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Impact of Inquiry-Based, Question-and-Answer Instruction in High-Enrollment Classes

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ABSTRACT

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Lecturing is a common way to teach large classes, especially in mathematics. Other styles of instruction have been proven to be more effective in small classroom settings, but those styles are not always practical (or even feasible) in classes with 200+ students. The lecturing dialogue primarily exists at the intersection of Spectrum Theory and the Socratic Method, utilizing inquiry-based learning within the realm of active learning, experiential learning, and constructivism to appeal to learners with various learning styles, which helps students to discover new information (specifically, why things occur in the way that they do) and make connections between old and new material. Spectrum Theory outlines different teaching styles, based on how much a specific style is teacher-centered versus student-centered. The lecturing dialogue combines several of the different approaches from the Spectrum of Teaching Styles, specifically lecture, tutoring, instructional conversation, inquiry-based learning, and guided discovery. It utilizes the Socratic Method to turn students into active learners through constructivism and experiential learning. A high-enrollment, college algebra class using a standard lecture significantly outperformed a class using the lecturing dialogue on labs, surveys/questionnaires, and final grade, but not on final grades without the labs. The labs could be worked on outside of the class, so factors beyond the teaching style may have influenced the results. There was not significant difference between the classes in regards to attitude (enjoyment, motivation, value, and self-confidence). Overall, it appears as though the teaching style has no real impact on either student performance or student attitude in large college algebra classes.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

List of Abbreviations

ACT – American College Test

ATMI – Attitudes Toward Mathematics Inventory

MBTI – Myers-Briggs Type Indicator

SAT – Scholastic Aptitude Test

STEM – Science, Technology, Engineering, Mathematics

Chapter 1 – Introduction

The foundation for this study was laid in the late 1980s and early 1990s when my youngest sister was learning to read. At first, I simply told my sister what sounds certain letters made when she had difficulty. She was pretty crafty, so she learned fairly quickly that she did not have to know the sounds of the letters because I would provide the answers when she could not remember something. As a result, she did not appear to try very hard to learn anything. That prompted a change in my strategy to assist her. I asked my sister what sounds she associated with each letter, and later prompted her to sound out the words using phonetics. Her improvement was astounding. She still struggled at times, but she made a lot more progress than before. I occasionally had to correct her mispronunciations, and ask her to try again, but she learned to read. This was my first observation as a teacher of the power of active learning.

I was always a good student, so I often tutored other students when I was in high school. Some were in my classes or classes that I had taken (mainly math, chemistry, and physics), but a few asked me about classes that I had not taken yet (such as probability and statistics). My earlier observations when helping my sister were reaffirmed: students often learned better when they were required to think on their own, as opposed to me merely telling them how to do a problem. Some of these tutoring situations required me to ask the other students questions. When I tutored students in a lower math classes, I knew of more advanced methods to solve a problem, but those methods were either not addressed in their class at all or would not be addressed until later in the course. I had to probe the students to find out what methods they knew and did not know. This process of questioning became an even more vital technique when I helped students in classes that I had not taken. I knew how to do some of the material, but not all of it. I asked the students to go as far as they could go in solving a problem and had them explain why certain steps were done so that I could understand the problem more. From their responses and reading their book, I could usually figure out the next step (as well as why to take that step).

When I went to college, I tutored as a part-time job. As with high school, I asked the students what they had learned in order for me to explain the material on the appropriate level, and to sometimes determine what the correct procedures were. After tutoring some students a few times, I got to know a little about them. I used that interpersonal knowledge to better explain concepts, using both personal information and examples relevant to the students' majors. For instance, average could be explained by batting average for a baseball lover, completion percentage for a football fan, or points per game for a basketball aficionado.

Over the years, I developed a specific tutoring style. I asked a lot of questions, mainly to ascertain what the student knew, but also to get the student to think about the material. I tried to build a bridge from what the student knew well to the material that he/she could not grasp. By taking incremental steps, the student could make the necessary connections, and be comfortable with each step. Beyond the technical aspects of teaching/tutoring, I learned to read body language and facial expressions. With some students, I could tell when they were struggling by a look that they gave or a nervous habit that they exhibited. For example, one student bit his lip when he started to get overwhelmed. Another would look up at me and cross her eyes when I said something that she didn't understand.

I initially studied electrical engineering in college, not education. After my junior year, I realized that I did not want to be an engineer for the rest of my life. I finished the bachelor's degree in electrical engineering, but needed something else to do with my life. I had a minor in mathematics from my electrical engineering work, and I had been tutoring, so I got a second bachelor's degree in mathematics with a secondary education emphasis. My teaching style in the classroom largely mirrored my style as a tutor: I asked questions and let the students make the connections, accompanied by prompts from me when the class seemed perplexed. That strategy worked fairly well with high school students in classes of about 25-40 students.

While pursuing my master's degree in mathematics, I held a graduate teaching assistantship that required me to teach one class each semester. The classes each had about 30 students, so I used the same style that I had employed when teaching high school. After receiving my degree, I was hired to teach full time. The class sizes varied between 80 and 300 students (with the majority having between 150 and 200), but I did not change how I taught. The methods had worked before, so I figured that they would work again, despite the increase in enrollment. The students' grades and the comments on the student evaluations indicated that my assumption was correct. The students did relatively well, and many students liked the question-and-answer format of the class.

As a student in both high school and college, I took a number of discussion-based honors classes (which had anywhere from 10-30 students in them), that were much more engaging, entertaining, and enjoyable than the other lecture-based classes (which typically had enrollments of 25+). I found that the honors classes kept my attention because I was forced to think. I was expected to offer my opinion and justify it, so I had to pay attention to what was being said. Furthermore, I was given the opportunity to participate, which helped me to learn. By offering my opinions and ideas, I was able to get instantaneous feedback and immediately rectify any misconceptions that I held. These experiences were reinforced by the observations that I had as a teacher/tutor, and solidified my beliefs in what I was doing.

During my life, whether in school and outside of school, I always learned more through experience (often trial-and-error) than from merely being told something by someone. It might have taken more time, but I learned the lesson deeper and stronger when I did it myself than when I was just told it. These encounters with discovery learning had an impact on me. It did not matter what the situation was. Whether I was working through a calculus problem in which I was unsure about what to do (so I tried various possible procedures), building a radio in an engineering class (and creating solder bridges that caused the radio to malfunction), taking apart a broken VCR to try to fix it (I found the broken part, but could not repair it), or building a ramp for sled riding (sometimes too low, sometimes too high), learning through experience ingrained the lesson in my head better than other ways, mainly because I was able to observe why things worked in a certain way and why they would not work in a different way (or how they worked differently). This was especially true in math, engineering, and science (mainly physics and chemistry).

Throughout my college career, I took many classes in various subjects, including philosophy, psychology, and education. We learned about different educational theories and techniques, along with the philosophical and psychological justifications for them. However, my teaching style was largely established by the time I learned about them. I later found validation for what I did, but everything developed organically and naturally. My teaching style was solely based on observations of what worked and what did not work (again, discovery learning played a part). Over time, I revised my strategies and techniques, but those changes were dictated by the students' reactions, comments, and grades.

After some contemplation and reflection, I realized that I needed to further analyze what I did in the classroom. It was an amalgamation of my experiences as a teacher, tutor, student, and budding engineer, a teaching style that I dubbed the "lecturing dialogue." It combined various teaching styles, most prominently lecturing, tutoring, instructional conversation (Goh, Yamauchi, & Ratliffe, 2012), guided discovery (Ashworth, 2008; Bruner, 1962), and a form of the Socratic Method called inquiry-based learning (Pasch, Sparks-Langer, Gardner, Starko, & Moody, 1991), and incorporated aspects of experiential learning and constructivism (types of active learning). I lectured at times (mostly when introducing more difficult new material that was fairly complex or when students were perplexed), but mainly relied on questioning the students. I wanted students to make the connections between old and new material, trying to take the next step before I did. The questions that I asked varied in specificity, ranging from vague and open-ended to pointed and precise (depending on the responses that the students offered).

From both my time as a student and observations of fellow instructors (as well as a few conversations), I noticed that many other instructors opted to use a standard lecture, rather than the interactive teaching style that I employed. Research confirmed my impressions that for high-enrollment classes (40 students or more), lecturing is the most common form of instruction (Tolley, Johnson, & Koszalka, 2012), particularly in postsecondary education and health care education (Lake, 2001).

Despite being the most efficient method of imparting knowledge to a large number of students while using minimal resources (Lake, 2001), lectures are not always effective because they do not engage students, who are not usually asked to think much on their own, and come to expect this lack of thinking during class (Udovic, Morris, Dickman, Postlethwait, & Wetherwax, 2002). Too often, students simply become scribes, copying notes from PowerPoint slides or the chalkboard. As a result, learning becomes passive (Swaak, de Jong, & van Joolingen, 2004; Lake, 2001). Ausubel (1977) referred to this as reception, in which the students only internalize what is presented. Sadly, many college students “expect to sit passively and listen to a professor ‘profess’” (Smith, 1996, p. 71). This is especially true in high-enrollment, college algebra classes that are required for many majors. Based on the researcher’s observations and student comments, of the students who take it, very few seem to want to be there. It appears as though the vast majority dislike mathematics and merely try to get through the class. They learn the minimum to get by, often barely knowing how to solve the problems, and never truly understanding why things work the way that they do. Students who come from smaller high schools (or high schools that had smaller classes) may face additional challenges. Oliver (2007) found that these students tend to be unfamiliar with the structure and atmosphere of high-enrollment classes, and may find them impersonal.

All of this led me to ponder the true effectiveness of the lecturing dialogue. All of my experiences as both a learner and a teacher suggested that active, constructive, inquiry-based learning built around some form of discovery learning worked better than passive learning. Were my observations correct? Were my thoughts verifiable? What were the tangible and measurable results? I sought a way to find out.

Statement of Research Questions

There were two main questions for this study: 1) Does an inquiry-based, question-and-answer format of instruction lead to higher grades than a standard lecture in a high-enrollment mathematics class? 2) Does an inquiry-based, question-and-answer format of instruction lead to improved attitudes towards mathematics in a high-enrollment mathematics class as compared to a standard lecture? Attitudes can be broken into the sub-categories of a) enjoyment, b) motivation, c) value, and d) self-confidence.

Statement of the Hypotheses

The lecturing dialogue is a teaching style that combines elements of several different styles of teaching (including lecture, instructional conversation, guided discovery, inquiry-based learning, and tutoring) and utilizes the inquiry aspect of the Socratic Method. The lecturing dialogue aims to mold students into active learners through constructivism and experiential learning, as opposed to the passive learners often seen with a classic lecture. It was hypothesized that students in a high-enrollment mathematics classroom (200+ students) who were taught using the lecturing dialogue would perform better in a college algebra class (as measured by final grades) than those who were taught using a standard lecture. Additionally, it was hypothesized that the students who were taught using the lecturing dialogue would experience greater enjoyment, more motivation, higher value, and increased self-confidence than students who were taught with a standard lecture.

History of Research in Mathematics Education

Before proceeding with the typical literature review, the framework of this study needs to be detailed, which will help to explain and define many of the terms and concepts that will be discussed. However, the history of research in mathematics education should be chronicled first to establish what has occurred in the past and how things have evolved. After all, in order to determine where one is going, one must know where he/she is and where he/she has been.

Mathematics education can be traced to ancient philosophy, dating from when the Socratic dialogues were first recorded in Plato's *Meno* around 380 B.C.E. (Schoenfeld, 2016). The Platonic concept of knowledge was that it already existed in the world as an ideal, waiting "to be revealed ('recollected' with guidance) but not discovered" (p. 498) by individuals who already had the knowledge within them. Others disagreed, believing that knowledge was discovered. Namely, Aristotle, John Locke, and Jean-Jacque Rousseau believed that humans were born as a *tabula rasa* (a blank slate), and that people need to acquire knowledge. The debate over discovering knowledge versus uncovering (or remembering) knowledge has raged over the years.

Despite the early beginnings of mathematics education, wide-spread research did not begin until the late-19th and early-20th centuries. Schoenfeld (2016) provided a very detailed summary of this period. At that time, mathematics education was primarily concerned with preparing students for a lifetime of work. High school was reserved solely for those who planned to attend college. Research did occur, and many studies even focused on mathematics. However, the research lacked the structure and organization that a professional organization (and the accompanying journals) could provide. This was soon to be remedied.

The American Mathematical Society (AMS), which focused on mathematics at the college level, was founded in 1888 (Archibald, 1938). The National Society for the Study of Education (NSSE) came into existence in 1901, and was founded, in part, by John Dewey. The purpose of the NSSE was a more general improvement of educational research (Schoenfeld, 2016). The Mathematical Association of America (MAA) formed after it broke from the AMS in 1915, largely to support *American Mathematical Monthly* (Mathematical Association of America, 2017). The American Educational Research Association (AERA) followed in 1916, and then the National Council of Teachers of Mathematics (NCTM) in 1920. These new scholarly organizations served to bring some structure and perspective to education, which led to standardized tests being administered by the College Entrance Examination Board in 1926 and a more regulated curriculum for training teachers (Schoenfeld, 2016).

In an attempt to capitalize on the procedural nature of arithmetic and algebra, and building on behavioralist psychology, Raleigh Schorling (NCTM president from 1924-1926) detailed 20 rules for mathematical drill (Schoenfeld, 2016). Apparently unconvinced by Schorling's list, Knight (1930) lamented in the NSSE Yearbook that "a mathematical description of an arithmetic process does not yield the kind of information about that process which is an essential basis for its instruction to children" (p. 162). Math requires more than merely memorizing rules; it demands thinking. While the general focus remained on procedure, Harold Fawcett took mathematics in another direction, seeking "to find a way not only to teach the important facts of geometry but also to acquaint the pupil with the kinds of thinking one needs in life situations which can best be learned by a study of geometry" (Reeve, 1938, p. v). As Schoenfeld (2016) elegantly put it:

> Geometry was not about Platonic truths handed down from generation to generation to be memorized or mastered; it was a rational human creation in which people made carefully considered definitions, from which certain conclusions followed. Fawcett saw his task as being the initiation of students into this culture of *doing* mathematics (p. 502).

If students belong to the culture of mathematics, then they will be more likely to become engrossed in the why's and how's of math, which will only intrigue them more and entice them to study it more. The more students learn, the more they want to know. It becomes the snowball gaining both momentum and mass as it rolls down a steep hill.

Following World War II, the demographics and purpose of education changed. The Servicemen's Readjustment Act of 1944 (more commonly known as the G.I. Bill) allowed more individuals to attend college. High school graduates among United States citizens rose from 25% in 1940 to 60% in 1974, and then 88% in 2014 (Schoenfeld, 2016). This forced a readjustment in the curriculum, with more advanced classes being offered earlier in students' educational careers. Instead of calculus being taught to college juniors, it was offered to high school seniors who were interested in science, technology, engineering, and mathematics (STEM) careers (Schoenfeld, 2016). Part of this was related to social issues, particularly international strife (cold wars, hot wars, economic wars, and technological wars). The Cold War and the space race prompted a reinvigorated focus on STEM classes (particularly, the National Defense Education Act of 1958 following the launch of the Soviet satellite Sputnik). The "New Math" of the 1960s was eventually discredited, to be "replaced by the 'back to basics' movement in the 1970s" (p. 503).

In the realm of research, George Polya (channeling the procedural focus of Schorling) introduced heuristic strategies (rules of thumb that work most of the time) in his 1945 work *How to Solve It* (Schoenfeld, 2016). However, change was imminent. Fawcett (1951) proposed that research be conducted in actual classrooms and focus on the thought processes of the students as they learned. Lahti (1956) also commented that a "step-wise solution is a highly stereotyped procedure and is probably not too effective" (p. 149). Rather, students needed more of an open mind to account for (and adjust to) different situations and circumstances. A fixed, step-by-step process was too limiting, and not adaptable.

Jean Piaget published *The Child's Conception of Number* in 1952 and *The Child's Conception of Geometry* in 1960. However, the ideas espoused in these works about how children conceptualize mathematics did not become mainstream in the United States until the late-1960s and early-1970s (Schoenfeld, 2016). Building on the idea that children develop mentally as they get older, Bruner (1960) introduced the "spiral curriculum," in which topics are consistently revisited in greater depth as students grow and become capable of understanding the topics on that deeper level. Despite the advancements made in cognitive study, both students and student thinking remained afterthoughts in the 1970 NSSE Yearbook (Schoenfeld, 2016). Overall, the research on mathematics education was wide-spread, with no discernable overarching focus (Schoenfeld, 2016). The *Journal for Research in Mathematics Education* (*JRME*) was also first publishing in 1970, and soon gained prestige as the leading journal in mathematics education.

The intellectual and research turmoil of the early-1970s calmed a bit by 1980. As described by Schoenfeld (2016), advances in computers and computer programming led to a greater focus on how the mind works, making the behaviorism of Skinner obsolete. Both humans and computers use similar problem-solving strategies, so research on such topics as self-regulation and metacognition were legitimized, even expanding beyond mathematics to reading and writing. Of particular interest was problem-solving failure: individuals obtaining an incorrect answer despite sufficient knowledge. Put simply, students should be able to solve a problem, but do not succeed.

Prior to the 1980s, research did not always agree with classroom results: “because of the differences in context, the results of laboratory research typically failed to apply meaningfully in classrooms” (Schoenfeld, 2016, p. 509). Studies began to investigate both teachers and teaching, with teachers gaining more respect as “problem solvers and decision makers” (p. 509), resulting in more reliable research in which the predicted results agreed with the actual results.

The end of the 1980s brought the beginning of the standards movement. Spurred, in part, by the reaction to (controversial and now-disputed) claims in *A Nation at Risk* (published in 1983) that the United States’ educational system was lagging behind other countries in the world, the NCTM published *Curriculum and Evaluation Standards for School Mathematics* in 1989, which reverted to the procedure perspective of the early-19th century (Schoenfeld, 2016). This paved the way for President George W. Bush's No Child Left Behind, President Obama's Race to the Top, and the Common Core State Standards. Both No Child Left Behind and Race to the Top produced the unintended consequences of a high-stakes competition for federal funds, resulting in the math wars, which were heated debates that arose from the standards movement about different mathematics curricula (Schoenfeld, 2016). Unfortunately, no data existed to support the claims of any of the curriculum makers in the math wars. Even worse, no curriculum was created for the Common Core State Standards, so no educational materials were available for instructors, which forced a furious scramble to assemble anything that was available (Schoenfeld, 2016).

Research in mathematics education changed along with the political policies in the 1990s and 2000s, accompanied by a general growth in education research (Schoenfeld, 2016). In particular, research has shown that a teacher's beliefs about both the students and the content that is taught can have an impact, regardless of the subject (Schoenfeld, 2016). The "social turn" (Lerman, 2000), also known as the "sociopolitical turn" (Gutierrez, 2013), recognized the impact that social and cultural issues can have on students. As Schoenfeld (2016) put it: "Different aspects of people's identities become manifest in different contexts, including mathematics classrooms" (p. 513). Basically, the experiences that a person has in life (both inside and outside of the classroom) play a part in shaping that individual's relationship with math. Prior knowledge, a sense of self, the classroom environment, and even family traditions can all impact whether one's experience is positive or negative (Schoenfeld, 2016).

By the mid-1990s, theories of mathematical learning had emerged and gained prominence (Steffe, Nesher, Cobb, Goldin, & Greer, 1996). An epistemological shift occurred, acknowledging that both the content and the process were important in mathematics, and metacognition became more widely accepted as a central part of cognition (Schoenfeld, 2016). An increase in the means for gathering and analyzing data naturally led to findings becoming more numerous, which allowed for more connections to be made among previous discoveries (Schoenfeld, 2016). Research shifted from laboratories to classrooms, with "the TeachingWorks project at the University of Michigan and the Teaching for Robust Understanding (TRU) framework developed by the Algebra Teaching Study and Mathematics Assessment Projects at the University of California at Berkeley, Michigan State University, and the University of Nottingham" (p. 515) being noted as examples.

One of the main concerns was building/rebuilding a corps of teachers that was equipped to handle the modern learning environment, in particular, the issues of diversity, assessment, and technology (Schoenfeld, 2016). Diversity is closely related to ensuring equal treatment for all students. Schoenfeld (2016) noted that the debate over assessment matched formative assessments (during a lesson) against summative assessments (at the end of a lesson). While summative assessment provides a means of ranking and comparing students, formative assessment affords a chance to correct any misconceptions and improve learning. Arguments over the usefulness and appropriate place for technology in education have largely been inconsequential because the technology changes so fast that the results from any study are quickly rendered antiquated by the latest advancements (Schoenfeld, 2016).

This brings us to the current day. Never before have researchers had so many assets and resources, with modern data-collection techniques, computer-based statistical analysis programs, and the most advanced and inclusive theories about the mind and learning. However, researchers also face a unique set of challenges, such as the ever-changing conditions produced by constantly evolving standards and policies that can impact what material is addressed and how it is taught, along with debates about which techniques are the most effective. Add in the pressure to make the next big discovery or find the magic quick fix for any perceived educational deficiencies, and a recipe for unprecedented stress is created. This is the price that is paid for progress.

To summarize its history, research in mathematics education has been through many changes since its beginnings in ancient Greece. Research was organized with the formation of professional societies in the late 1800s, but was stuck in behavioristic thinking until about the 1950s. Changes in society often governed the focus and advancement of mathematics education research (whether it was the GI Bill, the space race, the development of computers, the standards movement, or more recognition of diversity). Research seldom had a dominant focus and was very fluid. Advancements in technology brought new discoveries in research, but also caused problems due to pressure and complexity. The challenge of this study was to not get caught up in the trends or expectations of others. This study had a specific purpose, and addressing that purpose was its sole responsibility and focus.

Theoretical Framework

Now that the history of research in mathematics education has be explored, the framework for this study can be elucidated. This information is vital to understanding the prior research that will be presented in the literature review (Chapter 2).

Definition of learning. In order to study education, learning must be defined. Many authors have offered their versions of a definition, with varying results, potentially because of the complex nature of learning, which consists of several aspects and can be viewed from various perspectives, owing to "the multifaceted nature of understanding" (Hutchings, 2000, p. 12). Felder and Silverman (1988) offered a very succinct definition of learning: "a two-step process involving the reception and processing of information" (p. 674). The idea of learning being a process was echoed by Bain (2004), as well as Kolb and Kolb (2005), with new knowledge being constructed from either old knowledge or experiences. Students must form their own connections and ideas; it is not received or given to the students by the teacher. While Dewey (1938) noted this debate: "The history of educational theory is marked by opposition between the idea that education is development from within [student makes connections] and that it is formation from without [given by teachers]" (1938, p. 17), there is no doubt about how he felt: "the initiative lies with the learner" (Dewey, 1910, p. 29).

Biggs (1999) described learning as "a way of interacting with the world" (p. 60) because it can change one's conceptions of phenomena and, thus, how he/she sees the world. In order to learn, students must acquire the information and then make sense of it. "The acquisition of information in itself does not bring about such a change [in the conception of phenomena], but the way we structure that information and think with it does" (p. 60). Since a conceptual change is required, true learning occurs when a student's thoughts and actions are modified outside of the classroom (Bain, 2004). Hutchings (2000) also addressed this: "My questions would be about whether what happened in such-and-such a class influences the way students think in a next class or down the line somewhere" (p. 18). Kolb and Kolb (2005) wrote: "Learning is a holistic process of adaptation to the world" (p. 194), resulting from interactions with the environment that creates knowledge by resolving conflicts between opposing ideas. Taking this a step further, "learning is relearning" (p. 194), since experiences result in a re-evaluation of what one knows.

Adopting a performance-based perspective, Perkins and Blythe (1994) linked learning and understanding. The thinking is "that understanding is a matter of being able to do a variety of thought-demanding things with a topic – like explaining, finding evidence and examples, generalizing, applying, analogizing, and representing the topic in a new way" (pp. 5-6). The goal is to take the student beyond what he/she already knows by requiring a demonstrable act, what Perkins and Blythe (1994) dubbed an "understanding performance" (also called a "performance of understanding"). Tyler, Gagné, and Scriven (1967) elaborated on the distinction between knowledge and understanding: knowledge is a comprehension of information about various things (people, places, objects, concepts) and a grasp of how those things are interrelated, but understanding goes further, incorporating the use of that knowledge (potentially in situations that are new to the individual).

To foster the construction of knowledge (which is a prerequisite of learning), the right questions must be asked (Bain, 2004). The teacher needs to model this question-asking at first, helping the students to recognize which questions they should be asking. Questions allow people to index information, to store it in such a way that it can be retrieved by an individual. More questions allow the material to be indexed in more ways. "Better indexing produces greater flexibility, easier recall, and richer understanding" (p. 31). This questioning process leads to students learning the basic facts, but also promotes learning pertaining to how to think about and analyze those facts. Rather than just accepting the facts as true, students are encouraged to analyze why they are true (Bain, 2004).

Taking this all into consideration, learning will be defined for this study as the process of acquiring information or knowledge that results in a conceptual change. It must be noted that students make the choice to learn (Bain, 2004). The teacher cannot tell students that they must do something. In fact, the teacher cannot force the students to do anything.

Link between teaching and learning. Above and beyond having a mastery of the material being addressed, teachers must know their students, including "their learners' developmental stages, range of abilities, learning styles, needs, and interests" (Mueller & Mueller, 1992, p. 49). To do this, "[t]eachers must be skilled in a variety of instructional structures to meet the varying needs, abilities, and learning styles of their students" (p. 52), and then actually use those various approaches.

Lahti (1956) found that "teaching methods do differ significantly in their effectiveness in developing in the student the ability to use the scientific method" (p. 161). Roa and DiCarlo (2001) explored the phenomenon of different types of learners more deeply. Models and demonstrations help visual learners. Discussions, debates, and the accompanying questions are preferred by auditory learners. Kinesthetic learners favor hands-on activities, such as role playing and physical models. In general, active learning (which increases student involvement) reaches visual, auditory, kinesthetic, and tactile (hands-on) learners, whereas lectures cater only to auditory learners with good memories. Lectures also assume "that all students acquire the same information, presented orally at the same pace without dialogue with the presenter" (Roa & DiCarlo, 2001, p. 59), which clearly does not happen in real life.

Ertekin, Dilmac, and Yazici (2009) reported a direct correlation between teaching-learning style with both student performance and student attitude. Discrepancies between teaching style and preferred learning style can be detrimental to a student's academic achievement and attitude towards the subject matter, whereas consistency can benefit a student. Furthermore, based on the study by Ertekin et al. (2009), using specific teaching styles can help to ease different types of math anxiety in certain students. For example, teaching styles that appeal to either social-interaction learners (work with others) or authority learners (prefer expert guidance) can mitigate testing anxiety and math anxiety in everyday life. Using teaching styles preferred by tactile learners or visual learners can help to lessen anxiety about a math lesson and increase self-confidence in math.

Contradicting the recommendation by Ertekin et al. (2009) to match teaching style with learning style, other studies have found that (when provided a legitimate opportunity) students may benefit from exposure to different teaching styles, including styles that do not align with the students' natural abilities and common ways of thinking. Dunn, DeBello, Brennan, Krimsky, and Murrain (1981) quoted David Kolb as stating that "we should not deny students the opportunity to develop fully by only exposing them to educational environments that match their strengths" (p. 373). Rather than lowering the teacher's standards to meet the students' effort or expectations, Udovic et al. (2002) advocated helping students to amend their habits and actions to meet the new expectations that come with an unfamiliar teaching style. According to Ronald Schmeck, "periodically [and carefully] exposing students to contextual demands that do not precisely match their preferred styles" (Dunn et al., 1981, p. 373) can help "to avoid instilling in the student a feeling of incompetence" (p. 373). To accomplish this, the teaching style should approximate the students' preferred learning styles, but still provide some experience with the new style. This modicum of controlled chaos, as advocated by Luckie, Maleszewski, Loznak, and Krha (2004), can help the students to become more flexible in their learning and help to develop higher level skills, and can benefit students more than solely using their preferred styles (Dunn et al., 1981).

Barbe and Milone Jr. (1981) distinguished the differences between modality strengths and preferences: the preferred style is not always the student's strength, so it is not always the most effective style. However, Dunn and Carbo (1981) stated that increased academic achievement occurs when students are taught through their identified perceptual preferences. Jarrett (2018) found evidence that students mostly did not study in ways that aligned with their identified learning styles, as determined by VARK (Visual, Auditory, Reading, Kinesthetic), a popular instrument for that purpose. Even when students did use the appropriate study techniques, their grades were not significantly better than those of students who used other techniques (Jarrett, 2018). Of course, Jarrett (2018) did not clarify whether VARK identified the preferred learning style or the most effective learning style. Khazan (2018) went beyond doubting the effectiveness of matching teaching style and learning style to actually questioning whether learning styles exist at all. Rather, learning styles could more accurately be called "learning preferences" or "learning abilities" (Khazan, 2018). Instead of learning better by using pictures, the student merely liked pictures more than other representations.

Clearly, there is much debate and conflicting evidence surrounding teaching and learning styles. One confounding factor when using a new teaching style is the situational awareness of the class. "[S]tudents may not automatically recognize, and therefore may not immediately value, what they are learning, particularly when the learning goals and methods are unfamiliar" (Udovic et al., 2002, p. 280). Therefore, it is imperative that the reasoning, purpose, and goals of any new style be explained to the students, both at the beginning of the course and throughout the semester.

McDermott (1993) discussed six generalizations about teaching and learning, specifically as they applied to physics classes that were taught using a style in which general principles were presented in addition to the traditional applications. While the study focused on physics, similar results have been observed in other courses. The first was rather self-explanatory. "*Facility in solving standard quantitative problems is not an adequate criterion for functional* *understanding: Questions that require qualitative reasoning and verbal explanation are essential*" (p. 10). Basically, simply knowing how to solve a problem is not enough. Students must have a deeper comprehension of what is happening and why it is happening. Relationships between concepts must be recognized.

The second generalization mentioned by McDermott (1993) was: "*A coherent framework is not typically an outcome of traditional instruction: Students need to participate in the process of constructing qualitative models that can help them understand relationships and differences among concepts*" (p. 11). Learning quantitatively and learning qualitatively are not mutually exclusive. Aside from competition for valuable teaching time during class, qualitative and quantitative learning can coexist. In fact, the two may enhance each other, resulting in better performance on both qualitative and quantitative problems. When the qualitative topics are directly addressed, it is little surprise that there is an increase in ability concerning those types of questions. However, the better performance on the quantitative problems (despite the reduced number of examples done during class because some time is devoted to qualitative instruction) may derive from an enhanced understanding of the concepts.

McDermott's (1993) third topic was: "*Certain conceptual difficulties are not overcome by traditional instruction. Persistent conceptual difficulties must be explicitly addressed by repeated challenges in more than one context*" (pp. 12-13). In other words, certain misconceptions commonly occur when normal lectures are used. These mistakes must be emphasized and corrected. One strategy to accomplish this is to basically entrap the students by setting them up for failure. Put more elegantly, the teacher can "*elicit* a suspected difficulty by contriving a situation in which students are likely to make a related error. Once the difficulty has been exposed and recognized, the instructor must insist that students *confront* and *resolve* the issue" (p. 13). Unfortunately, students may need to experience this failure multiple times for a false impression to be identified and dispelled (McDermott, 1993). Students may still resist accepting the new concept, and could try to "simply memorize the answer for a particular case" (p. 14), missing the overall point of a new idea. Active learning is a vital part for this conceptual change to occur (McDermott, 1993).

Next on McDermott's (1993) list was: "*Growth in reasoning ability does not usually result from traditional instruction. Scientific reasoning skills must be expressly cultivated*" (p. 14). Reasoning skills must be taught outright. Students should not be expected to learning them indirectly as part of instruction that focuses on other topics. The fifth generalization is related to the fourth: "*Connections among concepts, formal representations, and the real world are often lacking after traditional instruction. Students need explicit practice in interpreting physics formalism and relating it to the real world*" (p. 15). A disconnect often occurs among different topics. This deficiency may be related to misconceptions or errors in reasoning (McDermott, 1993).

The final generalization discussed by McDermott (1993) stated: "*Teaching by telling is an ineffective mode of instruction for most students. Students must be intellectually active to develop a functional understanding*" (p. 17), as well as for meaningful learning to take place. "Those who learn successfully from lectures, textbooks[,] and problem solving do so because they constantly question their own comprehension, confront their difficulties, and persist in trying to solve them" (p. 18). Again, active learning is present, even if it is done internally. Although she did not name them, McDermott (1993) described the Socratic Method and either discovery learning or guided discovery (depending on the amount of guidance offered by the teacher, which she was not clear about), both of which require active learning and allow students to construct their own knowledge. McDermott (1993) did offer a defense of the traditional lecture/laboratory format, despite its disadvantages. Lectures might be necessary in high-enrollment classes, but it does not have to be passive, since other techniques can be used to promote active participation. While McDermott (1993) did not elaborate or offer examples, other authors have (as detailed throughout this paper).

Just as teachers can present information in different ways, students can learn in different ways. How well teaching styles match learning styles can have a profound impact on how well a student learns something. There are many aspects to consider in this relationship. A myriad of teaching styles exists, along with a plethora of nuances that can influence learning style.

Learning styles.

Levels of learning. Students can process (internalize, comprehend, handle) information on three main levels. Performance-avoiders only do enough to get by (Bain, 2004). These students fear failure if they try their best, which can adversely impact their self-esteem. As a result, they do not try hard, providing a ready excuse for any failure or lack of success. To help these students to prosper, teachers must construct assessments that are difficult enough to challenge students and provide a feeling of accomplishment when students successfully complete them, but easy enough that the tasks can be completed and build self-confidence (Bain, 2004). These small, doable tasks will show students that they are capable of doing the work, and should gradually progress to more and more complex assignments.

Surface-level processing "focuses on *the sign*" (Marton & Säljö, 1976, p. 9), or what the object is. Students on this level aim to memorize information (Marton & Säljö, 1976). Biology professor Craig Nelson called them "bulimic learners" (Bain, 2004, p. 40) because they "ingest" information and "regurgitate" it during tests (or on other assignments). Deep-level processing deals with "what is signified" (Marton & Säljö, 1976, p. 9), or what something means. One deep-level student who was interviewed by Marton and Säljö (1976) had this to say: "I tried to think what it is all about" (p. 9), getting to a concept's deeper, underlying essence.

These different learners vary in "*what is learned* instead of merely differing as regards *how much is learned*" (Marton & Säljö, 1976, p. 7). "How much is learned" refers to the overall quantity of material, whereas "what is learned" deals with quality (content difficulty and substance). Moreover, there is a "qualitative difference in what is learned" (p. 10), with deeper processing leading to better outcomes (Marton & Säljö, 1976). Bain (2004) stated that deep learning resulted in sustained influence, with students "making something their own, 'getting into it,' and 'making sense of it all'" (p. 9), in addition to how deep learning "'transformed their lives,' 'changed everything,' and even 'messed with their heads'" (p. 10). In general, deeper learning is more impactful and more influential than the less-committed types of learning.

Experiential Learning Theory. Experiential Learning Theory (more specific than the general concept of experiential learning) was mainly pioneered by David Kolb, and is based on "an individual's preferential resolution of the dual dialectics of experiencing/conceptualizing and acting/reflecting" (Kolb & Kolb, 2005, p. 199). Concrete experiences (CE) provide a basis for reflective observations (RO), which are used to formulate abstract conceptions (AC). Implications can be formed from the abstract conceptions that lead to active experimentation (AE). Active experimentation leads to new concrete experiences, completing the cycle.

According to Kolb, Boyatzis, and Mainemelis (2001), information is assimilated from an experience through either the actual, physical concrete experience during the interaction or abstract conceptualizations after contemplating the experience. In seeking new information, one can contemplate an experience through reflective observation or actively seek new experiences through active experimentation (Kolb et al., 2001). By choosing one form from each didactic, four possible learning styles are created (diverging, assimilating, converging, and accommodating), each with its own characteristics (Kolb et al., 2001). The diverging style combines concrete experience and reflective observation, and is preferred for generating ideas, gathering information, and working in groups. Assimilating aids in putting information into a concise, logical form and having time to think by combining abstract conceptualization and reflective observation. Abstract conceptualization and active experimentation form the converging style to make it easier to find practical uses for ideas, solve problems, and deal with technical tasks. The accommodating style is derived from concrete experience and active experimentation, with learning coming from hands-on experiences, acting on gut instinct, and working with others being the resulting traits.

These four learning styles can be examined at five different levels: early education specialization, professional career, current job role, adaptive competencies, and personality type (Kolb et al., 2001). The first three levels all deal with jobs in some way, but the quartet of different adaptive competencies are fairly obvious based on the corresponding occupation that one has, so the first four levels are related. As described by Kolb et al. (2001), those with diverging learning styles tended to pursue careers in the arts, history, English, political science, and psychology, and have valuing skills (relationships, helping others). The thinking skills (creating theory) of assimilating learning styles fit with jobs in economics, mathematics, sociology, and chemistry. Occupations in the physical sciences and engineering require the decision skills (analysis) embodied in converging learning styles. Acting skills (leadership) are needed in the business and management jobs favored by individuals with accommodating learning styles. It is easy to see the interdisciplinary nature of Experiential Learning Theory.

The personality types, which can be likened to the Myers-Briggs Type Indicator (MBTI), are of particular interest, with the introversion/extraversion pair in Myers-Briggs corresponding to the active/reflective component and feeling/thinking matching with concrete experience/abstract conceptualization (Kolb et al., 2001). Familiarity with the personalities of the students is an important aspect of the lecturing dialogue, so the Myers-Briggs Type Indicator will be explored in more detail when personality types are discussed in the next section.

Kolb et al. (2001) discovered that individuals may not display a preference for one of the four learning styles, straddling two of them. A northerner combines diverging and accommodating, with a preference for concrete experience and equal levels of reflective observation and active experimentation. Diverging and assimilating form an easterner, with a tendency to use reflective observation and a split for concrete experience and abstract conceptualization. Equality in reflective observation and active experimentation, with a preference for abstract conceptualization leads to straddling assimilating and converging to create a southerner. A westerner merges the characteristics of converging and accommodating, balancing concrete experience and abstract conceptualization, with a preference for active experimentation.

The final aspect of Experiential Learning Theory concerns the balance profiles. An individual can have an emphasis on the concrete experience/abstract conceptualization dimension, an emphasis on the reflective observation/active experimentation dimension, or a balance between the two. Experiential Learning Theory can be seen more clearly in Figure 1 (Kolb & Kolb, 2005, p. 198).

Personality types. Personality types are not learning styles, but they are very closely related to learning styles, so they will be discussed here.

Concrete Experience

Active Experimentation				Reflective Observation
	Feeling-Acting Accommodating	Feeling-Acting/Reflecting Northerner	Feeling-Reflecting Diverging	
Active Experimentation	Acting-Feeling/Thinking Westerner	Feeling/Thinking-Acting/Reflecting Balancing	Reflecting-Feeling/Thinking Easterner	Reflective Observation
	Thinking-Acting Converging	Thinking-Acting/Reflecting Southerner	Thinking-Reflecting Assimilating	

Abstract Conceptualization

Figure 1. Experiential learning theory visual.

In 1962, after 20 years of development, the mother-daughter team of Katherine Briggs and Isabel Briggs Myers published the Myers-Briggs Type Indicator (McCaulley, 1974). This (now) well-established instrument has been used in many different situations, studying the common traits and characteristics of different personality types, including “academic aptitude and achievement” (p. 1). The Myers-Briggs Type Indicator “was designed to show the direction of preference, more than the strength of preference” (p. 8) on four dichotomies: introversion-extraversion, feeling-thinking, sensing-intuition, and perceiving-judging. The dominant traits are identified from each duo, leading to one of sixteen personality types, which were originated by Carl Jung (McCaulley, 1974). Since strength of preference is not considered, slight preferences are treated the same as strong preferences.

Based on the descriptions of McCaulley (1974), introversion-extraversion refers to the “direction of interest and attention” (p. 2). Extraverts (denoted as E in the Myers-Briggs Type Indicator) focus more on “the outer world of object, people, and action” (p. 2). They learn better when the concept is presented after some experience with the material, and tend to prefer group learning. On the other hand, introverts (I) center on “the inner world of ideas and contemplation" (p. 2). They learn better when the concept precedes experience, and favor individual learning.

People can perceive things by sensing (S) or intuition (N). Those who lean towards sensing prefer the "immediate, the real, the tangible, the solid facts of experience" (McCaulley, 1974, p. 2), along with data, experimentation, and memorization (Felder & Silverman, 1988). They like to solve problems by using standard methods and dislike surprises and complications (Felder & Silverman, 1988). Furthermore, they are typically patient and careful, leading to a tendency to be slow (Felder & Silverman, 1988). Conversely, the intuition group is apt to see "the possibilities, meanings[,] and relationships of experience, often with only a passing interest in the facts themselves" (McCaulley, 1974, p. 2). Felder and Silverman (1988) also added principles and theory to the list of preferences for the intuition group, along with a capacity for innovation, grasping new concepts, and dealing with symbols. Members of the intuition group tend to be quick to learn and actually like complications, but they do not like repetition and may become bored by details, leading to carelessness (Felder & Silverman, 1988).

Decisions can be made by thinking (T) or feeling (F). Thinkers make "decisions objectively [and] impersonally, analyzing the facts and ordering them in terms of antecedents and consequences" (McCaulley, 1974, p. 3), and prefer to deal with logical principles. Feeling types use "a valuing process, weighing the importance of alternatives to oneself or others" (p. 3) to make decisions, and would rather work with people or study them.

As for malleability and rigidity, those who tend towards judging (J) prefer "living in a planned, decided, orderly way, aiming to regulate life and control it" (McCaulley, 1974, p. 3). They are more structured and organized. In contrast, people who are more inclined towards perception (P) are more flexible and spontaneous (McCaulley, 1974). They seek to understand life and are capable of adapting to changing conditions.

According to McCaulley (1976), all personality types attend college, but the purposes for going to college vary by type. Often, trends emerge for certain personality traits among the workers or students in specific fields, which explains why a cluster of a specific personality type may be found in a single major. In McCaulley's (1976) study of personality types, she found that students (in general) were about fairly evenly split between extraverts and introverts, but engineering majors were introverts by a ratio of two to one. On the sensing-intuition scale, sensing types are attracted to applied fields, but intuitive types tend to pursue jobs that involve theory, imagination, and interpretation. Thinkers frequently "score higher on tests of mathematics and science, [whereas feelers] score higher on tests of social sensitivity" (McCaulley, 1974, p. 3). While nearly every sample of teachers is dominated by feeling types, mathematics teachers are equally split between thinkers and feelers (McCaulley, 1974).

Outside of the traits identified by the Myers-Briggs Type Indicator, Felder and Silverman (1988) listed four additional dichotomies related to learning styles. The first of these was a preference for having material presented in a visual versus an auditory manner. "[B]oth visual and auditory modalities reinforce learning for all students" (p. 677), but there is a continuum of how much is learned.

> [S]tudents retain 10 percent of what they read, 26 percent of what they hear, 30 percent of what they see, 50 percent of what they see and hear, 70 percent of what they say, and 90 percent of what they say as they do something (p. 677).

Therefore, hearing is good, seeing is better, a combination of the two is even better than that, and doing things (while vocalizing the steps) is the most effective.

The second set of learning styles compared inductive and deductive learning. Felder & Silverman (1988) observed that people naturally learn through induction (a progression from the specific to the general), but naturally teach using deduction (moving from the general to the specific). With induction, students examine a particular example, note the details, and build a general rule from those observations. Felder and Silverman (1988) argued that even though it is not the "natural" teaching style, inductive instruction will produce more benefits than deduction, such as promoting effective learning, aiding academic achievement, enhancing abstract reasoning skills, retaining information longer, improving the ability to apply principles, promoting confidence in problem-solving abilities, and increasing the capability for inventive thought.

As for deduction, it is apt to encourage some misconceptions. For example, the neat and tidy explanation that is presented gives the false impression that the material was discovered in that same, neat fashion, leading students to believe that the course, curriculum, and professor are far beyond their abilities (Felder & Silverman, 1988). In reality, the knowledge was often gained through painstaking trial and error, in an undertaking similar to induction. However, despite these problems, deduction does have its uses. It is an "efficient and elegant way to organize and present material *that is already understood*" (p. 677). Therefore, a wise teacher should use induction to present new material, and then cement and solidify the understanding by using deduction. The overall process will start with an example, move to a general rule, and return to examples.

Felder and Silverman (1988) next contrasted sequential and global learners. Sequential learners (as the name implies) learn the material in the order that it is presented, and utilize "linear reasoning processes when solving problems" (p. 679). These students do not have to understand the material fully, and can work with it even "when they understand it partially or superficially" (p. 679). Sequential thinkers tend to be convergent thinkers who are strong analysts and learn best when the material progressively gets more complex or difficult. At the other end of the spectrum, global learners struggle until they reach the "Ah-Ha Moment" (a great revelation or epiphany) and suddenly "get it" (Felder & Silverman, 1988). They have difficulty partially or superficially understanding a topic. They advance in intuitive leaps, but often cannot explain how they reached their solutions. In contrast to the sequential learners, "global learners sometimes do better by jumping directly to more complex and difficult material" (p. 679), and are adept at synthesis and divergent thinking.

Most often, curricula are designed with sequential learners in mind, thereby neglecting the needs of global learners (Felder & Silverman, 1988). As a result, teachers must make adjustments to reach those overlooked students. Felder and Silverman (1988) made several suggestions for how to do this. Since global learners suddenly comprehend material, they will benefit from knowing the ultimate goal or big picture at the beginning of each lesson. Additionally, real-life applications (uses) and "what-ifs" (hypothetical situations) may be beneficial. Periodic exposure to advanced concepts earlier than they would normally be presented is also an option because global learners tend to grasp the more complicated material sooner. Global learners often make intuitive leaps, so explaining the learning process may help them to understand new topics. Teachers may also allow students to devise their own methods to solve creativity exercises (activities that can benefit both global and sequential learners). However, the teacher must ensure that these methods are procedurally sound.

The final comparison was between active experimentation and reflective observation (Felder & Silverman, 1988). Active experimentation resembles kinesthetic learning (learning by doing or experiment), and “involves doing something in the external world with the information” (p. 678) beyond just listening to and watching the teacher. This could include experimentation (as the name suggests) or discussions (including explaining the material to someone else). Offering more details, Felder and Silverman (1988) noted that most engineers (experimentalists) fall into this category. These active learners work well in groups (particularly in groups of three to four, especially when they stay on task), and do not learn as much when they are forced to be passive (such as during most lectures).

“[R]eflective observation involves examining and manipulating the information introspectively” (Felder & Silverman, 1988, p. 678). These theoreticians need time to think and reflect on ideas, and work better alone or with one other person (Felder & Silverman, 1988). Similarly, Lake (2001) discussed reflective practitioners, commenting that they must read and analyze new information before making “judgments about the relative merit of conflicting information within the pre-existing knowledge framework” (p. 897). To succeed, reflective students must evolve into autonomous and self-directed learners who accept personal responsibility for what they learn (Felder & Silverman, 1988).

Instruction based on active participation (doing more than just listening) uses techniques that utilize both active experimentation and reflective observation, thus benefiting both types of learners (Felder and Silverman, 1988). Citing several studies, Lake (2001) noted the following active learning techniques: interactive lecture, lecture by questioning (Socratic Method), whole class/group discussion, experiential/activity-based learning, role-playing/simulation, interactive computer-based learning, and problem-based learning. All of those are potential alternatives to a traditional lecture.

In contrast, constant passive learning benefits neither group, making effective learning impossible (Felder & Silverman, 1988). According to Lake's (2001) discussion of cognitive theory, active processing must be present for learning to occur, so lectures can fall short in reaching their goal of imparting knowledge. Studies by Stuart and Penner found that medical students can only maintain a high level of attention for about the first 10-15 minutes of a lecture, followed by an abrupt decline (Lake, 2001). Medical students are commonly perceived to be among the best and brightest students. If they can only stay focused for 10-15 minutes, then logic dictates that regular students would have shorter attention spans. Either different styles of instruction must be used or measures must be taken to keep the students' attention.

Drawing on the work of Perry and Clinchy, Bain (2004) listed four different types of learner based on the learner's motives, which also parallel the developmental stages of Perry (1999). The received knower (lowest level) accepts truth from a teacher or expert. Knowledge (or truth) is something that they cannot create or evaluate on their own; they must rely on others. The process is similar to Paulo Freire's (1921-1997) Banking Model of Education, "in which teachers deposit the correct answers into students' heads" (Bain, 2004, p. 42), only to have it "withdrawn" for assignments. It also resembles the surface-level processing of Marton and Säljö (1976), along with Nelson's bulimic learner.

The second level is the subjective knower (Bain, 2004). At this stage, feeling is used to make judgments, and everything is based on opinion. Level three is the procedural knower, who learns to "play the game" (p. 43). These students do what must be done to succeed in a class, separating school from the outside world. The highest level is what Perry called "Commitment." These "students become independent, critical, and creative thinkers, valuing the ideas and ways of thinking to which they are exposed[,] and consciously and consistently trying to use them.

They become aware of their own thinking and learn to correct it as they go" (Bain, 2004, p. 43). Students at the Commitment level can be divided into two subgroups: separate knowers and connected knowers (Bain, 2004). The former are more objective and detached, remaining skeptical and ready for debate. The latter are deliberately biased towards one view, but keep an open mind when listening to the points of view of others.

