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Evaluation of basal area projection models for unthinned and thinned central Appalachian hardwood forest stands

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EVALUATION OF BASAL AREA PROJECTION MODELS FOR UNTHINNED AND THINNED CENTRAL APPALACHIAN HARDWOOD FOREST STANDS

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ABSTRACT

Evaluation of basal area projection models for unthinned and thinned central Appalachian hardwood forest stands.

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Several basal area projection models, originally developed for pine plantations, were proposed for potential use in central hardwoods. The data came from five different studies that were established throughout the central Appalachian region. Analysis of variance was performed to detect significant differences among the different treatments and studies. The results indicated that there is a significant difference in basal area growth between unthinned and thinned plots as well as between the different studies. Each selected model was then fitted to the re-measured hardwood plot data, using a non-linear procedure. An assessment was perform on the analysis of variance outcome, residual distribution, and average bias separately for unthinned and thinned stands, in order to find the best equation for the data. The results indicated that all selected equations for this study can be applied in central hardwood stands for basal area growth modeling. Some model forms contained non-significant coefficient estimates that were excluded. The equations with a thinning modifier did not always exhibit better characteristics when compared to the model forms lacking an explicit thinning modifier. The best basal area projection model contained the smallest root mean square error and the highest coefficient of determination for the two treatments, the smallest bias when fit to the data for thinned plots, and a relatively small bias when fit to the data for unthinned plots. When tested for projection indifference by study, the best basal area projection model showed no major projection flaws and was finally approved.

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Table of Contents

List of Tables

List of Figures

CHAPTER 1: INTRODUCTION AND REVIEW OF THE EXISTING LITERATURE

1.1 Introduction

Forest managers monitor stand growth and prescribe silvicultural treatments to improve the potential value of their forest stands. Yet, effective stand management is hindered when accurate models of stand dynamics are unavailable. Basal area growth projection is an essential factor and the foundation for model systems that predict stand growth and yield. Basal area growth projection models attempt to describe forest stand dynamics over time (i.e. growth, mortality, reproduction, and associated changes at the stand level) and hence are widely used in forest management for their ability to update inventories, predict future yield, and to explore management alternatives. Compared with such measures as stand diameter, tree height, or crown diameter, basal area possesses a high degree of exactness in measurement or prediction. As a concept, it is applicable to a wide range of conditions. Basal area is of interest for forest inventories because it is highly correlated with volume growth of forest stands. Many silvicultural and forest management considerations, such as thinning intensity, are based on basal area measurements. In addition, the mean annual basal area increment is a useful tool for the appropriate management of forest stands and facilitates the timing of intermediate and final harvests (Zhang *et al*. 2007).

Growth and yield modeling has a long history in forestry. As early as the 1920s, the U.S. Forest Service published its first widely used yield tables for southern pines (Office of Forest Experiment Station, Forest Service, and Cooperating Agencies 1929). Similar tables for Douglas-fir, based on considerably more restricted samples, were also prepared by McArdle and Meyer (1930). The methods for measuring the growth of stand basal area have evolved from those developed in the United States and Germany during the last century. Stand basal area modeling has progressed rapidly since the first widely used model was published by the U.S. Forest Service. Over the years, a variety of models have been developed for predicting the growth and yield of uneven and even-aged stands using stand-level approaches. The modeling methodology has not only moved from an empirical approach to a more ecological process-based approach but also accommodated a variety of techniques such as simultaneous equation methods, difference models, artificial neural network techniques, linear or nonlinear regression models, and matrix models (Zhang 2007).

Basal area projection models are an important part of contemporary forest management. Such equations are of great interest in many areas of forestry practices. The existence of basal area projection models is essential for determining the right approaches for solving various issues regarding forest management. Some of these issues is to capture the long term effect of thinning practices in Appalachian hardwood forest stands and to evaluate their contribution in stand basal area growth. The ability to perform such tasks requires an obvious need for locally developed models that can project the changes in stand basal area growth over stand age and to answer the many questions regarding thinning practices.

Thinning is a process of removing some trees in an immature forest stand to increase the growth of the remaining trees and the total yield or value of usable wood (Emmingham and Elwood 1983). Theoretically, thinning redistributes the growth potential of a site to the quality trees so they can grow faster. Some of the main objects of thinning are to redistribute the growth of the stand to fewer trees of higher quality, thereby increasing the value of the stand and to use or sell trees that otherwise will decay and die. Other authors suggest that the main objects of thinning are to control and regulate the growth of the forest stand according to the priority functions and the stated management goals (Rafailov and Kostov, 2001).

There are two methods of thinning: commercial and pre-commercial. In commercial thinning, the immediate value of the removed trees at least pays for the cost of thinning. If the value of the removed trees is less than the cost of thinning, the practice is called pre-commercial thinning (Emmingham and Elwood 1983). The types of thinning are row, selective and combined thinning. Different approaches of tree removal can be used in each type of thinning, such as thinning from above (crown thinning), thinning from below (low thinning), and thinning from middle (a combination of the former two approaches).

The main goals of this study are:

1. To select a proper basal area projection model from some of the existing basal area projection equations, originally developed for pine plantations, that can be applied to unthinned and thinned central Appalachian hardwood forest stands, regardless of species, in order to accurately project their long term basal area growth,
2. To evaluate the overall basal area projection performance of the selected model.

1.2 Review of the Existing Literature

In general, basal area projection equations are functions of measurable variables such as current basal area (B_1), average stand age (A_i), trees per acre (N_i), average dominant height (H_i), and site index (SI), or:

$$B_2 = f\left(B_1, A_1, A_2, N_1, N_2, H_1, H_{2,} SI\right)$$

The diversity of the models is large and it depends on the methods used for their development and the types of variables that they include. Some of them are simple equations that involve only a few variables while others are more complex and involve many variables.

Since this study concentrates on projecting the long term basal area growth in central Appalachian unthinned and thinned hardwood stands, the equations that contain explicit thinning modifiers will be of primary interest. These modifiers can account for the type of thinning, thinning intensity, or a combination of both. One of the main interests in this study will be to examine the role of these modifiers and to evaluate their effectiveness and accuracy. Since the thinning type applied in Appalachian hardwood forests is selective only, the modifiers that account for the type of thinning are restricted to a single option.

Several basal area projection models were developed for slash (*Pinus elliottii* Englem.) and loblolly (*Pinus taeda* L.) pine plantations in southeastern United States and other countries. Some of the models assume that post thinning basal area growth is the same for both unthinned and thinned stands of the same initial conditions. The later has been rejected by several authors (Bailey and Ware 1983, Pienaar *et al*. 1985, Pienaar and Shiver 1986, Pienaar *et al*. 1989, Pienaar *et al*. 1990, Brooks 1992). Each of these authors has incorporated an explicit thinning modifier to provide for increased basal area growth as a result of thinning. The expectation is that the growth in thinned stands would asymptotically approach that of unthinned stands starting with the same initial conditions (Brooks 1992)

1.2.1 Clutter and Jones (1980)

Clutter and Jones (1980) provided the first regional study of thinned slash pine plantations in southeastern United States. They developed a basal area projection equation for thinned stands as a function of initial basal area, initial age and projection age. The model form is exhibited as Equation [1]:

$$B_2 = Exp\left[\left(\frac{A_1}{A_2}\right)^{\alpha_1} Ln(B_1) + \alpha_2\left(1-\left(\frac{A_1}{A_2}\right)^{\alpha_1}\right)\right] \quad [1]$$

where:

Exp	=	exponential function,
Ln	=	natural logarithm,
B_i	=	basal area per acre at age A_i,
A_i	=	stand age at time i,
α_i	=	coefficients to be estimated from the available data.

This model contains some desirable properties:

- $\lim_{A_2 \to A_1} (B_2) = B_1$,
- is path invariant,
- is asymptotic as $A_2 \to \infty$.

This equation form does not account for differences in site productivity and stand age at time of thinning. It also assumes that unthinned and thinned stands grow at the same rate if they start the growth period at the same age and initial basal area (Brooks, 1992).

1.2.2 Burkhart and Sprinz (1984)

A single whole stand basal area model was developed for repeatedly thinned, old-field loblolly pine plantations in the Virginia Piedmont and Coastal Plain. Plantations were operationally thinned from below up to three times. This equation was first presented by Cao *et* *al.* (1982) and later developed in compatible form by Burkhart and Sprinz (1984). The model form is exhibited as Equation [2]:

$$B_2 = Exp\left[\left(\frac{A_1}{A_2}\right)LnB_1 + \alpha_1\left(1-\frac{A_1}{A_2}\right)+\alpha_2 SI\left(1-\frac{A_1}{A_2}\right)\right] \qquad [2]$$

where:

Exp	=	exponential function,
Ln	=	natural logarithm,
B_i	=	basal area per acre at age A_i,
A_i	=	stand age at time i,
SI	=	site index,
α_i	=	coefficients to be estimated from the available data.

This model contains some desirable properties:

- $\lim_{A_2 \to A_1} (B_2) = B_1$,
- is path invariant,
- has site specific upper asymptote.

This model is sensitive to differences in site productivity, but does not contain a thinning modifier even though it was fit to thinned stands. If applied for both thinned an unthinned stands, it assumes that they have the same growth rate if they have the same initial characteristics (Brooks, 1992).

1.2.3 Bailey and Ware (1983)

Two basal area projection models that include a measure of thinning type have been developed for thinned loblolly pine stands. The first model was presented by Bailey and Ware (1983) for thinned and unthinned, even-aged natural stands in central Georgia. It incorporates the effects of age at the most recent thinning, site quality and a function of the type of thinning performed. The model form is exhibited as Equation [3]:

$$B_2 = B_1^{\left(\frac{A_1}{A_2}\right)} Exp\left[\alpha_1\left(1-\frac{A_1}{A_2}\right)+\alpha_2 X_t \frac{\left(\frac{1}{A_2}-\frac{1}{A_1}\right)}{A_t A_2}+\alpha_3 SI\left(1-\frac{A_1}{A_2}\right)\right] \qquad [3]$$

where:

Exp	=	exponential function,
Ln	=	natural logarithm,
B_i	=	basal area per acre at age A_i,
A_i	=	stand age at time i,
X_t	=	$1-R_t$, if $R_t \neq 0$,
		0, if $R_t = 0$,
R_t	=	ratio of the quadratic mean diameter of the trees removed in thinning to the quadratic mean diameter of the stand before thinning,
α_i	=	coefficients to be estimated from the available data.

This model form has the following desirable properties:

- $\lim_{A_2 \to A_1} (B_2) = B_1$
- $\lim_{A_2 \to \infty} (B_2) = Exp(\alpha_1+\alpha_3 SI)$
- projection is path invariant
- if $\alpha_2 < 0$ then B_2 is a monotonic increasing function of X_t
- if $X_t = 0$, the model reduces to the commonly used functional form for basal area growth suggested by Clutter (1963)

The last property implies that basal area growth for diameter-indifferent thinning would be same as unthinned stands having the same initial basal area since X_t would equal zero. The model also implies that basal area for stands thinned from above ($X_t<0$) will grow less, and basal area for stands thinned from below ($X_t>0$) will grow more than unthinned stands of the same initial basal area. The use of ratios of quadratic mean diameters have been used by others to characterize thinning though this was the first model to include such a variable to define the type of thinning (Brooks and Bailey, 1992).

1.2.4 Souter (1986)

The second model that includes a measure of thinning type was developed by Souter (1986) for naturally-regenerated loblolly pine stands in the Georgia Piedmont. This model form is exhibited as Equation [4]:

$$B_2 = \left(\frac{N_2}{N_1}\right)^{\left(\frac{A_1}{A_2}\right)^{\alpha_4}} B_1^{\left(\frac{A_a}{A_2}\right)} Exp\left[\left(\alpha_1 + \alpha_3 SI\right)\left(1 - \frac{A_1}{A_2}\right) + \alpha_2\left(\frac{D_r}{D_b * SDI_b}\right)\left(\frac{\left(\frac{1}{A_2} - \frac{1}{A_1}\right)}{A_t A_2}\right)\right] \quad [4]$$

where:

Exp	=	exponential function,
B_i	=	basal area per acre at age A_i,
N_i	=	number of trees per acre at age A_i,
A_i	=	stand age at time i,
D_r	=	the quadratic mean diameter of all trees removed during thinning,
D_b	=	the quadratic mean diameter of all trees before thinning,
SDI_b	=	Reineke's stand density index before thinning ($SDI_b = N_b*(10 / D_b)^{-1.605}$),
N_b	=	number of trees per acre before thinning,
α_i	=	coefficients to be estimated from the available data.

This model form includes an explicit thinning modifier which is a modification of the X_t variable employed by Bailey and Ware (Equation [3]). Both model forms attempt to capture the effects of thinning in a single modifier term that places more emphasis on thinning type than thinning intensity.

1.2.5 McTague and Bailey (1987)

McTague and Bailey (1987) developed an equation form for thinned loblolly pine plantations in Santa Catarina, Brazil where past thinning history was unknown. This projection model involves the concept of diameter distribution and attempts to capture the effects of thinning through the change in the 10^{th} and 63^{rd} percentiles. This model form is exhibited as Equation [5]:

$$B_2 = Exp\left[\begin{array}{l}\alpha_0\left(\frac{1}{A_2}-\frac{1}{A_1}\right)+\alpha_1 Ln\left(\frac{N_2}{N_1}\right)+\alpha_2 Ln\left(\frac{D63_2}{D63_1}\right)+\alpha_3 Ln\left(\frac{D10_2}{D10_1}\right)+ \\ \alpha_4\left(\frac{1}{D63_2-D10_2}-\frac{1}{D63_1-D10_1}\right)+Ln(B_1)\end{array}\right] \quad [5]$$

where:

Exp	=	exponential function,
Ln	=	natural logarithm,
B_i	=	basal area per acre at age A_i,
A_i	=	stand age at time i,
N_i	=	number of trees per acre at age A_i,
$D10_i$	=	the 10^{th} diameter percentile at age A_i,
$D63_i$	=	the 63^{th} diameter percentile at age A_i,
α_i	=	coefficients to be estimated from the available data.

The model exhibits the following desirable properties:

- path invariance,
- $\lim_{A_2 \to A_1} (B_2) = B_1$,
- unbounded as $A_2 \to \infty$.

However, some authors report that the basal area values are reasonable within the range of observed plot values from 9 to 253 (ft^2/ac) (Brooks and Bailey 1992).

1.2.6 Pienaar *et. al.* (1985)

Pienaar *et al.* (1985) developed a whole stand basal area projection model for thinned slash pine plantations in South Africa that include an explicit function of thinning intensity based on a basic basal area growth model suggested by Schumacher (1939) and generalized by Clutter and Jones (1980). The model form is exhibited as Equation [6]:

$$B_2 = B_1\left(\frac{A_1}{A_2}\right)^{\alpha_0+\alpha_1 X} Exp\left\{\alpha_2\left[1-\left(\frac{A_1}{A_2}\right)^{\alpha_0+\alpha_1 X}\right]\right\} \quad [6]$$

where:

Exp	=	exponential function,
B_i	=	basal area per acre at age A_i,
A_i	=	stand age at time i,
X	=	N_t / N_a,
N_t	=	trees per acre removed during thinning,
N_a	=	trees per acre after thinning,
α_i	=	coefficients to be estimated from the available data.

This model form has some desirable properties, such as:

- it reduces to the general Clutter and Jones (1980) form when thinning is not applied,
- both thinned and unthinned stands are assumed to have the same asymptotic basal area on any given site. The rate at which it is approached depends on thinning intensity.

Although this model explicitly accounts for differences in thinning intensity, it does not allow for differences in thinning response due to different types of thinning.

1.2.7 Pienaar *et al*. (1989)

Pienaar and Shiver (1986) utilized a stand-level based basal area growth projection equation for thinned and unthinned slash pine plantations in South Africa based on a Schumacher-type equation form. Pienaar *et al*. (1989-1990) further developed this model form using thinned slash pine plantation re-measurement data in the Atlantic Coast Flatwoods region. It involves additional variables such as number of trees per acre and stand average dominant height. It also explicitly includes a thinning modifier that accounts for thinning intensity and stand age at the time of thinning. Pienaar *et al*. (1989-1990) basal area projection equation was presented in two forms. The first form (Pienaar *et al*. 1989) is exhibited as Equation [7]:

$$B_2 = Exp\left[\begin{array}{l} Ln(B_1) + \alpha_1\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \alpha_2\left(Ln(H_2) - Ln(H_1)\right) + \alpha_3\left(Ln(N_2) - Ln(N_1)\right) + \\ \alpha_4\left(\frac{Ln(H_2)}{A_2} - \frac{Ln(H_1)}{A_1}\right) + \alpha_5\left(\frac{Ln(N_2)}{A_2} - \frac{Ln(N_1)}{A_1}\right) + \alpha_6\left(\frac{NtAt}{NbA_2} - \frac{NtAt}{NbA_1}\right) \end{array}\right] \quad [7]$$

where:

Exp	=	exponential function,
Ln	=	natural logarithm,
B_i	=	basal area per acre at age A_i,
A_i	=	stand age at time i,
H_i	=	average dominant height at age A_i,
N_i	=	number of trees per acre at age A_i,
A_t	=	stand age at the most recent thinning,
N_t	=	number of trees per acre removed during thinning,
N_b	=	number of trees per acre before thinning,
α_i	=	coefficients to be estimated from the available data.

This form has an explicit thinning modifier that accounts for the thinning intensity and the age of thinning.

1.2.8 Pienaar *et al*. (1990)

In the second projection form developed by Pienaar *et al*. (1990) the thinning term cancels and is no longer explicitly defined in the equation. It is exhibited as Equation [8]:

$$B_2 = Exp\begin{bmatrix} \frac{A_1}{A_2} Ln(B_1) + \alpha_1\left(1 - \frac{A_1}{A_2}\right) + \alpha_2\left(Ln(H_2) - \frac{A_1}{A_2} Ln(H_1)\right) + \\ \alpha_3\left(Ln(N_2) - \frac{A_1}{A_2} Ln(N_1)\right) + \alpha_4 \frac{(Ln(H_2) - Ln(H_1))}{A_2} + \\ \alpha_5 \frac{(Ln(N_2) - Ln(N_1))}{A_2} \end{bmatrix} \qquad [8]$$

where:

Exp	=	exponential function,
Ln	=	natural logarithm,
B_i	=	basal area per acre at age A_i,
A_i	=	stand age at time i,
H_i	=	average dominant height at age A_i,

N_i = number of trees per acre at age A_i,

α_i = coefficients to be estimated from the available data.

1.2.9 Brooks (1992) form 1

Some of the previously suggested models have attempted to project basal area growth after thinning utilizing a single thinning variable approach. Brooks (1992) suggested that both thinning type and thinning intensity affect subsequent basal area growth and that two separate variables could be utilized in a single basal area projection equation. The projection equation developed by Pienaar *et al.* (1989) (Equation [7]) was modified by Brooks (1992) to explicitly account for thinning type, thinning intensity, and stand age at initial thinning for the slash pine plantations in the Lower Atlantic Coastal Plain. The general form of Brooks (1992) basal area projection equation is shown as Equation [9]:

$$B_2 = Exp\left[\begin{array}{l} Ln(B_1) + \alpha_1\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \alpha_2\left(Ln(H_2) - Ln(H_1)\right) + \\ \alpha_3\left(Ln(N_2) - Ln(N_1)\right) + \alpha_4\left(\frac{Ln(H_2)}{A_2} - \frac{Ln(H_1)}{A_1}\right) + \\ \alpha_5\left(\frac{Ln(N_2)}{A_2} - \frac{Ln(N_1)}{A_1}\right) + \alpha_6\left(\frac{T_t}{A_t}\right)\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \\ \alpha_7\left(\frac{T_i}{A_t}\right)\left(\frac{1}{A_2} - \frac{1}{A_1}\right) \end{array}\right] \quad [9]$$

where:

Exp = exponential function,

Ln = natural logarithm,

B_i = basal area per acre at age A_i,

A_i = stand age at time i,

H_i = average dominant height at age A_i,

N_i = number of trees per acre at age A_i,

T_t = a function of thinning type,

T_i = a function of thinning intensity,

A_t = stand age at last thinning ($A_1 \leq A_t$),

α_i = coefficients to be estimated from the available data.

This model form has the following desirable properties:

- path invariance,
- $\lim_{A_2 \to A_1} (B_2) = B_1$,
- implicit sensitivity to differences in site productivity,
- sensitivity to differences in age at thinning,
- sensitivity to both type and intensity of thinning,
- reduction to the Pienaar *et al*. (1989) form when there is no thinning ($T_t = T_i = 0$),
- projections for both thinned and unthinned stands.

1.2.10 Brooks (1992) form 2

Utilizing the same approach employed in his first basal area projection model form (Equation [9]) for slash pine plantations, Brooks (1992) developed another explicit model form that includes two separate modifiers for thinning type and thinning intensity for loblolly pine plantations in the Piedmont and the Upper Atlantic Coastal Plain. It is a modified form of the Borders *et al*. (1990) basal area projection model and it is shown as Equation [10]:

$$B_2 = Exp\begin{bmatrix} \left(\frac{A_1}{A_2}\right)Ln(B_1) + \alpha_1\left(1-\frac{A_1}{A_2}\right) + \gamma_1 Z\left(1-\frac{A_1}{A_2}\right) + \\ \alpha_2\left(Ln(H_2) - \left(\frac{A_1}{A_2}\right)Ln(H_1)\right) + \alpha_3\left(Ln(N_2) - \left(\frac{A_1}{A_2}\right)Ln(N_1)\right) + \\ \alpha_4\left(\frac{Ln(H_2) - Ln(H_1)}{A_2}\right) + \alpha_5\left(\frac{Ln(N_2) - Ln(N_1)}{A_2}\right) + \\ \alpha_6\left(\frac{T_i}{A_t}\right)\left(1-\frac{A_1}{A_2}\right) + \alpha_7\left(\frac{T_t}{A_t}\right)\left(1-\frac{A_1}{A_2}\right) \end{bmatrix} \qquad [10]$$

where:

Exp = exponential function,

Ln = natural logarithm,

B_i = basal area per acre at age A_i,

A_i = stand age at time i,

Z = 0, if the stand is thinned,

Z = 1, if the stand is unthinned,

H_i = average dominant height at age A_i,

N_i = number of trees per acre at age A_i,

T_t = a function of thinning type,

T_i = a function of thinning intensity,

A_t = stand age at last thinning ($A_1 \leq A_t$),

α_i = coefficients to be estimated from the available data.

