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Design Considerations for Variable Power Split Hydraulic Drives for Industrial Applications

**By
Michael J.C. Shellenberger**

A THESIS

**Submitted to the College of Engineering and Mineral Resources
at
West Virginia University**

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Kenneth H. Means, Ph. D.
W. Scott Wayne, Ph. D.**

Abstract

Continuously variable transmissions (CVTs) offer some advantages over conventional transmissions in industrial applications. During discrete shifts, conventional transmissions suffer from momentary power transmission lapses, which may produce undesirable effects in the system. Currently, the types of CVTs utilized in industry are hydrostatic drives and hydrodynamic couplings. However, these drives are heavy, presenting an obstacle in applications where weight is a concern. In a power split continuously variable transmission (PS-CVT) the power that flows through the hydraulic element is reduced. This allows the overall size and weight of the transmission to be reduced.

The objective of this thesis is to develop a conceptual design procedure for development of a power split hydraulic variable transmission to be used in industrial applications. The procedure consists of a 15 step process which is outlined by means of a flow chart. The results of the procedure can be used to estimate the overall size and efficiencies of the conceptual transmission and to produce 2D and 3D scale drawings of the design. This procedure offers a means to improve the efficiency of the conceptual design stage of a PS-CVT.

An industrial pump was used as an example application to illustrate the conceptual design procedure. Two conceptual designs were completed utilizing different hydraulic variable elements. The two variable elements considered were a hydrostatic drive and a hydrodynamic coupling. The two examples illustrated the procedure that can be used to determine whether a PS-CVT is a viable alternative for an industrial pumping system.

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Nomenclature

F_c	Force on arm
F_h	Force on idler carrier
F_i	Force on idler
F_R	Force on ring gear
F_S	Force on sun gear
N_c	Number of teeth on control gear
N_o	Number of teeth on the variable element output shaft gear
N_R	Number of teeth on Ring gear
N_S	Number of teeth on Sun gear
N_1	Number of teeth on first gear
N_2	Number of teeth on second gear
P	Power
P_{cir}	Circulating power
P_d	Diametral pitch
P_f	Power transferred through element f
P_{in}	Power transferred through the input shaft
P_{loss}	Power loss from variable element
P_{oi}	Power transferred through variable element
P_{out}	Power transferred through the output shaft
P_{sun}	Power transferred through the sun gear
r_a	Radius of arm
r_c	Radius of control gear
r_o	Radius of N_o
r_R	Radius of ring gear
r_S	Radius of sun gear
R	Range of ratios for the power split unit
R_{tot}	Total range of ratios for transmission
R_{T1}	Range of ratios of stage 1
R_{T2}	Range of ratios of stage 2
R_{T3}	Range of ratios of stage 3
$Span_i$	Span of stage i
T	Torque
T_d	Torque on element d
T_e	Torque on element e
T_f	Torque on element f
T_{in}	Torque on input shaft
T_{oi}	Torque on stator
T_{or}	Torque on rotor
T_{out}	Torque on output shaft

Greek Letters

γ_c	N_c/N_o
γ_g	N_R/N_S
γ_h	Angular velocity ratio of variable element
γ_{hH}	Highest angular velocity ratio of variable element
γ_{hL}	Lowest angular velocity ratio of variable element
γ_n	N_1/N_2
γ_w	Worm gear ratio
γ_1	Gearbox ratio for stage 1
γ_2	Gearbox ratio for stage 2
γ_3	Gearbox ratio for stage 3
Γ	Fraction of power which flows through variable element
η	Variable element efficiency
ω_c	Angular velocity of control gear
ω_d	Angular velocity of element d
ω_{driven}	Angular velocity of the driven gear
ω_{driver}	Angular velocity of the driver gear
ω_e	Angular velocity of element e
ω_f	Angular velocity of element f
ω_{in}	Input angular velocity
ω_o	Angular velocity of N_o
ω_{out}	Output angular velocity
ω_p, ω_m	Angular velocity of the pump
ω_A	Angular velocity of arm relative to fixed link
ω_F	Angular velocity of the first gear in train relative to fixed link
ω_{FA}	Angular velocity of the first gear in train relative to arm
ω_L	Angular velocity of the last gear in train relative to fixed link
ω_{LA}	Angular velocity of the last gear in train relative to arm
ω_R	Angular velocity of ring gear
ω_S	Angular velocity of sun gear
ω_T	Angular velocity of the output of power split unit

Chapter 1 Introduction

1.1 Introductory Remarks

Transmissions are used to transmit torque and rotary motion from a power source to an output device. Two main types of transmissions are generally used for automotive and industrial applications. These are broadly classified as, geared and continuously variable. Geared transmissions consist of several fixed gear ratios and use discrete shifts to switch between them. With this type of transmission, the input/output angular velocity ratio is fixed for each gearshift. Therefore, to change the output velocity, the speed of the input device (power source) must be varied. On the other hand, continuously variable transmissions (CVTs) cover a continuous range of ratios, allowing the output speed to be varied while the input speed remains constant. Continuous ratios can offer an advantage over discrete shifts in some industrial applications. An example, is in high pressure pumping operations. In this application, the power lapses that occur during discrete shifts introduce the possibility of high pressures stalling the pump. This can result in severe damage to the pumping unit and power source.

For industrial applications where weight is a major concern, a power split continuously variable transmission (PS-CVT) would have advantages over shaft to shaft applications of a hydraulic CVT. Hydraulic CVTs that can handle high power requirements are very heavy. By reducing the power that flows through the hydraulic element, the overall size and weight can be reduced.

In the design exercise, a transmission designer is given a specific power source and an output device, and is given the task of developing a PS-CVT to suit the industrial application.

In the design process, many obstacles need to be overcome. These include selecting the appropriate configuration of planetary gear trains and continuously variable element. Development of a design procedure will allow the designer to determine the feasibility of a PS-CVT concept for a specific application.

1.2 Continuously Variable Transmissions

Continuously variable transmissions can be classified into two major groups; shaft to shaft drives and power split (PS-CVT) continuously variable transmissions. Shaft to shaft CVTs are transmissions in which 100% of the power being transmitted flows through the variable element. These can be sub-classified into the five groups listed in Table 1.1, below (Lu 1998, Kluger and Fussner 1997). Several applications have used these types of transmissions.

CVT Types	Examples
All Gear	Epicyclic
Traction	Toroidal and Nutating
Hydraulic	Hydrostatic, Torque converter, and Coupling
Chain	Chain
Belt	Flat, Rubber V, and Metal Pushing

Table 1.1: Classification of shaft to shaft continuously variable transmissions.

1.2.1 Shaft to shaft CVTs

Shaft to shaft continuously variable transmissions have also been used in a wide range of applications ranging from automotive and utility vehicles (with relatively low power requirements) to large power plants. CVTs with rubber V belts have been used successfully in utility vehicles such as golf carts, snowmobiles, and SAE Mini-Baja cars. Metal pushing

belts, toroidal, and nutating transmissions have been used in automobiles. These CVTs, however, have not yet been widely accepted although development for automotive applications continues. Hydrostatic drives have been used on construction equipment and farm equipment such as: cranes, dump trucks, front end loaders, and tractors. However, hydrostatic drives are generally heavy and are used where weight is not a concern and high power needs to be transmitted at relatively low speeds. Hydrodynamic couplings and torque converters have been placed in high power applications such as power plant generators and work best at high speed ranges.