Personality plays an important role in education. By learning about one's students, a teacher can tailor his/her explanations to meet the needs of the students, as well as their interests. A class of extroverts would lend itself to a discussion (whether as a whole class or in small groups), but a class of introverts would benefit from quiet reflection. Sequential learners benefit from a step-by-step process involving examples, but global learners need to know a more general, over-arching background of a subject. As mentioned, personality can provide some insights into likes and dislikes (as indicated by the use of the MBTI to identify probably job interests). Commonalities among the personalities of students can allow a teacher to use one example to relate to many students. For example, the researcher once taught a class in which half of the students were on the football team. Given that football was also very popular in the school overall, any football reference was understood by most of the class.

All of this establishes the intricate and tricky job that teachers have in dealing with students. With so many different aspects to learning, teachers have a lot to deal with and consider. They must be conscious of the level of learning that the students are engaged in, the various factors that encompass Experiential Learning Theory, and the myriad components of personality types that students may possess. Whether considering the 16 combinations detailed by the MBTI, the four pairs proposed by Felder and Silverman (1988), or the four types of learners based on motive (Bain, 2004), teachers must create lesson plans to reach as many of these different learners as possible. It is difficult to determine the direction of influence between personality and Experiential Learning preference, but a correlation exists (as is obviously seen by the descriptions of the MBTI dichotomies and Experiential Learning aspects). The Experiential Learning preferences then align with different learning styles (whether preferred or most effective). In short, with so much to consider, this is not a simple task. However, research shows that active learning is more beneficial than passive learning, so engaging the students is extremely important.

Teaching styles. Note: Active learning, inquiry-based learning, experiential learning, and constructivism are learning styles, but they are promoted and stimulated by the proper teaching styles, so they will be discussed here.

Cognitive differences. Implicit cognitive processes are “a set of learning and memory processes that operate primarily outside of the realm of awareness” (Woltz, 2003, p. 96), so they function in one’s subconscious. Many general intellectual aptitudes exhibit distinct developmental patterns and are impervious to short-term intervention strategies, so they cannot be affected by training or practice in these areas (Woltz, 2003). However, differences in implicit cognitive processes were observed among individuals that were “related to some forms of complex learning” (p. 102), so it appears as though implicit cognitive processes are subject to the influences of different teaching styles. As a result, these implicit cognitive processes can be targeted in an attempt to improve learning. This is a fortunate discovery because students are not completely limited by natural abilities and innate intelligence.

Personalistic teaching. Personalistic teaching is distinguished by the formation of a professional relationship between the teacher and students that allows for the exchange of some personal information (when appropriate). This added level of familiarity lets the teacher get to know a little more about the students, which can aid in developing lessons that the students are able to understand and relate to more. Many of the excellent teachers (with excellence based on surveys and recommendations from students and professors) that Bain (2004) interviewed recommended getting to know the students, which can lead to "insights into students' thinking" (p. 158). This personal connection may also motivate students to work harder, for fear of letting the teacher down (Bain, 2004). In other words, it establishes a sense of accountability for the students.

Ausubel (1968) noted that it is imperative to determine what the student already knows, which can be used to tailor an explanation accordingly. However, it is not enough to know what the student knows; the teacher must also know how the student thinks with what he/she (the student) knows. Therefore, a "teacher who is attuned to students' thinking will make different decisions about what to tell students and how to support the development of their understanding than a teacher who simply lectures according to a pre-planned and inalterable syllabus" (Hutchings, 2000, p. 17). Long and Coldren (2006) found that a personalistic teaching style allows the teacher to model/demonstrate methods (both internal thinking and external procedures) to engage and approach the material. By asking so many questions, the lecturing dialogue provides exactly this type of modeling.

Kember and Wong (2000) concluded that student-teacher "interaction and rapport were particularly susceptible to class size effects" (p. 70). In high-enrollment, lecture-based classes, personal interaction must be explicitly and deliberately fostered, conveyed through a personalistic style of instruction (Long & Coldren, 2006). The teacher should check on the students' comprehension throughout the class to make sure that everyone understands (Bain, 2004). Of course, it can be difficult to actually connect with or include each student in a large class, but feedback about the overall comprehension can be obtained by observing the students' reactions (facial expressions, eye contact, body language, etc.).

The attention and interest of the students must be retained by engaging them in the lesson, potentially with "some provocative act, question, or statement" (Bain, 2004, p. 109). This can be accomplished in several ways, as described by Long and Coldren (2006). First, the teacher can explain his/her thinking. Second, a team atmosphere can be created, with students being encouraged to work cooperatively, rather than competitively. Third, the teacher can get excited about the material. Enthusiasm is often contagious, so it can spread to the students. Fourth, the teacher should talk to the students, as opposed to talking at them. A "conversational atmosphere" (p. 242) can be established, which can aid the other methods used to convey a personal teaching style. Bain (2004) also advised finding a way to get students to talk, and Lake (2001) noted that active learning can lead to more vocal students. Fifth on the list by Long and Coldren (2006), students can be engaged using nonverbal cues, such as eye contact. Respect, interest, and involvement can also be expressed by using a warm tone of voice. Sixth, personal anecdotes can be shared (while ensuring that they are professionally relevant, of course). This lets the teacher appear to be human to the students. This advice was also offered by Bain (2004). The final method also helps to enhance the humanity of the teacher: laughing at one's own mistakes. No one is perfect, and these can be used as learning opportunities by emphasizing common mistakes.

Teachers who are recognized as being effective or good often go beyond teaching just the subject matter of a class (Bain, 2004). They still cover the material that they are supposed to cover, "but in the context of focusing on the intellectual, and often ethical, emotional, and artistic, development of their students" (p. 46). They teach overall thinking skills and integrate other subjects into their lessons, showing how different topics are related, along with explaining why the material is useful and relevant. To accomplish this, mere expertise in the field of study is not enough; teachers must know “the histories of their disciplines” (p. 25), which can also enable teachers to explain how and why theories, ideas, and procedures developed in their disciplines. Throughout the learning process, students are given the opportunity to ask questions, probing the subject in ways that are guided by their own senses of curiosity.

Bain (2004) suggested that teachers go beyond general thinking skills and engage the students in disciplinary (subject-related) thinking. The purpose is “to help students think about information and ideas the way scholars in the discipline do” (p. 114), which Long and Coldren (2006) also advised. The overall goal is to foster metacognition: thinking about one’s own thinking. Teachers should aim to “make students explicitly aware of that [thinking] process, constantly prodding them to do the same” (p. 115). To achieve this, teachers may use a form of inquiry-based learning, sometimes the Socratic Method (Bain, 2004). Ultimately, students should develop reasoning skills, which will help them to understand concepts rather than simply being able to solve problems robotically. Some teachers drew attention to the reasoning behind specific processes. Bain (2004) noted that the best teachers provided explanations, analogies, and questions to help students comprehend fundamental concepts and solve problems. Others contended that students must learn (memorize?) information before reasoning can be used, but the professors that Bain (2004) studied felt that learning facts must occur simultaneously with reasoning about those facts. In other words, learning involves more than merely knowing facts; learning involves understanding, which can (and should) occur from the beginning.

Good oral skills can make a difference in making the teacher appear to be more personable (Bain, 2004). Echoing Long and Coldren (2006), Bain (2004) recommended that teachers use warm language (inviting, emotional, descriptive) as opposed to cool language (detached, less emotional, less descriptive). Warm language tells a story, including the fine details, whereas cool language tells about the story, but does not reveal the full details of the plot (Bain, 2004). As for delivery, an analogy can be made to Jazz: The notes that you don't play are as important as the ones that you do play. To steal a cliché, silence is golden. As Bain (2004) put it, good teachers "know how to make silence loud" (p. 120). These teachers strategically use silence to emphasize key points, or to give the students time to think after asking a question. Along those lines, the volume of the teacher's voice can be altered to highlight specific words, concepts, or ideas. Similarly, pace can be varied to stress different notions. Speaking faster or slower can draw attention to certain ideas, and help to avoid monotony. Humor can also be used to break up the rhythm of a class, and keep students engaged in the material. Finally, teaching must be genuine. Students are smart, and they can usually tell when they are being deceived or deluded. "Teaching is not acting, yet good teachers do expect to affect their audience when they talk: to capture their attention, to inspire, to provoke thoughts and questions" (p. 121).

The lecturing dialogue encompasses many of the traits that were identified as being common in good teaching. Through the question-and-answer process, the teacher will learn about the students, which will help the teacher to identify indicators related to student comprehension. Students will also be given an opportunity to ask questions. As the teacher explains his/her thinking, he/she will model the appropriate disciplinary thinking, which will allow the students to emulate that thought process. Connections among topics and the history of a discipline will also be revealed as this occurs because experts in a field use those relationships when evaluating problems. If the teacher truly loves his/her subject, then the enthusiasm will show. Good oral skills allow the teacher to talk to the students, rather than at them. Most important, the familiarity that develops will make it easier the for the teacher to laugh at his/her mistakes in class because the students will recognize him/her as a normal human who makes mistakes, rather than some infallible demigod who has all of the answers. Mistakes by the teacher show the students that mistakes occur and are a normal part of the learning process. Laughing at mistakes and learning from them is vitally important. That sense of humor can extend to other parts of the class, as well, making the class more enjoyable overall. All of these characteristics appear in the lecturing dialogue.

Spectrum Theory. Bain (2004) noted that one teaching style will not work for all students, but Barbe and Milone, Jr. (1981) countered with the fact that it is impractical for a teacher to provide individualized education for every student without sacrificing valuable class time. Given this, the lecturing dialogue aims to address as many different learning styles as possible by combining several teaching styles, creating a hybrid style. In doing so, it utilizes what is known as Spectrum Theory (Spectrum Institute for Teaching and Learning, 2011).

In 1966, Muska Mosston developed the Spectrum of Teaching Styles for physical education. After teaming with Sara Ashworth in 1969, it was expanded to all types of education (not just physical education). Ashworth was the inspiration for establishing the Spectrum Institute for Teaching and Learning, a part of the Lanker Family Foundation, which continues to research Spectrum Theory. According to the Spectrum Institute for Teaching and Learning (2011), the theory is based on a continuum (or a spectrum) of teaching styles that range from fully teacher-centered to fully student-centered. Beginning with the most teacher-centered style, Spectrum Theory describes eleven different teaching styles: command, practice, reciprocal, self-check, inclusion, guided discovery, convergent discovery, divergent discovery, learner-designed individual program, learner-initiated style, and self-teaching (Figure 2).

Style	Teaching Style	Decision Maker
Reproduction Styles	Command	Teacher: Max Learner: Min
	Practice	↑
	Reciprocal	
	Self-Check	
	Inclusion	
Discovery Threshold		
Production Styles	Guided Discovery	
	Convergent Discovery	
	Divergent Discovery	
	Learner-Designed Individual Program	↓
	Learner-Initiated	Teacher: Min Learner: Max
	Self-Teaching	

Figure 2. Spectrum Theory visual.

These styles each have their strengths and weaknesses, and vary based on the roles that the teacher and students take. Even though the spectrum of styles exists as a continuum, the styles are unique, with specific descriptions, and do not overlap (Ashworth, 2008). However, several teaching styles may be used during a single lesson.

According to Ashworth's (2008) descriptions, command largely mirrors a lecture. Practice is similar to command, but includes students practicing the new skills. Reciprocal adds to practice by having students work together to check each other's work. Self-check is comparable to reciprocal, but students check their own work against a set of teacher-mandated criteria. Inclusion is the nearly identical to self-check, but students can select requirements from several different levels of difficulty for the tasks. As part of all of these styles, the teacher circulates throughout the classroom, checking on the students' progress and answering questions. Ashworth (2008) stated that these styles "promote reproduction cognitive operations while engaged in the task" ("Classroom Description" chart, para. 1), and can be used either when learning new material or reviewing old material. Of course, reproduction refers to the ability to complete a task, not necessarily the capacity to understand why the task is done or how things work.

For strictly new material, Ashworth (2008) recommended that the more student-centered styles be used to "promote different discovery cognitive operations while engaged in the task" ("Classroom Description" chart, para. 7). The least student-centered teaching style on this end of the spectrum is guided discovery, which consists of the teacher asking a student (or group of students) a series of specific questions, leading to the discovery of the desired knowledge (in a step-by-step process). Convergent discovery gives more autonomy to the students, but still has a specific desired outcome. Divergent discovery is identical to convergent discovery in practice, but can have several possible outcomes. The learner-designed individual program is just what its name indicates: an individualized program that is designed by the learner. The student selects the goals and criteria for learning a teacher-selected topic. The learner-initiated style mimics the learner-designed individual program, but allows the student to choose the topic, as well. In self-teaching, the student becomes the teacher, too. The actual teacher only becomes involved at the request of the student. The lecturing dialogue primarily relies upon command, guided discovery, convergent discover, and divergent discovery, but may employ any of the other styles as needed.

Active learning and inquiry-based learning. Ingrained in the plethora of teaching styles that the lecturing dialogue uses are active learning and inquiry-based learning. Active learning invites students to become an integral part of the educational process because students "learn more when they are actively involved in learning than when they are passive recipients of instruction. Students must do more than just listen: they must read, write, discuss, and be engaged in solving problems" (Rao & DiCarlo, 2001, p. 55). To truly tap into deep understanding and learning, students must engage in higher-order thinking, developing "cognitive skills, creative thinking, judgment, interpretation, and problem-solving skills" (p. 59).

Hake (1998) also noted that interactive engagement could help to improve problem-solving skills.

Sadly, Rao and DiCarlo (2001) found that despite active learning leading to better performance in the classroom by students when compared to a more traditional lecture, few teachers of high-enrollment classes used active-learning strategies, with many employers of passive techniques citing a lack of class time, an increase in the time needed to prepare to teach, and a dearth of in-class resources as reasons for not using active learning when teaching. Countering the preparation time argument, the free-flowing nature of the lecturing dialogue is so open-ended that less preparation is needed for specific examples, as long as the teacher is comfortable with improvisation and has a strong background in the subject.

Oliver (2007) explored inquiry-based learning in high-enrollment classes taken by first-year college students, finding that students benefited from becoming actively involved in the learning process as they solved problems. Engaging in the problem-solving process can help students develop transferable skills and knowledge. Furthermore, the majority of the students reported that they were satisfied by the inquiry-based approach with regards to the amount of material learned, learning success, and support for their preferred learning style. However, over 20% of the students were dissatisfied with the approach, how much was learned, success, and interest aroused.

Inquiry-based learning is closely related to discovery learning, with some teachers using the terms interchangeably (Volkmann, Abell, & Zgagacz, 2005). The inquiry/discovery teacher, as described by Volkmann et al. (2005), must guide students through the inquiry/discovery process, offering insights and explanations when needed, whereas the students must explore the phenomenon under investigation and interact with others (both fellow students and the teacher). "Thus, the teacher is a partner in the social construction of knowledge" (p. 864).

Steffe and Gale (1995) identified two different approaches that teachers can use for inquiry-based learning. Using the productive approach, "the teacher's role is to listen to and probe the learners in a way that only indirectly promotes their construction of increasingly powerful conceptual operations" (p. 437). This is basically a minimalist version of guided discovery. The (contrasting) nonproductive approach does not mean a lack of results. Rather, it refers to the fact that the teacher does not lead the students to produce new knowledge, but helps them to avoid mistakes. In other words, "the teacher's role is to be more directive in helping children avoid 'blind alleys'" (p. 437), which is closer to ensuring a more efficient discovery learning by avoiding false starts or errant paths.

Often (especially when learners are stuck in surface-level processing), students expect to be told what they need to know (to have information delivered to them), rather than seeking and discovering knowledge. Lev Vygotsky (1896-1934) developed a sociocontextual theory of instruction based on content, method, and context (Long & Coldren, 2006). To acquire knowledge, learners need "instruction in the form of guidance" (p. 238), such as that provided by a teacher. In addition, imitation is imperative. When true understanding is lacking, students must be able to mimic what they see and hear. Of course, mere imitation is not ideal, but it is adequate. The interactions between students and the teacher occur in what Vygotsky termed the zone of proximal development. The teacher provides scaffolding for the student, in which guidance is provided in the form of "a series of educational steps or prompts, eventually leading the student to reach a higher level of understanding" (p. 238). This interpersonal context offers the requisite support needed for students to succeed academically. Therefore, based on "a

Vygotskian framework, an instructor must be aware not only of the dominant styles at play, but also how they fit together" (p. 238), an idea that many other researchers have echoed.

Using inquiry-based learning demands more from the teacher than the teacher of a traditional lecture. Since students are encouraged to pose questions (which may be extremely diverse), the teacher must have a deeper understanding of mathematics, a better grasp of mathematical reasoning, a more developed ability to understand the reasoning of others, and an enhanced command of mathematical language and notation in order to help students in their explorations (Slavit & Lesseig, 2017). Hersh (1998) noted the common trait of many mathematics lectures of expecting detailed proofs, but diminishing the reason why that proof is used. The hypocrisy of expecting details in a proof but ignoring the proof (reasoning) for using that proof is evident and incomprehensible. Slavit and Lesseig (2017) observed that the more advanced abilities of the inquiry-based teacher allow him/her "to understand, explain, and facilitate discussions as to why the algorithm works" (p. 61), as well as to recognize conceptual errors, identify both mathematical potential and pitfalls in students' questions, and decide which ideas are worthwhile from an educational perspective. In essence, these enhanced skills can result in better learning.

Slavit and Lesseig (2017) detailed the specific knowledge requirements for a teacher who employs inquiry-based learning. Knowledge of content and teaching allows the teacher to identify which ideas will be the most beneficial to explore, or if an idea is even feasible. The ability to "anticipate student problem-solving strategies, potential errors or confusions, or strengths to build upon" (p. 62) was called knowledge of content and students. That is, beyond being aware of how they (the teachers) think, teachers must be cognizant of how the students think. Horizon content knowledge is an overarching familiarity with how different areas of mathematics are interrelated and interwoven. The awareness of educational materials and resources, as well as their quality and usability, was classified as knowledge of content and curriculum. A teacher must be able to use all of these types of knowledge to transfer the focus from the content to the students in order to maximize the effectiveness of inquiry-based learning.

It is important for teachers to continually probe students to ask more and more hypothetical "what if" questions to foster deeper understanding and flexibility in mathematics (Slavit & Lesseig, 2017). Answering the one question that is asked is admirable, but not enough. Students should use their findings to look for more questions to pose or more uncertainties to explore.

Unfortunately, many teachers misuse or misinterpret the purpose of inquiry-based learning. One teacher did not grasp the necessity of students discovering connections and knowledge on their own: "Listen, I found out already, so you don't have to go through[,] you know, sitting over the books and, and trying to figure out. I can tell you right away" (Volkmann et al., 2005, p. 858). This individual overlooked the distinction between teaching (drawing information out of students) and telling (simply providing information). Teachers were deterred from using inquiry-based learning by the frustration seen in students, mainly in relation to the amount of work students were required to do, the length of the activities, and students not getting the answers that they wanted (Volkmann et al., 2005). Others noted that "students were not used to thinking during class" (p. 859), and that students often only wanted to get the correct answers, rather than fully learn the material.

When teaching younger students, a limited attention span was used as an excuse to not use inquiry-based learning. Compounding this type of erroneous thinking, Bressoud and Rasmussen (2015) found that 46% of instructors at "successful universities" (those with high retention rates and a belief that mathematics is sensible, useful, and worthwhile) mildly agreed, 23% agreed, and 0% strongly agreed that “calculus students learn best from lecture, provided they are clear and well organized” (p. 146). The numbers were 35%, 21%, and 8%, respectively, for instructors at “other universities.” This devotion to lectures prevails despite clear evidence to the contrary from many studies.

The question-and-answer format of the lecturing dialogue creates the perfect environment for both active learning and inquiry-based learning. Students are constantly asking or answering questions, so they are being engaged frequently. The nature of the questions that are asked to the students trend towards why things occur the way that they do, so a deeper understanding is encouraged. Also, the teacher’s questions serve as a model for what the students should ask themselves when they are working outside of class.

Experiential learning. Experiential learning derives its name from the “role that experience plays in the learning process” (Kolb et al., 1999, p. 2), with aspects taken from “Dewey’s philosophical pragmatism, Lewin’s social psychology, and Piaget’s cognitive-developmental genetic epistemology” (p. 2). Dewey (1938) advocated learning through experience. While experience and education are related, they are not equal, since “some experiences are mis-educative” (p. 25), which can stifle learning and hinder the educative effectiveness that an experience can have in the future. Collateral learning involves the formation of attitudes (either good or bad) about school (Dewey, 1938), and must be taken into consideration. Kolb and Kolb (2005) offered an observation similar to that of Dewey (1938): some experiences can be counter-productive, leading to negative feelings and interfering with learning. After all, “[t]o learn something that one is not interested in is extremely difficult” (Kolb & Kolb, 2005, p. 208).

Dewey (1938) even viewed experiences as lying on a continuum based on their educational worth, which encompassed many aspects and traits. Dewey (1938) defined an educative experience is one that leads "both to knowledge of more facts and entertaining of more ideas and to a better, a more orderly, arrangement of them" (p. 82). Along these lines, intellectual activity includes both analysis (breaking down ideas) and synthesis (putting together ideas), two components necessary in reorganization and categorization (Dewey, 1938).

Additionally, Dewey (1938) proposed a few other related principles. One of these, the principle of habit, portends that every experience changes each person involved in it, which subsequently affects the quality of all successive experiences. Another, the principle of continuity states that the future must always be considered at every stage of the education process. However, the purpose of education is not to merely train students to acquire a specific skill, but to prepare them for a changing world (Dewey, 1938) and create thinkers (Dewey, 1910), who can be productive participants in a democracy. As students grow, they must be able to adapt what they have learned to new situations.

Dewey (1910) advocated inquiry as a means of discovery. The teacher should present the material in such a way as to foster this discovery by providing stimuli. Furthermore, teachers must not underestimate the impact that novel ideas may have on students, who may be intrigued and energized by something new (Dewey, 1910). A valuable feature of the lecturing dialogue is its flexibility, which allows new ideas to organically emerge during class. While the predominant question-and-answer format may become familiar and mundane, the questions constantly change, creating new situations during every class. Other styles are also used when necessary to break the monotony.

Interaction between the teacher and students is vital, specifically in what Dewey (1910) dubbed "recitation," which is "the period of most intimate intellectual contact of teacher with pupil and pupil with pupil" (p 201). This interaction is an unending trait of the lecturing dialogue, with the frequent student-teacher communication deriving from the question-and-answer format. An advantageous side benefit of the lecturing dialogue is vicarious learning. Students do not only learn from their own questions and direct experiences, but from the questions and responses of fellow students, as well (Dewey, 1910; Lewin, 1939). Additionally, by seeing classmates respond (as opposed to just the teacher), students may realize that the material is not beyond their capabilities (Felder & Silverman, 1988).

Learning is a social process (Dewey, 1938; Lewin, 1939). It involves not only the student and teacher, or even just the other students in a classroom, but the entire community. Tied to this idea and the purpose of education, Dewey (1938) believed that students should have some control over what was learned because "democratic social arrangements promote a better quality of human experience, one which is more widely accessible and enjoyed, than do non-democratic and anti-democratic forms of social life" (p. 34). Bain (2004) also mentioned students having some control over their education and collaboration as elements of good teaching. Kolb (1984) noted the semi-cyclical chain-reaction that occurs in experiential learning. Every student has the freedom to make choices and decisions in life, which determine what experiences he/she has, which then influence future decisions.

The teacher must retain some control over the learning environment, but should not be a dictator in the classroom. Rather, the teacher should serve as a director of learning, connecting prior knowledge to new material (Dewey, 1910, 1938). To accomplish this, Dewey (1938) noted that the teacher must be able to read the students' reactions. Since a universal style of instruction does not exist for all situations, the teacher must be able to adapt a style or switch to a more appropriate style to reach the students (Dewey, 1938), and present the material in an understandable way. Dewey (1910) described this as "adjusting the subject-matter to the nature of thought" (p. 204) of the students. Beyond merely connecting the old and the new, the teacher must also create interest in the new material (Dewey, 1910). When successful, a cycle is created. New discoveries lead to new questions about those discoveries, which lead to more discoveries. Dewey (1897) saw education as "a continuing reconstruction of experience" (p. 79), making the goal and process of education identical.

Jean Piaget (1896-1980) also commented on learning through experience and the teacher's role in the classroom. His theory of cognitive development "posits that the learner actively constructs knowledge through interaction with the environment to create cognitive operations and underlying mental structures" (Long & Coldren, 2006, p. 238). The act of inventing (or re-inventing) an idea or concept is vital in Piaget's theory. "Real comprehension of a notion or a theory implies the re-invention of this theory by the subject" (Piaget, 1977, p. 731). The onus is placed on the individual learner, who is solely responsible for learning the material. The teacher is absolved of any liability, and can actually be detrimental to a student's development. "Each time one prematurely teaches a child something he could have discovered himself[,] the child is kept from inventing it and consequently from understanding it completely" (Piaget, 1970a, p. 715).

To be clear, even though the teacher has no accountability when it comes to a student's learning, the teacher still has a role, serving as a guide to learning and presenting "situations that will give rise to curiosity and solution-seeking in the child, and … support[ing] such behavior by means of appropriate arrangements" (Piaget, 1977, p. 731). Additionally, the teacher must identify conflicts that exist through limitations in a students' knowledge (Steffe & Gale, 1995). However, teachers need to be careful not to exert too much influence on their students, which can interfere with how students construct knowledge (Steffe & Gale, 1995). The ultimate goal is to promote autonomous thinking, which occurs "when the individual can use the intellectual tools of his or her culture – including sign systems, models, and theories – so expertly as to produce new understandings or force a reshaping of those tools" (p. 443). In other words, direct, external manipulation is to be avoided (even if indirect control is present in the form of upbringing and cultural norms). By allowing students the chance to answer questions before being given the answers by the teacher, the lecturing dialogue can avoid this manipulation and corruption of learning.

Piaget (1970b) stated that "the subject is aware of the object only through its own activities, but can learn to know itself only through its action upon the object" (pp. 69-70). Said another way, the student can learn about certain material through experience, but the student can also learn about himself/herself through that same experience. Both the sciences and mathematics aim to understand what is happening and explain why it is happening (Piaget, 1970b), and students should emulate that perspective.

Echoing Dewey (1910, 1938), Nouwen (1975) advocated open communication between students and teachers, with each group learning from the other (specifically, each other's experiences). Along these lines, Kegan (1994) and Bain (2004) noted that a combination of challenge and support was needed. Machina (1987) commented that teachers "can and should ask students to stretch. But we [teachers] can reasonably expect them to stretch only so far at any given time" (p. 21). To summarize, students should be challenged, but not challenged beyond their abilities. Bain (2004) acknowledged the necessity of an environment in which nonjudgmental failure was accepted. In the lecturing dialogue, the challenge is present in the questions that the teacher asks, with the support coming as guidance and feedback. Familiarity with the students will enable the teacher to ask questions that are appropriately challenging.

The lecturing dialogue exhibits many of the characteristics described in experiential learning. By simply asking questions, the teacher takes on the minimalist role of a facilitator or guide. Students are expected to steer the class with their responses, so there are plenty of opportunities for experiences to be gained. The process of working through problems is detailed, with an emphasis on why the problems work the way that they do, often incorporating connections to other topics (which have varying amounts of commonalities).

Constructivism. The idea that students must build their own understandings is known as constructivism, since students actively construct their own meanings from their experiences (Buell, Greenstein, & Wilstein, 2017; Steffe & Gale, 1995). Learning and understanding must be done by the students; "no one else can do it for them" (Steffe & Gale, 1995, p. 434). The teacher is relegated to the background, acting as a facilitator who provides the "appropriate situations, tasks, and conditions" (p. 434). The "transmission" approach to education (in which the teacher transmits knowledge by telling things to the students) is replaced by a "discovery" approach (Steffe & Gale, 1995).

According to Perry (1999), people tend to make sense of or interpret experiences meaningfully based on some form of orderliness in that experience. Constructivism is a way "to organize the experiential world, not to discover an ontological reality" (Glasersfeld, 1995, p. 10). This act of organization and making sense requires a balance between assimilation and accommodation (Perry, 1999). Assimilation is the process of integrating new experiences with prior experiences. Accommodation consists of adapting previous ideas to new information "by means of recombinations and transformations which result in new forms of expectancy" (p. 42). Basically, this is a procedure for dealing with new data. Students (and others) can add the new knowledge to old knowledge (assimilation), or they can replace (or adjust) old knowledge with what is learned from new experiences (accommodation).

Similar to experiential learning, constructivism must take on an inquiry orientation, in which the students use their current knowledge to explore "the mathematical world, asking questions, solving problems, testing conjectures, validating ideas, and explaining relationships" (Buell et al., 2017, p. 78). Students need to think about their experiences and reflect upon them to foster a deeper understanding, allowing the students to comprehend why something is true, as opposed to only knowing how to do it (Buell et al., 2017). To make a distinction, "teaching" works towards understanding, but "training" only aims to modify behavior (Buell et al., 2017). Therefore, the students' thinking must be paramount, outweighing mere behaviors. The purpose is to connect ideas, creating the interwoven tapestry of an overall concept, rather than a collection of isolated ideas that can be easily forgotten or lead to computational errors (Buell et al., 2017).

The goal of constructivism is to raise "the questions that will help them [students] reason through the process, to see the nature of the questions and to think about how to answer them" (Bain, 2004, p. 102). This goes beyond merely finding the answers to actually understanding the answers (and being able to judge their validity and legitimacy). This deeper understanding will allow the students to answer the initial question and leave the students asking more (and deeper) questions.

Students may need assistance in this process of formulating relationships among ideas. This is where the teacher's job begins. "Any effective pedagogy must be responsive to students in the sense that instructional practices are informed and reformed by students' knowledge and experiences" (Buell et al., 2017, p. 76). Zandieh, Wawro, and Rasmussen (2017) observed that the teacher can act as a broker (a liaison or translator) between the students and the mathematical community (or even the subject matter itself). Buell et al. (2017) referred to this job as a guide.

To be an effective guide, the teacher must listen to the students to ascertain their thinking, and then increase their knowledge by offering support and assistance to connect the concepts and ideas to real-life situations, helping "to formalize their informal ideas" (Buell et al., 2017, p. 88). The environment should be open, welcoming, and safe, reinforcing the idea that "errors are opportunities for learning" (p. 80). In addition, one student's question may benefit other students (who may have the same question, but be too timid to ask it).

The lecturing dialogue supports the concepts and requirements of constructivism with its interactive style. The teacher can gauge the students' thinking through the responses from the many questions that are asked. Students can learn from both the teacher and other students. The constant questioning also gives students many opportunities to construct their own understandings, with the teacher guiding the students away from any misconceptions. The novelty of the lecturing dialogue may also enthrall some students, leading them to pay more attention in the class.

Socratic Method. The lecturing dialogue combines several of the teaching styles detailed in the Spectrum of Teaching Styles, but it is based on the Socratic Method. In his study of its use in political science and political theory, Meckstroth (2012) did a superb job of outlining the background, process, usefulness, and benefits of the Socratic Method, as well as its limitations. Meticulously recorded in Plato's *Dialogues*, the Socratic Method has been alternately referred to as the Socratic *elenchus* – a Greek term meaning "'cross-examination,' 'testing,' or 'refutation'" (p. 646) and "'proof' or 'examination' in a court of law" (p. 646).

The overall course of action in *Dialogues* unfolded between Socrates (the questioner) and the "interlocutor" (the respondent), and was in the form of a question-and-answer dialogue between the two. Essentially, the interlocutor stated an opinion/belief, which Socrates questioned/challenged. However, Socrates "does not present some counterargument" (p. 646). The interlocutor then justified or modified his (always a "he" in Plato's writings) original statement in response to the question, with Socrates questioning that response. The process was repeated until a paradox or contradiction was discovered (Figure 3). "Ultimately, it becomes

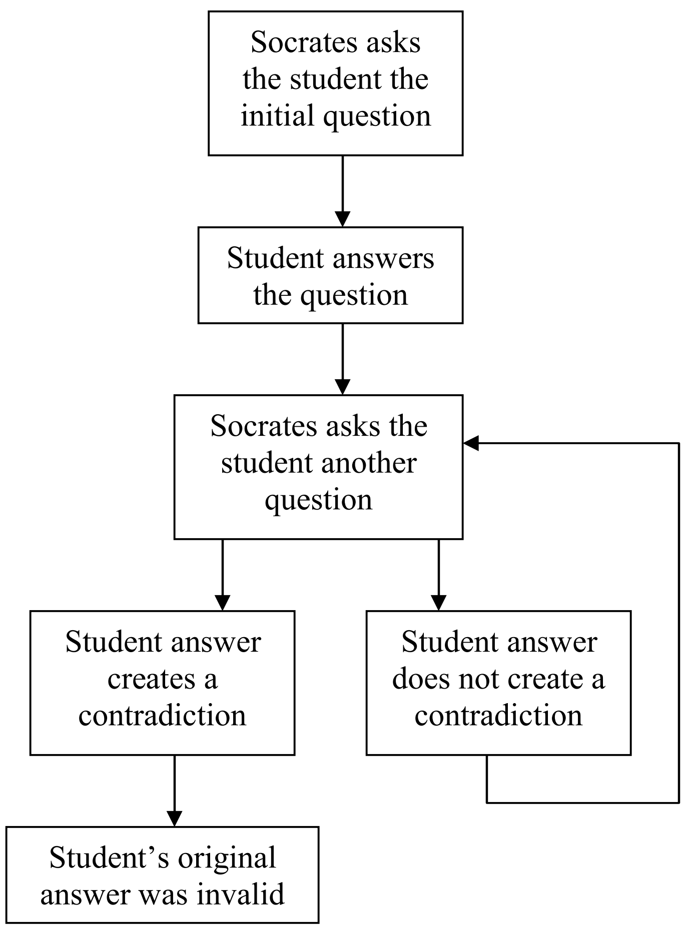

Figure 3. Socratic Method visual.

clear that some of these consequences contradict others, and this shows the interlocutor's position to be incoherent" (p. 646). Again, a bit of the chaos that was mentioned by Luckie et al. (2004) appears, particularly within the interlocutor's mind concerning his original beliefs. Also, Luckie et al. (2004) noted that "it is best to answer a student's question with another question" (p. 207), thereby corroborating the basis of the Socratic Method.

The beauty of the Socratic Method is its versatility. Meckstroth (2012) pointed out that the Socratic Method is antifoundational: "It does not depend on accepting as true any positive claim that cannot be justified strictly in and through *elenchus*" (p. 652). Since it does not rely on any foundational beliefs, the Socratic Method can be used in virtually any situation in which one expounds a conviction. In addition, every Socratic dialogue is unique. No two discourses are the same because the interplay depends on the beliefs/justifications of the interlocutor and the questions of the questioner/teacher (both of which can vary greatly). As a result, "Socrates' refutations cannot be universal" (p. 648), and are only specific to those exact instances/moments because of "their immanent and dialogical character" (p. 648). Therefore, any refutation relies upon the particular position of the interlocutor. This uniqueness of and variation within each dialogue makes the Socratic Method so valuable for teaching. Through the question-and-answer process, the teacher can discover what misconceptions a certain student has, and then address those specific areas (similar to what occurs during private tutoring).

In Bain's (2004) investigating into the traits of good teaching, he recommended starting with the student, not with the discipline (or subject). To accomplish this, the teacher should start with what, "as Sandel put it, 'students care about, know, or think they know, rather than just lay out a blueprint or an outline or tale or theory or account of our own'" (p. 110). This is precisely the strategy that Socrates employed in the Socratic dialogues described by Plato. "Using

Socratic questioning, [one] begins with what ‘common sense’ might suggest to the students; then, through additional probing, he helps them add the ‘muscle’ that disciplinary discoveries can give them” (p. 110). By asking questions and investigating the situation, problems with the proposed (common sense) solution may be identified. This process occurs often in mathematics. Rather than simply giving the students the answer, the teacher guides them to it through purposeful questioning. Of course, this progression must begin at the students’ level.

Theoretically, the Socratic Method will always find the contradiction in a false argument (depending on the questions that are asked, of course). The drawback to using the Socratic Method is that one can never prove a truth, since the question that will lead to a contradiction simply might not have been asked yet. “[O]ne can never eliminate every possible alternative view (or, at least, one could never know for certain that one had already accomplished this task), [so] there can be no end to the critical search for new alternatives to challenge” (Meckstroth, 2012, p. 649). However, the inability to demonstrate truth relies on the assumption that the questioner does not know the answer and is merely questioning the interlocutor to receive justification for the answers given. When the Socratic Method is used by a teacher (especially in mathematics) to inquire about the beliefs of a student, the teacher can verify that the student has adequately justified/supported/verified a true statement. Additionally, the teacher will be able to discover where any misconceptions lie, and then ask the appropriate questions to reveal those misconceptions.

Another important aspect of the Socratic Method lies in the fact that “knowledge is inherently method dependent” (Meckstroth, 2012, p. 645). As Meckstroth (2012) eloquently stated, “the pursuit of knowledge necessarily concerns the validity not only of our beliefs but also of the ‘way’ by which we arrive at them” (p. 645). This is also very important to teaching and learning. When a student knows why an equation or formula works, and how it is derived, he/she gains a deeper understanding of the material. The notion that knowledge and the method used to gain that knowledge are connected has been supported by such philosophers as Immanuel Kant (in *Critique of Pure Reason*, specifically, The Discipline of Pure Reason in the Transcendental Doctrine of Method) and Francis Bacon (in *Novum Organum*). The fundamental premise is that what is learned can be greatly affected by how it is learned. For example, students who are simply given the quadratic formula or change of base formula for logarithms can learn how to use them, but students who learn how those formulas were derived may gain a deeper understanding of quadratics and logarithms beyond being able to apply the formulas.

In addition to Meckstroth (2012) presenting evidence that the Socratic Method can be used in political science and political theory, Skordoulis and Dawson (2007) further demonstrated the variety of settings in which the Socratic Method can be employed, studying its use in organizational change. Basically, the question-and-answer aspect of the Socratic Method can be used to share ideas and perspectives as a manner of conflict resolution. By asking and answering questions, a specific party/group can learn what another party/group is thinking. From that information, a consensus can be reached, maximizing the results of any change made within an organization, and minimizing any conflicts or misunderstandings. Since all individual stakeholders can offer opinions, win-win situations can be reached (or mutually agreed upon compromises, at least), benefiting everyone.

Because the process of conflict resolution is based on learning what the other is thinking, the situation is reduced to one of learning and education. Each person/group endeavors to share his/her/its own point of view and learn the perspective of the other person/group. In the process, misinterpretations are clarified and knowledge is gained (Zandieh et al., 2017). As a result, the effectiveness of the Socratic Method as an educational tool is emphasized and established.

Areeda (1996) provided a comprehensive outline for what the Socratic Method is not, debunking common misconceptions and misuses. He also included reasons for both the resistance that students often offer and the hesitance that they exhibit. The Socratic Method is often misconstrued, stemming from false impressions about its nature. First and foremost, it is not a case study. While the Socratic Method does vary on a case-by-case basis, its purpose is not to examine individual instances. Rather, the goal is to seek truth (or reveal contradictions). Although it is based on an individual's thoughts and beliefs, the Socratic Method is more than just a student's opinion. It must be based on facts that are supported by logical reasoning and rational thinking.

The Socratic Method is not a "recitation of assignment" (Areeda, 1996, p. 911). It should not be used as a way to review a prior lecture with students reciting previously learned ideas. On a similar note, it is also not an "antiphonal catechism" (p. 911). That is, it is not a scripted question-and-answer session along the lines of some religious lessons or rituals. It is also not simply a brief interjection in a lecture. As Areeda (1996) put it, the Socratic Method is designed to be more than a mere "non-followed-up vague or big picture ramble" (p. 913) or a "token mid-lecture pause" (p. 913). When used, the Socratic Method should be employed for an extended period of time to reveal the truth (or background and development) about a topic or concept.

In legal studies, the Socratic Method should be used to do more than "demonstrate indeterminacy" (Areeda, 1996, p. 913). Although the Socratic Method often leads to a contradiction and the refutation of a commonly held belief, it is not meant to only show that generalizations or other maxims are not universal. This may occur, but it is nothing more than a side-effect of the process of the Socratic Method, not the main purpose.

Students often dislike the Socratic Method, particularly when they are first exposed to it. The source of this displeasure may stem from students being unfamiliar with the Socratic Method, particularly with its goals and procedures, as espoused by Udovic et al. (2002), but one cannot overlook the possibility that students will not comprehend the benefits that may be derived from its usage. As Udovic et al. (2002) and Volkmann et al. (2005) noted, students are not always used to thinking on their own and actually learning during class, so students resist the new, alien teaching style, preferring the recognizable, more comfortable lecture instead.

The inefficiency of the Socratic Method is another complaint from students (Areeda, 1996), who do not appreciate the educational advantages of discovery learning (or guided discovery, depending on the amount of teacher guidance), and do not understand why the teacher conceals the desired information and does not simply reveal it. The individual nature of each dialogue using the Socratic Method may also be frustrating (Areeda, 1996). Since each student may offer different arguments, a specific *elenchus* may not be transferable. This is important because all of the students in a high-enrollment class will be unable to verbally participate in their own dialogue with the teacher (Areeda, 1996). A solution to this is to ask all non-vocal students to become silent participants (active participants in the class who think through the teacher's questions along with the student who is directly engaged in the Socratic dialogue). Another alternative is to allow different students to become involved in any given dialogue. A particular student may answer one or more questions, but other students may answer follow-up questions. This can be done when the original student reaches an impasse and cannot answer a question, or merely to directly engage more students in the class.

Areeda (1996) advocated randomly calling on students to keep them focused on the class. However, that may inadvertently cause the next common complaint, that the Socratic Method "intimidates, entraps, and humiliates students" (p. 916). Students will most likely be uncomfortable speaking in front of 200 peers. Add the prospect of receiving unexpected questions from the teacher and vocally participating in class becomes a terrifying proposition. Failure (in the form of an incorrect answer) is equated with weakness and imperfection, opening the student to ridicule (whether real or simply imagined by an egocentric adolescent).

Students must constantly be reminded that mistakes are to be expected (Areeda, 1996). "They wouldn't need to be here if they already knew it all" (p. 917). Beyond learning from their own mistakes, students should also learn from the errors of others. Even if a student knows how to do a problem, a deeper investigation of a situation can often lead to insight that otherwise would not have been gained. However, students may feel that they have been set up to fail when they are led down a path that results in a contradiction (Areeda, 1996). It may not matter to a student who offers an incorrect answer, but a contradiction was the standard outcome of the original Socratic dialogues, so it is a proper use of the teaching technique. Additionally, the process of revealing a contradiction may both identify an erroneous method and reveal the correct procedure, benefiting a student in the end. Often, the error will be a common mistake that many students are likely to make, so addressing the frequent pitfall during class will draw attention to it, helping the students to be vigilant and avoid it on an actual assessment that is worth a grade.

Boredom and confusion were also listed by Areeda (1996) as being reasons why students dislike the Socratic Method. Boredom is likely the byproduct of either non-participation in the class (waiting for the final result instead of actively seeking the result during the dialogue) or the student being overqualified for the class (the student already knows the how and why of what is being discussed, so no discovery occurs). Neither situation can be avoided by the teacher because they are out of his/her control. Teachers cannot force students to participate, and class placement is an administrative concern.

Confusion, like the chaos of Luckie et al. (2004), is necessary and even constructive, providing incentive for students to continue to work through a problem. One objective of the Socratic Method is to help students to "reason from what they know to the solution of problems they didn't know they could solve. That object[ive] could not be achieved by professorial syntheses before the students have tried to solve the problem on their own" (p. 918). In other words, students will be incapable of learning a certain subgroup of material (regardless of what the teacher does) until they have reached a certain stage of progression that has been attained through their own endeavors.

Ideally, the Socratic Method will produce an overarching, long-term transformation in each student. The questions that the teacher asks should serve as a model for questions that students can ask themselves (Areeda, 1996). After reaching solutions with the guidance and assistance of the teacher, students gain confidence for reasoning through problems on their own. Eventually (and with enough practice and modeling), students will recognize which questions they should ask themselves (the same questions that the teacher would have asked), and can then carry out the process without any external aid. Essentially, the process of the Socratic Method becomes internalized, with the student carrying out the dialogue within himself/herself (and outside of any formal class). This is all fostered in the structure of the lecturing dialogue.

Lecturing Dialogue

The lecturing dialogue primarily exists at the intersection of Spectrum Theory and the Socratic Method, utilizing inquiry-based learning (in the form of instructional conversation and guided discovery, with elements of lecturing and tutoring) within the realm of active learning, experiential learning, and constructivism to appeal to learners with various learning styles, which helps students to discover new information (specifically, why things occur in the way that they do) and make connections between old and new material. Spectrum Theory outlines the different teaching styles (based on how much a specific style is teacher-centered versus student-centered). Each style has its own unique characteristics, strengths, weaknesses, and most-appropriate/most-useful setting. The lecturing dialogue combines several of the different approaches from the Spectrum of Teaching Styles (covering the material in many different ways) to reach as many students as possible (and maximize learning). As part of the teaching process, the lecturing dialogue relies heavily on the Socratic Method. The question-and-answer format of inquiry is ideally suited to identify any misunderstandings or misconceptions that the students might have, which can then be clarified and corrected. The overall goal is to engage students in the subject, making them active learners. The natural results are a more interesting and enjoyable class (intellectually stimulating, with a large focus on why things occur and how they are related), and a more effective learning experience for the students (learning more material, learning the material better, and earning higher grades).

Active learning, experiential learning, and constructivism are all interrelated and inseparable perspectives of the same aspect of learning. Experiential learning requires students to actively construct knowledge based on experiences. Constructivism also requires the active construction of knowledge, but relies on experiences to serve as a foundation for that construction. As active learners, students construct knowledge, and need experiences to gain a basis for that construction. Inquiry-based learning is a specific type of active, experiential, constructive learning that relies on questioning, either internally (wondering to oneself) or externally (another posing the questions). The Socratic Method is a particular type of external inquiry that occurs between the students and the teacher (and sometimes among the students). By asking questions within the context of the Socratic Method, the teacher uses various teaching styles from Spectrum Theory, choosing a specific teaching style based on a myriad of factors, including the knowledge, abilities, and personalities of the students. The teacher must have a vast and deep understanding of the content material in order to ask the appropriate questions to the students to facilitate learning, and to provide sufficient and detailed responses to the students' queries. A diagram for the lecturing dialogue can be seen in Figure 4.

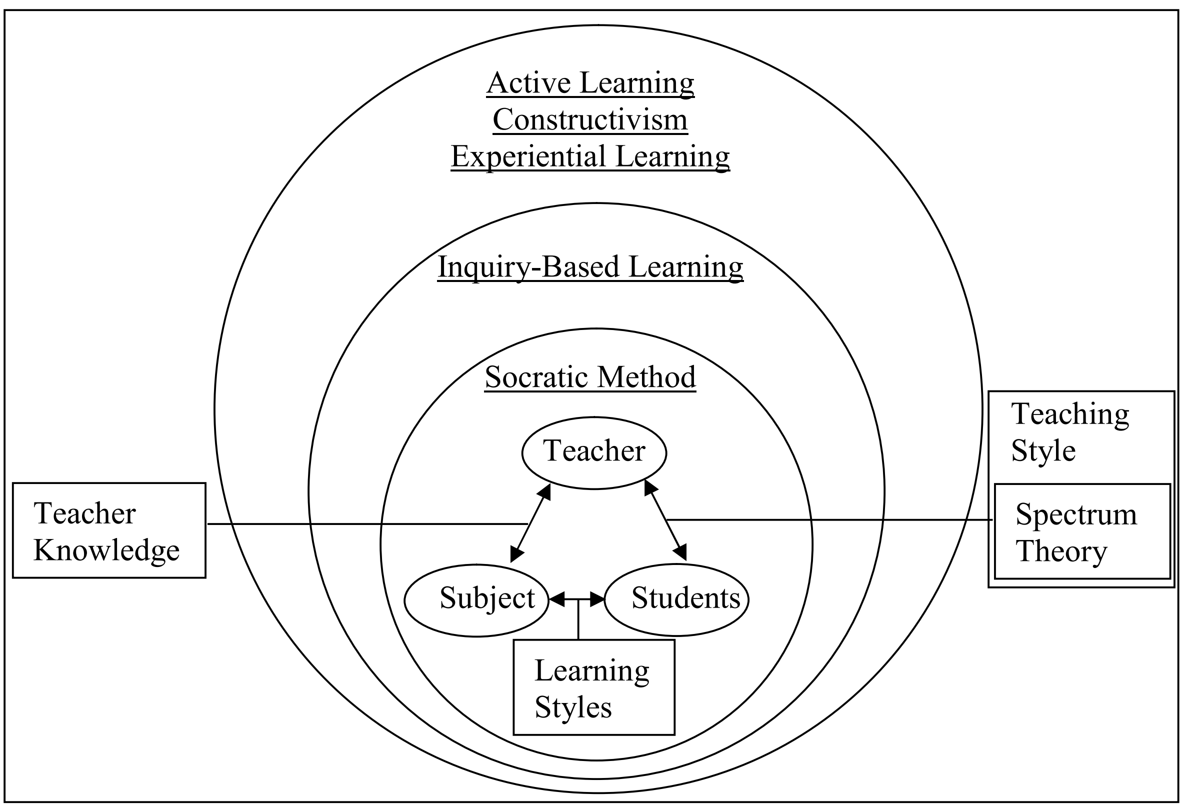

Figure 4. Lecturing dialogue visual.

