The intervention was started with pairs of students initially because of the late February start to the study. After the second pair began the intervention, the researcher noted that progress and attendance of the students was uneven and all other lessons for the students were implemented individually. Student 5 was the only student included in the final implementation of the intervention because one student moved during the study. Because the intervention was introduced at different times to pairs or individual students, the effects of outside factors on student performance were reduced (Kazdin, 2011).

The independent variable for each research question was an instructional routine that combined the CRA sequence and schema-based instruction to help students connect concrete manipulatives, virtual manipulatives, representational schemas, and abstract equations when completing word problems with fractions. Research on mathematical problem solving indicates that using a graduated sequence that connects concrete manipulations of objects, representational drawings of those objects, and the abstract equation results in improved student performance on problem solving tasks for students with learning disabilities or who struggle with mathematics (Maccini & Hughes, 2000; Allsopp et al., 2007; Gagnon & Maccini, 2001). Additionally, instruction on helping students identify specific problem components related to types of word problem schemas has been shown to be effective for students who struggle with mathematics (Xin et al., 2005; Xin & Zhang, 2009) and students with learning disabilities in mathematics (Maccini et al., 2007; Jitendra et al., 2002; Jitendra et al., 1999).

The dependent variable for each question was related to these key components of the instructional routine. In question one, the dependent variable was the students' ability to connect concrete and virtual manipulatives to the correct schematic diagram. In question two, the dependent variable was the students' ability to use the schematic diagram to write the correct equation needed to solve the word problem. The dependent variables for questions three and four were the students' ability to solve word problems with fractions from the group, change, and compare schemas. For question three, the students' ability to solve word problems that were similar to the problems used in the intervention was assessed. For question four, the ability to generalize the strategies to correctly solve problems that include irrelevant information, difficult vocabulary, or different conceptualizations of problems was measured.

Participants

Participants were five students from the fifth grade level from a public middle school. Four students were males and one student was female. Similar to the county and school demographics, all students in the study were White. All students were either currently receiving special education services and had been diagnosed with a specific learning disability, or had been identified as a struggling student by teachers because of their mathematics performance on state testing, benchmark assessments, and curriculum-based measures. The students were chosen in consultation with the special educator and general educators who provided services to the students. To qualify as a participant under the category of specific learning disabilities in math, the students were required to have an average to above average score on the WISC-IV (Wechsler, 2003) or similar standardized assessment and demonstrate a severe discrepancy between achievement and intellectual ability in math. The severe discrepancy formula was used because this formula was supported by state policies and used by local education agencies to determine eligibility at the middle school level until the 2011-2012 school year (WV Department of Education, 2010). Students with learning disabilities were also required to score at the partial mastery or novice on the state assessment, Westest II. They were also assessed using the Problem Solving subtest of the Key Math-3 Diagnostic Assessment (2007). To qualify for the study, students with learning disabilities needed to score at or below the 16th percentile (i.e., one standard deviation below the mean). An additional qualification of this study for students with learning disabilities included math instruction in a self-contained special education or inclusive general education setting.

To qualify as struggling student in mathematics, the students were required to score at the 50th percentile or lower at the fourth grade level on the Number and Operations subtest from the easyCBM curriculum-based measures (University of Oregon, 2010). In addition, struggling students could only qualify for the study if they scored at the partial mastery or novice on the state assessment, Westest II. Students who struggled with math were also were assessed using the Problem Solving subtest of the Key Math-3 Diagnostic Assessment (2007) and needed to score at or below the 16th percentile to qualify for the study.

Table 3.1
Student Demographic Information

Characteristics	Students				
	One	Two	Three	Four	Five
Age	11.1	12.3	11.2	12.2	11.10
Grade	5	5	5	5	5
Gender	M	M	M	M	F
Ethnicity	White	White	White	White	White
Disability	none	LD	OHI	none	none
Westest II Math Level (2011)	PM	PM	PM	PM	N
Key Math Problem Solving Subtest Percentile Ranking	16%	5%	2%	16%	2%
EasyCBM Number and Operations Gr. 4 Percentile Ranking	20%	50%	40%	50%	20%

Sampling. A purposive sample of fifth grade students with learning disabilities or who struggle in math was selected for this study. Originally seven students were selected as possible participants for the study based on the eligibility criteria and teacher recommendation. One student did not meet the qualifying score on the Key Math Problem Solving Test and was disqualified. Of the six remaining students, two students with learning disabilities were selected. These two students were the only students with learning disabilities who met the criteria and had a 30 minute instructional period that could be used for instruction. Three of the four remaining students who met the eligibility criteria for struggling students were selected based on teacher recommendation. After the third probe was administered, however, one student with learning disabilities moved unexpectedly. The final eligible student became a part of the study at this time and probes were administered with the other students to establish a baseline of performance.

Fifth grade students were selected for this study because according to the Common Core Standards (2010) that have been adopted in 43 states, students at this grade level are expected to solve addition and subtraction word problems with fractions. Yet, many fifth grade students with learning disabilities and learning problems in math still have difficulty due to lack of conceptual understanding of fractions or difficulty with understanding the underlying structures of the problems. Additionally, the fifth grade students that were selected for this study were from a public middle school where the researcher taught special education for six years. The researcher now acts as a liaison to this school from a local public university and works in the school one morning a week to provide support for students who struggle in mathematics and reading. Because of this relationship, the principal and teachers were comfortable with the researcher working with teachers and students to provide interventions. The dual role of the researcher as liaison did not impact privacy or confidentiality because confidential information on students was not reported to the university or to students from the university. Additionally, no students from the university were working with the students during the research study.

Setting

The middle school selected for this study is located in a rural town in north central West Virginia. According to the US Census Bureau (2010), the population of this town is over 95% White and the median income of residents is below the state median income. The town has an 80% high school graduation rate, but only approximately 13% of adults have a bachelor's degree or higher (US Census Bureau, 2010). Similar to community statistics, over 95% of students in the school are White. Unlike the statistics from the community, the teachers in the school are highly educated with over 70% reporting having a master's degree or above. Teachers also are very experienced and average over 20 years of teaching. Despite the education and experience of the teachers, the school has struggled to meet adequate yearly progress (AYP) for the past five years because of reading or math scores in special education (West Virginia Department of Education, 2011).

The school in this study has about 700 students in grades five through eight with approximately 175 students at the fifth grade level. The students with learning disabilities received services in math from one highly qualified special educator who had 5 years of experience teaching special education. The students who had been identified as at-risk in reading and mathematics received instruction in all subjects by one general educator with 32 years of teaching experience. This teacher taught students with disabilities for 12 years of those 32 years.

Instructional Intervention

The researcher in this study provided instruction on group, change, and compare word problems schemas that were adapted from Griffin and Jitendra (2009). According to these researchers, the group, change, and compare schemas are three common problem structures for addition and subtraction word problems. These types of problems are often found in textbooks or problem solving units.

Group Problems. According to Gurganus (2007), group problems include two discrete amounts that are combined to equal a total amount. The unknown amount in a group problem could be the total amount or the amount in either part or subset. The following is an example of a problem from the group schema: *James brought ¾ of a cheese pizza to the school party. Cindy brought ½ of a pepperoni pizza to the party. How much pizza did the two students bring to the party?* In this example, there are two different types of pizza that are combined to make a total amount of pizza.

Change Problems. A change problem involves a quantity or amount that is increased or decreased. The increase or decrease results in a new total amount (Gurganus, 2007, Griffin & Jitendra, 2009). The following is an example of a problem from the change schema: *Matt had 1/3 cup of sugar, but he gave ¼ of a cup to his mother to cook some brownies. How much sugar did Matt have left?* In this type of problem, there is an initial amount of one item, the sugar, which was decreased to a new amount of sugar.

Compare Problems. According to Gurganus (2007), compare problems involve comparisons between two distinct sets. The comparison is in terms of bigger and smaller or more and less. For example, *Sarah has 4 ½ dollars less than Rachael. If Rachael has 6 ½ dollars, how much money does Sarah have?*

Concrete-Representational-Abstract Sequence. The researcher provided instruction on problems from each schema through the CRA sequence. When using this sequence during the intervention, students were given instruction on how to manipulate concrete models of fractions to show a mathematical problem at the concrete level; use the virtual fraction manipulatives and schematic diagrams to represent the concrete models at the representational level; and then tie the virtual manipulatives and schematic diagram to the abstract numerals that could be used to solve the problems at the abstract level. For example, when providing instruction on the example group problem: *Jim had 3/4 of a pie. Todd had 2/4 of a pie. How much pie did they have all together?,* the researcher began by defining group problems as two parts that can be combined to make a whole. Next, the researcher helped the student identify the fractional parts (3/4, 2/4) and the unknown total in the problem and then showed the student how to model the parts of the problem by using concrete fraction circles and tiles. The researcher represented the unknown total with a ***W***.

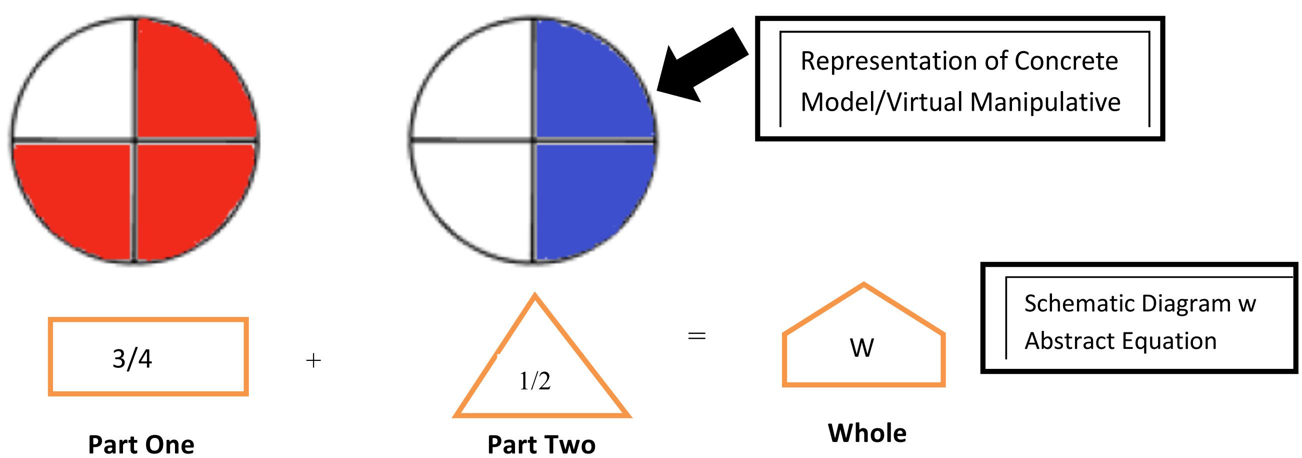


***Figure 3.1*. Model Problem of Schema-based Instruction with CRA Sequence.** The model includes a representation of a virtual manipulative and schematic diagram with abstract numbers from a sample group problem.

After guided and independent practice with concrete fraction circles and tiles, the researcher demonstrated how to model the same parts of the problem using virtual manipulatives of fraction circles and tiles. The researcher then helped students connect the virtual manipulative to a schematic diagram and the schematic diagram to the abstract equation. (See Table 3.2)

Table 3.2
Sample of problem types and models at each stage of the CRA sequence

Problem Type	Sample Problems for Probes or Practice	Concrete (Virtual Manipulative)	Representational (Schematic Diagram)	Abstract (Equation)
Group (Part 1 + part 2 = total)	Jim had 2 ½ cheese pizzas. Todd had 3 ¼ pepperoni pizzas. How many did they have all together?	Area model (fraction circles or tiles)	P1 P2 T + =	P1 + P2 = T 2 ½ + 3 ¼ = T
	Cindy had 2/3 of a bag of m & ms. The bag only has red and yellow m & ms. If ¼ of the bag is red m & ms. How much of the bag is yellow m & ms?	Area model	P1 P2 T + =	P1 + P2 = T ¼ + P2 = 2/3
Change (Starting amount +/- change = ending amount)	Erin was cleaning her room and found 1 ¼ packs of crayons. When she looked in her backpack she found another 1 ½ packs of crayons. How many packs of crayons does she have now?	Linear model (fraction bars)	St C E + =	St + C = E 1 1/4 + 1 1/2 = E
	Paula needed 2 ½ cups of flour to make a pie for her mom's party. If she already has 3/5 of a cup of flour, how much more flour will she need to make her pie?	Area model	St C E + =	St +/- C = E 3/5 + C = 2 1/2
	Kendall used 2 1/6 pieces of poster board for his science fair project. If he had 1/2 of a piece of poster board left, how much poster board did he have when he started the project?	Linear model	E C St + =	E + C = S 1/2 + 2 1/6 = S
Compare (Bigger – smaller = difference)	The new pen for the tigers at the zoo was 11 1/2 feet tall. The pen for the lions was 11 3/4 feet tall. How much taller was the pen for the lions than the pen for the tigers?	Area model or Set model	B s D - =	B – s = D 11 3/4 - 11 1/2 = D
	Mrs. Weaver had a shelf for her books that was 2 1/5 feet tall. Mrs. McCarthy had a shelf that was 1 1/5 feet taller than Mrs. Weaver's shelf. How tall was Mrs. McCarthy's shelf?	Linear model	B s D - =	B – s = D B – 2 1/5 = 1 1/5

Adapted from Fuchs, Zumeta, Schumacher, Powell, Seethaler, Hamlett, & Fuchs (2010), Xan, (2008), & Griffin & Jitendra, (2008)

When implementing the intervention, the researcher began with problems from the group schema and then moved to problems from the change schema. Problems from the compare schema were introduced last. This sequence began with the simplest type of problems, the group fraction problems, because most instruction on fraction concepts focuses on fractions as parts of a whole or a group (Misquitta, 2011). The group problems were modeled using fraction tiles and fraction circles which were familiar to students and easier for them to understand. After completing group problems, more difficult change and compare problems were introduced. Although there could have been order effects related to introducing problem types in the same order, this is unlikely because specific conceptual understanding of each problem type was necessary to model and solve problems.

The researcher provided instruction on problems from all schemas using concrete manipulatives, virtual manipulatives, schematic diagrams, and abstract equations. At each stage of instruction, the researcher modeled problems, provided guided practice, and gave students the opportunity for independent practice. Throughout this intervention process, students continued with math instruction in their regular classrooms. The students were given instruction on understanding and comparing fraction concepts, but they did not receive instruction on any of the problem schemas from this study.

Procedures

Interventions were conducted three to four times a week during either a 30 minute re-teach/enrichment block that was incorporated into the daily routine at this public middle school, or during morning instructional support time for math or reading in the general education classroom. Due to time constraints while awaiting IRB approval, local school board approval, and consent forms, the researcher initially implemented the intervention with the first two students together during the lessons on the group schema. The next two students began the intervention as soon as a stable baseline was established for the first two students. After the second pair of students began the intervention, the researcher noted that progress and attendance of the students was uneven and all other lessons for the students were implemented individually. Student 5 was the only student included in the final implementation of the intervention because one student moved during the study.

All interventions took place in an empty classroom that contained no windows and minimal decorations to limit distractions. The students sat beside the researcher and faced the chalkboard. The classroom was at the end of the hallway with some traffic to the stairs, so the door was shut to limit possible noise from hallway. Most interventions took place right after homeroom (8:05 – 8:35 a.m.) daily or during morning instructional support time in the general education classroom for math or reading. The instructional support time varied daily depending on the schedule of the general educator. The schedule of interventions did vary in lessons 11-13 for student 5. Because multiple probes were needed to demonstrate changes in performance for student 1and 2, student 5 did not begin the intervention until later in the school year. Interruptions from an extended spring break, state testing, and a mild winter that led to an early end of school all affected the implementation schedule for this student. As a result, some of the last intervention sessions for student 5 occurred in the afternoon as well.

Student performance on word problems with fractions was assessed every other day in the baseline phase using probes that included nine fraction word problems involving three different word problem schemas (See Appendix A). In addition, students were given one 12 question test of their ability to generalize knowledge of problem schemas to novel problems with unfamiliar vocabulary, irrelevant information, or different conceptualizations than the ones presented in the intervention (See Appendix B). During both of these assessments, instructions and problems were read aloud and all students were assessed at the same time. Students were allowed to use calculators because calculators helped students with difficult computations so they could focus on problem concepts (Montague, 2005). Furthermore, research indicates that the use of calculators helps students with disabilities or learning problems in math access higher-level problem solving skills that they could not access without calculators (Center for

Implementing Technology in Education, 2007). The use of calculators was necessary during the intervention because of the complex procedures necessary for computations with fractions (Misquitta, 2011).

After baseline was established, the intervention began with the first two students using problems from the group schema. Instruction on relating the information in the word problems to the concrete manipulatives was provided by the researcher using scripted, 30-40 minute lessons that occurred daily (See Appendix C). At the beginning of the intervention, the researcher showed students how to model problems from the schema using concrete manipulatives. The researcher modeled two complete problems that include the fractional parts and the solution so students could see how each component of the problem was modeled. According to Xin et al. (2005), the complete modeling of problems is important because it helps students who struggle with math "develop a mental representation of the problem schema" (p. 269). Additionally, using this process helps students understand the schema and retrieve it when solving problems. After students worked with complete problems, the researcher modeled two problems with unknown information to help students apply their understanding of schemas to solve problems with unknown information.

Table 3.3
Sample of complete group problems and problems with unknown information

Problem Type	Sample of Complete Problems for Instruction	Sample of Problems with Unknown Information
Group (Part 1 + Part 2 = Total)	Paul had ½ of a pizza. Susan had 1/3 of a pizza. Together they had 5/6 of a pizza.	Paul had ½ of a pizza. Susan had 1/3 of a pizza. How much pizza do they have altogether?
	Jake had ¼ bag of skittles. The bag only has red and yellow skittles. If 1/8 of the bag is red skittles, then 1/8 of the bag has yellow skittles.	Jake had ¼ bag of skittles. The bag only has red and yellow skittles. If 1/8 of the bag is red skittles, how much of the bag has yellow skittles?

After the researcher explicitly modeled and used a think aloud process to show students how to complete four problems from the first schema, the student were given two word problems that included all components. Using guided practice, the researcher helped the student model these problems. The student then practiced modeling two problems with missing information with feedback from the instructors. After guided practice with the instructor, the student were given six story situations to model. Two of these problems included complete information on the fractional parts and solutions and four problems contained unknown information. If the student correctly modeled 4 out of 6 problems, the intervention continued to the next phase. If the student was unable to model 4 out of 6 problems correctly, then the researcher modeled and provided guided feedback until the student reached 66% or greater mastery. This mastery level was adjusted from an original planned mastery level of 5 out of 6 (83%) because the students in the study were at the initial acquisition levels when modeling fractions. In other words, three students were unable to model basic problems with like denominators using manipulatives and no students were able to model problems with unlike denominators. According to Allsopp et. al. (2007), when developing understanding at the initial level of acquisition to move to an advanced level of acquisition, the expectation for mastery should range from approximately 50-95%. Because of the complexity of modeling equivalent fractions and the focus on understanding problem schemas, the researcher adjusted the mastery criteria to correspond more closely with the average of this range (approximately 72%). However, during the independent practice using the schematic diagram and writing the equations, most students solved problems at the original mastery level of 5 out of 6 (83%).

Table 3.4
Instructional sequence of problems from group, change, and compare schemas

Order of Instruction at Each Stage of CRA Sequence	Number/Types of Problems Used During Instruction	Mastery Level
Model	-Instructor uses the think aloud process to model or write the equation for two problems with complete information - Instructor uses the think aloud process to model or write the equation for two problems with unknown information	NA
Guided Practice	-Instructor guides and supports the students as they model or write the equation for two complete problems -Instructor guides and supports the students as they model or write the equation for two problems with unknown information	4/4 (100%)
Independent Practice	-Students independently model or write the equation for two complete problems -Students independently model or write the equation to solve four problems with unknown information	4/6 (66%)

Once the student was at mastery on independent practice problems with concrete manipulatives, the concrete-representational instruction occurred. The teacher used the same process of modeling, guided instruction, and 66% mastery on practice problems to help the student connect the concrete manipulatives to virtual manipulatives from Conceptua Fractions software that was designed to support students who have difficulty with fractions. (See *Figure 3.2*)

The teacher modeled complete examples and problems with unknown amounts with concrete manipulatives to help students make the connection between concrete models of manipulatives and the virtual models on the computer. The teacher then guided the students on how to model the problems using virtual manipulatives. The students were then given six story situations to model with virtual manipulatives. Two of these problems included complete information on the fractional parts and solutions and four problems contained unknown information. If the student correctly modeled 4 out of 6 problems, the intervention continued to the next phase. If the student could not model 4 out of 6 problems correctly, then the researcher modeled and provide guided feedback until the student reached 66% or greater mastery.