This model form contains the following desirable properties:

- path invariance,
- $\lim_{A_2 \to A_1} (B_2) = B_1$,
- implicit sensitivity to differences in site productivity,
- sensitivity to differences in age at thinning,
- sensitivity to both type and intensity of thinning,
- reduction to the Borders *et al*. (1990) form when there is no thinning ($T_t = T_i = 0$),
- projections for both thinned and unthinned stands.

CHAPTER 2: DATA AND METHODS

2.1 Locations of the study area

Measurement plots were established throughout central Appalachian region. The central Appalachian hardwood region encompasses a 205,000-square-mile area that includes all of West Virginia and parts of Kentucky, Maryland, Mississippi, New York, North Carolina, Ohio, Pennsylvania, Tennessee, and Virginia. It is a part of a vast system of mountains in eastern North America. The average elevation of the range is around 3,000 feet.

The Appalachians contain the largest temperate hardwood forest region in the world. Occurrences of plant species and forest types in the central Appalachians are related to elevation, soil acidity and moisture content (Mueller 1996). In general, the plants form a floristic province of The North America Atlantic Region. At lower elevations, the drier rocky sites are occupied by oak-chestnut type of forest that are dominated by a variety of oaks, hickories, and a few patches or individual trees of American chestnut (*Castanea dentata* Marsh.). The oak composition contains mainly red oak (*Quercus rubra* L.), white oak (*Quercus alba* L.), black oak (*Quercus velutina* Lam.), and chestnut oak (*Quercus prinus* L.). The more mesic sites are occupied by maple-cherry-yellow-poplar type that is dominated by sugar maple (*Acer saccharum* Marsh.), red maple (*Acer rubrum* L.), black cherry (*Prunus serotina* Ehrh.), yellow-poplar (*Liriodendron tulipifera* L.), white ash (*Fraxinus americana* L.), and American beech (*Fagus grandifolia* Ehrh.). Other species that are part of this type include basswood (*Tilia americana* L.), cucumber magnolia (*Magnolia acuminata* L.), sassafras (*Sassafras albidum* Nutt.), yellow birch (*Betula alleghaniensis* Brit.), black gum (*Nyssa sylvatica* Marsh.), and others. At higher elevations, the dominant conifer is red spruce (*Picea rubens* Sarg.), that is typical for subalpine sites. Other species include balsam fir (*Abies balsamea* L.), Fraser fir (*Abies fraseri* Pursh), and eastern hemlock (*Tsuga canadensis* L.).

The data for this study was collected from five different studies that were located in central Appalachian hardwood region, including West Virginia, Pennsylvania, Kentucky, and Ohio (Figure 1). Each study location falls into a specific eco-region (Figure 2) that has its own forest composition and specific site characteristics.

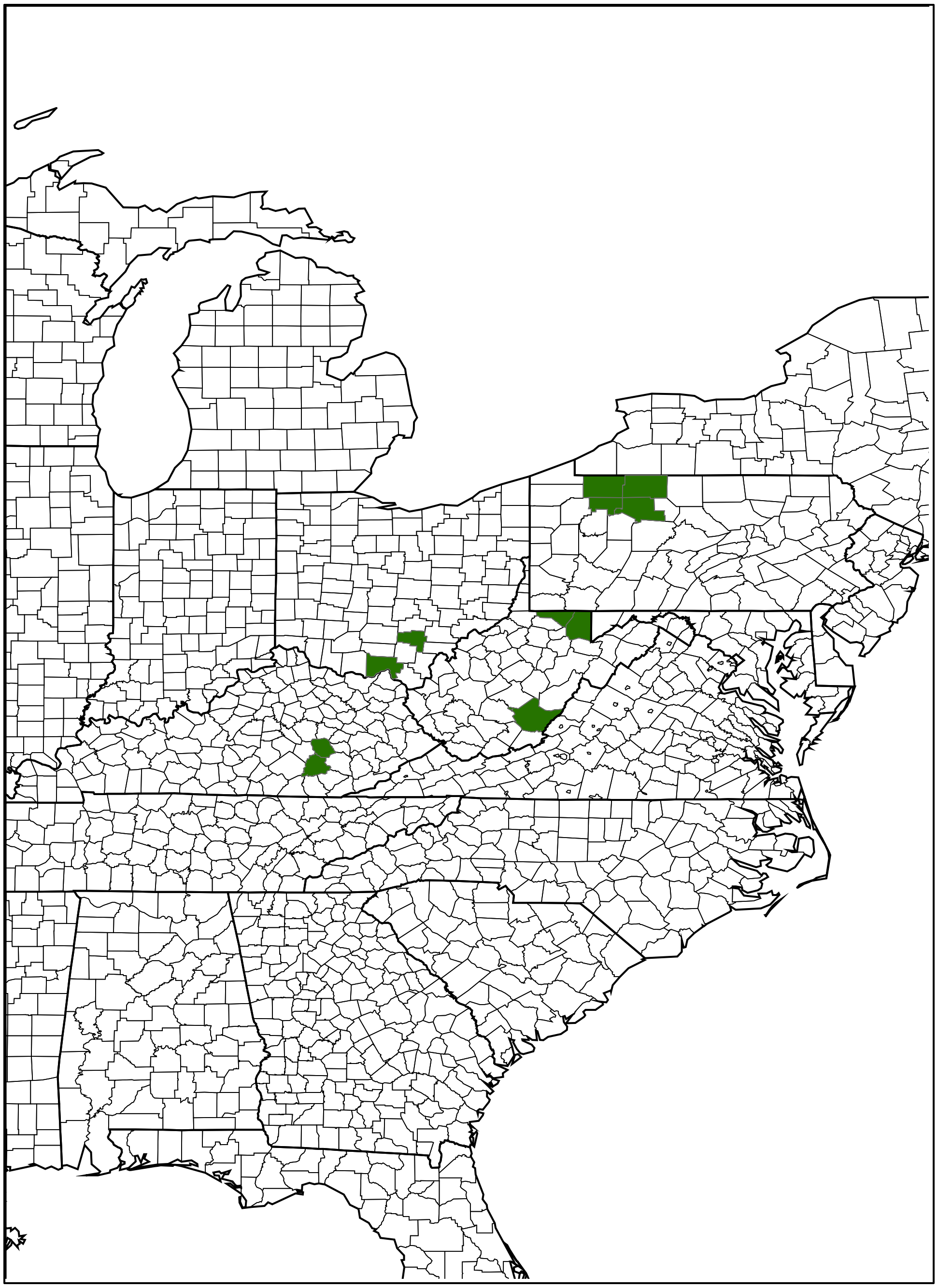

Figure 1. Locations of the study area by county and state. (ESRI ArcGIS Ver. 10).

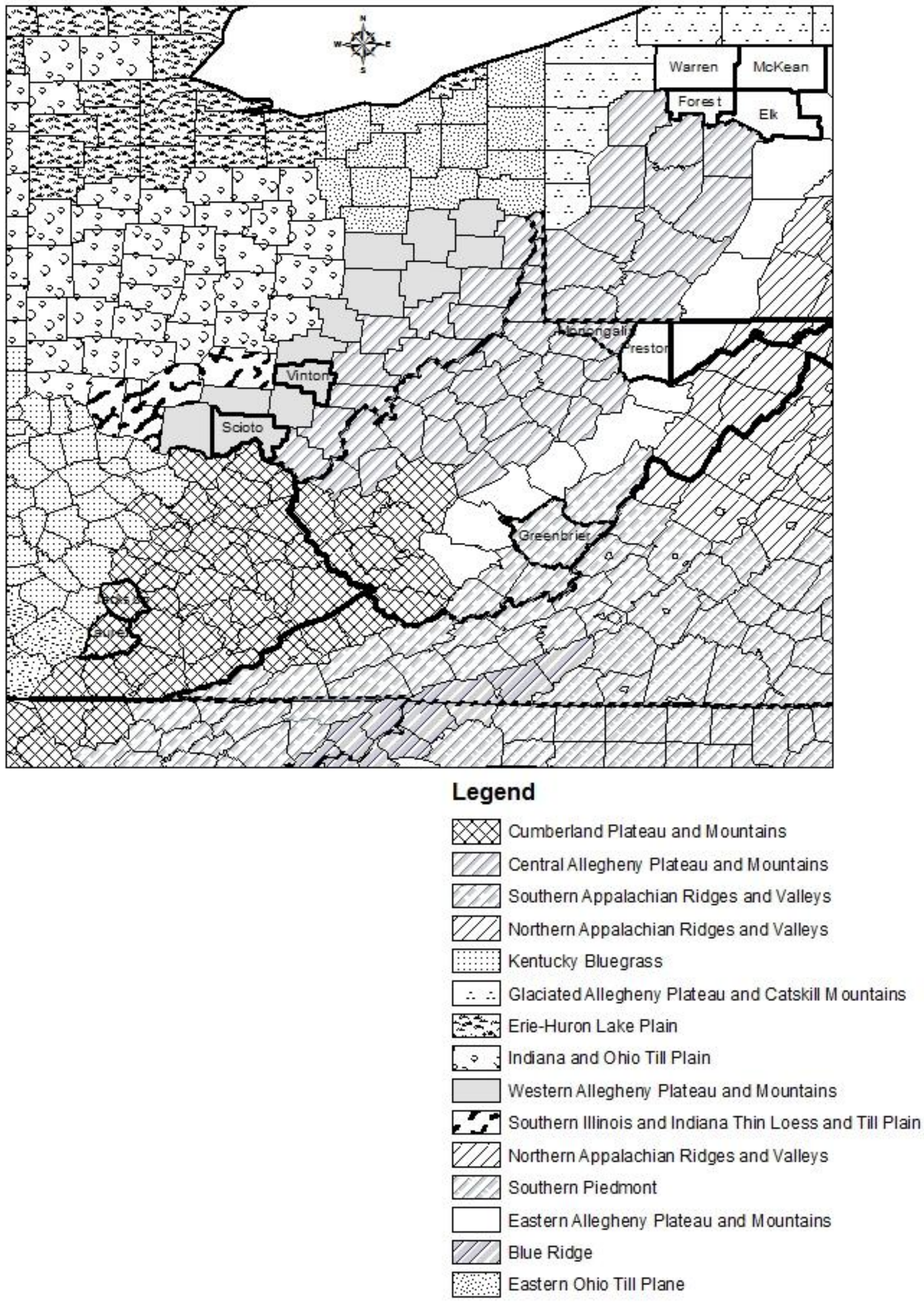

Figure 2. Location of the study area by eco-region. (ESRI ArcGIS Ver. 10).

2.1.1 West Virginia

Three different studies were established in even-aged stands, throughout various locations within West Virginia. The Hatch Thinning Study was conducted by West Virginia University in West Virginia University Research Forest (WVURF) located in Monongalia and Preston County, 10 miles east of Morgantown. It is a 7,664-acre forest that is managed by the West Virginia University Division of Forestry and Natural Resources under terms of a 100-year lease from the West Virginia Division of Natural Resources. The elevation on the forest varies from 1,042 ft. to 2,600 ft. and the average annual precipitation for the last twenty seven years was 47 inches of rainfall and 71 inches of snowfall. The forest consists of 60 to 80-year old stands of mixed oak and mesophytic hardwood types (WVU DFNR). There is a limited area of conifer plantations as well. Tree species include black cherry, white oak, red oak, red maple, sugar maple, yellow-poplar, and other hardwood species. It also contains a limited amount of eastern white pine (*Pinus strobus* L.), eastern hemlock, and planted species such as Norway spruce (*Picea abies* L.) and red pine (*Pinus resinosa* Aiton).

The 1987 Thinning of Large Plots for Yield Determination and Response Demonstration Study and the Tree Dynamics of Young Natural Hardwoods Study were established by the MeadWestvaco Timberland Division in a 62,000-acre forestland in Greenbrier County, WV. The forest composition includes black cherry, fire cherry (*Prunus pensylvanica* L.), black birch (*Betula lenta* L.), yellow birch, beech (*Fagus grandifolia* Ehrh.), cucumber magnolia, red maple, sugar maple, yellow-poplar, sassafras, and others.

2.1.2 Pennsylvania

A thinning study in a 50- to 55-year-old, even-aged, mixed species was conducted by the USDA Forest Service in the Allegheny National Forest, which is located in the north -western part of Pennsylvania, covering parts of Warren, McKean, Forest, and Elk counties. This national forest encopasses about 513,325 acres of hardwood forest and was established in 1923 by Presidential Proclamation under authority of the 1911 Weeks Act. The Allegheny National Forest is one of 155 National Forests managed by the U.S. Department of Agriculture - Forest Service, and the only one in Pennsylvania. The topography is relatively flat and the average elevation is 2,000 ft. Soils are stony, sand loams and the rainfall in the area averages 42 inches (USDA Forest Service 2006).

About 90% of the National Forest's area is covered in forests. The most widely distributed forests are upland hardwood forests of red maple, American beech, black cherry, and black birch. Other tree species include white ash and yellow- poplar, growing mostly in the middle and eastern parts of the forest. In the western and southern parts of the forest, especially along major river drainages and on steep, drier slopes are oak forests of northern red oak, white oak, black oak, and scarlet oak. In the north are northern hardwood forests of sugar maple, American beech, yellow birch, eastern hemlock, and eastern white pine (USDA Forest Service 2006).

2.1.3 Kentucky and Ohio

A stand density study (OAKSIM) was performed in even-aged stands, located in parts of Kentucky and Ohio. The Kentucky plots were established in central parts of Daniel Boone National Forest. The Daniel Boone National Forest was established in 1937. It is located along the Cumberland Plateau in the Appalachian foothills of eastern Kentucky and it covers parts of sixteen counties, including Clay, Wayne, Rowan, Jackson, Laurel and others. The forest encompasses over 707,000 acres of mostly rugged terrain. The land is characterized by steep forested ridges dissected by narrow ravines and over 3,400 miles of sandstone cliffs (USDA Forest Service). Forest composition within location of the plots include hemlock, white pine, yellow-poplar, American beech, cucumber magnolia, chestnut oak, red maple, basswood, sweet birch, and others.

The Ohio plots were established in the Wayne National Forest, located in the hills of southeastern Ohio. Established in 1911, it covers an area of 241,004-acres in federal ownership within a proclamation boundary of 833,990-acres and it is administrated by USDA Forest Service. The forest includes three administrative units: Athens, Marietta, and Ironton. A large part of the forest covers former coal-mining lands (USDA Forest Service). A variety of vegetation constitutes the forest composition in the rugged, non-glaciated hills of the forest. Several species of oaks and hickories, sassafras and native pitch pine (*Pinus rigida* Mill.) and shortleaf pine (*Pinus echinata* Mill.) populate the ridges. Mid-slope areas support oaks and hickories as well as soft and hard maple, basswood, yellow-poplar, buckeye (*Aesculus sp*.), blackgum, white ash, red elm (*Ulmus rubra* Muhl.), hackberry (*Celtis sp*.), and aspen (*Populus sp*.). Sweetgum (*Liquidambar styraciflua* L.), beech, black cherry, black walnut (*Juglans nigra*

L.), sycamore (*Platanus occidentalis* L.), birch, and butternut (*Juglans cinerea* L.) can also be found in bottom land areas and coves.

2.2 Data

The data for this project was gathered through the assistance of USDA Forest Service, West Virginia University, and the Appalachian Hardwood Research Alliance. Diameter at breast height (DBH) was measured and number of trees was determined in all locations, and an average stand age was also defined. Tree heights were measured or estimated for all studies that were conduct in West Virginia. Site index was determined for all studies, except for the thinning study in Allegheny National Forest. Thinning was applied in all locations, except for the MeadWestvaco Tree Dynamics of Young Natural Hardwoods, where all the plots represent unthinned stands for the entire measurement period. The thinning type was always selective for all thinned plots.

2.2.1 MeadWestvaco Study

The data were collected from two different studies conducted on MeadWestvaco Timberland Division property in Greenbrier County, West Virginia. The 1987 Thinning of Large Plots for Yield Determination and Response Demonstration Study (No 402019) was performed to evaluate the diameter growth due to thinning treatments in young hardwood forests. During the summer of 1987, thirty 0.5-acre plots were established in 12-15-year-old, even-aged natural hardwood stands in the Rupert District of Appalachian Woodlands (Dasher and Johnson 1988). Six different locations in the northern hardwood forest type were selected as replication sites. Each location contained five plots that represented five different treatments. The treatments were no thinning, two foot crop release, two foot non-crop release, thin to 90 percent stocking level, and thin to75 percent stocking level. Crop trees were selected based on species, size, form and position. The data was collected from 1987 through 1999, except for one location, where the last measurements were taken in 1992. Most of the measurements were taken annually, while the last one (in 1999) was taken after a five or six year interval. Measurements included diameter at breast height (DBH), total tree height, and merchantable height for some of the larger trees. All measurement plots were square-shaped and 0.25-acre in size except for one location, where two-foot crop and two-foot non-crop release treatments were conducted on plots measuring 0.21 acre in size. Some of the tree heights were originally estimated from height curves or equations.

The Tree Dynamics of Young Natural Hardwoods Study (No 402017) was initiated by C. H. Pham to examine the growth and stand dynamics of young hardwood stands. Twelve, 0.5-acre square plots were located in young, even-aged stands. Seven of them were located in northern hardwood forest types, and five were located in Appalachian hardwood types. Plots were installed between 1985 and 1989. A total of four measurements were taken every three years, except the last measurement, which was taken after five years. Tree measurements included DBH, total tree height and height to the base of the life crown which was measured on a subsample of trees at each plot. Crown class, presence in main canopy, and tree damage were also recorded. All plots were fully stocked and undisturbed throughout the entire measurement period. Thinning was not applied. A total of 42 plots were located throughout the MeadWestvaco property.

Unthinned plots

There were eighteen unthinned plots, located in the MeadWestvaco property, and distributed among the two different studies. The initial age of the stands was between 12 and 42 years. Basal area at initial age ranged from 43 to 134 square feet per acre, the initial stand density ranged from 744 to 2,644 trees per acre, the initial average dominant height ranged from 24 to 62 feet, and site index was between 53 and 101 feet. The check plots characteristics are displayed in Table 1.

Initial Age	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
12	3	43	65	1,595	2,644	78	101
13	3	56	76	1,703	2,218	75	96
15	2	53	57	1,683	1,739	60	73
16	1	84	84	1,888	1,888	83	83
20	1	82	82	2,422	2,422	77	77
24	1	97	97	744	744	85	85
31	1	107	107	930	930	82	82
34	1	127	127	786	786	90	90
35	1	130	130	1,024	1,024	60	60
37	1	101	101	862	862	53	53
40	2	134	134	796	796	68	68
42	1	118	118	834	834	59	59

Table 1. Stand characteristics of the MeadWestvaco unthinned plots at initial age.

Table 1 indicates that the unthinned plots within the MeadWestvaco property covered young to mid-aged stands with highly diverse characteristics.

Thinned plots

There were twenty-four thinned plots located on the MeadWestvaco property. The Stand Dynamics of Young Natural Hardwoods Study did not receive any thinning treatments. The age at first thinning was between 12 and 15 years. Basal area immediately after first thinning ranged between 6 and 37 square feet per acre, trees per acre ranged from 200 to 944, average dominant height was between 22 and 37 feet, and site index was between 60 and 102 feet. The thinned plots characteristics are displayed in Table 2.

Age at First Thinning	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
12	4	9	37	200	568	90	102
13	12	6	37	200	944	60	99
15	8	11	37	200	671	60	79

Table 2. Stand characteristics of the MeadWestvaco thinned plots immediately after the first thinning.

Table 2 indicates that the MeadWestvaco thinned plots were established in very young stands with low age difference and relatively diverse characteristics. It also indicates that heavy thinnings were applied when compared to the characteristics of the unthinned plots (Table 1).

2.2.2 Hatch Thinning Study

This thinning study was performed in late 1940s and early 1950s in young, even-aged hardwood stands to examine the long term effect of thinning practices. Thirty five 0.5-acre plots were established between 1949 and 1953 throughout the West Virginia University Research Forest and measurements were taken until 1990 in five to nine years intervals. Measurements included diameter at breast height (DBH), tree heights, and live crown heights, and the number of trees was recorded. Only 19 plots were selected for this study as some plots exhibited irregular data trends. The initial age of the stands in the plots was between 21and 24 years.

<u>Unthinned plots</u>

There were only three unthinned plots included in the Hatch study. The initial age was between 21 and 24 years. Basal area per acre at initial age ranged from 66 to 90 square feet per acre, trees per acre at initial age ranged from 752 to 964, and site index ranged between 61 and 80 feet. The unthinned stand characteristics are displayed in Table 3.

Initial Age	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
21	1	66	66	752	752	61	61
23	1	66	66	896	896	79	79
24	1	90	90	964	964	80	80

Table 3. Stand characteristics of the Hatch unthinned plots at initial age.

Table 3 indicates that the Hatch unthinned plots were established in young stands with relatively uniformed characteristics.

<u>Thinned plots</u>

Sixteen of the Hatch plots were thinned. They had different thinning intensities and some of them were thinned more than once. The age at first thinning was between 21 and 42 years. Immediately after the first thinning, basal area ranged from 62 to 97 square feet per acre, the stand density after the first thinning ranged from 310 to 1,174 trees per acre, and site index ranged from 62 to 95 feet. The stand characteristics are displayed in Table 4.

Age at First Thinning	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
21	3	65	75	1,004	1,044	67	70
22	2	62	68	1,026	1,174	64	65
23	3	73	77	728	802	86	93
24	2	88	95	618	828	80	80
31	2	70	71	366	404	67	75
33	2	80	97	310	490	82	95
34	1	87	87	464	464	91	91
42	1	86	86	486	486	62	62

Table 4. Stand characteristics of the Hatch thinned plots immediately after the first thinning.

Table 4 indicates that the Hatch thinned plots encompassed young to mid-aged stands. It also indicates that low thinning intensity was applied since there is a little difference in change of basal area per acre and trees per acre between unthinned (Table 3) and thinned stands.

2.2.3 Allegheny National Forest Study

This dataset was provided by USDA Forest Service based on a study to examine the effects of stand structure after thinning on the growth and timber volume increment in mixed hardwoods in the Allegheny National Forest. Experimental units (plots) were randomly installed in 1973-1976. Measurement plots were established in a pole-size, even-aged cherry-maple stands on the Allegheny National Forest, located in north-western Pennsylvania. At the time of thinning in 1975 and 1976, the stands were 53-54 years old and fully stocked (Marquis and Ernst 1991). The data presented here was collected from 11 blocks. Each block contained a different number of plots and the plots within each block received different treatments. The total number of plots was 60. Measurements were taken between 1972 and 2003 as the blocks had different time periods and frequency of re-measurements. Plots were 2 acres in size and rectangular in shape. The central 0.6-acre portion of each plot represented the actual measurement area. Diameter at breast height was measured and the number of trees was recorded before and after each treatment.