As a disclaimer, teachers cannot simply copy the styles of other high-quality teachers (Bain, 2004). Techniques can be adopted, but they must also be adapted to fit the teacher. Bain (2004) compared teachers to great artists: “each [artist] had to find his own genius. So, too, must teachers adjust every idea to who they are and what they teach” (p. 21). Self-reflection and self-assessment can aid in this task, allowing teachers to contemplate how each lesson was received. Therefore, while the lecturing dialogue is being described here, and its benefits are being detailed, it may not be the best teaching style for every teacher. However, the lecturing dialogue can be modified as appropriate for each teacher’s individual personality.

Chapter 2 – Review of Related Literature

Teaching and Learning Methods

In high-enrollment classes (over 200 students for the purposes of this study, but defined for various class sizes by different studies), the options for teaching styles are limited, especially in mathematics. Most large classes are conducted as lectures (Tolley et al., 2012), mainly because of practicality, logistics, and basic ease. The question arises as to whether another style would lead to students understanding mathematics better (along with higher grades and better attitudes for the students) than a typical lecture style, all while minimizing the necessary in-class resources. Note: The terms style and method were both used in articles to describe ways of teaching. For consistency, style will be used, other than for the Socratic Method and direct quotes.

Little research could be found devoted to which styles mathematics professors use or prefer. There were several studies about individual teaching styles in mathematics (which were the same as the styles used in general for all subjects), but nothing that investigated the frequency with which the individual styles were used. The closest study was by Bressoud and Rasmussen (2015), who found that 50% of the calculus instructors at successful universities (those with high retention rates and a belief that mathematics is sensible, useful, and worthwhile) rated their teaching style as a three (with one being "very innovative" and four being "very traditional") and 37% as a two, but only 9% selected a one. At the other universities, only 5% were a one, 29% a two, and 56% a three. Also, limited research was found about different question-and-answer techniques (other than the Socratic Method).

Lectures, also referred to as expository teaching (Ausubel, 1963), expository instruction (Webb, Metha, & Jordan, 2000), or command instruction (Ashworth, 2008), are very practical for teaching high-enrollment classes while using few resources because they can get information to a large number of students in an efficient, cost-effective manner. According to Webb et al. (2000), lecturing is the "best method for students grouped homogeneously by ability" (p. 510). However, that homogeneity is subjective. In any group of students with similar abilities, differences will still exist, since no two students are exactly alike. Some will be at the higher end of the ability range of the group, but others will be at the lower end. Furthermore, lectures are not very engaging for the students, and can even be boring. The students become passive learners (Swaak et al., 2004) and student-teacher interaction is limited. This becomes more significant when considering the findings of Smith (1996) that student-student interaction and student-teacher interaction were among the most significant factors that impact education.

Discussions, the instructional conversation studied by Goh et al. (2012), work well with smaller classes, especially those that deal with controversial topics, subjective material, or items that can be debated, such as religion, art, or politics. However, it is difficult (if not impossible) to try to conduct an all-inclusive discussion in a classroom of 200 students. If every student had an opportunity to speak, then nothing would get accomplished; there is just not enough time during class. Also, it can be very difficult to keep that many students focused and on task. Side discussions start too easily, and disruptions become commonplace. To be fair, discussions can be used in mathematics, but they are often limited to which technique to use to solve a problem or why a theorem is valid, topics that generally have minimal relevancy in a college algebra class. Additionally, mathematical discussions (like most discussions in other subjects) are typically conducted in classes with smaller enrollments.

Discovery learning (Ausubel, 1977; Bruner, 1961), also referred to as inquiry instruction (Webb et al., 2000), inquiry-based learning (Pasch et al., 1991), convergent discovery

(Ashworth, 2008), or divergent discovery (Ashworth, 2008), is excellent for gaining hands-on experience, but requires self-motivated students and mentoring from the teacher (and maybe a few others). The students must be able to work (largely) by themselves (Bruner, 1961), with guidance from the mentors (Webb et al., 2000). The average student demonstrates neither the discipline nor the curiosity necessary to succeed in discovery learning. Oliver (2007) described this deficiency as “limited self-regulated learning skills” (p. 4). Keeping all of the students on track becomes a daunting chore. On top of that, the number of mentors that would be needed for a high-enrollment class makes this type of instruction prohibitive (Oliver, 2007).

Guided discovery (Ashworth, 2008; Bruner, 1962), also called critical thinking instruction by Webb et al. (2000), is similar to discovery learning, but requires more mentoring and guidance. Students must still work without immediate supervision for much of the time, but they have the benefit of several mentors steering the students’ studies in the desired direction (Webb et al., 2000). To adapt this to a large class, an even larger number of mentors would be needed than for discovery learning, creating a stronger disincentive for using guided discovery.

Guided discovery has been modified to be used in high-enrollment classrooms by incorporating supplemental practice (Miller & Schraeder, 2011), which is an extra day of class that combines cooperative learning (Webb & Palincsar, 1996), worked examples (Miller & Schraeder, 2011; Sweller & Cooper, 1985), and mentor guidance (Webb et al., 2000). Citing prior work with Johnson and Johnson, Smith (1996) defined cooperative learning as “the instructional use of small groups so that students work together to maximize their own and each other’s learning” (p. 71), which is similar to reciprocal instruction (Ashworth, 2008). A worked example is comprised of a worked-out example (along with some basic notes about the techniques and properties used) followed by an example for the students to do (Miller &

Schraeder, 2011). The mentors were present to answer any questions, as well as to help to keep the students on task. This style mainly benefited average students (when disregarding motivational aspects), having little effect on low-level or high-level students (Miller & Schraeder, 2011). Also, a total of at least four mentors were needed for a class of 200 students, and keeping all of the students on task was still a struggle.

Private tutoring is an excellent way to teach, since the student receives one-on-one attention. Lessons can be tailored to meet a student's individual needs, learning style, and prior knowledge. Unfortunately, the resources are not available (neither the number of tutors nor the necessary funding) to give every student a private tutor. Instead, one teacher must be able to reach many students at once, all of whom have different backgrounds and learning styles (Webb et al., 2000).

Several different types of question-based instruction have been defined as inquiry-based learning, usually ranging from guided discovery to self-teaching (any of the production styles in Spectrum Theory), but any teaching style with the appropriate characteristics can be included in the group. Flores, Phelps, and Jansen (2017) defined inquiry as "students [posing] their own mathematical questions and problems instead of being presented with teacher-prepared material to be learned or only solving problems posed originally by others" (p. 47). In their study of a college math student's experiences with inquiry-based learning, Flores et al. (2017) found that inquiry learning helped the student "develop a deeper idea of the nature of problem-posing and research, including the inherent frustrations and tensions of the process" (p. 52). Furthermore, the student enjoyed the problem-posing process, which involved asking why things occur the way that they do. Along the lines of discovering reasons and asking why, Cook and Borkovitz (2017) found that inquiry learning led to more confidence in mathematics and an increased ability when solving problems for a mathematics major. The student attributed the improved learning to exploration (or inquiry).

Firkins Nordstrom and Sumner (2017) studied a style similar to a learner-designed individual program, in which the teacher suggested targets for inquiry. In most inquiry-based learning situations, students are asking questions that many scholars and students have asked and answered in the past, but it is important to remember that these questions are new for the students (Firkins Nordstrom & Sumner, 2017). Until students "feel that they can ask and answer questions in a field [of study], the inquiry model will not work" (p. 10). The real challenge came from trying to reach students who lacked the background knowledge or self-confidence to succeed in an inquiry-based learning situation. As a potential remedy for the lack of confidence (or a lack of interest), Firkins Nordstrom and Sumner (2017) recommended using popular culture references to pique the students' curiosity and get their attention.

Stasis (the lack of a definitive answer) and disagreement can be used to engender an environment of discovery in the classroom by keeping the students on the edge of stability (Firkins Nordstrom & Sumner, 2017), similar to the chaos of Luckie et al. (2004). When students are unsure, they begin to ask questions, and are likely to seek the answers. Elbow (1998) reaffirmed this idea: "Doubting an assertion is the best way to find an error in it. You must assume it is untrue if you want to find its weakness" (p. 148). Students must often believe that a statement is true in order to understand it (Elbow, 1998), but they can then question the statement to provide verification of its correctness.

Once students gain some experience with inquiry, they can learn to ask their own questions, and gain the confidence to debate and be wrong (Firkins Nordstrom & Sumner, 2017). These chances to ask questions are rarely available during a lecture, but discussion-based classes provide ample opportunities (Firkins Nordstrom & Sumner, 2017). With enough practice, students may learn to ask the correct questions when they are alone (which is the desired end result of inquiry-based learning, and should be emphasized during class). Formulating ideas as a community by employing discussions can help students cultivate a feeling of ownership of both the material and the process used to develop that material (Firkins Nordstrom & Sumner, 2017). Learning in a community setting can also provide the opportunity for students to receive feedback from other students, not just from the teacher (Firkins Nordstrom & Sumner, 2017).

For inquiry-based learning to succeed, students must be self-motivated and want to learn (von Renesse & Ecke, 2017). Asking questions and collaborating with other students is also beneficial, and a pursuit of mathematics outside of the classroom is a definite advantage (von Renesse & Ecke, 2017). It is important to foster a "culture of asking" (p. 148) in which curiosity is abundant and questions flow naturally, especially because "curiosity improves learning" (p. 149). However, the curiosity must be managed so that students do not attempt (or are not asked) to span too large of a knowledge gap (von Renesse & Ecke, 2017). Additionally, an environment must be created "in which mistakes and 'wondering questions' are celebrated" (p. 159), with the teacher modeling the desired communication by asking questions. Piercey and Cullen (2017) also acknowledged the importance of allowing mistakes, whether computational or conceptual. Instead of adopting a fixed mindset (which is more likely to lead to quitting in the face of adversity), students must cultivate a "growth mindset, [in which] the challenge of finding the answer outweighs any stigma of making a mistake" (p. 23).

Brown and Walter (2014) noticed that inquiry can improve students' ability to explore slight alterations to situations that the students are already familiar with by reposing a question (altering the task) or reframing a situation (changing either the assumptions or the context to create a different setting). Slavit and Lesseig (2017) referred to this as asking "what if?" questions, as in "What if something was different?" This allows students to be more productive and solve new problems and can also diminish the reliance on textbooks and other instructional materials by eliminating the traditional educational boundaries (Slavit & Lesseig, 2017).

Not all of the research involving inquiry-based learning was positive, however. Kirschner, Sweller, and Clark (2006) found that pure-discovery teaching styles (some extreme implementations of inquiry-based learning, such as self-teaching) may be counterproductive because a lack of feedback allows students to make too many errors that lead to frequent misconceptions. Students may become frustrated with repeated false starts and the accompanying inefficiency (Kirschner et al., 2006). Another disturbing development was the dearth of material addressed in some inquiry classes. Cook and Borkovitz (2017) noted that some inquiry-based courses addressed less material than the same classes at other institutions (which used more traditional teaching styles). Other inquiry-based classes addressed topics that were different from those addressed in comparable non-inquiry-based classes. Some variation is not too alarming because differences commonly exist between schools, but covering less material is a disservice to the students, since they may not be prepared for future classes.

As a hybrid teaching style, the lecturing dialogue combines the typical lecture (which is hard to avoid to some extent in many subjects, especially mathematics) with a dialogue between the students and teacher (instructional conversation). This dialogue is a conversation in a question-and-answer format (inquiry-based learning), with both the students and the teacher asking and answering questions, and utilizing the Socratic Method, in which the teacher questions students as to why they hold certain beliefs and points of view (Meckstroth, 2012). This forces the students to think about why formulas work the way that they do, and (hopefully) leads to a deeper understanding of the nature of mathematics. Dialogue can help to reveal the historical developments in a subject (Piaget, 1970b), particularly those of mathematical formulas and techniques. Along with a lecture and instructional conversation, the lecturing dialogue contains elements of guided discovery (the teacher asking leading, pointed questions to guide the students' thinking), discovery learning (asking open-ended questions to facilitate thinking, utilizing both convergent discovery and divergent discovery), and tutoring (gaining insight from students' questions and answers about what they are thinking to tailor responses to their inquiries). The overall format of the lecturing dialogue is similar to practice instruction (Ashworth, 2008), which combines lecture with students practicing new skills. However, the lecturing dialogue does not use only a straight lecture as a means of explanation.

Unfortunately, the same difficulty exists for the lecturing dialogue as for many of the other non-lecture teaching styles: the need for student-teacher interaction. In high-enrollment classes, students are often reserved and seem reluctant to speak up in front of their peers. This can be overcome by relying on a group of frequent participants (questioners and/or responders) in the class, but the existence of that group is never guaranteed. The intent is that, by asking questions, the teacher can help the students to overcome their unwillingness to speak. By engaging the students in a conversation, the students are (ideally) drawn into the subject, making it more interesting and enjoyable, and causing the students to want to learn more. Also, because the students know that they will be asked questions (and be expected to respond), they will be more likely to read ahead in the textbook and come to class prepared for the day's lesson. All of this is designed to increase the students' knowledge, value, and enjoyment of mathematics, along with elevating motivation and self-confidence. As a side benefit, the students' grades should also improve.

Attitude and Grades

A working definition for attitude was not provided in any of the articles used for this study, so the definition provided by *Webster's Dictionary* will be used: "posture of a person; mental or moral disposition." In light of that, Tapia (1996) drew upon a combination of the statistical results from her research and a review of literature to identify four sub-categories of attitude: enjoyment, motivation, value, and sense of security. A closer examination of Tapia's (1996) categories revealed that "sense of security" could more aptly be called "self-confidence." Among other feelings, the questions in the survey referred to comfort and anxiety in regards to mathematics, which could easily describe self-confidence (or a lack of it).

Grades and attitudes are often related to one another. Specifically, higher grades can lead to greater self-confidence, and vice versa. Students who enjoy classes or see value in them are more likely to be motivated to work harder. Based on observations, frustration can harm both enjoyment and value, which can lead to a lack of motivation, known as amotivation (Deci & Ryan, 1985).

Lijun (2011) studied three different learning approaches (surface, deep, and achieving) as they applied to college students in China. Each learning approach was associated with its own strategy and motivation. Surface motivation refers to learning material simply to pass a test, and involves students using "passive, superficial[,] and dealing strategies" (p. 127); deep motivation is "aiming to understand and master knowledge" (p. 127), which is attained by using specific, active strategies; and achieving motivation is trying to gain the approval of others (usually teachers and/or parents), and involves strategies to please others. The results revealed that achievement motivation had a significant, positive correlation with each of the three strategies (surface – weak, deep – medium, achieving – strong), and the achievement strategy had a significant, positive correlation with each of the three motivation styles (surface – weak, deep – medium, achieving – strong). In other words, those who had achieving motivation used all three strategies to reach their goals, and the achieving strategy was used by those with all three types of motivation. While the lecturing dialogue aims to foster deep motivation, the student-teacher relationships that develops may help to support achieving motivation, since students will be enticed to impress a teacher with whom they have a personal (although professional) relationship (Bain, 2004).

While Lijun (2011) did not measure success rates with his study of motives and strategies, he did look deeper into the data to investigate whether the year in school or gender of the participants was related to the learning motives and strategies. More females had achieving motivation than males, presumably, as Lijun (2011) speculated, because the females were more worried about pleasing others. Deep motivation was relatively high in freshmen, dropped in sophomores, and then rose drastically for juniors and seniors. Lijun (2011) did not offer a reason for this, but a plausible explanation is that freshmen were excited about going to college, so they worked hard. The novelty of college wore off by the second year, so motivation waned, only to increase as students prepared for life after college and sought jobs.

Goodman, Jaffer, Keresztesi, Mamdani, Musariri, Pires, and Schlechter (2011) investigated the relationship between motivation (both intrinsic and extrinsic) and academic performance (GPA), with effort (an internal response that leads to specific actions) as a mediator. The data for motivation and effort were collected from participant responses to a questionnaire, using a five-point Likert-scale. The study showed that the relationship between motivation (both intrinsic and extrinsic) and GPA was significant (when mediated by effort), but weak, with intrinsic motivation having more of an influence on GPA than extrinsic motivation. Motivation (again, both intrinsic and extrinsic) was positively correlated with effort (more motivation meant more effort), which then impacted academic performance (Goodman et al., 2011). Therefore, motivation had both a direct and an indirect influence on GPA.

Lau (2009) studied motivation and self-efficacy changes (as they applied to reading) over time (different grade-levels in school, spanning elementary school and high school), between genders, and among different levels of academic achievement (Band 1 – highest 33.3% of students, Band 2 – middle 33.3%, and Band 3 – lowest 33.3%) in China. In addition to intrinsic motivation (doing something for its own enjoyment) and extrinsic motivation (doing something to achieve another goal), Lau (2009) included social motivation (doing something to fit in with others). The values for self-efficacy (similar to self-confidence) and all three types of motivation were obtained from participant responses to a survey utilizing a four-point Likert scale. Lau (2009) found that significant differences existed for self-efficacy and all three types of motivation across the achievement levels (Band 1 students were significantly more motivated and had significantly higher self-efficacy than Band 3 students) and across the grade levels (all four measures declined over time).

To distinguish between the two terms, self-confidence is “whole-hearted reliance on one’s own powers and resources,” whereas self-efficacy is “a belief in one’s power to produce effects” (according to *Webster’s Dictionary*). Put another way, self-confidence is an overall belief in one’s abilities, specifically doing something on one’s own. Self-efficacy (which Lau did not explicitly define) is the belief that one will be effective (or successful), but leaves open the possibility that an individual may use the help of others. Otherwise, the two beliefs are virtually identical.

In addition, Lau (2009) found that significant differences in intrinsic motivation and social motivation existed between genders (with females being more motivated than males). As for the interaction effects, none of the four measures was significantly different for the interaction between gender and achievement level, so self-efficacy and all three types of motivation had similar variations for both males and females at each achievement level. On the other hand, the interaction between gender and grade level showed significant differences for self-efficacy and social motivation. These results were similar to those found by Lijun (2011): females had lower self-efficacy and were more apt to do things for reasons related to social motivation as the students progressed through school. Lau (2009) attributed this to two tendencies of females that are not as prevalent in males: conforming to a group's ideals and wanting to please others.

Lau's (2009) study found that the interaction between grade level and achievement level was significantly different for extrinsic motivation (Band 2 students had the most variation in motivation as they progressed through school). The combined interaction among grade level, gender, and achievement level showed significant differences for extrinsic motivation and social motivation, so males and females reacted differently at different grade levels, depending on the achievement level. The interaction effects that included the interaction between gender and grade level had significant differences in social motivation, and the interaction effects that included the interaction between grade level and achievement level had significant differences in extrinsic motivation. Basically, the results from Lau (2009) illustrate the complicated and complex relationships among grade level, gender, motivation, and achievement level. Students with different backgrounds will react in dissimilar ways.

Attitude and Teaching Style

Innovative teaching styles (defined as any teaching style that was different from what the students were used to) had a significant positive effect on overall attitude, general interest, and career interest in several math and science classes (Savelsbergh, Prins, Rietbergen, Fechner, Vaessen, Draijer, & Bakker, 2016). Furthermore, innovative teaching had a significant and strong impact on achievement (Savelsbergh et al., 2016). Other findings included no correlation "between the effect of an intervention on attitude and its effect on achievement" (p. 168), so those who did better in the class did not necessarily like it better, and vice versa. Also, attitude was harder to influence with older children (in studies dealing with grades 5-12). As a disclaimer, Savelsbergh et al. (2016) acknowledged the importance of "quality of the content and implementation" (p. 168). Depending on a myriad of factors, the same technique can be stimulating or boring, intriguing or confusing, profound or superficial (Savelsbergh et al., 2016). Context also plays a big part. Based on their results, Savelsbergh et al. (2016) recommended that teachers experiment with innovative teaching styles that fit their personalities.

A study by Trevena and Clarke (2002) investigated small, student-led groups in health care classes. Based on both performance and feedback from the students, this style was effective and enjoyable. Also using small groups, Li and Demaree (2010) studied the development of "scientific discourse" (p. 28) within a "Community of Practice" (p. 25) in a large-lecture physics class. By modeling question-and-answer techniques (based on the Socratic Method), first in a whole-class environment and later in small groups, students did, indeed, acquire the desired skills. However, Li and Demaree (2010) only observed the results in small-group discussions. As a result, a precedent was established for using the Socratic Method in large-lecture classes, and confidence was built for its effectiveness, but whether the Socratic Method will truly work in whole-class settings was not tested.

Elbert-May, Brewer, and Allred (1997) took both active, inquiry-based instruction and cooperative groups to the next level (in terms of class size, at least) by studying their implementation in large lectures. Building on the findings of Glasson and Lalik that "[l]earning science at any level is a constructive process that requires active participation by both the student and teacher" (Elbert-May et al., 1997, p. 601), Elbert-May et al. (1997) found that a combination of cooperative learning and question-and-answer instruction helped to produce more effective learning (based on the results of a National Association of Biology Teachers exam) and increased self-confidence in students when compared to a traditional lecture. Specifically, the confidence was in doing science, analyzing data, and explaining biology to other students. Also, being more active led students to take more ownership of the course and their own learning (higher value and enjoyment). Overall, Elbert-May et al. (1997) found that "a cooperative learning classroom emphasizing inquiry and depth of knowledge is one way to begin the process of reaching more students, especially in large-enrollment courses" (p. 607).

Udovic et al. (2002) modified a traditional hands-on, follow-the-directions biology lab into a discovery learning workshop, in which students were encouraged to make their own observations. Students were uncomfortable at first, and frustrated with the experience (a lack of value and enjoyment). Udovic et al. (2002) speculated that the students (who were commonly asked to merely memorize and regurgitate facts in other classes) were unfamiliar with situations that required them to truly learn, so they were not used to having to do so. Despite the early misgivings, at the end of the course, students stated that they felt that (even though it meant extra work) the "workshop provided a better learning experience" (p. 277), indicating that the students learned to value their overall experience. Students were willing to work, but would prefer doing less work when the same grades could be earned by taking an easier path (Udovic et al., 2002), so their enjoyment could have been higher.

Investigative laboratories (dubbed I-Labs, in which students created experiments after some initial instruction) were used by Sundberg and Moncada (1994) in a biology class, leading to better understanding by students, including difficult topics. However, the students were not always cognizant of their achievements. “Instead, they were frustrated by their apparent inability to learn” (p. 703), mainly when the standard technique of memorization proved to be ineffective. This frustration implies lower levels of enjoyment and value.

Similar to the I-Labs (following the same format of students designing their own experiments), inquiry Teams and Streams (TS) laboratories were studied by Luckie et al. (2004), also in biology. Students from the TS labs were more positive in their comments about the lab than students in a traditional lab, but those from the traditional lab did not offer many comments to begin with. Luckie et al. (2004) speculated that the engagement experienced in the TS labs contributed to “a feeling of ownership in the course” (p. 205), and, subsequently, more comments. Kolb and Kolb (2005) described this feeling of ownership as “self-authorship,” which “is impossible unless students are able to connect learning with their lived experiences; self-authorship requires making meaning of one’s own experience” (Baxter-Magolda, 1999, p. 13). This ownership should help to increase both the value felt towards the subject and the enjoyment of learning the material.

Additionally, students who used the TS labs scored higher on a standardized test (similar to the MCAT) administered at the end of the semester. Despite students’ natural inclination towards easy assignments, the TS labs utilized an element of controlled chaos by increasingly requiring students to think more in order to succeed. As a result, the TS labs provided the desired safe haven for students to learn and grow, moving "students from a comfortable zone existing as anonymous passive 'receivers of facts' to a less comfortable domain where they are active 'investigators of ideas'" (Luckie et al., 2004, p. 208).

Guided inquiry labs were used in the study of biology classes by Gormally, Brickman, Hallar, and Armstrong (2009). Students who were inadequately prepared for the inquiry process (whether due to insufficient knowledge, experience, or cognitive development) were provided with help in the form of prompts and guiding questions. The goal was to reduce frustration, but still retain the desired level of intellectual challenge. Gormally et al. (2009) found mixed results when researching the effectiveness of inquiry instruction in terms of affecting either learning or attitude towards science. Despite a better understanding of the material, frustration and resistance (low value and enjoyment) were common reactions from the students, largely due to an aversion to the extra work required to think things through during the inquiry process. Students preferred memorization to deep understanding.

In summary, many studies have investigated group work, discovery learning, and inquiry-based learning. Collaboration (in the form of group work) was both enjoyable and effective (Trevena & Clarke, 2002), so the interactive nature of the lecturing dialogue seems promising. Hands-on discovery, investigative, or inquiry labs have been used (Udovic et al., 2002; Sundberg & Moncada, 1994; Luckie et al., 2004; Gormally et al., 2009), but always utilized small groups, not an entire class. The study by Elbert-May et al. (1997) did involve high-enrollment classes, but studied biology, not mathematics.

Although mixed results have been found, the general trend has been increased learning, but a dislike for the techniques by the students. An adjustment period to get used to the new teaching style was also common. Of course, different teaching styles have been shown to benefit or be preferred by different students in different situations (Ertekin et al., 2009). To maximize students' learning, the lecturing dialogue (although based on the Socratic Method) combines different aspects of various styles in an attempt to engage students in the material and help them to become active learners. The Socratic Method has been successfully used to model scientific discourse (Li & Demaree, 2010), but not as an instructional tool for a whole-class situation. As such, this aspect needed to be investigated.

Chapter 3 – Method

Participants

The participants for this study were chosen from the non-STEM college algebra classes at a large 4-year research institution located in the eastern United States. The enrollment of each class is typically between 150 and 300 students, with three to five classes offered each semester. The researcher taught three of the four classes during the Fall 2017 semester: 200 students in a class meeting at 10:30 AM, 279 students meeting at 11:30 AM, and 251 students meeting at 12:30 PM. The 11:30 and 12:30 classes met in the same room, and were closer in enrollment size, so they were used for the study. The 11:30 class was randomly chosen to be the experimental group (using the lecturing dialogue), with the 12:30 class serving as the control group (standard lecture). At the end of the semester (after some students dropped the class), there were 223 students in the experimental group and 220 in the control group. Note: There were college algebra classes specifically for STEM majors, but they had roughly 40 students in each class, so they were not considered for this study.

The students were placed into college algebra based on their performance on some assessment (whether from an ACT or SAT score, a university-administered placement test, or by completing the pre-requisite course sequence), so the two classes were comparable in terms of the abilities. A retired ACT (a version of the test that is no longer used for college admissions purposes) was administered at the beginning of the semester to verify that the two classes did, indeed, begin with comparable background knowledge. An independent *t*-test showed that the scores were not significantly different $(p = .940)$ between the experimental and control groups.

Both classes were predominantly composed of freshmen, but the percentages of students in each year in college (freshman, sophomore, junior, senior) were fairly evenly distributed

(Table 1). The overwhelming majority of freshmen was unavoidable because college algebra serves as an entry-level class. Furthermore, most high-enrollment classes are entry-level classes, so completing this study with a larger number of upperclassmen was unlikely, since those students have most likely moved on to more advanced classes that have smaller enrollments.

Table 1

Class Standing Demographics

Year in School	Group	Number	Percent
Freshmen	Control	196	78.088
	Experimental	209	74.910
Sophomores	Control	42	16.733
	Experimental	50	17.911
Juniors	Control	10	3.984
	Experimental	15	5.376
Seniors	Control	3	1.195
	Experimental	5	1.792

Note. 279 students in the Experimental Group. 251 students in the Control Group.

Instruments

The effectiveness of the lecturing dialogue (when compared to a lecture style of teaching) was measured in several ways: lab grade, quiz grade, homework, surveys/questionnaires, participation, absences, each of the four tests, final exam, final grades without extra credit, final grades without extra credit or labs, Pre-ACT, Post-ACT, ACT change, and attitude (broken into enjoyment, motivation, value, and self-confidence) both at the beginning and end of the semester.

All assessments have been used in the course during previous semesters, so they were not developed independently or altered for this study. All quizzes and tests (including the final exam) were taken on a computer, and were multiple-choice. The 10-question quizzes were taken from any computer (and students could use any available resources), but the 20-question tests were given during the normal laboratory sessions in a controlled environment (closed notes, closed book, no formula sheet, and only the basic windows calculator on the computer allowed). The questions that each student was asked on the quizzes and tests were randomly chosen from a question bank that was created by the course coordinator based on the assigned homework. Each problem had between five and 15 questions that could be selected, with each group of questions being similar in content and difficulty. The possible questions in each question set were often identical other than the numbers used, so they tested the same skills. Because of this, while no two quizzes or tests were identical, they were all comparable.

Students worked in groups of two or three for the collaborative laboratory reports, and were free to choose their own group members from among the other students in their lecture and lab, and could change lab groups at any time. The lab reports were completed on paper and submitted during class, but utilized a grapher and various applications on a computer in a computer lab. Beyond working in groups, students could seek outside help. Homework assignments were done on a computer, as well, but on the students' own time outside of class, with no restrictions on the help that students could receive. The grade for surveys/questionnaires was based on the completion of two surveys and four questionnaires, with each being worth five points. Details for the content and administration of the surveys/questionnaires are provided below. Participation was based on homework and the completion of the surveys and questionnaires throughout the semester. Homework (worth 80 points total) and surveys/questionnaires (30 points) were combined to account for at most 100 points, although 110 points could be earned. The final grades were calculated by the distributions listed in Table 2. The assignments were due according to the schedule in Table 3.

A retired version of the ACT was administered at the beginning and end of the semester,

Table 2

Final Grade Composition

	Assessment	Number	Points		Percent of Grade	
1	Surveys/Questionnaires	6	30	At most	3%	At most
2	Homework	40	80	100 points	8%	10%
3	Quizzes	6	100		10%	
4	Laboratory Reports	8	200		20%	
5	Tests	4	400		40%	
6	Comprehensive Final Exam	1	200		20%	
TOTAL			1000		100%	

and was used in several ways. First, the Pre-ACT was used to verify that the two classes had comparable prior knowledge in college algebra. Second, the Post-ACT scores were compared to gauge the end knowledge of the students. Third, the change in ACT score measured the improvement (or decline) of the students over the course of the study. Extra credit was given to the students based on how well they performed on each ACT, which helped to ensure that the students tried to do their best and took the assessments seriously. Although the extra credit was included in the final grades during the class, it was excluded for the purposes of this study. Students were given 50 minutes to complete each ACT, which were administered during the normal lab sessions (and were closed notes and closed book, like a typical ACT). The questions were in the standard ACT booklets, but the answers were entered on a computer.

Miller and Schraeder (2015) analyzed each of the 60 questions on the ACT to classify their content by comparing each question to the material in the course's textbook: Sullivan and Sullivan (2017). Fifty of those questions were deemed to be directly related to the content in the college algebra course. The remaining 10 questions from the ACT pertained to trigonometry and probability/statistics. As a result, the ACT was deemed to be an accurate assessment of the content knowledge of the college algebra students.

Table 3

Tentative Class Schedule

Date	Day	Text Sections	Topic(s)	Assignments Due
8/16	W	Syllabus, R.1 – R.2	Introduction	
8/18	F	R.3 – R.4	Review Sections	HW (Sun 11:59 PM)
8/21	M	R.5, R.7 – R.8	Review Sections	
8/22	T	**Extra Credit**	**Pre-ACT Quiz**	
8/23	W	1.1 – 1.2	Graphs and Solving Equations	
8/25	F	1.3	Quadratic Equations	HW (Sun 11:59 PM)
8/28	M	1.5	Radical, Absolute Value, and Factorable Equations	
8/29	T	**Lab 1a**	**Intro to eCampus; Intro to Basic Graphs Lab**	ATMI due (11:59 PM)
8/30	W	1.7	Solving Inequalities (Cut Off for Test 1)	
9/1	F	2.1 – 2.2	Graphing Key Equations and Lines	Quiz Release&Quiz1(8:00AM),HW(Sun11:59
9/4	M		**Labor Day – No Classes**	
9/5	T	**Lab 1b**	**Power Functions Lab**	
9/6	W	2.3	Circles	Lab 1 Due (Intro. to Basic Graphs and Power
9/8	F	Review		HW (Sun 11:59 PM)
9/11	M	Review		
9/12	T		**Test 1**	Quiz 2 Due (8:00 AM), Questionnaire Due (8:00
9/13	W	3.1	Functions	
9/15	F	3.2	Graphs of Functions	HW (Sun 11:59 PM)
9/18	M	3.3	Properties of Functions	
9/19	T	**Lab 2**	**Lemonade Stand Lab**	
9/20	W	3.4	Library of Functions and Piece-wise Functions	Lab 2 Due (Lemonade Stand)
9/22	F	3.5	Transformations (Cut Off for Test 2)	HW (Sun 11:59 PM)
9/25	M	Review		
9/26	T	**Lab 3**	**The Box Lab**	
9/27	W	6.1	Composite Functions	Lab 3 Due (The Box)
9/29	F	Review		
10/2	M	Review		
10/3	T		**Test 2**	Quiz 3 Due (8:00 AM), Questionnaire Due (8:00
10/4	W	3.6, 4.4	Math Models	
10/6	F	4.3	Quadratic Functions	HW (Sun 11:59 PM)
10/9	M	1.4	Complex Numbers	
10/10	T	**Lab 4**	**Falling Ball Lab**	
10/11	W	5.1	Polynomials	Lab 4 Due (Falling Ball)
10/13	F	5.1, R.6	Polynomials and Synthetic Division	HW (Sun 11:59 PM)
10/16	M	5.5	Real Zeros of a Polynomial (Cut Off for Test 3)	
10/17	T	**Lab 5**	**Polynomial Functions Lab**	
10/18	W	5.6	Complex Zeros and the Fundamental Thm of Alg	Lab 5 Due (Polynomial Functions)
10/20	F	Review		
10/23	M	Review		
10/24	T		**Test 3**	Quiz 4 Due (8:00 AM), Questionnaire Due (8:00
10/25	W	5.2 – 5.3	Rational Functions	
10/27	F	5.3	Rational Functions	HW (Sun 11:59 PM)
10/30	M	6.2	Inverses	
10/31	T	**Lab 6**	**Rational Functions Lab**	
11/1	W	6.3	Exponential Functions	Lab 6 Due (Rational Functions)
11/3	F	6.4	Logarithmic Functions	HW (Sun 11:59 PM)
11/6	M	6.5	Properties of logarithms (Cut Off for Test 4)	
11/7	T	**Lab 7**	**Exponential Functions Lab**	
11/8	W	6.6 – 6.7	Exponential and Logarithmic Models	Lab 7 Due (Exponential Functions)
11/10	F	6.8	Exponential Growth and Decay	
11/13	M	Review		
11/14	T		**Test 4**	Quiz 5 Due (8:00 AM), Questionnaire Due (8:00
11/15	W	12.1	Systems of Linear Equations	
11/17	F	12.2	Matrices	HW (Sun 11:59 PM)
			Thanksgiving Break – No Classes	
11/27	M	Review		
11/28	T	**Lab 8**	**Logarithmic Functions**	
11/29	W	Review		Lab 8 Due (Logarithmic Functions)
12/1	F	Review		HW (Sun 11:59 PM)
12/4	M	Review		
12/5	T	**Extra Credit**	**Post-ACT Quiz/Make-Up Test (By**	Quiz 6 Due (8:00 AM)
12/8	F		**Final Exam**	

To measure attitude, the Attitudes Toward Mathematics Inventory (ATMI) Survey, which was created by Tapia (1996), was used. The 40 questions (Appendix A) comprise four sub-categories: enjoyment, motivation, value, and self-confidence. Tapia (1996) found that the ATMI was both reliable (Cronbach alpha of .9667) and valid (correlation greater than .49). The eight items in the enjoyment category had a reliability of .88, with motivation (nine items) at .89, value (eight items) at .86, and self-confidence (15 items) at .95 (Tapia, 1996). The questions in the ATMI were answered on a 5-point Likert scale: Strongly Disagree (one point), Disagree (two points), Neutral (three points), Agree (four points), and Strongly Agree (five points). Eleven items had their scores reversed (Appendix A). The scores for the individual sub-categories were obtained by adding the scores for the questions in that sub-category. The ATMI was administered twice during the semester: during the first two weeks of classes and during the final week of classes. Students took the ATMI surveys on a computer outside of class.

To help to verify that the classes were taught differently, a short questionnaire (Appendix B) was administered before each test to gauge the students' opinions about involvement and engagement in the class. Additionally, the researcher was videoed on five occasions teaching each class. The videos were watched by one of the course assistants, who tabulated the number of times that the instructor asked a question during class and a student responded, as well as the number of questions that the students asked. Any response to an instructor-posed question was tallied, whether the answer was right or wrong. Similarly, any question asked to the instructor counted as a question asked. The researcher also watched the videos to corroborate those results. The values for the control group were identical, and the totals for each video of the experimental group differed by at most two, so the values were considered to be valid.

Design

Two college algebra classes were studied, one taught using the lecturing dialogue (experimental group) and the other taught utilizing a traditional lecture (control group). The experimental group started with 279 students but finished with 223 after students dropped the course. The control group dropped from 251 to 220. Both classes were given the same assessments, including laboratory reports, quizzes, homework, surveys/questionnaires, participation, and tests. At the end of the semester, the statistics for the final grades of the students from each class were compared using two-tailed independent *t*-tests (the non-parametric Mann-Whitney U Test was needed for most of the comparisons because the assumptions for the parametric test were not met). Additionally, the results from the ACT and ATMI (pre-assessment, post-assessment, and change in score) were also compared using independent *t*-tests (again, the Mann-Whitney U Test was required the majority of the time).

Both classes met Monday, Wednesday, and Friday for lecture, and Tuesday for lab. The experimental class met at 11:30 AM for the lecture, while the control group met at 12:30 PM. Both classes met in the same room. For lab, 200 students from the experimental group met at 11:30 AM, and were split between two computer labs (120 in one and 80 in the other). The remaining 79 students met at 8:30 AM in the computer lab that that held 120 students. The control group was similarly split, with 200 students being divided between the two computer labs at 12:30 PM, and 51 attending the 8:30 AM lab. The students in the 8:30 lab were separated by class, so the students from the experimental and control groups did not intermingle. As a note of clarification, the larger computer lab held 120 students, but 130 were enrolled in the 8:30 AM lab. Coincidentally, some students always missed the lab, so the lab never reached capacity.

The Pre-ATMI was used verify that the students in the two classes begin with comparable attitudes (in all four sub-categories), while the Post-ATMI gauged the attitudes after nearly completing the course. The change in attitude was also studied. The Pre-ACT, Post-ACT, and ACT change were used in a fashion similar to that of the ATMI. The various grades were compared between the two groups to determine if differences existed. The questionnaire results were analyzed to ensure that the classes were taught differently, and supplemented (and verified) the results from the videos.

Procedure

Of the three non-STEM college algebra classes taught by the researcher during the Fall 2017 semester, two were deemed to be similar based on enrollment size, classroom location, and time that the classes met. One of the two classes was randomly chosen to be the experimental group (using the lecturing dialogue), with the other being the control group (using a standard lecture). Both classes addressed the same material, which was taught in the same order and at roughly the same pace (slight variations in pace occurred, but no more than one class period). The same assessments were used for both classes, with the same course policies. The two classes were identical in regards to how they were conducted, except for the student-teacher interaction during class, and the time of day that the classes were offered (11:30 AM for the experimental group, 12:30 PM for the control group). The time difference could not be avoided because two college algebra classes are seldom offered at the same time. Also, concurrent classes would not allow the same instructor to teach both classes.

To demonstrate the differences in teaching style, refer to the following typical scripts from each type of class.

<u>Lecture</u>

Instructor: "We need to solve this equation: $[x^2 + 3x = 10]$. Since we are dealing with a quadratic equation, we need to set the equation equal to zero. Subtract the 10 from each side to get $[x^2 + 3x - 10 = 0]$. We should try to factor it. This one factors easily into $[(x+5)(x-2) = 0]$. Since we have two chunks that are multiplied to get zero, we can apply the Zero-Product Property and set each factor equal to zero. $[x + 5 = 0]$ or $[x - 2 = 0]$, so $[x = -5]$ or $[x = 2]$. Any questions?"

<u>Lecturing Dialogue</u>

Instructor: "We need to solve this equation. How?" $[x^2 + 3x = 10]$
Student 1: "Get everything on one side."
Instructor: "Correct. How and why?"
Student 2: "You need 0 on one side, so move the 10 over."
Instructor: "How?"
Student 2: "Subtract."
Instructor: "Ok, subtract the 10 from each side. Now what?" $[x^2 + 3x - 10 = 0]$
Student 3: "FOIL."
Instructor: "FOIL?"
Student 3: "I mean, factor."
Instructor: "Right. Into what?"
Student 1: "Five and two."
Instructor: "Two?"
Student 1: "Negative two." $[(x+5)(x-2) = 0]$
Instructor: "Now what?"
Student 2: "Solve."
Instructor: "How?"
Student 2: "Set them both equal to zero." $[\, x + 5 = 0 \qquad x - 2 = 0 \,]$
Instructor: "And?"
Student 1: "Solve for x."
Instructor: "And what are your answers?"
Student 3: "Negative five and two." $[\, x = -5 \qquad x = 2 \,]$
Instructor: "Yes. Any questions?"

The instructor wrote the corresponding steps on the chalkboard during this exchange, as shown in brackets. Note the key comment at the end of each script. Students were allowed to ask questions in both classes. Not doing this would be an injustice to teaching. Special care was taken to present the material in the same way, using the same words. Variations occurred due to student questions and responses, but the initial presentations were as near to identical as possible, including using the same examples.

The Pre-ACT was given during the first laboratory session (day five of classes overall), and the Post-ACT was administered during the final laboratory session (on the last day of classes). Giving the ACTs at such extreme dates in the semester was an attempt to obtain true pre- and post-tests. During the first few days of classes, only the review material was addressed, and not even all of the review material for the class. This material should have been addressed in a previous class, so its impact on any measurement of the prior knowledge of the students was considered to be minimal. All of the course material was addressed before the Post-ACT was given (with a few days for students to practice the material, and become familiar with it), so none of the college algebra-related questions from the Post-ACT should have been new or unfamiliar to the students.

Similar to the ACT, the Pre-ATMI and Post-ATMI served as appropriate bookends for the class. For logistical reasons, a little over two weeks were allowed for students to complete the Pre-ATMI. Students often register for the class late (as late as the end of the first week) due to a variety of reasons: changes in schedules, testing into the class on a second or third try, difficulties securing financing for tuition. To allow all students an opportunity to complete the Pre-ATMI, it was open until the second Tuesday of classes (the end of the tenth day of classes). This also accounted for students neglecting to do assignments with the confusion of a new semester (the first semester in college for most of the students in college algebra). However, the six days of instruction that the students received before the Pre-ATMI was due were unlikely to influence the students' feelings towards mathematics very much. Students were given the final week of classes to complete the Post-ATMI.

Chapter 4 – Results

The experimental group finished with 223 students, and the control group had 220. Some of those students did not complete all of the assignments. Many missed tests, and some did none of the labs, none of the quizzes, none of the homework, or none of the surveys/questionnaires. A total of 112 students were not present for either the Pre-ACT or Post-ACT, both of which were given during class and could not be made up. Any student missing a grade for one of the measures used for a comparison was excluded from all of the grade comparisons. This decision was made to provide a common population for all of the grade comparisons. Furthermore, if a grade of zero was assigned due to a missing assignment, then that could artificially impact the results of the comparison test. Missing an assignment counted as a zero in the grades, but that zero was not earned, and did not accurately represent the student's knowledge. Fifty-eight students were excluded from the experimental group (165 of the 223 were used) and 57 were not used from the control group (163 out of 220 were used), so no group gained a perceivable advantage. Granted, omitting these grades would still affect the results of the comparisons, but a common population was deemed to be preferable because of the reasons stated above. To clarify, missing one quiz, one lab, one homework assignment, or one survey or questionnaire did not cause a student to be excluded. Students were only omitted from the study if they missed a test or ACT, or had a zero for one of the comparison categories, meaning that they missed all of the assignments in that category. That is, they did none of the quizzes, none of the labs, none of the homework, or none of the surveys or questionnaires.

For the attitude comparisons, students needed to complete both the Pre-ATMI and Post-ATMI because the change in each of the four sub-categories was calculated. A missing score would have falsely inflated the results. A total of 128 students from the experimental group completed both the Pre-ATMI and Post-ATMI, and 165 from the control group did both.

The questionnaires were all examined independently and a change was not calculated, so missing one questionnaire did not preclude inclusion in the other comparisons. Additionally, the researcher decided that too few students did all four questionnaires (81 from the experimental group and 91 from the control group) to merit a tactic employing exclusion. These totals were below 50% of the total enrollment. Even though 81 and 91 are sizable numbers by themselves, they constituted too small of a percentage to accurately represent each group as a whole.

Independent *t*-tests were used to compare the results between the experimental and control groups for the following: lab grade, quiz grade, homework grade, surveys/questionnaires, participation points, absences, Test 1, Test 2, Test 3, Test 4, Final Exam, Pre-ACT, Post-ACT, ACT change, final grade without extra credit, Pre-ATMI for each of the four sub-categories, Post-ATMI for each of the four sub-categories, ATMI change for each of the four sub-categories, and all four questionnaires. Before the *t*-tests could be run, the assumptions needed to be checked to determine which test could be used in each instance: the parametric test or the non-parametric Mann-Whitney U Test.

The assumptions that must be met to use a parametric *t*-test are that the data must be independent, be a random sampling from a defined population, be at least interval level, be normally distributed, and have equal variances. The data from the two groups were independent in that no student was in both groups, so the students were only exposed to one style of teaching. Also, those who were in one class had no influence on those who were in another class. Students may have talked and interacted, but that interaction had no impact on the teaching style in the class.

The two classes that were chosen to be in the study were deliberately picked based on their similar characteristics (enrollment size, location, time, instructor), so they were not randomly chosen from all of the available college algebra classes. However, the classes were randomly chosen among the two options to be either the control group or the experimental group.

Data can be on a nominal, ordinal, interval, or ratio level. The nominal and ordinal levels are categorical because they are used to divide the data into groups (or categories). The interval and ratio levels are numerical identifiers because they rely on the true value of the numbers. The nominal level is only used as a classifier to put data in different groups, and value of the specific number that is assigned to an entity is meaningless (such as one for male and two for female). Ordinal data puts the data in some sort of order, so the number has actual value. However, the distance between any two rankings may be different than the distance between any other two rankings. For example, the first and second entries maybe be very close, with number one barely being better than number two, but number three could be far worse. When the distance between all numbers is the same, the data are on the interval level. The distance between a one and two is the same as the distance between a fifty and fifty-one. The highest level is ratio data, which is similar to interval level, but zero is also included and ratios have meaning. A zero at the ratio level truly means that the entry has no value. Since ratios hold, a 20 is twice as good as a 10, just like a 40 is twice as good as a 20.

To use a parametric test, the data must be on at least an interval level. That requirement was met for this study. The grades were at least interval. The surveys and questionnaires used five-point Likert scales, which can be treated like interval-level data.

To determine whether the data are normally distributed (that is, whether the distribution resembles a bell curve), one may visually inspect the distribution curve. In situations when the data points are spread out among many options (such as grades that vary by increments of .25), there might not be enough entries for every value to make the distribution meaningful, so a histogram can be plotted instead that covers intervals of scores. For example, only two students scored an 83.75 on an assessment, but 25 students scored between 80 and 85. The histograms for each of the data sets are in Appendix C.

Another way to get a sense of whether the data are normally distributed is to analyze the Quantile-Quantile Plot (Q-Q Plot), which graphs the quantiles of the first set of data against the quantiles of the second set of data (Appendix D). A quantile is the fraction or percentage of points that are below a specific value. For example, the .25 quantile is the point at which 25% of the data fall below that value. In a Q-Q Plot, this information is plotted along with the reference line $y = x$ (a 45-degree line). Data with a distribution curve close to a normal distribution have points that lie close to the reference line, but data that do not have a normal distribution have points that do not align with the reference line.

A visual examination is not very exact, so more precise statistical measurements can be used, such as skewness (how far left or right of center the peak of the distribution curve leans) or kurtosis (how pointy or flat the distribution curve is). A negative skewness indicates that the curve leans to the right (although it is called left-skewed or left-tailed because the left-side tail is longer than the right-side tail), while a positive skewness leans to the left (although it is called right-skewed or right-tailed because the right-side tail is longer than the left-side tail). A skewness value of zero means that the data are not skewed at all. A positive kurtosis is pointier than usual (called leptokurtic), whereas a negative kurtosis is flatter than usual (called platykurtic). A normal distribution is mesokurtic.