1.2.2 Power Split CVTs

Power split transmissions use differentials in combination with a variable element, such as a CVT. As the name implies the power is split so that a fraction of the power circulates through the CVT element to the differential while the remaining power circulates directly through the differential. This type of CVT is a compromise between the ratio coverage of the variable element and the amount of power that is transferred through the variable element. Power split transmissions have the advantage of transferring powers that are greater than the capacity of the variable element, which is the limiting factor in most CVT applications.

To overcome the limitations of the narrower ratio coverage, the concept of a stepless multistage PS-CVT has been developed in this thesis. This type of transmission uses an external gearbox to amplify the ratios that the PS-CVT covers. Though this type of transmission incorporates shifts, these are considered stepless since the transmission ratio remains the same before and after the shift and the impact on engine speed is minimal. When the gearshifts occur the engine experiences only a slight decrease in velocity due to inertial

loss of the driven device during the power lapse. The engine, however, does not have to reduce its velocity through its complete operating range, as it would with a discrete gear shift transmission.

As with shaft to shaft continuously variable transmissions, PS-CVTs have been developed for several applications. Conceptual designs have been developed for motorcycles. Single and multistage PS-CVTs have been developed for automotive applications using metal push belts and hydrostatic drives for the variable element. Multistage PS-CVTs have been developed for farm tractors using hydrostatic drives. Finally, hydrodynamic couplings and torque converters have been used to develop PS-CVTs for power plants, steel mills, and offshore and onshore pipeline systems.

1.3 Objectives

The main objective of this study is to develop a conceptual design procedure for a power split continuously variable transmission for an industrial application using a hydraulic variable element. The objective is to be accomplished through the completion of the following tasks:

1. A literature review on continuously variable transmissions with emphasis on power split multistage continuously variable transmissions.
2. Development of a design procedure for a conceptual design of a multistage power split continuously variable transmission.
3. Development of an example of an industrial application, to illustrate the design procedure.

Chapter 2 Review of Relevant Literature

2.1 Power Split Continuously Variable Transmission Configurations

A search for relevant literature pertaining to power split continuously variable transmissions revealed that substantial work has been done in this area of transmission design. Several combinations of components and differentials have been used to develop power split continuously variable drives. Noted examples are illustrated below.

Lemmens (1972) described a power split continuously variable transmission that consists of a V-belt drive and a differential. The input shaft delivers the input power directly from the driving device to the driving gear of a chain drive and the driving pulley of the V-belt drive. The chain drive connects the input shaft to the planetary carrier of the differential. The driven pulley of the V-belt drive is connected to the sun gear. The output shaft of the transmission is connected to the ring gear. The chain drive carries the larger percentage of input power to the differential, while the lower percentage is transferred through the V-belt. Infinitely variable speed ranges in both forward and reverse directions, and a geared neutral condition can be achieved by varying the V-belt drive ratio.

Kumm (1991) presented a regenerative transmission that uses a flat belt CVT and a normal planetary mechanism. This transmission increased the efficiency and capacities of a flat-belt CVT by reducing the power which flows through it and permits the use of a CVT over a full speed range with two modes of operation. A high and low speed mode. In low speed mode, the input power is divided into two paths. The first path is through the planetary gearing to the output while the second path is from the planetary gearing through the CVT back to the input shaft. In high speed mode, the clutch that connects the output shaft to the

ring gear of the planetary gear train is disengaged. Then, another clutch is engaged which permits the input power to be transmitted directly through the CVT to the output shaft. Reverse output speeds are made available by changing the velocity ratio control of the CVT in the low speed range without actuating clutches. This regenerative PS-CVT arrangement eliminates the capacity limits of the flat belt, but the power transmitted by the gear train is larger than the input power.

Takayama et al. (1991) combined a V-belt and a two way clutch to create a PS-CVT. Once the power from the prime mover is transferred to the input shaft of the transmission, two paths are able to transmit the power from the shaft. The main transmitting path is through the V-belt drive. The sub-power transmitting path consists of a two way clutch. When the V-belt is near the maximum drive ratio, the rotational speed of the output gear of the two way clutch becomes slower than its input gear. This results in power transfer from both the main path and sub-path to the output shaft of the transmission. This makes it possible to increase driving power and torque and also to decrease the load carried by the V-belt. The benefit of this transmission over previous PS-CVTs is that the overall power capacity is increased while the weight is decreased due to a reduced number of clutches. This is a less optimum PS-CVT, because most of power is transmitted through the V-belt.

Cowen (1992, 1993) presented a PS-CVT design that combines a variable speed transmission unit connected with a planetary gear train. Cowen's design allowed the use of any type of variable speed unit. In this configuration the input was connected to the input of the variable speed unit and the sun gear of the planetary gear train. The output of the variable speed unit was connected to a secondary shaft. This shaft is connected to the ring gear of the planetary gear train by a control gear or gears. The output shaft is connected to

the planetary carrier. When the secondary shaft and the ring gear are connected such that they rotate in the same direction, the input power is divided into two paths. The first is directly through the sun gear to the output shaft. The second is through the variable speed unit, secondary shaft, control gears, and the planetary gear train to the output shaft. With this transmission configuration, it is possible to achieve a function such that only a fraction of the total power load is transmitted through the variable speed transmission unit.

A family of configurations based on a simple planetary-CVT presented by Cowen (1992, 1993) was presented by Mucino et al. (1995). This set of eight PS-CVT configurations contained two planetary gear trains and a CVT unit. The combination of these three main sub-systems does not require the use of clutches or chain drives. The corresponding equations for the transmission velocity ratio and belt forces were derived for each configuration. These systems can be adjusted with relative ease to match the input/output requirements for various applications. A particular feature that this system offers is the possibility of designing the mechanism in such a way that the power circulation is directed to take place through the gear sets at low speeds, just when the requirements for torque are greatest.

2.2 Extended range PS-CVTs

A power split continuously variable transmission represents a compromise design since the variable speed range of the transmission is narrowed to reduce the amount of power that flows through the variable element. Much research has been devoted to extending the range of PS-CVTs. The various methods that have been used are discussed below.

Orshansky and Weseloh (1972) investigated the characteristics of a multiple range hydromechanical transmission. The transmission consisted of a variable displacement pump, fixed displacement motor, two planetary gear trains, and several clutches. The transmission presented had two hydrostatic modes and three power split modes. In hydrostatic modes, the input power was transmitted through the hydrostatic drive. The output was coupled to one of the two planetary gear trains by clutching. Two power split modes were accomplished by clutching the input shaft to a shaft that contained the sun gears of both planetary gear trains. As in the hydrostatic modes the output was connected to one of the planetary gear trains by clutching. A third power split mode occurred when the second planetary gear train was connected to the output element, and the hydrostatic drive was taken back and used through its full range. It was concluded that multirange transmissions have the advantage of high efficiency and the ability to transmit large horsepower with hydraulic units of relatively small size. They are also increasingly advantageous where conditions require wider torque variation at full power and a high efficiency over a wider range than single stage PS-CVTs.