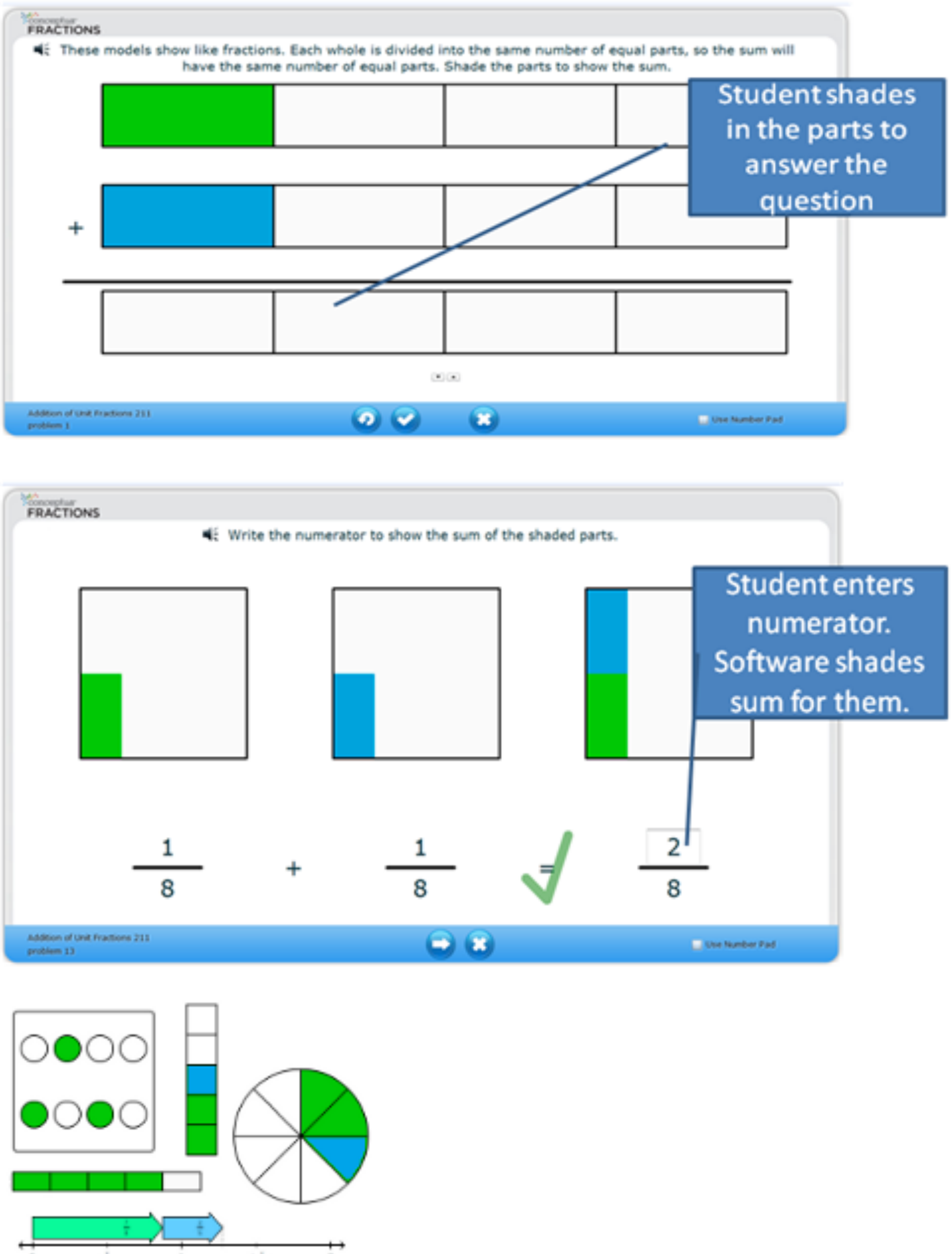
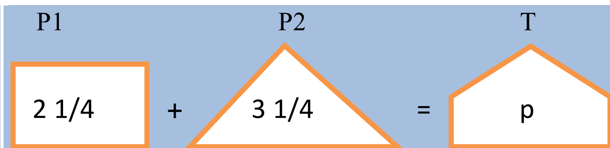


***Figure 3.2*. Screenshots of Virtual Manipulatives from Conceptua Fractions Website.** Screenshots show the possible models that could be used by students (linear, area and set models). Adapted from *Conceptua Fractions* by A. Khalsa, 2010. Retrieved from http://conceptuamath.appspot.com/fractions.html#AddingCD

Next, the student was given a diagram of the schema with a word problem. The instructor modeled how to identify critical problem components and mapped them on the appropriate schematic diagram. The student then practiced identifying information and placing it into the schematic diagram with feedback from the instructors. After guided practice with the instructor, the student was given six story situations to map onto the schematic diagrams. The student was asked to place the information in the schematic diagram with 66% mastery required before moving to the next phase.

Table 3.5
Sample problem with completed schematic diagram

Problem Type	Sample Problems for Probes or Practice	Representational (Schematic Diagram)
Group (Part 1 + Part 2 = Total)	Jim had 2 ¼ pizzas. Todd had 3 ¼ pizzas. How many did they have all together?	

The same process was used to complete the representational-abstract phase of the intervention. The teacher used modeling and guided practice to show students how to connect the information in the schematic diagram to the appropriate mathematical equation. The students were given a cue card to help with the steps for connecting the schematic diagram to the equation. The original cue cards included only the initial diagram. After the first two students had difficulty reversing operations and identifying procedures for unknown addends, the cue cards were modified to include reverse operations for group and compare problems and difficult conceptualizations needed for change problems. The students then practiced connecting information from their previously completed diagrams to mathematical equations with 66% mastery required before moving to the next phase.

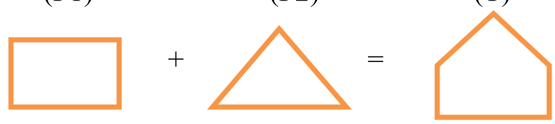
Figure 3.3. Sample Cue Cards

Group Problems – Cue Card

Two distinct parts combine to form a new group or total (Griffin & Jitendra, 2008)

*If you know the two parts, **add** to find the total or whole.

Part One	**Part Two**	**Total**
(P1)	(P2)	(T)



 Which sentence tells about a part of a group? Find the amount and write it in the rectangle.

 Which sentence tells about another part of a group? Find the amount and write it in the triangle.

 Which sentence tells about the total number of items? Find that amount and write it in the pentagon (house).

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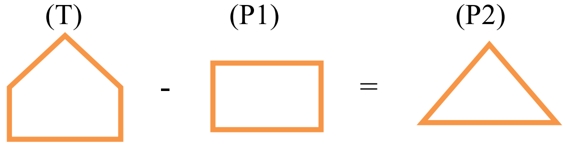
*If you know the total (whole) and one part, **subtract** to find the other part.

Total	**Part One**	**Part Two**
(T)	(P1)	(P2)



 Which sentence tells about the total number of items? Find that amount and write it in the pentagon (house).

 Which sentence tells about a part of a group? Find the amount and write it in the rectangle.

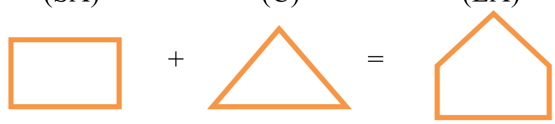
 Which sentence tells about another part of a group? Find the amount and write it in the triangle.

Change Problems – Cue Card

Increase or decrease an amount to find a new amount (Griffin & Jitendra, 2008, p. 188)

*If you know the starting amount and how much you will increase that amount, **add** to find the ending amount.

Starting Amount (SA) | **Change** (C) | **Ending Amount** (EA)



 Which sentence tells you the starting amount? Find the amount and write it in the rectangle.

 Which sentence tells how much the starting amount will increase (+change)? Find the amount and write it in the triangle.

 Which sentence tells about the ending amount? Find that amount and write it in the pentagon (house).

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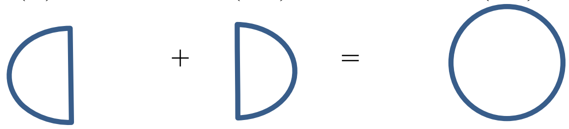
*If you know the change amount and how much your ending amount will be, **add** to find the starting amount.

Change (C) | **Ending Amount** (EA) | **Starting Amount** (SA)



 Which sentence tells the change amount? Find that amount and write it in the half circle that begins with a C.

 Which sentence tells about the ending amount? Find that amount and write it in the rectangle.

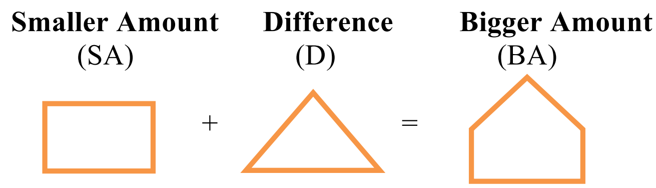
 Which sentence tells about the starting amount? Find the amount and write it in the blue circle.

(adapted from Y.P. Xin & D. Zhang, 2009)

Compare Problems – Cue Card

Compare a bigger amount and a smaller amount to find the difference (Griffin & Jitendra, 2008)

*If you know the smaller amount and the difference, **add** to find the bigger amount.



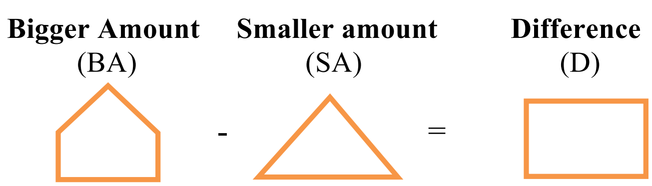
Which sentence tells you the smaller amount? Find the amount and write it in the rectangle.

Which part tells what the difference is between the bigger and smaller amount? Find the amount and write it in the triangle.

Which sentence tells about the bigger amount? Find that amount and write it in the pentagon (house).

---

*If you know the bigger amount and the smaller amount, **subtract** to find the difference between the two amounts.



Which sentence tells about the bigger amount? Find that amount and write it in the pentagon (house).

Which sentence tells about the smaller amount? Find the amount and write it in the triangle.

Which sentence tells about the difference between the two amounts? Find that amount and write it in the box.

(adapted from Y.P. Xin & D. Zhang, 2009)

***Figure 3.3*. Sample Cue Cards.** *Cue cards include schematic diagram and steps for schema instruction.* (Adapted from Y.P. Xin & D. Zhang, 2009*)*

Throughout the instructional sequence, the instructor taught students how to solve the equations with missing information in any of the three positions (either part or total amount) using the mnemonic LISTS strategy designed by the researcher. The LISTS strategy helped students remember to 1.) Locate key terms; 2.) Identify the problem type; 3) Show the model with concrete or virtual manipulatives; 4.) Tie the model to the diagram; and 5.) Select the correct equation and solve for the unknown amount. Combining the mnemonic LISTS strategy with the CRA sequence and schema-based instruction helped students remember the steps in this routine when solving word problems.

LISTS Checklist

- **L**ocate key terms
- **I**dentify the problem type and model
- **S**how the model with concrete or virtual manipulatives
- **T**ie the model to the diagram
- **S**elect the correct equation and solve for the unknown amount

***Figure 3.4* – LISTS Checklist for Students.** Checklist describes the students that the students should use when using the CRA sequence plus schema-based intervention.

During the intervention students were assessed after every two intervention sessions using the nine problem probe that contains all three problem types. After students mastered the group problems, the same process and schedule of probes was used during instruction on the change problems and the compare problems. When student performance stabilized, the maintenance phase began. Unfortunately, because of the interruptions only one maintenance probe was administered to students 1, 3, and 4. This probe was administered 10 days after the completion of the intervention. Additionally, at the beginning of the maintenance phase, students 1,3, 4, and 5 were given one twelve problem post-test on their ability to generalize the intervention strategies to novel problems with irrelevant information, unfamiliar vocabulary, and different conceptualizations of fraction concepts. Student 2 was an exception. He did not meet the mastery criteria on lesson 10 prior to the end of the school year, but he was given the transfer test after instruction on lesson 10 to see if there were any changes in his performance from the pre-test. The transfer posttest given was the same assessment as the pretest. This test was used because of the limited number of questions answered correctly on the pretest and the length of time between pre-test and post-test (3 months).

When conducting the intervention, the researcher implemented one to two lessons per week. Most lessons required two 30 to 40 minute sessions resulting in some weeks with two days of instruction, a probe, and then two more days of instruction. During some weeks there were interruptions in the intervention schedule due to school events. In these cases, the researcher provided interventions on three days and administered a probe on one day. The LISTS strategy was taught after instruction on the group schema was implemented at the concrete, concrete-representational (virtual manipulatives), representational (schematic diagram), and abstract levels for problems in the group schema. The LISTS strategy was taught at this point in the instructional sequence because the students had an understanding of one schema and the strategy could be taught within the context of that schema. For example, the researcher showed students how to locate key terms in a group problem, identify that the problem is from the group schema, model the group problem with manipulatives, tie the model to the schematic diagram for group problems, select the correct equation, and solve for the unknown amount. The implementation schedule continued with the same schedule for problems in the change and compare schemas as long as the student met the mastery criteria of 66% when solving problems independently. If the student did not meet the mastery criteria then further instruction occurred.

Table 3.6
Schedule of Implementation for Students who met Mastery Criteria for All Lessons

Phase		CRA Level	Instruction *	Assessments
Baseline (Pre-Intervention)				Probes every other day, Pre-test Transfer
Intervention Sessions				
Sessions 1-2	Lesson 1	Concrete	Group Schema	Probes administered after every two sessions Independent practice problems after each lesson
Sessions 3-4	Lesson 2	Concrete-Representational		
Sessions 5-6	Lesson 3	Representational		
Session 7-8	Lesson 4	Abstract		
Session 9	Lesson 5		LISTS Strategy	
Sessions 10-11	Lesson 6	Concrete	Change Schema	
Sessions 12-13	Lesson 7	Concrete-Representational		
Session 14	Lesson 8	Representational		
Session 15	Lesson 9	Abstract		
Session 16-17	Lesson 10	Concrete	Compare Schema	
Session 18-19	Lesson 11	Concrete-Representational	Compare Schema	
Session 20	Lesson 12	Representational		
Session 21	Lesson 13	Abstract		
Maintenance (Post-Intervention)				Probe after 10 days for students 1,3,4; Transfer post-test for all students

*Regular classroom instruction for fifty minutes in mathematics continued through all phases of the intervention. Instruction in this chart refers to instruction that took place in addition to daily math instruction.

Additional instruction was necessary for students 1 and 2 during instruction with the concrete manipulatives and the schematic diagram lessons of the group schema. Further instruction was provided for each lesson and the both students were able to reach the mastery criteria. Student 2 did not meet the mastery criteria for compare problems at the concrete manipulative level before the completion of the school year and therefore did not complete the sequence of lessons. Even when the students did not meet the mastery criteria, however, the assessment probes were implemented after every two sessions for the duration of the intervention.

Experimental Controls

One threat to this study was to fidelity of implementation of the intervention. To reduce this threat the researcher standardized instruction during the intervention as much as possible. To ensure fidelity of implementation, the researcher used scripted lessons and followed the same administration and testing procedures for all probes and tests. To ensure that the treatment was implemented consistently, a faculty member at local university with expertise in schema-based instruction listened to audiotapes of 25% of the lessons using a checklist that contained key components of the intervention (See Appendix D). The faculty member marked the parts of the lesson that were taught during the observation and recorded that 98 % of the key components were implement throughout the lessons.

A second threat to internal validity is developing valid measures and maintaining consistency when scoring measures. To address these threats, a fourth grade math teacher with 32 years of teaching experience and expertise in schema-based instruction, the researcher, and faculty member reviewed the measures of problem solving performance for this study to ensure that the methods are valid. These measures included the following: (1) a series of word-problem probes that include nine fraction word problems involving three different word problem schemas: and (2) a twelve problem test that requires generalization of problem schemas to novel problems. The math teacher, researcher, and faculty member determined that the word problems on the probes were representative of each schema by comparing the definition and examples of problems from each schema to the problems in the probes. The same individuals reviewed the transfer pre- and post-test. Again, they compared the definition and examples of problems from each schema to the problems in the transfer test. They also checked to see if the problems would require generalization of strategies to problems that included vocabulary that would be difficult for fifth graders, irrelevant information that was not required to solve problems, and different conceptualizations through analysis of information in tables in all three problem schemas. The researcher, math teacher, and faculty member had 100% agreement in all areas.

To ensure internal validity, baseline levels of performance on group, change, and compare problems were established by administering assessments of problem solving using different versions of the word problem probes until a baseline was developed. The implementation of the interventions was staggered to show controlled replications or lack of replication. Inter-scorer agreement checks were conducted for probes by having the fourth grade math teacher and researcher check student responses with 100% of those assessments checked for agreement on right and wrong responses. The inter-scorer agreement was 100%..

Instrumentation

A series of word-problem probes that include nine fraction word problems involving three different word problem schemas (change, compare, and group) were constructed for this study. Each probe contained three word problems from each of the three schemas. Students were given one point for each problem answered correctly and zero points for each incorrect response. These probes were given every other day during the baseline phase and after every two sessions during the intervention phase. A single probe was given 10 days after completion of the intervention during the maintenance phase for students 1,3, and 4. Student 2 did not complete the intervention lessons and student 5 completed the intervention two days before the end of the school year, so they did not complete this probe. Additionally, to determine students' abilities to generalize to novel problems, a separate test that included twelve problems was created. This test included four problems from each of the three schemas. The problems from each schema included at least one problem with irrelevant information, one problem that contained difficult vocabulary, and one problem that require a different conceptualization because the information in the problem was presented in a different way than the problems presented in the intervention (i.e. - through charts). This test was given during the baseline phase and at the end of the intervention phase for all students except student 2. This student did not complete the intervention, but he was still given the transfer post-test after lesson 10 to see if any changes in performance occurred. The responses of all probes and the transfer tests were assessed separately to determine if the students were able to use the schematic diagram to write the correct equation. Students were given one point if the correct equation was recorded from the schematic diagram and zero points if the incorrect equation was recorded.

Data Collection and Analysis

Data on research questions one and two related to strategy use were collected by evaluating the student's use of models to record information on the schematic diagrams, and the student's use of the schematic diagram or labeling of schema parts in the word problem to write correct equations. To answer research question three, data of performance on problems similar to those taught during the intervention were collected and analyzed using visual inspection and mathematical calculations of graphed data. Data on student performance on transfer problems were collected through a pre-assessment during the baseline phase and a post-assessment during the maintenance phase to answer research question four. Additionally, throughout the study the researcher kept a daily journal to record student responses to the intervention, probes, and transfer tests as well as daily events at the school that affected student performance during the interventions or assessments. The information in this journal was used to support or extend understanding of the quantitative data collected to answer each of the research questions.

Research question one. To determine if students could connect the concrete manipulatives and virtual fraction manipulatives to the representational change, compare, and group schemas when using the CRA sequence, the researcher recorded the number of times that each student correctly modeled a problem with concrete or virtual manipulatives. Data were recorded for all problems that each student solved independently during the intervention phase. The researcher determined the percentage of times that each student used a model to help record information on a schematic diagram by dividing the number of times that the problem was modeled prior to completing the schematic diagram by the total number of word problems attempted during each phase of the intervention.

Research question two. To determine if students could connect the change, compare, and group schemas at the representational level to the abstract equations when using the CRA sequence, the researcher recorded the number of times that each student correctly recorded problem information on the schematic diagram and used that information to write an abstract equation to solve the problem. The data were recorded for probe problems completed during the pre-intervention, intervention, and maintenance phases. During the intervention phase, data were also recorded separately for the problems solved independently on the group, change, and compare schemas. The researcher determined the percentage of times that each student used information on a schematic diagram to write an abstract equation by dividing the number of times that a student wrote an equation from information in a schematic diagram by the total number of word problems attempted during each phase of the intervention.

Table 3. 7
Phases of Instruction and Type of Problems Assessed when Connecting the Schematic Diagram and Equation

Intervention Phases	**Baseline (Pre-Intervention)**	**Group Component of Intervention**	**Change Component of Intervention**	**Compare Component of Intervention**	**Maintenance (Post-Intervention)**
Problems Evaluated	Probes	Problems Solved During Independent Practice, Probe Problems			Probe

Research question three. To answer question three and determine whether using a CRA sequence to connect problem solving schemas and equations would improve student performance on problems similar to the problems used during the intervention, data from the probes were analyzed using visual inspection and mathematical calculations of graphed data. The graph for each student included the total number of questions answered correctly for each probe and the number of correct responses on problems from each schema (group, change, and compare). Data were analyzed to assess gains in overall student performance. Additionally, data were analyzed to determine if student performance only improved on problems from the schema taught or on problems from other schemas that had not been taught as well. Each probe contained problems from all three schemas, but the number of problems correctly answered from each schema was indicated by bar graphs to determine how instruction affected problems from each schema.

The analysis of data for each student was conducted using the same process. In this process, the mean for each phase was calculated for each student and graphed to determine increases and decreases between means during each phase. Visual inspection and comparison of the number correct at the end of one phase and the beginning of the next phase was used to determine if there were changes in the level of the number of correct responses between the baseline and intervention phases or between the intervention and maintenance phases for each student. Visual inspection was also be used to determine if there was a change in the trend from baseline to intervention or intervention to maintenance on these problems. The slope of the trend line was calculated by dividing the rise by the run for each phase for each student to provide a specific mathematical value to use when comparing trend data. Additionally, the variability was calculated by subtracting the lowest score from the highest score to determine the range of each phase. The range of each phase was compared to determine if variability increased or decreased between phases.

Research question four. To answer question four and determine whether using a CRA sequence with schema-based instruction improved student performance on problems that required generalization from the models provided during the intervention, the researcher gave a pre-test of transfer skills during the baseline phase and the same test as a post-test of transfer skills during the maintenance phase. The data on the number of correct responses for each of the three types of transfer problems used to assess generalization of problem solving skills were collected and analyzed for each student. The individual performance of each student and the mean percentage correct on transfer problems that included the pre-test and post-test transfer characteristics of irrelevant information, unfamiliar vocabulary, and problems that require different conceptualizations were calculated and visually compared using bar graphs. Additionally, the pre-test and post-test individual scores and mean scores by problem type (change, compare, group, and all) were calculated and visually compared using bar graphs. This process of analysis indicated the types of transfer problems that students could successfully solve.