Unthinned plots

There were fourteen unthinned plots in this study. The initial age was between 50 and 63 years. Basal area per acre at initial age ranged from 102 to 181 square feet per acre and trees per acre at initial age ranged between 304 and 1,051. The stand characteristics are displayed in Table 5.

Initial Age	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
50	2	102	125	710	993	-	-
52	2	128	132	944	1,051	-	-
53	2	115	129	754	886	-	-
57	1	181	181	304	304	-	-
62	5	109	167	408	654	-	-
63	2	138	143	531	559	-	-

Table 5. Stand characteristics of the Allegheny National Forest unthinned plots at initial age

Table 5 indicates that the Allegheny National Forest unthinned plots were established in mid-aged stands with similar stand characteristics.

Thinned plots

There were forty-six thinned plots that have received different types of thinning. Age at first thinning ranged between 50 and 63 years. Basal area immediately after the first thinning ranged between 45 and 118 square feet per acre and the number of stems per acre after thinning ranged between 91 and 742. Stand characteristics are displayed in Table 6.

Age at First Thinning	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
50	9	45	110	91	742	-	-
52	8	59	113	194	646	-	-
53	9	59	99	194	667	-	-
57	3	101	118	193	305	-	-
62	10	55	112	143	546	-	-
63	7	52	113	144	420	-	-

Table 6. Stand characteristics of the Allegheny National Forest thinned plots immediately after the first thinning.

Table 6 indicates that the age at first thinning for the Allegheny National Forest thinned plots is identical to the initial age for the unthinned plots (Table 5). It also reveals that moderate to heavy thinning were applied within the thinned plots since there is some noticeable reduction of basal area per acre and trees per acre within the thinned plots.

2.2.4 OAKSIM Stand Density Study

Eighty-one plots of size 0.25, 0.3, 0.5, and 1 acre were established in even-aged stands by US Forest Service across southern Ohio (Vinton and Scioto counties) and southeastern Kentucky (Jackson and Laurel counties), receiving different thinning treatments. Measurements were taken between 1959 and 2006 and the initial measurement year for each location ranged between 1959 and 1977.

Unthinned plots

There were thirty-three unthinned plots in this study. The initial stand age ranged from 22 to 110 years. Initial basal area was between 15 and 116 square feet per acre, initial trees per acre were between 74 and 798, and site index ranged from 60 to 75 feet (based on black oak). The stand characteristics are summarized in Table 7.

Initial Age	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
22	1	70	70	760	760	60	60
31	1	36	36	222	222	71	71
32	4	31	82	268	798	66	73
33	3	15	58	94	734	62	73
34	9	25	66	220	766	66	70
50	1	57	57	157	157	75	75
51	3	56	59	137	153	68	74
52	2	57	58	150	150	71	74
62	3	55	65	132	160	60	64
64	1	48	48	76	76	71	71
109	4	99	116	74	103	65	71
110	1	107	107	135	135	67	67

Table 7. Stand characteristics of the OAKSIM unthinned plots at initial age.

Table 7 indicates that the OAKSIM study unthinned plots covered a variety of stands with highly diverse characteristics. There is an obvious presence of many different initial ages, especially the old stands (109-110 years old at initial age) that were not presented in the previous studies.

Thinned plots

There were forty-eight thinned plots in this study. The stand age at first thinning was between 42 and 93 years. Basal area per acre immediately after the first thinning ranged from 21 to 103 square feet per acre, trees per acre after the first thinning ranged from 38 to 464, and site index ranged from 60 to 77 feet. The stands characteristics are summarized in Table 8.

Age at First Tinning	Number of Plots	Basal Area Per Acre [ft^2/ac]		Trees Per Acre		Site Index	
		Min.	Max.	Min.	Max.	Min.	Max.
42	1	78	78	464	464	71	71
43	6	23	76	68	406	67	77
44	7	27	73	88	308	63	76
48	1	78	78	192	192	72	72
49	1	70	70	210	210	62	62
55	1	77	77	140	140	74	74
57	1	73	73	146	146	71	71
58	2	42	67	50	110	70	73
61	1	54	54	124	124	67	67
62	1	103	103	194	194	74	74
70	1	58	58	104	104	63	63
71	1	79	79	158	158	67	67
72	2	21	72	47	165	60	64
74	2	48	59	68	141	62	69
75	1	22	22	38	38	69	69
77	2	63	66	68	172	60	64
78	1	66	66	68	68	60	60
79	2	23	71	40	94	65	65
80	1	41	41	103	103	60	60
81	3	27	64	46	158	62	65
82	1	77	77	156	156	68	68
83	2	74	82	76	177	64	67
84	1	43	43	94	94	61	61
90	4	68	100	50	116	60	66
91	1	56	56	38	38	65	65
93	1	44	44	100	100	63	63

Table 8. Stand characteristics of the OAKSIM thinned plots immediately after the first thinning.

Table 8 indicates that the OAKSIM stand density study thinned plots covered stands with diverse characteristics. The first thinning was applied in mid-aged to old stands and encompassed many different age groups.

2.3 Methods

All plot measurements from all different locations were combined and organized in a single hardwood data file. The file contains two categorical variables that account for the different treatments (unthinned and thinned plots) and the different studies (locations). The effects of the categorical variables on basal area growth were tested for significant difference with general analysis of variance procedure using SAS 9.1. Some of the existing basal area projection models for pine plantations were selected for further analysis with the hardwood data, according to the variables they include and the available data. The selected equations were fitted to the hardwood data using SAS 9.1 Proc NLIN (SAS Institute, Inc. 2002-2003). Evaluation of the models goodness of fit was a three phase process based on residual analysis, models root mean square error (RMSE), and average bias (BIAS) (Brooks *et al*. 2008). This process encompassed all necessary procedures for comprehensive and detailed examination in order to select the best projection equation for the hardwood data. The phases are ordered as follows:

Phase 1. Examination of the analysis of variance (ANOVA) table and coefficient estimates for the selected basal area projection models.

The first step of model evaluation is to examine the results from the analysis of variance table in terms of the mean square error and the estimates for the coefficients, after fitting the selected models to the hardwood data using non-linear procedure. The models with lower mean square error and more significant coefficients are preferable.

Phase 2. Diagnostics of the selected basal area projection models.

The estimation and inference from the models depend on some important assumptions. These assumptions need to be checked using regression diagnostics. Diagnostics are applied for assessing residual distribution and checking the assumptions that the errors are independent, normally distributed, and have homogeneous variance. They are also used to find unusual observations, or outliers.

Phase 3. Calculating average bias and root mean square error for the selected basal area projection models.

Bias concerns the center of error distribution. An error distribution is unbiased when the mean equals zero, or $E(\mu) = (0, \sigma^2)$. It can also be positively biased ($\mu>0$) or negatively biased ($\mu<0$). Root mean square error (RMSE) is a measure of variability of the sampled data, and it tends to be small in large sample size and low variable data. The general forms for calculating average bias and root mean square error for this study are defined as in Brooks *et al*. (2008):

$$BIAS = \frac{\sum_{i=1}^{n}\left(B_2^{*} - B_2\right)}{n} \qquad [11]$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(B_2^{*} - B_2\right)^2}{n}} \qquad [12]$$

where:

B_2^{*} = estimated basal area at projection age,

B_2 = observed basal area at projection age,

MSE = mean square error from ANOVA table,

n = number of observations.

After the analysis of each selected model, a final comparison among all equations was performed for selecting the best basal area projection model for the hardwood re-measured data. The newly selected model was then tested for indifference in the outcome of its characteristics when fit with partial and full dataset.

The two partial datasets were created by randomly excluding twenty percent (validation data) of each study and treatment from the complete dataset, since significant difference was detected among the two categorical variables. The two forms of the best basal area projection model for hardwood unthinned and thinned stands were then fit using the remaining eighty percent of the data, separately by treatment and study. Then the two forms of the best model were fit again with the twenty percent validation dataset, by treatment and study, for testing purposes. Finally, the two forms of the best model were fit with the complete dataset, separately by treatment and study. For a final evaluation of the best basal area projection equation for hardwood stands overall performance, the analysis of variance outcomes from the partial and complete datasets were compared.

CHAPTER 3: RESULTS AND ANALYSIS

3.1 Analysis of Variance for the Categorical Variables

The result for the treatment categorical variable indicated that there is a significant difference between unthinned and thinned plots in terms of basal area growth. This suggests that two different coefficient estimates are needed to account for the two treatments for each proposed basal area projection model. Each equation in this study was then examined separately by treatment. The analysis of variance also indicated a significant difference among the different studies which impose that separate coefficients are also needed for each study location. The results from the analysis of variance for the categorical variables are summarized in Tables A-1, A-2, and A-3 (Appendix A).

3.2 Selection of the Proposed Basal Area Projection Equations for Further Analysis

Two of the existing basal area projection models in chapter two (Souter 1986 and McTague and Bailey 1987) were ignored due to lack of essential data for the variables they include. The rest of the proposed models were selected for further analysis. Some of the selected models were modified under the conditions of this study. Table 9 summarizes the process of selection the proposed basal area projection equations.

Equation	Status	Reason
Clutter and Jones (1980) [1]	Selected	All necessary variables available
Burkhart and Sprinz (1984) [2]	Selected	All necessary variables available
Bailey and Ware (1983) [3]	Selected and modified	No data for the X_t variable
Souter (1986) [4]	Ignored	No data for D_r, D_b, SDI_b variables
McTague and Bailey (1987) [5]	Ignored	No data for D10 and D63 variables
Pienaar *et. al.* (1985) [6]	Selected	All necessary variables available
Pienaar *et al*. (1989) [7]	Selected and modified	Not enough data for H_1 and H_2 variables
Pienaar *et al*. (1990)[8]	Selected and modified	Not enough data for H_1 and H_2 variables
Brooks (1992) form 1 [9]	Selected and modified	Not enough data for H_1 and H_2 variables and no need of thinning type modifier
Brooks (1992) form 2 [10]	Selected and modified	Not enough data for H_1 and H_2 variables, no need of the Z variable and the thinning type modifier

Table 9. Selection process of the proposed basal area projection models for further analysis.

3.3 Analysis of the Selected Whole Stand Basal Area Projection Equations

The selected models are ordered from the simplest, which contain the least number of explanatory variables, to the most complex, which contain the most number of explanatory variables. The thinning modifiers can be categorized in two general forms. The first form involves ratios that attempt to capture the change in number of trees per acre or basal area per acre following thinning and is a measure thinning intensity. The second form involves the concept of capturing the change in stand quadratic mean diameter and is a measure of thinning type, but was ignored, as previously noted in the selection process, due to the lack of essential data and the fact that only selective thinning was applied in all thinned plots.

The selected models were fit to the hardwood data separately for unthinned and thinned stands throughout all different studies since a significant difference was found among the treatments. Even though a significant difference was also detected among the different studies, the selected models were not fitted by study. The reason for this is that in this part of the analysis the main interest is to assess how much variation is explained by the selected models when fit to unthinned and thinned hardwood data separately, since many of them have an explicit thinning modifier. Also the main goal of this project is to select only one basal area projection equation that can be applied in any unthinned or thinned central hardwood stand. The effect of study (location) will then be applied on the best whole stand basal area projection model.

3.3.1 Clutter and Jones (1980) Whole Stand Basal Area Projection Equation

This model was selected for assessment as the simplest equation, involving only three explanatory variables. It was not modified since all variables involved were measured or recorded for all plots. The model form is exhibited as Equation [1].

<u>Phase 1.</u> Examination of ANOVA table and coefficient estimates

Clutter and Jones (1980) basal area projection model was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned and thinned plots throughout all different studies. The output is displayed in Tables 10a and 10b.

Source	DF	Sum of squares	Mean square	*F*-value	*p*-value
Model	2	9,501,622	4,750,811	183,229	<.0001
Error	1,214	31,476.9	25.9282		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard error	95% Confidence Limits	
α_1	1.0866	0.0958	0.8987	1.2744
α_2	5.2977	0.0625	5.1752	5.4203

Table 10a. Analysis of variance and coefficient estimates for Clutter and Jones (1980) whole stand basal area projection equation for unthinned stands.

Source	DF	Sum of squares	Mean square	*F*-value	*p*-value
Model	2	7,569,893	3,784,946	186,356	<.0001
Error	1,117	22,686.7	20.3104		
Uncorrected Total	1,119	7,592,579			

Coefficient	Estimate	Standard error	95% Confidence Limits	
α_1	0.5928	0.0734	0.4488	0.7367
α_2	6.2766	0.2170	5.8509	6.7022

Table 10b. Analysis of variance and coefficient estimates for Clutter and Jones (1980) whole stand basal area projection equation for thinned stands.

The output indicates a very large *F*-value and, respectively, small *p*-value for all plots, which indicates that Clutter and Jones (1980) basal area projection model is statistically significant. This is due to the small value of mean square error as a result of the large sample size. Noticeably, the mean square error for thinned plots is smaller than that for unthinned plots. All coefficients are significant (do not contain zero in their 95% confidence intervals) and stable, having small standard errors and narrow confidence limits. All of the analysis indicates that this model explains a large amount of variation of basal area growth for central hardwood unthinned and thinned stands.

<u>Phase 2.</u> Diagnostics

The residuals here are defined as:

$$r = (B_2^{*} - B_2) \quad [13]$$

where:

r = residuals,

B_2^{*} = projected basal area, estimated from the Clutter and Jones (1980) equation,

B_2 = projected basal area, observed from the inventory data.

Graphical methods were used to assess error distributions. The distribution of the residuals for unthinned and thinned plots is displayed in Figures 3a and 3b.

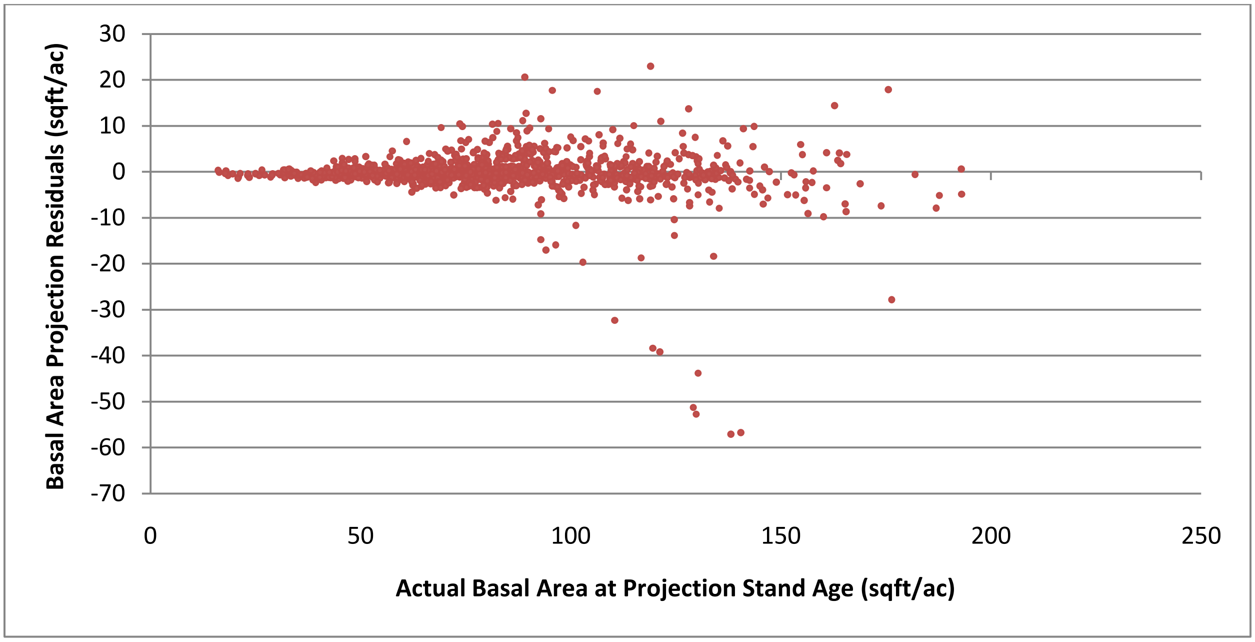

Figure 3a. Basal area residual distribution for Clutter and Jones (1980) whole stand basal area projection equation for unthinned plots.

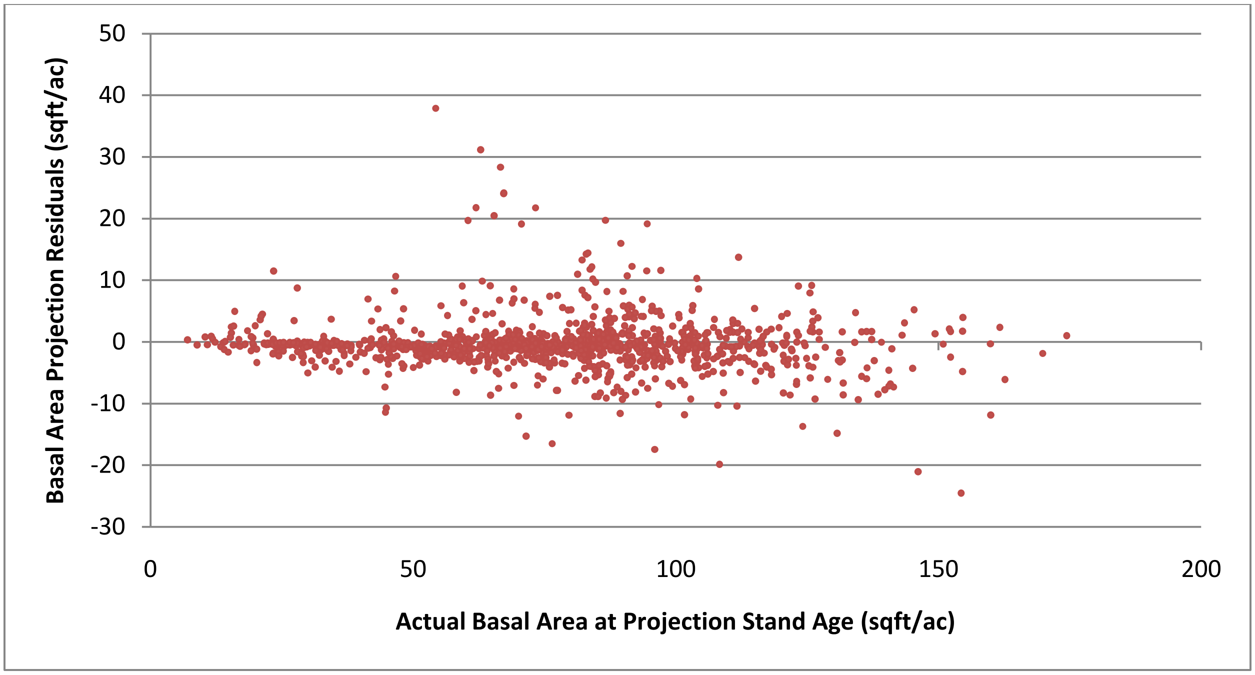

Figure 3b. Basal area residual distribution for Clutter and Jones (1980) whole stand basal area projection equation for thinned plots.

The residual error distribution for Clutter and Jones (1980) basal area projection equation reveals some important points. The residuals appear to be equally distributed among the neutral line without any signs of non-constant variance or unusual patterns for both treatments. This strengthens the assumption of independent and normally distributed errors. Most of them fall within ±10 square feet per acre. The variance for unthinned plots slightly increases as the actual basal area at projection stand age increases, with very low variability for plots containing less than fifty square feet per acre of observed basal area at projection stand age. The variance appears to be more stable for thinned plots. Some noticeable error values are obvious for both residual distributions. The unthinned stands distribution contains some extreme negative residuals, while the thinned stands distribution contains less extreme, positive errors. After a closer examination of the data, these extreme error values are due to some unusual sharp changes in basal area growth for some measurement plots, but since they represent actual stands, they were not considered as outliers and were not removed.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for Clutter and Jones (1980) basal area projection model are displayed in Table 10.

Treatment	Bias	RMSE
Unthinned plots	-0.2859	5.0920
Thinned plots	-0.4227	4.5067

Table 11. Average bias and root mean square error values for Clutter and Jones (1980) basal area projection equation.

This model has small overall negative bias which indicates a small underestimation of the actual basal area at projection age. The table indicates that the bias for thinned plots is larger than the bias for the unthinned plots, while the root mean square error value for the thinned plots is smaller than that for unthinned plots.

3.3.2 Burkhart and Sprinz (1984) Whole Stand Basal Area Projection Equation

This model was selected since it includes an additional variable of site index (SI) that accounts for site characteristics. The full form of this model was fit and assessed, since site index was recorded for all studies, except for the Allegheny National Forest dataset. The model form is exhibited as Equation [2].

Phase 1. Examination of ANOVA table and coefficient estimates

Burkhart and Sprinz (1984) basal area projection model was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned and thinned plots throughout all different studies. The output is displayed in Tables 12a, 12b, and 12c.

Source	DF	Sum of Squares	Mean Square	*F*-value	*p*-value
Model	2	7,797,296	3,898,648	154,305	<.0001
Error	1,127	28,474.7	25.2659		
Uncorrected Total	1,129	7,825,771			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	5.5121	0.2026	5.1146	5.9096
α_2	-0.00167	0.00271	-0.00698	0.00364

Table 12a. Analysis of variance and coefficient estimates for Burkhart and Sprinz (1984) whole stand basal area projection equation for unthinned stands.

Source	DF	Sum of Squares	Mean Square	*F* -value	*p*-value
Model	1	9,501,602	9,501,602	366,526	<.0001
Error	1,215	31,496.9	25.9234		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	5.3546	0.0263	5.3030	5.4061

Table 12b. Analysis of variance and coefficient estimates for Burkhart and Sprinz (1984) whole stand basal area projection equation for unthinned stands after removing the non-significant coefficient.

Source	DF	Sum of Squares	Mean Square	*F* -value	*p*-value
Model	2	4,214,734	2,107,367	105,960	<.0001
Error	789	15,691.9	19.8883		
Uncorrected Total	791	4,230,426			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	4.6540	0.1654	4.3293	4.9787
α_2	0.0112	0.00211	0.00711	0.0154

Table 12c. Analysis of variance and coefficient estimates for Burkhart and Sprinz (1984) whole stand basal area projection equation for thinned stands.