White (1974) derived two families of variable-ratio transmission mechanisms. Each of these mechanisms employ two coaxial differential gears and an input coupled variable ratio unit. The two families of mechanisms are classified as two path twin differential and two path cross coupled differential gear trains. The transmissions provide variable speed-ratios from a minimum number of geared units and allow transfer between two drive-ranges by means of clutches engaged at zero speed-difference. A method was presented to determine the speed ratios for each stage. Since the shift between the two stages occurs at a zero speed difference, the feasibility of each arrangement could be determined. A table was developed

which ensures an arrangement can be chosen from these two families with confidence that all solutions have been covered.

Hausenbauer (1976) presented computer models that investigated the effects of adding a PS-CVT in series to a 4-speed manual gearbox in automobiles. The power split transmission was of the input coupled type utilizing a hydrostatic drive as the variable element. Stepless shifts of the manual gearbox extended the range of the PS-CVT. The computer simulation work resulted in projected fuel economy improvements of approximately 50% over that of the conventional 4-speed gearbox. The model was also used to assess the ease with which the vehicle could be driven.

White (1977) illustrated another configuration in which a gearbox was used to extend the range of a PS-CVT. The two-speed gearbox connects to the variable element output shaft to the differential. The shift between the two gear ranges occurs when the velocity of the hydrostatic drive is zero. Therefore, there is no discontinuity in the transmitted torque and there is no wear on the clutch. This unit required further gear trains to allow zero and reverse output speeds to be achieved through synchronous shift clutches. This imposes a penalty in the form of a decreased power/weight ratio. The mechanism represents one of a number of possible approaches to the problem of improving the performance of stepless variable transmissions while widening the spread of speed ratios.

2.3 PS-CVT Design for Applications

Power split continuously variable transmissions have been designed for several applications. Most of the literature found was for vehicles including automobiles, buses, and

motorcycles. Several methods of designing and analyzing PS-CVTs for specific applications are presented below.

Beachley et al. (1984) evaluated a power split continuously variable transmission for automotive applications. The study focused on PS-CVT systems using a single differential with parameters such that the ratio coverage is extended beyond that of the continuously variable unit. Both single regime and two regime systems were considered. It was not necessary to consider the mechanical details of either the variable unit or the differential because the work was based on mathematical relationships. The data developed in this study can serve as a guide to help determine whether transmissions of this type would be suitable, from efficiency considerations for use in conventional automobiles. The results of this study suggest that no split path extended range CVT should be considered for automotive applications without first performing a realistic analysis of the efficiency aspects. The analysis should be based on the efficiency characteristics of the particular continuously variable unit to be used, as the efficiency penalty can be high in many cases.

Dorey and Vaughan (1984) investigated the design of the power split transmission for a vehicle drive and illustrated how the transmission characteristics could be matched to the particular requirements of the vehicle duty. This design study was accomplished using computer aided design due to the number of design alternatives available. A unified approach to the modeling procedure was used to enable computer programs, which describe the behavior of the complex transmissions, to be easily constructed and altered. This study stressed the importance of computer models in the design of PS-CVTs since they reduce the time required to develop a new transmission and greatly facilitate evaluation of modifications to the transmission configuration.

Sheu (1996) presented a procedure for the conceptual design of hybrid transmissions for motorcycle applications. The hybrid transmissions are developed using combinations of fixed gear ratio, split power, and continuously variable transmission systems. Four design concepts were synthesized. These included: split power, split power and continuously variable, fixed gear and continuously variable, and fixed gear and split power transmission systems. Using a V-belt drive continuously variable unit and simple differential gears, topological structures of the hybrid transmissions were created. The feasibility of the proposed procedure was illustrated with a design example.

Lu et al. (1999) presented a simulator program that aids in the design and development of a power split continuously variable transmission. The simulator can be used to optimize the transmission and compare it with other transmissions. The objective of the design simulator was to develop a PS-CVT design to minimize the loading on the belt while providing for increased power transfer compared to existing designs. An optimized design was presented. A vehicle acceleration simulation indicated that the optimized PS-CVT could provide a continuously variable ratio change. It also illustrated that the fuel economy of a vehicle with a PS-CVT is better than that of a standard or a multi-ratio automatic transmission.

Although the literature review revealed several designs for specific applications, these configurations are typically for low power systems. For larger power industrial applications, PS-CVT units are not well documented. Units that fulfill these larger power requirements are address in this thesis.

Chapter 3 PS-CVT Concepts and Components

3.1 Overview

Before the process of designing a split-power drive for heavy duty applications can be described, the main components that comprise the design are explained. These components include the input and output gear arrangements, the velocity ratio, power flow through the variable element, and the variable element itself. For each of these components, the kinematic relationships and performance curves are presented for the overall design of a power split transmission. These components are then used to describe the design process with an example application.

3.2 Input and Output Configurations Gear Arrangements

Variable elements and differentials can be arranged in one of three ways. The three combinations are illustrated in Figure 3.1. These three combinations can further be classified into two groups: variable shunt (Figure 3.1 A and B), and variable bridge (Figure 3.1 C). Variable shunts consist of a variable element connected to a single differential. The other end of the variable element is coupled to the input or output shaft. These configurations are referred to as input coupled variable shunt and output coupled variable shunt respectively. The variable bridge consists of a variable element bridged between two differentials. This design study focuses on the input coupled variable shunt configuration. There are two reasons for this. First, the input and output coupled variable shunts contain only one differential and are assumed lighter than the variable bridge. Second, based on the work of Hsieh and Yan (1990), input coupled variable shunt can be arranged to be mechanically more efficient than the output coupled variable shunt.

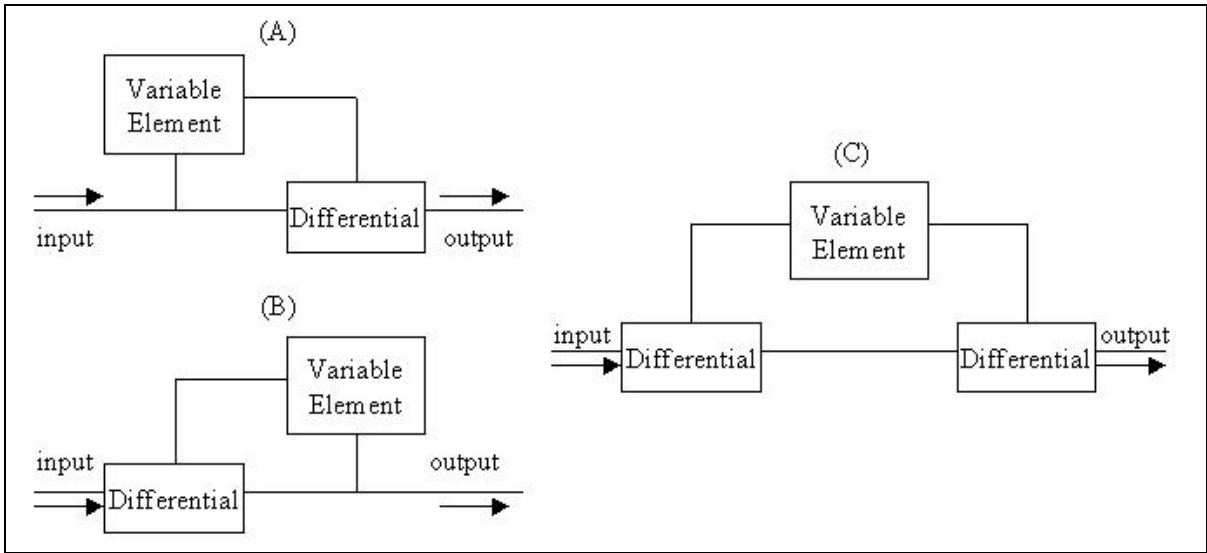


Figure 3.1: The three basic variable element and planetary gear arrangements.