Supporting Qualitative Data. To enhance understanding of the results of the quantitative data the researcher kept a daily journal. After each probe or intervention session, the researcher recorded student behavior, student comments, or events that might have affected student performance. Then, based on the results and analysis of the quantitative data for each question, the researcher analyzed the journal data to identify possible explanations for student performance. The analysis of qualitative data focused on the following issues: 1.) Possible reasons why some students connected the concrete or virtual manipulatives to the schematic diagrams and other students did not connect the manipulatives to the diagram; 2.) Possible reasons why some students connected problem solving schemas to abstract equations and others did not connect schemas to equations; 3.) Possible reasons why students were able or not able to solve problems similar to those presented in the intervention; and 4.) Possible reasons why students were able or not able to solve transfer problems. An analysis of student performance and behavior during the intervention, probes, and transfer tests was conducted to identify specific themes, patterns of behavior, and patterns of performance related to each of these issues. Because this information did not directly answer the research questions and provided possible explanations for student performance related to each question, it is included in the discussion of the findings in chapter five.

Summary

This chapter described how an instructional routine that combines the CRA sequence and schema-based instruction was implemented to address the research questions in this study. Information on how the students were selected, the setting for the study, and the specific procedures that were used was also provided. Finally, a description of the measures of problem solving performance and strategy usage were outlined with specific information on how the data from these measures were analyzed.

Chapter Four

Results

Introduction

The purpose of this chapter is to describe all data collected on the effects of using a concrete-representational-abstract (CRA) sequence on the problem solving performance of students who struggle with mathematics or have been identified with learning disabilities in mathematics. For each research question, the data for each student is described and an overall summary is provided. First, the data on the number of times that students used a concrete or virtual model to draw diagrams is presented to answer research question one. Next, student use of diagrams or labeling to write equations during independent practice for each condition and on assessment probes given throughout the study is presented to answer research question two. Overall probe performance on problems by schema; probe performance by overall score and by each problem type (group, change, compare); and data on mean, trend, level, and variability of probe performance for each student is reported to answer research question three. Finally, student gains in performance from transfer pre-test to transfer post-test is given to answer research question four.

Research Question One

When using the concrete-representational-abstract (CRA) sequence, can students connect the concrete manipulatives and virtual fraction manipulatives to the representational change, compare, and group schemas?

Research question one examined whether students could connect the concrete and virtual models to the schemas presented during the intervention. The concrete and virtual models were taught during the first two lessons related to each schema. Students could use the models during the guided practice and independent practice components of the last two lessons for each schema and during the LISTS strategy instruction. The number of problems where students used the concrete or virtual models to help draw the schematic diagrams during the independent practice for each lesson was recorded. Each independent practice sheet included six word problems. Therefore, students who completed all lessons had the opportunity to use the strategy a total of 42 times. The data of model use is recorded in *Table 4.1.*

Table 4.1
Student use of models to draw diagrams during independent practice for each condition

	CRA + Group Intervention		LISTS Strategy	CRA + Change Intervention		CRA + Compare Intervention		Total Use by Number and Percent
Strategy Use	L. 3	L. 4	L. 5	L. 8	L. 9	L. 12	L. 13	
Student 1	2	x	2	x	6	6	6	22/42 (52%)
Student 2	1	x	2	2	x	NA	NA	5/30 (17%)
Student 3	x	x	x	6	2	x	x	8/42 (19%)
Student 4	6	x	x	6	2	x	x	14/42 (33%)
Student 5	6	6	6	6	6	6	6	42/42 (100%)

X = Strategy was not used
NA= Student did not receive instruction; no opportunity for strategy use

Student 1. Student 1 used either a concrete or virtual model to help draw diagrams on a total of 22 out of 42 possible problems. Initially, he was less likely to use a model. In lessons 3 and 5 he only modeled 2 out of 6 problems and he did not model any problems during lessons 4 and 8. In later lessons 9, 12, and 13, student 1 modeled all problems for each lesson for a total of

18 out of 18 problems modeled. Student 1 used a combination of the concrete and virtual models when completing the problems.

Student 2. Student 2 only modeled 5 out of 30 problems when completing the independent practice. He only attempted to model problems with like denominators. When he chose to model problems, he selected only the concrete fraction tiles to use to model the problems. Student 2 did not have the opportunity to model problems in lessons 12 and 13 because he did not obtain the mastery criteria for lesson 10 prior to the end of the school year.

Student 3. Only 8 out of 42 problems were modeled with concrete or virtual manipulatives by student 3. This student chose to model all problems during lesson 8 on the change schema and two problems from lesson 9 in the change schema. In lesson 9, the student only modeled problems with unlike denominators. He chose to model these problems after he completed the problem in the schematic diagram because he believed that he had made an error in his response.

Student 4. Student 4 modeled 14 out of 42 problems using concrete or virtual manipulatives. He modeled all problems from lesson 3 in the group schema and lesson 8 in the change schema. He also modeled two problems with unlike denominators from lesson 9. Student 4 chose to use the virtual manipulatives when he modeled problems. Similar to student 3, he modeled the problems with unlike denominators in lesson 9 after he completed the problems in the schematic diagrams because he believed that he had made an error in his responses.

Student 5. Student 5 modeled all 42 problems using models. In lessons 3, 4, 5, 8, and 9, she modeled all problems using concrete manipulatives because she had great difficulty understanding fraction concepts and was not comfortable using the virtual fraction tool without assistance. During lessons 12 and 13, student 5 became more confident with the virtual fraction tool and used it to model all problems.

Summary of results. Out of 198 possible problems, students used either a concrete or virtual model 91 times (46%) to model problems. Although overall this is not a high percentage, model use varied widely from one student to another. Student 5 used models when completing all problems, but student 2 used the models only 17% of the time and student 3 used the models only 19% of the time. Some students who used models infrequently selected different types of problems to model. Student 2 modeled only problems with like denominators. By lesson 9, students 3 and 4 chose to model only problems with unlike denominators after they were unsure of their responses using the schematic diagrams.

Research Question Two

When using the CRA sequence can students connect the representational change, compare, and group schemas to the abstract equations?

Research question two examined whether students used the schematic diagram or labeled parts of the schema in the word problems to write equations presented during the intervention and during the assessment probes that the students took after they were introduced to the schematic diagrams. In the third lesson on each schema, the students were taught the schematic diagram for the schema. Students could use the diagrams during the guided practice and independent practice components of the last two lessons for each schema and during the LISTS strategy instruction. *Figure 4.1* shows an example of student use of a diagram to write an equation.

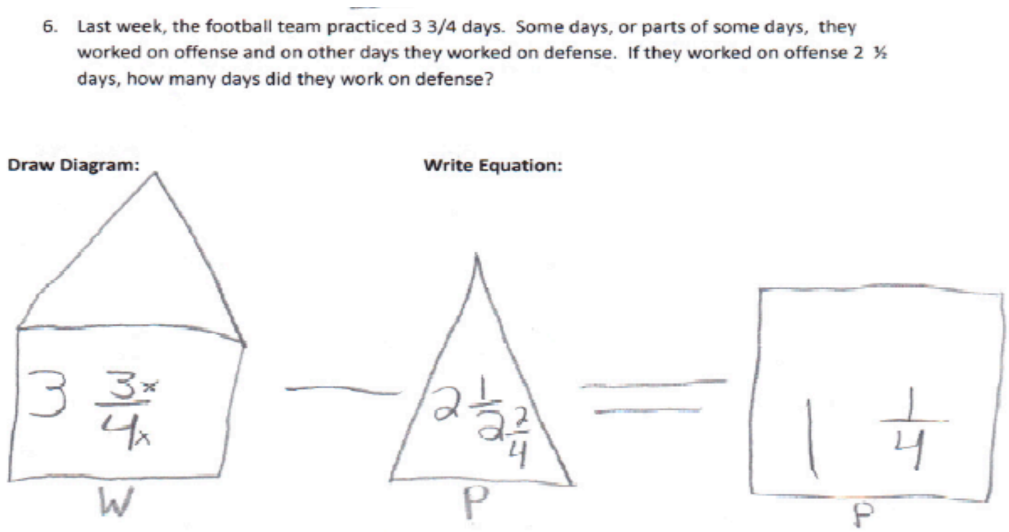


Figure 4.1. **Student work sample of group problem with diagram**

Some students began to label the parts of the schema within the context of the problem and then write an equation. This labeling process also was counted as data that students could connect the schema to the equation.

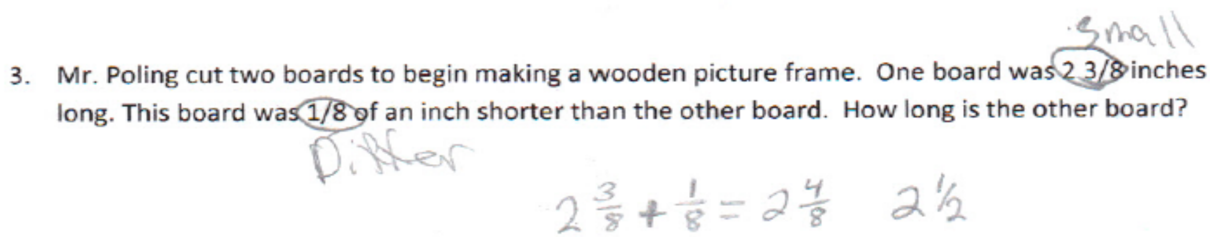


Figure 4.2. **Student work sample of compare problem with labeling**

The number of problems that the students used the schematic diagrams or labeling process during the independent practice for each lesson or during the probes was recorded as data to answer question 2. Each independent practice sheet included six word problems. Therefore, students who completed all lessons had the opportunity to use the strategy a total of 42 times during independent practice (See Table 4.2). The number of opportunities to use the schematic diagram or label problems varied during probes depending on when the student began the intervention.

Student 1. Student 1 used a schematic diagram to write an equation on 36 out of 42 (80%) of problems during the independent practice of the lessons, but he only used a schematic diagram on 9 out of 81 (10%) of possible probe problems. During independent practice sessions student 1 wrote the problems and the solutions on his paper for all lessons except the final lesson on the compare schema. Even though he was instructed to write the equation used to solve the problems during the probes, student 1 often only wrote his final answer on the paper.

Table 4.2
Student use of diagram or labeling to write equations during independent practice for each condition

	CRA + Group Intervention		LISTS Strategy	CRA + Change Intervention		CRA + Compare Intervention		Total Use by Number and Percent
Strategy Use	L. 3	L. 4	L. 5	L. 8	L. 9	L. 12	L. 13	
Student 1	6	6	6	6	6	6	x	36/42 (80%)
Student 2	6	6	6	6	x	NA	NA	24/30 (80%)
Student 3	6	x	6	6	6	6	6 (label)	36/42 (80%)
Student 4	6	x	6	6	6	6	6 (label)	36/42 (80%)
Student 5	x	6	6	6	6	6	6	36/42 (80%)

X = Strategy was not used
NA= Student did not receive instruction; no opportunity for strategy use
Label= Students did not use the diagram, but did label each component of the schematic diagram in the text of the problem.

Table 4.3
Student use of diagram or labeling to write equations during probes

Strategy Use	Possible Probes for Strategy Use	Probes with Strategy Use (Number of Problems)	Total Use (after strategy was taught)
Student 1	Probes 6-14	Probe 8 (all 9 problems)	9/81 (10%)
Student 2	Probes 6-14	x	0/81 (0%)
Student 3	Probes 9-14	x	0/54 (0%)
Student 4	Probes 9-14	-Probes 9, 10, and 11 (All problems; 27 total) -Probes 12, 13, 14 labeled schema parts (All problems- 27 total)	54/54 (100%)
Student 5	Probes 12-16	Probes 12, 13,14, 15, 16 (all problems- 45 total)	45/45 (100%)

Student 2. Student 2 used a schematic diagram to write an equation in 24 out of 30 (80%) of problems during the independent practice problems. He used the schematic diagram in all possible lessons except the final lesson on the change schema. Because he did not meet the mastery requirements to complete the lessons with modeling on the compare schema, he did not have the opportunity to use the diagrams during the compare lessons. Student 2 did not use the schematic diagram on any of the probe lessons. Like student 1, he often only recorded the answers to the problems even though he was instructed to write the equation and the solution to each problem.

Student 3. Student 3 used a schematic diagram or labeled problems for 36 out of 42 (80%) of possible problems during independent practice, but he did not use the process at all during the probes. During independent practice, student 3 did not use the schematic diagram during the final lesson in the group schema. During the final intervention lesson, student 3 changed to a procedure of labeling the parts of the word problems. Unlike students 1 and 2, student 3 did record all equations when completing probe problems, but provided no evidence of diagrams or labeling to complete the problems.

Student 4. The results for student 4 were consistent with the results for other students in the study during the independent practice component of the lessons. He used the schematic diagram or labeled schema parts in 36 out of 42 (80%) of the problems during these lessons. The results for student 4 were different than those for other students during the probes. He attempted to use a schematic diagram or labeling process for all problems when he completed the probes. In the first three probes after he learned how to use the schematic diagrams, he used the diagrams when solving problems. In the next three probes he modified his approach and labeled the components of the schema within the word problems.

Student 5. Student 5 also used a schematic diagram for 36 out of 42 (80%) of problems during independent practice. She did not use the diagram for the initial group lesson, but she used the diagram for all other possible lessons. Student 5 used the diagram for all 45 problems during the probes as well. She used a combination of labeling and schematic diagrams. In other words, she labeled the components of the schema within the problems and used the diagrams when solving all independent practice problems and probe problems.

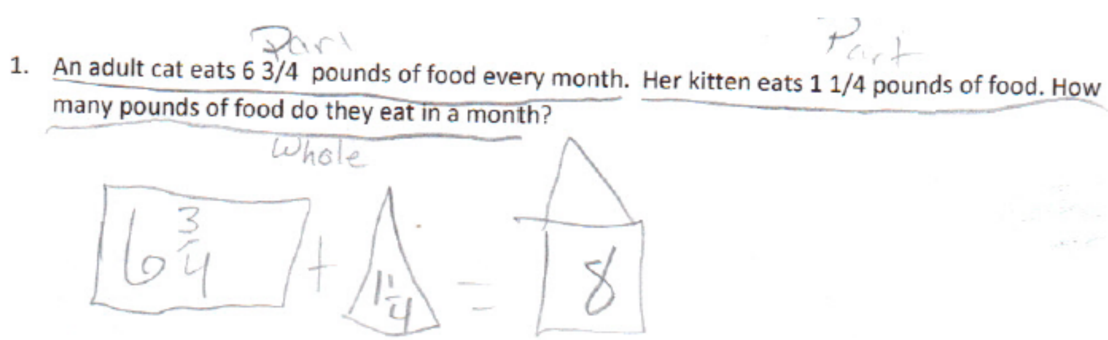


Figure 4.3. **Student 5 work sample of group problem with labeling and schematic diagram**

Summary of results. Out of a total of 198 independent practice problems, the students used a schematic diagram or labeling process on 168 (80%) of all problems. Student results were very consistent with all students using the strategy 80% of the time. Each student failed to use the strategy during one lesson, but this lesson varied from student to student. Strategy use during the probes was significantly lower and more inconsistent between students. Students 2 and 3 did not use the schematic diagram or a labeling process at all during the probes and student 1 only used the diagram during one probe. On the other hand, students 4 and 5 used a schematic diagram, labeling process, or both for all probes.

Research Question Three

Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems similar to the problems used during the intervention?

Student performance on problems similar to problems used during the intervention was examined in research question three. To assess student performance, students were given probes that included three problems from each schema. Data in *Figure 4.4* shows overall student performance when receiving intervention lessons from each schema (CRA + group, CRA + change, CRA + compare). These figures show the overall performance during baseline, intervention, and maintenance for all students on all problems in each probe. Student 2 did not complete the intervention, so no maintenance data was obtained. Student 5 finished the intervention on the day before school was completed for the year, so no maintenance data could be obtained for her either.

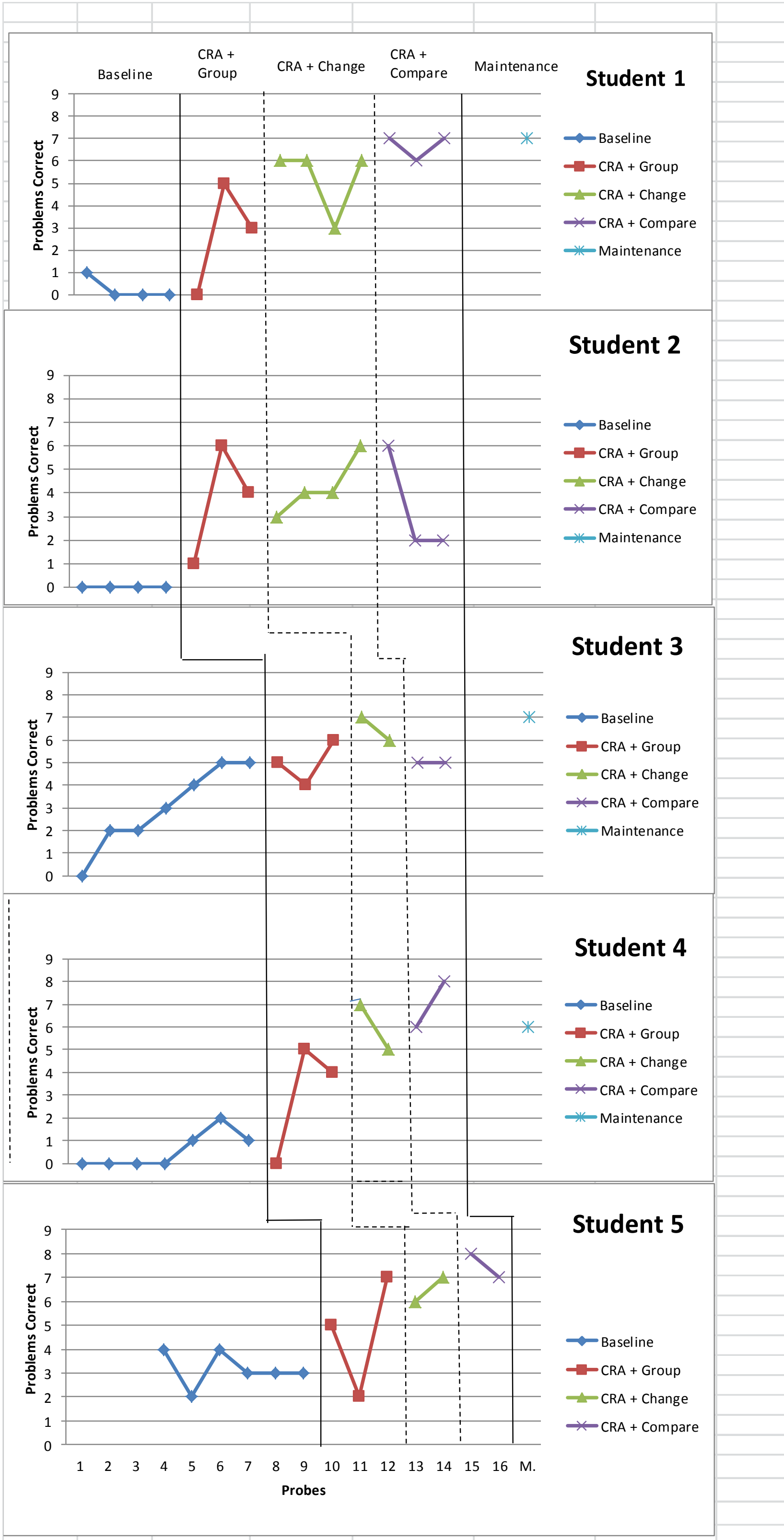


Figure 4.4. **Overall probe performance on problems by schema**

Figure 4.5 includes the performance on each type of problem (i.e. group, change, compare) in relation to the type of instruction during the intervention and overall performance. In other words, performance on group, change, and compare problems can be seen during baseline, the CRA + group lessons, CRA + change lessons, CRA + compare lessons, and during maintenance. The diamonds in the graphs represent the number of problems students answered correctly on each of the 9 problem probes. The bars under each diamond show the number of problems answered correctly by each problem type on each probe and add up to the total number of problems answered correctly on the probe. Since only 3 problems were included from each schema on each probe, the greatest number of problems that each student could answer correctly from each schema was three. For example, the graph shows that for probe 6, student 1 correctly answered five questions including three questions from the group schema, one question from the change schema, and one question from the compare schema. Student 2 answered six questions correctly on the sixth probe including two from the group schema, two from the change schema, and two from the compare schema.