The ANOVA table indicates a large *F*-value and small *p*-value, rejecting the null hypothesis and indicating the significance of Burkhart and Sprinz (1984) basal area projection model for both treatments. The mean square error is very similar to the Clutter and Jones (1980) projection equation. Some degrees of freedom have been lost due to the lack of site index values for Allegheny National Forest dataset. Even though, the model has a smaller mean square error for both treatments than the Clutter and Jones (1980) projection equation. Table 12a indicates that the coefficient estimate for site index (α_2) for unthinned stands is non-significant, since it contains zero in its confidence limits. This indicates that adding site index variable for unthinned stand did not explain a significant amount of variation in basal area growth and was removed from this form. However, Table 12c reveals that α_2 is significant, indicating that site index explains a substantial amount of variation in basal area growth for thinned stands. The other coefficient estimate (α_1) remained significant for both treatments with small standard errors and relatively narrow 95% confidence intervals.

Phase 2. Diagnostics

All residuals are as previously defined. The distribution of the residuals for unthinned and thinned plots is displayed in Figures 4a and 4b.

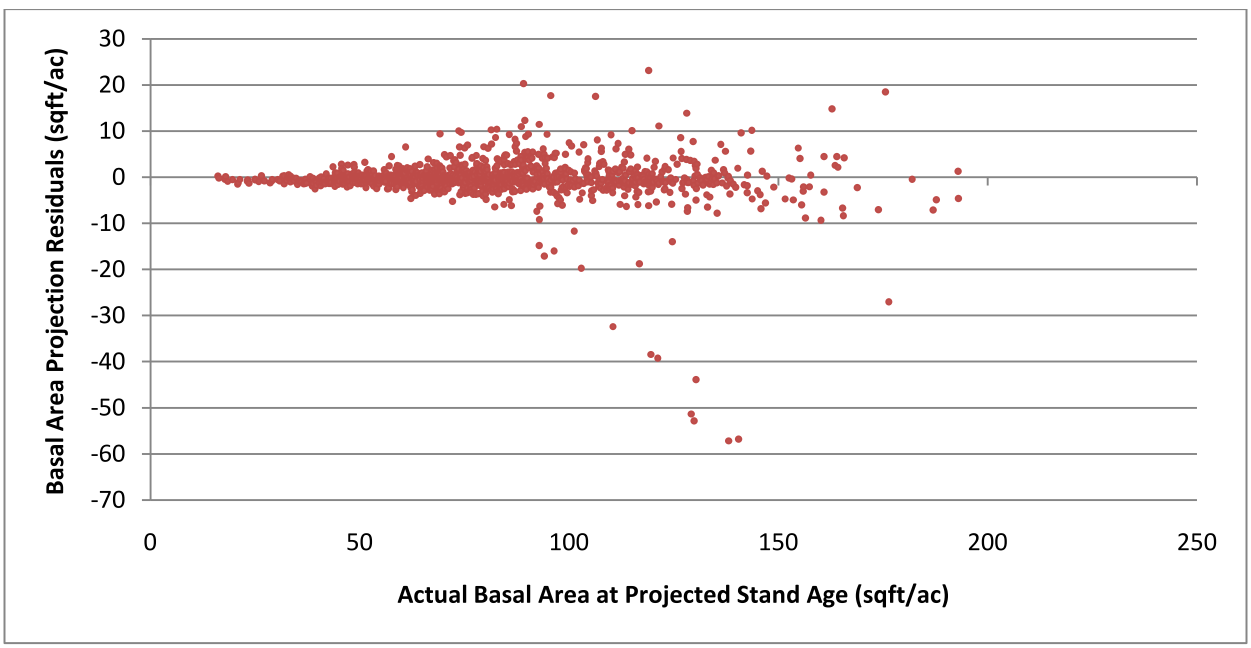

Figure 4a. Basal area residual distribution for Burkhart and Sprinz (1984) whole stand basal area projection equation for unthinned plots.

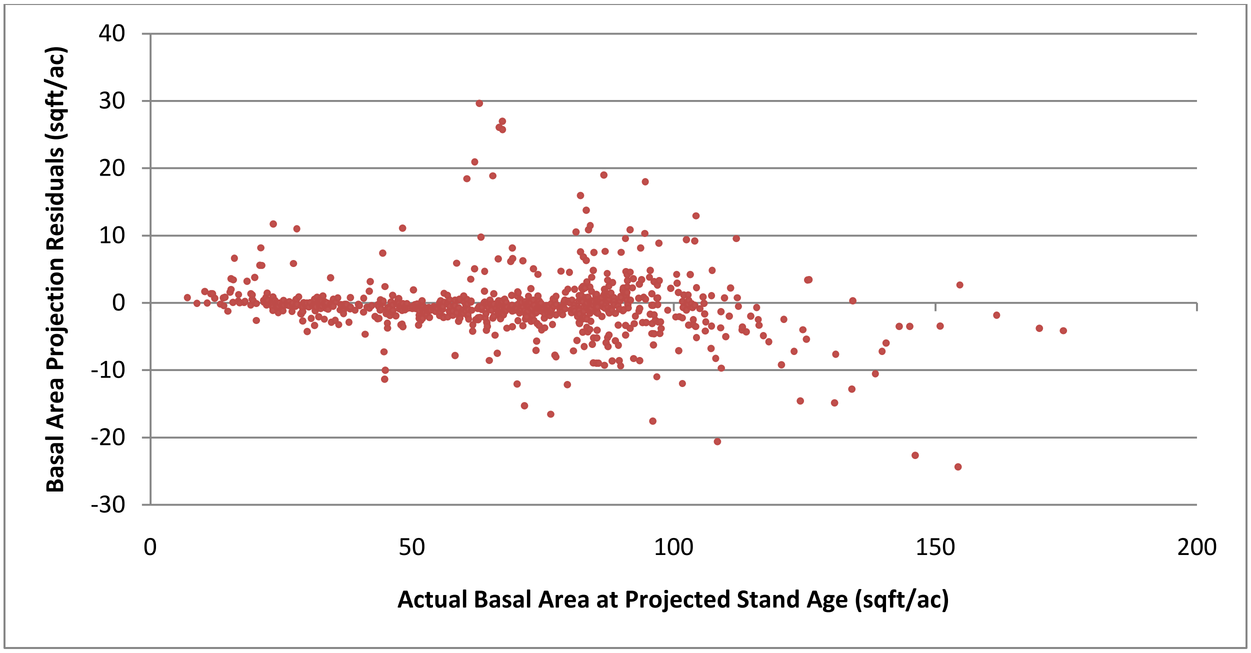

Figure 4b. Basal area residual distribution for Burkhart and Sprinz (1984) whole stand basal area projection equation for thinned plots.

The distribution of the residuals for Burkhart and Sprinz (1984) basal area projection model has similar shape to the Clutter and Jones (1980) basal area projection equation residual distribution for unthinned and thinned stands. There is no sign of unusual pattern of the residuals. The variance is relatively constant for both treatments, with little increase as the actual basal area at projection age increases for the unthinned plots. The residuals are equally distributed among the neutral line for both treatments. The main density of the residuals falls within ± 10 square feet per acre. There are some extreme error values, especially for the unthinned plots, due to some extreme changes in basal area growth, but they were not considered as outliers and were kept as they represent actual stands. Also, there is a noticeable low variance for the unthinned plots containing less than 50 square feet per acre of actual basal area at projection stand age, which indicates an excellent projection power for untreated stands containing less basal area per acre. The graphs support the assumption of uncorrelated and normally distributed residual errors.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for Burkhart and Sprinz (1984) basal area projection model are displayed in Table 13.

Treatment	Bias	RMSE
Unthinned plots	-0.3257	5.0915
Thinned plots	-0.4683	4.4596

Table 13. Average bias and root mean square error values for Burkhart and Sprinz (1984) basal area projection equation.

The results indicate smaller bias for Burkhart and Sprinz (1984) basal area projection model when fited to the data for unthinned stands and smaller root mean square error when fited to the data for thinned stands. The negative sign of the bias indicates a slight underestimation of the actual basal area growth. Table 13 also reveals that the average bias for the unthinned plots after removing the non-significant coefficient representing site index (α_2) is larger than the bias from Clutter and Jones (1980) basal area projection model for unthinned stands (-0.2859 ft^2/ac), indicating a reduction of projection precision for Burkhart and Sprinz (1984) model for these stands.

3.3.3 Clutter (1963) Whole Stand Basal Area Projection Equation

Bailey and Ware (1983) basal area projection equation was also selected for further assessment. It also contains site index (SI) explanatory variable as the Burkhart and Sprinz (1984) equation. Due to the lack of diameter measurements for the removed trees after thinning for all datasets, the X_t variable in Equation [3] cannot be utilized and this model was reduced to the Clutter (1963) form:

$$B_2 = B_1^{\left(\frac{A_1}{A_2}\right)} Exp\left[\alpha_1\left(1-\frac{A_1}{A_2}\right)+\alpha_3 SI\left(1-\frac{A_1}{A_2}\right)\right]$$

[14]

<u>Phase 1.</u> Examination of ANOVA table and coefficient estimates

Equation [14] was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned and thinned plots throughout all different studies. The results are summarized in Tables 14a, 14b, and 14c.

Source	DF	Sum of Squares	Mean Square	F-value	p-value
Model	2	7,797,296	3,898,648	154,305	<.0001
Error	1,127	28,474.7	25.2659		
Uncorrected Total	1,129	7,825,771			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	5.5121	0.2026	5.1146	5.9096
α_3	-0.00167	0.00271	-0.00698	0.00364

Table 14a. Analysis of variance and coefficient estimates for Clutter (1963) whole stand basal area projection equation (Equation [14]) for unthinned plots.

Source	DF	Sum of Squares	Mean Square	F-value	p-value
Model	1	9,501,602	9,501,602	366,526	<.0001
Error	1,215	31,496.9	25.9234		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	5.3546	0.0263	5.3030	5.4061

Table 14b. Analysis of variance and coefficient estimates for Clutter (1963) whole stand basal area projection equation (Equation [14]) for unthinned plots after removing the non-significant coefficient.

Source	DF	Sum of Squares	Mean Square	F-value	p-value
Model	2	4,214,734	2,107,367	105,960	<.0001
Error	789	15,691.9	19.8883		
Uncorrected Total	791	4,230,426			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	4.6540	0.1654	4.3294	4.9787
α_3	0.0112	0.00211	0.00711	0.0154

Table 14c. Analysis of variance and coefficient estimates for Clutter (1963) whole stand basal area projection equation (Equation [14]) for thinned plots.

The results from the analysis of variance table for Clutter (1963) basal area projection equation are exactly the same as the once for the Burkhart and Sprinz (1984) basal area projection model for both treatments. The site index coefficient (α_2) is again non-significant for unthinned plots and was removed and the model was refit. This suggests a similar outcome of basal area growth estimation for both projection equations. Even though, this model has different structure than the Burkhart and Sprinz (1984) projection model and further analysis was performed.

<u>Phase 2.</u> Diagnostics

All residuals are as previously defined. Graphical methods were used for error assessment. The distribution of the residuals for unthinned and thinned plots is displayed in Figures 5a and 5b.

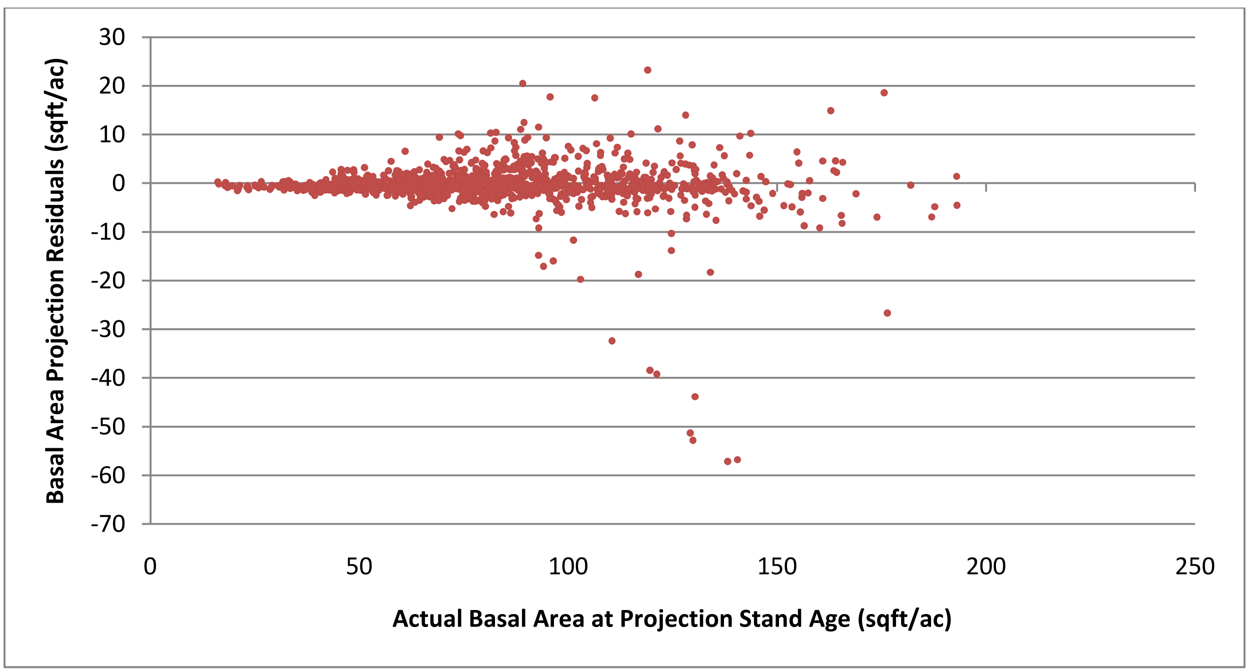

Figure 5a. Basal area residual distribution for Clutter (1963) whole stand basal area projection equation (Equation [14]) for unthinned plots.

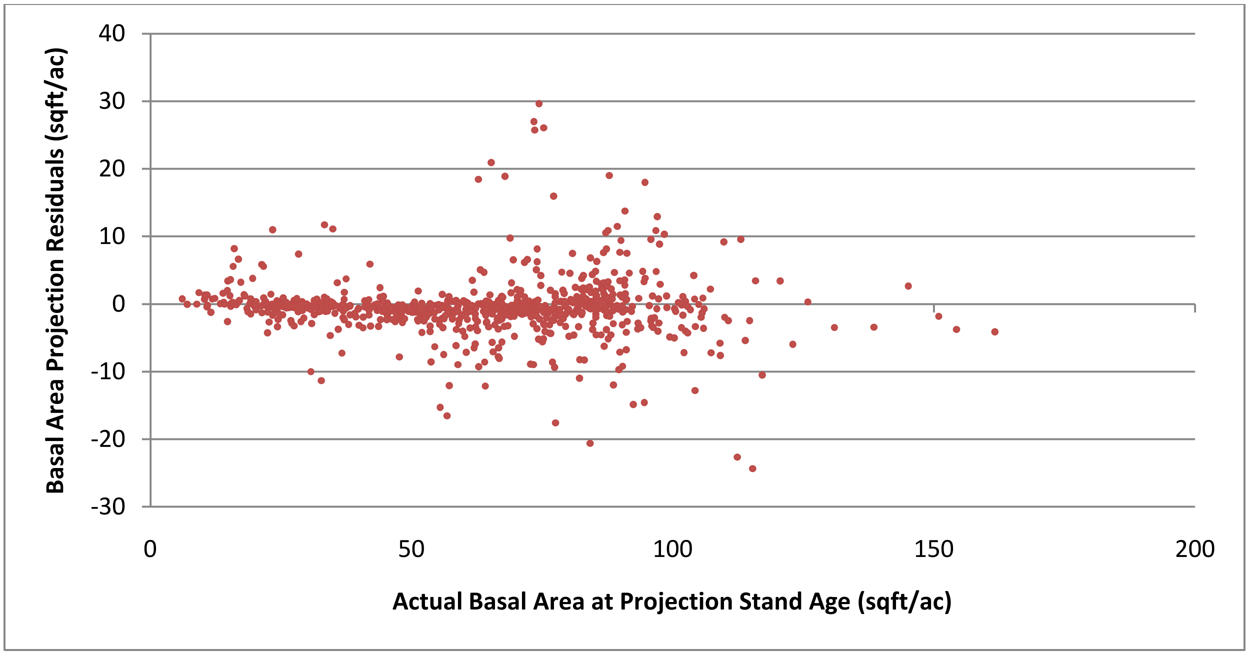

Figure 5b. Basal area residual distribution for Clutter (1963) whole stand basal area projection equation (Equation [14]) for thinned plots.

The distribution of the residuals for Clutter (1963) basal area projection model has very similar shape to the Burkhart and Sprinz (1984) basal area projection equation residual distribution for unthinned and thinned stands. The graphs reveal no sign of any unusual patterns. The variance is relatively stable, with little increase as the actual basal area at projection age increases for the unthinned plots. The residuals are equally distributed among the neutral line for both treatments. The main density of the residuals falls within ± 10 square feet per acre. There are again some noticeable errors, especially for the unthinned stand, having some extreme values. The reason for such extremes is due to some sharp changes in basal area growth but they were not considered as outliers, since they represent actual plots, so they were not ignored. Also, there is a noticeable low variance for the unthinned plots containing less than 50 square feet per acre of basal area at projection stand age, which indicates lower errors and better projection precision for untreated stands containing less basal area per acre at projection stand age. The graphs support the assumption of uncorrelated and normally distributed residual errors.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for Clutter (1963) basal area projection model are displayed in Table 15.

Treatment	Bias	RMSE
Unthinned plots	-0.3257	5.0915
Thinned plots	-0.4683	4.4596

Table 15. Average bias and root mean square error values for Clutter (1963) basal area projection equation (Equation [14]).

The results indicate a smaller bias for Clutter (1963) model when fitted to the data for unthinned stands and smaller root mean square error when fitted to the data for thinned stands. The negative sign of the bias for both treatments indicates a slight underestimation of the actual basal area growth. They are exactly the same as the previously selected model by Burkhart and Sprinz (1984), indicating that even with different structure, Clutter (1963) projection model behaves exactly the same way.

3.3.4 Pienaar *et al.* (1985) Whole Stand Basal Area Projection Equation

Pienaar *et al*. (1985) basal area projection model was also selected for this study. It contains an explicit thinning modifier (X) and is of greater interest because of the two forms for unthinned and thinned stands. All necessary variables were available in the dataset. The model form is exhibited as Equation [6].

Phase 1. Examination of ANOVA table and coefficient estimates

Pienaar *et al.* (1985) basal area projection model was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned and thinned plots throughout all different studies. The results are summarized in Tables 16a, 16b, and 16c.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	2	9,427,324	4,713,662	164,776	<.0001
Error	1,214	345,56.6	28.6065		
Uncorrected Total	1,216	9,461,881			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_0	0.0284	0.7897	-1.5211	1.5778
α_2	25.1159	669.1	-1287.7	1338.0

Table 16a. Analysis of variance and coefficient estimates for Pienaar *et al*. (1985) whole stand basal area projection equation for unthinned plots.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	2	8,347,949	4,173,974	147,696	<.0001
Error	1,109	31,341.0	28.2606		
Uncorrected Total	1,111	8,379,290			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_0	2.9877	0.4558	2.0933	3.8820
α_2	1.3569	0.0228	1.3122	1.4016

Table 16b. Analysis of variance and coefficient estimates for Pienaar *et al*. (1985) whole stand basal area projection equation for Allegheny National Forest and OAKSIM unthinned plots.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	3	7,570,515	2,523,505	127,638	<.0001
Error	1,116	22,064.3	19.7709		
Uncorrected Total	1,119	7,592,579			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_0	0.7618	0.0527	0.6583	0.8653
α_1	0.3047	0.0448	0.2167	0.3927
α_2	2.2293	0.0664	2.0990	2.3596

Table 16c. Analysis of variance and coefficient estimates for Pienaar *et al*. (1985) whole stand basal area projection equation for thinned plots.

The results from ANOVA table indicate again that Pienaar *et al*. (1985) whole stand basal area projection model is statistically significant for both forms (*p*-value <0.0001). However, the coefficient estimate table for the unthinned stands (Table 16a) indicates that none of the estimates are significant since all of them contain zero in their confidence intervals. The reason for this was that the model was fit with the unthinned data across all different studies and failed to converge using Proc NLIN. When fitted by study, stable coefficient estimates were obtained from Allegheny National Forest and OAKSIM datasets. The rest of the studies failed to converge. This model was then refit until convergence was met. The output is summarized in Table 16b. The analysis for unthinned plots will continue using only Allegheny National Forest and OAKSIM datasets combined. The form for thinned stand converged for all studies.

After the convergence criteria were met, all coefficient estimates are significant for both treatments, contain relatively narrow 95% confidence limits. The mean square error is higher for unthinned stands but smaller for the thinned stands than all previously proposed models. This indicates that this thinning modifier is able to improve the projection precision for the thinned plots.

Phase 2. Diagnostics

All residuals are as previously defined. Graphical methods were used for error assessment. The distribution of the residuals for unthinned and thinned plots is displayed in Figures 6a and 6b.

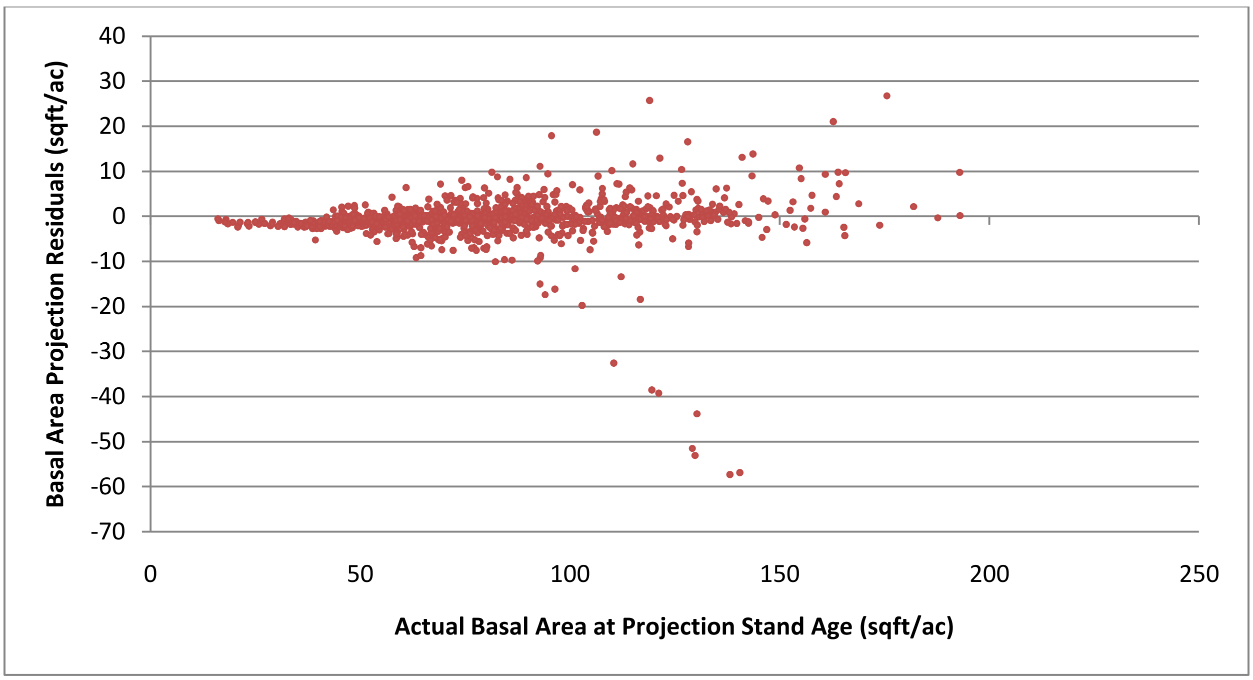

Figure 6a. Basal area residual distribution for Pienaar *et al*. (1985) whole stand basal area projection equation for unthinned plots.