A variable shunt drive consists of two main components: the branch control circuit and the differential. In all of the configurations presented, a third connecting element is used. This element connects the branch control circuit to the input shaft. It can be described as a fixed planetary, or simply a multiplier or reducer, depending on the arrangement of gears. The differential is a single planetary gear train consisting of a ring gear, sun gear, and planetary carrier in all of the configurations presented.

Two examples of typical configurations are illustrated in Figure 3.2, in which a variable transmission element is used with a through-shaft. The variable element divides the input power between the branch control unit (secondary shaft) and the primary shaft (through-shaft). Each transfers the power to the output shaft. In example 1, the input power is branched between the planet carrier and the sun gear, and the output power is transferred from the ring gear. Another possible configuration is example 2, where the output comes from the planet carrier and the input power is branched between the other two elements. These two examples also illustrate two possible multiplier/reducer elements, which connect

the variable element to the planetary gear train. Example 1 contains a reducer, while example 2 contains a multiplier.

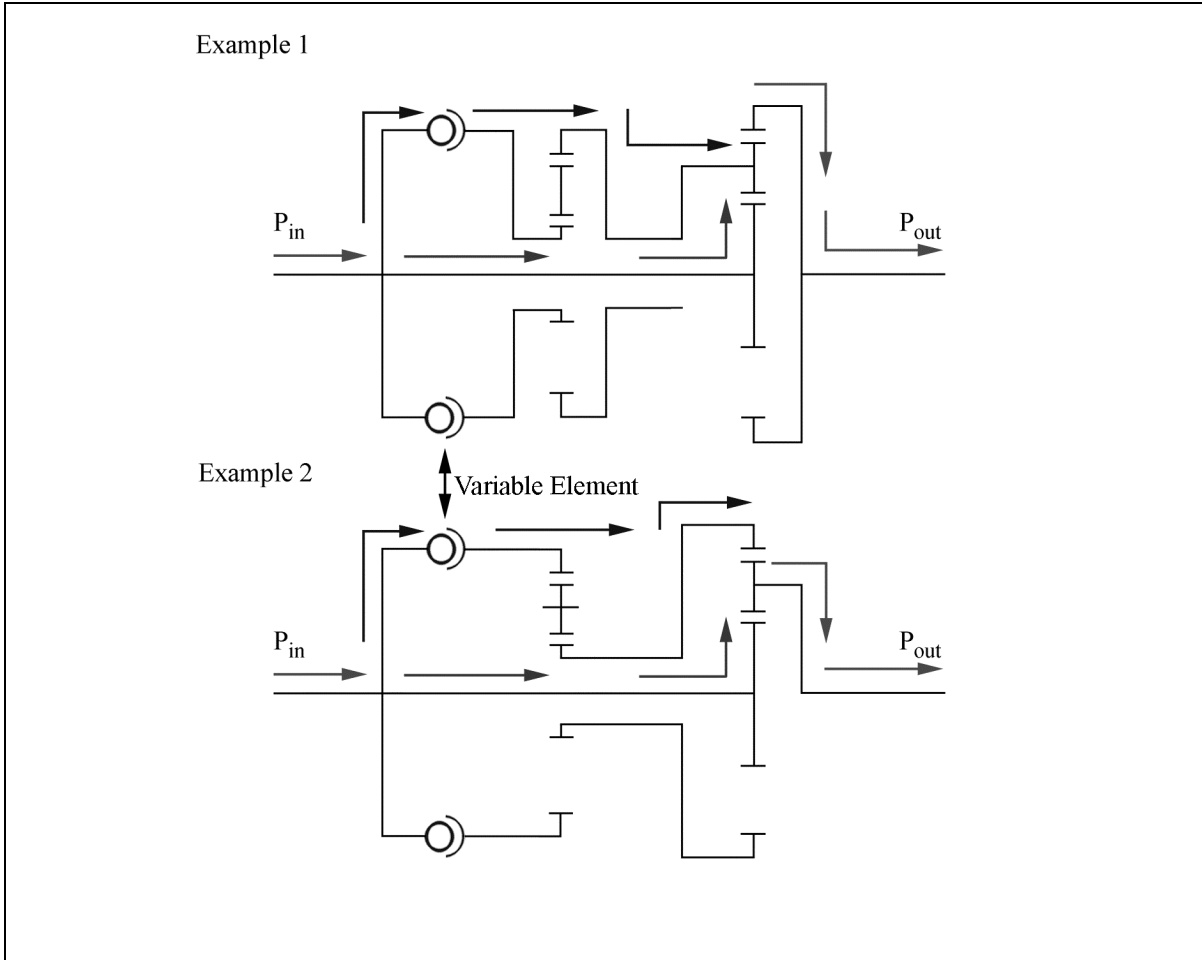


Figure 3.2: Example configurations of split power transmission.

A continuously variable transmission provides a continuous speed ratio change with a constant power flow. This is accomplished in this system by the variable element, which varies the angular velocity between the primary and the secondary shaft. The ratio of input angular velocity divided by the output angular velocity of the variable element is designated as γ_h . By varying the angular velocity of one of the shafts, the angular velocity of one of the differential input elements is also varied. With one of the inputs of the differential constant

and the other variable, the output angular velocity is varied. The variable element supplies a continuous flow of power through the transmission while providing a varying velocity ratio.

The configurations considered in this thesis have two main segments: one designated as input and a second one as output. The input segment contains the variable element and the multiplier/reducer, while the output segment contains the planetary gear train. Five different configurations were found for the multiplier/reducer. They can be connected in two ways to the variable element, for a total of ten possible input combinations. These combinations are illustrated in Table 3.1, which is continued on the following page. The secondary shaft can be connected to the variable element through the multiplier/reducer so that its rotation varies with γ_h , while the primary shaft rotation is equivalent to the input angular velocity. These are configurations 1 through 5 in the table. In the other five configurations, the primary shaft's angular velocity is varied with γ_h , and the secondary shaft rotation is equivalent to the multiplied or reduced input angular velocity.