Instead of analyzing these features individually, one may use the Kolmogorov-Smirnov Test or the Shapiro-Wilk Test, among others. These two tests return the likelihood that the given distribution curve is similar to a normal curve. A *p*-value of less than .05 (or whatever value is chosen as significant) means that the distribution curve is significantly different from a normal curve. The two groups were analyzed separately, so even if only one group had a distribution that was significantly different from a normal distribution, then the assumption was violated. In other words, both distributions had to be fairly normal (or not significantly different from normal) in order to satisfy the assumption.

Technically, an α-value is the chosen significance value for a statistical study. It sets the probability that the null hypothesis was rejected when it was actually true. In other words, it is the probability that the differences were shown to be significantly different by the test, but really were not. So, when $\alpha = .05$ (as with this study), and the *p*-value is less than that, there is less than a 5% chance that the two sets of data were really not significantly different. Looked at another way, there is more than a 95% chance that the two sets of data were significantly different, so there is a good deal of confidence that the test's results (showing that the data sets were different beyond mere chance) were correct.

Variance (σ^2) is a measure of how much the individual data scores differ from the mean (average) of the scores. Specifically, it is the sum of the squares of the individual differences: $\sigma^2 = \sum_{i=1}^{n} (\bar{x} - x_i)^2$, for n data entries. More individual difference (whether positive or negative) produces a larger variance. The equality of variances can be measured by using Levene's Test of Equality of Error Variances, which produces the likelihood that two sets of data have similar (or equal) variances. A *p*-value smaller than the chosen significance level (.05 for this study) means that the variances are significantly different. That is, the difference is greater than any difference that can be explained by chance.

All of the data satisfied the assumptions of independence, random selection, and being at least interval level. Most of the comparisons (all but two) also upheld the assumption of equal variances (Table 4). However, only three to six comparisons (depending on the test used)

Table 4

Levene's Test of Equality of Error Variances

Comparison	F	Degrees of Freedom 1	Degrees of Freedom 2	Significance
Lab Grade	.003	1	326	.959
Quiz Grade	1.468	1	326	.226
Homework	.059	1	326	.808
Surveys/Questionnaires	3.346	1	326	.068
Participation Points	.140	1	326	.708
Absences	1.839	1	326	.176
Test 1	.010	1	326	.920
Test 2	.682	1	326	.409
Test 3	.012	1	326	.913
Test 4	.107	1	326	.744
Final Exam	5.224	1	326	.023*
Pre-ACT	1.585	1	326	.209
Post-ACT	1.273	1	326	.260
ACT Change	1.052	1	326	.306
Final Grade without Extra Credit	.238	1	326	.626
Final Grade without Extra Credit or Labs	.211	1	326	.646
Pre-ATMI – Enjoyment	1.133	1	291	.288
Pre-ATMI – Motivation	.009	1	291	.924
Pre-ATMI – Value	.605	1	291	.437
Pre-ATMI – Self-Confidence	3.773	1	291	.053
Post-ATMI – Enjoyment	.299	1	291	.585
Post-ATMI – Motivation	1.128	1	291	.289
Post-ATMI – Value	1.543	1	291	.215
Post-ATMI – Self-Confidence	2.299	1	291	.131
ATMI Change – Enjoyment	.797	1	291	.373
ATMI Change – Motivation	.006	1	291	.941
ATMI Change – Value	17.300	1	291	.000*
ATMI Change – Self-Confidence	.281	1	291	.597
Questionnaire 1	3.876	1	314	.050
Questionnaire 2	.538	1	308	.464
Questionnaire 3	.323	1	335	.570
Questionnaire 4	.556	1	320	.457

Note. Tests the null hypothesis that the error variance of the dependent variable is equal between groups.
For significance, $\alpha = .05$.
*Significant difference, $p < .05$.

satisfied the requirement for a normal distribution (Table 5): Post-ATMI enjoyment, Pre-ACT, Post-ACT, and maybe ACT change, Post-ATMI self-confidence, and Questionnaire 3. Because the data for ACT change, Post-ATMI self-confidence, and Questionnaire 3 were deemed to be significantly different from a normal distribution by one of the tests, the three sets of data were treated as non-parametric. As a result, only three comparisons (Post-ATMI enjoyment, Pre-ACT, and Post-ACT) could use the parametric *t*-test, while the others had to use the non-parametric Mann-Whitney U Test.

Table 5

Tests for Normal Distribution

Comparison	Group	Skewness	Kurtosis	Kolmogorov-Smirnov Test			Shapiro-Wilk Test		
				Statistic	N	Sig	Statistic	N	Sig
Lab Grade	Control	-1.853	3.959	.187	163	.000	.805	163	.000
	Experimental	-1.106	.983	.107	165	.000	.915	165	.000
Quiz Grade	Control	-2.302	7.898	.186	163	.000	.797	163	.000
	Experimental	-1.147	1.470	.118	165	.000	.916	165	.000
Homework	Control	-1.259	1.337	.137	163	.000	.880	163	.000
	Experimental	-1.444	2.031	.142	165	.000	.859	165	.000
Surveys/ Questionnaires	Control	-1.370	1.725	.254	163	.000	.791	163	.000
	Experimental	-.891	-.138	.224	165	.000	.842	165	.000
Participation Points	Control	-1.431	1.372	.223	163	.000	.779	163	.000
	Experimental	-1.347	1.205	.203	165	.000	.814	165	.000
Absences	Control	2.314	5.677	.232	163	.000	.708	163	.000
	Experimental	2.139	4.868	.218	165	.000	.736	165	.000
Test 1	Control	-.674	.332	.116	163	.000	.957	163	.000
	Experimental	-.355	-.605	.115	165	.000	.966	165	.000
Test 2	Control	-.657	.031	.126	163	.000	.951	163	.000
	Experimental	-.365	-.356	.110	165	.000	.970	165	.001
Test 3	Control	-.574	-.424	.142	163	.000	.951	163	.000
	Experimental	-.325	-.802	.107	165	.000	.961	165	.000
Test 4	Control	-.785	.216	.140	163	.000	.943	163	.000
	Experimental	-.420	-.602	.118	165	.000	.963	165	.000
Final Exam	Control	-.126	-.582	.078	163	.017	.984	163	.062
	Experimental	.066	-.518	.112	165	.000	.981	165	.025
Pre-ACT	Control	-.230	.236	.063	163	.200*	.992	163	.511*
	Experimental	-.007	-.022	.067	165	.064*	.991	165	.400*
Post-ACT	Control	-.091	.368	.059	163	.200*	.990	163	.323*
	Experimental	.176	-.168	.053	165	.200*	.990	165	.337*
ACT Change	Control	-.341	.242	.073	163	.033	.985	163	.073*
	Experimental	.437	.279	.077	165	.017	.984	165	.050*
Final Grade without Extra Credit	Control	-.745	.105	.097	163	.001	.952	163	.000
	Experimental	-.216	-.652	.049	165	.200	.983	165	.044
Final Grade	Control	-.770	.350	.082	163	.009	.956	163	.000

without Extra Credit or Labs	Experimental	-.246	-.516	.052	165	.200	.983	165	.036
Pre-ATMI –	Control	-.259	-.766	.097	165	.001	.976	165	.005
Enjoyment	Experimental	.106	-.426	.060	128	.200	.990	128	.503
Pre-ATMI –	Control	-.080	-.660	.068	165	.059	.982	165	.033
Motivation	Experimental	.042	-.551	.085	128	.023	.981	128	.073
Pre-ATMI –	Control	-.871	1.113	.120	165	.000	.951	165	.000
Value	Experimental	-.423	.226	.077	128	.063	.982	128	.092
Pre-ATMI –	Control	-.358	-.846	.118	165	.000	.962	165	.000
Self-Confidence	Experimental	-.009	-.260	.059	128	.200	.988	128	.342
Post-ATMI –	Control	-.185	.168	.064	165	.095*	.989	165	.259*
Enjoyment	Experimental	.295	-.370	.068	128	.200*	.984	128	.135*
Post-ATMI –	Control	-.035	.314	.084	165	.006	.980	165	.015
Motivation	Experimental	.511	-.549	.127	128	.000	.956	128	.000
Post-ATMI –	Control	-.522	.655	.096	165	.001	.974	165	.003
Value	Experimental	-.588	.424	.091	128	.012	.974	128	.016
Post-ATMI –	Control	-.181	-.719	.061	165	.200	.982	165	.031
Self-Confidence	Experimental	.047	-.433	.067	128	.200	.983	128	.122
ATMI Change –	Control	-.541	2.553	.075	165	.024	.964	165	.000
Enjoyment	Experimental	.583	.981	.102	128	.002	.972	128	.010
ATMI Change –	Control	.143	.892	.117	165	.000	.977	165	.007
Motivation	Experimental	-.148	-.062	.086	128	.021	.989	128	.416
ATMI Change –	Control	-.899	2.977	.086	165	.004	.955	165	.000
Value	Experimental	-.779	1.044	.099	128	.003	.956	128	.000
ATMI Change –	Control	-.054	1.421	.086	165	.004	.977	165	.007
Self-Confidence	Experimental	.421	1.547	.098	128	.004	.976	128	.021
Questionnaire 1	Control	-.249	-.449	.070	168	.042	.986	168	.080
	Experimental	-.623	.776	.097	148	.002	.971	148	.003
Questionnaire 2	Control	-.467	-.040	.103	157	.000	.977	157	.011
	Experimental	-.367	-.557	.088	153	.006	.973	153	.004
Questionnaire 3	Control	-.040	-.694	.065	167	.078	.983	167	.041
	Experimental	-.507	-.112	.067	170	.063	.972	170	.001
Questionnaire 4	Control	-.105	-.554	.074	165	.027	.981	165	.020
	Experimental	-.229	-.520	.067	157	.083	.980	157	.022

Note. Tests the null hypothesis that the data are normally distributed.
For significance, $\alpha = .05$.
*Significant difference, $p < .05$.

After applying the appropriate *t*-tests, only three of the grade comparisons (and none of the attitude comparisons) were significant at the $\alpha = .05$ level: lab grade, surveys/questionnaires, and final grade without extra credit (Table 6). Because the lab grade was significant and a large portion of the final grade (20%), the final grades were compared again, but without the extra credit or the labs. Like most of the other comparisons, this was non-parametric and not significant. However, the non-significance was a change from the final grade without extra credit. Even though the surveys/questionnaires were significantly different, they only accounted

Table 6

Independent t-test Results

Comparison	Significance	*t*-value⁺	Degrees of Freedom⁺
Lab Grade	.000*		
Quiz Grade	.188		
Homework	.240		
Surveys/Questionnaires	.020*		
Participation Points	.054		
Absences	.588		
Test 1	.109		
Test 2	.840		
Test 3	.683		
Test 4	.112		
Final Exam	.541		
Pre-ACT	.940	.076	427
Post-ACT	.658	-.443	336
ACT Change	.673		
Final Grade without Extra Credit	.007*		
Final Grade without Extra Credit or Labs	.124		
Pre-ATMI – Enjoyment	.571		
Pre-ATMI – Motivation	.422		
Pre-ATMI – Value	.785		
Pre-ATMI – Self-Confidence	.574		
Post-ATMI – Enjoyment	.786	-.272	291
Post-ATMI – Motivation	.417		
Post-ATMI – Value	.546		
Post-ATMI – Self-Confidence	.218		
ATMI Change – Enjoyment	.672		
ATMI Change – Motivation	.513		
ATMI Change – Value	.512		
ATMI Change – Self-Confidence	.057		
Questionnaire 1	.000*		
Questionnaire 2	.002*		
Questionnaire 3	.000*		
Questionnaire 4	.006*		

Note. Tests the null hypothesis that the means of the data are equal between groups.
⁺Available for the parametric *t*-test only.
For significance, $\alpha = .05$.
*Significant difference, $p < .05$.

for at most 3% of the final grade, so they were left in for the new final grade comparison. The control group had higher averages than the experimental group on most of the assessments, but not all of them (Table 7). And, as noted, only three of the averages were significantly higher.

The comparisons (*t*-tests) of the four questionnaires that were used to gauge interaction in the classrooms helped to verify that the classes were taught differently. Questionnaire 1 violated

Table 7

Comparison Descriptive Statistics

Comparison	Group	N	Mean	Variance	Standard Deviation
Lab Grade	Control	163	176.145	321.242	17.923
	Experimental	165	160.982	336.576	18.346
Quiz Grade	Control	163	83.476	223.951	14.965
	Experimental	165	83.152	156.166	12.497
Homework	Control	163	65.464	176.077	13.269
	Experimental	165	63.704	230.677	15.188
Surveys/Questionnaires	Control	163	24.91	37.183	6.098
	Experimental	165	23.33	49.492	7.035
Participation Points	Control	163	88.416	231.404	15.212
	Experimental	165	85.466	317.212	17.810
Absences	Control	163	4.71	40.305	6.349
	Experimental	165	4.55	33.127	5.756
Test 1	Control	163	70.58	223.887	14.963
	Experimental	165	68.97	222.712	14.924
Test 2	Control	163	74.02	233.907	15.294
	Experimental	165	74.64	212.977	14.594
Test 3	Control	163	70.00	333.333	18.257
	Experimental	165	70.21	303.461	17.420
Test 4	Control	163	72.457	313.053	17.693
	Experimental	165	71.158	265.900	16.306
Final Exam	Control	163	61.41	316.515	17.791
	Experimental	165	60.91	233.010	15.265
Pre-ACT	Control	163	25.78	38.914	6.238
	Experimental	165	25.77	31.117	5.578
Post-ACT	Control	163	29.23	45.414	6.739
	Experimental	165	28.97	35.237	5.936
ACT Change	Control	163	3.45	38.114	6.174
	Experimental	165	3.20	33.563	5.793
Final Grade without Extra Credit	Control	163	75.798	121.825	11.037
	Experimental	165	73.613	105.366	10.265
Final Grade without Extra Credit or Labs	Control	163	58.183	102.348	10.117
	Experimental	165	57.515	86.944	9.324
Pre-ATMI – Enjoyment	Control	165	29.94	66.423	8.150
	Experimental	128	29.61	62.051	7.877
Pre-ATMI – Motivation	Control	165	13.61	17.460	4.178
	Experimental	128	13.23	17.283	4.157
Pre-ATMI – Value	Control	165	37.12	35.632	5.969
	Experimental	128	37.52	39.260	6.266
Pre-ATMI – Self-Confidence	Control	165	44.84	187.821	13.705
	Experimental	128	44.59	153.109	12.374
Post-ATMI – Enjoyment	Control	165	28.23	69.508	8.337
	Experimental	128	27.96	72.731	8.528
Post-ATMI – Motivation	Control	165	12.42	20.063	4.479
	Experimental	128	12.22	21.448	4.631
Post-ATMI – Value	Control	165	34.72	60.312	7.766
	Experimental	128	35.09	69.024	8.308
Post-ATMI – Self-Confidence	Control	165	44.39	215.838	14.691
	Experimental	128	42.67	179.970	13.415
ATMI Change – Enjoyment	Control	165	-1.71	40.720	6.381

	Experimental	128	-1.65	32.387	5.691
ATMI Change – Motivation	Control	165	-1.18	13.040	3.611
	Experimental	128	-1.02	11.590	3.404
ATMI Change – Value	Control	165	-2.40	36.388	6.032
	Experimental	128	-2.43	79.649	8.925
ATMI Change – Self-Confidence	Control	165	-.44	79.456	8.914
	Experimental	128	-1.92	90.340	9.505
Questionnaire 1	Control	168	28.74	28.147	5.305
	Experimental	148	31.35	22.270	4.719
Questionnaire 2	Control	157	27.75	35.957	5.996
	Experimental	153	29.92	31.841	5.643
Questionnaire 3	Control	167	26.22	41.688	6.457
	Experimental	170	28.64	47.630	6.901
Questionnaire 4	Control	165	26.55	46.762	6.838
	Experimental	157	28.63	40.786	6.386

both the equality of variances and normal distribution. Questionnaires 2 and 4 violated the normal distribution requirement. Questionnaire 3 did not violate the normal distribution assumption using the Kolmogorov-Smirnov Test, but did violate it according to the Shapiro-Wilk Test. At least one assumption was violated for each questionnaire, so all four were treated as non-parametric.

The four questionnaires each had a significant difference between the two groups (Table 6), with the experimental group being considered to be more engaging than the control group each time (Table 7). The questionnaires were not meant to provide a definitive verification that the classes were taught differently, so they were not tested for reliability and validity. Rather, they were designed to be short and offer a glimpse into how the students perceived the class. Their purpose was to establish the fact that the classes were taught differently, and act as a supplement to the video tabulations.

Both classes were videoed on the following days: August 30, September 20, October 13, November 3, and November 13. The first four dates were normal classes in which new material was addressed, but the last date was a review session, in which students submitted written problems to be worked out during class. One of the course assistants and the researcher independently watched the videos and recorded the number of times that students responded to a question from the instructor and the number of times the students asked questions, with only slight variations between the two tabulations (Table 8). The experimental class clearly had more interaction than the control group.

Table 8

Classroom Interaction Tabulation

Date of Recorded Class	Group	Student Responses	Student Questions
August 30, 2017	Control	2	5
	Experimental	166	21
September 20, 2017	Control*	1	1
	Experimental	155	7
October 13, 2017	Control	2	4
	Experimental	160	10
November 3, 2017	Control	1	1
	Experimental	94	8
November 13, 2017	Control	2	4
	Experimental	154	6

Note. Totals in the table were tabulated by the course assistant.

Both classes were ahead of the planned schedule, so the topic for the November 3 class was Section 6.8: Exponential Growth and Decay. That section involved doing several real-world examples, which required more exposition by the instructor to provide the necessary details for each problem, so the students had less time (and, therefore, fewer opportunities) to respond to questions. As a result, the total for that day was lower than the other classes for the experimental group.

Chapter 5 – Discussion

As described earlier, the experimental group was taught using the lecturing dialogue, and relied on a question-and-answer format that encouraged student-teacher interaction. Conversely, the control group was taught using a standard lecture, and featured the instructor providing the explanations and information. The results of the Mann-Whitney U Test for the questionnaires, in conjunction with the tabulations from the videos, helped to establish that the two classes were taught differently, with the experimental group being more interactive. The differences between the groups on the Mann-Whitney U Test were significant, so there is confidence that the classes were taught differently.

It was hypothesized that students in a high-enrollment mathematics classroom (200 students) who were taught using the lecturing dialogue would perform better in a college algebra class (as measured by final grades) than those who were taught using a standard lecture. Additionally, it was hypothesized that the students who were taught using the lecturing dialogue would also experience greater enjoyment, more motivation, higher value, and increased self-confidence than students who were taught with a standard lecture. Both hypotheses were shown to be incorrect. The experimental group did not significantly outperform the control group on any measure (neither grades nor attitude), but the control was significantly higher than the experimental group for a few grade measures.

The independent *t*-tests (whether parametric or non-parametric) only showed a significant difference (at the .05 level) for the lab grade, surveys/questionnaires, and final grade without extra credit, with the control group outperforming the experimental group in each instance. None of the other grades were significantly different. While the control group averaged higher scores than the experimental group on most assignments (Table 7), that trend was not absolute, and those differences were mainly not significant. Beyond the few grades that were significantly different, the results were fairly similar, so it does not appear as though the teaching style had much impact on the grades (either individual assignments or the overall grade). When the lab grades were removed from the calculations, the final grades were not significantly different, adding evidence that the other grades were similar.

The lack of a significant difference for all three comparisons involving the ACT was important. The Pre-ACT was used to gauge prior knowledge, so a lack of a significant difference indicated that the two groups started the class at similar levels, with the control group having a very slightly higher average (25.78 vs. 25.77). While the control group's average increased more than the experimental group's average (3.45 vs. 3.20), that difference in increase was not significant. Furthermore, the Post-ACT averages were still not significantly different (29.23 vs. 28.97), so the control group did not benefit enough to substantiate a claim that a lecture was superior to the lecturing dialogue.

The surveys and questionnaires were based only on opinion, so they did not require any mathematical competence to complete them. The questionnaires were very short (only eight questions) and could be completed fairly quickly, so whether students did them was mainly a matter of effort (not ability). Throughout the semester, the control group seemed to display a higher level of maturity than the experimental group (based on general observations during class), so the fact that the control group did more of the surveys and questionnaires was not completely surprising.

To be specific about maturity, aspects were observed in several situations, including punctuality, responsibility, respect, attention span, and preparation. Attendance was taken each day using a portable card reader. Students were required to swipe their university ID cards to register their attendance. If a student came in late and did not get the card reader or forgot his/her ID card, then that student was required to sign in after class to get recognition for attendance. Students who arrived more than 15 minutes late did not receive the attendance credit. The students in the experimental group arrived late or forgot their ID cards more frequently than the students in the control group (3.1 per day vs. 2.6 per day). Punctuality and bringing the required materials to class are signs of personal responsibility, which is a trait of maturity.

Also related to attendance, students who left class early did not receive attendance recognition, unless they informed the instructor beforehand of the situation and were present for at least 35 minutes of the class. If a student left class early without informing the instructor, but after attendance was taken, then a short quiz was given at the end of the class. The quiz was graded on completion, not correctness. Five of these attendance quizzes were given to the experimental group (including on the first day of classes), but only two to the control group. Leaving class early without notifying the instructor is a sign of disrespect, and an indication that the student lacks maturity.

On a similar note, students in the experimental group frequently started to pack their belongings with about two or three minutes remaining in class. At least once per week, the experimental class needed to be reminded that class was not over, and that their behavior while preparing to leave was disruptive. This occurred with the control group at times, but the frequency was much less (once every two or three weeks). Again, self-control, respect for others, and attention span demonstrated a higher level of maturity for the control group than the experimental group.

Other disruptions (in the form of side conversations among students) were also more prevalent in the experimental group than the control group. Of course, this may have been a result of the different atmospheres of the classes. The teacher-led classes of the control group could have encouraged a more no-nonsense, business-like attitude than the more casual, laid-back nature of the experimental group. While there appeared to be a correlation, causality was not as easy to establish.

A final example involves preparation for class. At the beginning of every class (for both groups), students were given the opportunity to ask questions about anything that they wanted to ask, including course policies, upcoming assignments, and material from previous classes. Students in the control group submitted questions at the beginning of class on an almost daily basis (at least twice per week). Other than on designated review days, the students in the experimental group rarely did this. It is unclear why this pattern emerged, but several aspects could have contributed to it. The students in the control group might have attempted the homework sooner than the students in the experimental group and were able to ask questions before the homework was due each Sunday night. There was no significant difference in homework grade, so the students in the experimental group completed the homework, but they may not have started the homework until after the Friday class, so they would not have had a chance to ask questions during class before the due date. The students in the experimental group could have asked the questions on Monday, but the students could not change their grades at that time, so many did not bother. Of course, the students in the experimental group could have merely asked their questions to others (tutors or classmates) outside of class, so they did not need to ask the instructor.

One could claim that the students in the experimental group had a better understanding of the material, so they did not need to ask questions at the beginning of class, but the grades contradict that assertion. However, the students in the experimental group could have believed that they understood the material better, even though the students' collective performance indicated otherwise. Another possibility is that the students in the control were reluctant to disrupt the monologue of the lecture and just held their questions until the next class. As a note of clarification, the questions asked at the beginning of the class were written on paper, so they were not counted when the videos were examined.

The question arises as to whether this potential advantage in maturity influenced any other aspects of the study, such as study habits, seeking a tutor, or work ethic. Unfortunately, these details (which could have also been influenced by upbringing and habits formed during high school) were beyond the scope of this study. Adding to the supposition that the control group was more mature than the experimental group, a greater number of students in the control group completed both the Pre-ATMI and Post-ATMI than the experimental group (165 vs. 128). Also, 91 students in the control group submitted all four questionnaires, as opposed to only 81 in the experimental group. The significant difference for the lab grades could possibly be related to the maturity issue. Since they were completed in groups of two or three, and could be done outside of the classroom, a higher level of maturity may have impacted the observed results, whether resulting from simply completing the labs on time, paying attention to details, or seeking the necessary help.

The significant difference in the lab grades must be investigated beyond factors that were unrelated to the teaching style. While maturity, outside help, work ethic are possible reasons for the observed differences, an inherent quality of the lecture could have been responsible. The labs were designed to address conceptual understanding, so lectures may prove to be advantageous in that realm. Although more engaging, the lecturing dialogue's reliance on student responses could have led to misinformation being absorbed by the students. Incorrect answers were always identified as such, and correct answers were eventually given, but the sometimes-meandering journey to reach that correct answer may have left some students bewildered. Contrarily, the lecture typically provided a more direct path. The instructor occasionally made an intentional error to emphasize a common pitfall, but that was the exception to the standard. However, it is curious why such a distinction did not produce similar results on the other assessments. This conundrum merits further investigation.

Although unlikely, mere luck and coincidence might have played a part in the differences in labs grades. Students formed their own lab groups and were free to change groups at any time. It is possible that stronger students partnered with weaker students in the control group, with the abilities of the stronger students leading to higher grades for each group. If students with equal abilities formed groups, then groups with stronger students would get higher grades, but the weaker groups would get lower grades, resulting in a lower overall average for the class.

In terms of attitude, there were only non-significant differences in all four sub-categories (enjoyment, motivation, value, and self-confidence), both at the beginning and the end of the semester. The changes in each sub-category were also similar for both groups. Surprisingly, the scores for all four sub-categories declined for both groups, meaning that the enjoyment, motivation, value, and self-confidence all decreased during the semester. It is difficult (if not impossible) to identify the causes of this overall deterioration with the available data. It could have been the nature of the college algebra class, which was the first college math class for most of the students. The greater expectations, faster pace, and larger enrollment could have negatively affected the students' opinions. Even in a class that encourages interaction, it is challenging (if not impossible) to replicate the student-teacher relationship that develops in a small high school classroom. The overall morale of both classes could also have been affected by the students being tired and stressed at the end of a long semester (the first semester in college for the vast majority of them). The excitement and optimism of a new school year (coupled with the novelty of college) may have produced artificially inflated values at the beginning of the semester. Further study must be conducted to determine whether a similar decline occurs during the Spring, or in other classes (including those with more upperclassmen).

Chapter 6 – Conclusions

The results of this study were not those that were predicted, but they are nonetheless important and impactful. Although this study was primarily quantitative in nature, the comments from the student evaluations (although not intended to be included in the study) proved to be insightful. The evaluations probed what the students liked best, liked least, and would change, as well as providing a chance for students to offer comments. The evaluations did not solicit any specific type of information, so those who opined on the teaching style did so of their own accord. Both teaching styles had advocates and detractors. Ten of the students in the control group applauded the use of the standard lecture, but seventeen lamented it. Sixteen students in the experimental group enjoyed the lecturing dialogue, while only three despised it. While more students offered positive perspectives of the interactive style of teaching than the lecture, the split results mirrored those observed by Savelsbergh et al. (2016). Specifically, students who enjoyed the lecturing dialogue cited the interaction and engagement, which was also lamented as missing from the lecture. Fans of the lecture appreciated that the instructor "got right to the point" and stated that they "don't like interacting in class."

As was discussed in the literature review, each student has his/her own preferred learning style and most effective learning style, which may or may not be the same (Barbe & Milone, Jr., 1981). When introducing a new teaching style, it may be met with opposition if the students are not willing to accept that teaching style and give it a fair chance to succeed (Dunn et al., 1981; Udovic et al., 2002). Most students (being freshmen) were unlikely to be prepared for the levels of interaction and thinking demanded by the lecturing dialogue, as mentioned by Volkmann et al. (2005), so they may have rebelled against the foreign experience. On the other hand, the students who did not like the lecture apparently craved the benefits provided by more interaction.

While the results of this study were not the same as those that were predicted, they were not entirely unexpected. The review of literature revealed that students often do not recognize or appreciate what they have learned (Sundberg & Moncada, 1994), beyond simply just not liking a class. The students may have merely been unfamiliar with the teaching style of the lecturing dialogue (Dunn & Carbo, 1981; Udovic et al., 2002) or classes with large enrollments (Oliver, 2007), and teaching with the lecturing dialogue in a subsequent class could produce different results now that the students have had some experience with it. In terms of attitude, Gormally et al. (2009) also observed the same decrease in value and judgment as seen in this study. Elbert-May et al. (1997) saw different results in learning and self-confidence, but they investigated biology majors, as opposed to the non-STEM majors of this study.

Although the grades on the assessments did not show it, the lecturing dialogue could have had a real impact on the students. Not all changes and differences can be detected immediately, or even detected at all. The questions on the assessments that were given for the college algebra class used in this study were largely computational. The exercises that did delve into theory only required a comprehension of what the theory said, but not why it worked. Essays were asked on the labs, but (again) they required observations of changes in graphs or procedural descriptions.

To be clear, the labs were developed to utilize guided discovery to address conceptual understanding through interactive computer-based learning, a form of active learning recognized by Lake (2001). While the labs did not require an understanding of why things occurred in a certain way, they were designed to encourage an exploration of why by using multiple representations of a problem (verbal description, table of values, graph, equation) to understand the situation better. Students could answer the questions based on the observed changes, but a true demonstration of why would have proven to be prohibitively complicated for the nature of the assignment. Additionally, the level of explanation of why can be subjective, varying on an order of depth. For example, why do the lights go on when you flip the switch? Because the light bulb is in the socket. Because of electricity. Because flipping the switch caused the completion of a circuit that allowed electrons to flow through the conductive wires, encountering resistance when they reach the tungsten filament, causing heat to build, which made the tungsten glow, producing light. The required level of detail cannot always be described in a question without giving too much of a hint for how to correctly answer the question. The labs also were not afforded the opportunity to use the back-and-forth nature of the Socratic Method to draw out the deeper explanation of why when a student offered too shallow of an explanation. The result is a question that can be answered with observations of what happens, and not a full understanding of why.

On another note, since the labs were collaborative projects, it would have been impossible to know if all of the lab group's members understood the answers provided or just one member of the group. Realistically, none of the members of a group could have understood the answer that was provided (and they may have merely received help from someone else). Such essay questions could have been asked on tests, but grading them would have been prohibitive without more resources. Each class had a course assistant who graded the labs, but even that would not have been enough help to grade 200+ essays in a timely manner.

Beyond the results seen in the college algebra class during this study, it would be interesting to see how these students perform in future classes. The lecturing dialogue may not have produced instantaneous results, but a deeper understanding could have developed that will aid the students in more advanced math classes. A longitudinal study was beyond the scope of this dissertation, but is a possibility for subsequent investigation.

Just as the students might be more receptive to the lecturing dialogue if they are exposed to it in future classes, different results might have been observed for this study if students had been exposed to a teaching style similar to the lecturing dialogue in high school (or even earlier). The subsequent familiarity and accompanying comfortability would have reduced the required adjustment period needed for students who are introduced to new teaching styles, and students are less likely to resist teaching styles that they are acquainted with. While some high school teachers do use a more interactive teaching style, it would be prohibitive to identify which students were exposed to which styles (even using self-reporting surveys) because interactive styles can vary greatly, and students often do not have the knowledge, experience, references, or perspective to identify the level of interaction present in any given teaching style.

Another intriguing alteration to this study (and possibly one for future examination) would be to allow students to choose which class they would like to attend (lecture or lecturing dialogue) and observe the results. This would mitigate any preconceived prejudice or bias that the students might have against a specific teaching style, which Ertekin et al. (2009) and Dunn and Carbo (1981) discussed. Unfortunately, the students (especially college freshmen) are unlikely to know enough about the different teaching styles or their own learning styles make informed decisions about which class to take. Additionally, this would prove to be a logistical nightmare. Students often have difficulty scheduling the overall courses that they want, let alone the specific sections that they desire. Extra resources would be required to ensure that enough options were available to meet the desires of the students.

Ideally, a population would be obtained in which the individuals have had experience with multiple teaching styles during high school, so that they could make informed decisions. The participants would also have a well-developed understanding of both their preferred and most effective learning styles. However, given the intellectual abilities and emotional development of college freshmen, let alone the resources necessary to ensure that they have the necessary background knowledge about teaching and learning styles, this situation is unlikely to occur.

The overall conclusion is that in a class of 200+ students, the actual teaching style is virtually inconsequential. Some student will enjoy a standard lecture (maybe because it matches the teaching style that they are used to or prefer), but others will enjoy the lecturing dialogue (maybe because they find the interaction stimulating). In a class as large as the two that were investigated for this study, an interactive style may engage some, but will turn off others because of its variation from what is expected and what the students are used to. Similarly, a lecture may appeal to some students because of its familiarity, but it will turn off others of its coldness and formality. Ultimately, a teacher should use the teaching style that he/she feels the most comfortable with and which matches his/her personality the best, as advised by Bain (2004) and Savelsbergh et al. (2016). The results might be different for smaller classes, but large classes do not afford such a distinction, rendering the teaching style largely immaterial.

Limitations

There were several possible limitations for this study. The population was somewhat one of convenience. The researcher was restricted as to who could be studied, so he had to use the students that were at his disposal. As stated earlier, the resources for testing were also limited. More in-depth essay questions could not be asked on the tests to fully gauge how much the students understood the reasons behind why things occurred the way that they did (which was one of the outcomes that the researcher had hoped for).

The impact that the personalities, genders, and majors of the students who took part in this study had on the grades and attitudes was not explored (beyond all of the students being non-STEM majors). Again, these questions were beyond the scope of this study, but they could potentially yield insightful results and merit further study. On the lines of personality, the instructor's personality could have had an impact on the results that were observed. The personalistic style of teaching and warm speech (Bain, 2004) were discussed in the theoretical framework. However, that style was prevalent for both groups in this study. A total of 57 students (30 from the control group and 27 from the experimental group) commented on the personalistic teaching style (although not using that phrase to describe it) of the researcher on the evaluations. Primarily, students mentioned the researcher sharing some small details about himself in the form of true-or-false questions at the end of class, a sense of humor, and enthusiasm while teaching as favorite aspects of the class. As with the comments about liking or disliking the teaching style, the comments about the personalistic teaching style (in particular) were unsolicited.

A comparison of a more formal, unpersonalistic lecture with the lecturing dialogue could produce different results, as would a comparison of a formal lecture with a more personalistic lecture. The personalistic style of teaching (while more obvious in the lecturing dialogue than the lecture) could not be avoided in the lecture without resulting in different explanations of the material. The personalistic style was ingrained in the way that the ideas and concepts were presented using the lecturing dialogue, so that same personalistic style had to be used in the lecture to keep the explanations as similar as possible.

Given unlimited resources (time, money, control), the researcher would repeat this experiment with multiple classes of various sizes: 200+ students in each class, 100 students, 50 students, and 20 students. The background and demographic information for each student would be collected and analyzed. The students would also be followed through future classes and interviewed in an attempt to learn exactly how much of an impact each teaching style had on conceptual understanding and attitude.

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Appendix A – ATMI Details

ATMI Questions

Rate all statements as Strongly Agree, Agree, Neutral, Disagree, or Strongly Disagree.

1. Mathematics is a very worthwhile and necessary subject.
2. I want to develop my mathematical skills.
3. I get a great deal of satisfaction out of solving a mathematics problem.
4. Mathematics helps develop the mind and teahes a person to think.
5. Mathematics is important in everyday life.
6. Mathematics is one of the most important subjects for people to study.
7. High school math courses would be very helpful no matter what I decide to study.
8. I can think of many ways that I use math outside of school.
9. Mathematics is one of my most dreaded subjects.
10. My mind goes blank and I am unable to think clearly when working with mathematics.
11. Studying mathematics makes me feel nervous.
12. Mathematics makes me feel uncomfortable.
13. I am always under a terrible strain in a math class.
14. When I hear the word "mathematics," I have a feeling of dislike.
15. It makes me nervous to even think about having to do a mathematics problem.
16. Mathematics does not scare me at all.
17. I have a lot of self-confidence when it comes to mathematics.
18. I am able to solve mathematics problems without too much difficulty.
19. I expect to do fairly well in any math class I take.
20. I am always confused in my mathematics class.

21. I feel a sense of insecurity when attempting mathematics.

22. I learn mathematics easily.

23. I am confident that I could learn advanced mathematics.

24. I have usually enjoyed studying mathematics in school.

25. Mathematics is dull and boring.

26. I like to solve new problems in mathematics.

27. I would prefer to do an assignment in math than to write an essay.

28. I would like to avoid using mathematics in college.

29. I really like mathematics.

30. I am happier in a math class than in any other class.

31. Mathematics is a very interesting subject.

32. I am willing to take more than the required amount of mathematics.

33. I plan to take as much mathematics as I can during my education.

34. The challenge of math appeals to me.

35. I think studying advanced mathematics is useful.

36. I believe studying math helps me with problem solving in other areas.

37. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.

38. I am comfortable answering questions in math class.

39. A strong math background could help me in my professional life.

40. I believe I am good at solving math problems.

Reversed Questions

9, 10, 11, 12, 13, 14, 15, 20, 21, 25, 28

Sub-Categories by Question

Enjoyment – 3, 24, 25, 26, 27, 29, 30, 31, 37, 38

Motivation – 23, 28, 32, 33, 34

Value – 1, 2, 4, 5, 6, 7, 8, 35, 36, 39

Self-Confidence – 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 40

Appendix B – Engagement Questionnaire Questions

Rate all statements as Strongly Agree, Agree, Neutral, Disagree, or Strongly Disagree.

1. The instructor asked questions to the class.

2. The teacher only lectured.

3. The teacher explained the "why" behind formulas and equations.

4. The teaching was effective.

5. I feel involved in the class.

6. I feel the need to pay attention in class.

7. I am encouraged to participate in class.

8. I think that there is a lot of student-teacher interaction in the class.

Appendix C – Histograms

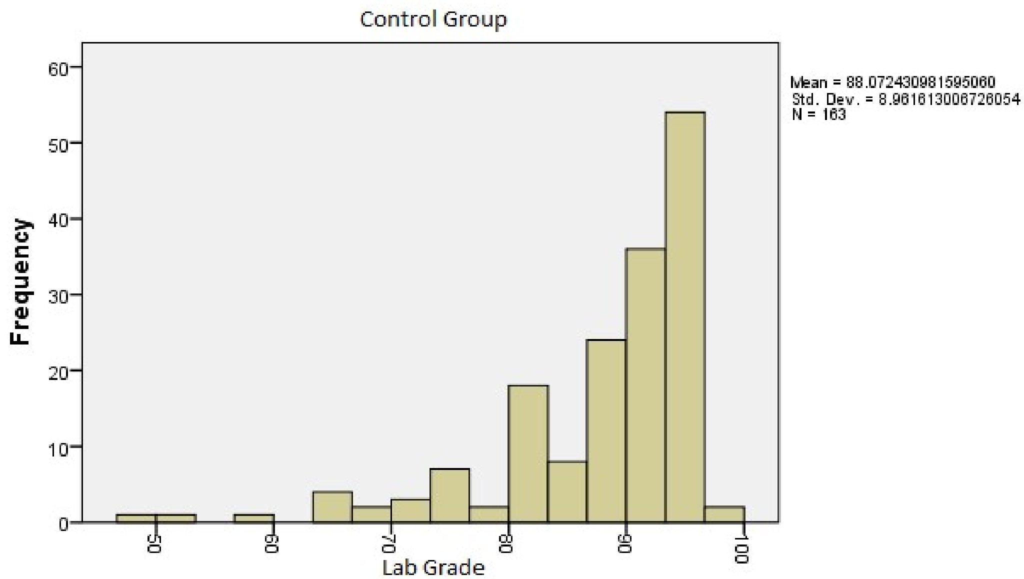

Figure C1. Histogram for control group lab grades.

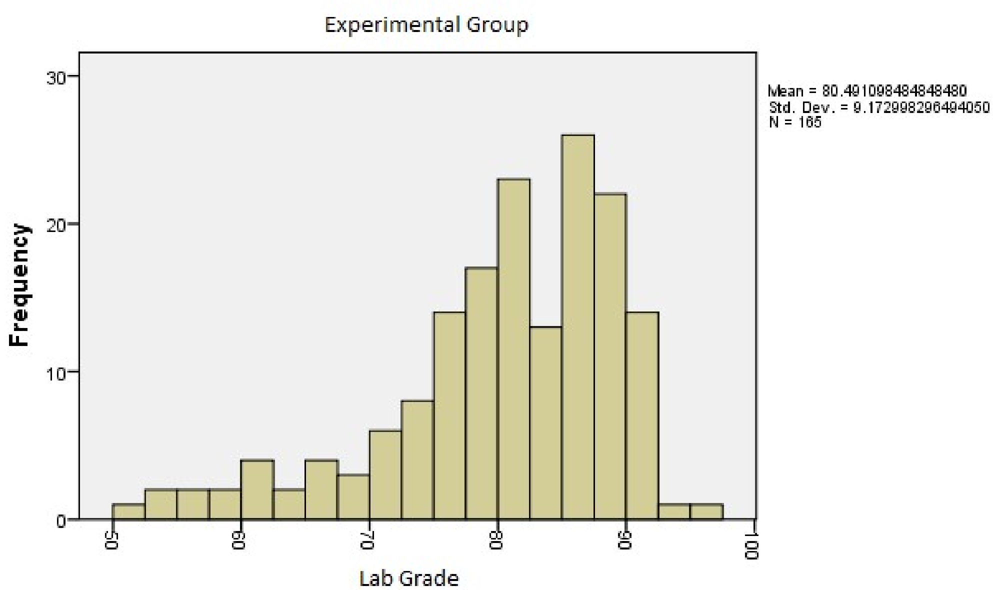

Figure C2. Histogram for experimental group lab grades.

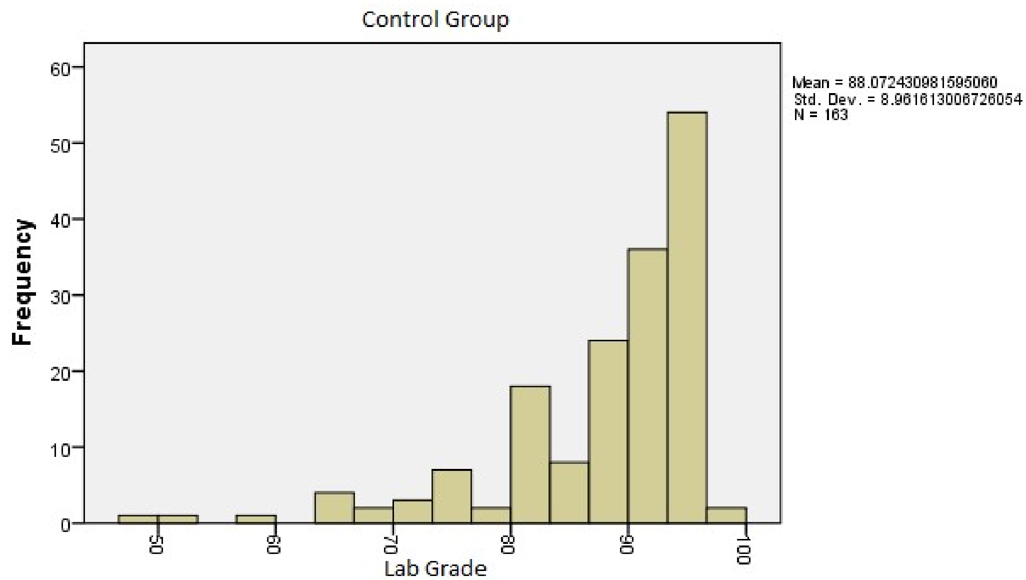

Figure C3. Histogram for control group quiz grades.

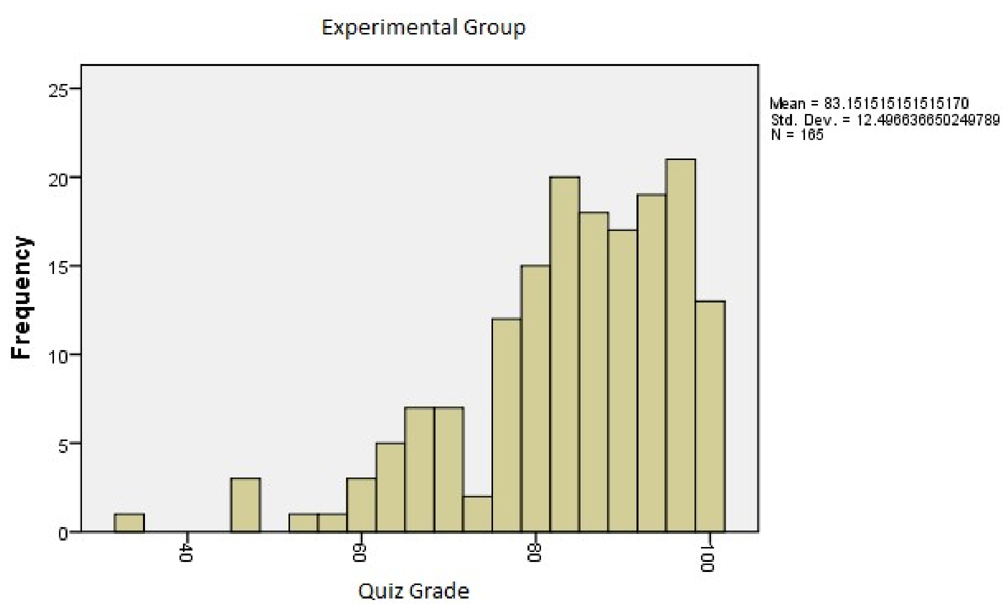

Figure C4. Histogram for control group quiz grades.

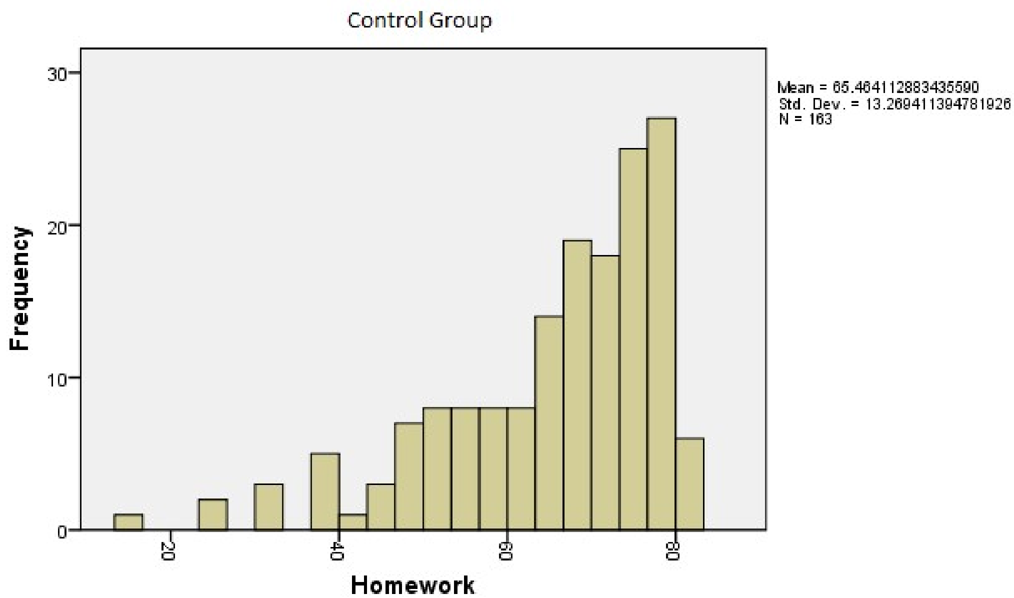

Figure C5. Histogram for control group homework.

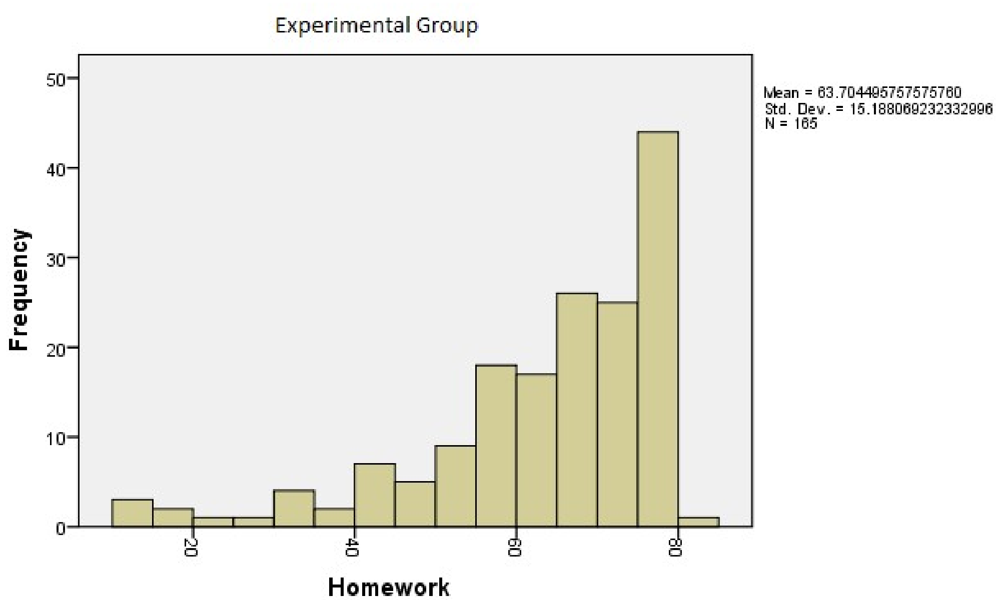

Figure C6. Histogram for experimental group homework.