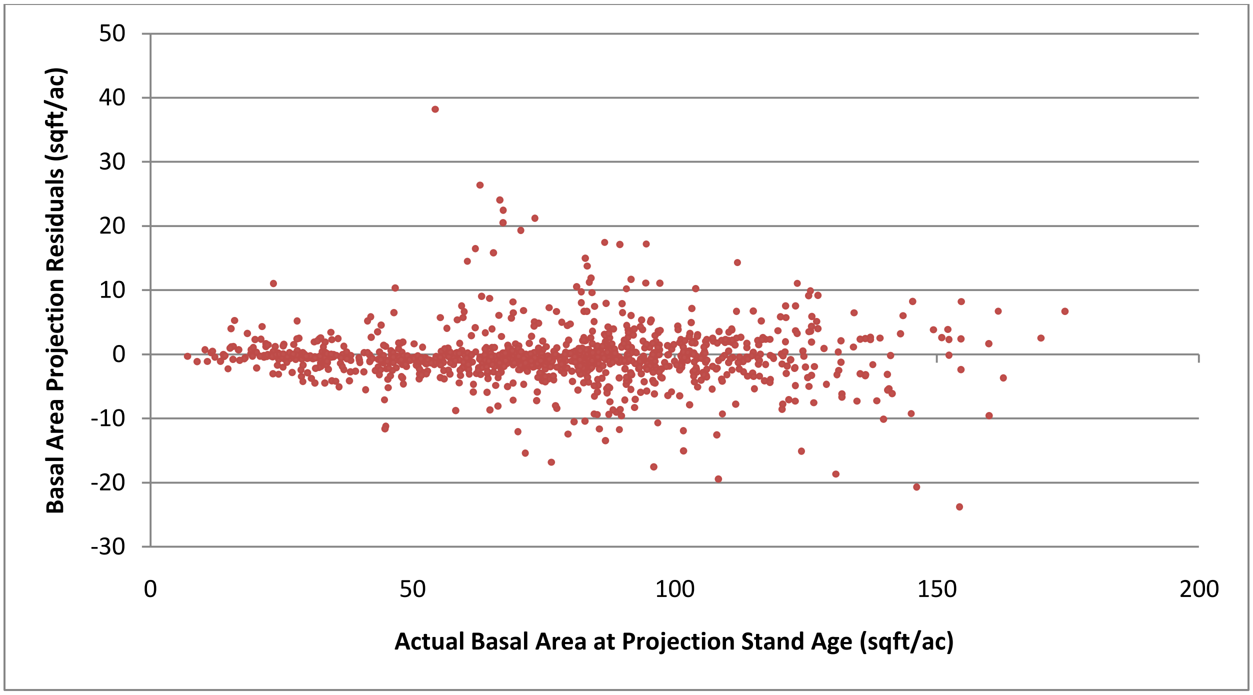

Figure 6b. Basal area residual distribution for Pienaar *et al*. (1985) whole stand basal area projection equation for thinned plots.

The residual distribution for the Pienaar *et al*. (1985) basal area projection equation shows no indications of unusual patterns or non-constant variance. The main density of the residuals again falls within ± 10 square feet per acre, as the previously selected models. The graph for unthinned stands indicates a slight increase in variance as the basal area at projection age increases. There are some extreme negative residuals for stands with higher basal area per acre at projection age due to extreme changes in basal area growth in some plots. Since they represent actual stands, they were not considered as true outliers and were not ignored. There is an obvious sign of a slight non-normality of the errors for the unthinned stands, containing less than fifty square feet per acre of basal area at projected age, since most of the residuals are below the neutral line.

The residual distribution for the thinned stands reveals more stable variance with less extreme residuals, which were kept as they were not considered as outliers. The residuals are better distributed among the neutral line even for the plots containing less basal area per acre at projected age compared to the graph for the unthinned stands.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for Pienaar *et al*. (1985) basal area projection model are displayed in Table 17.

Treatment	Bias	RMSE
Unthinned plots	-0.6761	5.3161
Thinned plots	-0.3472	4.4464

Table 17. Average bias and root mean square error values for Pienaar *et al*. (1985) basal area projection equation.

The summarized results in Table 17 reveal a smaller average bias and root mean square error for Pienaar *et al*. (1985) model form for thinned stands, indicating better performance. Also the form for thinned stands contains a smaller average bias than all previously selected forms, while the form for unthinned stands has higher average bias than the previous models. The negative sign of the bias for both forms indicates a slight underestimation of the actual basal area growth.

3.3.5 Pienaar *et al.* (1989) Whole Stand Basal Area Projection Equation

The first Pienaar *et al*. (1989) basal area projection form, based on the Pienaar and Shiver (1986) general form, was also selected for this study. This model form contains number of trees per acre as additional variable, which was recorded for all plots. The thinning modifier is more complex than the Pienaar *et al*. (1985) basal area projection model thinning variable, including age of thinning, that was also recorded for all thinned stands. Due to the lack of tree height measurements in most of the locations, average dominant height variable was excluded and Equation [7] was modified to Equation [15]:

$$B_2 = Exp\left[\begin{array}{l} Ln(B_1) + \alpha_1\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \alpha_2\left(Ln(N_2) - Ln(N_1)\right) + \\ + \alpha_4\left(\frac{Ln(N_2)}{A_2} - \frac{Ln(N_1)}{A_1}\right) + \alpha_6\left(\frac{NtAt}{NbA_2} - \frac{NtAt}{NbA_1}\right) \end{array}\right] \quad [15]$$

Phase 1. Examination of the ANOVA table and parameter estimates

This modified form of Pienaar *et al*. (1989) basal area projection model (Equation [15]) was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned and thinned plots throughout all different studies. The output is displayed in Tables 18a, 18b, and 18c.

Source	DF	Sum of Squares	Mean Square	*F*-value	*p*-value
Model	3	9,498,733	3,166,244	111,757	<.0001
Error	1,213	34,366.2	28.3316		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	-55.8107	4.2270	-64.1038	-47.5176
α_2	-0.0343	0.0235	-0.0804	0.0118
α_4	4.4543	0.5964	3.2842	5.6244

Table 18a. Analysis of variance and coefficient estimates for the modified Pienaar *et al*. (1989) whole stand basal area projection equation (Equation [15]) for unthinned plots.

Source	DF	Sum of Squares	Mean Square	*F*-value	*p*-value
Model	2	9,498,671	4,749,336	167,471	<.0001
Error	1,214	34,428.0	28.3591		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	-51.4805	3.0831	-57.5294	-45.4317
α_4	3.7643	0.3739	3.0308	4.4979

Table 18b. Analysis of variance and coefficient estimates for the modified Pienaar *et al*. (1989) whole stand basal area projection equation (Equation [15]) for unthinned plots after removing the non-significant coefficient.

Source	DF	Sum of Squares	Mean Square	*F*-value	*p*-value
Model	4	7,572,628	1,893,157	105,799	<.0001
Error	1,115	19,951.7	17.8939		
Uncorrected Total	1,119	7,529,579			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	-127.2	8.0340	-142.9	-111.4
α_2	-0.0846	0.0300	-0.1433	-0.0258
α_4	15.4584	1.1284	13.2444	17.6725
α_6	-1.0597	0.1744	-1.4019	-0.7176

Table 18c. Analysis of variance and coefficient estimates for the modified Pienaar *et al*. (1989) whole stand basal area projection equation (Equation [15]) for thinned plots.

The results from ANOVA table indicate a good fit for the modified Pienaar *et al*. (1989) projection model form with small mean square error and *p*-value for both treatments. One of the coefficients from the unthinned form that accounts for change in trees per acre (α_2) is not significant since it contains zero in its confidence interval. This model form was refit without this coefficient and the variables that it modifies and the results of this reduced model are summarized in Table 18b. After removing α_2 from the form, the mean square error remained about the same but the *F*-value has noticeable increased, indicating that very small amount of variation was explained by the this coefficient and the variables it modifies.

The model form for thinned stands contains a smaller mean square error than the form for unthinned stands and all coefficients are significant. The former statement indicates that the thinning modifier in this form explains a substantial amount of variation in basal area growth in thinned stands, since the coefficient that accounts for the thinning variable (α_6) is significant. The results from these tables also indicate that the mean square error for the unthinned stands is higher than all previously selected forms, while the model form for the thinned stands contains the smallest mean square error than all previously suggested forms.

<u>Phase 2.</u> Diagnostics

The residuals are as previously defined. Graphical methods were used for error assessment. The residual distribution for unthinned and thinned plots is displayed in Figure 7a and 7b.

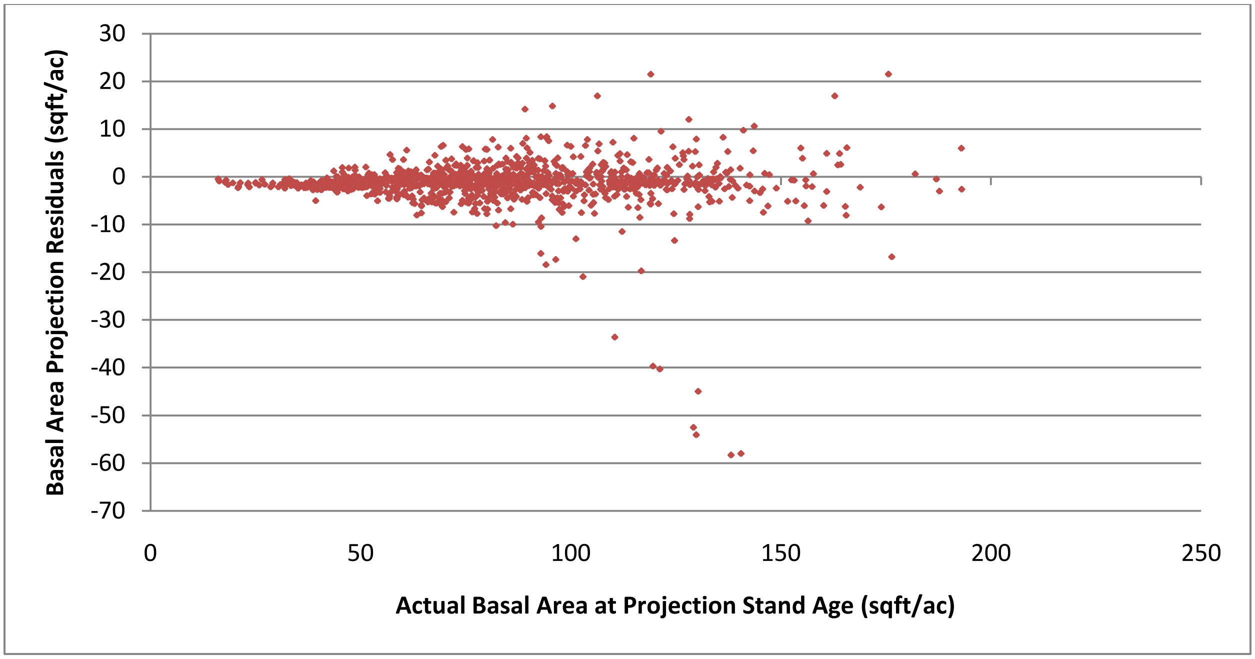

Figure 7a. Basal area residual distribution for the modified Pienaar *et al*. (1989) whole stand basal area projection equation (Equation [15]) for unthinned plots.

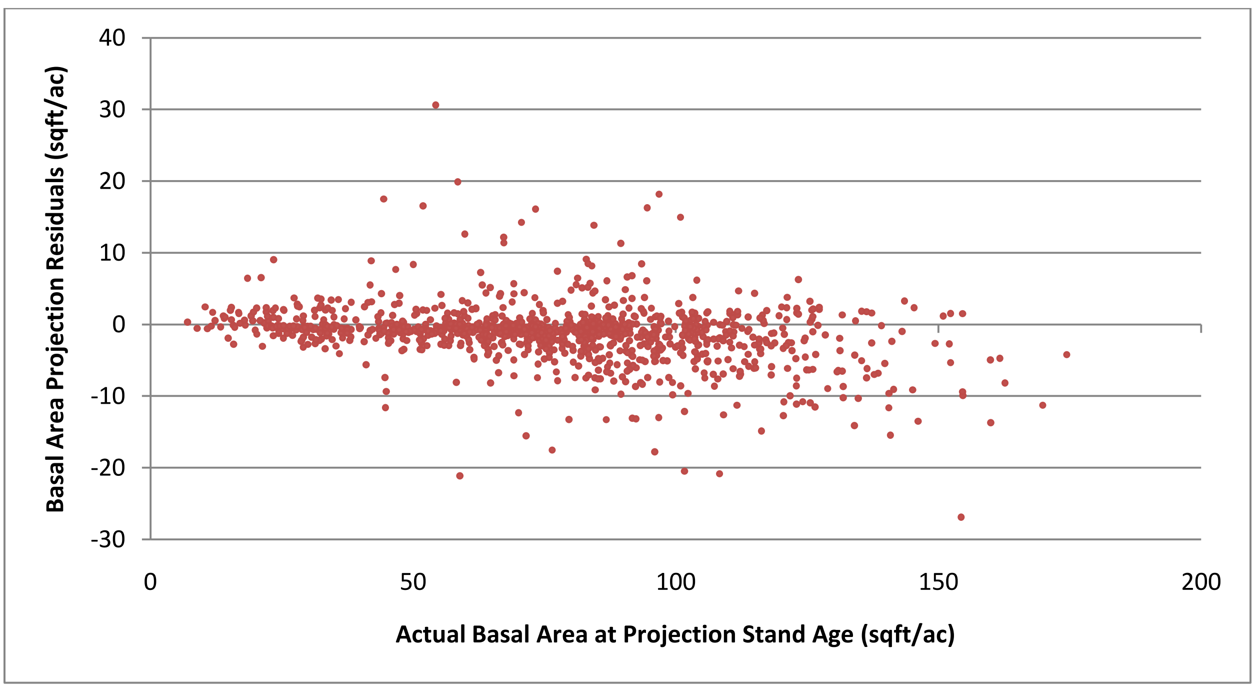

Figure 7b. Basal area residual distribution for the modified Pienaar *et al*. (1989) whole stand basal area projection equation (Equation [15]) for thinned plots.

The distribution of the residuals for the modified Pienaar *et al*. (1989) basal area projection model form reveals some important points. The graphs indicate no sign of any unusual pattern or non-constant variance. The variance is relatively stable for both treatments. There is a little increase of the variance for the unthinned stands as the actual basal area at projection stand age increases. Some extreme negative error values are presented as a result of some extreme changes in basal area growth in some measurement plots, but they were not considered as outliers and were not removed, since they represent actual stands. The residuals are a bit unevenly distributed among the neutral line for the unthinned plots with more negative than positive values. It is more noticeable for stands containing basal area at projection age less than fifty square feet per acre.

The residuals are more equally distributed among the neutral line for the thinned stands with less extreme error values. The variance is quite constant regardless of the amount of basal area at projection age that the stands contain. The main density of the residuals falls within ± 10 square feet per acre for both treatments. The graphs support the assumption of uncorrelated and normally distributed residual errors.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for the modified Pienaar *et al*. (1989) basal area projection form are summarized in Table 19.

Treatment	Bias	RMSE
Unthinned plots	-1.3026	5.3253
Thinned plots	-1.2060	4.2301

Table 19. Average bias and root mean square error values for the modified Pienaar *et al*. (1989) basal area projection equation (Equation [15]).

Table 19 indicates that the modified Pienaar *et al*. (1989) model form for unthinned stands has higher average bias and root mean square error than the form for thinned stands. In fact, these two forms have the highest average bias than all other previously selected forms. The negative sign of the bias indicates a small underestimation of the actual basal area growth.

3.3.6 Pienaar *et al*. (1990) Whole Stand Basal Area Projection Equation

The second Pienaar *et al*. (1990) basal area projection form, based on the Pienaar and Shiver (1986) general form was also selected for analysis, even though there is no longer an explicit thinning modifier. This model form also contains the number of trees per acre as additional variables, which were recorded for all plots. Average dominant height (H_i) was again ignored due to lack of tree height measurements for most of the plots. Pienaar *et al*. (1990) basal area projection model (Equation [8]) was modified to a form, exhibited in Equation [16].

$$B_2 = Exp\left[\begin{array}{l}\frac{A_1}{A_2}Ln(B_1)+\alpha_1\left(1-\frac{A_1}{A_2}\right)+\alpha_3\left(Ln(N_2)-\frac{A_1}{A_2}Ln(N_1)\right)+ \\ +\alpha_5\frac{(Ln(N_2)-Ln(N_1))}{A_2}\end{array}\right] \qquad [16]$$

<u>Phase 1.</u> Examination of the ANOVA table and parameter estimates

This modified form of Pienaar *et al*. (1990) basal area projection model (Equation [16]) was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned and thinned plots throughout all different studies. The output is displayed in Tables 20a, 20b, and 20c.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	3	9,503,721	3,167,907	130,917	<.0001
Error	1,213	29,378.7	24.2199		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	4.7455	0.1194	4.5113	4.9796
α_3	0.1278	0.0209	0.0868	0.1687
α_5	0.3304	0.5674	-0.7829	1.4436

Table 20a. Analysis of variance and coefficient estimates for the modified Pienaar *et al.* (1990) whole stand basal area projection equation (Equation [16]) for unthinned plots.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	2	9,503,712	4,751,856	196,299	<.0001
Error	1,214	29,387.6	24.2073		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	4.6887	0.0761	4.5393	4.8381
α_3	0.1371	0.0146	0.1084	0.1657

Table 20b. Analysis of variance and coefficient estimates for the modified Pienaar *et al*. (1990) whole stand basal area projection equation (Equation [16]) for unthinned plots after removing the non-significant coefficient.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	3	7,578,109	2,526,036	194,810	<.0001
Error	1,116	14,470.8	12.9667		
Uncorrected Total	1,119	7,592,579			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	5.4158	0.1119	5.1963	5.6354
α_3	0.0928	0.0198	0.0539	0.1317
α_5	17.6678	1.0022	15.7014	19.6342

Table 20c. Analysis of variance and coefficient estimates for the modified Pienaar *et al*. (1990) whole stand basal area projection equation (Equation [16]) for thinned plots.

The results from the ANOVA table indicate a very high *F*-value and, respectively, small *p*-value for both treatments. However, the coefficient estimates for the unthinned stands (Table 20a) indicates that one of the coefficients (α_5) is non-significant, containing zero in its confidence interval. It was removed and Equation [16] was refit. The reduced form contains only significant coefficients (Table 20b) with a slight reduction of the mean square error. The coefficients for thinned stands (Table 20c) are all significant. All the coefficients have relatively narrow confidence intervals.

The output also indicates that the mean square error for thinned stands (Table 20c) is about half the size the mean square error for the unthinned stands. In fact, this modified form of Pienaar *et al*. (1990) basal area projection model for thinned stands provides a noticeable reduction of the mean square error (12.9667) compared to the previously selected forms for thinned stands (17.8939 or higher), even without an explicit thinning modifier.

Phase 2. Diagnostics

The residuals here are as previously defined. Graphical methods are used for error assessment. The residual distribution for the unthinned and thinned stands is displayed in Figures 8a and 8b.

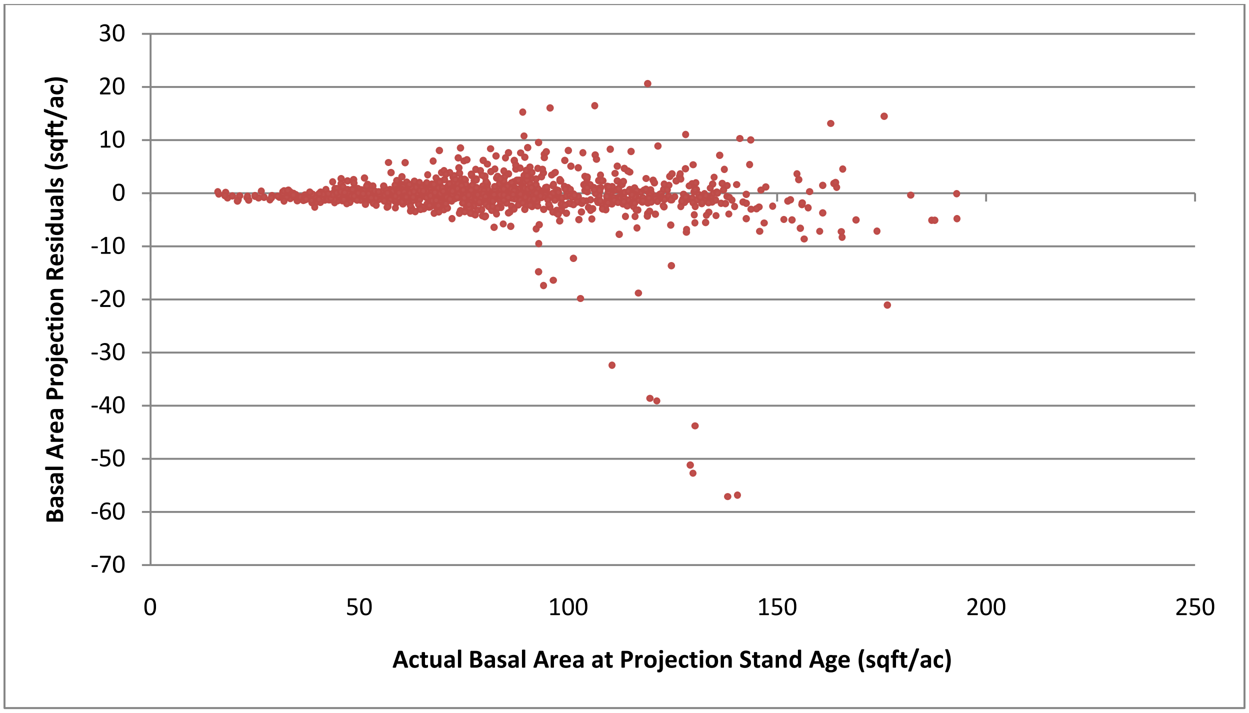

Figure 8a. Basal area residual distribution for the modified Pienaar *et al*. (1990) basal area projection equation (Equation [16]) form for unthinned plots.