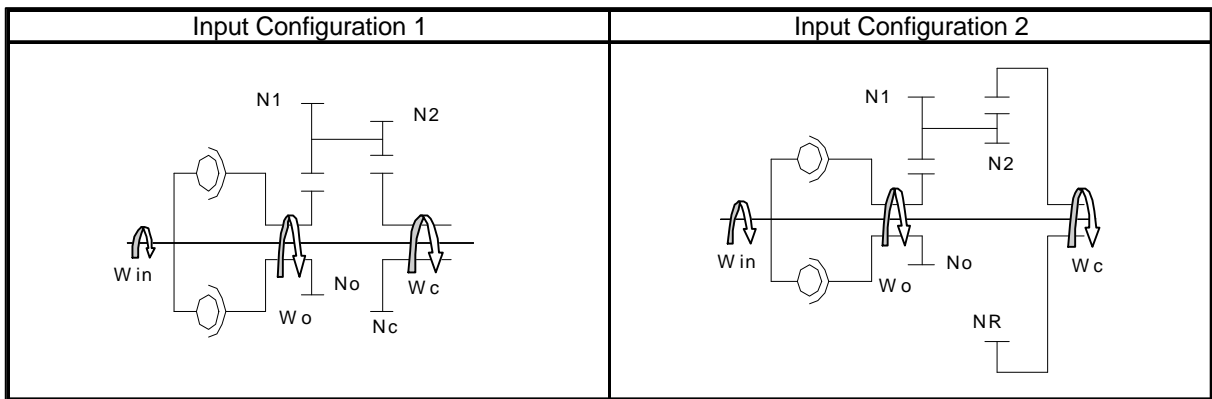


Table 3.1: Input Configurations.

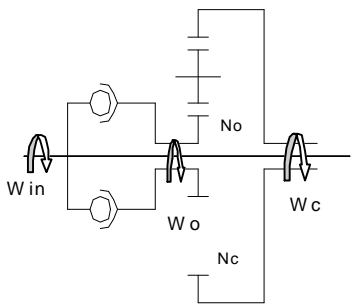
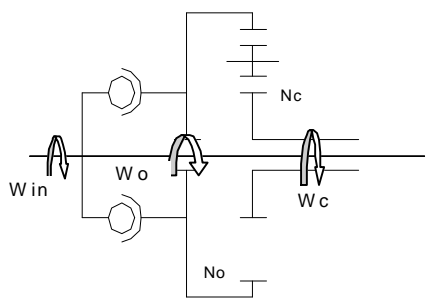
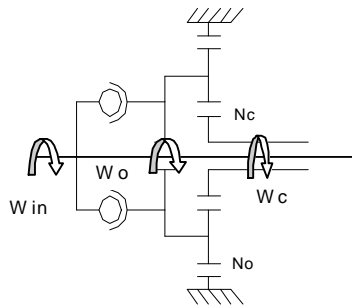
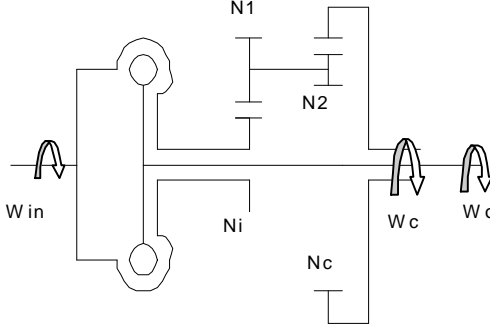
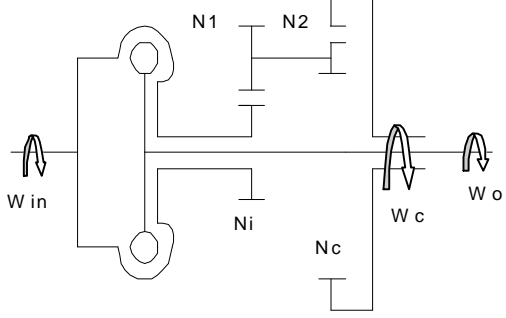
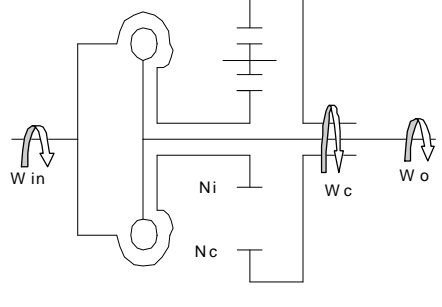
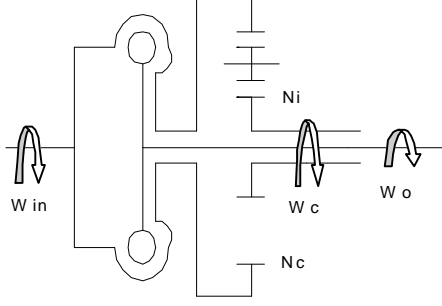
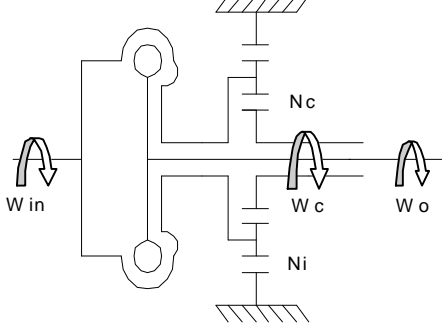
<p style="text-align: center;">Input Configuration 3</p> 	<p style="text-align: center;">Input Configuration 4</p> 
<p style="text-align: center;">Input Configuration 5</p> 	<p style="text-align: center;">Input Configuration 1'</p> 
<p style="text-align: center;">Input Configuration 2'</p> 	<p style="text-align: center;">Input Configuration 3'</p> 
<p style="text-align: center;">Input Configuration 4'</p> 	<p style="text-align: center;">Input Configuration 5'</p> 

Table 3.1 continued: Input configurations.

There are six possible combinations for two input elements and one output element of the planetary gear train. The six possible output configurations are illustrated in Table 3.2. These six, along with the ten inputs, combine for sixty different configurations. Each is designated by the name configuration, followed by the input number and then the output number. For example, the configuration that combines the input configuration 1 and output configuration 1 is designated configuration 1.1.