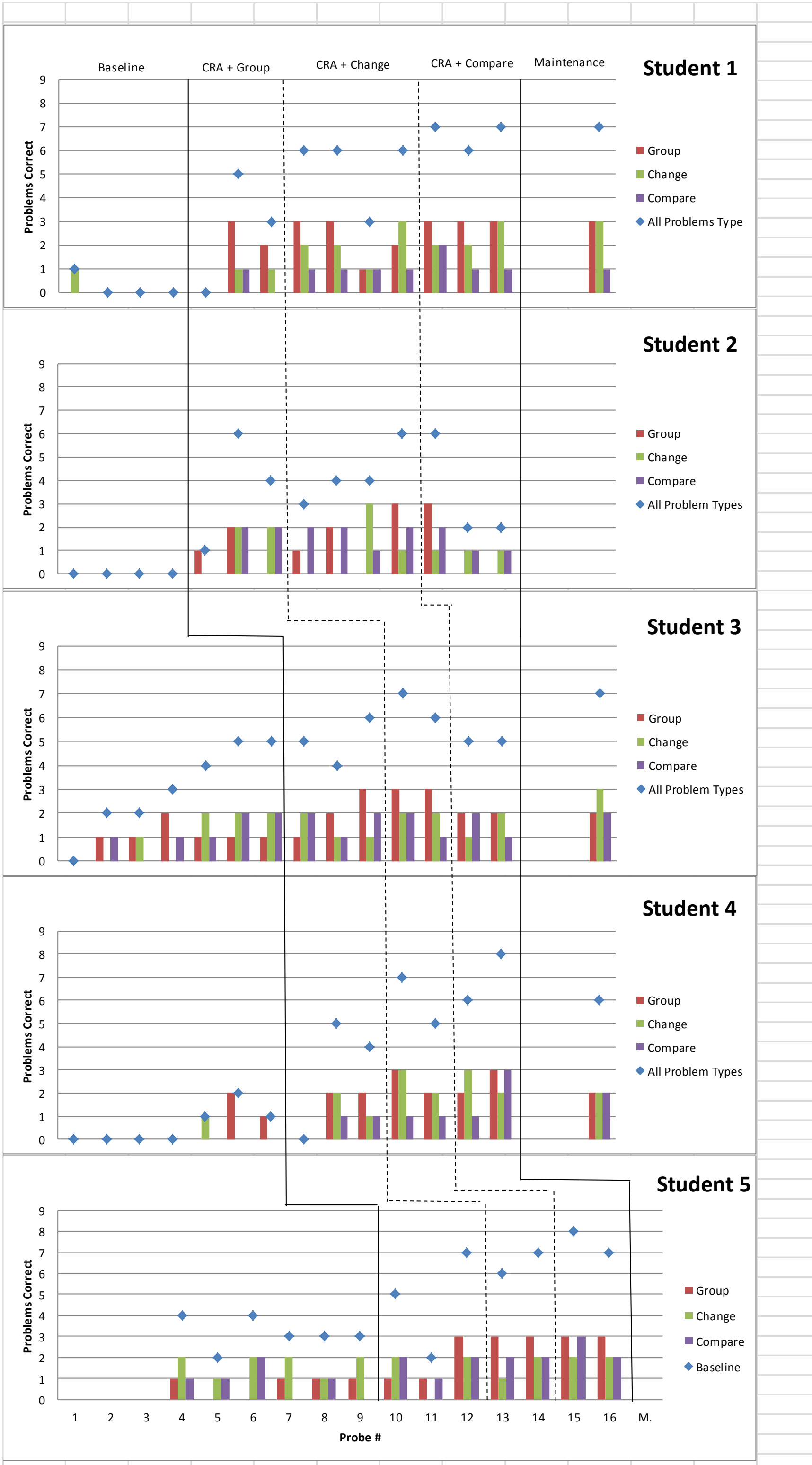


Figure 4.5. **Probe performance by overall score and by each problem type (group, change, compare)**

Student 1. Overall performance on all problems in the probes rose from an average of .25 correct in the baseline phase to an average of 6.66 problems correct during the compare component of the intervention for student 1. His score of seven correct in the maintenance phase was consistent with his final probe performance during intervention. Visual inspection and review of mathematical calculations show that the largest gains for student 1 occurred on probes given during the CRA + group instruction and the CRA + compare instruction where he improved from an average of .25 correct in baseline to 5.25 correct by the end of the lessons on the change schema. Changes in level (+3) and variability (+5) also support strong performance on probes given during the group instruction. On the probes given during the CRA + group instruction, student 1 showed a positive trend of +1.66, but he did show a negative trend (-.75) on the probes during the change lessons due to poor performance on one probe (See Table 4.4).

A review of data in *Figure 4.5* on student performance on the different types of problems shows that student 1 initially improved on problems from the group schema during instruction on this schema. In probe 6, he correctly answered all three problems from the group schema and only one problem from both the change and compare schema. In probe 7 his performance declined, but he still correctly answered two problems from the group schema and only one from the change schema. During instruction on the change schema, strong performance on problems from the group schema continued and performance on problems from the change schema improved as well. In both probes 8 and 9, student 1 correctly answered all three problems from the group schema, two problems from the change schema, but only one problem from the compare schema. In probe 11, he correctly answered all three problems from the change schema and two from the group schema. The exception to this pattern occurred during probe 10 when student 1 only answered one problem correctly from each schema.

The pattern of improvement on specific problems types did not continue on probes conducted during the compare lessons of the intervention. In the initial probe given after this schema was introduced student one continued the pattern of three correct group problems and two correct change problems. The number of compare problems answered correctly rose to two, but this change did not continue in the final two probes or on the maintenance probes. On each of these probes, student 1 only answered one compare problem correctly.

Table 4.4
Data on mean, trend, level, and variability for each student

Word Problems	Baseline to Intervention	Group to Change	Change to Compare	Compare to Maintenance	Maintenance
Student One					
Mean	.25	2.66	5.25	6.66	7
Trend	-.25	1.66	-.75	-.33	na
Level	0	+3	+1	0	na
Variability	1	5	3	1	na
Student Two					
Mean	0	3.66	4.25	3.33	na
Trend	0	1.66	.75	-1.33	na
Level	1	-1	0	na	na
Variability	0	5	3	4	na
Student Three					
Mean	3	5	6.5	5	7
Trend	.71	.66	-.50	0	na
Level	0	+1	-1	+2	na
Variability	5	2	1	0	na
Student Four					
Mean	.57	3	6	7	6
Trend	.29	1.66	-1	1	na
Level	-1	+3	+1	-2	na
Variability	2	5	2	2	na
Student Five					
Mean	3.16	4.66	6.5	7.5	na
Trend	.33	1.66	.50	-.50	na
Level	+2	-1	+1	na	na
Variability	2	5	1	1	na

Student 2. Performance on probes for student 2 rose from an average of zero correct during the baseline phase to a high of 4.25 problems correct on probes given during the change lessons of the intervention. This average number correct on probes actually decreased during the compare lessons of the intervention to just 3.33 problems correct. Student 2 showed the greatest gains on probes given during the group intervention. His scores showed a positive trend of 1.66 and slight improvement in level (+1). Student 2 did show a slight improvement in the average number of probe problems answered correctly during the change lessons with his mean correct increasing from 3.66 to 4.25 and a positive trend of .75. Visual inspection of graphed data and an analysis of variability data indicate that his performance on probes was inconsistent. Student 2 went up to a high of six problems answered correctly on probe 6 down to only three correct in probe 8 with consistent gains on probes given during the change intervention. Student 2 recorded a sharp drop in the number of correct responses on probes given during the compare intervention. The average number of problems answered correctly (3.33) dropped to below the average number of problems answered correctly during the group intervention (3.66) and a sharp negative trend of -1.33 was recorded on probe performance. Student 2 did not have maintenance data because he did not meet the mastery criteria to complete the lessons prior to the end of the school year.

There was significant variability in the types of problems that student 2 answered correctly on the probes. When receiving the group component of the intervention, his performance on group problems did not show a pattern. He initially answered one group problem correctly on probe 5 and two group problems correctly in probe 6, but he did not answer any group problems correctly in probe 7. During lessons from the change component of the intervention, student 2 showed a slight increase in the number of group problems correctly answered on probes with the exception of probe 10. Toward the end of the intervention, student 2 also correctly answered three change problems on probe 11, but then only answered one change problem correctly in probe 12. During the compare component of the intervention lessons, he answered three group problems correctly in probe 12, but no group problems correctly in probes 13 or 14. He consistently answered one change problem correctly in probes 12-14. Throughout the intervention phase, student 2 answered one to two compare problems correctly. There was no change in this pattern during the compare lessons of the intervention. Therefore, analysis of this data indicates that there were no patterns of improvement on probe problems from the change and compare schemas after instruction on problems from each of these schemas.

Student 3. Similar to student 2, student 3 also recorded his greatest gains in mean performance correct from probe problems given during the baseline phase (3.00) to probe problems given during the change component of the intervention (6.5). His performance on probe problems decreased when receiving instruction on the compare schema. The mean number of five problems with correct responses on probes during the compare intervention was the same as the mean number of five problems answered correctly during the group component of the intervention. Student 3 did show an increase to seven correct responses on the maintenance probe however. Student 3 recorded positive trends during baseline (+.71) and during the probes administered during the group intervention lessons (.66). Although there were only two sets of probes with consistent scores in the baseline phase prior to the intervention, the researcher began the intervention because of the large number of probes (7) that had been administered.

A review of performance on individual problem types indicates that performance on group problems improved on probe problems completed during the group intervention and on probe problems completed during the change intervention. During the baseline phase, student 3 answered zero to three group problems on probes correctly, with an average of one group problem answered correctly across the seven probes. During the group intervention, student 3 correctly answered one to three group problems correctly with an average of two correct responses, while during the change intervention he correctly completed all three group problems on both probes. Performance on group problems did decrease on probes taken during the compare intervention and maintenance with only two correct responses on probes 13 and 14 and the maintenance probe.

Visual inspection of performance on the change and compare probe problems indicates that there were no significant changes in performance on these types of problems from baseline to the CRA + group, CRA + change, or CRA + compare components of the intervention. In the baseline phase, student 3 ranged from zero to three problems correct from the change schema, but by the end of the baseline phase in probes 5-7, student 3 only completed two change problems correctly on each probe. During instruction on the group, change, and compare schemas, student 3 correctly completed one to two problems from the change schema on each probe. His performance did increase to three correct problems during the maintenance probe, but this level of accuracy was only demonstrated in this one probe. Performance on compare probe problems was similar to that of change problems. Student 3 ranged from 0-2 correct problems on probes given during the baseline phase and on probes during the group phase. During the change lessons, compare lessons, and maintenance, his performance on compare probe problems remained stable with 1-2 compare problems answered correctly on each probe.

Student 4. Overall performance on all problems in the probes rose from an average of .57 correct in the baseline phase to an average of seven problems correct during the compare component of the intervention for Student 4. Student 4 demonstrated consistent gains during probes given during the group lessons and change lessons with a mean of three correct during the probes given during the group lessons and six correct during the change lessons. Visual inspection of graphs and data on trend (+1.66), level (+3) and variability (5) also indicate that student 4 made significant gains in probe performance during the group intervention phase. Although the trend on probe performance during the change intervention was a -1, overall performance was higher during this component of the intervention. Performance continued to increase up to a high of eight correct on probe 14 during the compare intervention, but the number of problems answered correctly was only six on the maintenance probe.

Student 4 correctly answered 0-2 group problems during probes given during the baseline phase. His performance on probe problems from the group schema remained in the 0-2 range during the group intervention, but he correctly answered 2-3 group problems on probes administered during the change and compare lessons of the intervention. The number of correct responses to group problems during the maintenance phase was two problems as well. Out of seven probes administered in the baseline phase, student 4 correctly answered one change problem on probe 5, but did not obtain correct answers on any other change problems during baseline. During the group component of the intervention, student 4 averaged just one correct change problem on each probe. The average increased to 2.5 change problems correct on probes given during the change and compare components of the intervention. During the maintenance probe, he correctly answered two change problems. Student 4 did not answer any compare problems correctly during the baseline probes. He answered 0-1 compare problems correctly during the group component of the intervention and one problem on each probe of the change component of the intervention. Student 4 did not show a strong pattern of performance on probe compare problems during the compare component of the intervention. He answered one compare problem correctly on the first probe administered during the compare intervention, but he answered three problems correctly on the second probe administered during the intervention. On the maintenance probe he answered two compare problems correctly.

Student 5. Performance on probes for student 5 rose from a mean of 3.16 correct during the baseline phase to a mean of 7.5 correct for probe problems during the compare component of the intervention. Participant 5 demonstrated significant variability during the group component of the intervention by obtaining five correct responses on probe 10, two correct responses on probe 11 and then seven correct responses on probe 12. Student 5 demonstrated less variability (1) and consistently strong scores (mean=6.5) on probes administered during the change component of the intervention. This high performance and decrease in variability remained consistent during probe performance in the compare component of the intervention.

Student 5 did show a pattern of improvement on probe group problems after instruction in the group component of the intervention. She correctly answered only 0-1 problems from the group schema during the six probes given during the baseline phase. On the initial two probes administered during the group component of the intervention, she answered one group problem correctly, but on the third probe she answered three group problems correctly. On the probes administered during the change and compare components of the intervention her scores remained high. She correctly answered three group problems on both probes administered during the change component and both probes given during the compare component of the intervention. Student 5 correctly answered 1-2 change problems on the six probes given during the baseline phase. There were no significant changes in this pattern on probes administered during the group phase of the intervention (0-2 correct) the change phase of the intervention (1-2 correct), or the compare phase of the intervention (2 correct). Performance on compare problems increased slightly during instruction on the compare component of the intervention. Student 5 ranged from 0-2 problems correct on compare problems on the probes given in baseline and 1-2 problems correct on the probes given during the group component of the intervention. Her performance became more stable with two problems correct during instruction on the change schema and it increased slightly to an average of 2.5 problems correct on probes given during instruction on the compare schema.

Summary of results. All students demonstrated some gains in the mean number of questions answered correctly on the probes from baseline to intervention, but the performance of student 3 returned to baseline levels at the end of the intervention and the number of problems student 2 answered correctly dropped significantly during the last two probes of the intervention. Student 1, 4, and 5 showed positive trends in performance throughout the intervention. Student 1 started with a mean performance of .25 in the baseline phase and finished with seven correct (75% gain) on the final probe during intervention. Student 4 obtained an average of .57 correct in the baseline phase and completed eight problems correctly (82% gain) on the final probe of the intervention. Student 5 had a higher average of 3.16 correct during the baseline phase, but still improved to seven problems answered correctly (43% gain) in the final intervention probe. All three of these students recorded positive overall trends in performance with the following measures of slope calculated from a line of best fit for intervention performance: student 1 = 0.52; student 4= .96; and student 5 = .64. Student performance varied on the maintenance probes. Student 3 performed better on the maintenance probe than he did on his final probes during the intervention. Student 1 performed at the same level as his final probe, while student 4 had a decrease in the total number correct on the maintenance probe (See *Figure 4.6*).

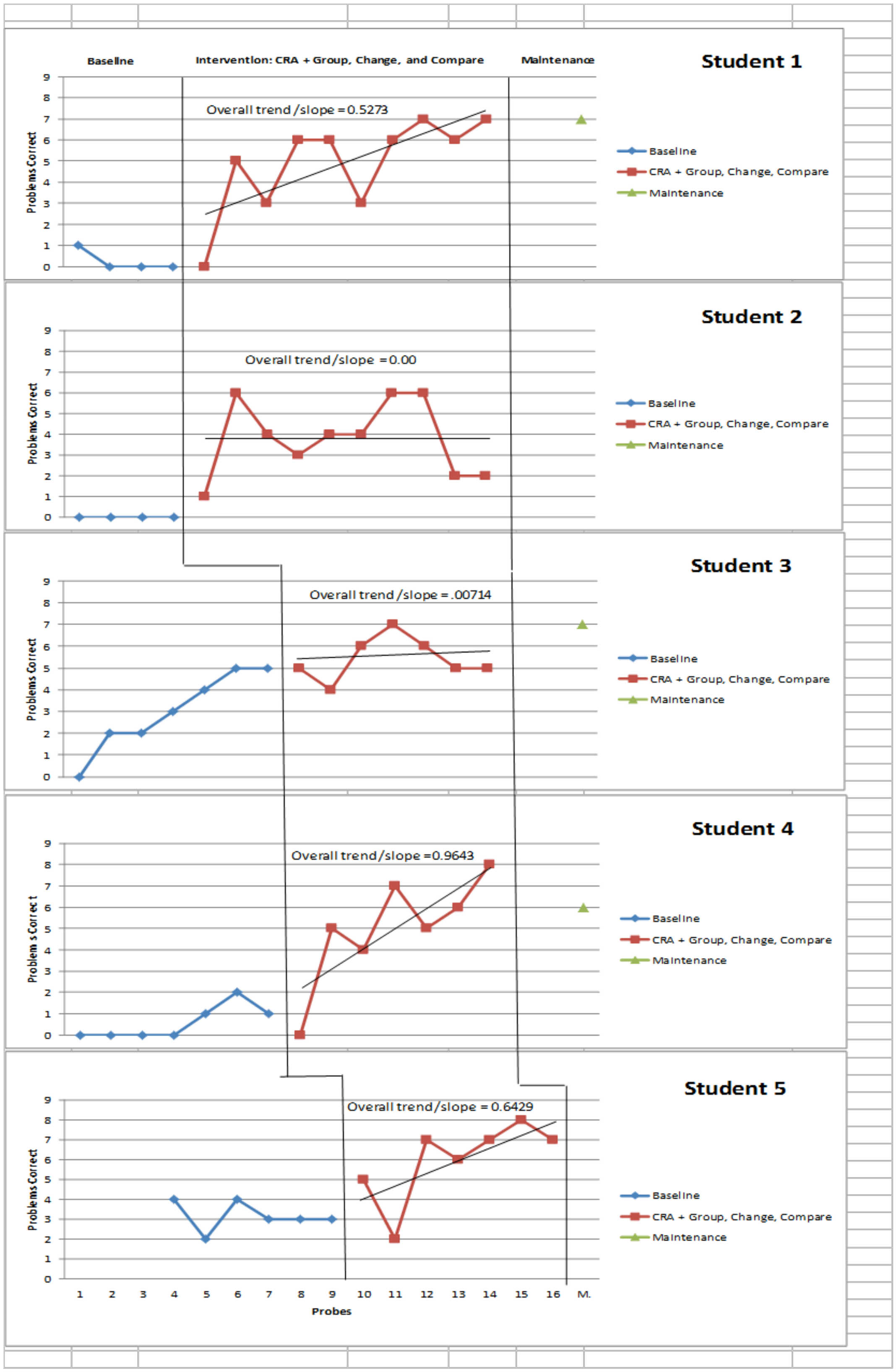


Figure 4.6. **Summary of overall performance on probe problems across entire intervention**

Students 2 and 3 showed the biggest average gains in performance on probes that were administered during instruction on the CRA + group component of the intervention, while students 1, 4, and 5 demonstrated the greatest average gains in performance on probes that were administered during instruction on the CRA + change component of the intervention. Students 1,4, and 5 all showed slight gains in average performance on the probes administered during the CRA + compare component of the intervention, but students 2 and 3 actually saw decreases in performance on probes given during lessons on the compare schema.

It is difficult to make summary statements on the relationship between instruction on a specific schema and change in student performance due to the small number of problems from each schema on each probe and the small number of probes given during instruction on each schema. However, there were some general trends in performance noted across students. First, all students made some gains in performance in the number of group problems answered correctly after instruction on the group schema. Student 1 and 5 demonstrated a clear pattern of stronger performance on group problems, while students 2, 3, and 4 made smaller or more inconsistent gains in performance on group problems. Some students appeared to make gains in performance on change problems after instruction on the change schema. Student 1 and student 4 both increased in the average number of change problems completed correctly during the CRA + change and CRA + compare components of the intervention. On the other hand, no students showed significant improvements in performance on compare problems during or after instruction on the compare schema. Student 4 and Student 5 did each have one probe given during the compare schema where they correctly answered all 3 problems from the compare schema, but this performance was not sustained in subsequent probes.

Research Question Four

Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems that require generalization from the models provided during the intervention?

Research question four examined whether students could apply the information from the intervention to novel problems that included difficult vocabulary, irrelevant information, and different conceptualizations (i.e. - through charts) of information. Students were given a transfer pre-test of 12 problems that included four problems from each of the three schemas during the baseline phase of the study. The problems from each schema included at least one problem with irrelevant information, one problem that contained difficult vocabulary, and one problem that required a different conceptualization of information. The students were given the transfer post-test after completion of all lessons during the intervention. Student 2 was an exception. He did not meet the mastery criteria on Lesson 10 prior to the end of the school year, but he was given the transfer test to see if there were any changes in his performance from the pre-test. Table 4.5 shows overall gains on the number of problems answered correctly and overall percent gains in performance for each student, as well as gains in the number of problems answered correctly for each student by problem type and transfer characteristic. Figure *4.7* includes graphs of performance for each student by problem type and transfer characteristic.

Table 4.5
Student gains in performance from transfer pre-test to transfer post-test

	Gains by Problem Type				Gains by Transfer Characteristic		
	Overall Pre- to Post-Test (% change)	Group	Change	Compare	Difficult Vocabulary	Irrelevant Information	Different Conceptualization
Student 1	5 (42%)	1	2	2	4	1	0
Student 2	1 (8.3%)	1	0	0	1	0	0
Student 3	2 (16.7%)	0	1	1	2	0	0
Student 4	1 (8.3%)	0	0	1	1	0	0
Student 5	2 (16.7%)	1	0	1	2	0	0

Student 1. On the transfer pre-test, student 1 did not answer any problems correctly, but on the transfer post-test he correctly answered five questions. This was an overall gain of 42%. On the post-test, student 1 answered one group problem correctly, two change problems correctly, and two compare problems correctly. Analysis of student performance by transfer characteristics shows that student 1 had the greatest increases on the post-test on problems with difficult vocabulary (4 correct). He also correctly responded to one problem with irrelevant information, but he did not answer any problems correctly that required different conceptualizations.

Student 2. Student 2 did not answer any problems correctly on the pre-test and he answered just one problem correctly on the post-test. The problem that was answered correctly on the post-test was from the group schema and included difficult vocabulary.