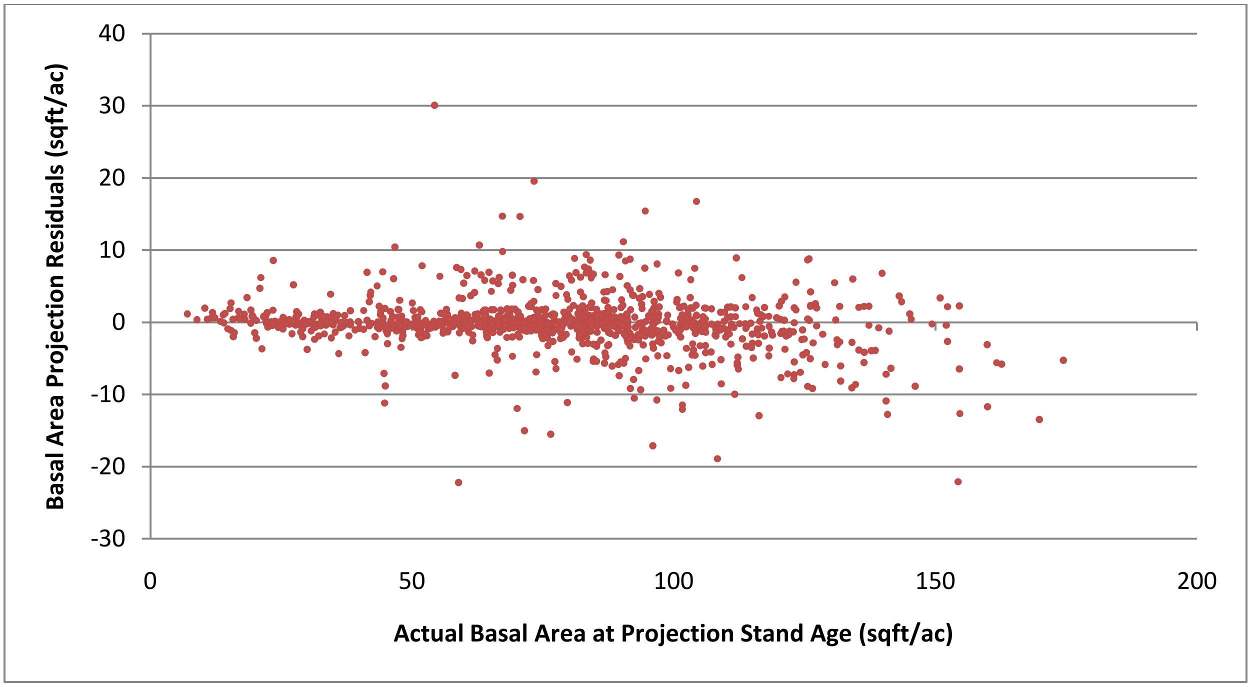

Figure 8b. Basal area residual distribution for the modified Pienaar *et al*. (1990) basal area projection equation (Equation [16]) form for thinned plots.

The residual distribution for this modified form of Pienaar *et al*. (1990) basal area projection model is similar to the modified Pienaar *et al.* (1989) basal area projection form (Equation [15]). The distribution does not reveal any signs of unusual patterns and the variance is relatively constant for both treatments. The main density of the residuals falls within ± 10 square feet per acre of basal area for both treatments, which indicates relatively high projection precision.

The graph for the unthinned stands (Figure 8a) again indicates some noticeable negative error values due to some extreme changes in basal area growth for some unthinned plots, but they were not considered as outliers since they represent actual stands and were not excluded from the data. There is a slight increase of the variance as the basal area at projection stand age increases, indicating higher variability in stands containing more basal area per acre. The residual distribution for the thinned stands (Figure 8b) contains a more stable variance without any extreme residuals, indicating that even without an explicit thinning modifier this modified Pienaar *et al*. (1990) form expresses a good basal area projection precision for thinned stands.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for the modified Pienaar *et al*. (1990) basal area projection equation are displayed in Table 21.

Treatment	Bias	RMSE
Unthinned plots	-0.4398	4.9201
Thinned plots	-0.2008	3.6009

Table 21. Average bias and root mean square error values for the modified Pienaar *et al*. (1990) basal area projection equation (Equation [16]).

The results indicate that the modified Pienaar *et al*. (1990) basal area projection model contains a smaller bias and root mean square error when fit to the data for thinned plots than when fit to the data for unthinned plots. Moreover, this model contains a smaller bias and root mean square error for thinned stands than all previously selected model forms for thinned stands. The negative value of the average bias for both treatments indicates a slight underestimation of the actual basal area growth.

3.3.7 Brooks (1992) Whole Stand Basal Area Projection Equation Form 1

This basal area projection model form developed by Brooks (1992) and based on Pienaar *et al*. (1989) form, was also selected for this study for further analysis. The specific feature of this model form is that it utilizes two separate variables of the thinning modifier that account for thinning type and thinning intensity. This basal area projection model was modified under the conditions for this project. Since the thinning type applied in all studies is selective, there is no need for a variable that accounts for the type of thinning and it was ignored. Also, due to lack of tree height measurements for most of the plots, the variables that account for average dominant height were ignored as well. The modified model is shown as Equation [17]:

$$B_2 = Exp\left[\begin{array}{l} Ln(B_1) + \alpha_1\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \alpha_3\left(Ln(N_2) - Ln(N_1)\right) + \\ \alpha_5\left(\frac{Ln(N_2)}{A_2} - \frac{Ln(N_1)}{A_1}\right) + \alpha_6\left(\frac{T_{ii}}{A_t}\right)\left(\frac{1}{A_2} - \frac{1}{A_1}\right) \end{array}\right] \quad [17]$$

Several modifiers accounting for thinning intensity were tested to determine which form best fits the existing data. In general, they can be separate in two groups, where the first group involves the concept of change in basal area per acre before and after thinning and the second group involves the concept of change in trees per acre before and after thinning. The variables are described in Table 22.

Thinning Intensity by Basal Area	Thinning Intensity by Trees per Acre
$T_{b1} = B_a / B_b$	$T_{n1} = N_a / N_b$
$T_{b2} = B_t / B_b$	$T_{n2} = N_t / N_b$
$T_{b3} = B_t / B_a$	$T_{n3} = N_t / N_a$

Table 22. Thinning intensity modifiers used in the modified first form of Brooks (1992) basal area projection equation (Equation [17]) development.

The symbols of the thinning variables are described as:

T_{bi} = a measure of thinning intensity using basal area,

T_{ni} = a measure of thinning intensity using trees per acre,

B_a = basal area after thinning,

B_t = basal area removed during thinning,

B_b = basal area before thinning,

N_a = number of trees per acre after thinning,

N_t = number of trees per acre removed during thinning,

N_b = number of trees per acre before thinning.

All combinations of these thinning variables were used in the modified model and fit to the hardwood data for all different studies with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN. The values of root mean square error, average bias, and coefficient estimates with their 95% confidence intervals are displayed in Table 23.

Intensity variable	RMSE	BIAS	α_i	95% CI	
T_{b1}	4.0865	-0.7462	α_1 = -147.9 α_3 = 0.0372 α_5 = 12.4568 α_6 = 988.4	-159.1 -0.0255 10.3256 806.8	-136.6 0.0998 14.5880 1170.0
T_{b2}	4.0586	-1.1158	α_1= -179.7 α_3 = -0.0820 α_5 = 20.3464 α_6 = 450.9	-191.5 -0.1350 18.5455 373.5	-168.0 -0.0290 22.1474 528.3
T_{b3}	4.1215	-0.7077	α_1= -180.8 α_3 = -0.1354 α_5 = 21.3957 α_6 = 102.4	-193.0 -0.1873 19.5322 81.3462	-168.7 -0.0834 23.2591 123.6
T_{n1}	4.2975	-1.1718	α_1= -161.4 α_3 = -0.1716 α_5 = 19.6775 α_6 = -43.0945	-175.1 -0.2356 16.9682 -259.3	-147.7 -0.1076 22.3868 173.1
T_{n2}	3.9780	-0.9259	α_1= -186.5 α_3 = -0.0703 α_5 = 21.1596 α_6 = 417.3	-198.1 -0.1221 19.3915 356.3	-174.9 -0.0185 22.9278 478.4
T_{n3}	3.9422	-0.3753	α_1 = -201.0 α_3 = -0.1482 α_5 = 24.1937 α_6 = 71.8489	-213.3 -0.1976 22.3262 61.4580	-188.7 -0.0988 26.0613 82.2398

Table 23. Comparison of the thinning intensity modifiers used in the modified first form of Brooks (1992) basal area projection equation (Equation [17]).

The summarized results from the analysis of variance for all types of thinning intensity modifiers of Equation [17] indicate that two of the forms contain a non-significant coefficient – T_{b1} (α_3 is non-significant) and T_{n1} (α_6 is non-significant). In the case of T_{n1}, the thinning modifier turned out to be non-significant, since the coefficient that accounts for the thinning effect is not significant (α_6 contains zero in its confidence limits). The rest of the forms contain only significant coefficients. Table 23 also reveals that the form with the thinning modifier that incorporates the ratio of trees per acre removed during thinning and trees per acre after thinning (T_{n3}) has the smallest root mean square error (3.9422 ft^2/ac) and the smallest average bias (-0.3753 ft^2/ac) among all other forms. Also, the coefficient that accounts for the thinning variable (α_6) for this particular form has the narrowest confidence limits. After this analysis, the form that contains T_{n3} as a variable for the thinning modifier has been selected as a part of Equation [17] for further analysis, or:

$$B_2 = Exp\left[\begin{array}{l} Ln(B_1)+\alpha_1\left(\frac{1}{A_2}-\frac{1}{A_1}\right)+\alpha_3\left(Ln(N_2)-Ln(N_1)\right)+ \\ \alpha_5\left(\frac{Ln(N_2)}{A_2}-\frac{Ln(N_1)}{A_1}\right)+\alpha_6\left(\frac{T_{n3}}{A_t}\right)\left(\frac{1}{A_2}-\frac{1}{A_1}\right) \end{array}\right] \quad [18]$$

Phase 1. Examination of ANOVA table and coefficient estimates

This modified form of Brooks (1992) basal area projection equation (Equation [18]) was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned (without α_6) and thinned (with α_6) plots throughout all different studies. The output is displayed in Tables 24a, 24b, and 24c.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	3	9,498,733	3,166,244	111,757	<.0001
Error	1,213	34,366.2	28.3316		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	-55.8108	4.2270	-64.1039	-47.5177
α_3	-0.0343	0.0235	-0.0804	0.0118
α_5	4.4543	0.5964	3.2842	5.6244

Table 24a. Analysis of variance and coefficient estimates for the modified first form of Brooks (1992) whole stand basal area projection model (Equation [18]) for unthinned plots.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	2	9,498,671	4,749,336	167,471	<.0001
Error	1,214	34,428.0	28.3591		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	-51.4802	3.0831	-57.5290	-45.4313
α_5	3.7643	0.3739	3.0307	4.4979

Table 24b. Analysis of variance and coefficient estimates for the modified first form of Brooks (1992) whole stand basal area projection model (Equation [18]) for unthinned plots after removing the non-significant coefficient.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	4	7,575,251	1,893,813	122,366	<.0001
Error	1,115	17,328.5	15.5413		
Uncorrected Total	1,119	7,592,579			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	-201.0	6.2646	-213.3	-188.7
α_3	-0.1482	0.0252	-0.1976	-0.0988
α_5	24.1937	0.9518	22.3262	26.0613
α_6	71.8489	5.2957	61.4580	82.2398

Table 24c. Analysis of variance and coefficient estimates for the modified first form of Brooks (1992) whole stand basal area projection model (Equation [18]) for thinned plots.

The results indicate that the modified first form of Brooks (1992) basal area projection equation (Equation [18]) is significant, containing large *F*-value and very small *p*-value for the unthinned and thinned stands. The coefficient estimates for unthinned stands (Table 24a), however, indicates that there is a non-significant coefficient (α_3) since it contains zero in its confidence interval. This coefficient was removed and the model was refit. The new results, summarized in Table 24b which are identical to the results in Table 18b since this modified Brooks (1992) form for unthinned stands is identical to the modified Pienaar *et al*. (1989) form for unthinned stands, indicate that all coefficients are significant with a very small increase of the mean square error. When applied for thinned stands, the model does not contain any non-significant coefficients (Table 24c). The results also reveal that the mean square error for thinned stands (15.6218 ft^2/ac) is almost twice as small as the one for unthinned stands (28.1830 ft^2/ac). This important point indicates a good basal area projection precision of the form for thinned stands. Also the mean square error for thinned stands is one of the smallest among all previously selected models, while the mean square error for the unthinned stands is one of the largest.

<u>Phase 2.</u> Diagnostics

The residuals here are as previously defined. Graphical methods are used for error assessment. The residual distributions for unthinned and thinned stands are displayed in Figures 9a and 9b.

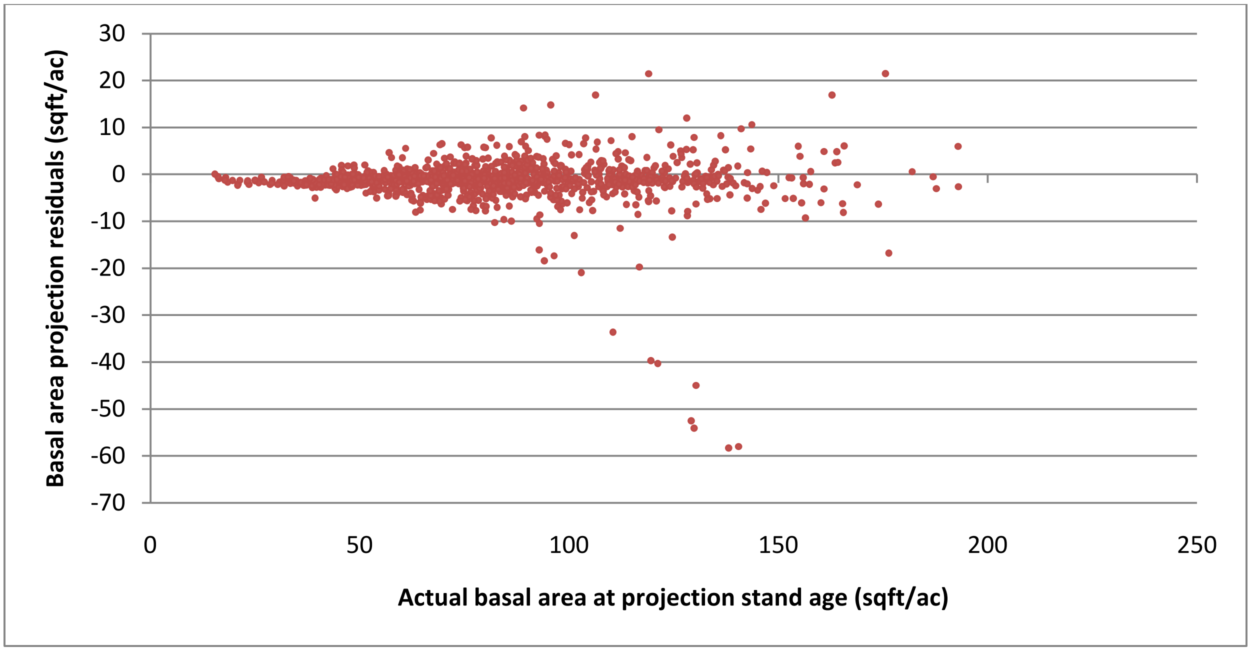

Figure 9a. Basal area residual distribution for the modified first form of Brooks (1992) basal area projection equation (Equation [18]) for unthinned plots.

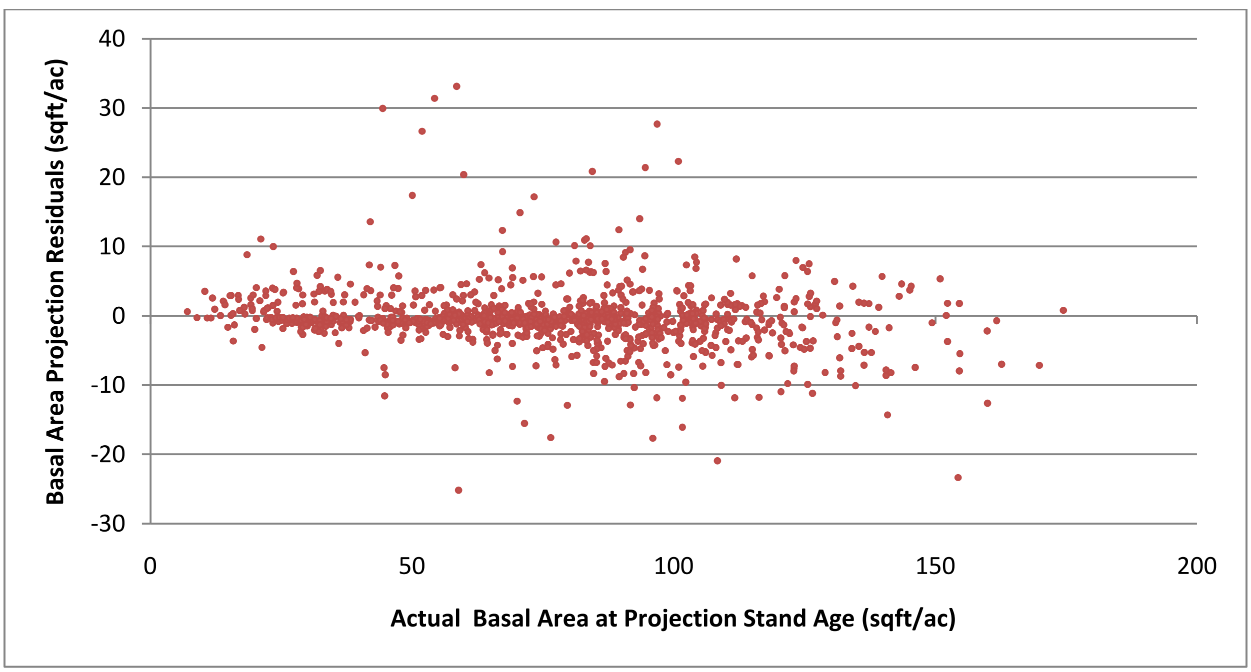

Figure 9b. Basal area residual distribution for the modified first form of Brooks (1992) basal area projection equation (Equation [18]) for thinned plots.

The residual distribution for the modified first form of Brooks (1992) whole stand basal area projection equation (Equation [18]) does not reveal any unusual pattern or any obvious signs of non-constant variance for both treatments. The residuals are evenly distributed among the neutral line, which supports the assumption that they are uncorrelated and normally distributed. The main density of the residuals falls within ± 10 square feet per acre of basal area, indicating good performance of this model.

The graph for the unthinned stands (Figure 9a) indicates some noticeable negative error values for stands containing more basal area per acre at projection stand age. These extremes are due to some abrupt changes of basal area growth for some unthinned plots. Since they represent actual stands, they were not considered as outliers and were not removed from the data. There is a slight increase of the variance as the basal area per acre at projection stand age increases. Figure 9a also indicates that most of the residuals are situated below the neutral line for check plots containing less than fifty square feet of basal area at projection age, indicating a sign of a slight underestimation of basal area growth for these stands.

The graph for thinned plots (Figure 9b) indicates more constant variance than the one for the unthinned plots, with no extreme error values. Also the residuals are more evenly distributed among the neutral line, without obvious signs of bias.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for the modified first form of Brooks (1992) basal area projection equation (Equation [18]) are displayed in Table 25.

Treatment	Bias	RMSE
Unthinned plots	-1.3026	5.3253
Thinned plots	-0.3753	3.9422

Table 25. Average bias and root mean square error values for the modified first form of Brooks (1992) basal area projection equation (Equation [18]).

Table 25 reveals smaller bias and root mean square error for the modified Brooks (1992) form for thinned plots compared to the form for unthinned plots, indicating a better fit and good performance of the thinning modifier for the thinned plots. The bias and the average root mean square error for the form for unthinned plots here are identical to the Pienaar *et al*. (1989) form for unthinned stands. The negative value of the average bias for both treatments indicates a slight underestimation of the actual basal area growth.

3.3.8 Brooks Whole Stand Basal Area Projection Equation Form 2

The second form of Brooks (1992) stand basal area projection model, based on Borders *et al*. (1990) projection model, was also selected for this study. It involves the same variables as the previous Brooks (1992) basal area projection model form (1), but has a different structure. The thinning modifier also utilizes two separate variables that account for thinning type and thinning intensity.

This proposed basal area projection model has been modified under the conditions for this project. The Z variable was removed because of repetition and lack of stability of the γ_1 coefficient for unthinned stands. Since the thinning type is selective for all datasets in this study, there is no need for a variable that accounts for the type of thinning. Due to lack of tree height measurements for most of the plots in this project, the variables that account for average dominant height were removed as well. The reduced model is displayed as Equation [19]:

$$B_2 = Exp\left[\begin{array}{l}\left(\frac{A_1}{A_2}\right)Ln(B_1)+\alpha_1\left(1-\frac{A_1}{A_2}\right)+\alpha_3\left(Ln(N_2)-\left(\frac{A_1}{A_2}\right)Ln(N_1)\right)+ \\ \alpha_5\left(\frac{Ln(N_2)-Ln(N_1)}{A_2}\right)+\alpha_6\left(\frac{T_{ii}}{A_t}\right)\left(1-\frac{A_1}{A_2}\right)\end{array}\right] \qquad [19]$$

Several modifiers accounting for thinning intensity were tested to determine the best variable that fits the existing data. These modifiers involve the same concept as of Brooks (1992) first basal area projection form (Equation [18]). The modifiers are summarized in Table 26.