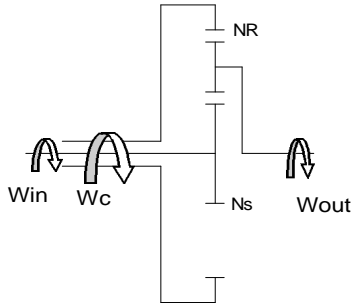
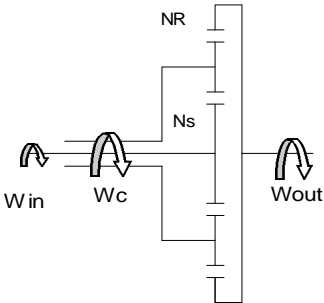
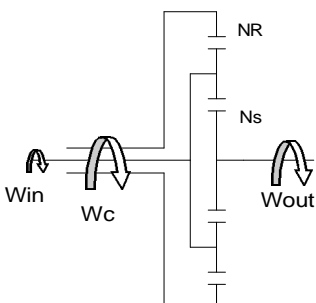
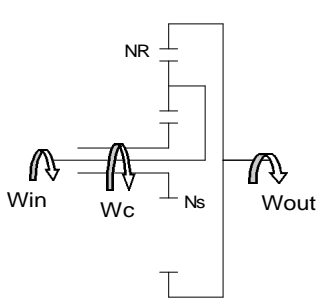
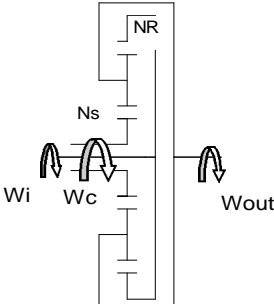
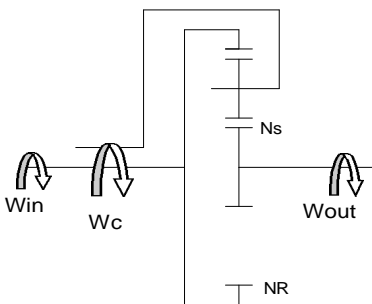
Output Configuration 1	Output Configuration 2
	
Output Configuration 3	Output Configuration 4
	
Output Configuration 5	Output Configuration 6
	

Table 3.2: Output configurations.

To determine the best configuration for a given application, both the velocity ratio span and the fraction of power which flows through the variable element, gamma (Γ) must be considered. The velocity ratio span, hereafter referred to simply as the span, is equal to the overall low-speed transmission ratio of the transmission divided by the overall high-speed ratio. Each configuration has its own unique span and Γ , the process for deriving these properties is discussed in the next few sections.

3.3 Velocity Ratio

The velocity ratio is defined as the input angular velocity divided by the output angular velocity, or ω_{in}/ω_{out} . The variable element and the input/output configuration of the transmission each have an effect on its velocity ratio. The velocity ratio of a configuration can be derived by kinematics or force analysis. Example derivations of the velocity ratios are shown for configuration 3.2 using first kinematics, then force analysis. The two derivations are shown to provide a check. The example derivations illustrate the method used to derive the equation for the velocity ratios for all the configurations.

3.3.1 Analysis of the Velocity Ratio using Kinematics

In this section, kinematics is used to derive the velocity ratio, ω_{in}/ω_{out} , for configuration 3.2. To derive the velocity ratio, the angular velocities of all the planetary gear train elements are used. The angular velocities of the sun and ring gears from inspection of Figure 3.3 are ω_{in} and ω_{out} respectively. The only angular velocity not known in this configuration is that of the planet carrier. To find this, an equation is derived for the input to output angular velocity ratio of the variable element. Using the notation provided by Figure 3.2 this equation is:

$$\frac{w_{in}}{w_o} = g_h \quad (3.1)$$

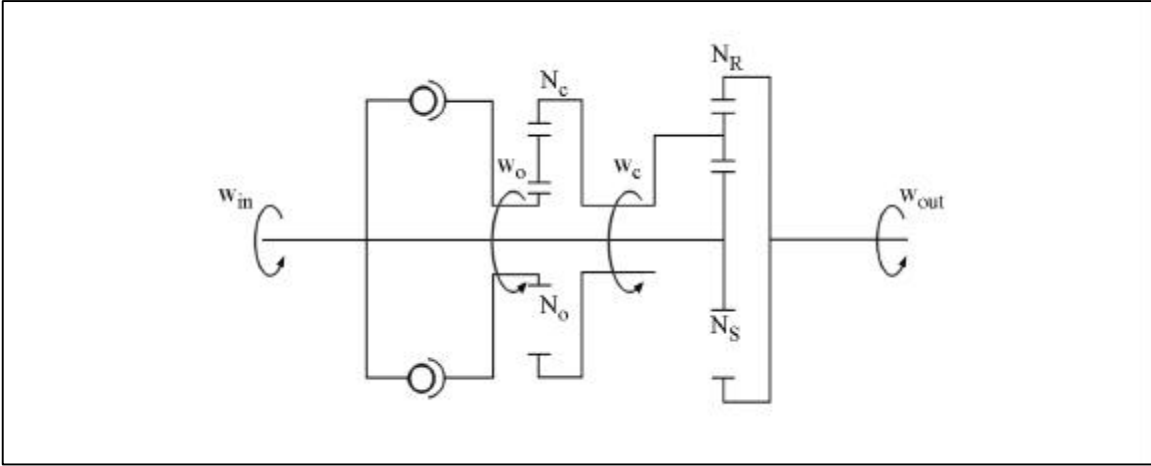


Figure 3.3: Configuration 3.2.

The relationship between ω_o and ω_c is then determined. This is done using the relation that $\omega_{driven}/\omega_{driver}$ is equal to the product of teeth of driving gears divided by the product of teeth of driven gears (Mabie and Reinholtz 1987).

$$\frac{w_o}{w_c} = -g_c \quad (3.2)$$

Where g_c is equal to:

$$g_c = -\frac{N_c}{N_o} \quad (3.3)$$

The result of solving equation 3.1 for combining with equation 3.2 may be written in terms of ω_o as follows:

$$w_c = -w_{in} \frac{g_c}{g_h} \quad (3.4)$$

The angular velocities of all the planetary gear train elements are now known. Using the following equation for planetary gears (Mabie and Reinholtz 1987) and the previous derivation, an expression for ω_{in}/ω_{out} can be determined.

$$\frac{\mathbf{w}_{LA}}{\mathbf{w}_{FA}} = \frac{\mathbf{w}_L - \mathbf{w}_A}{\mathbf{w}_F - \mathbf{w}_A} = -\frac{N_F}{N_S} = -\mathbf{g}_g \quad (3.5)$$

where

$\frac{\mathbf{w}_{LA}}{\mathbf{w}_{FA}}$ = velocity ratio of last gear to first gear both relative to arm

\mathbf{w}_L = angular velocity of last gear in train relative to fixed link

\mathbf{w}_A = angular velocity of arm relative to fixed link

\mathbf{w}_F = angular velocity of first gear in train relative to fixed link

For this derivation, the ring gear is selected as the first gear in the planetary gear train and the sun gear is selected as the last gear. Substituting these in the rearranged equation 3.5 for the angular velocities of the first and last gears yields:

$$\mathbf{w}_S - \mathbf{w}_A = \mathbf{g}_g (\mathbf{w}_A - \mathbf{w}_R) \quad (3.6)$$

From inspection of Figure 3.2:

$$\mathbf{w}_R = \mathbf{w}_{out}$$

$$\mathbf{w}_A = \mathbf{w}_c$$

$$\mathbf{w}_S = \mathbf{w}_{in}$$

Substituting these and equation 3.4 for ω_c into equation 3.6 and simplifying results in the following equation, which is the angular velocity ratio of configuration 3.2:

$$\frac{\mathbf{w}_{in}}{\mathbf{w}_{out}} = -\frac{\mathbf{g}_g \mathbf{g}_h}{\mathbf{g}_h + \mathbf{g}_c (\mathbf{g}_g + 1)} \quad (3.7)$$

3.3.2 Analysis of Velocity Ratio using Force Analysis

The ratio of the input to output angular velocities can also be calculated using force analysis. The force analysis of configuration 3.2 is presented here as an alternative method to calculate the velocity ratio and as a check to the previous derivation using kinematics. Figure 3.4 is the free body diagram of configuration 3.2. This figure illustrates the forces and torques present in this configuration. By determining the relation between the input to output torque and using the assumption that there are no losses in the transmission the input to output velocity ratio is derived.