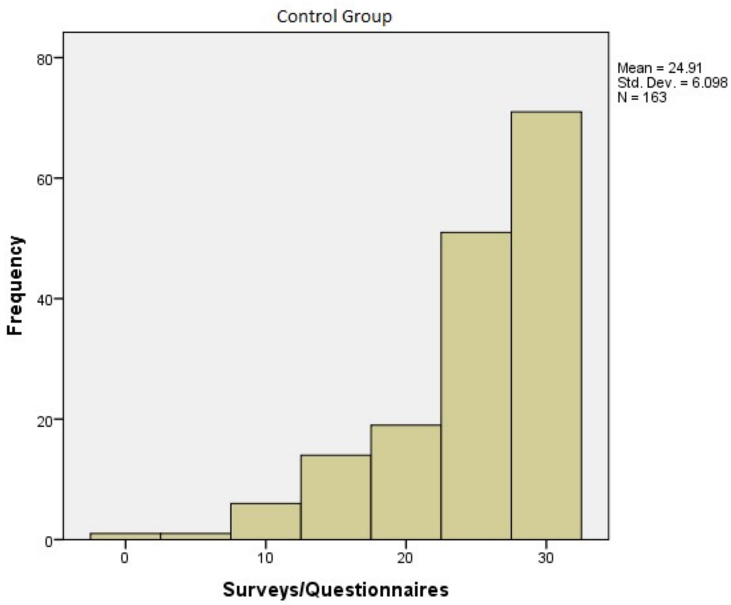

Figure C7. Histogram for control group surveys/questionnaires.

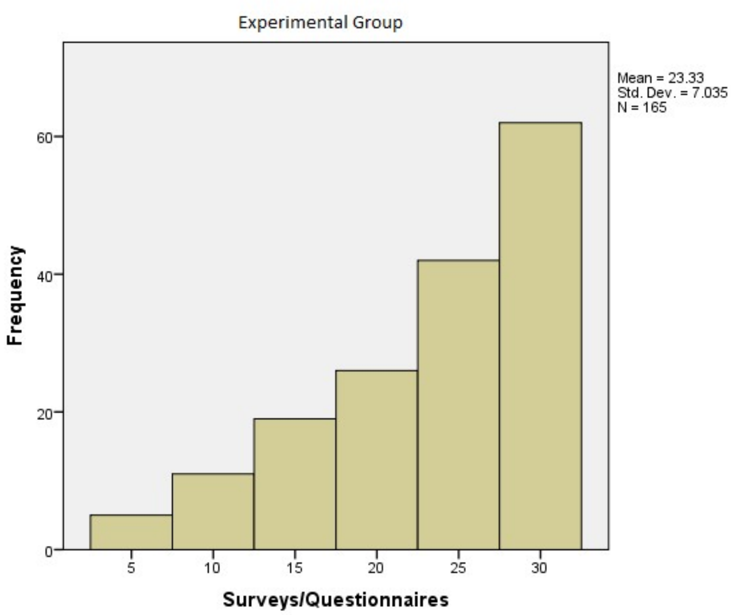

Figure C8. Histogram for experimental group surveys/questionnaires.

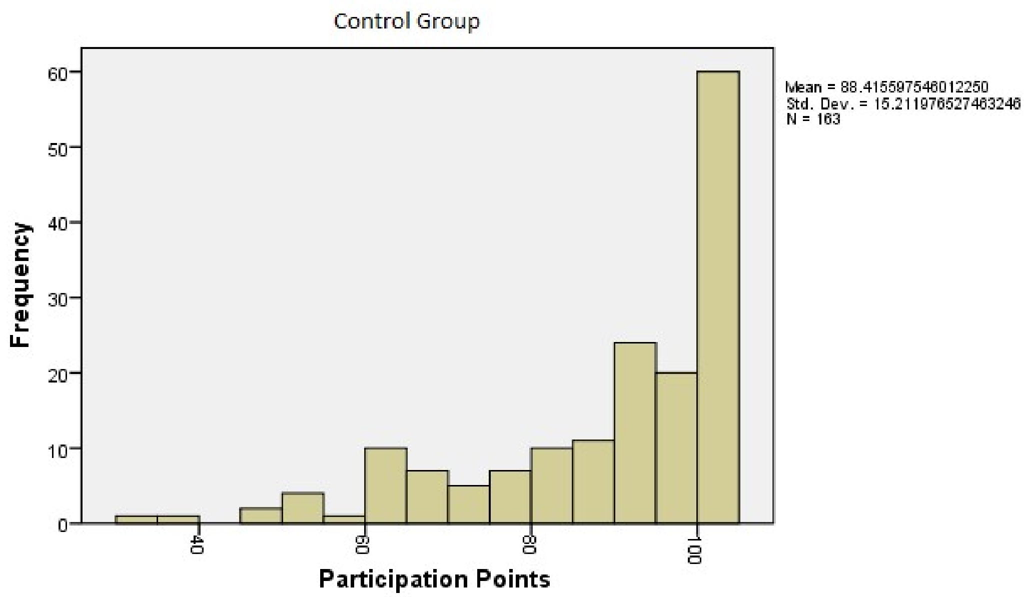

Figure C9. Histogram for control group participation points.

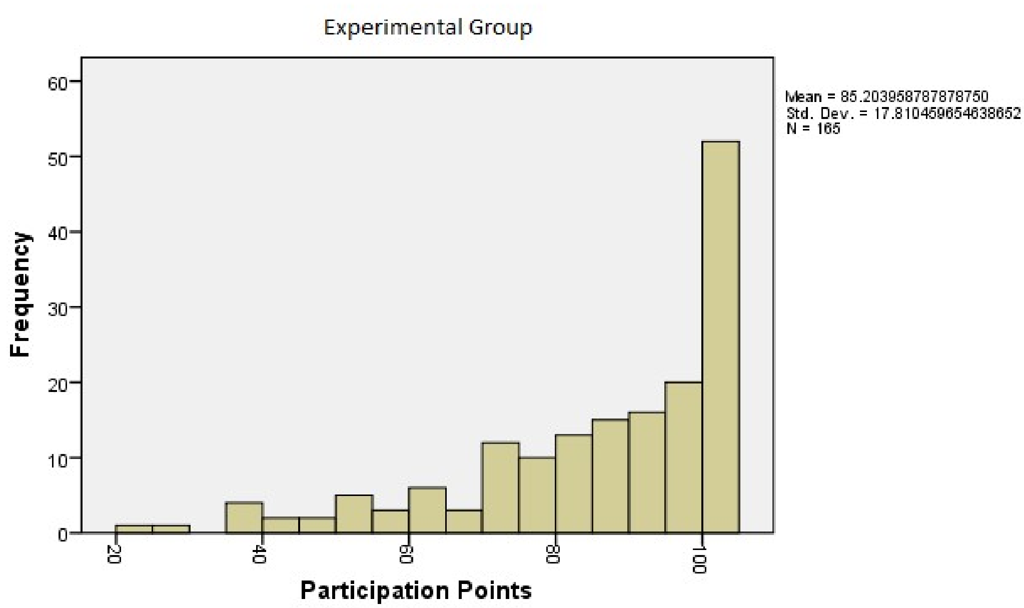

Figure C10. Histogram for experimental group participation points.

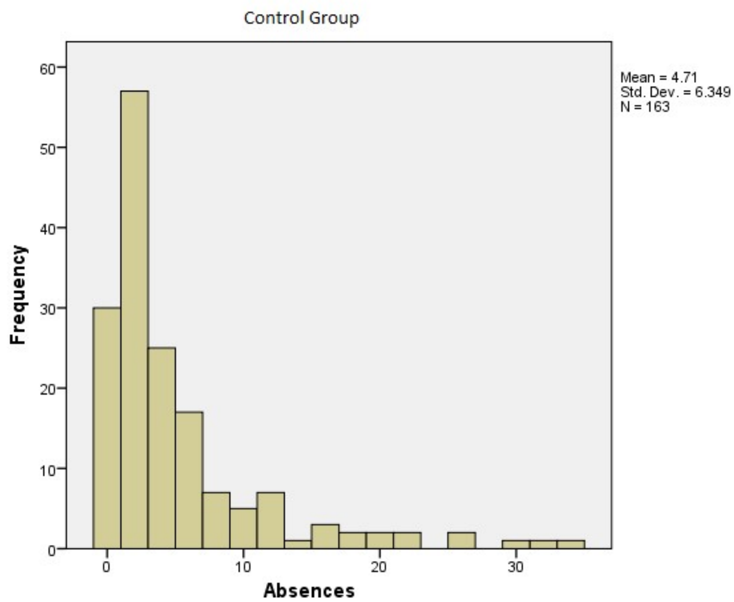

Figure C11. Histogram for control group absences.

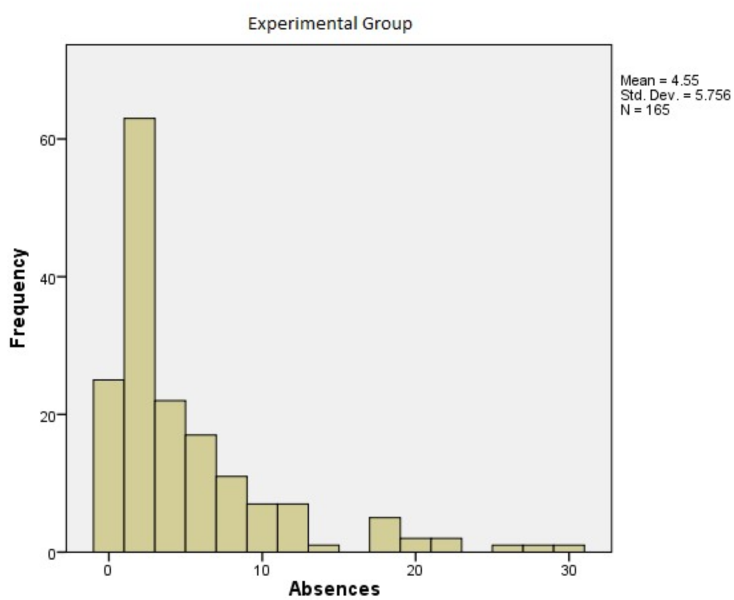

Figure C12. Histogram for experimental group absences.

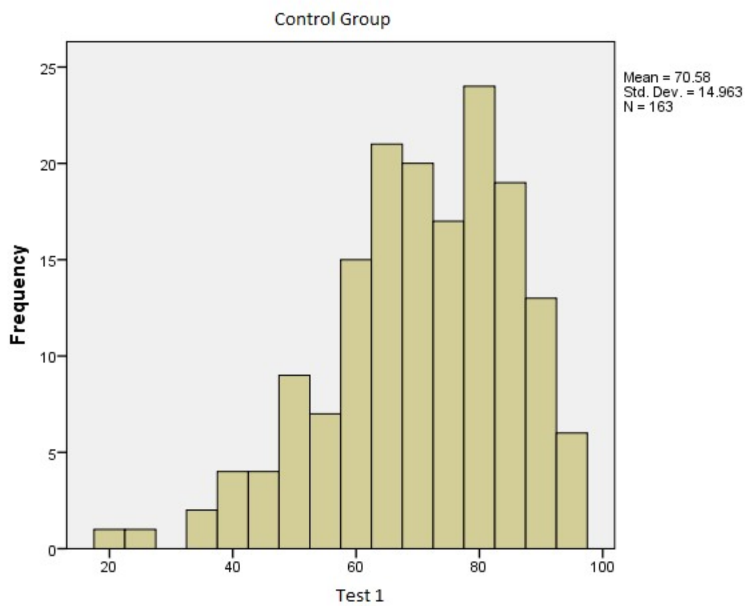

Figure C13. Histogram for control group test 1.

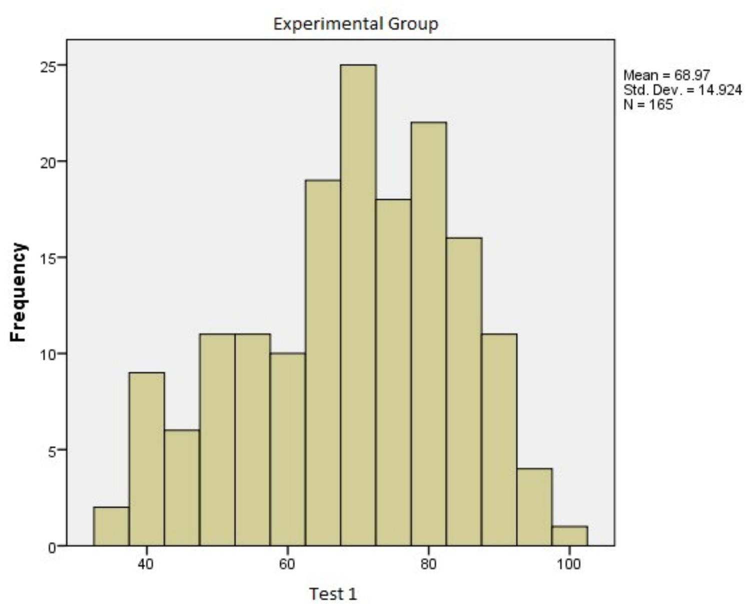

Figure C14. Histogram for experimental group test 1.

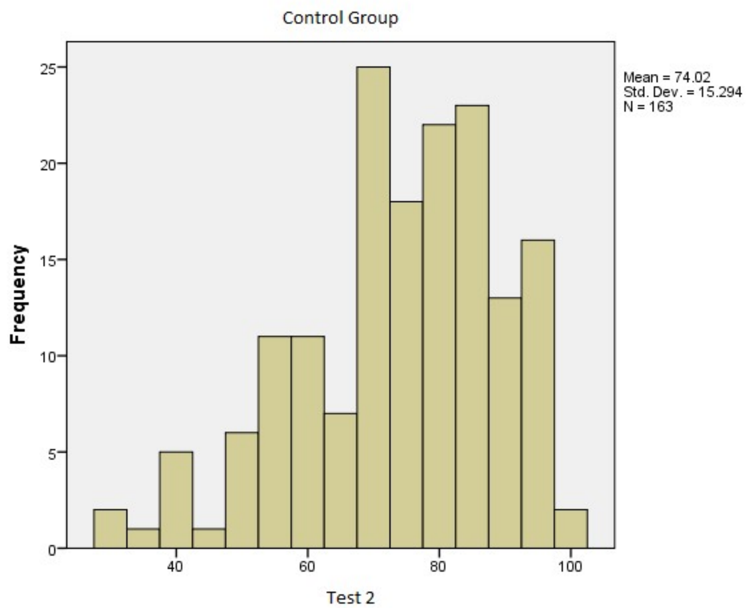

Figure C15. Histogram for control group test 2.

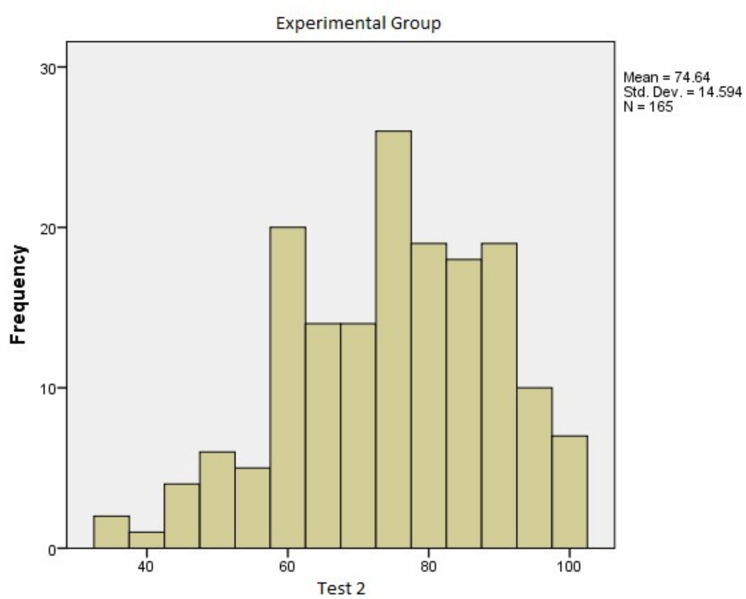

Figure C16. Histogram for experimental group test 2.

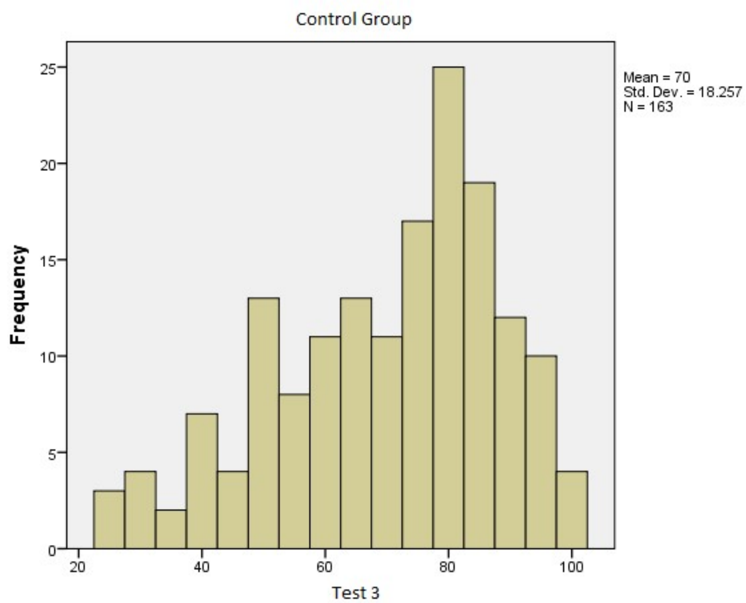

Figure C17. Histogram for control group test 3.

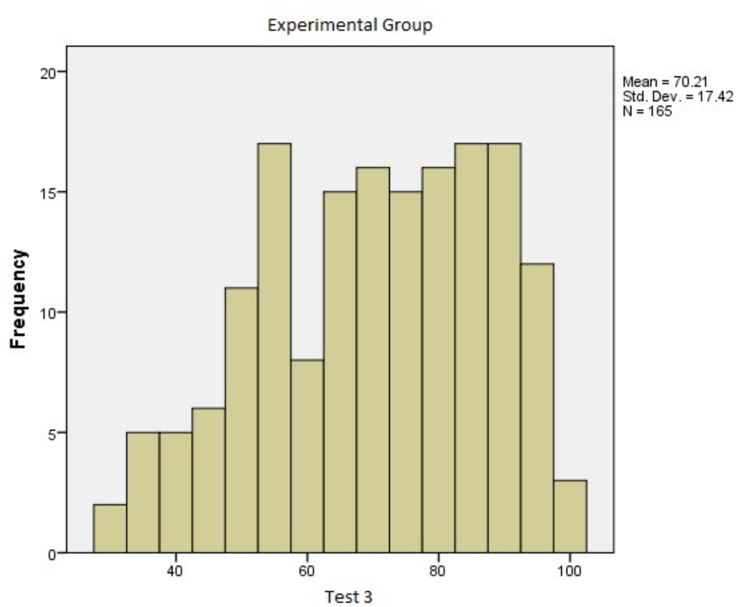

Figure C18. Histogram for experimental group test 3.

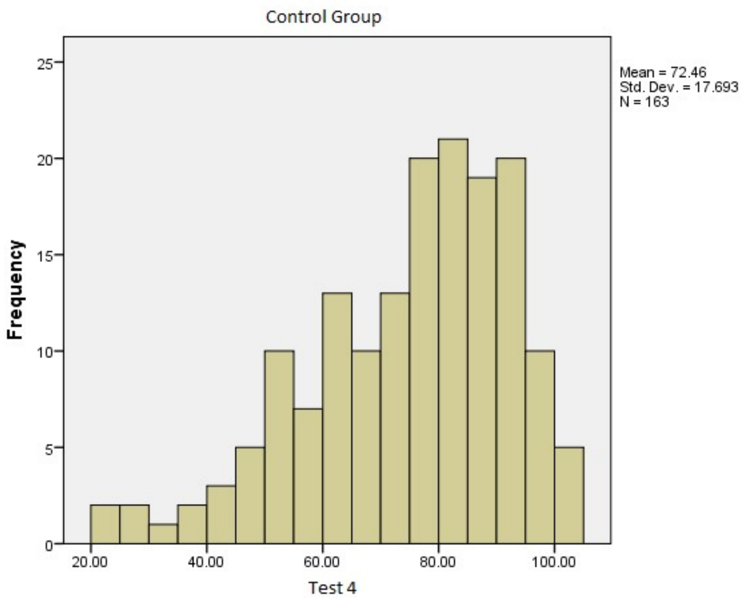

Figure C19. Histogram for control group test 4.

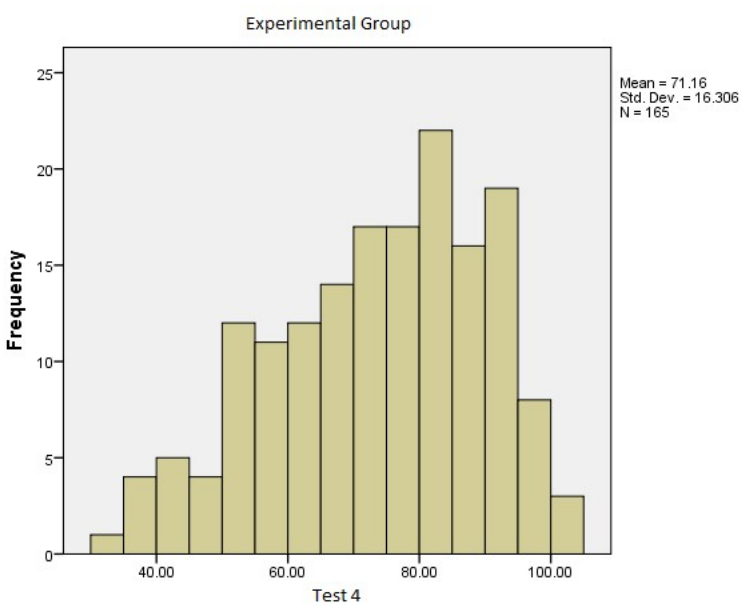

Figure C20. Histogram for experimental group test 4.

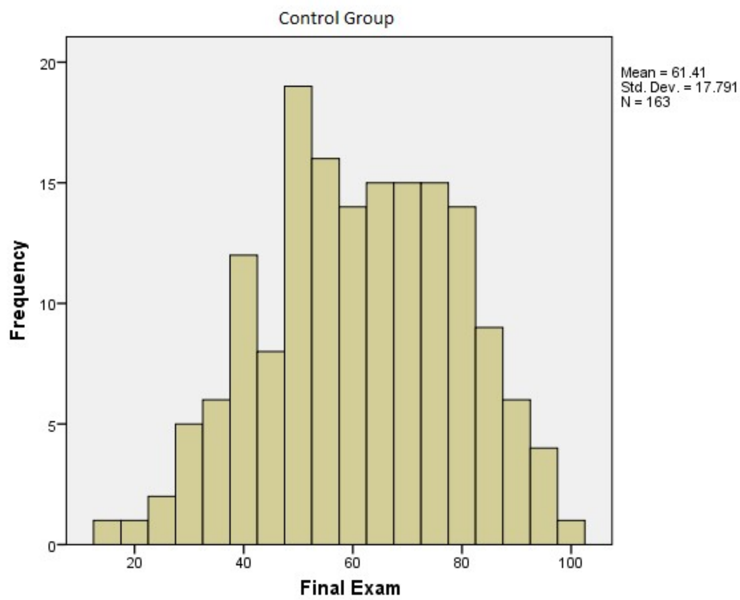

Figure C21. Histogram for control group final exam.

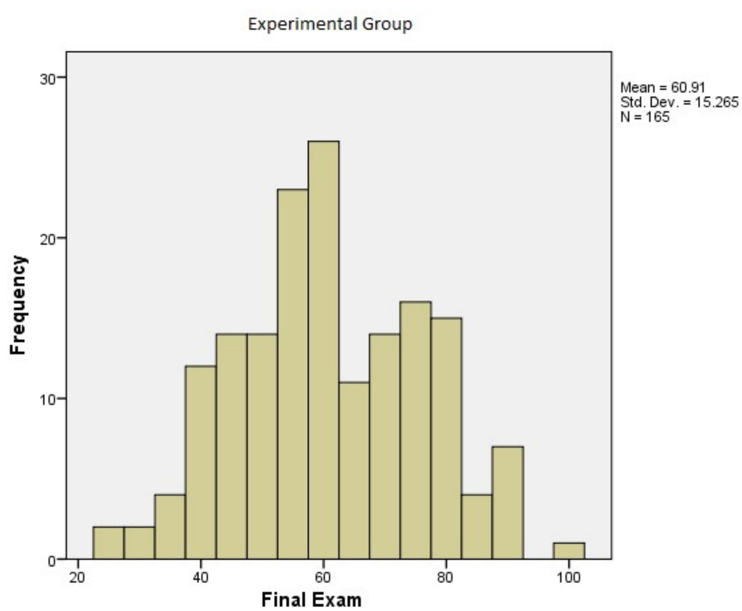

Figure C22. Histogram for experimental group final exam.

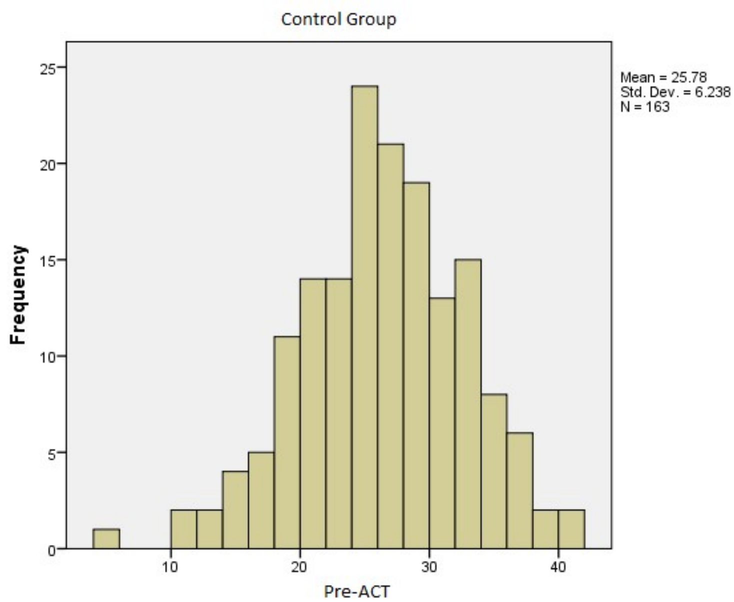

Figure C23. Histogram for control group pre-ACT.

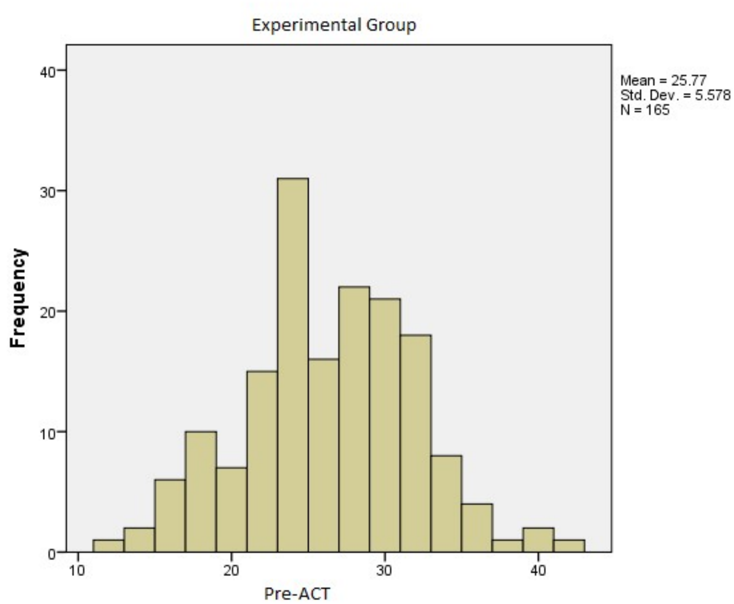

Figure C24. Histogram for experimental group pre-ACT.

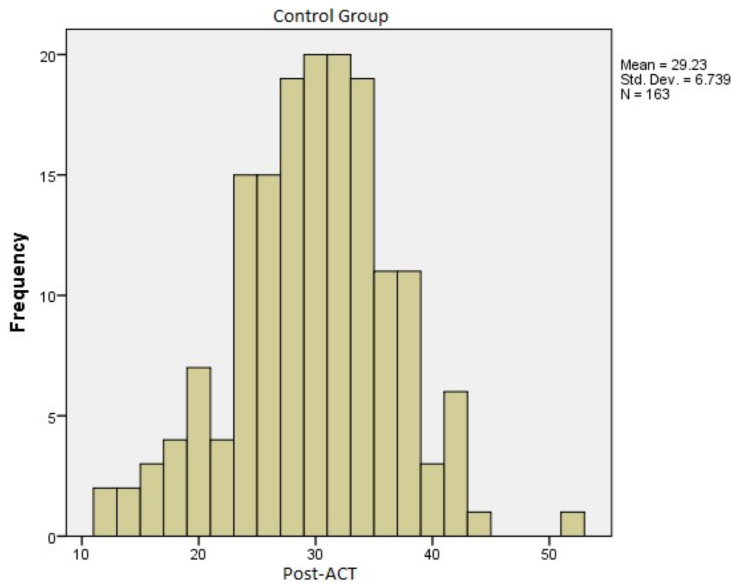

Figure C25. Histogram for control group post-ACT.

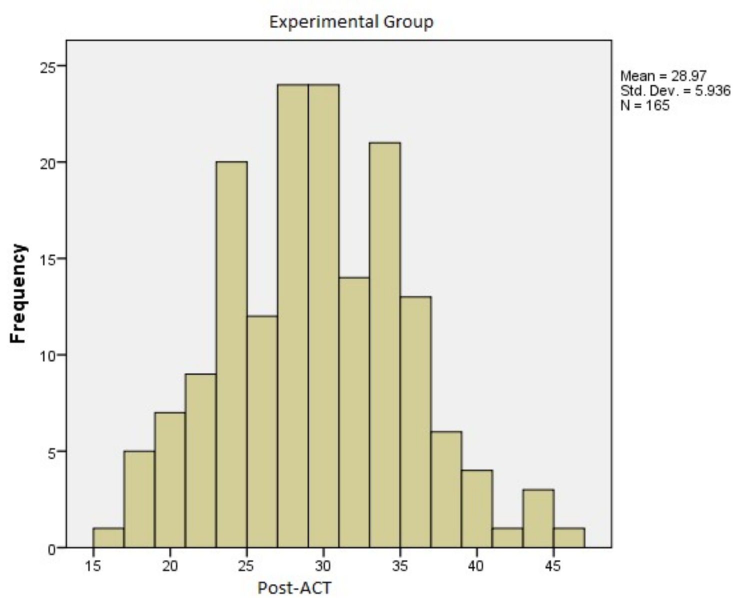

Figure C26. Histogram for experimental group post-ACT.

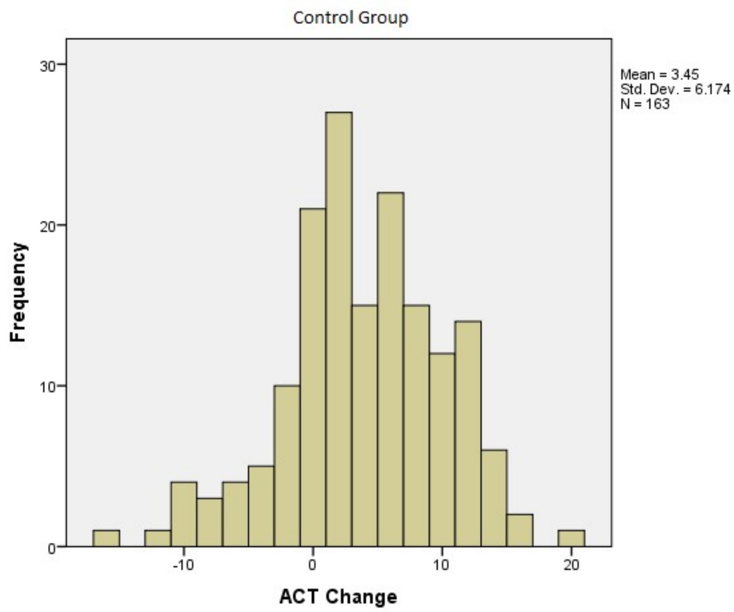

Figure C27. Histogram for control group ACT change.

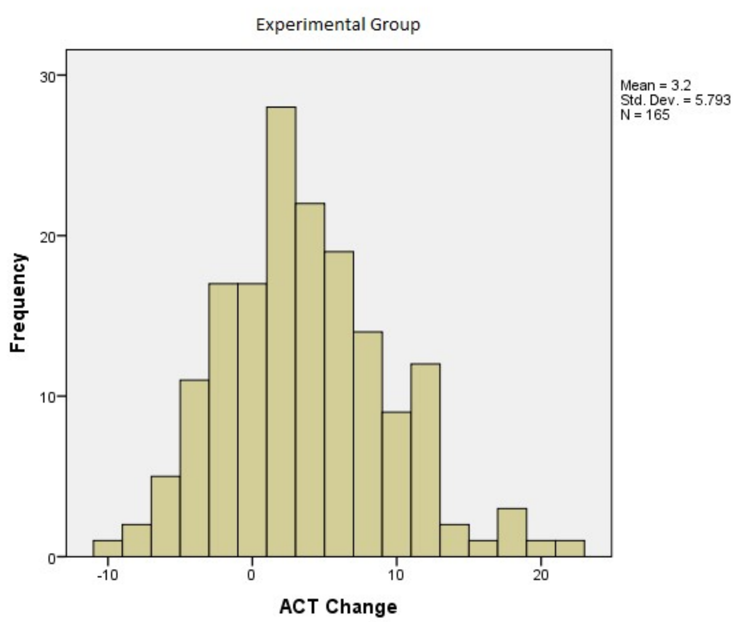

Figure C28. Histogram for experimental group ACT change.

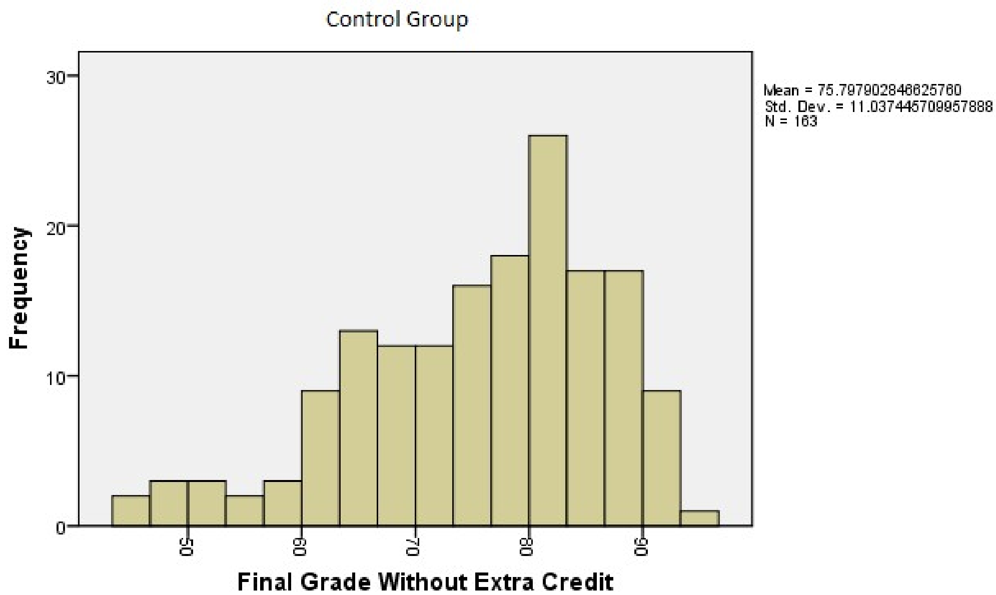

Figure C29. Histogram for control group final grade without extra credit.

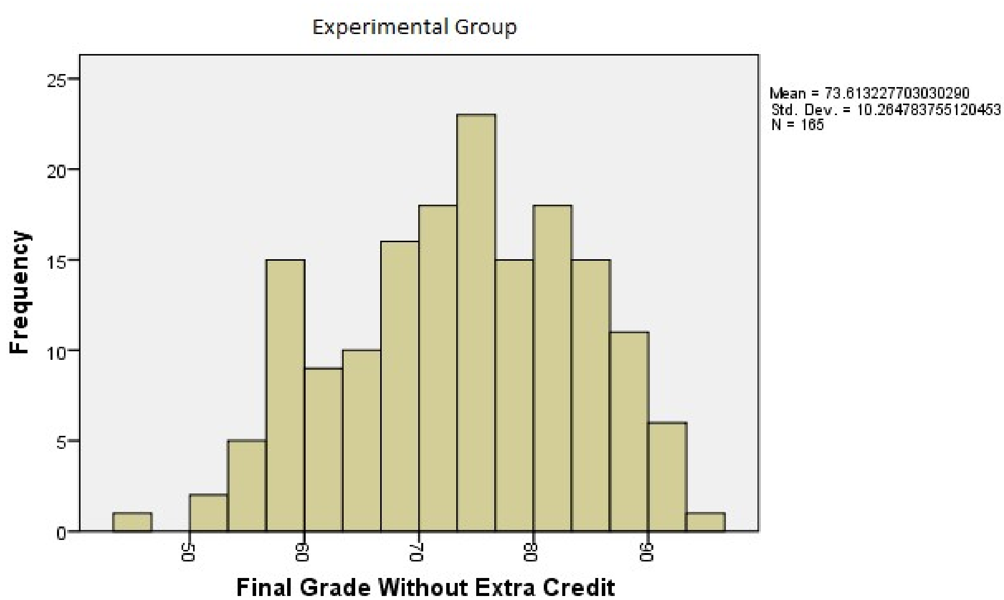

Figure C30. Histogram for experimental group final grade without extra credit.

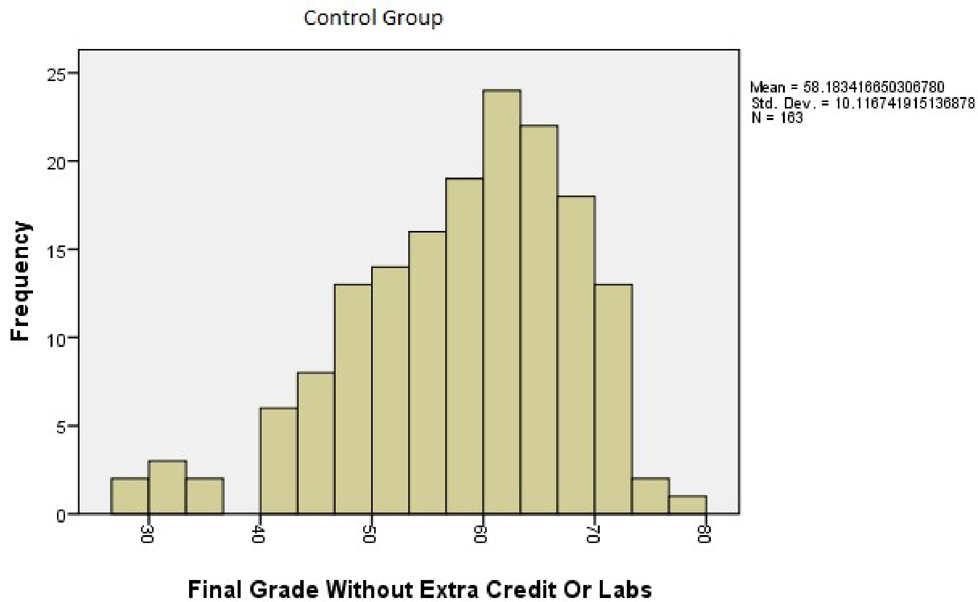

Figure C31. Histogram for control group final grade without extra credit or labs.

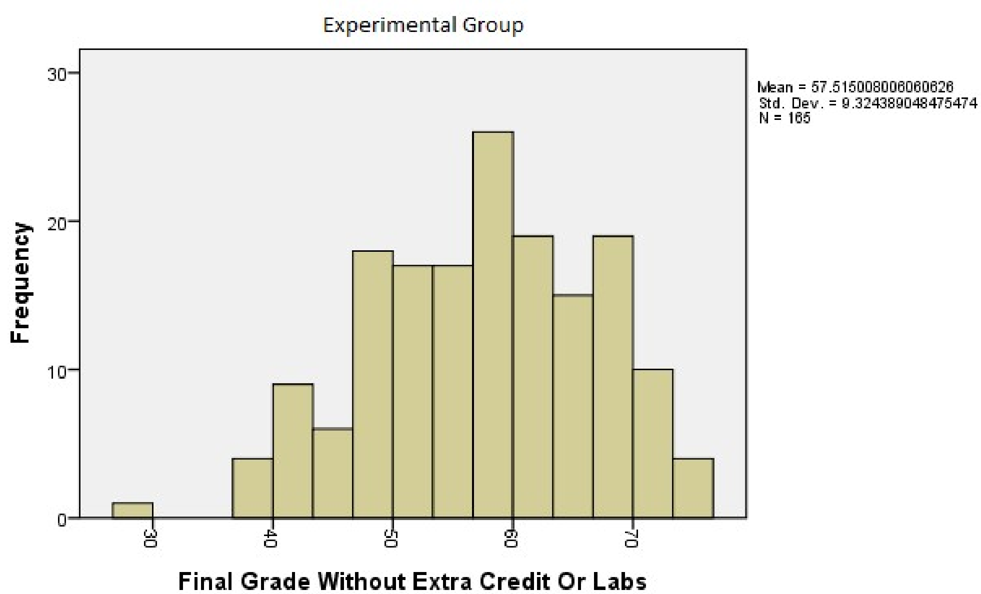

Figure C32. Histogram for experimental group final grade without extra credit or labs.

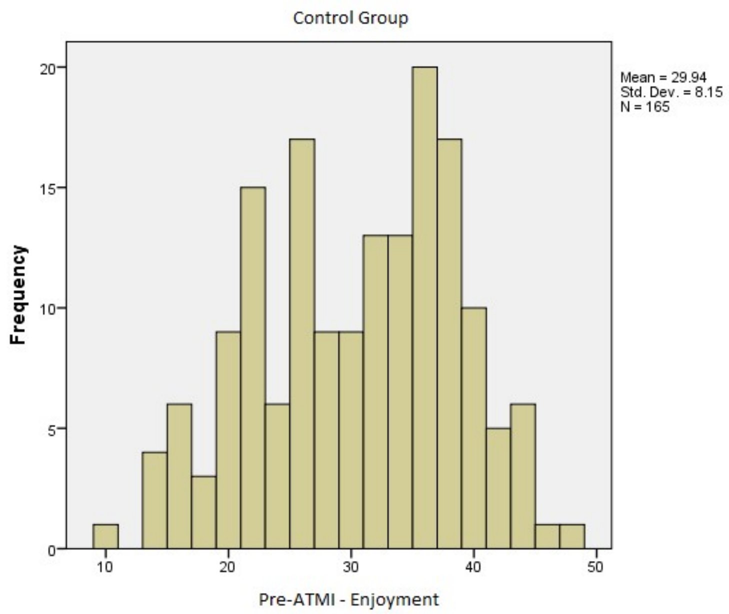

Figure C33. Histogram for control group pre-ATMI enjoyment.

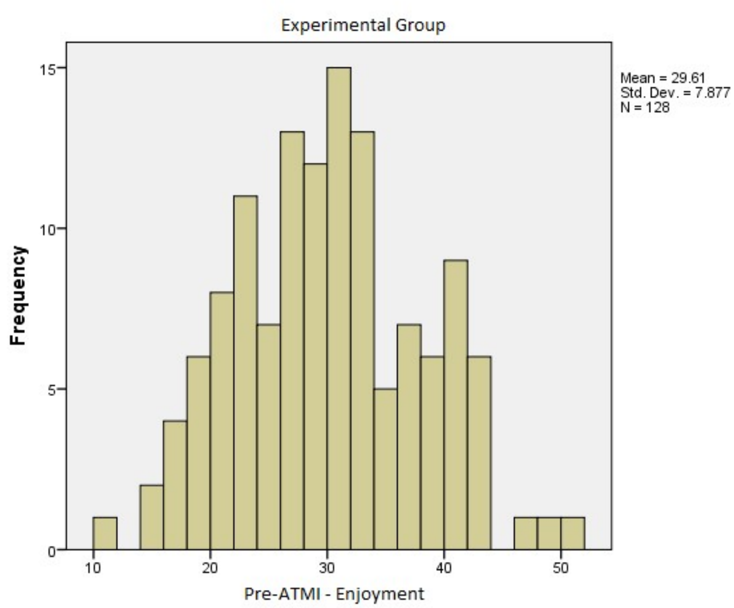

Figure C34. Histogram for experimental group pre-ATMI enjoyment.

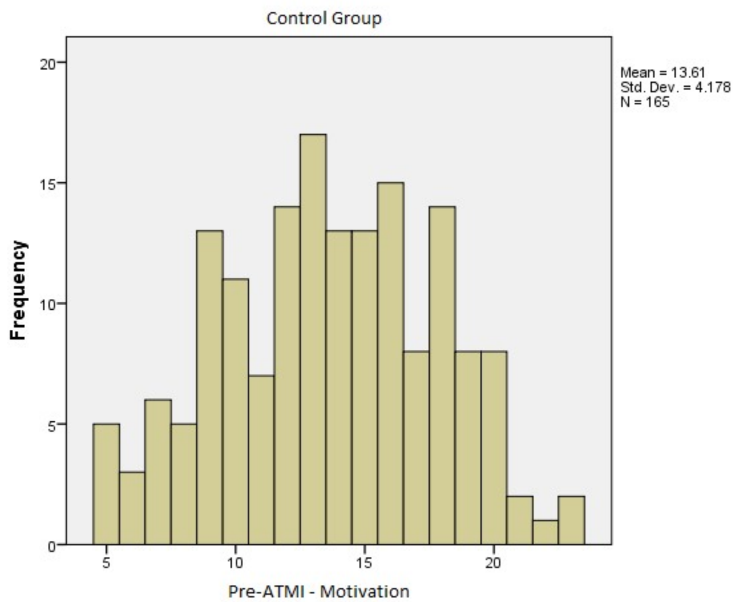

Figure C35. Histogram for control group pre-ATMI motivation.

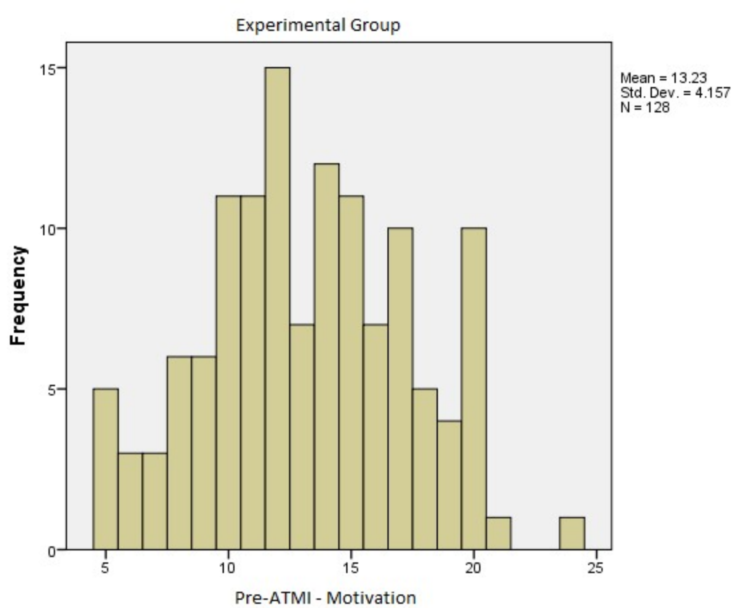

Figure C36. Histogram for experimental group pre-ATMI motivation.

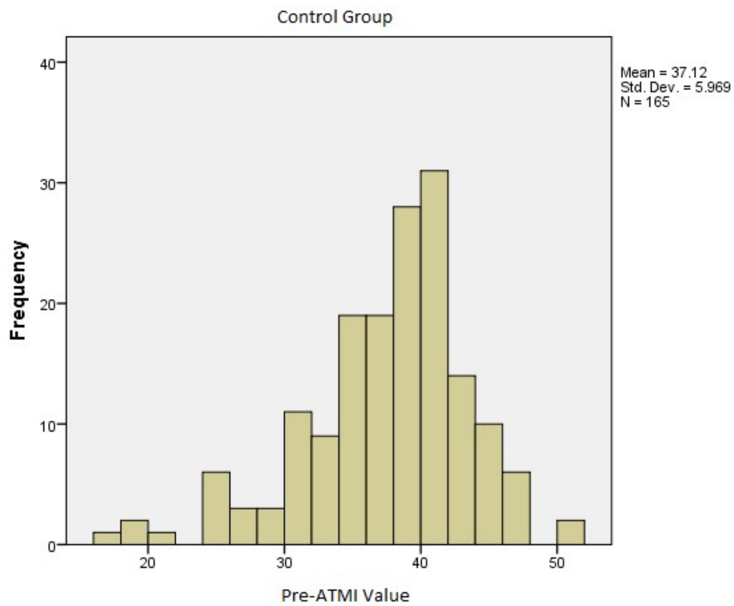

Figure C37. Histogram for control group pre-ATMI value.