Number Correct by Problem Type **Number Correct by Transfer Characteristic**

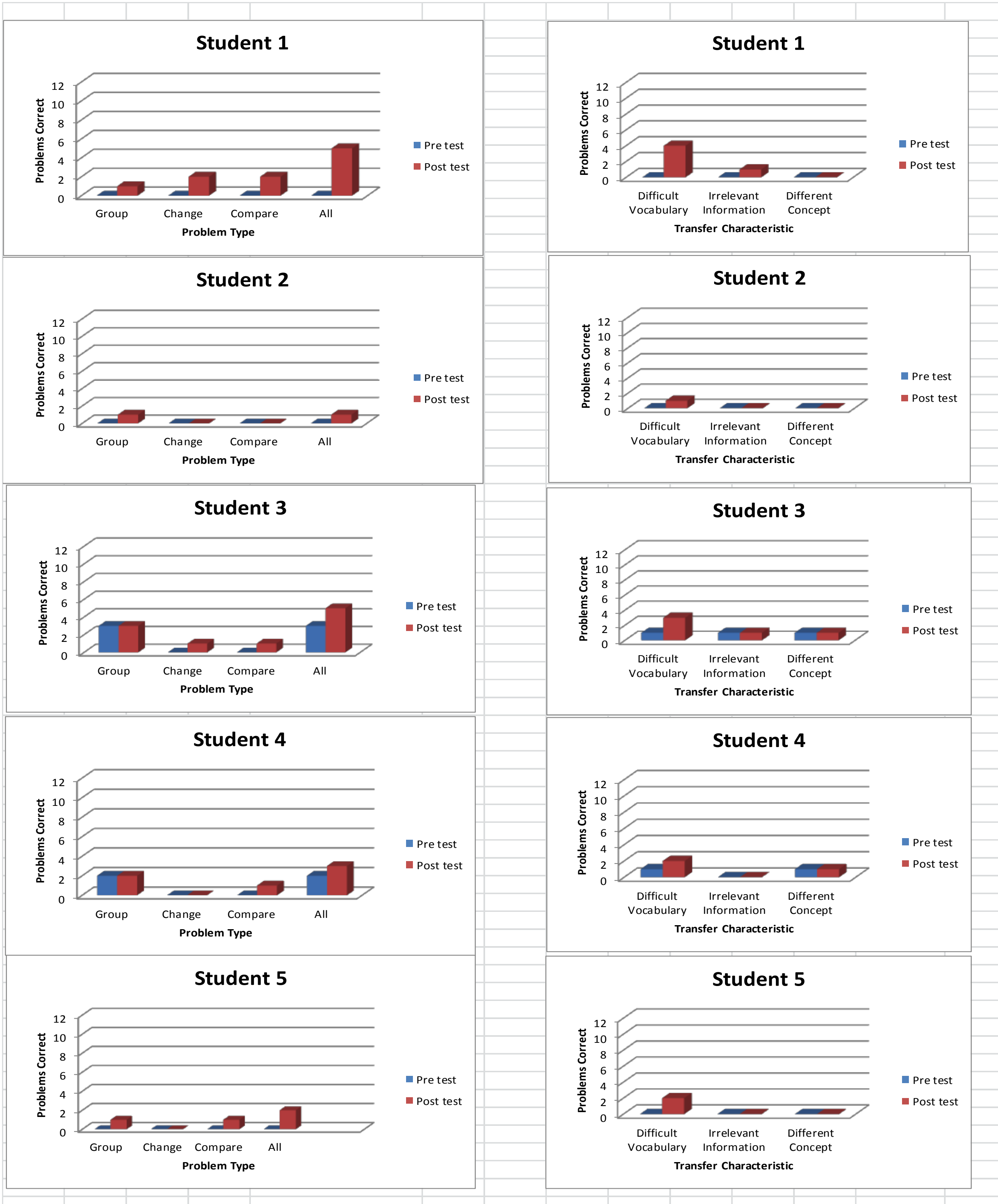


Figure 4.7. **Pre- and Post-test Transfer Data by Problem Type and by Transfer Characteristics**

Student 3. On the transfer pre-test, student 3 correctly responded to three problems. He was able to answer five problems correctly on the post-test (16.7% gain). Student 3 determined the correct solution to the same number of group problems on the pre-test and post-test. He correctly answered one more change and one more compare problems on the post-test than he did on the pre-test. From pre-test to post-test student 3 showed no gains on the number of problems solved correctly with irrelevant information or with different conceptualizations. On the post-test, he did answer two more questions with difficult vocabulary correctly than he did on the pre-test.

Student 4. Student 4 answered two questions correctly on the pre-test and three questions correctly on the post-test (8% gain). He recorded the correct response to two group questions in both the pre- and post-tests and did not answer any change questions correctly on either test. He did not answer any compare questions correctly on the pre-test, but he did obtain the correct response to one compare question on the post-test. An analysis of problems answered correctly by transfer characteristics show that student 4 did not get the correct response to any questions with irrelevant information on either the pre or post-test and he responded correctly to one question with different conceptualizations on both the pre- and post-test. He correctly responded to one question with difficult vocabulary on the pre-test. His performance on these questions improved to two correct responses on the post-test.

Student 5. Similar to students 1 and 2, student 5 did not respond correctly to any problems during the pre-test. She correctly responded to two questions on the post-test (16.7% gain). On the post-test, student 5 recorded correct responses to one group problem, zero change problems, and one compare problem. Student 5 did not answer any problems with irrelevant information and different conceptualizations correctly, but she did answer two problems with difficult vocabulary correctly.

Summary of results. Overall, students improved the number of problems correct an average of 2.2 problems from the transfer pre-test to the transfer post-test. Average gains by problem type were greatest for the compare problems. The average number of problems answered correctly increased by one problem overall. An average gain of 0.6 was recorded for both change and group problems (See *Figure 4.8*). There was some variability by student noted. Students 1, 2, and 5 correctly answered one more group problem on the post-test than they did on the pre-test. Students 3 and 4 had no changes in performance on group problems from pre- to post-test. On change problems, student 1 increased the number of change problems answered correctly by two problems and student 3 answered one more problem correctly on the post-test than on the pre-test. Students 2, 4, and 5 had no changes in performance on the change problems from pre- to post-test.

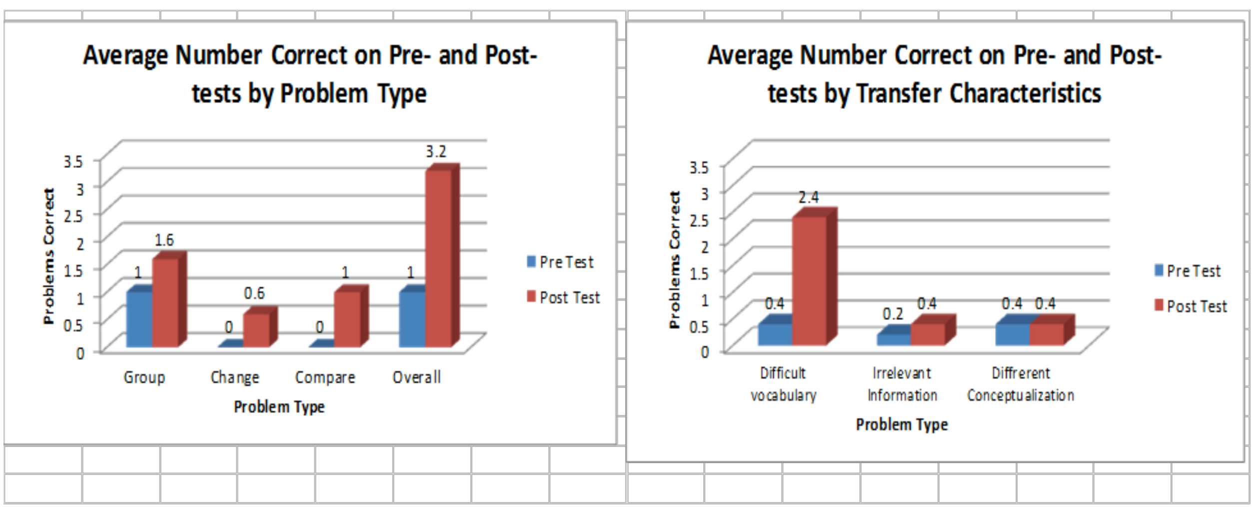


***Figure 4.8*. Average number correct for all students on pre- and post-tests by problem type and transfer characteristics**

On compare problems, students 3, 4, and 5 increased the number of problems correctly answered by one in the post-test and student 1 increased the number of problems answered correctly by two on the post-test. Student 2 did not record any changes in performance from pre- to post-test on compare problems.

Average gains by transfer characteristics were greatest on problems that included difficult vocabulary. The average performance rose from 0.4 correct on the pre-test to 2.4 correct on the post-test for an average gain of two problems answered correctly on problems with difficult vocabulary. Significant variability was noted in the gains on problems with difficult vocabulary by student. Students 2 and 4 increased the number of correct responses on these problems by one, students 3 and 5 increased the number of correct responses on these problems by two, and student 1 increased the number of correct responses on these problems by four. On the other hand, there was a very slight average gain in performance from pre- to post-test on problems with irrelevant information. Students only improved an average of 0.2 of a problem from pre-test to post-test. Only student 1 improved performance on these problems on the post-test and he only answered one more problem correctly than he did on the pre-test. There was no difference in performance from pre- to post-test on problems with different conceptualizations. Average performance on both tests was only 0.4 problems answered correctly.

Conclusion

In this chapter, the data collected to determine the effects of using a concrete-representational-abstract (CRA) sequence on the problem solving performance of students who struggle with mathematics or have been identified with learning disabilities in mathematics were reported. For each research question, the results were presented by student and an overall summary of results for all students was provided. Results on research question one show that models were used with about half of all possible problems during independent practice to help draw schematic diagrams, but that model use varied widely among students in the study.

Analysis of data on research question two indicates that all students used schematic diagrams or labeled parts of the schema to write equations on 80% of the problems given during the independent practice sessions, but three out of the five students did not apply this strategy to most or all probe problems. The two students that did apply the strategy to probe problems used the strategy when completing all probe problems. Results for research question three showed positive trends and significant improvements on probes given in the baseline phase to probes given in the intervention phase for students 1, 4, and 5. Students 2 and 3 made some improvements in probe performance during the group and change components of the intervention, but both students had decreases in performance during the compare component of the intervention. Data indicated that for most students, performance on group problems did increase after instruction on the group schema and some students did show increases in performance on change problems after instruction on the change schema, but no student saw consistent increases in performance on problems from the compare schema after instruction on this schema. Finally, analysis of data from transfer pre- and post-tests for research question four indicates that students only increased correct responses by an average of about two problems from pre-test to post-test. Greatest gains were seen on problems from the compare schema and problems with difficult vocabulary.

Chapter Five

Discussion

Introduction

This chapter includes the major research findings, conclusions, and recommendations based on results of the study. The chapter begins with a review of the purpose and research questions for the study. Each research question is then restated and the findings related to each research question are presented. In the following section, overall conclusions based on those findings are described. Next, limitations of the study are provided. These limitations are the basis for the final section which includes recommendations for future research and practice.

Review of Purpose and Research Questions

The purpose of this study was to examine the effects of using a concrete-representational-abstract (CRA) sequence on the problem solving performance of students who struggle with mathematics or have been identified with learning disabilities in mathematics. Using a single-case multiple baseline across participants design, the researcher provided an intervention to five students in the fifth grade that included instruction in three problem schemas for addition and subtraction (change, compare, and group). The intervention in this study included explicit connections between concrete manipulatives, virtual manipulatives, representational problem solving schemas, and abstract equations when solving word problems with fractions. The following questions guided this study:

1. When using the concrete-representational-abstract (CRA) sequence, can students connect the concrete manipulatives and virtual fraction manipulatives to the representational change, compare, and group schemas?
2. When using the CRA sequence can students connect the representational change, compare, and group schemas to the abstract equations?

3. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems similar to the problems used during the intervention?
4. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems that require generalization from the models provided during the intervention?

Discussion of Results

Research Question One. When using the concrete-representational-abstract (CRA) sequence, can students connect the concrete manipulatives and virtual fraction manipulatives to the representational change, compare, and group schemas?

Only two students used models to draw diagrams in more than half of all possible problems, and both students generally modeled problems to support their conceptual understanding of the fractions and fraction operations necessary to solve the problems. Student 5 used concrete or virtual manipulatives to draw diagrams for all independent practice sessions. Data and anecdotal records on her performance indicate that this student needed to see the models to understand the problems. Even though student 5 understood each of the problem solving schemas (group, change, and compare) when introduced to these schemas, she had a very weak understanding of fraction concepts. During the initial lessons on the group schema, she could not model mixed numbers with concrete manipulatives and had difficulty understanding how to model problems that required combining fractional parts to make whole numbers (i.e. – 1 ¾ + ¾). She was unable to see that 1 6/4 should be rewritten as 2 2/4 or 2 1/2. This student also had difficulty understanding how to model subtraction problems when it was necessary to change whole numbers to fractional parts to subtract (i.e. – 1 ¼ - ¾). She could not initially "see" that it was necessary to exchange 4/4 for one whole to model the subtraction in this problem. Although she could sometimes correctly put numbers in the diagram and solve the problems, she needed the concrete models to understand the fraction concepts and operations. This finding on the importance of concrete models for student 5 is similar to previous research which suggests that concrete models can improve performance on subtraction with integers (Maccini & Ruhl, 2000), area and perimeter word problems (Cass et al., 2003), and fraction word problems (Butler et al., 2003). Furthermore, Cramer and Wyberg (2009) suggest that concrete models can be used to help students create mental representations of ideas that can be used to facilitate understanding of abstract concepts. Using the concrete manipulatives appeared to support this student's understanding of abstract fraction concepts.

Student 5 did not have the same success when using only virtual fraction manipulatives. During the initial lessons on the group and change schemas, student 5 had to model each problem with the concrete manipulatives prior to modeling with the virtual manipulatives to understand the problems. This modeling process appeared to help her develop a visual representation of fractions and fraction operations. This student was highly motivated to understand the problems and continued with a combination of concrete and virtual manipulatives throughout the intervention. The adherence to this sequence of modeling by student 5 provides support for research which suggests that concrete and virtual manipulatives can support student learning of concepts in unique ways (Suh & Moyer, 2007; Olympiou and Zacharia, 2012). Furthermore, the positive results from the combination of concrete and virtual manipulatives used by student 5 is similar to the findings from recent research in science education. This research indicates that using a combination of concrete and virtual manipulatives can be more beneficial for students than using either concrete or virtual manipulatives alone (Olympiou & Zacharia, 2012; Jaakkola, Nurmi, & Veermans, 2010). However, the results from the current study contradict research that suggests that there is no significant difference in student performance when modeling math problems with virtual manipulatives or concrete manipulatives (Burns & Hamm, 2011). Student 5 was able to solve problems most effectively with the combination of concrete and virtual manipulatives, but she had greater success with using only the concrete manipulatives than she did with only using the virtual manipulatives.

Student 1 used the manipulatives in over half of the independent practice problems, but he primarily modeled problems from later lessons on the change and compare schemas. During the compare lessons, the concrete model was switched from fraction tiles to fraction tower blocks to illustrate comparisons. Student 1 was highly motivated to work with these tower blocks, so this could partially account for the increased use of concrete manipulatives when drawing the compare diagrams. However, student 1 also had more difficulty solving problems from the compare schema and often used the tower blocks to make sure that he was accurately labeling each component of the schema before writing the problems in the compare diagrams. Thus, both student 1 and student 5 used visual representations to support their understanding of information in the word problems. The findings related to these students support research which suggests that effective problem solvers are able to create good visual representations of problem information (Van Garderen & Montague, 2003).

The other students in the study were not able to consistently connect the concrete or virtual manipulatives to the schematic diagrams or chose not use the manipulatives to help draw schematic diagrams. Some students had difficulty understanding problem schemas, equivalent fractions, and the virtual manipulatives. Because of these difficulties, they were unable to accurately model some types of problems. Other students had a strong understanding of fraction concepts and were able to accurately model problems with little assistance. These students generally chose not to use models to draw diagrams unless they encountered more challenging problems.

Conceptual Understanding of Problem Schemas. Initially, lack of understanding of problem schemas could have limited some students' ability to connect the manipulatives to the diagrams. During instruction in the group schema, students 1 and 2 struggled with understanding the meaning of basic addition and subtraction word problems. This lack of understanding hampered their ability to model the problems with concrete fraction tiles. When asked to model a group problem such as, "*Ben has 1 ½ pepperoni pizzas and Todd has 1 ½ cheese pizzas. How many pepperoni and cheese pizzas do they have?"* they were initially unsure whether they needed to add or subtract. They had similar difficulties with subtraction problems that included the whole amount and one of the parts. In other words, a reversal of the problem above such as, "*Ben and Todd have 3 pepperoni and cheese pizzas altogether. If Ben has 1 ½ pepperoni pizzas, how many cheese pizzas does Todd have?"* was problematic for these students. When given these problems, students 1 and 2 would sometimes try to model a subtraction word problem as an addition word problem or an addition word problem as a subtraction word problem. Data on student use of models to draw diagrams show that during the group lessons, LISTS Strategy instruction, and early change lessons students 1 and 2 did not model more than two problems in any lesson. It is possible that student 1 may have needed more instruction on the schemas prior to modeling with concrete or virtual manipulatives because as he received more instruction on the use of the models, his use of models increased. This finding supports research on schema-based and schema-broadening instruction. In a review of eleven studies, Powell (2011) states that students showed significant positive gains in problems solving when researchers included 13 to 45 sessions on schema-based or schema broadening instruction. Powell argues that students needed many sessions to develop an understanding of problem schemas.

Finding equivalent fractions. Research on problem solving with fractions suggests that students must have a strong conceptual understanding of fractions to perform well on word problems with fractions (Hetcht et al. 2003). Several of the students in this study did not have a good conceptual understanding of fractions. This lack of conceptual understanding made it difficult for these students to find equivalent fractions which could have limited students' ability to use models to draw diagrams as well. Even though students had some prior instruction in equivalent fractions, only students 3 and 4 were able to find equivalent fractions to solve some word problems with unlike denominators. The word problems that required finding equivalent fractions caused so much difficulty for students that many of the word problems used in the guided practice and independent practice sessions had to be changed from the original planned lessons to include like denominators so students could understand the modeling process and focus on understanding the problem schema. As students became more competent in equivalent fractions in the later lessons, more practice problems with unlike denominators were included. In general, this gradual increase in problem difficulty was effective for students. However, the frustration with trying to find equivalent fractions while solving word problems may have negatively impacted student 2. He worked well with the concrete models during instruction in the group and change schemas, but commented that he did not like using the fraction tiles or tower blocks during instruction on the compare schema.

Previous research on finding equivalent fractions suggests that fraction equivalence is problematic for many students who struggle with math or who have math disabilities (Misquitta, 2011). Butler et al. (2003) found that students who were given instruction using the CRA sequence performed better on problems that required an understanding of equivalent fractions than those who were given instruction using only the representational-abstract (RA) components of the sequence. While the findings of the current study support previous research on the difficulty of fraction equivalence problems for students, the use of the CRA sequence in this study may have been beneficial for some students, but not for other students.

Virtual fraction tool. Difficulty modeling addition and subtraction problems using the virtual fraction tools also could have contributed to student's inability to connect the models to the schematic diagrams. Students 1, 2, and 5 had difficulty modeling addition and subtraction problems using the virtual manipulatives even when they understood the schema of the problem and how to solve it. For example, when solving the addition word problem example with pizzas using the virtual fraction tools, the students needed to model the 1 ½ pepperoni pizzas and the 1 ½ cheese pizza. The students would then have to move the tiles together to determine how many pizzas the two boys had altogether. Students would sometimes have trouble understanding that the fractional parts could be combined to make whole numbers or mixed numbers with the virtual manipulatives, because unlike concrete manipulatives, the virtual manipulatives could not be organized neatly into rows by whole number units. For example, in the sample problem in *Figure 5.1,* these students could not see that the green and blue bars could be added together to make 2 2/2 or 3. Although the answer appears in step 2 of *Figure 5.1*, the answer is not provided until the student types it in on the website.

Model of Addition Problem from Conceptua Fraction's Website

Step 1 – *Model the 1 ½ pepperoni pizzas and the 1 ½ cheese pizzas*.