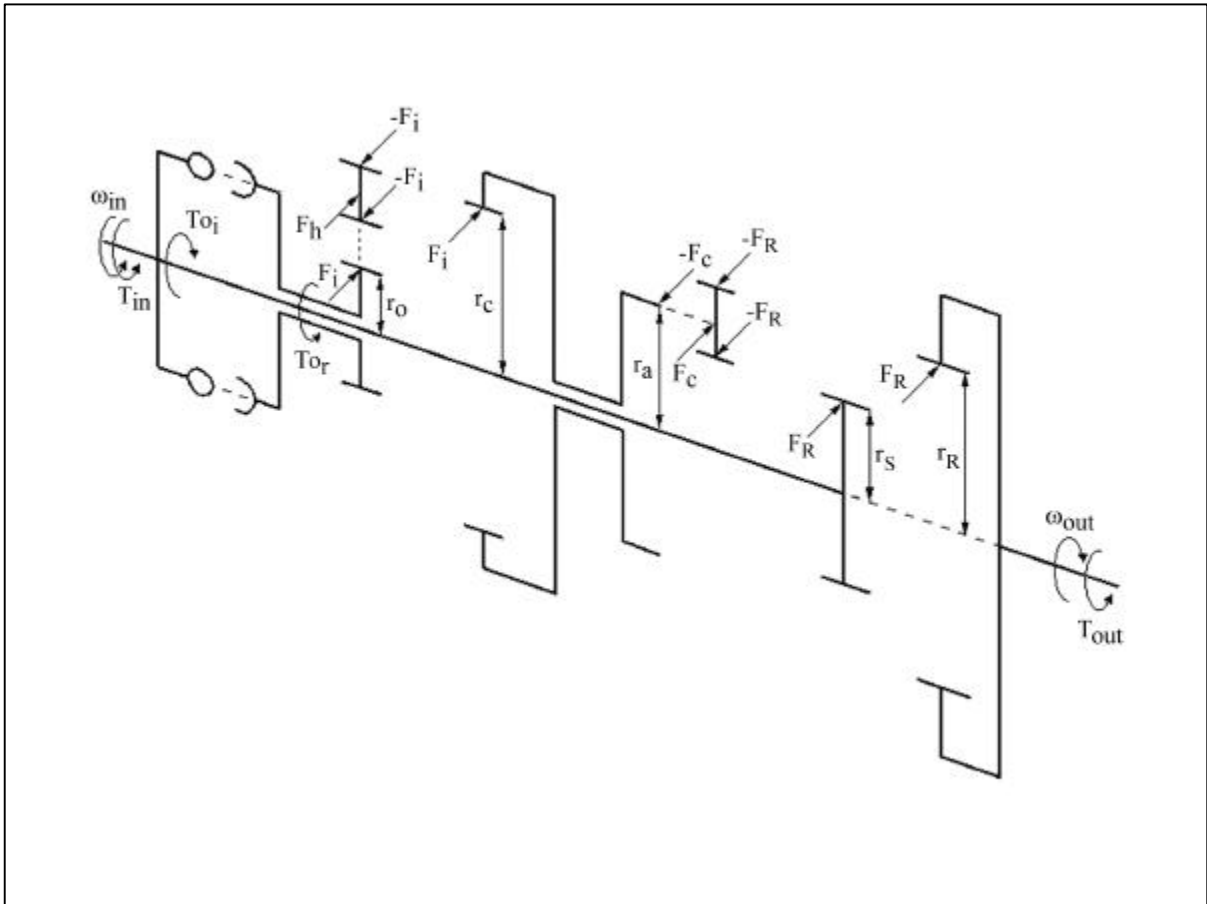


Figure 3.4: Free body diagram of configuration 3.2.

The first step in the derivation of angular velocity ratio using force analysis is to sum the torques on the input shaft. This summation yields the following equation:

$$\sum T = T_{in} - T_{O_i} - F_R r_S = 0 \quad (3.8)$$

The torques can also be summed on the output shaft of the variable element or secondary shaft. This produces the following equation:

$$\sum T = T_{O_r} - F_i r_o = 0 \quad (3.9)$$

Next, the forces on the idler gear are summed, this yields the following equality:

$$F_h = 2F_i \quad (3.10)$$

The torques on the planet gear carrier are summed, resulting in the following equation:

$$\sum T = F_c r_a - F_i r_c = 0 \quad (3.11)$$

Finally, the forces on the planet gear are summed, this yields:

$$F_c = 2F_R \quad (3.12)$$

Now, equations 3.9 through 3.11 can be substituted into equation 3.12 and simplified to produce:

$$T_{O_r} = T_{out} \frac{r_o}{r_R r_c} (r_S + r_R) \quad (3.13)$$

Equation 3.8 can then be solved for T_{O_i} .

$$T_{O_i} = T_{in} - \frac{T_{out}}{\mathbf{g}_s} \quad (3.14)$$

Assuming that the variable element has an efficiency of 1, the power into the element equals the power out. The effect of the variable element's efficiency on the velocity ratio will be examined in a later section. The power in and power out can be expressed as a product of angular velocity and torque, which yields the following equation:

$$\mathbf{w}_{in} T_{O_i} = \mathbf{w}_o T_{O_r} \quad (3.15)$$

Solving this for ω_{in}/ω_o and substituting in equation 3.1 yields:

$$\frac{T_{o_i}}{T_{o_r}} = \frac{1}{g_h} \quad (3.16)$$

Equation 3.13 and 3.14 can now be substituted into Equation 3.16 and simplified to:

$$\frac{T_{in}}{T_{out}} = \frac{g_c g_g}{g_h + g_c (g_g + 1)} \quad (3.17)$$

Since it was assumed that the variable element has an efficiency of 1, the total system is assumed to have the same efficiency. The total power in equals the total power out which is expressed as the following equation:

$$\frac{W_{in}}{W_{out}} = -\frac{T_{out}}{T_{in}} \quad (3.18)$$

Substituting equation 3.17 into equation 3.18 results in equation 3.7. The two derivations match. This method was repeated to calculate the velocity ratio for the other configurations. The results using either of the methods can be used to calculate the span of velocity ratios and Γ , which is covered in the following sections.