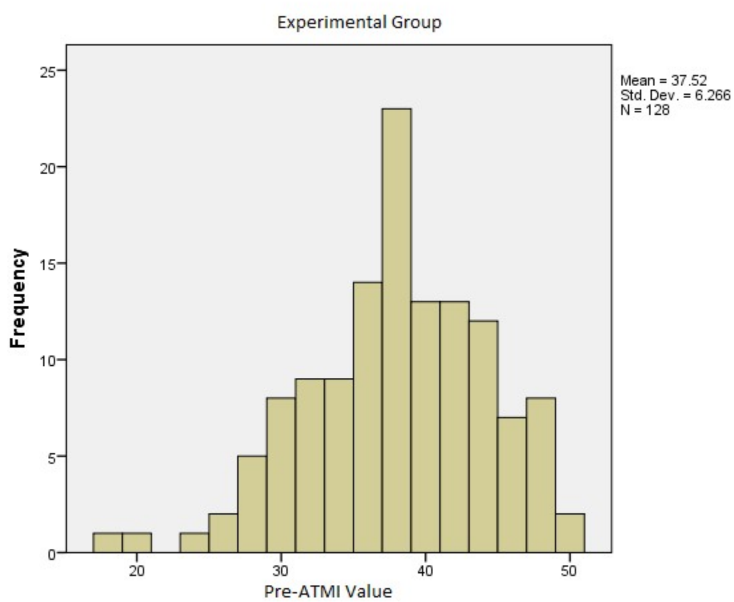

Figure C38. Histogram for experimental group pre-ATMI value.

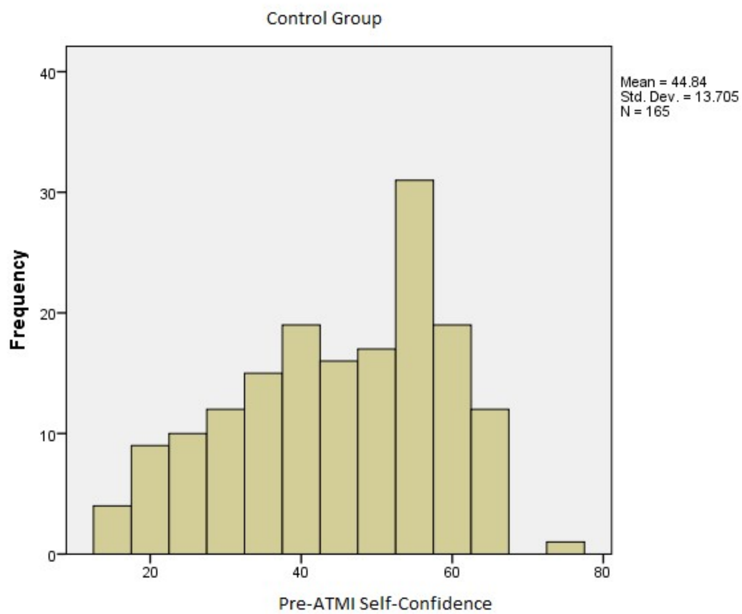

Figure C39. Histogram for control group pre-ATMI self-confidence.

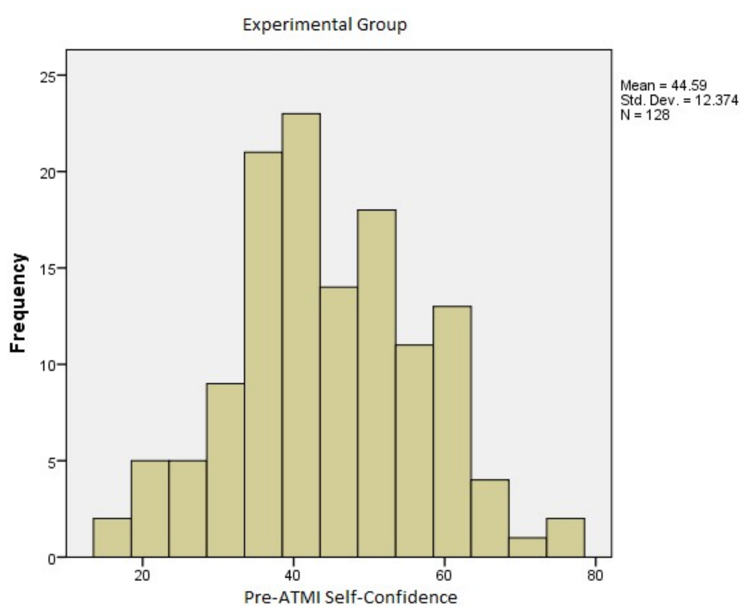

Figure C40. Histogram for experimental group pre-ATMI self-confidence.

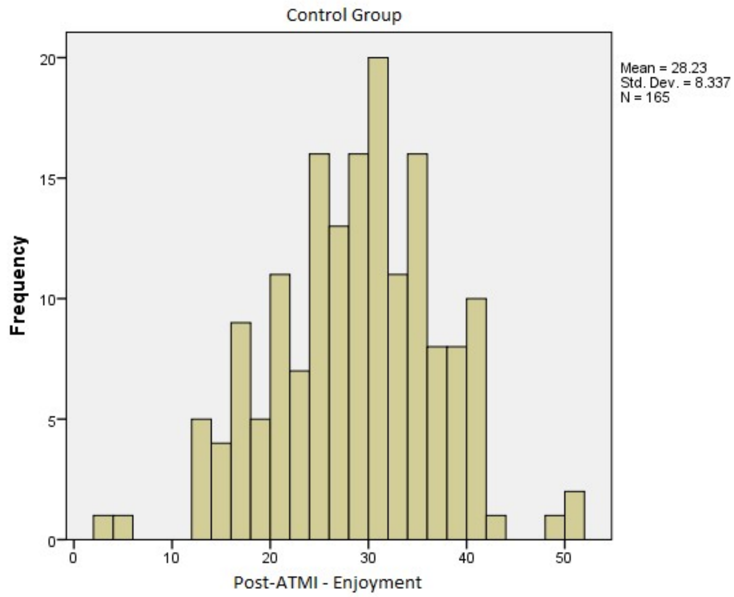

Figure C41. Histogram for control group post-ATMI enjoyment.

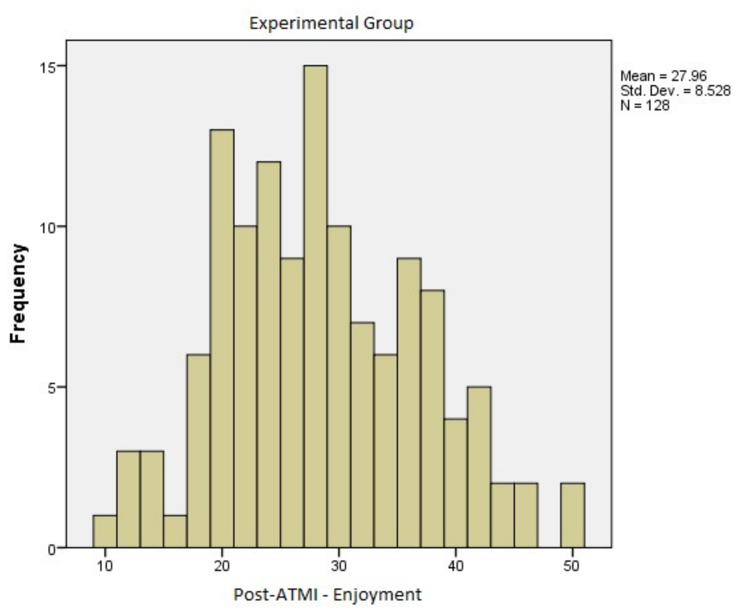

Figure C42. Histogram for experimental group post-ATMI enjoyment.

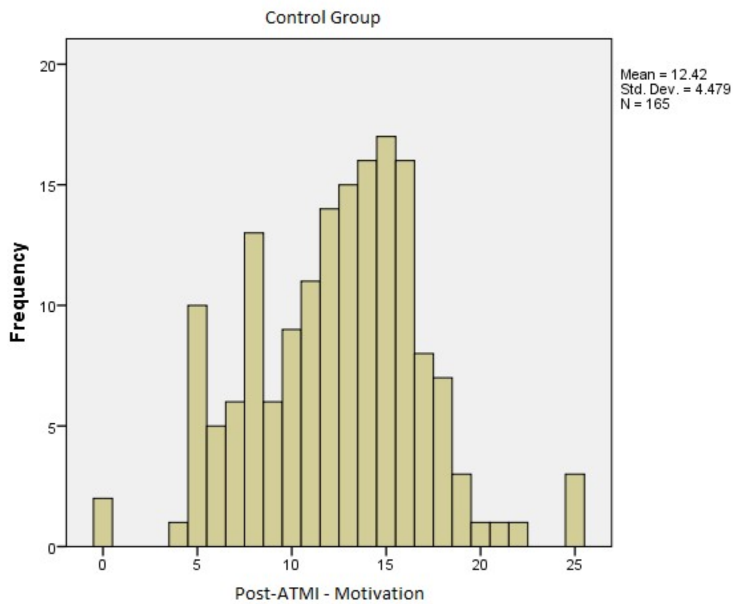

Figure C43. Histogram for control group post-ATMI motivation.

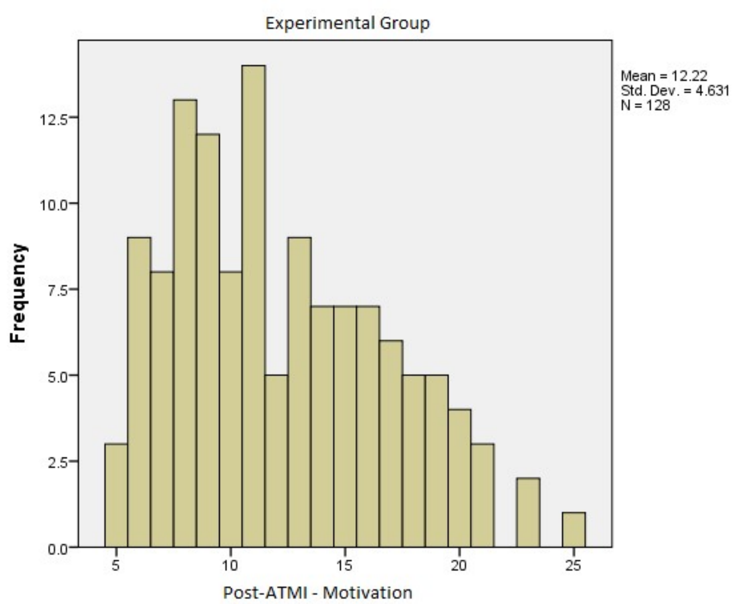

Figure C44. Histogram for experimental group post-ATMI motivation.

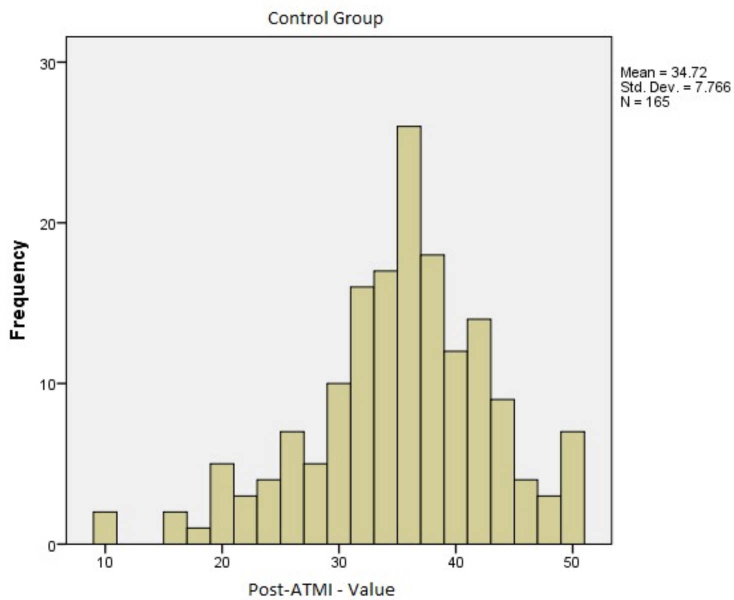

Figure C45. Histogram for control group post-ATMI value.

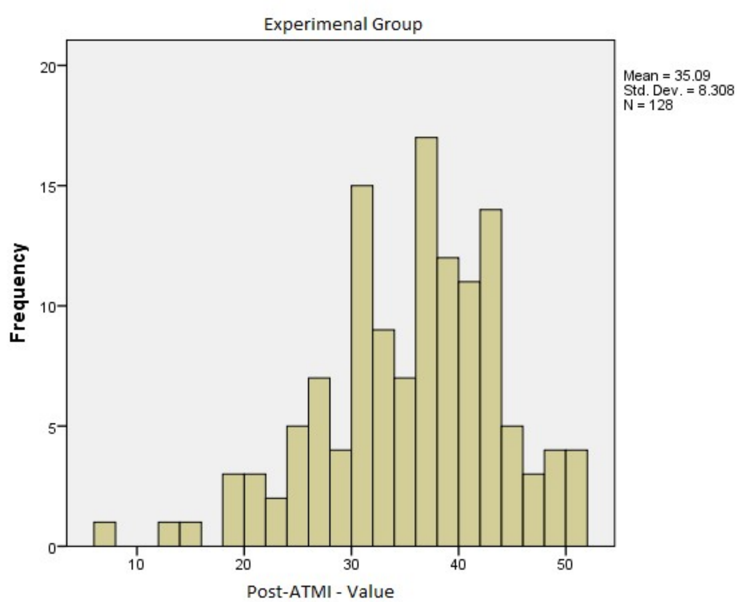

Figure C46. Histogram for experimental group post-ATMI value.

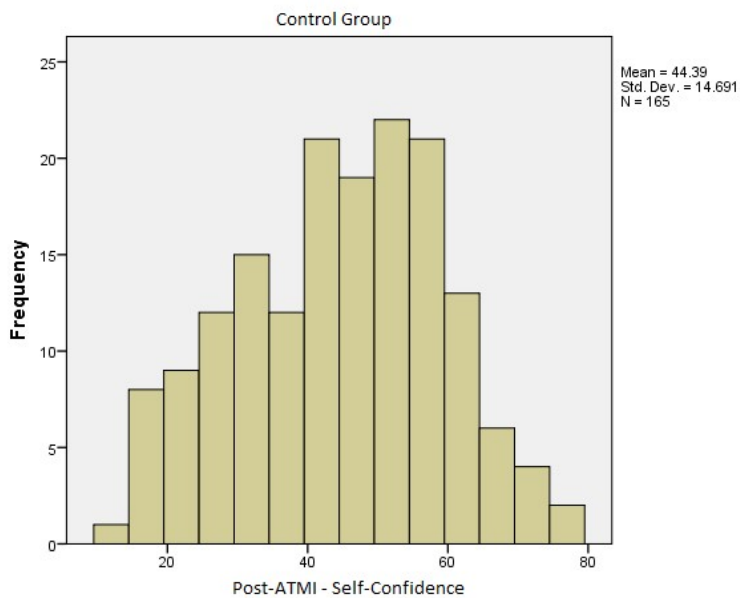

Figure C47. Histogram for control group post-ATMI self-confidence.

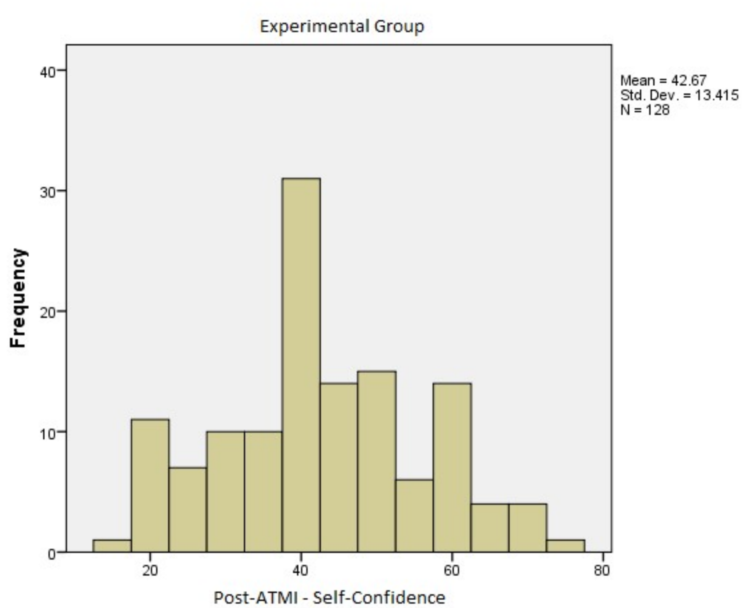

Figure C48. Histogram for experimental group post-ATMI self-confidence.

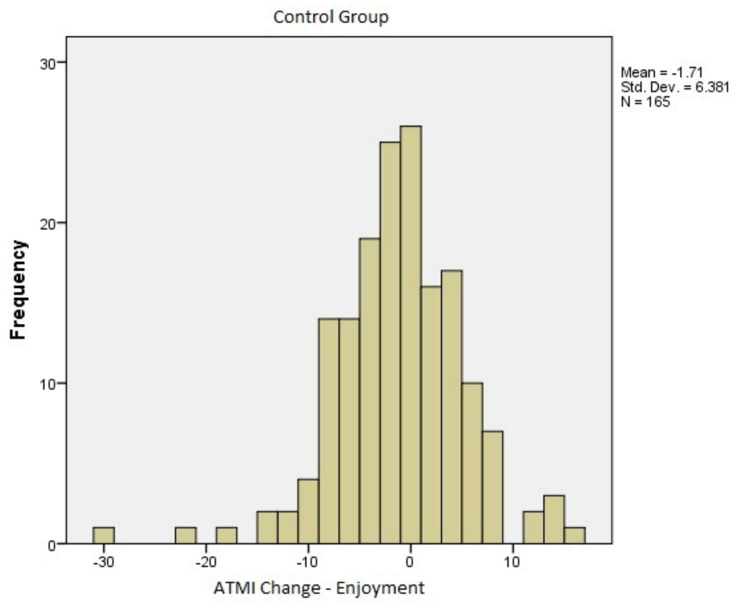

Figure C49. Histogram for control group ATMI change – enjoyment.

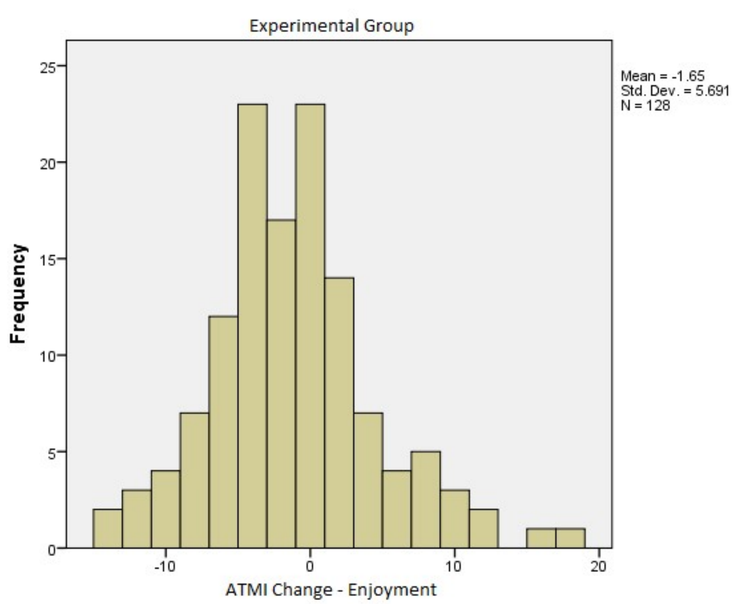

Figure C50. Histogram for experimental group ATMI change – enjoyment.

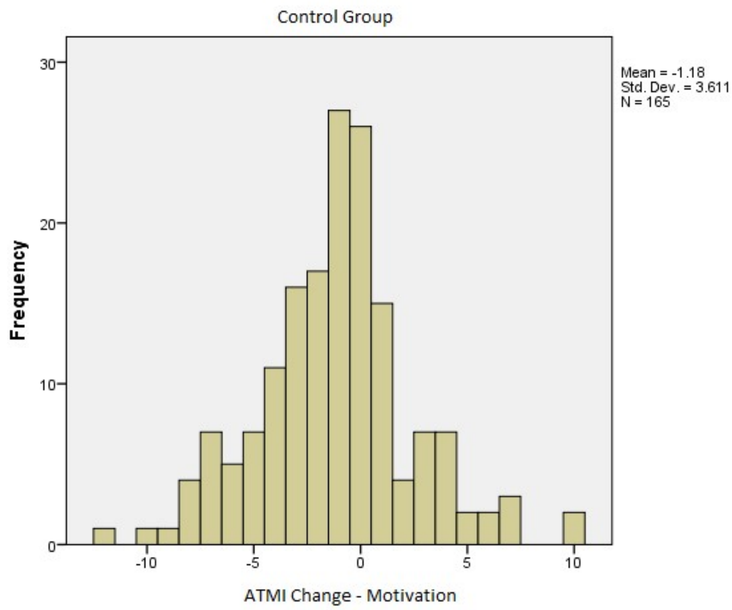

Figure C51. Histogram for control group ATMI change – motivation.

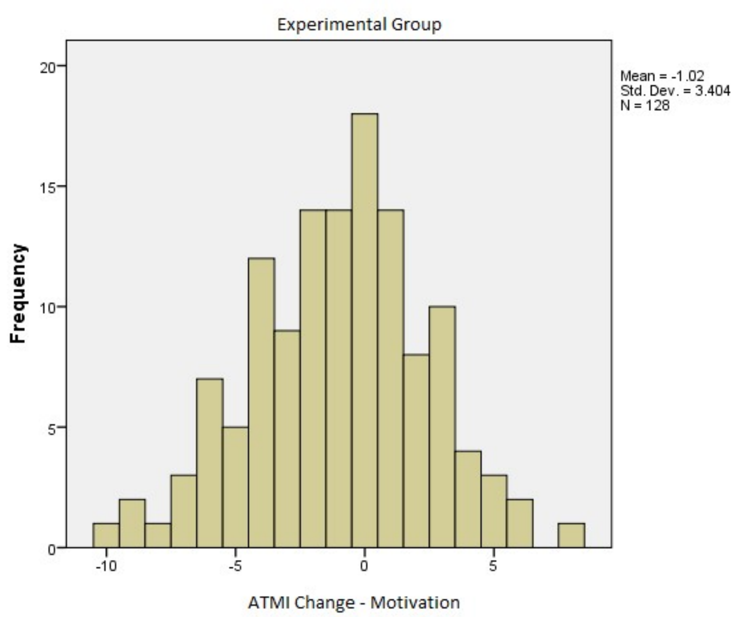

Figure C52. Histogram for experimental group ATMI change – motivation.

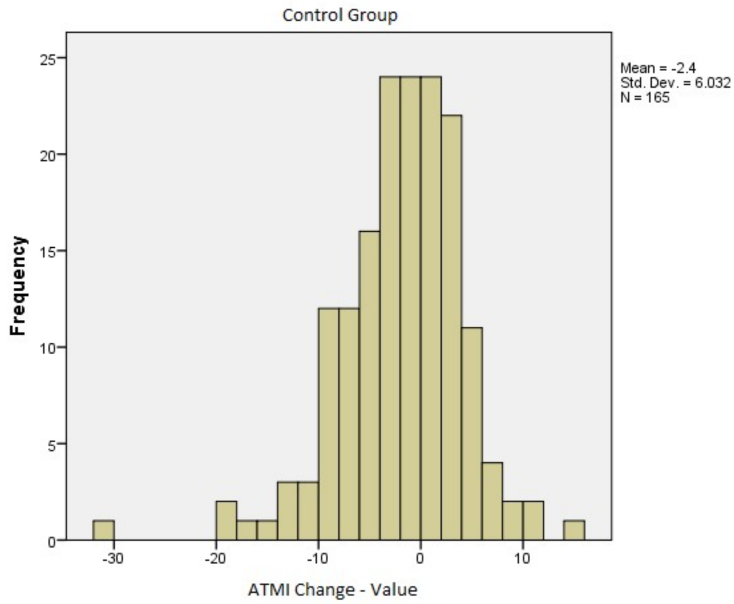

Figure C53. Histogram for control group ATMI change – value.

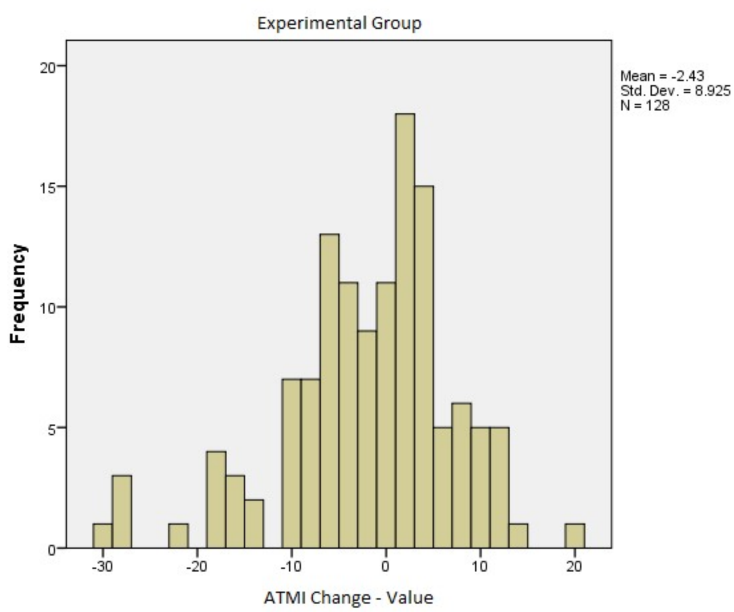

Figure C54. Histogram for experimental group ATMI change – value.

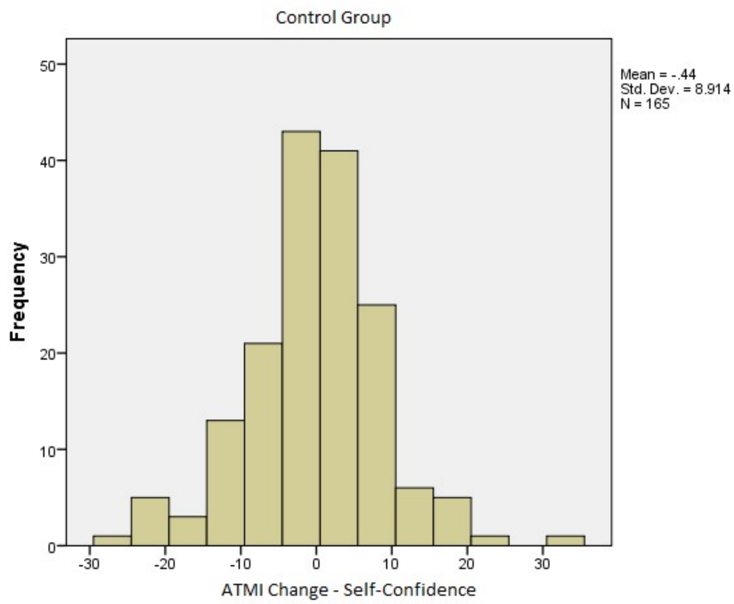

Figure C55. Histogram for control group ATMI change – self-confidence.

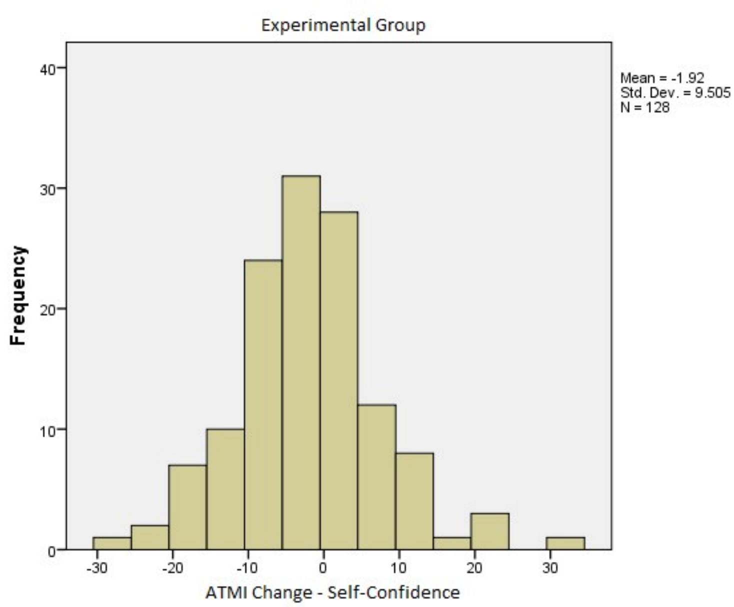

Figure C56. Histogram for experimental group ATMI change – self-confidence.

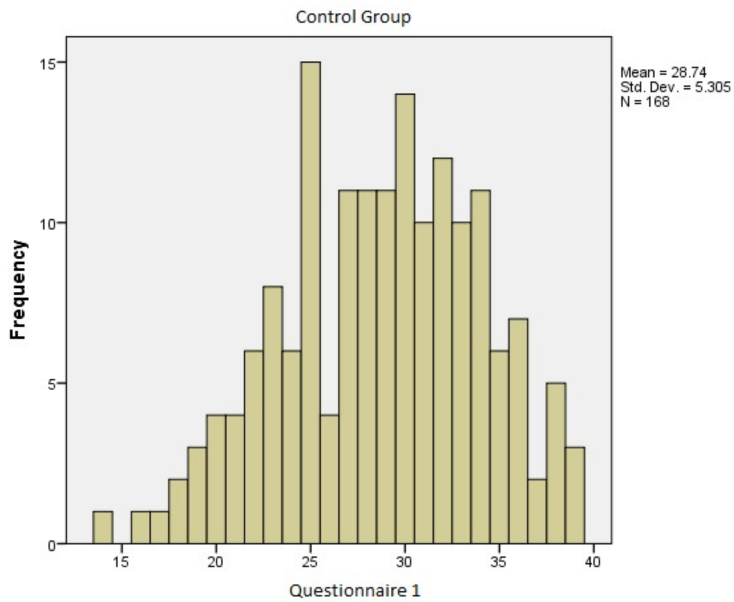

Figure C57. Histogram for control group questionnaire 1.

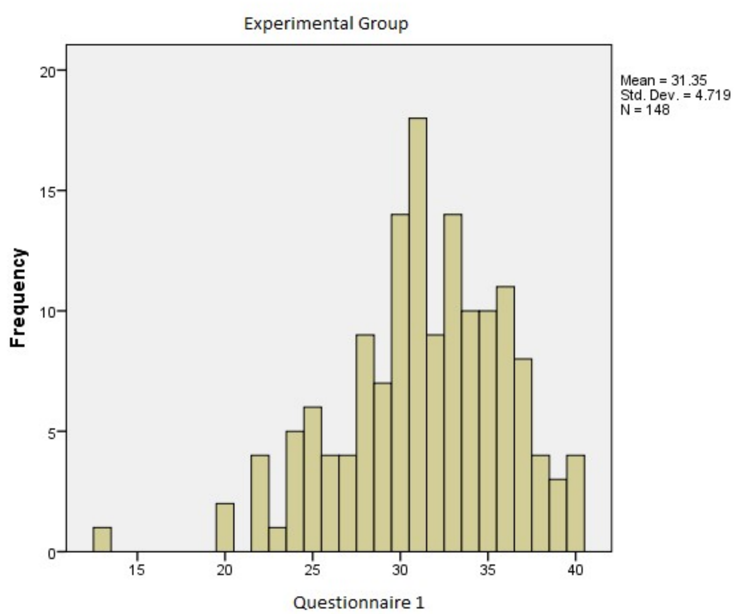

Figure C58. Histogram for experimental group questionnaire 1.

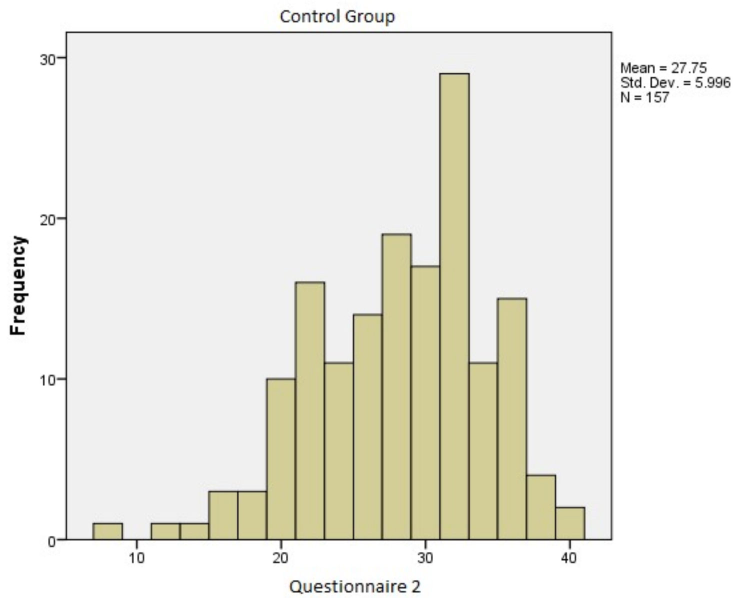

Figure C59. Histogram for control group questionnaire 2.

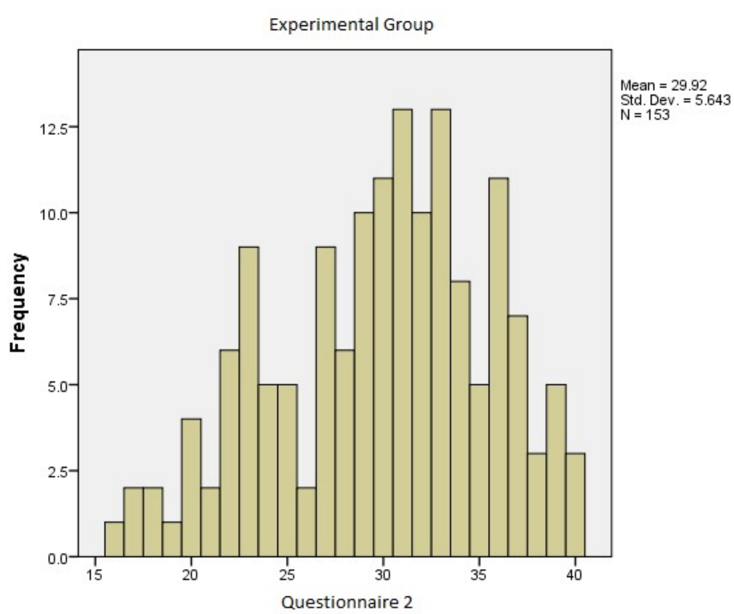

Figure C60. Histogram for experimental group questionnaire 2.

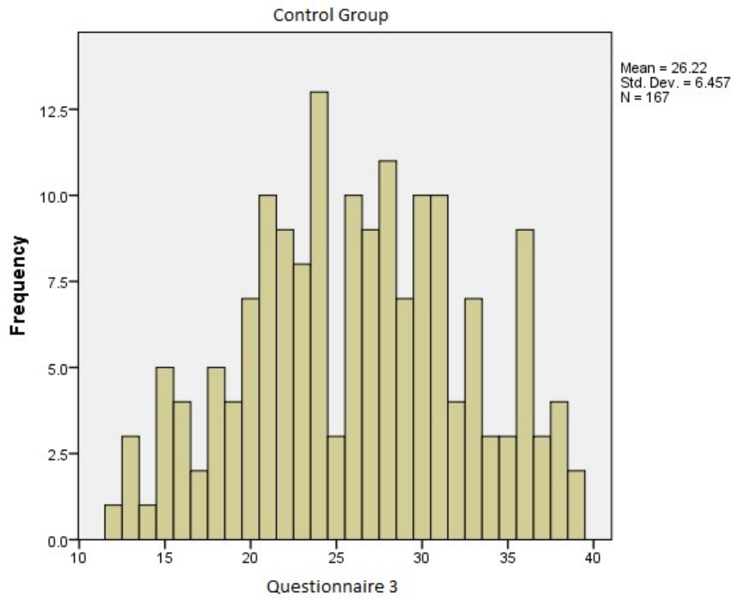

Figure C61. Histogram for control group questionnaire 3.

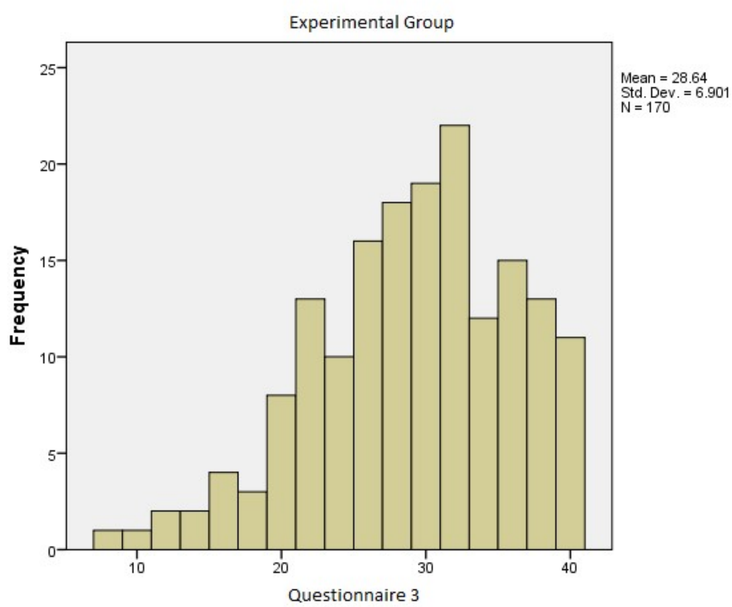

Figure C62. Histogram for experimental group questionnaire 3.

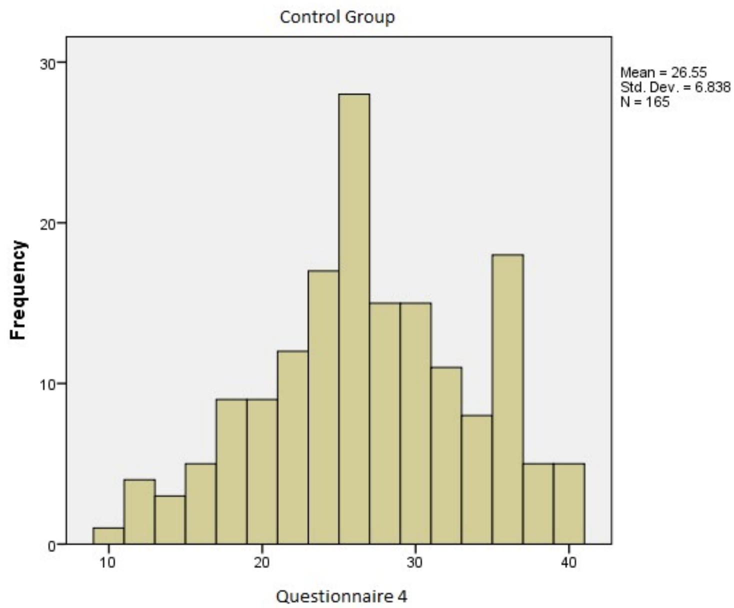

Figure C63. Histogram for control group questionnaire 4.

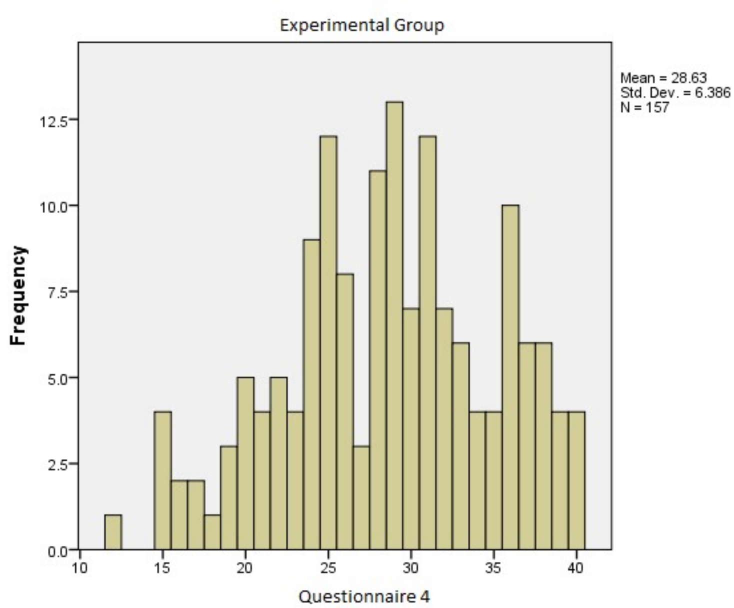

Figure C64. Histogram for experimental group questionnaire 4.

Appendix D – Q-Q Plots

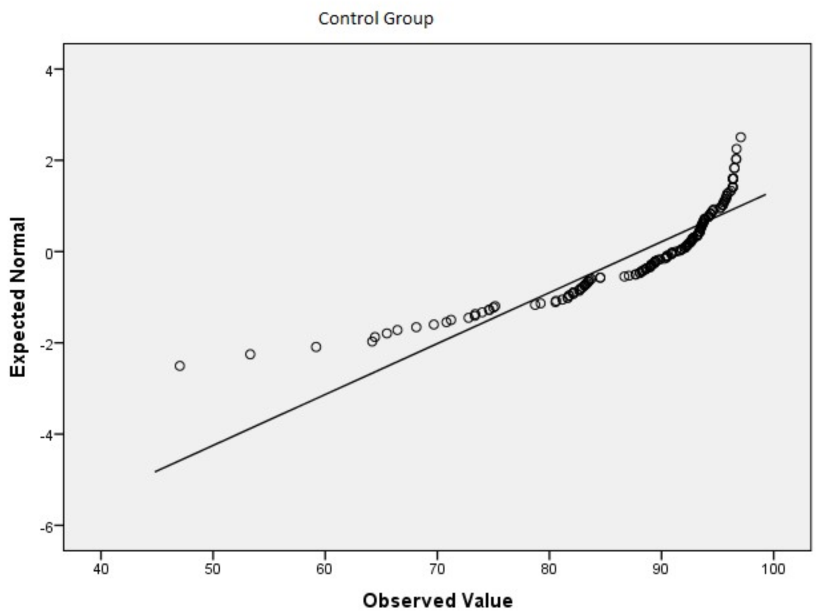

Figure D1. Q-Q plot for control group lab grade.

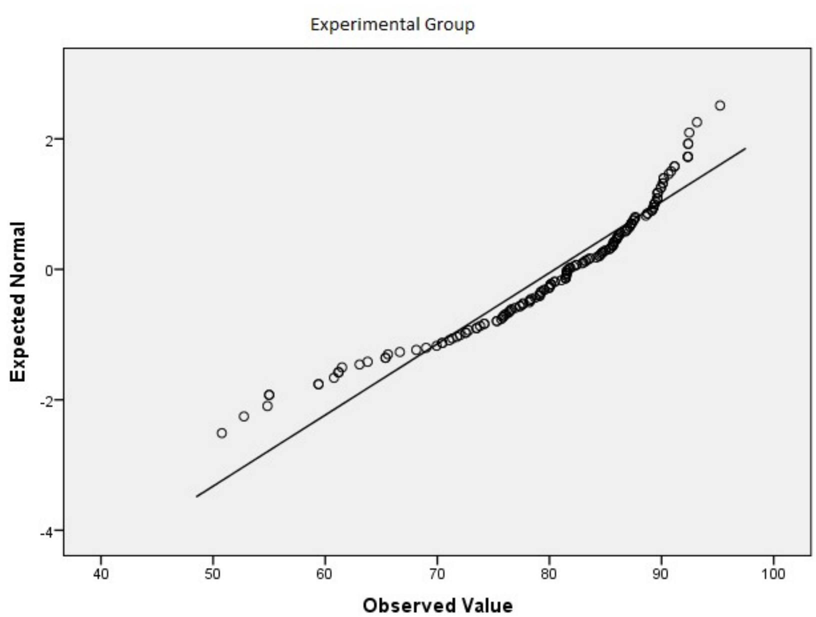

Figure D2. Q-Q plot for experimental group lab grade.

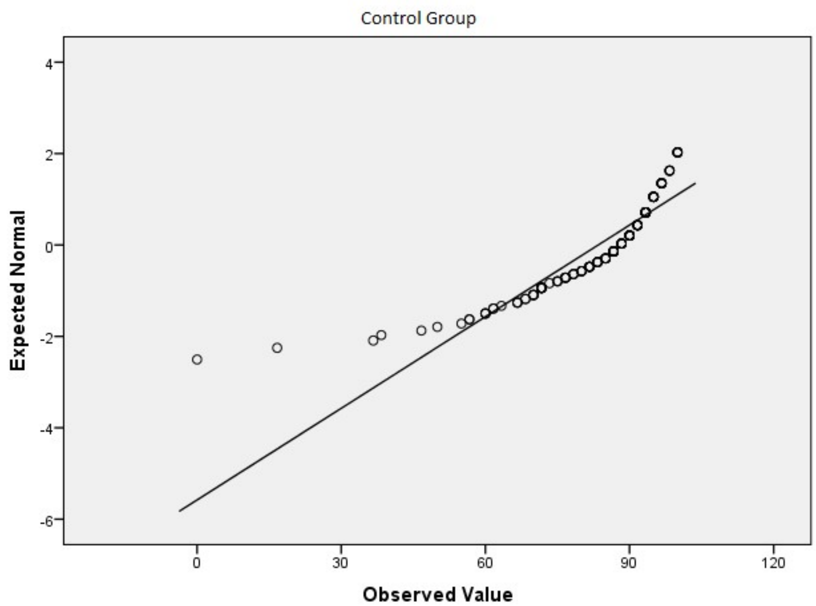

Figure D3. Q-Q plot for control group quiz grade.

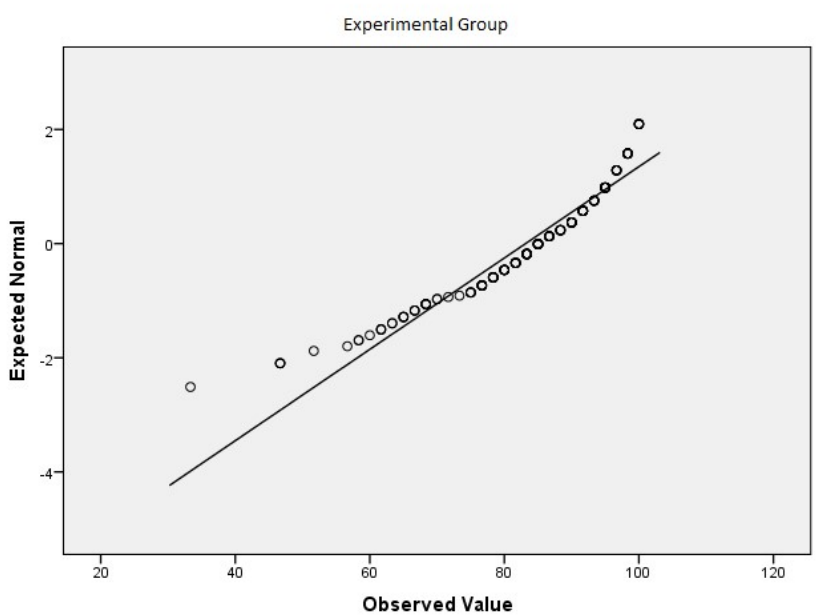

Figure D4. Q-Q plot for experimental group quiz grade.

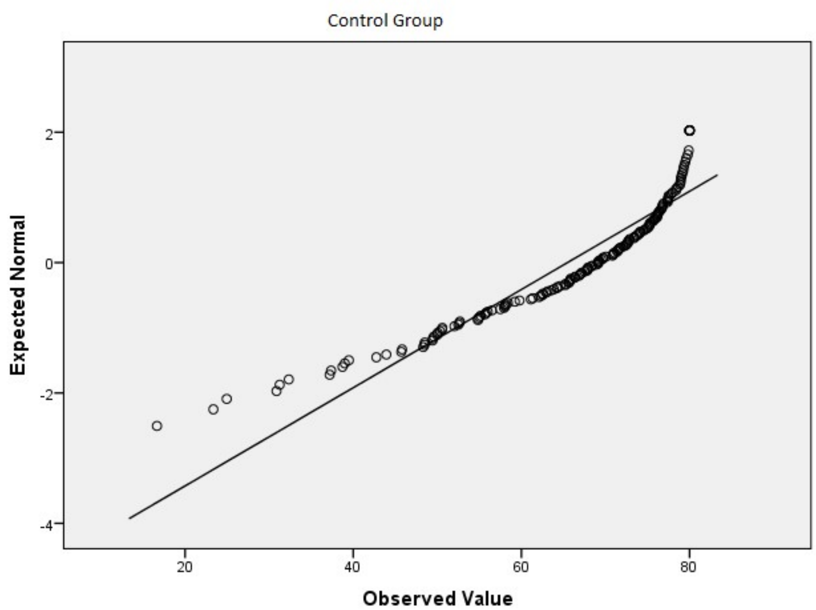

Figure D5. Q-Q plot for control group homework.