Step 2 – *Move the second fraction modeled next to the first model to show the combined amount and add the parts to find the total.*

***Part One**: Slide tiles* ***Part Two**: Count combined tiles and find total as a mixed number*

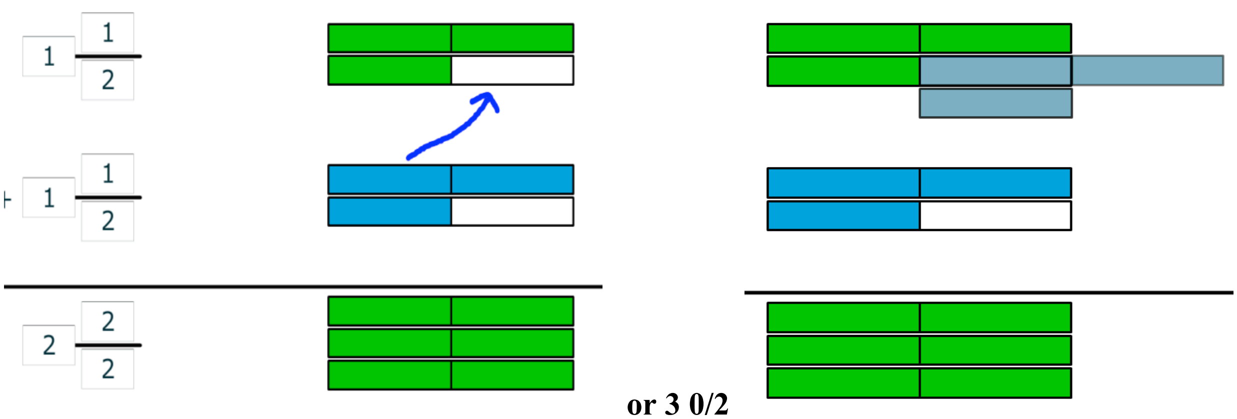


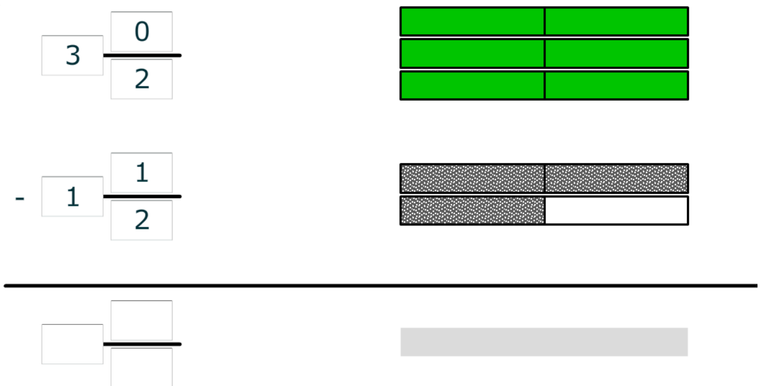
Figure 5.1. **Sample of process of addition using the virtual manipulative tool on the Conceptua Fractions website.** To complete an addition problem the student would slide the model of the second fraction next to the model of the first fraction to determine the solution.

Students 1, 2, and 5 also had difficulty with the virtual subtraction tool. In the subtraction problem example with pizzas, the students needed to model the total of 3 pizzas using halves so they could take away the 1 ½ pepperoni pizzas. The concept of modeling 3 as six halves was initially very challenging for these students because they did not understand that they needed to change three whole tiles to three tiles with two halves each, so they could take 1 ½ away. Additionally, to "take away" Ben's 1 ½ pepperoni pizzas, the students had to move the 1 ½ fraction tiles to "cover" the part taken away to determine the other part (i.e., the amount of cheese pizza). Students 1, 2, and 5 had difficulty understanding this process of covering meant subtraction because they were not actually taking a part away. (See *Figure 5.2*) While students 1 and 5 developed an understanding of this process by modeling problems with both concrete and virtual manipulatives during the lessons on the change and compare schemas, student 2 continued to struggle with more complex problems using the virtual fraction tool.

Student difficulties with the virtual fraction manipulatives extends the research on concrete manipulatives which suggests that there may be features of certain concrete models that limit or enhance student understanding of fraction concepts (Keijzer & Terwel, 2003). In a study that compared the efficacy of different concrete models for teaching fractions, Cramer and Wyberg (2009) found that some concrete models did not show the action of adding or subtracting with fractions clearly or required some prior understanding of fraction equivalence concepts. In the current study, the process of adding and subtracting with virtual manipulatives required some clarification for some students. Students 1, 2, and 5 had to develop understanding of the virtual fraction tool by comparing the process of adding and subtracting fractions with the concrete manipulatives to the process of adding and subtracting fractions with the virtual manipulatives. For example, when subtracting fractions, these students had to connect the process of taking away the fraction tiles using concrete manipulatives with the process of covering fraction tiles using the virtual manipulatives.

Model of Subtraction Problem from Conceptua Fraction's Website

<u>Step 1</u> – *Model the total of 3 pizzas and show the 1 ½ pepperoni part that would be subtracted.*



<u>Step 2</u> – *Slide the 1 ½ over the total amount to cover the amount taken away and subtract.*

Part A*: Slide bars* ***Part B****: Cover to take away and record green bars "left" as mixed number*

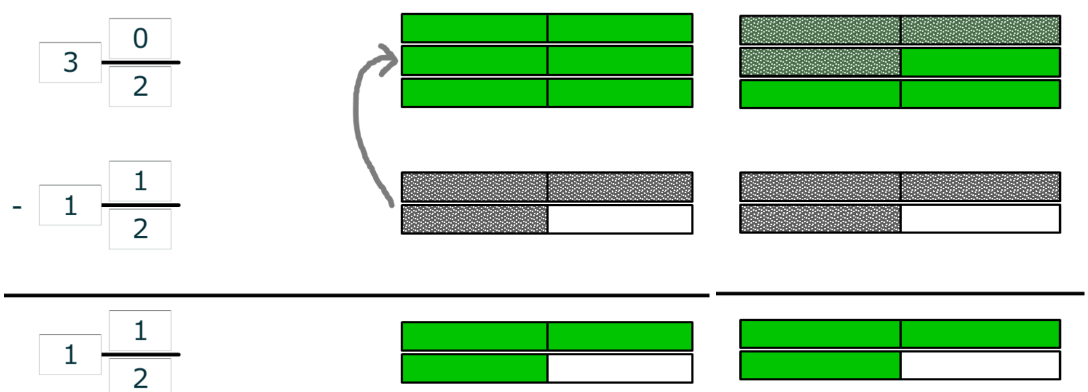


Figure 5.2. **Sample of process of subtraction using the virtual manipulative tool on the Conceptua Fractions website**. To complete a subtraction problem the student would slide the model of the second fraction over to the model of the first fraction to cover the part that would be taken away when using concrete manipulatives.

Student perceived need. Some students were able to effectively model problems with the concrete models or virtual models, but chose not to use these models when drawing the schematic diagrams. These students only used models when they encountered more difficult problems or when they wanted to "check" answers that they had obtained when using the schematic diagrams. For example, students 3 and 4 were able to model the addition and subtraction problems during the lessons, but stated that they did not feel it was necessary to model most of the problems with the manipulatives when using the schematic diagrams. Both of these students had some initial difficulty with problems from the change schema. The only time that student 3 used models to draw diagrams was during lessons on this schema. Student 4 used models initially during the group lesson, but was more likely to use the models when solving problems from the change schema as well.

Research Question Two. When using the CRA sequence can students connect the representational change, compare, and group schemas to the abstract equations?

Data on the use of diagrams or labeling to write equations show that students consistently used diagrams to write equations during 80% of the independent practice problems, but only students 4 and 5 consistently used diagrams to write equations for all probe problems. Analysis of the work on independent practice problems and probe problems suggests that the students that used the diagrams on the independent practice and the probe problems were able to apply their knowledge of each schema to the problems in the probes. In other words, students 4 and 5 could look at each probe problem, determine the correct problem schema, and use the schematic diagram to solve most problems. For example, during instruction in the group schema student 5 initially tried to draw the diagram for the group schema on all probe problems, but as she received instruction in the change and compare schemas she began to recognize the connections between the different diagrams and used those when labeling her work (*See Figure 5.3)*.

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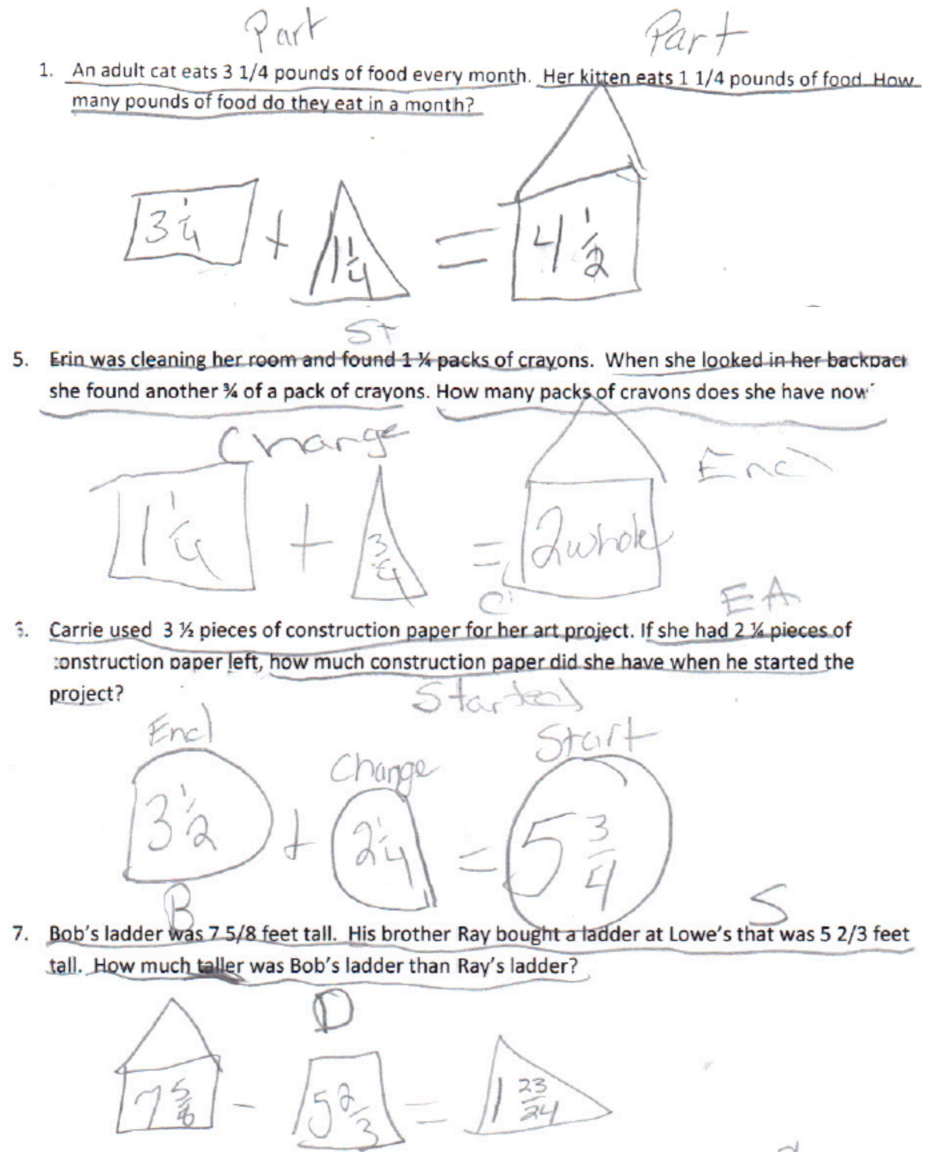


Figure 5.3. **Student 5 probe sample**. The sample demonstrates this student's ability to apply the schematic diagrams from the group, change, and compare schemas to solve probe problems.

Additionally, student 4 also understood the similarities and differences between the different schemas and schematic diagrams that were used to represent these schemas. For example, when solving a group problem that required addition such as "*Jim had 2 ½ bags of Doritos. Todd had 3 ¼ bags of Fritos. How many chips did they have altogether*?", student 4 was able to see that this problem used the same sequence of shapes in the schematic diagram as the following problem from the change schema: "*Cindy started with 2 ¼ cups of water in her chili. She poured another ¼ of a cup of water in her chili. How much water did Cindy pour into her chili*?" Student 4 could also see that an addition problem from the compare schema such as, "*Mrs. Weaver had a shelf for her books that was 2 1/5 feet tall. Mrs. McCarthy had a shelf that was 1 1/5 feet taller than Mrs. Weaver's shelf. How tall was Mrs. McCarthy's shelf*?" used the same sequence of shapes in the schematic diagram as addition problems from the group and change schemas.

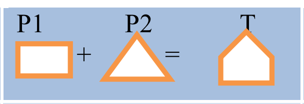
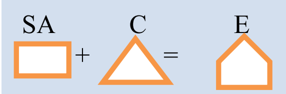
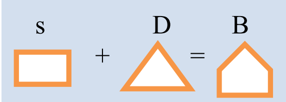
Problem Type	Sample Problems for Probes or Practice	Representational (Schematic Diagram)	Abstract (Equation)
Group (Part 1 + part 2 = total)	Jim had 2 ½ bags of Doritos. Todd had 3 ¼ bags of Fritos. How many chips did they have altogether?		P1 + P2 = T 2 ½ + 3 ¼ = T
Change (Starting amount +/- change = ending amount)	Cindy started with 2 ¼ cups of water in her chili. She poured another ¼ of a cup of water in her chili. How much water did Cindy pour into her chili?		St +/- C = E 2 ¼ + ¼ = E
Compare (Smaller + difference = Bigger)	Mrs. Weaver had a shelf for her books that was 2 1/5 feet tall. Mrs. McCarthy had a shelf that was 1 1/5 feet taller than Mrs. Weaver's shelf. How tall was Mrs. McCarthy's shelf?		S + D = B 2 1/5 + 1 1/5= B

***Figure 5.4.* Problems from the three different schemas that use the same sequence of shapes in the schematic diagram**

When working change problems student 4 commented that, "*When you add the change amount to the starting amount to get the ending amount it is like adding the parts together to make a whole.*" He also commented that all of these kinds of problems (See *Figure 5.3) "start with a rectangle plus a triangle to equal the house (pentagon)."* According to some researchers, this understanding of the relationship between schematic diagrams and equations of different schemas requires algebraic reasoning. This use of algebraic reasoning helped student 4 make connections between problem schemas and equations. As a result, student 4 was able to solve problems from different schemas correctly. This finding supports research which indicates that algebraic reasoning helps students understand the connections between different types of problems and leads to improved performance (Xin, 2008, Xin & Zhang, 2009, & Fuchs et al., 2010).

The other students in the study did not consistently connect the schematic diagrams to the equations when completing probe problems. Students 1 used diagrams to write equations on only 10% of problems on the probe and students 2 and 3 did not write diagrams when solving any probe problems. Analysis of student performance and anecdotal records indicate that student 2 may have been more likely to use the diagrams during independent practice because he only had to answer problems from one schema during the independent practice sessions. Student 2 appeared to have difficulty applying the schematic diagrams when problems from all three schemas were included on the probes. After failing to meet the mastery criteria on the group lesson using the schematic diagram and completing probe 8, Student 2 was still unsure how to tell the difference between the parts and the whole. During a conversation he asked, *"How can you tell when you are missing the whole or the parts?"* An additional remedial lesson had to be designed at this point to help him identify these components in the schema before he could effectively use the group schematic diagram. After this lesson, he was able to use the schematic diagram for group problems during independent practice, but he had difficulty identifying group problems when they were combined with change and compare problems. The findings related to student 2 support research by Fuchs et al. (2004) and Powell (2011) which indicate that students need practice identifying problems from different schemas when problems from different schemas are presented in assessment tasks. In a review of schema-based and schema broadening instruction, Powell (2011) discussed that studies that showed improvements in student problem solving performance included some type of practice with sorting problems by schema.

It was not clear from the data why students 1 and 3 chose not to use the schematic diagrams when completing the probes. Both students were able to identify different components of each schema in the word problems and use that information to effectively solve problems during the independent practice. During the limited examples from the LISTS lessons, both students seemed to be able to identify problems from different schemas when they were presented together. Informal conversations with student 1 indicate that he may have been using the information on the schemas to help solve the problems even though he did not label or record diagrams on his probe. He stated when working on the compare probe problems that he, "*thought of compare problems as bigger part – smaller part = difference part*" because that helped him remember how to solve the problems. This statement demonstrated application of information from the group schema to the compare schema.

Research Question Three. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems similar to the problems used during the intervention?

Overall Findings. Results for research question three indicate that using a CRA sequence to connect problem solving schemas and equations can improve performance on problems similar to problems used during the intervention for some students. In this study, students 1, 4, and 5 recorded positive trends and significant improvements on the mean number correct on probes given in the baseline phase to probes given in the intervention phase. Results for students 2 and 3 did not show strong gains in performance. The overall mean correct from the baseline phase to the intervention phase did increase for each of these students. Specifically, students 2 and 3 made some improvements in probe performance during the group and change components of the intervention. However, both students had decreases in performance during the compare component of the intervention. Additionally, trend lines show that there was minimal growth from the start of the intervention to the end of the intervention for these students. Several features related to the intervention may have affected student performance on probe word problems. Although there was not enough data to show that these features caused changes in performance, several features were correlated with higher or lower performance.

Intervention features. The use of schema-based instruction appeared to have a positive impact on the students who used the schematic diagrams to write equations for independent practice and probe problems. These students recorded strong mean gains in probe performance from baseline to intervention and strong positive trends. Two students who did not use the schematic diagrams to write equations showed little improvement in performance. Student 1 was an exception to this pattern, but while he did not specifically write the schematic diagrams to solve equations, anecdotal evidence suggests that he was applying his knowledge of the different schemas to the probe problems. The improved performance by the students who used their understanding of the schemas to solve problems is supported by the research on schema-based instruction which suggests that schema-based instruction does lead to increases in performance on problems similar to those used during the intervention (Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Xin et. al., 2005; Jitendra, Hoff, & Beck, 1999).

Researchers investigating schema-based instruction and schema-broadening instruction have incorporated mnemonic strategies with interventions to help students follow a general problem solving process to effectively solve word problems (Fuchs et al., 2008; Griffin & Jitendra, 2009; Xin et al., 2008). Use of the mnemonic LISTS strategy may have affected student performance in this study as well. Students 1, 4, and 5 all used this strategy or a modified version of this strategy to solve at least half of the independent practice problems. These students showed increases in mean performance and trend during the intervention. When using this strategy students followed these general steps: locate key terms; identify the problem type and model; show the model with manipulatives; tie the model to the diagram; and solve the equation. Student 5 consistently used this strategy for all lessons. She even used a modified version of this strategy on her own prior to receiving formal instruction on the strategy in lesson five. Student 4 also used his own modified version of this strategy. After receiving instruction on the strategy in lesson five he chose to modify step 3 to "show the model with manipulatives *as needed*". Findings in this study suggest that the mnemonic LISTS strategy was beneficial for those students who used it during the intervention. This finding supports previous research which suggests that instruction in mnemonic strategies can help students use effective problem solving strategies (Montague, Enders, & Dietz, 2011; Maccini & Hughes, 2000; Witzel, Riccomini, & Schneider, 2008).

Research on the CRA sequence suggests that using this sequence can increase student understanding of mathematical concepts and lead to better performance on word problems with fractions (Butler et al., 2003). The use of the CRA sequence of instruction also appeared to have some positive impact on performance for some students in the study. Those students (students 1 & 5) who used the modeling process to help draw schematic diagrams for at least half of the problems during independent practice showed strong gains in overall mean performance from baseline to intervention and strong positive trends during the intervention. Two of the students who did not use the modeling process to draw diagrams (students 2 & 3) did not show strong gains in overall performance. Again, there was one exception to this pattern. Student 4 did show strong gains in overall performance, but he did not consistently use the modeling process. Data indicate that he chose not to use this process because he already had a good conceptual understanding of fractions and did not need to use this process to understand how to solve the problems.

Setting features and student characteristics. Features related to the timing of the study may have negatively impacted the performance of some students in the study. Because of the relatively late start of February for the study and the need for remedial lessons for the first two students, the intervention was not completed for students until mid-May or later. Since the last day of classes was at the end of May, the students were required to take Westest II during the second week of May and complete benchmark testing during the third week in May. Additionally, the student with learning disabilities also had to take extended school year testing during the third week in May. While this large number of tests did not seem to have a strong impact on students 1, 4, and 5, it did seem to have a negative impact on students 2 and 3. According to teacher reports, student 3 became very upset and frustrated during the Westest II on several occasions. He reported to the researcher after the test that he was "tired of tests and did not want to take any more". His performance on his final two probes which were given during the week after the Westest II was lower. However, on the maintenance probe 10 days later his performance did return to pre-Westest II levels. Student 2 also reported his frustration with testing and school during the two weeks after Westest II. On the Monday after Westest II he continued with the intervention, but he stated, "I don't need to know this anymore. I am done with testing." He also expressed his frustration with school in general and reported that he should be "watching movies and doing fun things now." Before each of the last two probes student 2 stated that he was just going to guess on his responses. His performance on these two probes was significantly lower than his performance on the probes prior to testing.

While frustration and negative attitudes may have negatively impacted performance on the final probes for students 2 and 3, positive attitudes and motivation seemed to positively affect probe performance for students 1, 4, and 5. Student 4 had a positive attitude and a strong work ethic throughout the intervention. Even though he had issues related to attention, he always wanted to attend lesson sessions and consistently worked hard during all sessions and probes. Student 1 reported that he needed to stay focused and work hard in May because he would be grounded for the summer if he did not have at least a “B” in math class. Although his performance during the study did not affect his grade, it was noted that he remained very focused through the final lessons of the intervention despite the numerous field trips, special activities, and final testing that occurred during May. Student 5 also remained very focused during the final intervention lessons in May. She even offered to give up recess time or activity time to work on math because she believed that it was very important to do well in math so she would be ready for sixth grade. These positive attitudes appeared to be correlated with higher performance. Performance on probes for students 1, 4, and 5 remained high on probes administered after testing, while probe performance dropped for the two students who had expressed difficulty or negative concerns after testing. These findings support prior research on students with math difficulties that suggests that motivation and attitude can positively or negatively impact student achievement (Sideridis, Morgan, Botsas, Padeliadu, & Fuchs, 2006; Woodward & Brown, 2006).

Findings on performance by schema.