Thinning Intensity by Basal Area	Thinning Intensity by Trees per Acre
$T_{b1} = B_a / B_b$	$T_{n1} = N_a / N_b$
$T_{b2} = B_t / B_b$	$T_{n2} = N_t / N_b$
$T_{b3} = B_t / B_a$	$T_{n3} = N_t / N_a$

Table 26. Thinning intensity modifiers used in the modified second form of Brooks (1992) basal area projection equation (Equation [19]) development.

The symbols of the thinning variables are described as:

T_{bi} = a measure of thinning intensity using basal area,

T_{ni} = a measure of thinning intensity using trees per acre,

B_a = basal area after thinning,

B_t = basal area removed during thinning,

B_b = basal area before thinning,

N_a = number of trees per acre after thinning,

N_t = number of trees per acre removed during thinning,

N_b = number of trees per acre before thinning.

All combinations of these thinning variables were used in the second form of Brooks (1992) projection model (Equation [19]) and were fit to the thinned hardwood data with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN throughout all different studies. The values of the root mean square error, average bias, and coefficient estimates with their 95% confidence intervals are displayed in Table 27.

Intensity variable	RMSE	BIAS	α_i	95% CI	
T_{b1}	3.5005	-0.0584	α_1 = 4.9898 α_3 = 0.2489 α_5 = 12.3689 α_6 = -19.4671	4.7522 0.1953 10.0854 -24.1855	5.2274 0.3025 14.6523 -14.7487
T_{b2}	3.5840	-0.3572	α_1 = 5.3276 α_3 = 0.1191 α_5 = 17.0686 α_6 = -4.6587	5.1016 0.0770 15.0711 -7.4768	5.5536 0.1612 19.0661 -1.8406
T_{b3}	3.5843	-0.1797	α_1 = 5.3677 α_3 = 0.1077 α_5 = 17.3764 α_6 = -1.2847	5.1467 0.0678 15.4061 -2.0604	5.5888 0.1476 19.3467 -0.5090
T_{n1}	3.5461	-0.1614	α_1 = 5.0956 α_3 = 0.2052 α_5 = 13.0907 α_6 = -14.7699	4.8551 0.1520 10.6577 -19.6209	5.3362 0.2585 15.5236 -9.9189
T_{n2}	3.5816	-0.3167	α_1 = 5.3381 α_3 = 0.1174 α_5 = 17.2951 α_6 = -4.0662	5.1148 0.0762 15.3252 -6.3167	5.5615 0.1585 19.2649 -1.8156
T_{n3}	3.5728	-0.1359	α_1 = 5.3877 α_3 = 0.1052 α_5 = 17.7467 α_6 = -0.8444	5.1691 0.0660 15.7928 -1.2427	5.6063 0.1443 19.7006 -0.4460

Table 27. Comparison of the thinning intensity modifiers used in the modified second form of Brooks (1992) basal area projection equation (Equation [19]).

The summarized results in Table 27 indicate that all coefficients are significant for all different combinations. This indicates that all types of thinning intensity modifiers can be a part of the second form of Brooks (1992) modified basal area projection model (Equation [19]) when applied to thinned hardwood stands. Further analysis indicates the form that incorporates the ratio of basal area after thinning and basal area before thinning (T_{b1}) as a variable for the thinning modifier has the smallest mean square error (3.5005 ft^2/ac) and average bias (-0.0584 ft^2/ac) among all other thinning intensity variables. After the analysis, Equation [19] was modified to include the T_{b1} thinning modifier for thinned stands and takes the form:

$$B_2 = Exp\left[\begin{array}{l}\left(\frac{A_1}{A_2}\right)Ln(B_1)+\alpha_1\left(1-\frac{A_1}{A_2}\right)+\alpha_3\left(Ln(N_2)-\left(\frac{A_1}{A_2}\right)Ln(N_1)\right)+ \\ \alpha_5\left(\frac{Ln(N_2)-Ln(N_1)}{A_2}\right)+\alpha_6\left(\frac{T_{b1}}{A_t}\right)\left(1-\frac{A_1}{A_2}\right)\end{array}\right] \quad [20]$$

Phase 1. Examination of ANOVA table and coefficient estimates.

This second form of Brooks (1992) modified basal area projection equation was fitted with SAS 9.1 (SAS Institute, Inc. 2002-2003) using Proc NLIN separately for unthinned (without α_6) and thinned (with α_6) plots throughout all different studies. The output is displayed in Tables 28a, 28b, and 28c.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	3	9,503,721	3,167,907	130,798	<.0001
Error	1,213	29,378.7	24.2199		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	4.7455	0.1194	4.5113	4.9796
α_3	0.1278	0.0209	0.0868	0.1687
α_5	0.3304	0.5674	-0.7829	1.4436

Table 28a. Analysis of variance and coefficient estimates for the modified second form of Brooks (1992) whole stand basal area projection equation (Equation [20]) for unthinned plots.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	2	9,503,712	4,751,856	196,299	<.0001
Error	1,214	29,387.6	24.2073		
Uncorrected Total	1,216	9,533,099			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	4.6887	0.0761	4.5393	4.8381
α_3	0.1371	0.0146	0.1084	0.1657

Table 28b. Analysis of variance and coefficient estimates for the modified second form of Brooks (1992) whole stand basal area projection equation (Equation [20]) for unthinned plots after removing the non-significant coefficient.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	4	7,578,917	1,894,729	154,626	<.0001
Error	1,115	13,662.8	12.2536		
Uncorrected Total	1,119	7,592,579			

Coefficient	Estimate	Standard Error	95% Confidence Limits	
α_1	4.9898	0.1211	4.7522	5.2274
α_3	0.2489	0.0273	0.1953	0.3025
α_5	12.3689	1.1638	10.0854	14.6523
α_6	-19.4671	2.4047	-24.1855	-14.7487

Table 28c. Analysis of variance and coefficient estimates for the modified second form of Brooks (1992) whole stand basal area projection equation (Equation [20]) for thinned plots.

The analysis of variance indicates that this modified form of Brooks (1992) basal area projection equation (Equation [20]) is statistically significant for both treatments, since the *p*-value is very small ($p < 0.0001$). However, the form for unthinned stands (Table 28a) has a non-significant coefficient (α_5), since its confidence interval contains zero. This indicates that the variables that this coefficient modifies could not explain a substantial amount of variation in basal area growth. This coefficient was removed, and the form was refit. After the removal of α_5, the reduced form for unthinned plots contains only significant coefficients (Table 28b) with almost no change in the mean square error.

The form for thinned stands (Table 28c) does not contain non-significant coefficients. The analysis also reveals that the form for thinned stands has about half the mean square error than the form for unthinned stands, indicating a good basal area projection precision. Further analysis indicates that the second form of Brooks (1992) basal area projection equation (Equation [20]) for thinned stands contains the smallest mean square error of all previously tested model forms for thinned stands.

Phase 2. Diagnostics

The residuals are as previously defined. Graphical methods are used for error assessment. The residual distribution is displayed in Figures10a and 10b.

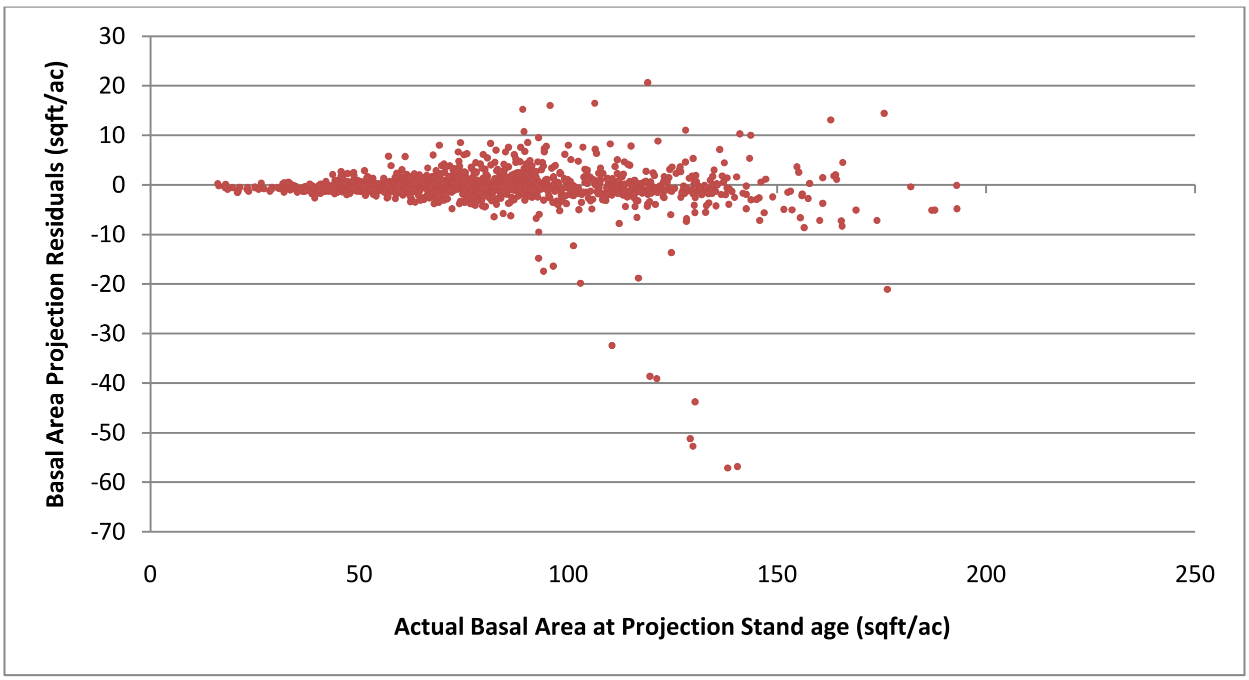

Figure 10a. Basal area residual distribution for the modified second form of Brooks (1992) basal area projection equation (Equation [20]) for unthinned plots.

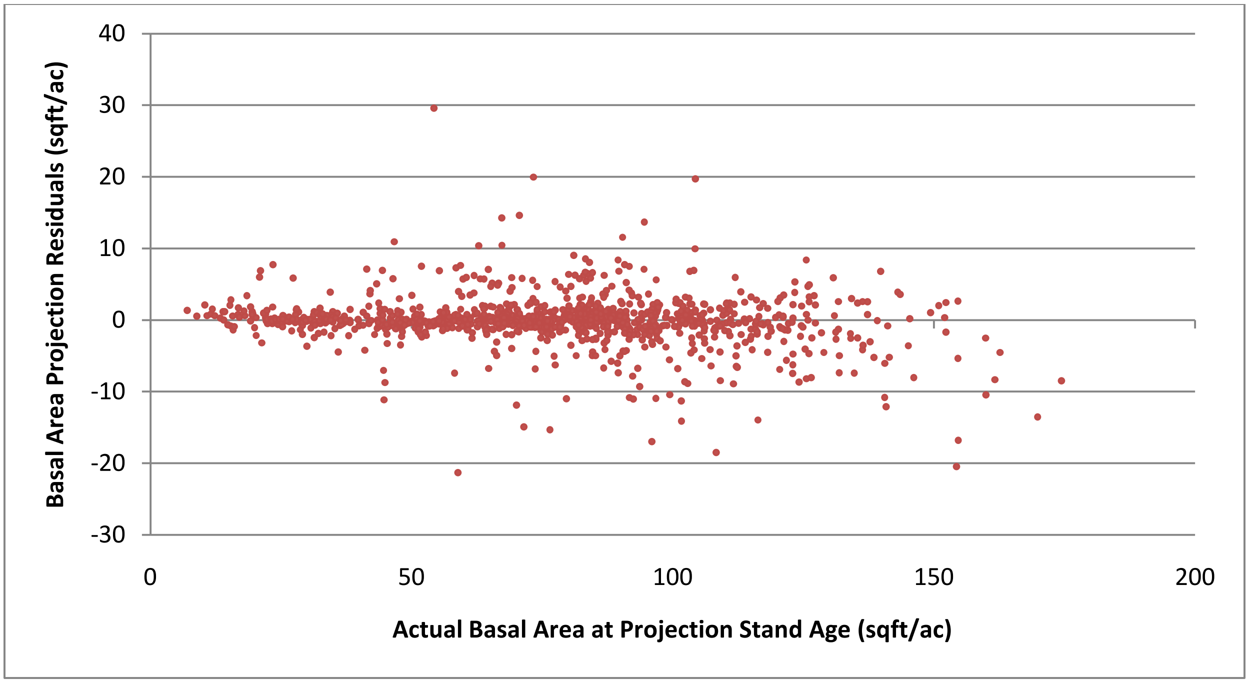

Figure 10b. Basal area residual distribution for the modified second form of Brooks (1992) basal area projection equation (Equation [20]) for thinned plots.

The residual distribution for the second form of Brooks (1992) modified whole stand basal area projection equation (Equation [20]) does not reveal any unusual pattern of the residual errors or signs of non-constant variance for both treatments. The residuals are evenly distributed among the neutral line, which supports the assumption that they are uncorrelated and normally distributed. The main density of the residuals falls within ± 10 square feet per acre of basal area. The residual distribution for unthinned stands (Figure 10a) indicates some noticeable negative error values for stand, containing higher basal area per acre at projection stand age. They are present because of some extreme changes in basal area growth in few unthinned plots. Since these extremes represent actual stands, they were not considered outliers and were not removed. The residual distribution for unthinned stands also indicates a slight increase of the variance as the basal area per acre at projected age increases. The graph for thinned stands (Figure 10b) does not reveal any extreme residuals and the variance is relatively constant, indicating a better fit.

Phase 3. Calculating average bias and root mean square error

The average bias and root mean square error for the modified second form of Brooks (1992) basal area projection equation (Equation [20]) are displayed in Table 29.

Treatment	Bias	RMSE
Unthinned plots	-0.4398	4.9201
Thinned plots	-0.0584	3.5005

Table 29. Average bias and root mean square error values for the modified second form of Brooks (1992) basal area projection equation (Equation [20]).

Table 29 indicates that the Equation [20] form for thinned plots has smaller average bias and root mean square error than the form for unthinned stands. The average bias for the form for thinned stands here is about seven times smaller than the average bias for the form for unthinned stands. The negative sign of the bias for both treatments indicates a slight underestimation of the actual basal area growth for this model.

3.4 Selecting the Best Whole Stand Basal Area Projection Equation for the Central Hardwood Stands

The detailed analysis of each proposed basal area projection model indicated that all forms can be applied for central hardwood forest stands, even though some equations have better characteristics than the others, depending on their structure and the variables they incorporate.

The proposed equation by Clutter and Jones (1980) (Equation [1]) for modeling pine plantations basal area growth in the southeastern United States exhibits high potential when applied in unthinned and thinned natural hardwood forest stands. Even with only three basic variables, this simple equation is able to explain a large amount of variation of basal area growth and provides satisfactory performance. A reduction of the average bias for thinned stands might be achieved if additional thinning variables were included in this model form.

The proposed basal area projection model by Burkhart and Sprinz (1984) (Equation [2]) for repeatedly thinned pine plantations in Virginia Piedmont and Coastal Plain has similar characteristics compared to the proposed model by Clutter and Jones (1980) when applied for central hardwood stands. Inclusion of a site index variable proved not to be significant for unthinned stand, and significant for thinned stands. The value of mean square error is very similar to Clutter and Jones (1980) model (Equation [1]) for both treatments, indicating that site index did not substantially improve the overall precision of this model. After all examination, Burkhart and Sprinz (1984) projection model can be a good choice for central hardwood forest stand as the Clutter and Jones (1980) projection model, but less desirable since the additional site index variable did not improve the overall projection precision.

The proposed basal area projection model by Bailey and Ware (1984) (Equation [3]) for thinned and unthinned, even-aged natural stands in central Georgia, which reduced to Clutter (1963) (Equation [14]) model form for the purpose of this study, has exactly the same characteristics and outcomes as the proposed model by Burkhart and Sprinz (1984) when applied to central hardwood stands. Inclusion of a site index variable again proved not to be significant for unthinned stand, and significant for thinned stands. All of the results indicate that Clutter (1963) basal area projection model can be equally applied for basal area growth projection purposes in hardwood forest stands as the projection model proposed by Burkhart and Sprinz (1984).

The proposed basal area projection model by Pienaar *et al.* (1985) (Equation [6]) for thinned slash pine plantations in South Africa can also be applied for central hardwood forest stands. It has some limitations when the form for unthinned stands is used to model hardwood basal area growth, compared to the previously proposed forms, since it requires more stable basal area growth rate and individual coefficients by eco-region. The form for thinned stands does not have such limitation and exhibits better characteristics, indicating a good performance of the thinning modifier. This indicates a better projection precision for thinned stand than for unthinned stands.

The two forms of the modified basal area projection model proposed by Pienaar *et al.* (1989-1990) for thinned slash pine plantation re-measurement data in the Atlantic Coast Flatwoods and based on Pienaar and Shiver (1986) stand-level basal area growth projection equation can be successfully applied for hardwood forest stands. The results indicate that each form can be used for basal area growth projection for unthinned and thinned stands. Both forms contained a non-significant coefficient, when applied for unthinned hardwood stands, indicating that some variables did not significantly add to the model. The thinning modifier in the modified first form of Pienaar *et al*. (1989) basal area projection model (Equation [15]) for thinned stands did improve the overall precision, indicating the usefulness of the variables that it incorporates. The results from the modified second form of Pienaar *et al*. (1990) basal area projection equation (Equation [16]) indicate that even without an explicit thinning modifier, this form contains better characteristics when applied for thinned hardwood stands. This important point suggests that the type of thinning modifier used in the first form may not have the desirable characteristics when used for thinned hardwood stands.

The modified first form of Brooks (1992) whole stand basal area projection equation (Equation [18]) can be successfully employed for central hardwood forest stands. The results from the previous analysis indicate that the form for thinned stands has better characteristics that the one for unthinned stands, possessing smaller root mean square error and average bias, and having a better residual distribution. The thinning modifier that this model form contains proved to explain a substantial amount of variation in thinned stands. The form for the unthinned plots (which is identical to the modified Pienaar *et al*. (1989) form for unthinned stands) contained a non-significant coefficient that was removed, indicating that some of the variables did not explain a significant amount of variation in basal area growth for these stands.

The modified second form of Brooks (1992) whole stand basal area projection equation (Equation [20]) can also be successfully used for central hardwoods basal area growth modeling. The results from the analysis indicate that the form for thinned stands has better characteristics that the form for unthinned stands, containing a smaller root mean square error and average bias, and having a better residual distribution. The form for the unthinned plots contained a non-significant coefficient that was removed, indicating that some of the variables did not explain a significant amount of variation in basal area growth for the unthinned stands. The thinning modifier was successfully employed to improve the overall precision of this form, reducing the bias and mean square error more than all previously proposed forms for thinned stands.

A final comparison was performed in order to select the most appropriate model form for the provided hardwood dataset. This process compares the main characteristics of all previously considered models separately for unthinned and thinned stands. The summarized results from the previous analysis with an addition of coefficient of determination (R^2) are displayed in Table 30a and 30b.

Model	Explanatory Variables	Number of Significant Coefficients	RMSE	R^2	Average Bias
Equation [1]	B_1,A_1, A_2	2	5.0920	0.9719	-0.2859
Equation [2]	B_1,A_1, A_2, SI	1	5.0915	0.9719	-0.3257
Equation [14]	B_1,A_1, A_2, SI	1	5.0915	0.9719	-0.3257
Equation [6]	B_1,A_1, A_2	2	5.3161	0.9686	-0.6761
Equation [15]	B_1,A_1, A_2, N_1, N_2	2	5.3253	0.9691	-1.3026
Equation [16]	B_1,A_1, A_2, N_1, N_2	2	4.9201	0.9735	-0.4398
Equation [18]	B_1,A_1, A_2, N_1, N_2	2	5.3253	0.9691	-1.3026
Equation [20]	B_1,A_1, A_2, N_1, N_2	2	4.9201	0.9735	-0.4398

Table 30a. Fit statistics for all selected basal area projection models fitted to the provided central hardwood re-measurement data for unthinned stands.

Model	Explanatory Variables	Number of Significant Coefficients	RMSE	R^2	Average Bias
Equation [1]	B_1, A_1, A_2	2	4.5067	0.9779	-0.4227
Equation [2]	B_1, A_1, A_2, SI	2	4.4596	0.9744	-0.4683
Equation [14]	B_1, A_1, A_2, SI	2	4.4596	0.9744	-0.4683
Equation [6]	B_1, A_1, A_2, N_t, N_a	3	4.4464	0.9786	-0.3472
Equation [15]	$B_1, A_1, A_2, N_1, N_2, N_t, N_b$	4	4.2301	0.9806	-1.2060
Equation [16]	B_1, A_1, A_2, N_1, N_2	3	3.6009	0.9860	-0.2008
Equation [18]	$B_1, A_1, A_2, A_t, N_1, N_2, N_t, N_a$	4	3.9422	0.9833	-0.3753
Equation [20]	$B_1, A_1, A_2, A_t, N_1, N_2, B_a, B_b$	4	3.5005	0.9869	-0.0584

Table 30b. Fit statistics for all selected basal area projection models fitted to the provided central hardwood re-measurement data for thinned stands.

Table 30a indicates that for unthinned stands Equations [16] and [20] have the smallest root mean square error (4.9201 ft^2/ac) and, respectively, the highest coefficient of determination (0.9735), while Equation [1] has the smallest average bias (-0.2859 ft^2/ac). Table 30b reveals that for thinned stands Equation [20] has the smallest root mean square error (3.5005 ft^2/ac) and average bias (-0.0584 ft^2/ac), and the highest coefficient of determination ($R^2 = 0.9869$). It is obvious that the proposed models have different characteristics when applied for the two types of treatments. After all the analysis, the results suggest that Equation [20] contains the best characteristics when applied for central hardwood forest stands for both treatments. Even though the form for unthinned stands contains a higher average bias than some of the selected models, Equation [20] contains the smallest mean square error (same as Equation [8.1]) and the highest coefficient of determination for both treatments as well as the smallest average bias for thinned stands.

After the final comparison, Equation [20] has been selected as the best and most appropriate for the central Appalachian hardwood stands. Table A-4 (Appendix A) summarizes the coefficient estimates from all previously analyzed models.

3.5 Evaluation of the Best Basal Area Projection Model Overall Performance

Equation [20] was tested for basal area projection indifference when fit with partial and full datasets. Since a significant difference was found among the different studies throughout the central Appalachian hardwood region, separate coefficients were obtained by study and Equation [20] was evaluated for projection performance separately for each study. The evaluation test was performed in order to detect any major basal area growth projection flaws when Equation [20] was fit with partial and complete datasets. The results are summarized in Tables 31a through 34b.