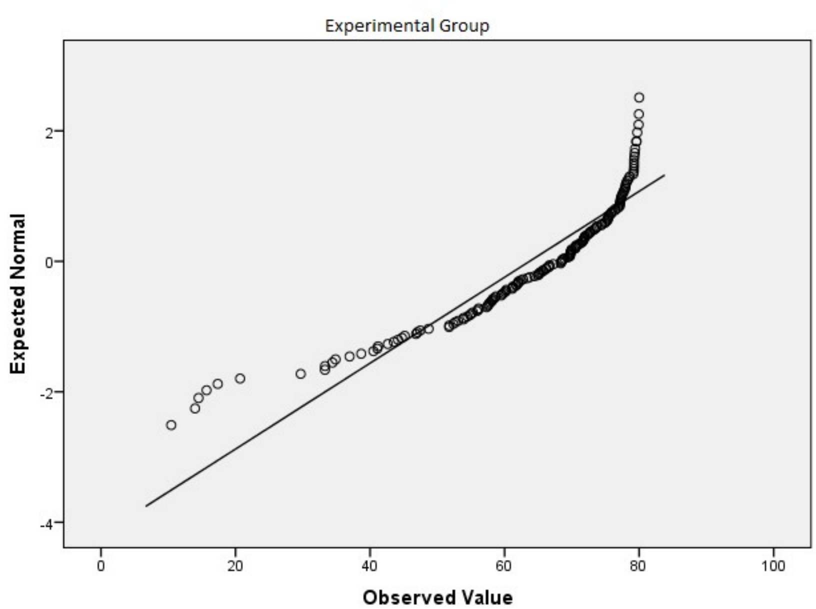

Figure D6. Q-Q plot for experimental group homework.

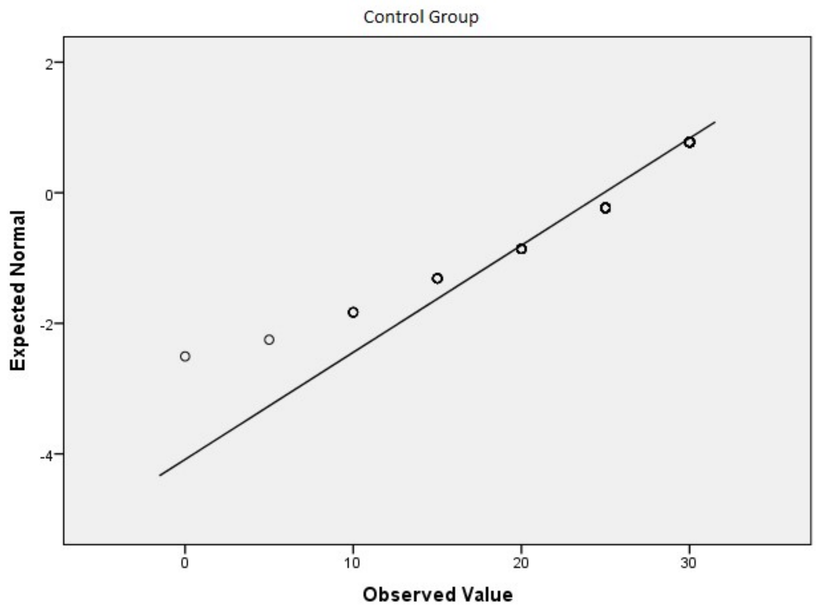

Figure D7. Q-Q plot for control group surveys/questionnaires.

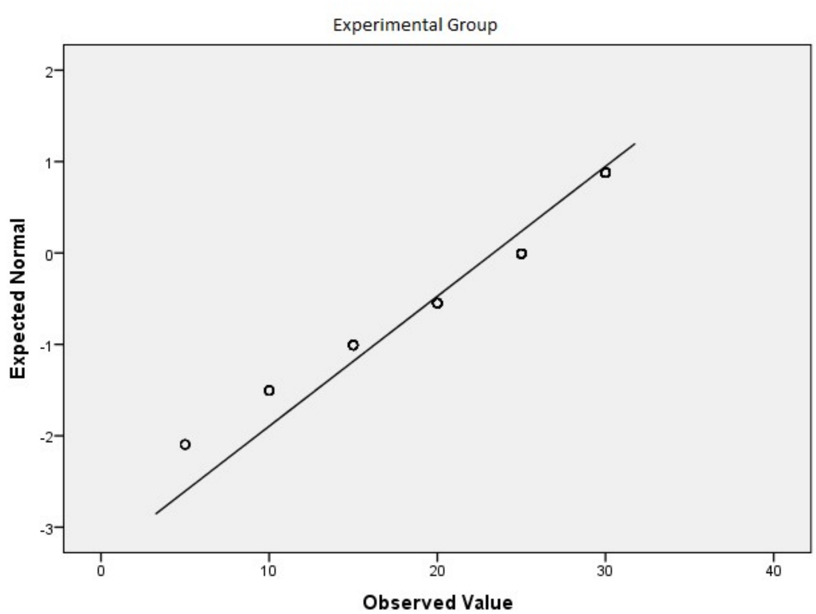

Figure D8. Q-Q plot for experimental group surveys/questionnaires.

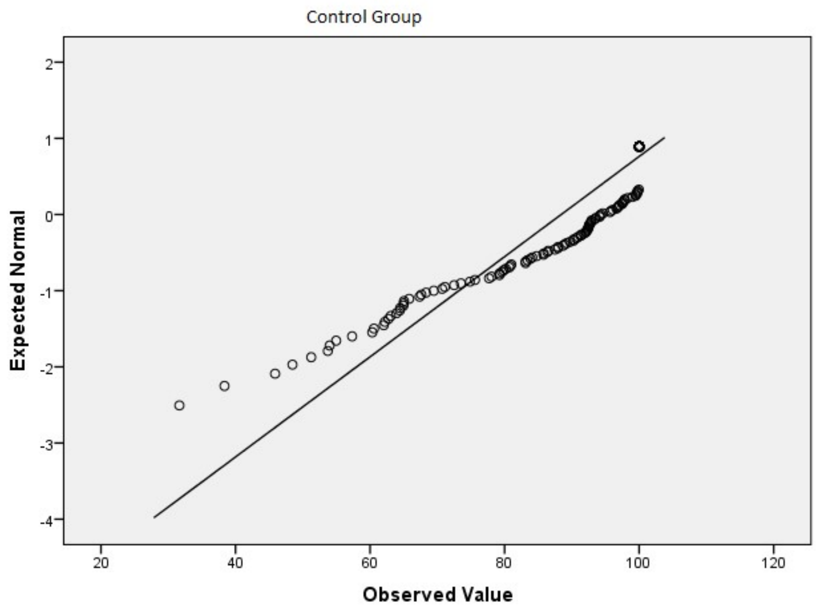

Figure D9. Q-Q plot for control group participation points.

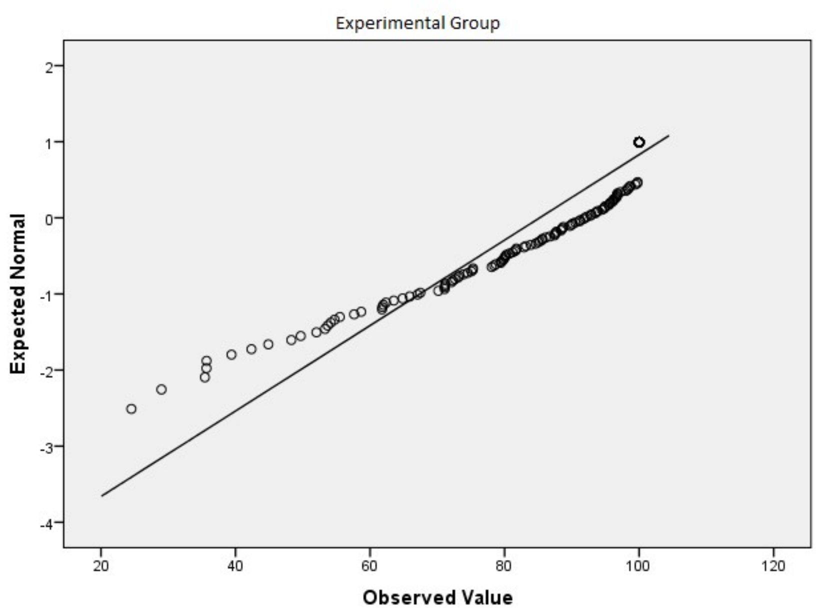

Figure D10. Q-Q plot for experimental group participation points.

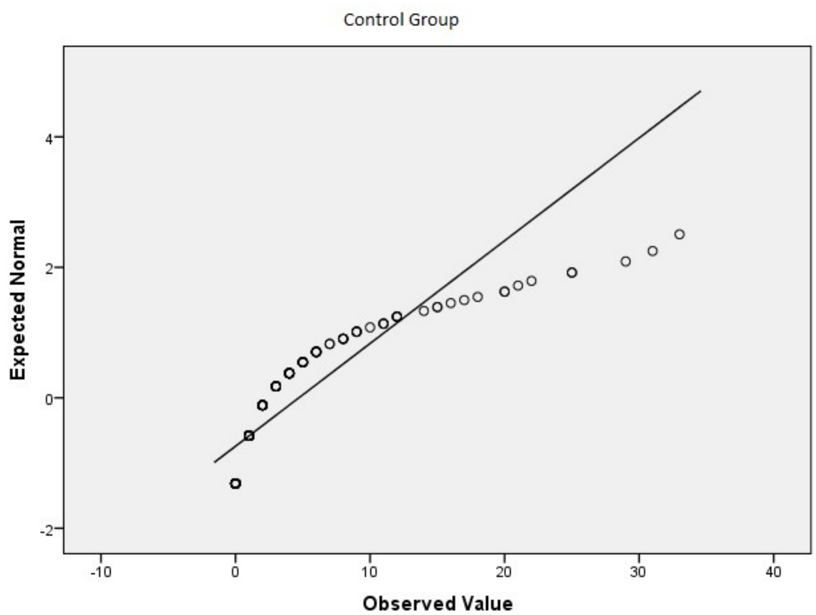

Figure D11. Q-Q plot for control group absences.

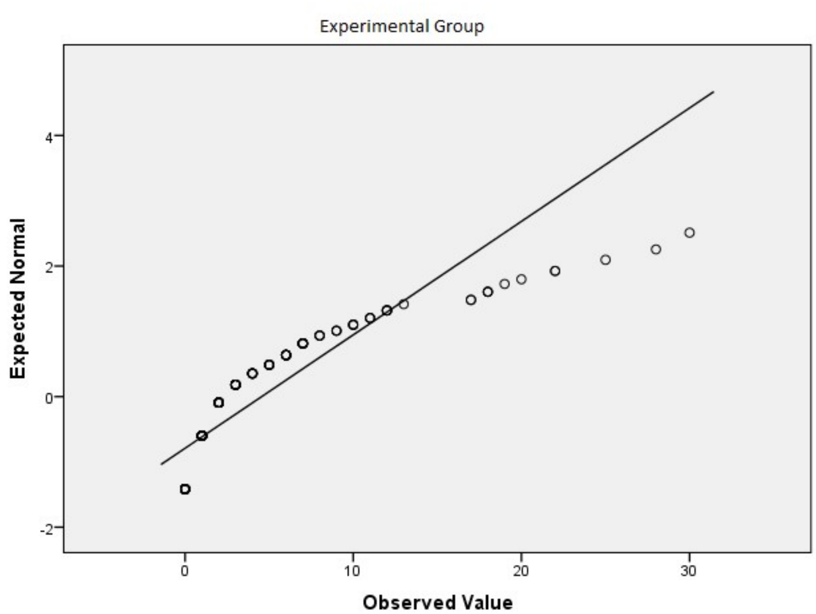

Figure D12. Q-Q plot for experimental group absences.

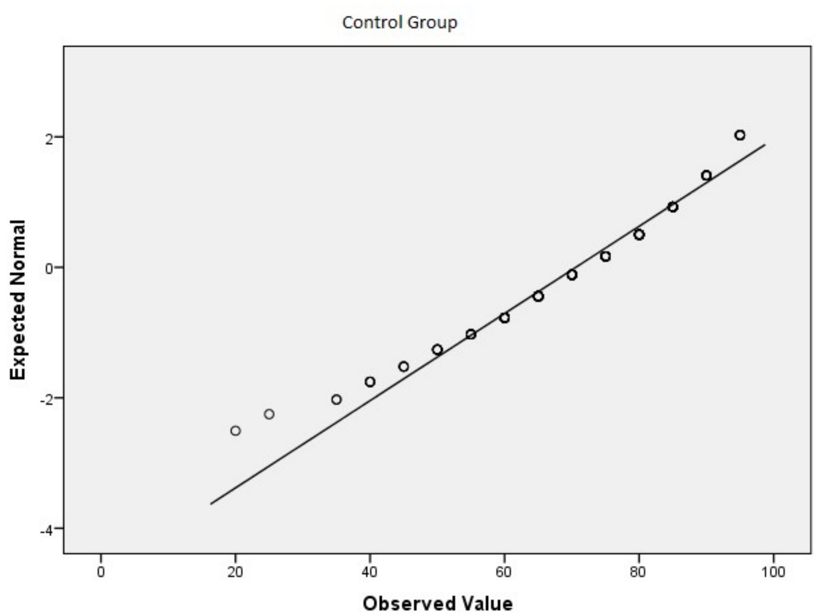

Figure D13. Q-Q plot for control group test 1.

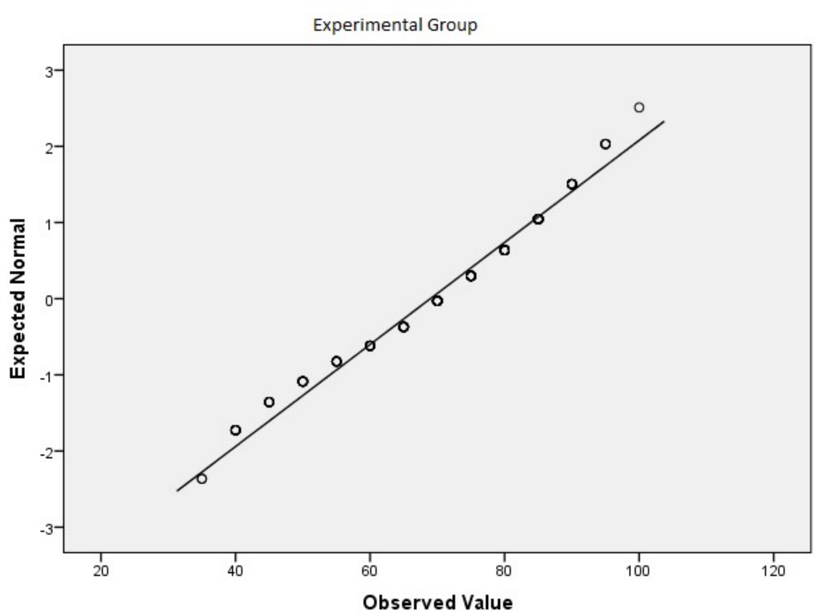

Figure D14. Q-Q plot for experimental group test 1.

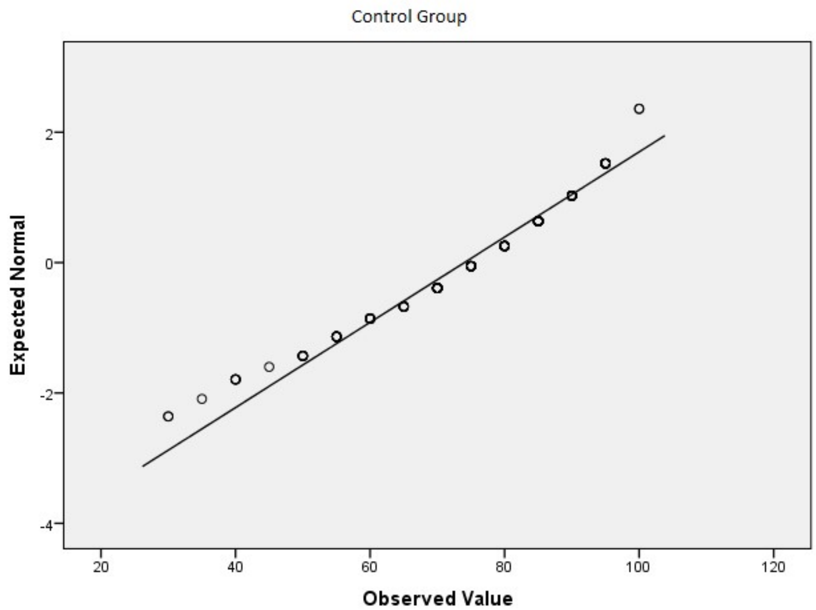

Figure D15. Q-Q plot for control group test 2.

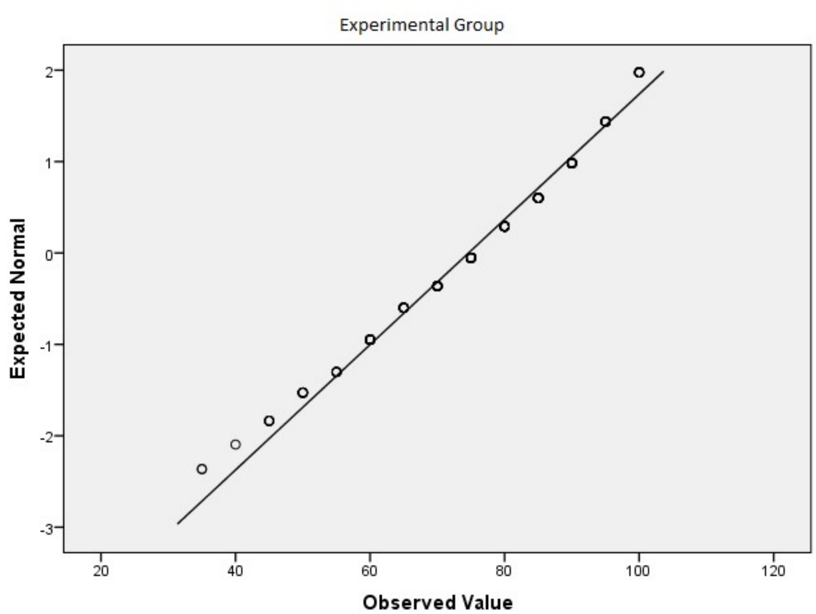

Figure D16. Q-Q plot for experimental group test 2.

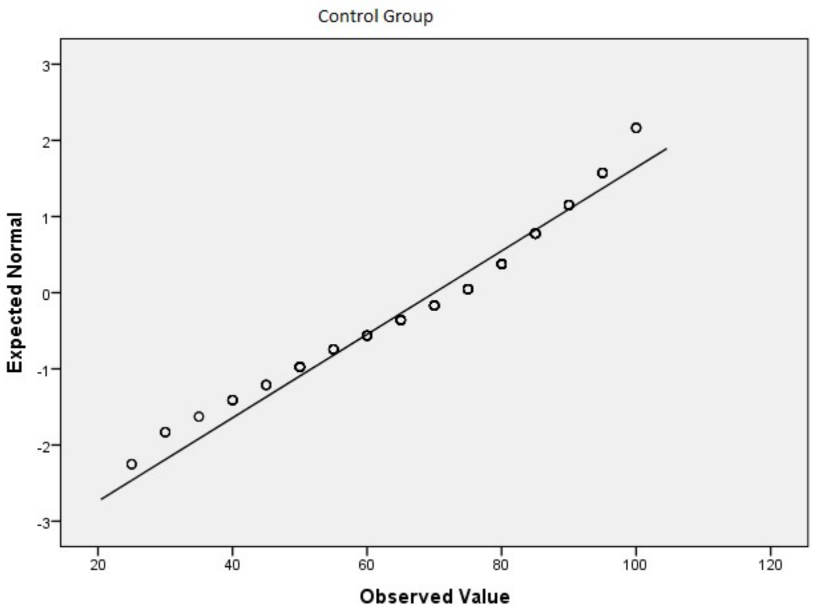

Figure D17. Q-Q plot for control group test 3.

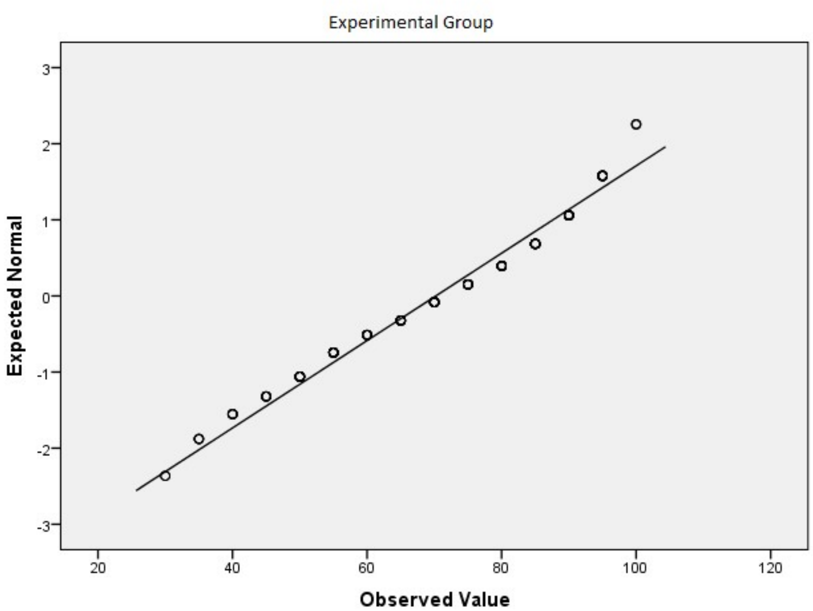

Figure D18. Q-Q plot for experimental group test 3.

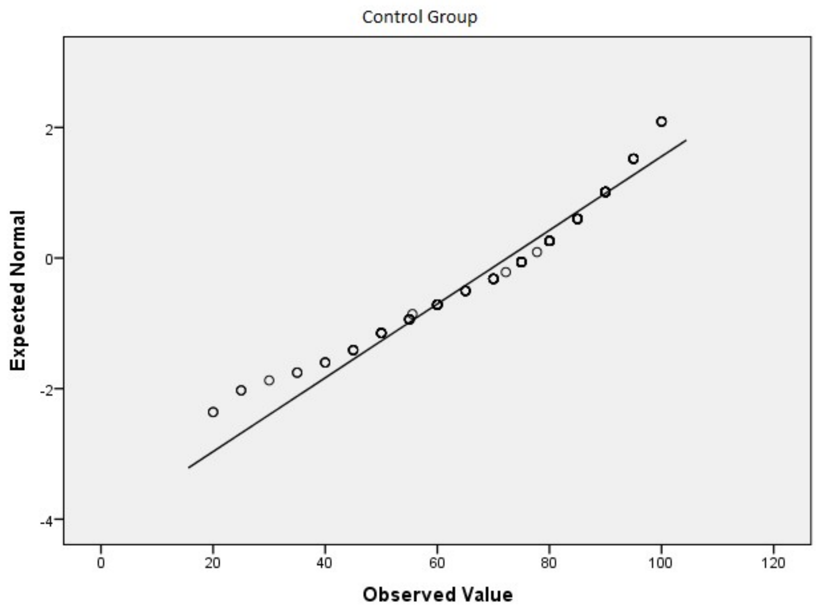

Figure D19. Q-Q plot for control group test 4.

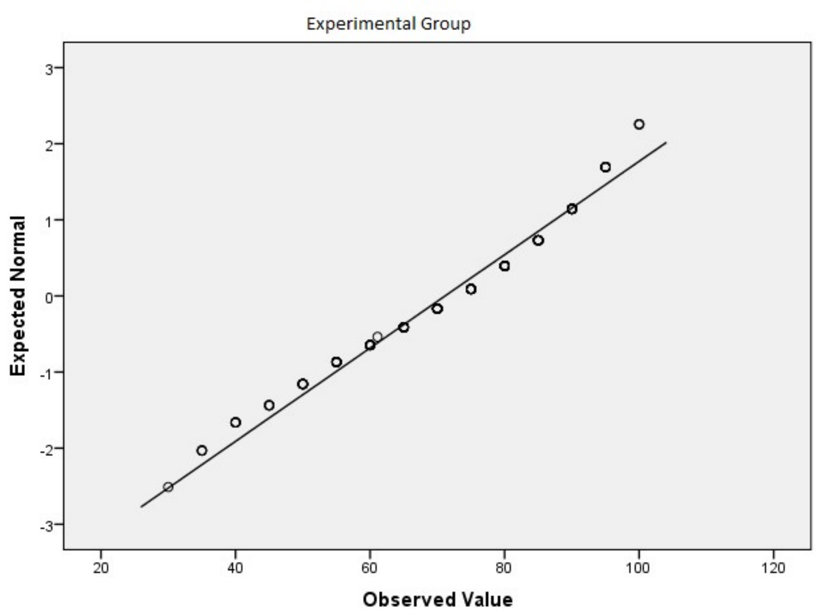

Figure D20. Q-Q plot for experimental group test 4.

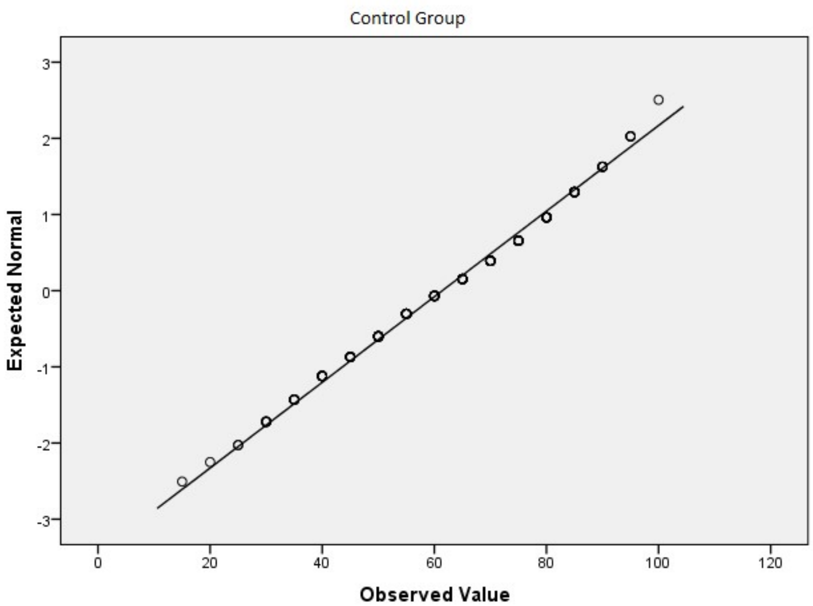

Figure D21. Q-Q plot for control group final exam.

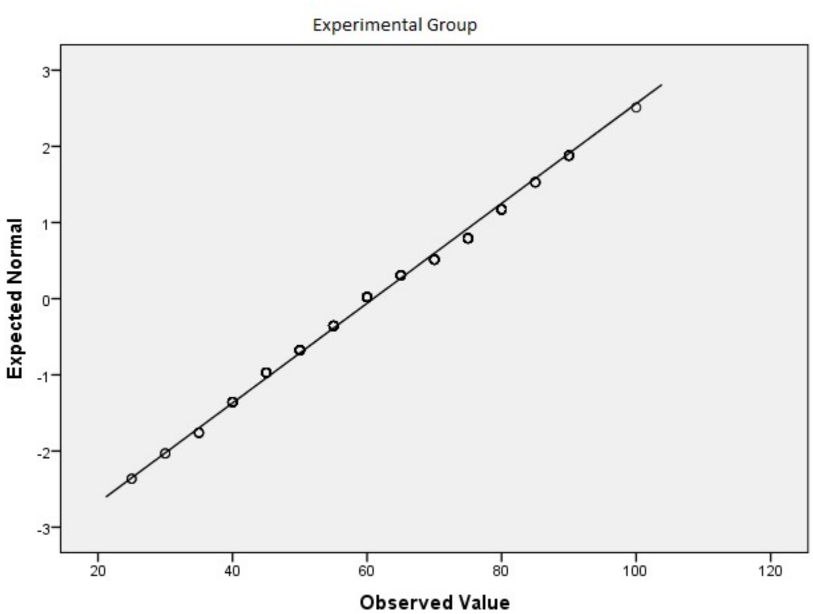

Figure D22. Q-Q plot for experimental group final exam.

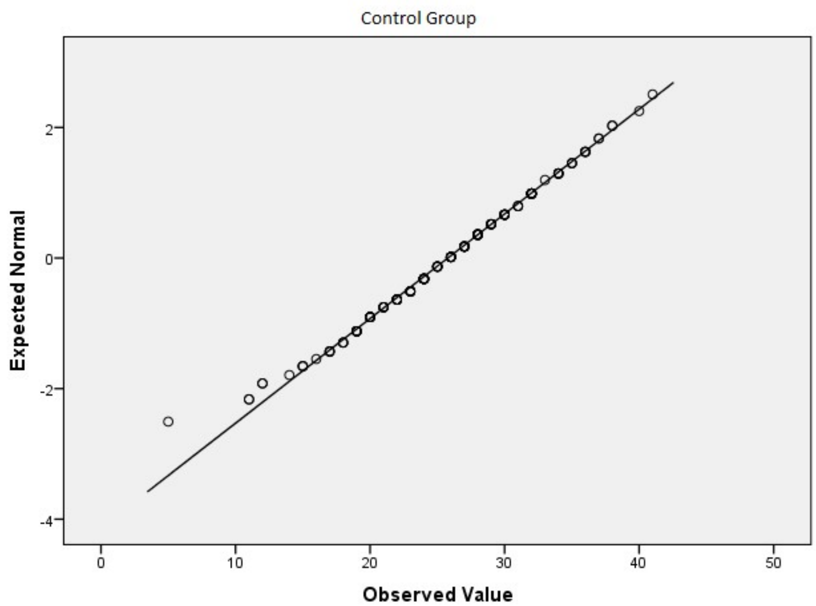

Figure D23. Q-Q plot for control group pre-ACT.

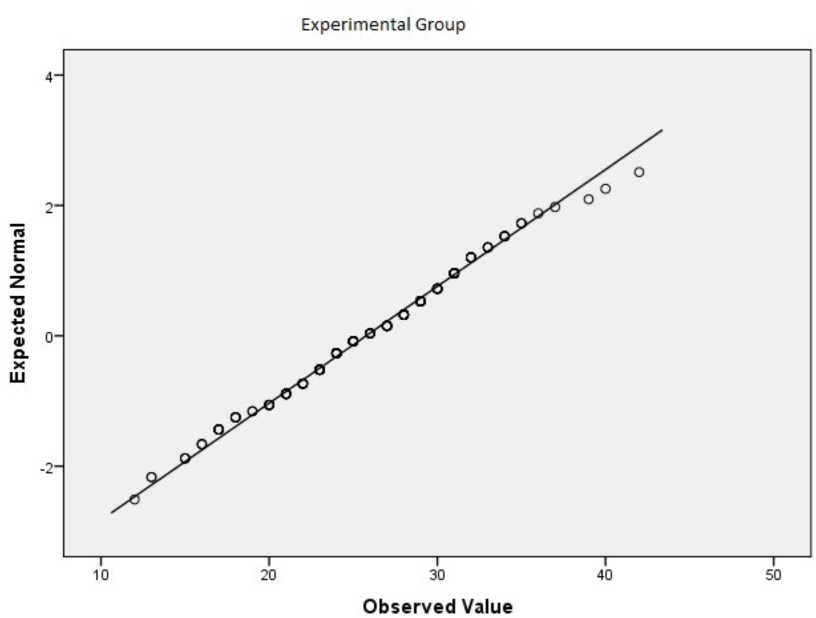

Figure D24. Q-Q plot for experimental group pre-ACT.

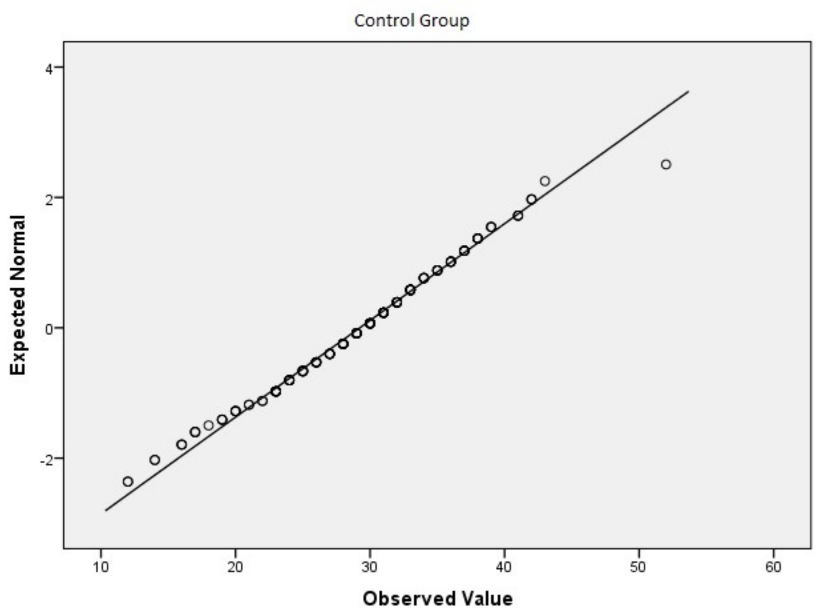

Figure D25. Q-Q plot for control group post-ACT.

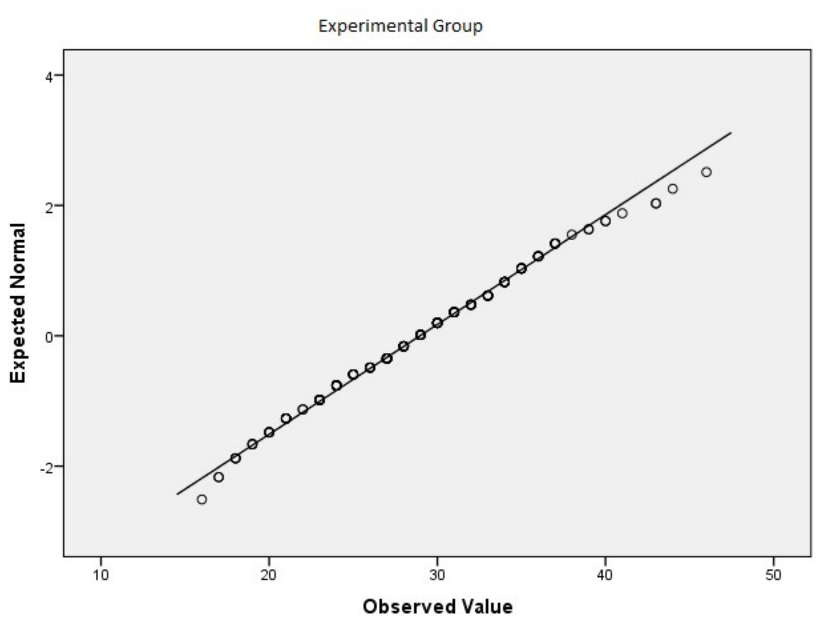

Figure D26. Q-Q plot for experimental group post-ACT.

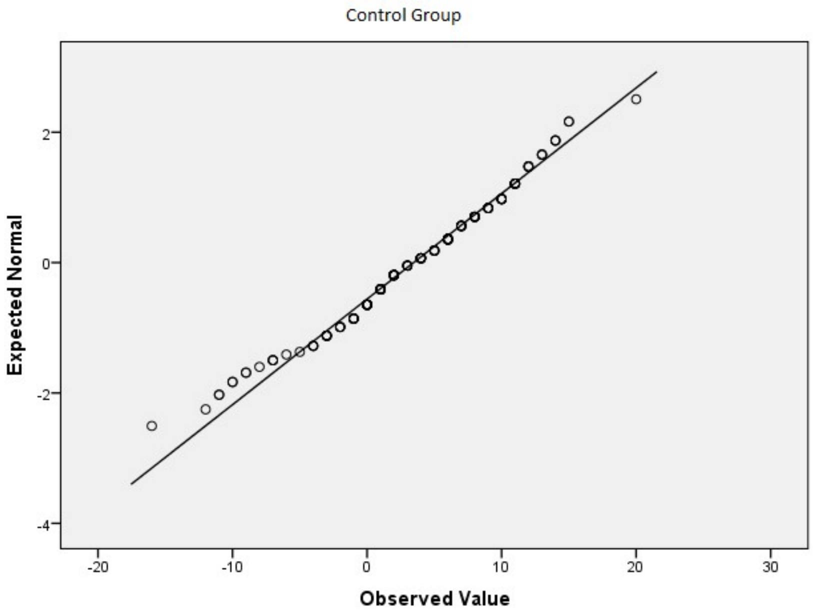

Figure D27. Q-Q plot for control group ACT change.

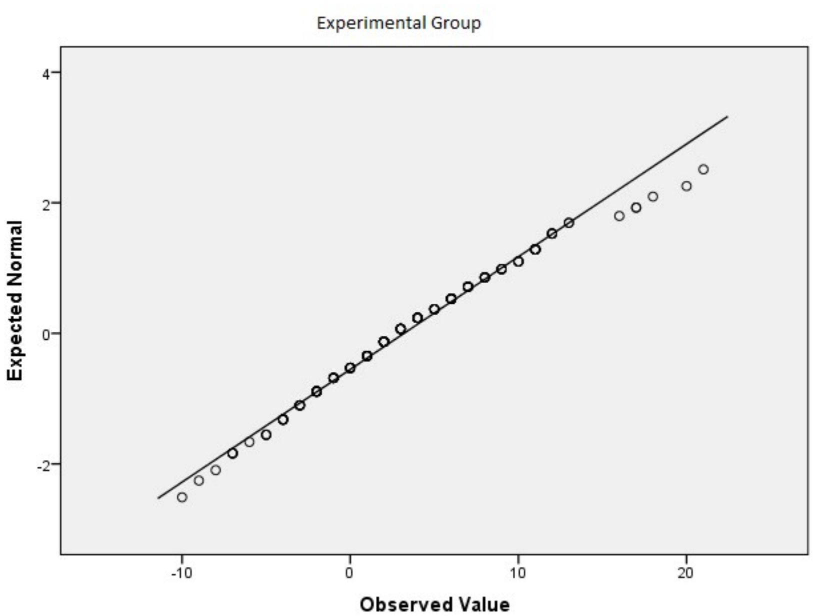

Figure D28. Q-Q plot for experimental group ACT change.

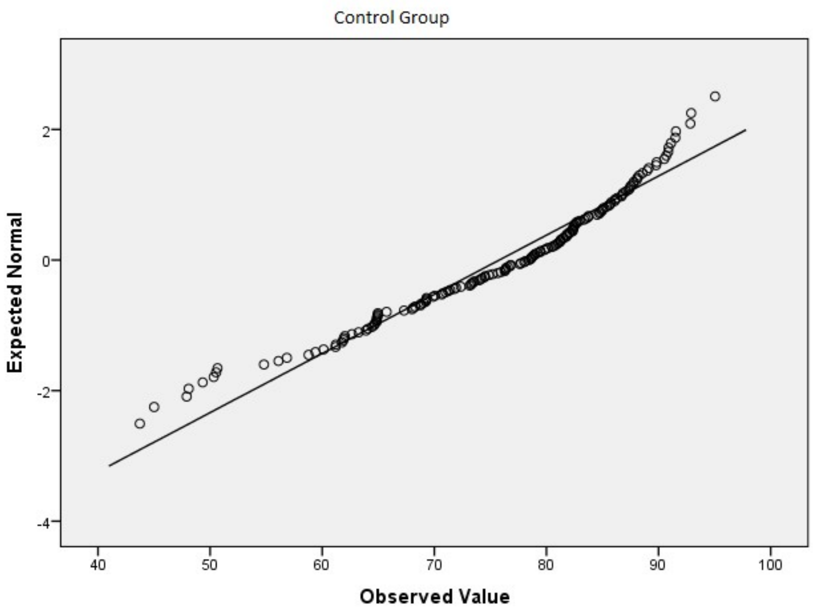

Figure D29. Q-Q plot for control group final grade without extra credit.

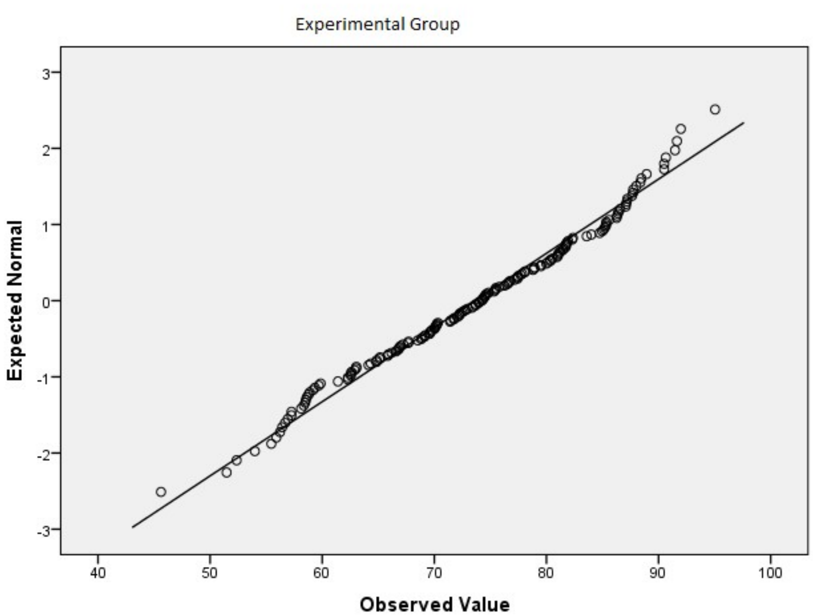

Figure D30. Q-Q plot for experimental group final grade without extra credit.

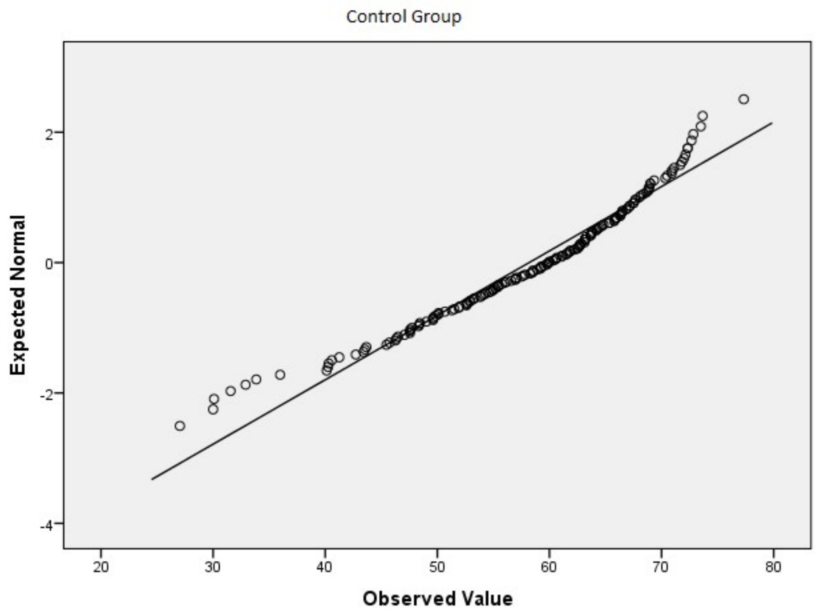

Figure D31. Q-Q plot for control group final grade without extra credit and labs.

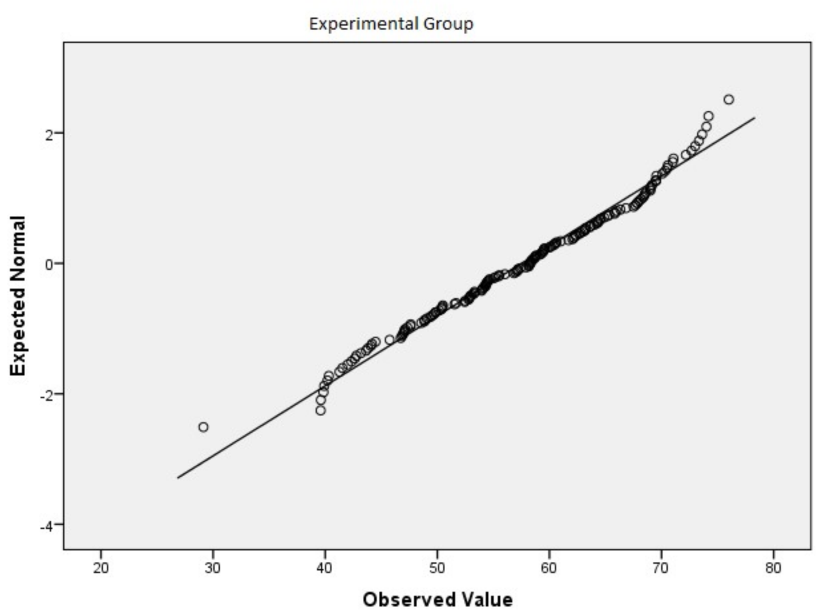

Figure D32. Q-Q plot for experimental group final grade without extra credit and labs.

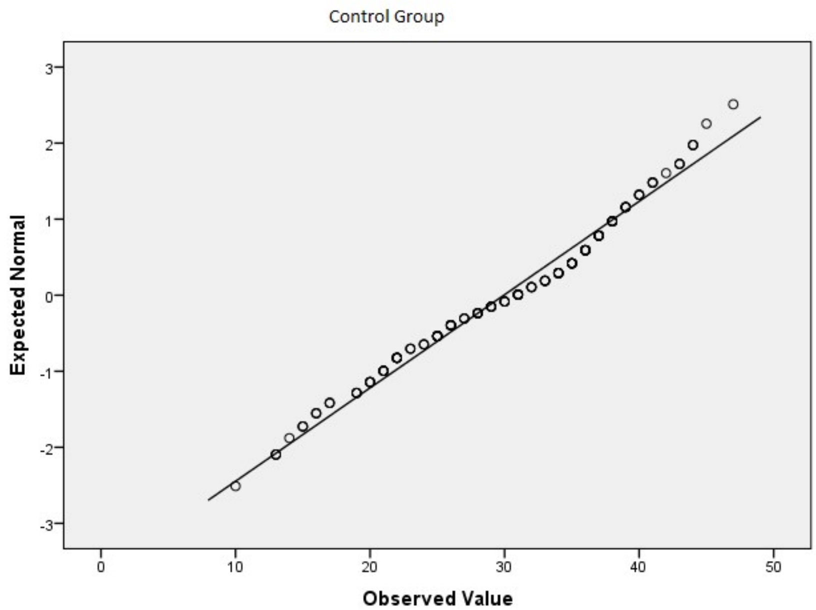

Figure D33. Q-Q plot for control group pre-ATMI enjoyment.

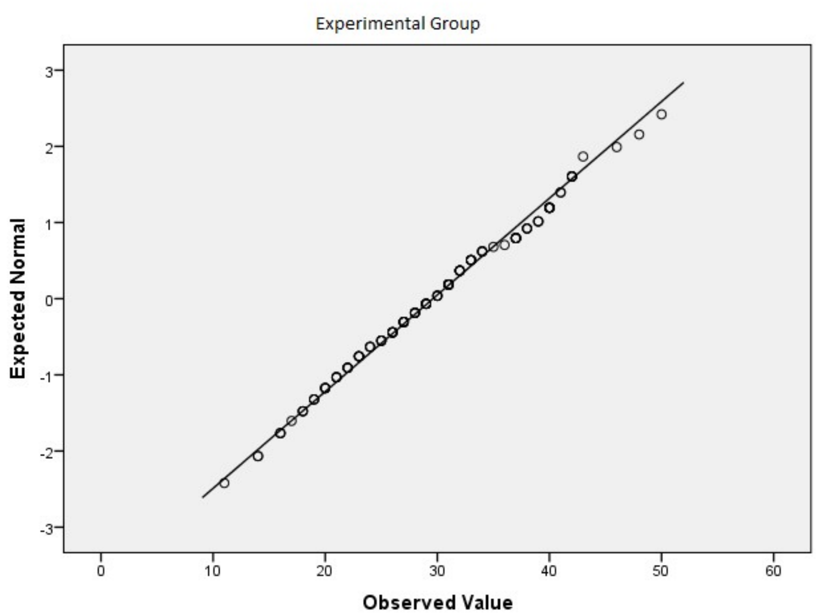

Figure D34. Q-Q plot for experimental group pre-ATMI enjoyment

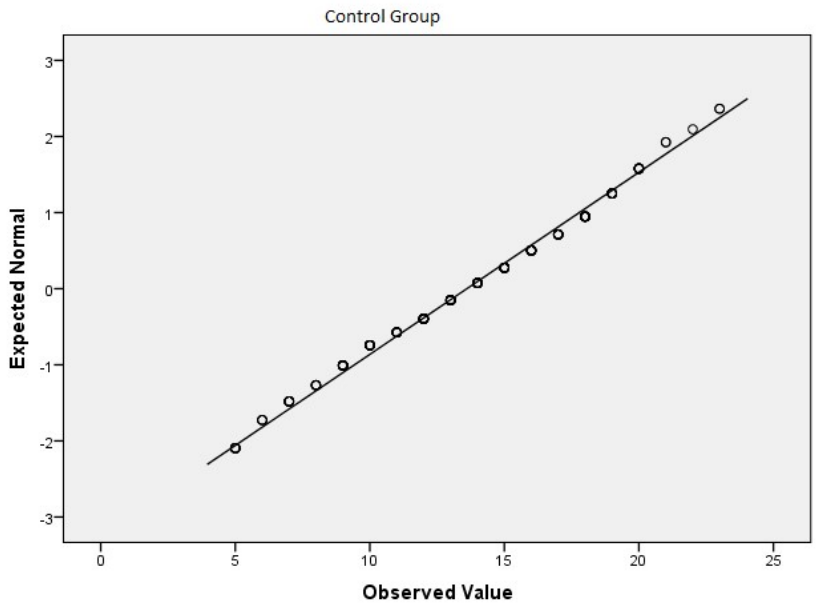

Figure D35. Q-Q plot for control group pre-ATMI motivation.