Group Schema. The results also indicated that instruction by schema can improve performance on problems from that specific schema, but that students seemed to benefit most from the initial instruction on the group schema. This schema appeared to be the easiest for students to understand because they only had to make distinctions between two schema components, the parts and the whole, when solving these types of word problems. When solving change and compare problems students needed to determine three schema components before they could solve the problems. Furthermore, the schematic diagram for the group schema was easy for students to understand. They were able to see how a part place in a rectangle added to another part placed in a triangle could be combined to obtain a total or whole that would be placed in a house (pentagon). It is possible, however, that the increased performance on group problems was related to order effects or to the instruction on problems from that specific schema. Even though students had the same number of lessons on each schema, they did have more opportunities to apply their knowledge of the group schema to probe problems since the lessons on the group schema were the first lessons taught to all students.

Change Schema. Some students did show increases in performance on change problems after instruction on the change schema, but complexities related to the change schema may have limited student gains in performance. When solving problems from the group schema students only had to consider two possible relationships in the schema; part + part = whole or whole – part = part. Change problems required consideration of four types of possible relationships between components in the schema. Students had to determine whether they needed to add or subtract from the initial change amount and then understand the reverse procedure for each type of equation. They also had to understand how to solve problems with missing change amounts.

Analysis of responses to probe questions indicated that students were most successful solving change problems that included a missing ending amount, but they had the most difficulty solving problems where the change amount was missing (See *Figure 5.4: ex. 1*). Those students who showed gains in performance on change problems seemed to have a better understanding of how to solve change problems with missing change amounts.

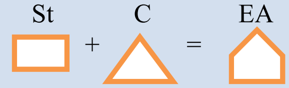
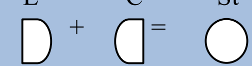
Change (Starting amount +/- change = ending amount)	Ex. 1: Paula needed 2 ½ cups of flour to make a pie for her mom's party. If she already has 3/5 of a cup of flour, how much more flour will she need to make her pie?	St C EA 	St + C = EA 3/5 + C = 2 1/2
	Ex. 2: Kendall used 2 1/6 pieces of poster board for his science fair project. If he had 1/2 of a piece of poster board left, how much poster board did he have when he started the project?	E C St 	E + C = S 1/2 + 2 1/6 = S

Figure 5.5. **Problems that show the different conceptualizations needed to solve problems from the change schema.**

Another factor that could have impacted student performance on change problems was related to the students' conceptual understanding of the schema and schematic diagram. For the change schema, conceptual understanding of adding a starting amount that was smaller to a change amount to determine an ending amount that was bigger was very similar to the understanding of the group schema (See *Figure 5.5: ex. 1*), but the conceptual understanding of adding an ending amount that was smaller to a change amount to get a starting amount that was bigger was difficult for students to understand. Furthermore, this type of problem did not visually match the original diagram which combined amounts placed in a rectangle and a triangle to obtain a total larger amount that was placed in a pentagon. (See *Figure 5.5: ex. 2*). A separate schematic diagram had to be added for this type of change problem because the original diagram could not be used to correctly model the relationship between the starting, change, and ending amount in these types of problems. A new diagram that was developed used a "D" for the smaller Ending Amount, a half- circle that started as with a “C” as the change amount and a circle for the larger starting amount. As a result, the use of the shapes in the schematic diagrams that seemed to help students with group problems may have hindered some students when solving the change problems. Those students who could understand when to use the different schematic diagrams for the change problems showed improvements in probe performance on problems from the change schema.

Compare Schema. No students saw consistent increases in performance on problems from the compare schema after instruction on this schema. Item analysis of missed responses on the probes given during intervention shows that the last two questions on every probe, which were two problems from the compare schema, were missed the most by all students throughout the intervention. In these two problems students had to have a strong understanding of the compare schema because the same word in a problem, such as shorter or taller, might require a different equation depending on the relationship between the components of the schema. For example, in *Figure 5.6,* both problems include the word shorter, but in one problem the smaller or shorter amount is added to the difference to get the bigger amount. In the other problems the difference is subtracted from the bigger amount to determine the smaller or shorter amount.

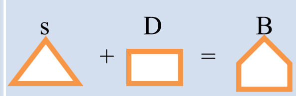
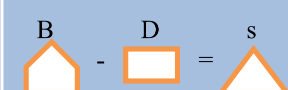
Compare (Bigger – smaller = difference)	Ex. 1. Tim cut two boards to begin making a wooden picture frame. One board was 7 9/12 inches long. This board was 1 ½ inches shorter than the other board. How long is the other board?		s + D = B 7 9/12 + 1 ½= B
	Ex. 2. Mrs. Weaver had a shelf for her books that was 2 1/5 feet tall. Mrs. McCarthy had a shelf that was 1 1/5 feet shorter than Mrs. Weaver’s shelf. How tall was Mrs. McCarthy’s shelf?		B – D= s 4 1/5 - 1 1/5 = s

***Figure 5.6.* Problems that show the different conceptualizations needed to solve problems from the compare schema.**

Furthermore, sometimes the information in the problems from the compare schema was not logical to the students. For some problems, the smaller component of the schema that needed to be identified in the problem might have actually been the larger number in the problem (See *Figure 5.6: Ex. 1)*. Some students may have confused the bigger and smaller components of the compare schema with the larger and smaller numbers in the word problems on the probes.
The lack of conceptual understanding or confusion with the language in these types of compare problems may have impacted student performance on these problems during the probes. This finding supports research by Fan, Mueller, Marini (1994) and Fuson and Carroll (1996) which indicates that compare problems are challenging for students because the conceptual and linguistic complexities in these problems do not cue students to the specific operation needed to solve these problems.

It also is possible that some students had difficulty applying the information on the different schemas and schematic diagrams by the time they received the final intervention lessons on the compare schema. Using the same shapes for all three schemas seemed to be helpful to some students and somewhat confusing to others. For example, some students would try to apply the components of the group schema to problems from the compare schemas or the components of the change schema to problems from the compare schema on the probes. This confusion might have led to more errors on probe problems from the compare schema.

Additionally, it is difficult to determine what effect the timing of the instruction on the compare problems had during this study. Students 2 received instruction on the first two lessons from the compare schema during the week after the Westest II when he having significant difficulty concentrating and did not want to complete the lessons. Because he was consistently not meeting the mastery criteria for the lessons, the intervention was discontinued at this time.

Students 3, 4, and 5 also received instruction on some lessons from the compare schema during the week following testing. If instruction on the compare schema had occurred earlier in the semester, they may have had more success on these types of probe problems.

Summary of Findings on Performance by Schema. The findings in this study suggest that in general, students had the most success solving problems from the group schema and that they had the most difficulty with problems from the compare schema. These findings contradict the findings of Garcia, Jimenez, and Hess (2006) which suggest that the semantic structure or problem schema alone was not a predictor of the difficulty of specific problems for students. However, student performance in the current study was also affected by the location of the missing information in the problem. These findings do support the findings of Garcia et al., (2006) and Powell, Fuchs, Fuchs, Cirino, and Fletcher (2008) which suggests that problems with missing information in the initial position (e.g. - starting amount) or second position (e.g. – change amount) are more difficult for students that problems with missing information in the third position (e.g. – whole, ending amount).

Research Question Four. Will using a CRA sequence that includes concrete and virtual manipulatives to connect problem solving schemas and equations improve student performance on problems that require generalization from the models provided during the intervention?

Overall, student performance on problems that require generalization did not improve significantly after instruction using the CRA sequence to connect problem solving schemas and equations. Average overall performance on the transfer test only increased by two problems from pre-test to post-test and only student 1 recorded a significant increase (+5) from pre-test to post-test. The lack of significant improvements for most students on the transfer post-test was not surprising because the students in the study did not receive direct instruction on any of the transfer characteristics. The benefits of direct instruction for students who struggle with math or who have learning disabilities have been well-documented for instruction on fractions (Bottge et al., 2010; Misquitta, 2010) and when implementing schema-based or schema broadening instruction (Powell, 2011). Specifically, this finding was consistent with previous research on problem solving instruction which states that students with disabilities or difficulties in math need direct instruction on how to solve problems that require generalization from the problems modeled during instruction to improve performance on these transfer problems (Fuchs et. al., 2004 Fuchs et. al., 2006).

Another key finding from the transfer test was that all students improved on the number of problems with difficult vocabulary that they answered correctly on the post-test. For students 2, 3, 4, and 5 the only gains that were made between pre- and post-test were on problems with difficult vocabulary. This finding suggests that while students were not able to improve performance on novel problems with irrelevant information or different conceptualizations on the post-test, they did make some improvements on problems with this specific transfer characteristic. Previous research on the different types of transfer characteristics suggests that students are more likely to solve problems that include transfer characteristics with more similarities to the original problem called "near" transfer problems, than those problems include transfer characteristics that include more novel characteristics (Fuchs et al., 2006; Fuchs et al., 2008). While the findings in this study support this research, they also indicate that even within the category of "near" transfer some specific transfer characteristics may be more challenging for students than others.

Perhaps one of the most intriguing findings of this study was the types of gains that students made on the transfer post-test. Even though students recorded the weakest gains on compare problems and the lowest average correct on problems from the compare schema during the probes, they recorded the highest gains on compare problems on the transfer post-test. Furthermore, 4 out of the 5 students in the study made at least some gains on the compare problems from pre-test to post-test. This may have been related to the types of compare problems that were included on the transfer test. One type of compare problem required the use of the equation: biggest – smallest = difference, and students needed to find the missing difference amount. This was the simplest type of compare equation for students to solve during the intervention. Additionally, two compare problems also included the transfer characteristic of difficult vocabulary. As previously stated, this transfer characteristic caused the least trouble for students when solving transfer problems.

Conclusions

The intervention implemented in this study appeared to improve problem solving performance on problems similar to those presented in the intervention for some students in the study, but not for all students in the study. Several factors may have affected student performance on probes including student use and understanding of manipulatives and schematic diagrams; student understanding of fraction concepts; and the students' ability to correctly identify problem schemas. However, several conclusions can be drawn about specific components of the intervention that led to improved performance on independent practice and probe problems. First, students who had weak conceptual understanding of fractions needed the instruction with the concrete manipulatives to understand how to model more complex problems that required equivalent fractions. If they had not had this experience with the concrete materials initially, it is unlikely that they would have developed the conceptual understanding to model problems correctly with the virtual manipulatives. These findings support the research by Suh and Moyer (2007) which suggested that there may be unique features of both concrete and virtual manipulatives that could be used to support student understanding of math concepts. The findings of this study are also similar to those of Gire et al. (2010) who concluded that kinesthetic activities with concrete or physical manipulatives may be more beneficial prior to instruction with virtual manipulatives when students are working with concepts that can be clearly demonstrated with concrete objects. However, for the students with the greatest difficulty with understanding mathematical concepts in this study, these findings do not support research which suggests that students perform as well or better when using virtual manipulatives than when using concrete manipulatives (Suh, 2005; Yuan et al., 2010).

Furthermore, the intervention implemented in this study appeared to lead to stronger conceptual understandings of fractions for all students. Developing a conceptual understanding of fractions seemed to help some students understand how to solve fraction problems from the different types of schemas and led to improved performance on probes. However, some students improved their conceptual understanding of fractions, but did not make significant gains in performance on probe word problems. These students may have encountered more difficulty related to their conceptual understandings of problem schemas. Students in the study who were able to identify the appropriate schema for each problem on the probe and use their understanding of the schema to solve the problems made significant gains on the probes in the study. The students who made gains in conceptual understanding, but not in probe performance appeared to have difficulty identifying the schemas of problems when problems from all three schemas were presented. Research on identifying schemas supports this conclusion. Fuchs et al. (2004) found that students who received schema-based instruction with practice sorting word problems into schemas performed better on problem solving tests than students who received schema-based instruction without this sorting practice.

In some cases, the type of the missing information in the problem may have caused difficulty for students as well. Certain problems from each schema were easier to solve than other problems in the same schema. Adding the parts to find the total in the group schema, adding the starting amount to the change amount to get the ending amount in the change schema, and subtracting the smaller amount from the bigger amount to get the difference in the compare schema were the easiest problems for students to solve. Problems from the change schema which asked student to find the change amount and problems from the compare schema where students had to find a bigger or smaller amount were the most challenging and required a deeper understanding of the schemas. Including more of these difficult example problems during the modeling and guided practice components of the lessons could help students improve their performance on these problems during the intervention and maintenance probes.

The intervention did not appear to be successful for student 2 who was diagnosed with learning disabilities and student 3 who was diagnosed as OHI. Data from the study suggest that there may be different reasons why these students did not make significant gains on the probes or transfer tests. Student 2 experienced challenges with multiple components of this study including his conceptual understanding of fractions, his ability to identify and apply information from each schema when problems from the three schemas were mixed together on the probes, and his motivation and attitude at the end of the year. Although the other students experienced some of these challenges as well, his difficulties in these areas were more severe than the other students in the study. The cognitive challenges this student faced may have resulted in a cognitive overload that affected performance. Research on students who struggle with math suggests that this type of cognitive overload can lead to poorer performance and more negative attitudes toward math (Woodward & Brown, 2006). Additionally, a review of multiple studies on cognitive, behavior, and affective deficits in students with learning disabilities suggests that the combination of factors that affected student 2 during this study is often detrimental to the performance of students with learning disabilities (Sideridis et al., 2006).

Although student 3 did not make significant gains in performance, he demonstrated a very different pattern of performance during the intervention, probes, and transfer tests. His performance improved during baseline conditions and was relatively strong when he started the intervention. His performance on the transfer pre-test was the highest of all students. During the intervention he appeared to have a good conceptual understanding of fractions and the three schemas presented in the lessons. While he did make some gains, these gains were not sustained on the probes presented during the lessons on the compare schema. It is possible that this student's performance was significantly affected by the testing during May. However, his pattern of performance indicates that this student may not have applied the information from the lessons when completing the probes or transfer post-test. This student seemed to use the same strategies during the intervention and transfer tests as he used prior to the intervention. Additionally, there was no evidence on the probes or from anecdotal data that this student applied the information on schemas when solving probe problems during intervention, maintenance, or the transfer post-test.

Finally, implementing the intervention did not lead to overall improvement on problems that required generalization. While the results from the transfer post-test in this study were disappointing, they did suggest that even transfer skills that would be considered "near" transfer skills are not equally difficult for students. In this study it appeared that problems that required students to determine which information was irrelevant or problems that required student interpretation of information in charts was more difficult for students than problems with difficult vocabulary. These transfer characteristics seemed to have more of an impact on student performance than the type or problem (group, change, or compare) that was solved. Although previous research has addressed immediate transfer, near transfer, and far transfer characteristics of word problems (Fuchs et al., 2004; Fuchs et al., 2006; Fuchs et al., 2008), no research could be found that considered the difficulty of specific transfer characteristics within each category.

Limitations

This study was conducted with only five students in the fifth grade, so the generalizability of the results to students in different grade levels is limited. Additionally, this study was conducted in a small rural town in the eastern United States with Caucasian students, so the results may not apply to students in cities, other locations, or students from diverse ethnic backgrounds. Since the intervention in this study was conducted with pairs or individual students, the results may not be applicable to pullout special education classrooms or the general education classroom. Furthermore, since the study included three students who struggled with math and two students with disabilities, the findings on the students without disabilities may not be generalizable to students with disabilities. On the other hand, the findings related to the students with disabilities may not apply to students who struggle with math.

There were several limitations related to the content of the study as well. First, only group, change, and compare problems that required addition and subtraction were addressed in this study. The results may not be applicable to other types of problems or problems that require multiplication and division. Second, this study used the CRA sequence with fraction problems. The results may not be applicable to problems with whole numbers because students may not need the CRA sequence to visualize problems with whole numbers. Finally, the problems in the transfer tests only addressed the near transfer characteristics of difficult vocabulary, irrelevant information, or different conceptualizations. The results may not apply to other problems with different near transfer characteristics or real world problems that would be include far transfer characteristics.

Limitations on the use of the concrete and virtual fractions could also affect the application of the findings. During the study, fraction circles, fraction tiles, and fraction tower blocks were the primary concrete manipulatives used during the intervention. Similar virtual manipulative were used from Conceptua Fractions to model these problems as well. Use of other concrete or virtual manipulatives, such as fraction squares or the number line, may have changed the results of the study. Furthermore, limited kinds of problems could be modeled with these concrete and virtual manipulatives. The concrete manipulatives only had up to twelve fractional parts and each virtual manipulative could only be divided into thirty parts. Therefore, the results may not be applicable to word problems with fractions with denominators larger than thirty.

Implications for Practice

There are several possible implications for practice from this study. First, findings from this study suggest that teachers and other personnel who work with students who have weak conceptual understandings of fractions should include instruction with concrete manipulatives and virtual manipulatives to improve student understanding of fraction concepts. The findings also indicate that instruction should start with the concrete manipulatives because the actual physical manipulation and modeling of fractions seems to help students with math difficulties develop an understanding of fraction concepts. Instruction using virtual manipulatives should follow instruction on concrete manipulatives because instruction with virtual manipulatives can extend student understanding by using a broader range of examples and models.

Second, analysis of results also indicates that teachers should explicitly connect the CRA sequence to problem schemas. Students who have conceptual difficulties with fractions can benefit from these explicit connections when solving word problems with fractions. Students who already have a strong conceptual understanding of fractions may not need to model all word problems with manipulatives, but they can use these models when they have difficulty understanding specific problems. Furthermore, analysis of data also suggest that teachers should provide explicit instruction on how to label the parts of the schemas in word problems and on how to use the schematic diagrams to write equations to solve word problems. Students should be given the opportunity to select the method of using the schema information to solve problems. Students who used the schematic diagrams to write equations in the study improved their performance on word problems, but students who developed their own process of labeling or applying information on schemas to word problems also improved performance.

Additionally, after teaching the three problem schemas, teachers should give students multiple opportunities to practice identifying the schema of problems when problems from all three schemas are combined. In this study, students who were able review problems from different schemas on the probes, select the appropriate schema for each problem, and apply the information on the schema to the problem demonstrated gains on probe performance. Multiple opportunities to sort problems by schema could help students identify the correct schema for each problem when all problems from all schemas are combined. This explicit instruction could also provide opportunities for discussion on the similarities and differences between problem schemas. Teachers could use these discussions to develop algebraic reasoning skills by helping students see the connections between similar problems and equations.

Furthermore, when teaching problems from the change and compare schemas, teachers should provide additional examples and practice with problems that are more challenging for students. Students may need more explicit modeling, guided practice, and independent practice with problems that include missing change amounts from the change schema or missing smaller or bigger amounts from the compare schema. When solving all types of problems a mnemonic strategy, such as the LISTS strategy used in this study, should be used to help students remember the steps of the problems solving process.

The findings of this study also suggest that teachers should provide explicit instruction on transfer skills. Specifically, students could benefit from direct instruction and practice with transfer problems that include irrelevant information and different conceptualizations through charts or graphs. Students may also need explicit instruction on transfer problems in each of the three schemas. Students may be able to solve problems similar to those during instruction from the group, change, and compare schemas, but they may need additional help solving transfer problems from each of these three schemas.

There are several specific implications for instruction of students with learning disabilities from this study. Results indicate that students with learning disabilities could need additional instruction and practice solving word problems from each schema. Students may benefit from a combination of easier and harder problems or distributed practice so they will maintain a positive attitude and not reach cognitive overload (Woodward & Brown, 2006). Students with learning disabilities may also need additional scaffolding with graphic organizers to help them apply the correct schematic diagram to word problems. Including a blank copy of the correct schematic diagrams on probe problems could help students with learning disabilities apply their knowledge of these schemas to the word problems. The students could identify schema components in the problems and fill in the diagrams until they were able to master this skill with problems from different schemas combined. At this point, the diagrams could be gradually withdrawn. This use of graphic organizers could also be beneficial for students who did not apply the strategy when problems from different schemas were combined.

Implications for Further Research

There are several implications for further research from this study. The suggestions below are based on the analysis of the results and the conclusions presented in this chapter.

1. Although the results of this study indicate that a sequence of instruction that includes concrete and virtual manipulatives improves conceptual understanding of fractions, more research needs to be conducted to determine if this sequence of instruction is most effective when developing an understanding of fraction concepts. Additionally, more research needs to be conducted to determine if this sequence of instruction is effective when providing instruction on other math concepts, to students at different grade level, or to students who perform at or above grade level in mathematics.
2. Most research on virtual manipulatives suggests that this type of manipulative can be effectively substituted for instruction using concrete manipulatives (Suh, 2005; Yuan et al., 2010). The findings of this study did not support this research. Additional research should compare interventions that use concrete manipulatives with interventions that use virtual manipulatives to determine if virtual manipulatives can be as effective as concrete manipulatives when working with different concepts, students at different grade levels, or students with disabilities.

3. Although both the concrete and virtual manipulatives in this study could be created and moved to solve problems, some students were not as successful when manipulating the virtual manipulatives on the computer screen. Research that considers the characteristics of concrete and virtual manipulatives should be conducted to determine how a concrete manipulative is defined and where virtual manipulatives fit in the CRA sequence.
4. Previous research indicates that providing schema-based instruction with opportunities to sort problems by schema improves student performance on word problems (Fuchs et al., 2004). The current study provided limited opportunities for students to identify problems schemas when problems from three schemas were combined. Research should be conducted that combines instruction on the CRA sequence with schema-based instruction that includes more opportunities for sorting problems by schema.
5. To clarify the connection between the intervention used in the current study and students' ability to generalize to problems with novel characteristics, additional research that includes explicit instruction by problem type and transfer characteristic should be conducted. This additional research could provide a better understanding of the relationship between the CRA sequence and schema-broadening instruction.
6. Research that includes additional supports to help students connect problem schemas to equations could provide a better understanding of the level of support needed to help students who struggle with math and students with learning disabilities solve word problems with fractions. This type of research could include blank copies of the schematic diagram with word problems to help students make the connection to the schema or drawings of concrete/virtual manipulatives to help students conceptually understand difficult problems.