3.5.1 MeadWestvaco Study

Unthinned plots

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits	
Partial dataset - 80%	9.9159	32,619.3	<.0001	$\alpha_1 = 5.6075$ $\alpha_5 = 5.0172$	5.4990 3.2236	5.7159 6.8109
Partial dataset - 20%	2.5061	17,722.6	<.0001	$\alpha_1 = 5.5702$ $\alpha_5 = 5.6957$	5.3951 4.3735	5.7453 7.0179
Complete dataset	8.5028	42,608.0	<.0001	$\alpha_1 = 5.6246$ $\alpha_5 = 5.5714$	5.5369 4.3761	5.7124 6.7668

Table 31a. Comparison of the analysis of variance outcomes for Equation [20] fit with the MeadWestvaco partial and complete dataset for unthinned plots.

The analysis of variance for the MeadWestvaco unthinned plots indicate that for the partial and complete dataset α_3 remained non-significant and it was removed. This indicates that little variation was explained by the variables that this coefficient accounted for. This could be due to the fact that α_3 and α_5 modify the same type of variables (change in stand density).

Table 31a also indicates that the significant coefficient estimates for the selected Equation [20] form for unthinned stands remained very stable across the partial and complete MeadWestvaco dataset, without any obvious changes. The mean square error is much smaller for the twenty percent partial dataset, indicating less unexplained variability of basal area growth. Since no major flaws were detected, this model form can be successfully applied for the MeadWestvaco unthinned stands basal area growth modeling.

Thinned plots

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits	
Partial dataset - 80%	3.8636	19,748.1	<.0001	α_1 = 2.4076 α_3 = 0.5372 α_5 = -3.8764	1.5283 0.4358 -7.4752	3.2870 0.7772 -0.2777
Partial dataset - 20%	4.9592	7,344.21	<.0001	α_1 = 2.0731 α_3 = 0.6118	0.5494 0.3601	3.5967 0.8636
Complete dataset	4.9592	24,901.3	<.0001	α_1 = 2.0742 α_3 = 0.5950 α_5 = -4.9693	1.2865 0.4678 -8.2380	2.8619 0.7221 -1.7006

Table 31b. Comparison of the analysis of variance outcomes for Equation [20] fit with the MeadWestvaco partial and complete datasets for thinned plots.

The analysis of variance for the MeadWestvaco thinned plots reveals an important point that the coefficient that accounts for the thinning modifier (α_6) remained not significant across the partial and complete datasets. The reason for this is most likely due to the fact that the thinned stands in this study were young (12 to 15 year old at the time of thinning) and the thinning modifier could not model the additional difference of basal area growth between the unthinned and thinned plots because of the lack of long term re-measurements.

Table 31b also indicates that the mean square error remained relatively constant among the partial and complete MeadWestvaco datasets for thinned stands with the smallest value when the selected model was fit with eighty percent of the partial dataset, and the largest when the model was fit with the twenty percent of the partial dataset. The significant coefficient estimates are also similar across the partial and complete datasets, without any signs of extreme changes in their values and relatively narrow confidence limits. The α_1 coefficient remained the most stable, while α_5 has the widest confidence interval. This indicates that the selected Equation [20] form for thinned stands can be successfully used to model basal area growth for the MeadWestvaco thinned plots.

3.5.2 Hatch Study

Unthinned plots

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits
Partial dataset - 80%	34.3852	5,119.46	<.0001	$\alpha_1 =$ 5.9213 $\alpha_5 =$ 15.7499	5.7127 6.1299 8.2473 23.2524
Partial dataset - 20%	24.4710	3,089.89	<.0001	$\alpha_1 =$ 5.3000	4.9949 5.6052
Complete dataset	33.1367	6,452.57	<.0001	$\alpha_1 =$ 5.8254 $\alpha_5 =$ 12.5675	5.6506 6.0001 6.9191 18.2158

Table 32a. Comparison of the analysis of variance outcomes for Equation [20] fit with the Hatch partial and complete datasets for unthinned plots.

The analysis of variance for the Hatch unthinned plots indicate that α_3 remained non-significant for all datasets and was removed. Table 32a also indicates that when the selected model form for unthinned stands was fit with the twenty percent of the Hatch dataset, α_5 also became non-significant. The reason is most likely due to the very small sample size, since only three plots were not thinned.

Table 32a also indicates that the significant coefficients remain relatively stable across all datasets. The α_1 coefficient changed very little with very stable confidence limits. The case of α_5 loosing significance in the twenty percent partial dataset can be ignored since the reason is most likely due to the very small sample size in this particular dataset. The mean square error change remained small across all datasets as well, having the smallest value when the selected form was fit with the twenty percent of the data. After the results, the selected Equation [20] form for unthinned stands can be successfully applied for modeling basal area growth in the Hatch study unthinned plots.

<u>Thinned plots</u>

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits
Partial dataset - 80%	38.0645	6,321.59	<.0001	α_1 = 8.1694 α_3 = -0.3800 α_5 = 28.6177	7.1258 9.2130 -0.5554 -0.2046 22.3865 34.8488
Partial dataset - 20%	87.0551	662.18	<.0001	α_1 = 5.8942 α_5 = 21.4143	5.4105 6.3779 5.3016 37.5270
Complete dataset	42.6584	6,541.81	<.0001	α_1 = 8.1400 α_3 = -0.3740 α_5 = 28.7027	7.0835 9.1965 -0.5508 -0.1971 22.5420 34.8634

Table 32b. Comparison of the analysis of variance outcomes for Equation [20] fit with the Hatch partial and complete datasets for thinned plots.

Table 32b indicates some important points. There is an extreme reduction of the *F*-value and, respectively, increasing the value of the mean square error when the selected Equation [20] was fit with twenty percent of the Hatch dataset. For the same partial dataset α_3 lost significance, α_1 gained quite different value and α_5 became less stable with wider confidence limits than the other two outcomes. The reason for this is most likely due to the small sample size and the previously noted high variability in this dataset.

The comparison of the selected Equation [20] form for thinned stands fit with the Hatch partial and complete datasets also reveals that the eighty percent partial dataset and the complete dataset have similar outcomes. The mean square error values for these two sets are not substantially different from each other. The coefficient estimates are also very similar (in fact, almost the same) with quite narrow confidence intervals.

Another important point is the non-significance of the coefficient for the thinning modifier (α_6) across all datasets. After a closer examination of the Hatch data, some of the thinned plots were measured only once after the initial thinning. This could be one of the reasons that α_6 did not gain significance. Another reason could be the highly variable dataset. Since no major flaws were detected among the analysis of variance outcomes for the partial and complete datasets, the selected Equation [20] form for thinned stands can be applied for modeling the Hatch thinned plots basal area growth.

3.5.3 Allegheny National Forest Study

Unthinned plots

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits
Partial dataset - 80%	22.0007	30,528.4	<.0001	α_1 = 3.3537 α_3 = 0.4469	2.6575 4.0500 0.2825 0.6112
Partial dataset - 20%	35.2935	10,256.2	<.0001	α_1 = 5.0273	4.6645 5.3902
Complete dataset	23.3096	36,580.4	<.0001	α_1 = 3.5668 α_3 = 0.3924	2.9911 4.1426 0.2548 0.5300

Table 33a. Comparison of the analysis of variance outcomes for Equation [20] fit with partial and complete Allegheny National Forest study datasets for unthinned plots.

Table 33a indicates that the mean square error values for Equation [20] fit with eighty percent partial and complete datasets are very similar, while the mean square error for Equation [20] fit with twenty percent partial dataset is higher. Across all datasets, α_1 remained significant while α_5 remained non-significant and was removed. This outcome is similar to the outcome of the selected Equation [20] form for unthinned stands fit across all different studies, when the same coefficient was not significant. When the selected Equation [20] was fit with twenty percent partial dataset, α_3 also lost significance and was removed.

Table 33a also indicates that the values of the coefficients obtained from the eighty percent partial and complete Allegheny National Forest study datasets are very similar with narrow confidence limits. After this analysis, the selected Equation [20] form for unthinned stands can be useful when applied for basal area growth modeling in the Allegheny National Forest unthinned plots since no major flaws were detected. The case of losing α_3 coefficient when the selected equation was fit with twenty percent partial dataset was not considered as a major drawback, since the reason for this was due to the small sample size and highly variable stand density rather than a reduced performance of the selected Equation [20] form for this particular dataset.

<u>Thinned plots</u>

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits	
Partial dataset - 80%	11.8439	55,528.1	<.0001	α_1 = 4.3015 α_3 = 0.4573 α_5 = 13.1343 α_6 = -37.6458	3.5430 0.3088 1.5959 -72.8835	5.0599 0.6058 24.6728 -2.4081
Partial dataset - 20%	7.6904	31,551.4	<.0001	α_1 = 5.9684 α_5 = 55.7820 α_6 = 63.1814	5.5946 41.5648 29.4874	6.3422 69.9992 96.8754
Complete dataset	11.2383	99,614.8	<.0001	α_1 = 4.2247 α_3 = 0.3981 α_5 = 19.3728	3.5517 0.2761 9.8169	4.8976 0.5201 28.9287

Table 33b. Comparison of the analysis of variance outcomes for Equation [20] fit with partial and complete Allegheny National Forest study datasets for thinned plots.

Table 33b indicates that the values of the mean square error for the selected Equation [20] form for thinned stands fit with the eighty percent partial and the complete Allegheny National Forest study datasets are about the same, while the outcome from the twenty percent partial dataset indicates much smaller value. The α_1 and α_5 coefficients remained significant across all different datasets, but α_1 is more stable than α_5. An important point here is the fact that when the selected model form was fit using eighty percent partial dataset all coefficients remained significant, indicating the significance of all variables that Equation [20] contains for this particular dataset. The other two outcomes indicated a non-significant parameter that was ignored.

In the case of α_6 becoming non-significant when the selected Equation [20] form for thinned stands was fit with the complete Allegheny National Forest dataset, the reason could be found from the fact that it has negative value in the outcome from the eighty percent partial dataset and positive value in the outcome from the twenty percent partial dataset. This indicates a substantial difference of the thinning intensity applied in the plots within each of these two partial datasets.

The results from the final comparison table (Table 33b) indicate that the selected Equation [20] form for thinned stands can be successfully applied for basal area growth modeling in the Allegheny National Forest study thinned plots since no major projection flaws were detected.

3.5.4 OAKSIM Study

Unthinned plots

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits
Partial dataset - 80%	15.7632	119,156	<.0001	α_1 = 3.4451 α_3 = 0.4367 α_5 = -8.8218	3.0159 3.8744 0.3436 0.5298 -11.1818 -6.4619
Partial dataset - 20%	47.9715	10,574.0	<.0001	α_1 = 6.2353 α_5 = 38.2065	5.7401 6.7304 21.5815 54.8314
Complete dataset	22.9333	96,636.0	<.0001	α_1 = 3.4940 α_3 = 0.4183 α_5 = -8.1075	3.0133 3.9746 0.3154 0.5212 -10.7455 -5.4695

Table 34a. Comparison of the analysis of variance outcomes for Equation [20] fit with partial and complete OAKSIM datasets for unthinned plots.

Table 34a indicates some noticeable fluctuation of the mean square error values across the three different datasets. The selected Equation [20] fit with the eighty percent partial dataset for unthinned plots contains the smallest mean square error, while it increases more than three times when fit with twenty percent partial dataset. Such increase reflected in some extreme changes in the *F*-value for the former two datasets. This could be due to the loss of one of the coefficients and hence higher amount of unexplained variation of basal area growth within the twenty percent partial dataset.

Table 34a also indicates that most of the coefficient estimates remained significant. Only the outcome from the twenty percent partial dataset indicates one non-significant coefficient (α_3) that was removed. Also the coefficient estimates for the eighty percent partial and complete datasets are almost identical, while the coefficient estimates for the twenty percent have different values where α_1 almost doubles and α_5 changes its sign. Even though, no major projection flaws were detected and the selected model form for unthinned stands behaves well across all datasets. Hence, the selected Equation [20] form for unthinned stands can be successfully applied for basal area growth modeling in the OAKSIM unthinned plots.

Thinned plots

Dataset	MSE	*F*-value	*p*-value	Coefficient estimates	95% Confidence Limits
Partial dataset - 80%	6.3358	92,054.3	<.0001	α_1 = 4.0965 α_3 = 0.6571 α_6 = -79.5650	3.5343 4.6587 0.4801 0.8342 -107.3 -51.8370
Partial dataset - 20%	3.4043	73,621.3	<.0001	α_1 = 3.2866 α_3 = 0.8964 α_6 = -99.1657	2.7802 3.7929 0.7709 1.0218 -127.6 -70.7773
Complete dataset	5.7850	177,747	<.0001	α_1 = 3.8527 α_3 = 0.7292 α_6 = -85.9820	3.5627 4.1427 0.6616 0.7968 -99.7485 -72.2156

Table 34b. Comparison of the analysis of variance outcomes for Equation [20] fit with partial and complete OAKSIM datasets for thinned plots.

The analysis of variance results for the selected Equation [20] fit with the OAKSIM partial and complete datasets for thinned plots indicate a good fit. The value of the mean square error for the selected model fit with twenty percent of the OAKSIM dataset for thinned stands is the smallest compared to the rest. It almost doubles when the model was fit with eighty percent of the OAKSIM dataset, indicating higher variability in basal area growth in this dataset. Otherwise, the values of the mean square error are low and the *F*-values are high for all datasets, indicating a very good fit.

Table 34b also indicates that one of the coefficient estimates (α_5) was non-significant across all datasets and was ignored. The thinning modifier coefficient (α_6) remained significant in all datasets. Each dataset has equal number of significant coefficients that are quite stable, where only the thinning modifier coefficient (α_6) contains wider confidence limits. The results above indicate that the selected Equation [20] form for thinned stands can be successfully applied in the OAKSIM study thinned plots for basal area growth modeling.

3.6 Final Conclusion for the Best Model Basal Area Growth Projection Performance

The evaluation of Equation [20] proved its usefulness for the purpose of basal area growth modeling across all different studies. Even though the analysis of variance indicated some noticeable differences between the partial and complete dataset outcomes, the selected model performed relatively well without any significant offsets. None of the forms fit with partial and complete datasets became statistically non-significant (since *p*-value remained very small for all forms) or without any significant coefficients. The loss of thinning modifier in some of the studies was due to the stand characteristic and the intensity of the thinning rather than a poor performance of the modifier itself. The biggest differences were found between the twenty percent partial datasets and the other two datasets. These differences were due to the small sample size in the twenty percent partial datasets rather than a drawback in the selected model performance.

The comparison of the OAKSIM dataset outcomes (Tables 34a and 34b) indicated better fit for Equation [20] then the rest of the studies. After closer examination of the OAKSIM dataset, the reason for such a good outcome is most likely due to the age of the stands (mid-aged to old growth), the large sample size, and the longer periods of re-measuring the sample plots characteristics with higher frequency (often annually) for both treatments than the rest of the studies.

Often the outcomes from the eighty percent partial datasets and the complete datasets were similar in most of the studies in terms of mean square error value and number of significant coefficients. Also some datasets have highly variable data that altered the outcomes between the partial and full datasets. The coefficient estimates for the selected Equation [20] are summarized by study in Table A-5 (Appendix A).

CHAPTER 4: SUMMARY

The examination of several basal area projection models, originally developed for pine plantations, indicated that all of them can be successfully applied to central hardwood re-measurement data, collected from a variety of different studies across the central Appalachian region. All equations proved to be useful when fit to hardwood data for the purpose of modeling basal area growth. The analysis indicated that some models have better characteristics than others when applied to unthinned and thinned stands. The inclusion of additional variables in some of these forms (such as site index) did not improve their overall projection precision, but in others (such as stand density) it substantially reduced the amount of unexplained variation for both treatments.

The models without an explicit thinning modifier (Clutter and Jones (1980), Burkhart and Sprinz (1984), Clutter (1963), and Pienaar *et al*. (1990)) were still able to explain a substantial amount of variation of basal area growth for the thinned stands. The models that contain an explicit thinning modifier (Pienaar *et al*. (1985), Pienaar *et al*. (1989), and Brooks (1992) both forms), had, in general, better characteristics than the forms that lack a thinning modifier, in terms of reducing the mean square error. In the case of Equation [16] (Pienaar *et al*. (1990)), however, lacking a thinning modifier did not necessarily reduced the basal area projection precision for the thinned stands. This indicates the importance of the thinned stands characteristics and the right approach of selecting the variables that define a particular thinning modifier.

Since some of the selected models were modified under the conditions of this project, including the best basal area projection model (Equation [20]), better forms could have been achieved if more data was available. The lack of long term tree height measurements for most of the plots resulted in removal of the average dominant height from all the models that include this variable. When fit with the part of the data that contains average dominant height, the selected basal area projection model (Equation [20]) form for unthinned stands had much smaller mean square error than the form that was used for this study (without the tree dominant height variables), even with a smaller sample size. It also contained only significant coefficients. This indicates that inclusion of the average dominant height variable could have produced a better model form for unthinned stands. On the other hand, after similar tests were performed, the rest of the basal area projection model forms that contain average dominant height as a predictor, did not exhibit any improvements. This indicates the complexity of basal area growth modeling within the data, used in this study, and the importance of selecting the proper type of basal area growth projection equation.

This study was performed to select a basal area growth projection model that can be applied to a variety of central Appalachian hardwood unthinned and thinned stands in order to facilitate some important forest management issues. One such issue is when forest managers and forest land owners want to determine the right thinning intensity and the age of thinning in order to achieve optimal basal area growth. The right thinning practices can improve the overall value of many hardwood stands by reducing the stand density in order to improve the growing space of the remaining trees. Basal area projection equations can also be used for developing volume projection equations for unthinned and thinned hardwood stands, which are another important part of the contemporary forestry. Such equations can be used for modeling volume growth and to evaluate the effect of thinning practices in order to achieve an optimal stand structure. Finally, the basal area and volume projection equations can be used to evaluate the cost-effectiveness of the thinning practices when applied in the central Appalachian hardwoods.

This study was also performed to compare the selected whole stand basal area projection model for central Appalachian hardwood stands (Equation [20]) with some of the existing, locally developed basal area projection equations and growth and yield simulators. Similar study was performed in order to evaluate and compare projection accuracy for three available, software based growth and yield simulators (SILVAH, the Forest Vegetation Simulator (FVS), and the Stand Damage Model (SDM)), that used the same hardwood data as for this study (Brooks and Miller 2011). The two studies can be further compared in order to assess the outcomes and to determine if the best basal area projection model (Equation [20]) for hardwood stands from this study exhibits better projection accuracy than the existing growth and yield simulators.

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Appendix A

Table A-1: Analysis of variance for the treatment categorical variable.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	182	37,290.2	204.89	10.15	<.0001
Error	2,152	43,442.6	20.19		
Uncorrected Total	2,334	80,733.5			

R-Square	Coeff Var	Root MSE	Dependant Variable	δB Mean
0.4619	110.554	4.493	δB	4.064

Source	DF	Anova SS	Mean Square	*F*-value	*p*-value
δA	13	20,967.5	1,612.88	79.90	<.0001
δN	168	15,080.3	89.76	4.45	<.0001
TRTM	1	1,243.1	1,243.10	61.58	<.0001

δB = B2 – B1

δA = A2 – A1

δN = N2 – N1

Table A-2: Analysis of variance for the study categorical variable.

Source	DF	Sum of Squares	Mean square	*F*-value	*p*-value
Model	184	49,592.9	269.53	18.61	<.0001
Error	2,150	31,140.7	14.48		
Uncorrected Total	2,334	80,733.5			

R-Square	Coeff Var	Root MSE	DB Mean
0.6143	93.64	3.806	4.064

Source	DF	Anova SS	Mean Square	*F*-value	*p*-value
δA	13	20,967.5	1,612.88	111.36	<.0001
δN	168	15,080.3	89.76	6.20	<.0001
Study	3	13,545.0	4,515.01	311.72	<.0001

Table A-3: Fisher's multiple comparison procedure (least significant difference) for the study categorical variable

Alpha	0.05
Error Degrees of Freedom	2,150
Error Mean Square	14.48
Critical Value of *t*	1.96

Comparisons significant at the 0.05 level are indicated by ***.

Study Comparison	Difference Between Means	95% Confidence Limits	
2 - 1	6.6994	5.8036	7.5952 ***
2 - 3	9.5068	8.6787	10.3349 ***
2 - 4	10.8275	10.0617	11.5932 ***
1 - 2	-6.6994	-7.5952	-5.8036 ***
1 - 3	2.8074	2.1868	3.4280 ***
1 - 4	4.1281	3.5935	4.6626 ***
3 - 2	-9.5068	-10.3349	-8.6787 ***
3 - 1	-2.8074	-3.4280	-2.1868 ***
3 - 4	1.3206	0.9094	1.7319 ***
4 - 2	-10.8275	-11.5932	-10.0617 ***
4 - 1	-4.1281	-4.6626	-3.5935 ***
4 - 3	-1.3206	-1.7319	-0.9094 ***

Table A-4: Coefficient estimates for the selected basal area projection models fit to the hardwood re-measurement data for all physiographic regions.

Equation	Treatment	Coefficient estimates						
		α_0	α_1	α_2	α_3	α_4	α_5	α_6
[1]	Unthinned		1.0866	5.2977				
	Thinned		0.5928	6.2766				
[2]	Unthinned		5.3546					
	Thinned		4.6540	0.0112				
[14]	Unthinned		5.3546					
	Thinned		4.6540		0.0112			
[6]	Unthinned	2.9877		1.3569				
	Thinned	0.7618	0.3047	2.2293				
[15]	Unthinned		-51.4805			3.7643		
	Thinned		-127.2	-0.0846		15.4584		-1.0597
[16]	Unthinned		4.6887		0.1371			
	Thinned		5.4158		0.0928		17.6678	
[18]	Unthinned		-51.4802				3.7643	
	Thinned		-201.0		-0.1482		24.1937	71.8489
[20]*	Unthinned		4.6887		0.1371			
	Thinned		4.9898		0.2489		12.3689	-19.4671

* the best basal area projection equation

Table A-5: Coefficient estimates for the best basal area projection Equation [20] fit separately by study.

Study	Treatment	Coefficient estimates			
		α_1	α_3	α_5	α_6
MWVCO	unthinned	5.6246		5.5714	
	thinned	2.0742	0.5950	-4.9693	
HATCH	unthinned	5.8254		12.5675	
	thinned	8.1400	-0.3740	28.7027	
ANF	unthinned	3.5668	0.3924		
	thinned	4.2247	0.3981	19.3728	
OAKSIM	unthinned	3.4940	0.4183	-8.1075	
	thinned	3.8527	0.7292		-85.9820