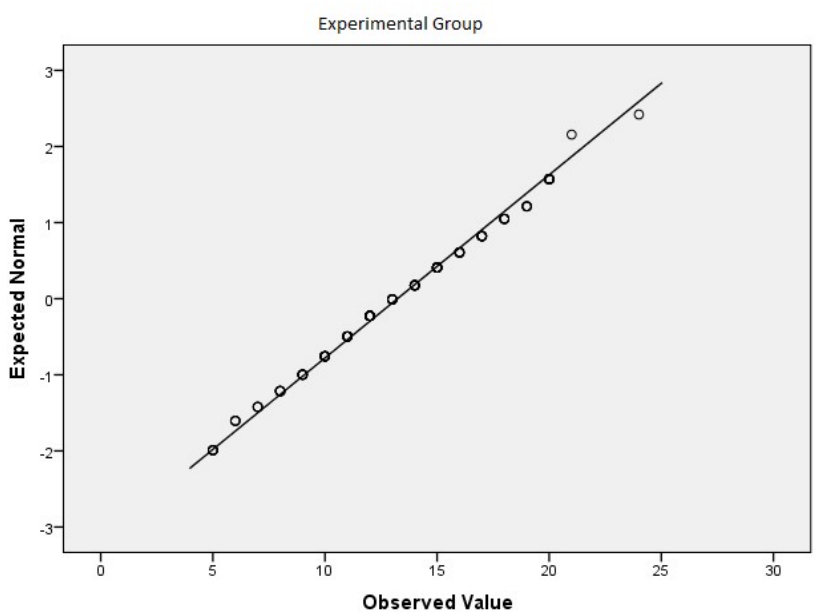

Figure D36. Q-Q plot for experimental group pre-ATMI motivation.

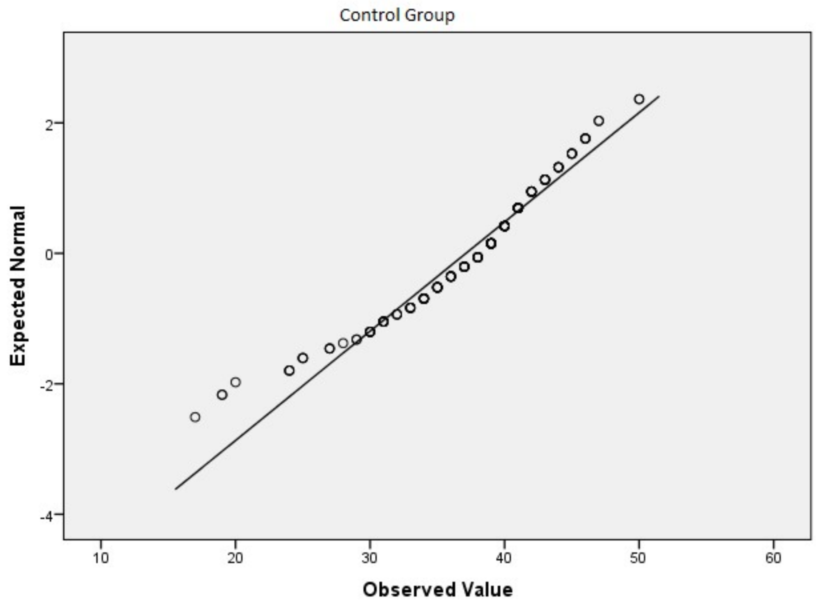

Figure D37. Q-Q plot for control group pre-ATMI value.

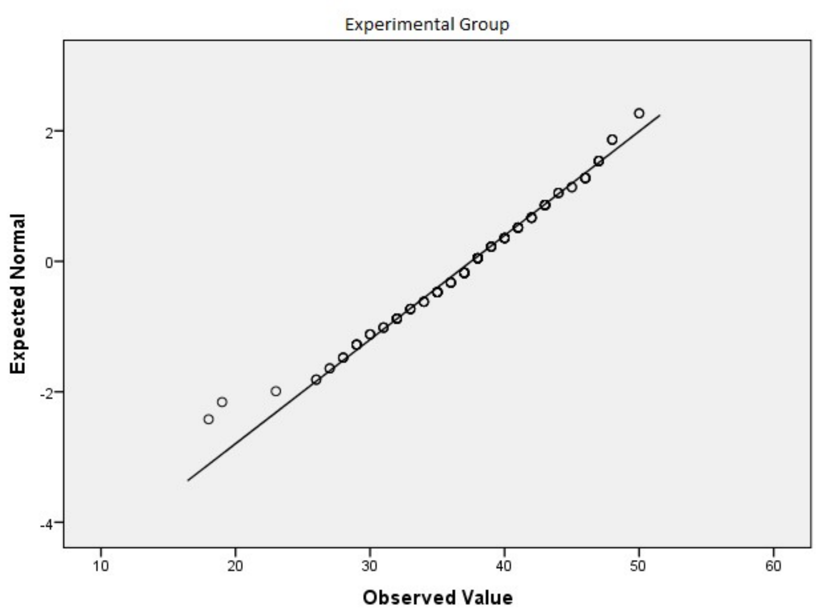

Figure D38. Q-Q plot for experimental group pre-ATMI value.

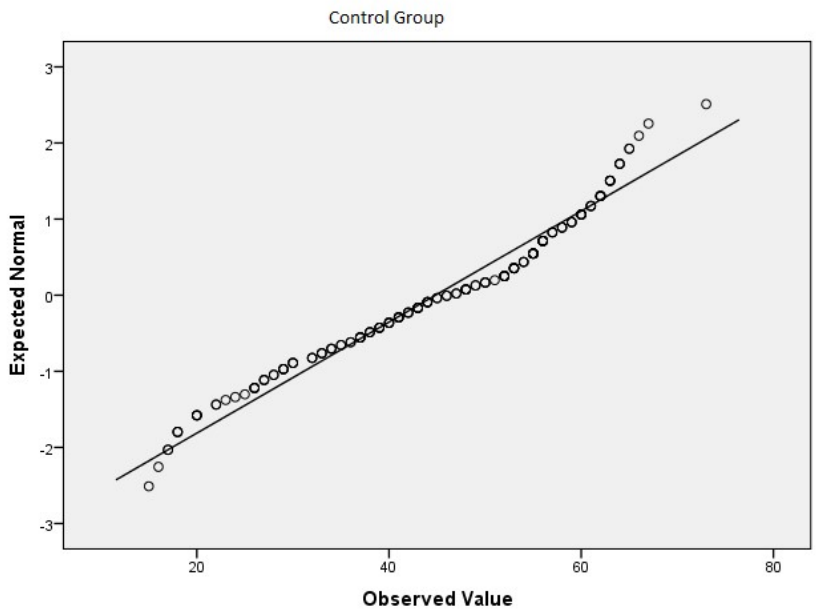

Figure D39. Q-Q plot for control group pre-ATMI self-confidence.

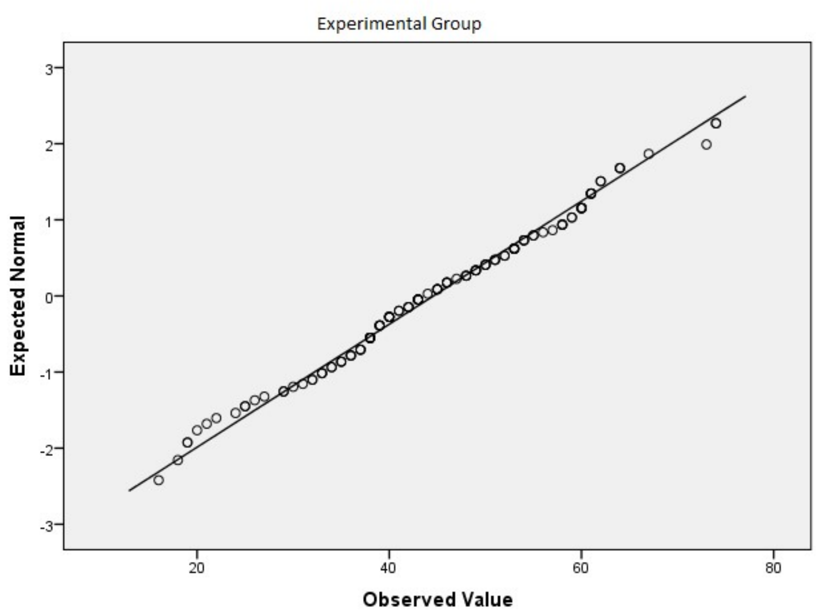

Figure D40. Q-Q plot for experimental group pre-ATMI self-confidence.

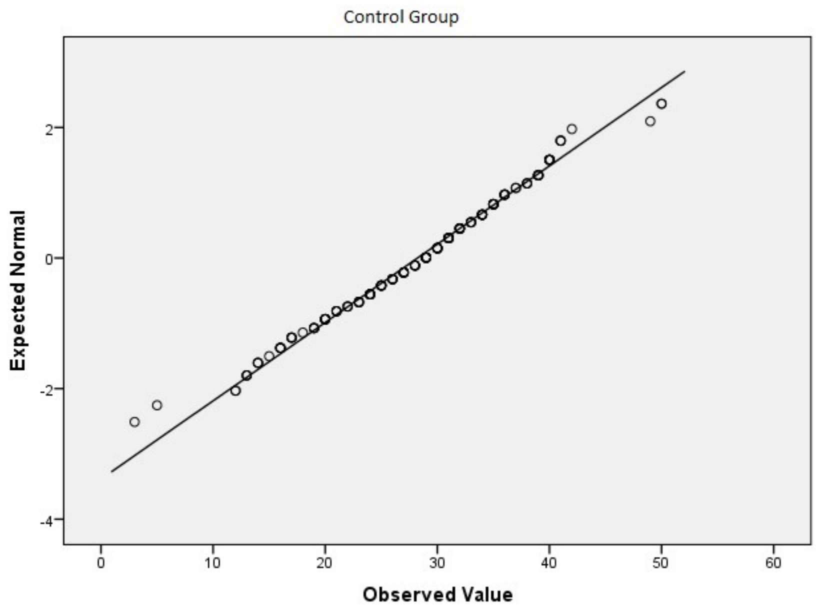

Figure D41. Q-Q plot for control group post-ATMI enjoyment.

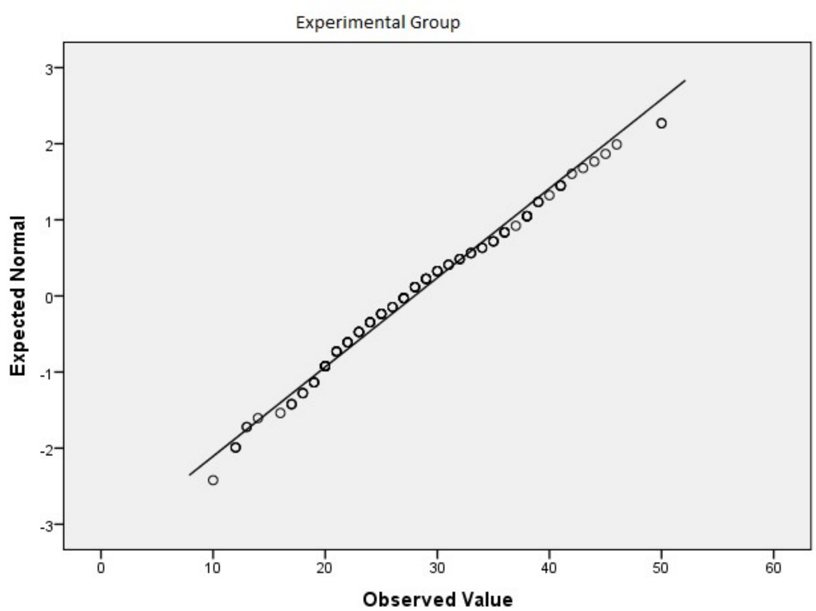

Figure D42. Q-Q plot for experimental group post-ATMI enjoyment.

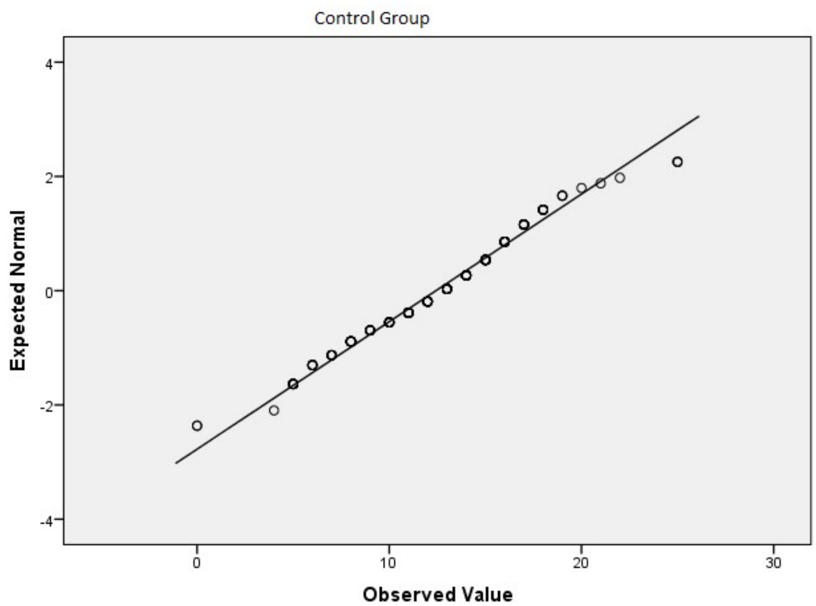

Figure D43. Q-Q plot for control group post-ATMI motivation.

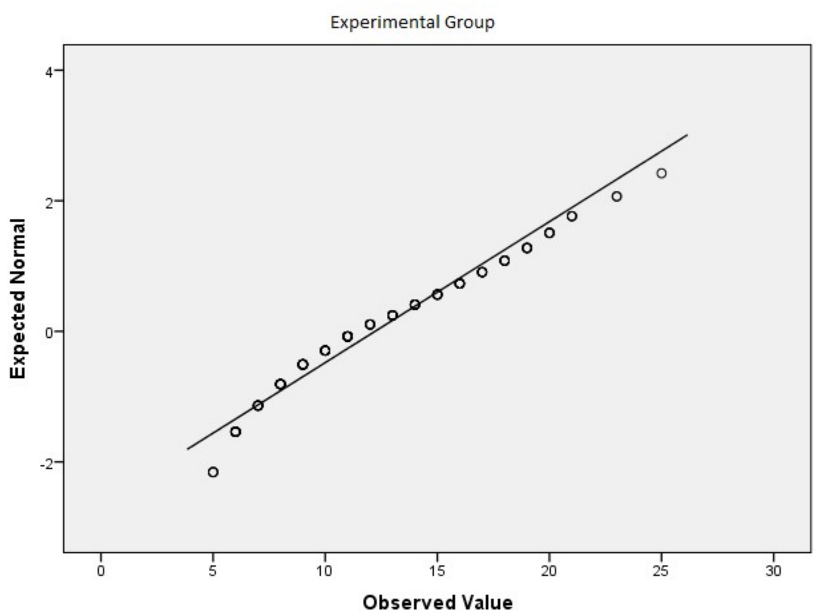

Figure D44. Q-Q plot for experimental group post-ATMI motivation.

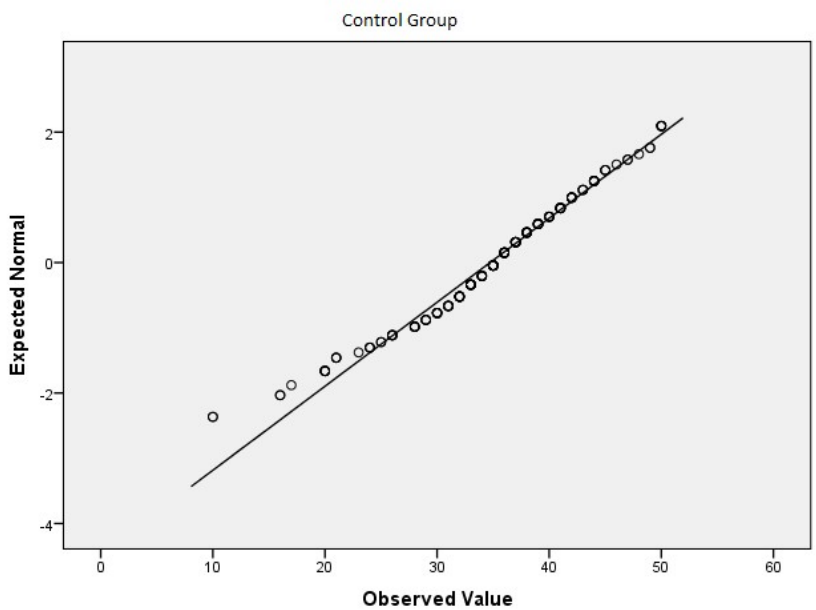

Figure D45. Q-Q plot for control group post-ATMI value.

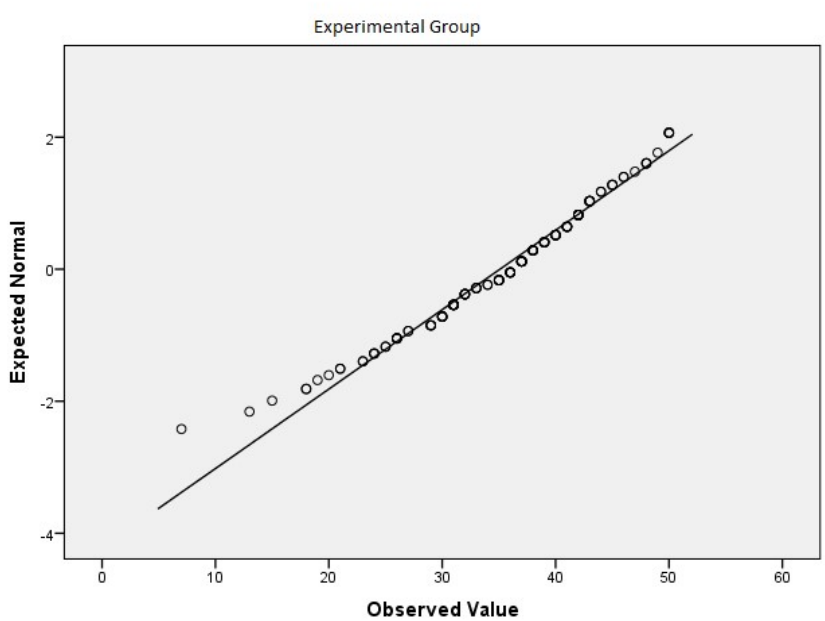

Figure D46. Q-Q plot for experimental group post-ATMI value.

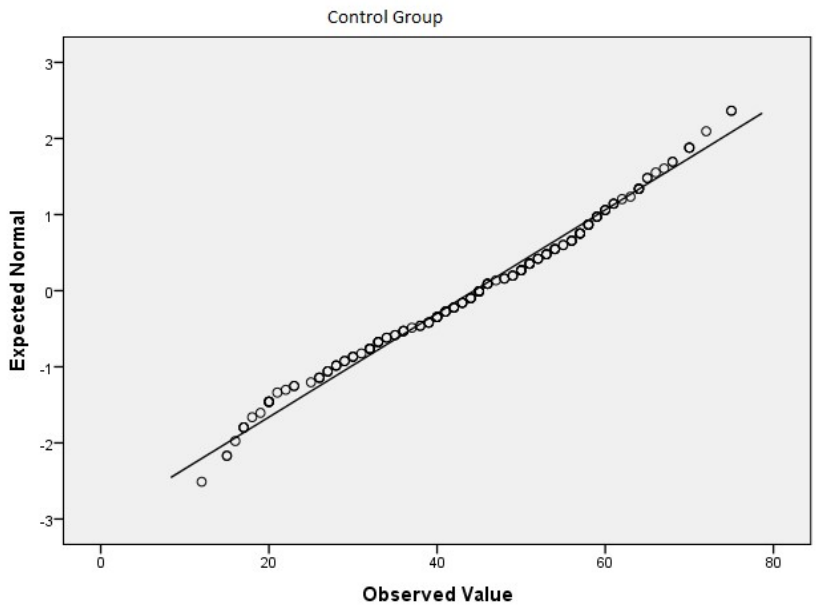

Figure D47. Q-Q plot for control group post-ATMI self-confidence.

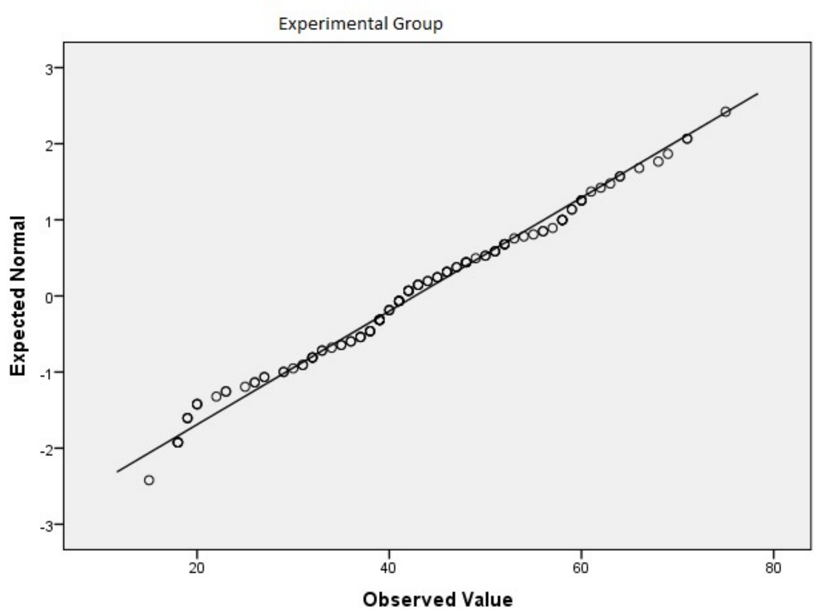

Figure D48. Q-Q plot for experimental group post-ATMI self-confidence.

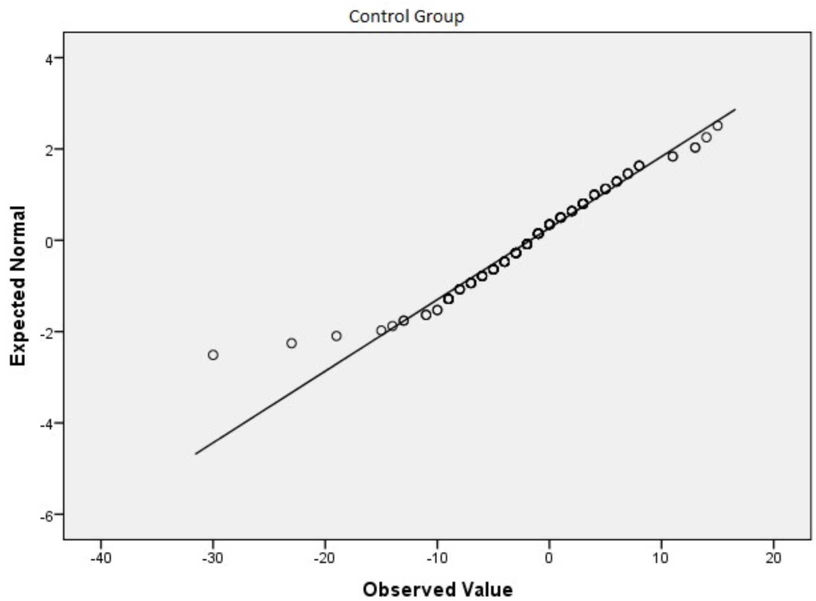

Figure D49. Q-Q plot for control group ATMI change – enjoyment.

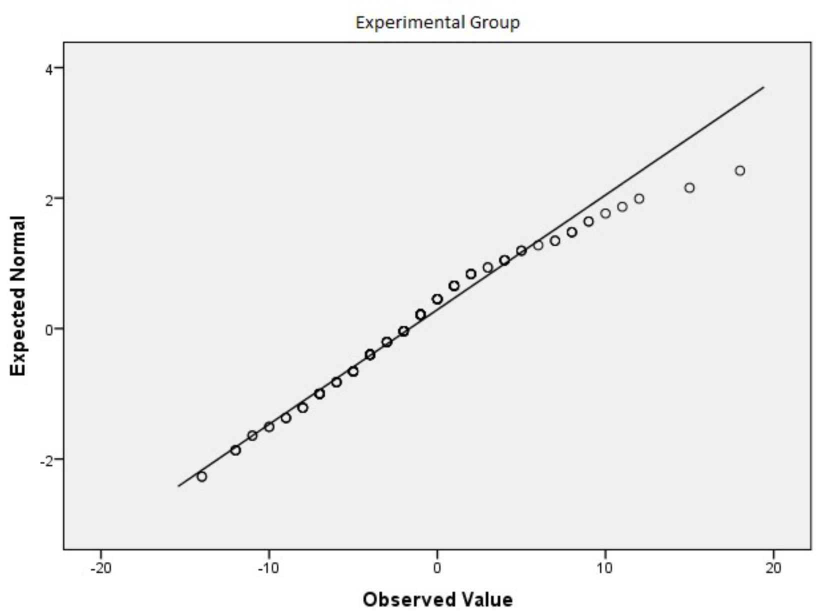

Figure D50. Q-Q plot for experimental group ATMI change – enjoyment.

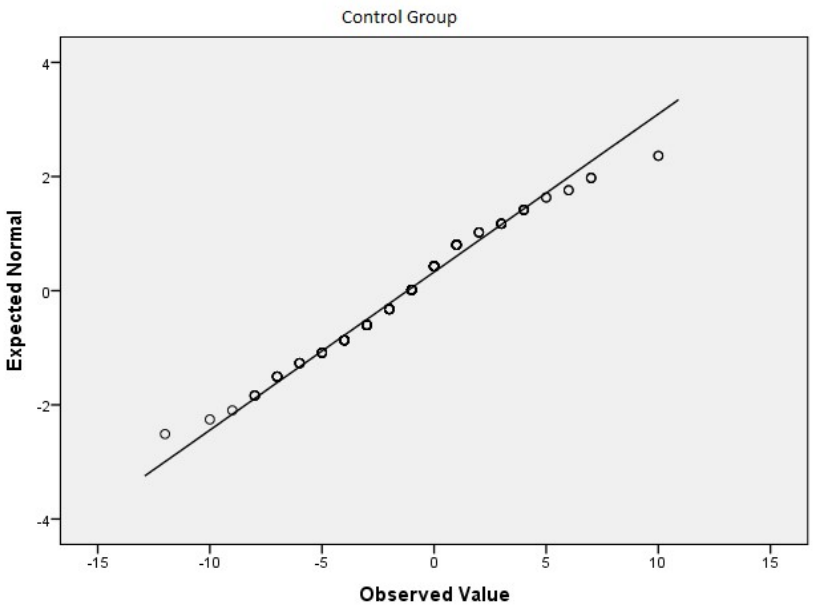

Figure D51. Q-Q plot for control group ATMI change – motivation.

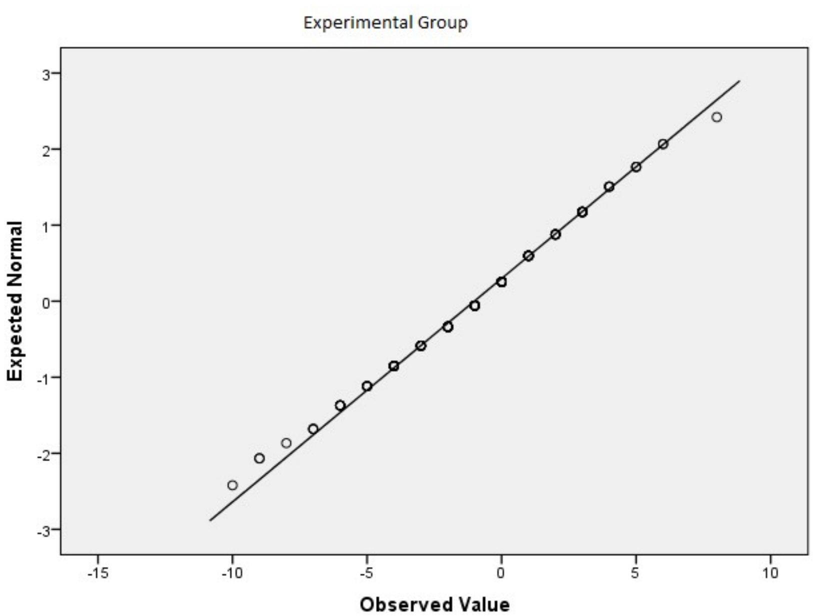

Figure D52. Q-Q plot for experimental group ATMI change – motivation.

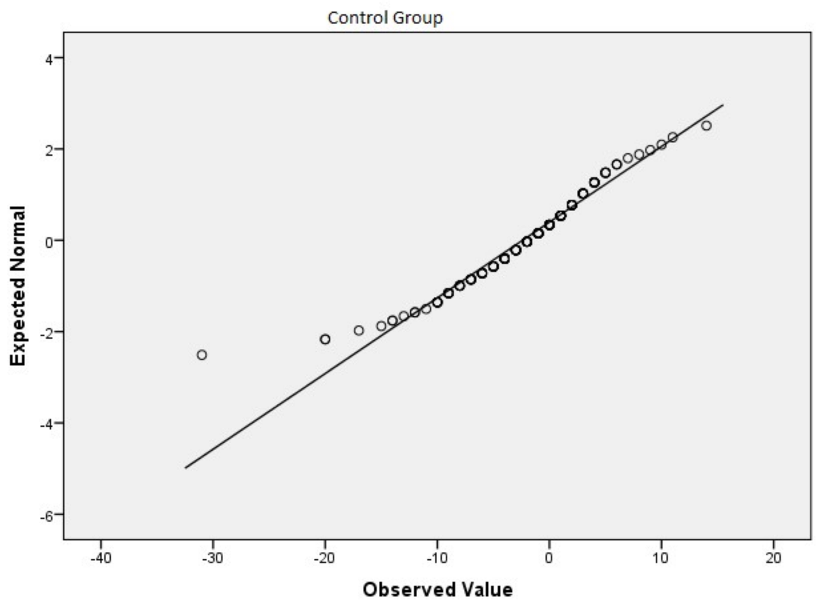

Figure D53. Q-Q plot for control group ATMI change – value.

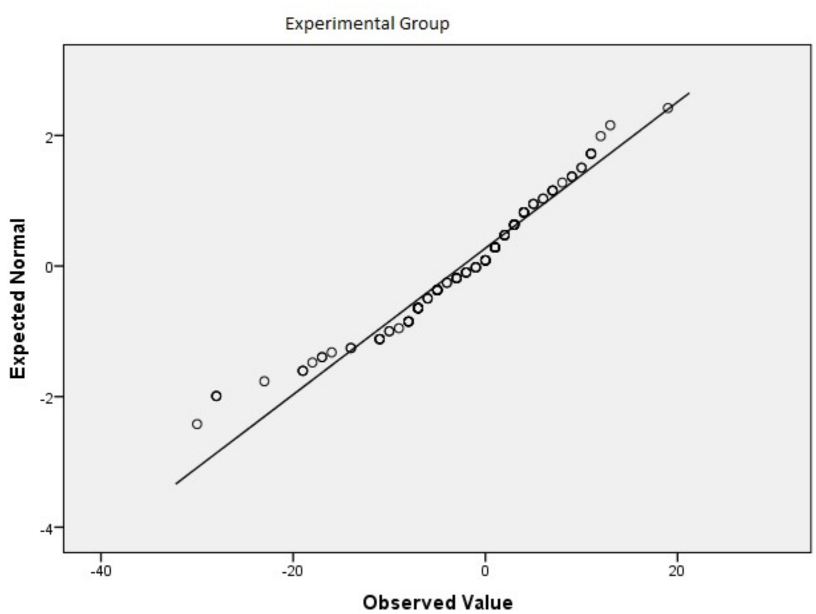

Figure D54. Q-Q plot for experimental group ATMI change – value.

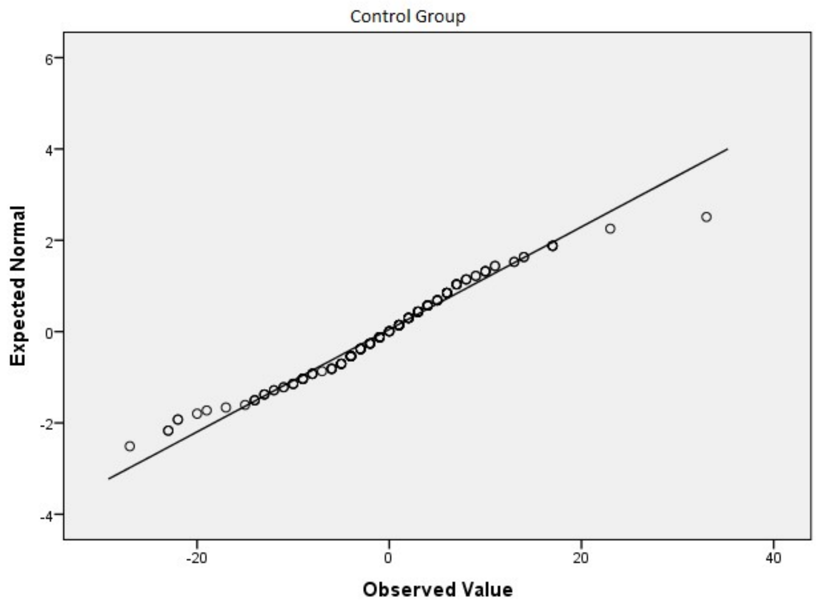

Figure D55. Q-Q plot for control group ATMI change – self-confidence.

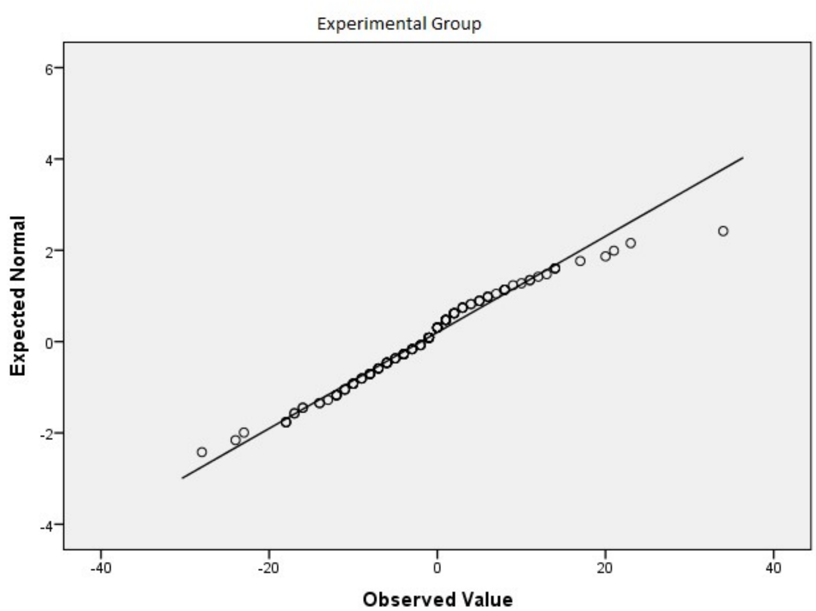

Figure D56. Q-Q plot for experimental group ATMI change – self-confidence.

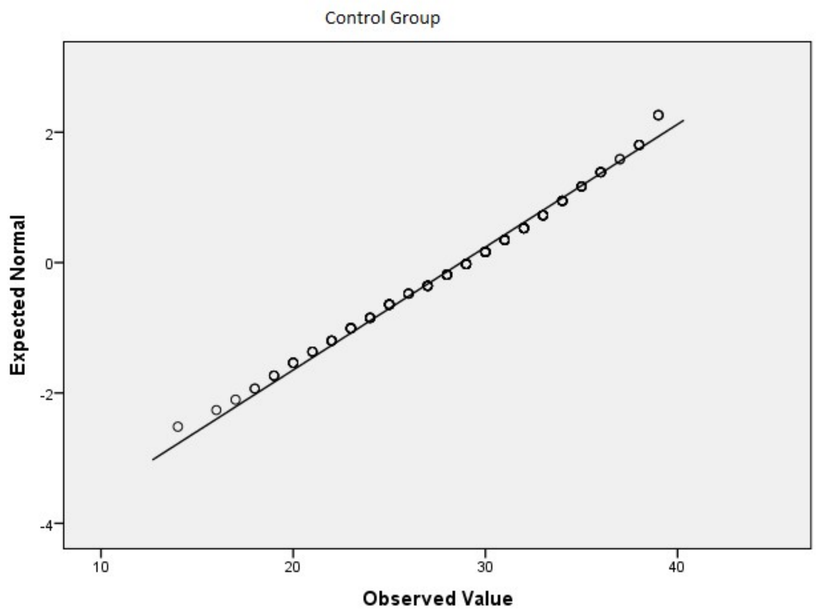

Figure D57. Q-Q plot for control group questionnaire 1.

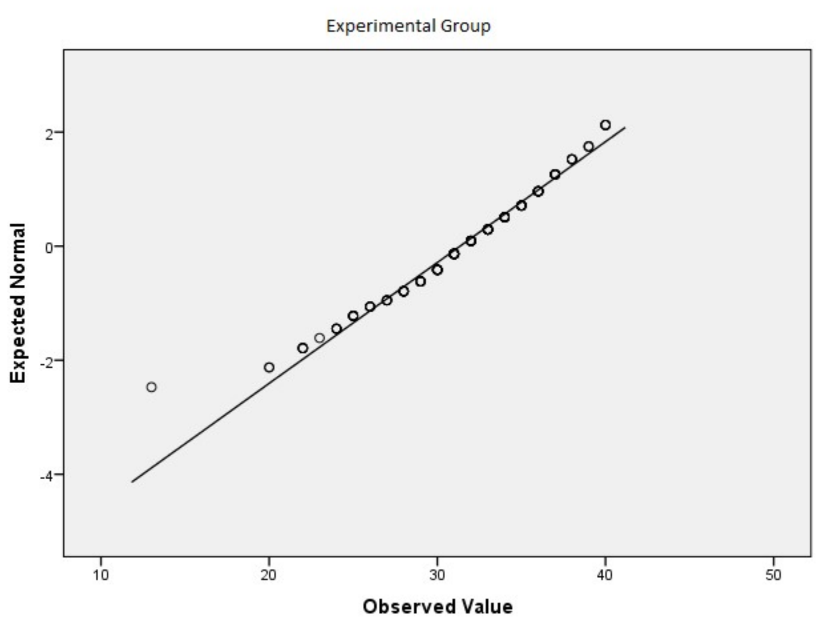

Figure D58. Q-Q plot for experimental group questionnaire 1.

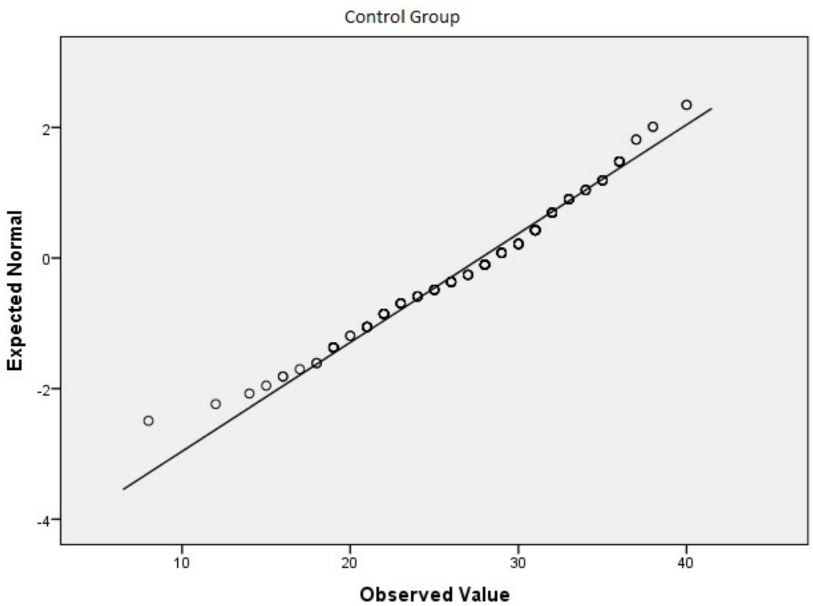

Figure D59. Q-Q plot for control group questionnaire 2.

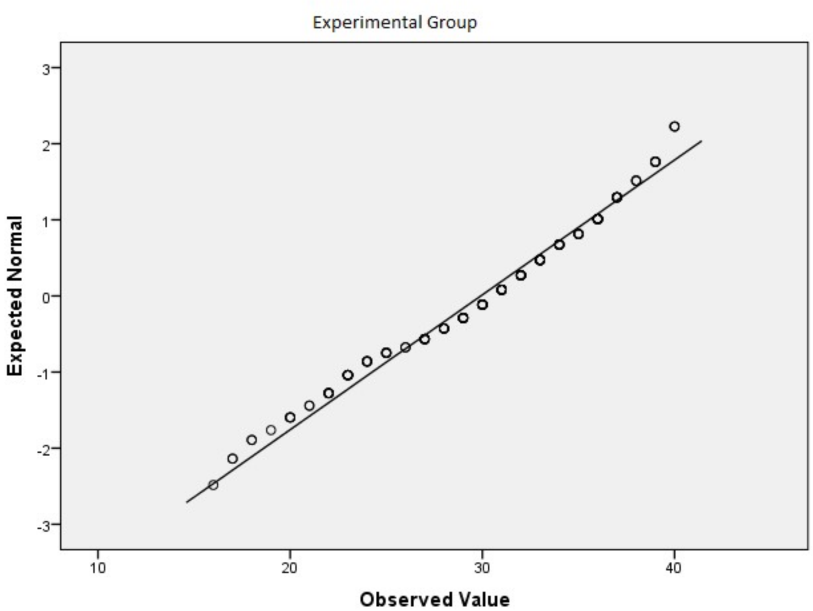

Figure D60. Q-Q plot for experimental group questionnaire 2.

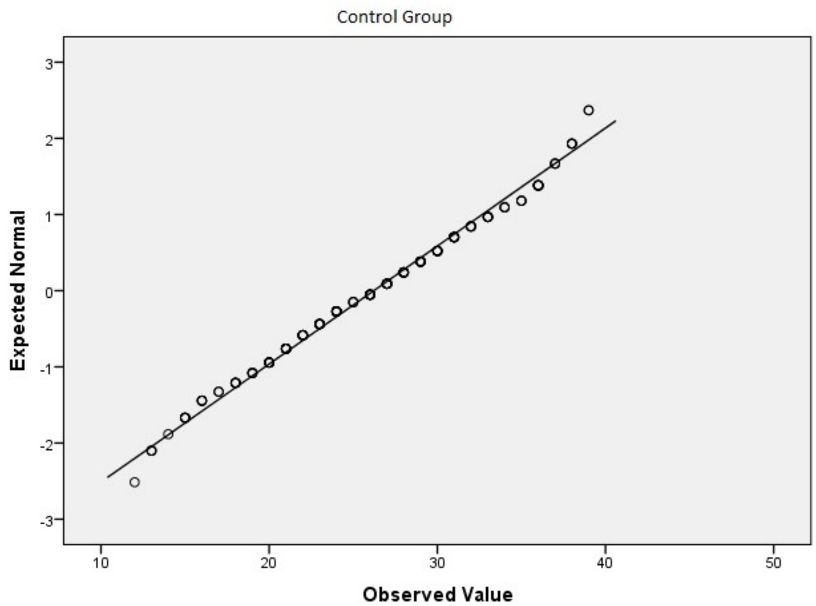

Figure D61. Q-Q plot for control group questionnaire 3.

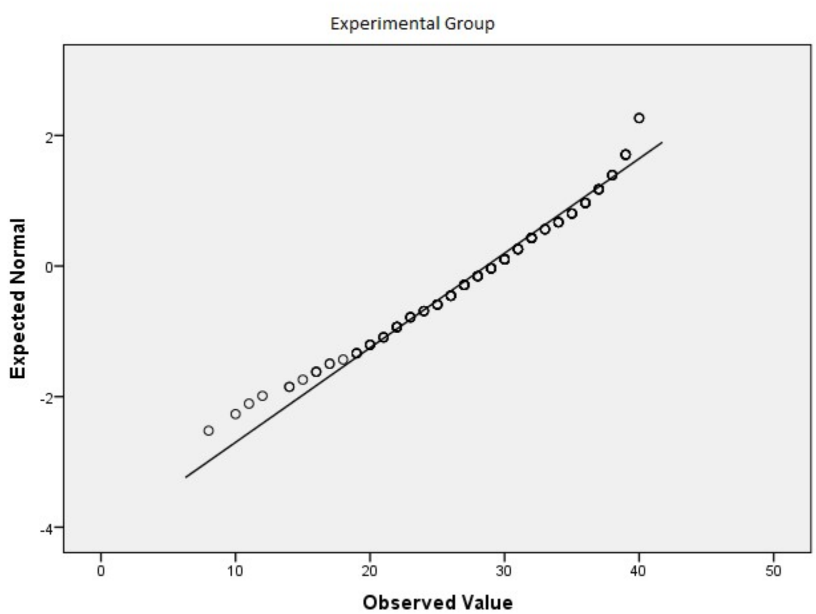

Figure D62. Q-Q plot for experimental group questionnaire 3.

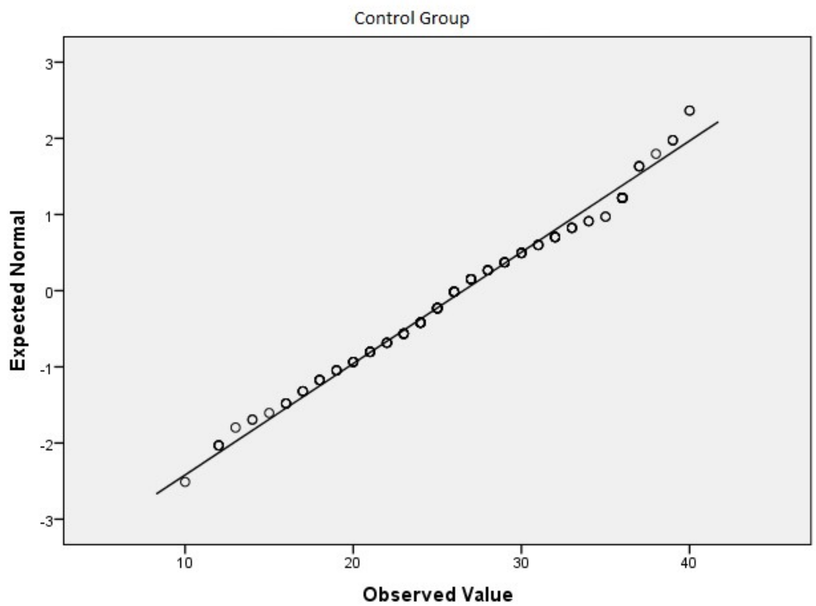

Figure D63. Q-Q plot for control group questionnaire 4.

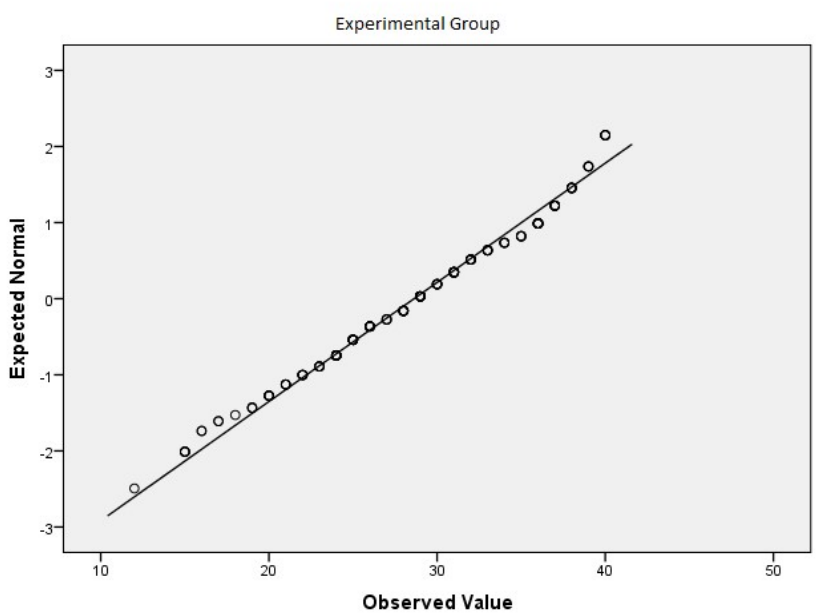

Figure D64. Q-Q plot for experimental group questionnaire 4.