7. To determine the connection between students' conceptual understanding of fractions and their ability to solve fraction word problems, further research on the CRA sequence with schema-based instruction should include pre- and post- assessments of conceptual understanding of fractions. Specific assessments of conceptual understanding may provide a better understanding of the relationship between a conceptual understanding of fractions and students' abilities to solve word problems with fractions.

Summary

Despite the mixed results of this study, this research suggests that a sequence of instruction that combines the CRA sequence with schema-based instruction can help some students solve word problems with fractions. However, this research also highlights the need for more research on the use of concrete and virtual manipulatives when providing this type of intervention for students who struggle or have learning disabilities in math. Additionally, the findings of this study indicate that problems from certain schemas and specific types of transfer characteristics may be more difficult for students. These problems may require more modeling by teachers and practice by students. This research also suggests that the intervention used in this study may be beneficial for students who struggle with math, but students with disabilities may need even more intense instruction and additional scaffolding to make connections between the CRA sequence and problem schemas when word problems from multiple schemas are combined.

Appendix A

Sample Math Probes

Directions for Probes and Transfer Tests

Today I would like for you to answer these word problems with fractions. I will read each problem to you and you may use a calculator to complete each problem. Even though you may use a calculator, you will still need to show your work on the paper. For example if you enter ½ + ¼ = __ or ½ x 1/3 = _ in the calculator, you will need to write the problem down with your answer on the page. If you use pictures or diagrams to help you with the problem, please draw these in the space below the problem. Are there any questions?

The researcher will read each problem out loud.

Math Probe #1

1. A mother elephant eats 2 ¼ tons of food every month. Her baby eats 1 ½ tons of food. How many tons of food do they eat in a month? (group)
2. Tiffany bought 3 ½ pounds of yellow and red m & ms to take to a friend's party. If 1 2/3 pounds of m & ms were yellow, how many pounds were red? (group)
3. Last week, John kept track of the weather for five days. In his town it was sunny some days and rainy other days. If it rained 1 ½ days, how many days were sunny? (group)
4. Sarah needed 6 1/3 cups of water to make her soup recipe. If she already has 2 2/3 cups of water, how much more water will she need to make her soup? (change)
5. Rachael was cleaning her room and found 1 ¼ packs of colored pencils. When she looked in her backpack she found another ¾ of a pack of colored pencils. How many colored pencils does she have now? (change)
6. Josh used 2 ½ pieces of construction paper for his art project. If he had 4 ¼ pieces of construction paper left, how much construction paper did he have when he started the project? (change)
7. Gary's basketball goal was 8 ¼ feet tall. His younger sister Cindy bought a goal at Wal-Mart that was 6 2/3 feet tall. How much taller was Gary's goal than Cindy's goal? (compare)
8. Paul cut two boards to begin making a wooden picture frame. One board was 5 1/8 inches long. This board was 1 ½ inches shorter than the other board. How long is the other board? (compare)
9. Julie built a tower out of blocks that was 4 ¾ feet tall. Angel built a tower that was 1 1/3 feet taller than Julie's tower. How tall was Angel's tower? (compare)

Math Probe #2

1. An adult dog eats 6 3/4 pounds of food every month. Her puppy eats 1 1/3 pounds of food. How many pounds of food do they eat in a month? (group)

2. Heather bought 2 3/4 pounds of yellow and red skittles to take to a friend's party. If 1 2/5 pounds of skittles were yellow, how many pounds were red? (group)
3. Last week, Chris kept track of the weather for five days. In his town it was sunny some days and snowy other days. If it snowed 1 1/4 days, how many days were sunny? (group)
4. Kim needed 4 2/3 cups of water to make her chili. If she already has 2 1/4 cups of water, how much more water will she need to make her chili? (change)
5. Erin was cleaning her room and found 1 ¼ packs of crayons. When she looked in her backpack she found another ¾ of a pack of crayons. How many packs of crayons does she have now? (change)
6. Carrie used 3 ½ pieces of construction paper for her art project. If she had 2 ¼ pieces of construction paper left, how much construction paper did she have when he started the project? (change)
7. Bob's ladder was 7 5/8 feet tall. His brother Ray bought a ladder at Lowe's that was 5 2/3 feet tall. How much taller was Bob's ladder than Ray's ladder? (compare)
8. Mary cut two ribbons to begin making a bow for her hair. One ribbon was 4 2/5 inches long. This ribbon was 1 1/3 inches shorter than the other ribbon. How long is the other ribbon? (compare)
9. Dorothy built a tower out of Legos that was 7 ¾ inches tall. Hannah built a tower that was 1 1/3 inches taller than Dorothy's tower. How tall was Hannah's tower? (compare)

<u>Math Probe #3</u>

1. Jared went to McDonalds and ate 3 1/2 Chicken McNuggets. His brother ate 4 1/2 Chicken McNuggets. How many Chicken McNuggets did they eat altogether? (group)
2. Tia bought 3 1/3 pounds of green and red lollipops to take to a friend's house. If 1 2/7 pounds of the lollipops were green, how many pounds were red? (group)
3. Last week, the football team practiced five days. Some days, or parts of some days, they worked on offense and on other days they worked on defense. If they worked on offense 2 1/3 days, how many days did they work on defense? (group)
4. Betty needed 3 5/8 cups of flour to make a cake for her mom's birthday. If she already has 2 2/3 cups of flour, how much more flour will she need to make her cake? (change)
5. T.J. was cleaning his room and found 1 ¼ packs of baseball cards. When he looked in his desk he found another 1/5 of a pack of baseball cards. How many packs of baseball cards does he have now? (change)
6. Brandon used 1 1/8 pieces of poster board for his science fair project. If he had 2 3/5 pieces of poster board left, how much poster board did he have when he started the project? (change)
7. Jay is 5 1/4 feet tall. His friend Sam is 5 1/2 feet tall. How much taller is Sam than Jay? (compare)

8. Joseph cut two pieces of wood to make a shelf for his room. One piece of wood was 2 7/8 feet long. This piece of wood was 6 1/3 feet shorter than the other piece of wood. How long is the other piece of wood? (compare)
9. Tim built a tower out of Legos that was 5 3/7 inches tall. Hunter built a tower that was 2 2/3 inches taller than Tim's tower. How tall was Hunter's tower? (compare)

Appendix B

<u>**Math Transfer Pre- and Post-Test**</u>

1. Elephants can communicate through low frequency infrasonic rumbles. Their sounds can travel from 1/8 km to 9 ½ km. How much farther can the longest sound travel than the shortest sound? p. 249 (difficult vocabulary, compare)
2. The escape velocity for a rocket to move out of the Earth's gravitational pull is 6 9/10 miles per second. The Moon's escape velocity is 5 2/5 miles per second slower. How fast does a rocket have to launch to escape the moon's gravity? p. 249 (difficult vocabulary, compare)
3. The two largest meteorites found in the U.S. landed in Canyon Diablo, Arizona, and Williamette, Oregon. The Arizona meteorite weighs 33 1/10 tons! Oregon's weighs 16 ½ tons. How much do the two meteorites weigh in all? p. 249 (difficult vocabulary, group)
4. The new president of the United States timed his inauguration speech at 5 1/6 minutes. The television producer informed him that he would only have 4 ½ minutes to complete his speech. How much time will the president have to remove from his speech to complete it in 4 ½ minutes? (difficult vocabulary, change)
5. Jack decreased his best time in the 100 meter race by ¾ of a second. His new best time is 8 ½ seconds. Jack's friend Tim's best time is 8 ¼ seconds in the 100 meter race. What was Jack's old time in the 100 meter race? (irrelevant information, change)
6. The average person in the United States chews 1 9/16 pounds of gum each year. The average person in Japan chews 7/8 pounds of gum and the average person in England chews 1 ¼ pounds of gum. How much more gum does the average American chew than the average person in Japan? (irrelevant information, compare)
7. Before she went to the hairdresser on Saturday, Sheila's hair was 7 ¼ inches long. When she left the salon on Saturday, it was 5 ½ inches long. Next time that she gets it cut, she hopes that it will only be 3 ½ inches long. How long was Sheila's hair when she left the salon on Saturday? (irrelevant information, change)
8. Grant bought 3 1/8 pounds of apples at Foodland. Hannah bought 2 1/3 pounds of bananas and Heather bought 4 ½ pounds of oranges. How many pounds of apples and bananas were bought by the students? (group, irrelevant information)

*The following chart will be used to answer questions 9-12.

Jenny's Gift Wrap Table	
Gift Size	**Paper Needed (yards)**
Small	11/12
Medium	1 5/9
Large	2 2/3
X-Large	3 1/9

9. Jenny is working at a gift wrap center. She has 2 ¼ yards of wrapping paper to wrap a small box. How much wrapping paper will be left after she wraps the gift? (different conceptualization, change)
10. Jenny needs to wrap one x-large box and one medium box. How much wrapping paper will she need to wrap both boxes? (different conceptualization, group)
11. Jenny has 3 1/3 yards of wrapping paper. She needs to wrap one small box and one medium box. How much wrapping paper will she have left after she wraps both boxes? (different conceptualization, group)
12. Jenny needs to wrap a large box and a small box. How much more wrapping paper will she need to wrap the large box than the small box? (different conceptualization, compare)

*Problems were adapted from Bennett, J. M., Chard, D. J., Jackson, A., Milgram, J., Scheer, J. K., Waits, B. K. (2004). *Holt Middle School Math: Course 1.* Austin, TX: Holt, Rinehart , and Winston.

Appendix C

Sample Scripted Lesson
Lesson 1 – Group Problems with Manipulatives

Objectives:

Given modeling and guided practice, the student will be able to solve fraction word problems from the group schema by modeling the problem with manipulatives.

Materials:

Practice Sheet 1

Rainbow Fraction Tiles

Fraction Circles

Pencils

*Note: In all lessons, the researcher will respond to questions from the student related to the lesson. The researcher may provide clarification or additional information if the student does not understand the examples or how they are modeled using manipulatives or the schematic diagram.

Advanced Organizer:

"Word problems in math are used to help you learn to solve math problems that you might have to figure out from the real world. You may be in situations when you are shopping, cooking, or hanging out with your friends that you need to use math to solve problems. Can you think of times when you have to solve word problems? Can you think of ways that you might need math to solve problems when you are not in school?

When you get older you may have to calculate how many miles for trips, how much gas you need for your car, or how much pizza you want to buy for your party so it is important to learn how to solve problems. During the next couple of weeks I will be teaching you some strategies to help you solve word problems. We will be focusing on word problems with fractions because this is an important skill for fifth grade, but you can use these strategies to help you solve word problems without fractions as well."

Pre-requisite practice with concrete manipulatives on equivalent fractions:

"*Before we start with the word problems, I want to show you how you might use some concrete materials to help you model different kinds of fraction problems. These materials are called manipulatives. Have you ever used these manipulatives before?* The researcher will show the students samples of fraction tiles, fraction tower blocks, fraction circles, and fraction squares. If the student responds with, yes, then the research will ask: *"What do you remember about using these manipulatives?* to determine the background knowledge of the student.

"Today, we are going to specifically focus on how we might use the fraction tiles to model fractions in word problems. Fraction tiles are color-coded. The first fraction tile is just the number one and it represents one whole. The researcher will pick up the red fraction tile of one and show it to the student. *So if we were talking about 1 pizza or 1 pound of food or 1 day of the week, we could use the red fraction tile of 1 to model that number. The other fraction tiles are different ways that we can break the number one into smaller parts. For example, the pink fraction tiles can each be used to model ½. So if we were trying to model ½ pound of m and ms or ½ of a pizza, we could use a pink ½ tile to model that amount.* The researcher will use the same format to introduce thirds, fourths, sixths, eighths, tenths, and twelfths and provide an example using pizza or m and ms for each.

*"The nice thing about fraction tiles is that we can compare different types of fractions to see if they are equivalent. Equivalent means that the amounts are the same. For example if I line up 2/4 on the table (*the researcher will line up ¼ and then another ¼*) and then put ½ below it* (the researcher will put the pink ½ tile below it) *then you can see that the 2/4 are the same length as the ½. The models show the same amount. These are called equivalent fractions. This is important to know because often times when you are solving problems with fractions you need to find an equivalent fraction so you can show the answer to the problem using the smallest fraction possible. Another example of equivalent fractions is 2/3 (*the researcher will line up an orange 1/3 tile and then another orange 1/3 tile) *and 4/6* (the researcher will line up four teal 1/6 tiles). *Do you have any questions about the fraction tiles or equivalent fractions?* (If not, then the researcher will continue to describe and model phase of the lesson. If so, the researcher will answer questions and provide additional examples if necessary.)

Describe and Model Group Problems:

"There are several different types of word problems. If you understand the characteristics of each type of word problem, then that will help you figure out how to solve similar problems. Today we are going to learn about one type of word problem called a "group" problem. When you solve group problems you combine two parts of something to make a whole. (The researcher will write P + P = W on the board for a visual reference). *For example, if you have 1 pepperoni pizza and your friend has 2 cheese pizzas, then how many pizzas do you and your friend have altogether? (3) In this problem the whole group or total number of pizzas is 3 pizzas, but you have two parts: 1 pepperoni pizza and 2 cheese pizzas. Or if you had 6 bags of m and ms: 2 were peanut and 4 were plain, then your whole group or total bags of m and ms is 6 bags, but one part of the group or total is the 2 bags of peanut m & ms and the other part of the group or total is the 4 bags of plain m & ms. Do you have any questions about group problems?*

Now, I'm going to model some examples of group problems with fractions using the fraction tiles that I showed you earlier. Even though we are using fractions, I will still be modeling problems where two parts can be combined to make a total or group.

(The researcher will then read each problem with complete information to the student and use the fraction tiles to model the problems. The researcher will identify each part of the problem and the whole or total amount and explain how sometimes a fraction can be shown using the smallest fraction possible or in "lowest terms".)

Model – Four Problems

Complete Information

10. A mother elephant eats 2 3/4 tons of food every month. Her baby eats 1/4 of a ton of food. Altogether they eat 2 4/4 or 3 tons of food.

"When we have these types of problems we combine or add the two parts together to find the total. So in this case we have one part, the 2 3/4 tons of food for the mother elephant and another part, the 1/4 of a ton of food for the baby, the total or whole is 2 and 4/4, but the 4/4 can be changed to 1 whole. We can then say that the total is 3 tons of food. (The researcher will write Part + Part = Whole on the board to reinforce this concept.)

(The researcher will then read the following problem.)

11. Tiffany bought 2 1/2 pounds of yellow and red m & ms to take to a friend's party. If 1 1/2 pounds of m & ms were yellow, then one pound is red. (group)

(The researcher will model the problem with fraction tiles while stating the following:)

"In this problem the two parts are the 1 ½ pounds of yellow m & ms and the 1 pound of red m & ms. We can model our yellow m & ms with the red fraction tile for 1 and the half with the pink ½ fraction tile. We can model the 1 pound of red m & ms with another red fraction tile for 1. The total or whole group is the 2 ½ pounds of yellow and red m & ms combined. This problem is still a group problem, but it is written in a different way. The total or group is at the beginning of the problem and the two parts are at the end of the problem. (The researcher will show the student on the written problem.)

Unknown Information

"Sometimes we have group problems and we know each of the parts, but we don't know the total or whole amount in the group. I'm going to model this type of problem. (The researcher models the following problem with fraction tiles.)

12. A mother elephant eats 2 2/3 tons of food every month. Her baby eats 2/3 of a ton of food. How many tons of food do they eat in a month?

"When we have these types of problems we combine or add the two parts together to find the whole. So in this case we have one part, the 2 2/3 tons of food for the mother elephant and another part, the 2/3 of a ton of food for the baby and we want to know how much we have as the total or "whole" amount. If we look at the tiles we can see that we have 2 and 4/3, but the 3/3 can be changed to one whole (the researcher will model with the blocks) *so our total or group amount is 3 1/3."*

13. Tiffany bought 3 2/3 pounds of yellow and red m & ms to take to a friend's party. If 1 1/3 pounds of m & ms were yellow, how many pounds were red?

"In this problem we know that the total or whole group is the 3 2/3 pounds of yellow and red m & ms combined. We know that one of the parts is the 1 1/3 pounds of yellow m & ms, but we don't know the other part (red m & ms). We can model the 3 2/3 pounds of m& ms with our fraction tiles and take away the part that we know (the 1 1/3 pound of yellow m & ms) to see what our other part or red m & ms would be. We can tell by the tiles that are left that there would be 1 1/3 pounds of red m & ms. Do you have any questions about these problems?"

Guided Practice:

"Now I'm going to help you model some group problems. The first two problems include all the information, so you will just practice modeling the two parts and the total or whole amount of the problem. (The researcher will then read each problem to the student and guide the student as he/she models the problem. The researcher will point out the different parts and the total in each problem.)

Guided Practice – Four Problems

1. Last week, John kept track of the weather for three days. In his town it was sunny some days and rainy other days. If it rained 1 ½ days, then 1 ½ days were sunny.
2. An adult dog eats 1 3/4 pounds of food every day. Her puppy eats 1 1/4 pounds of food. Altogether they eat 3 pounds of food each day.

The next two problems have missing information. I will help you model the known information. I will then help you use the model of the group problem to determine the missing information. (The researcher will then read each problem to the student and guide the student as he/she models the problem. The researcher will point out the known parts or total and guide the student in using the manipulatives to find the missing part or total.)

3. Last week, John kept track of the weather for three days. In his town it was sunny some days and rainy other days. If it rained 2 ½ days, how many days were sunny?
4. An adult dog eats 1 1/3 pounds of food every day. Her puppy eats 2/3 pounds of food. How many pounds of food do they eat in a day?

"Do you have any questions about the problems that we worked together? (The researcher will answer any questions and provide clarification at this point.) *Now I am going to let you model two problems that have all the information and I'm going to let you try to use the fraction tiles to solve four group problems similar to the ones that we worked together. I will read the problems to you, but I want you to describe how you are getting the answer and model the problem with the fraction tiles. I will read each problem to you.* (The researcher will read the following problems to the student. The researcher will take notes as the student models each problem and describes how he/she solves the problems.)

Independent Practice:

1. Isaiah bought 2 3/4 pounds of yellow and red skittles to take to a friend's party. If 1 1/4 pounds of skittles were yellow then 1 2/4 pounds were red.
2. Mrs. Poling brought 1 3/4 cheese pizzas to the fifth grade party and Mrs. Young brought 1 ¾ pepperoni pizzas to the party. Together they brought 3 2/4 pizzas to the class party.
3. Last week, Kola kept track of the weather for three days. In his town it was sunny some days and snowy other days. If it snowed 1 1/2 days, how many days were sunny?
4. Jared went to McDonalds and ate 1 2/3 Chicken McNuggets. His brother ate 1 2/3 Chicken McNuggets. How many Chicken McNuggets did they eat altogether?
5. Hannah bought 3 2/3 pounds of green and red lollipops to take to a friend's house. If 1 1/3 pounds of the lollipops were green, how many pounds were red?
6. Last week, the football team practiced three days. Some days, or parts of some days, they worked on offense and on other days they worked on defense. If they worked on offense 2 1/4 days, how many days did they work on defense?

Scoring:

The researcher will check to see if each problem is modeled correctly and the correct answer is given. If the student correctly answers 5/6 questions the researcher will continue to the next lesson. If the student does not correctly answer 5/6 questions then the researcher will review the modeling and group schema with the student.

Practice Sheet 1 Name_______________________________

For each problem you will need to model each problem with the fraction tiles, describe how you modeled the problem to Mrs. Reneau and record your answer on this sheet. The first two problems have all the information, in the rest of problems you will need to solve for missing information.

1. Isaiah bought 2 3/4 pounds of yellow and red skittles to take to a friend's party. If 1 1/4 pounds of skittles were yellow then 1 2/4 pounds were red.

2. Mrs. Poling brought 1 3/4 cheese pizzas to the fifth grade party and Mrs. Young brought 1 ¾ pepperoni pizzas to the party. Together they brought 3 2/4 pizzas to the class party.

3. Last week, Kole kept track of the weather for three days. In his town it was sunny some days and snowy other days. If it snowed 1 1/2 days, how many days were sunny?

4. Jared went to McDonalds and ate 1 2/3 Chicken McNuggets. His brother ate 1 2/3 Chicken McNuggets. How many Chicken McNuggets did they eat altogether?

5. Hannah bought 3 2/3 pounds of green and red lollipops to take to a friend's house. If 1 1/3 pounds of the lollipops were green, how many pounds were red?

6. Last week, the football team practiced three days. Some days, or parts of some days, they worked on offense and on other days they worked on defense. If they worked on offense 2 1/4 days, how many days did they work on defense?

Appendix D

Checklist for Review of Audiotaped Lessons

1. _____ The researcher included a description of the problem type in the introduction.

2. _____ The researcher included a review of the previous problem type (when appropriate) or review of the previous lesson in the introduction.

3. _____ The researcher modeled two problems with known amounts (this could be completed with help from the student).

4. _____ The researcher modeled two problems with unknown amounts (this could be completed with help from the student).

5. _____The student and researcher completed two guided practice problem(s) with known amounts.

6. _____The student and researcher completed two guided practice problems with unknown amounts.

7. _____ The student completed two independent practice problems with known amounts.

8. _____The student completed four independent practice problems with unknown amounts.

9. _____The researcher (or student) read each independent practice problem aloud.

10. _____The researcher responded to student questions when asked throughout the lesson.

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