3.4 Velocity Ratio Curve and Span of Velocity Ratios

In the previous sections the method used to derive the velocity ratio for each configuration was presented. The equation for the velocity ratio of each configuration can be found in the configuration charts in Appendix A. In every case γ_h is the only variable in the equation. The other terms are based on the geometry of the configuration and therefore remain constant. Because of this, the velocity ratio can be plotted against the variator's ratio (γ_h), producing a velocity ratio curve. Figure 3.5 is an example curve plotted for configuration 3.2. Arbitrary values for γ_g and γ_c , of 1.5 and .32 respectively, were used and γ_h was varied between 1 and 5. Inspection of the velocity ratio curves presented in the configuration charts, reveals two properties of the velocity ratio. The first, is that the sign of

the velocity ratio is either positive or negative. A positive velocity ratio signifies that the rotation of the output shaft is in the same direction as the input shaft, while a negative velocity ratio signifies the output rotates in the opposite direction as the input. The second property that is shown by the velocity curves is the relationship between the velocity ratio and γ_h . Depending on the configuration, the velocity ratio is either proportional to or inversely proportional to γ_h . This relationship is important in calculating the span of velocity ratios.

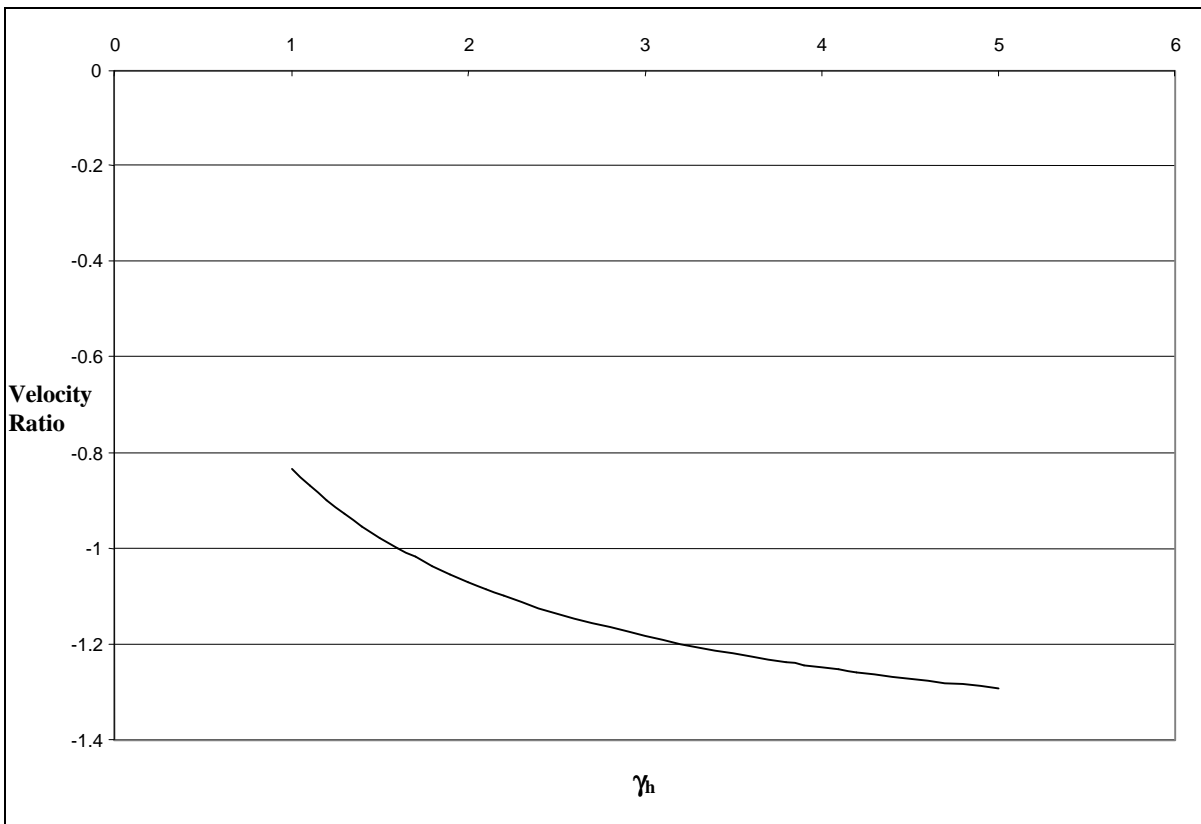


Figure 3.5: Velocity ratio curve for configuration 3.2 with arbitrary values of γ_h , γ_g , and γ_c .

The span for a configuration is a value that describes the range of velocity ratios that is covered by that configuration. The span of a configuration is defined as its low-speed velocity ratio divided by its high-speed velocity ratio. The larger the span is for a configuration, the more velocity ratios the configuration covers. The velocity ratio curves

can be used to compare the spans for different configurations. The larger the slope of a configuration's velocity ratio curve, the larger the span will be. The curves illustrate the trend of a configuration's velocity ratio, and allow the designer to compare the spans of different configurations without actually using specific numbers.

To calculate the span of a specific configuration, the velocity ratio equation and the relationship between the configuration's velocity ratio to the variable element's velocity ratio must be known. Equation 3.19 is the equation used to calculate the span for Configuration 3.2,

$$Span = \frac{\left[-\frac{\mathbf{g}_g \mathbf{g}_{hL}}{\mathbf{g}_{hL} + \mathbf{g}_c (\mathbf{g}_g + 1)} \right]}{\left[-\frac{\mathbf{g}_g \mathbf{g}_{hH}}{\mathbf{g}_{hH} + \mathbf{g}_c (\mathbf{g}_g + 1)} \right]} \quad (3.19)$$

where γ_{hL} is the low-speed velocity ratio of the variable element and γ_{hH} is the high-speed velocity ratio of the variable element. To determine the equation of the low and high-speed velocity ratios the velocity ratio curve was used. From inspection of the velocity ratio curve of configuration 3.2 it is determined that the velocity ratio is proportional to the velocity ratio of the variable element. Therefore, the low and high-speed velocity ratios occur at the low and high-speed velocity ratios of the variable element respectively. The method to determine the equation for the span of a different configuration, consist of simply using the derived equation of the velocity ratio for that configuration and the relationship of the velocity ratio to the velocity ratio of the variable element.

3.5 Power Flow through Variable Element

Gamma (Γ) is defined as the fraction of power that circulates through the variable element. The efficiency of the variable element and the input/output configuration of the transmission each have an effect on Γ . Γ can be calculated using an ideal case where the variable element's efficiency is 1 or using the actual efficiency of the variable element. Both cases are presented. The ideal case is used to determine the best configuration for a given application for two reasons. First, it is easier to derive. Second, the efficiency of the variable element will have an equal affect on all configurations. The second case, which takes into account the power loss due to the actual efficiency of the variable element, is used to specify the size of the variable element and to describe the transmission performance. The method of deriving the Γ for each case is shown in the following two sections.

3.5.1 Calculating Gamma in the Ideal Case

In this section, the ideal Γ for configuration 3.2 is derived. The example derivation describes the method used to derive the ideal Γ for all configurations. To derive Γ , the power flow through the differential must be analyzed. The following derivation of the power flow through a differential is from Mechanisms and Dynamics of Machinery (Mabie and Reinholtz 1987).

The first step in analyzing the power flow through the differential is labeling the rotating elements of the system as elements d, e, and f. These are illustrated in Figure 3.6. One of these elements is the arm that carries the planetary gears and the other two are gears on independent shafts. Element d is always the member that projects from the differential to a

point outside the system and connects to element e. Element e is always the member that transmits power to or from the differential to the branch control

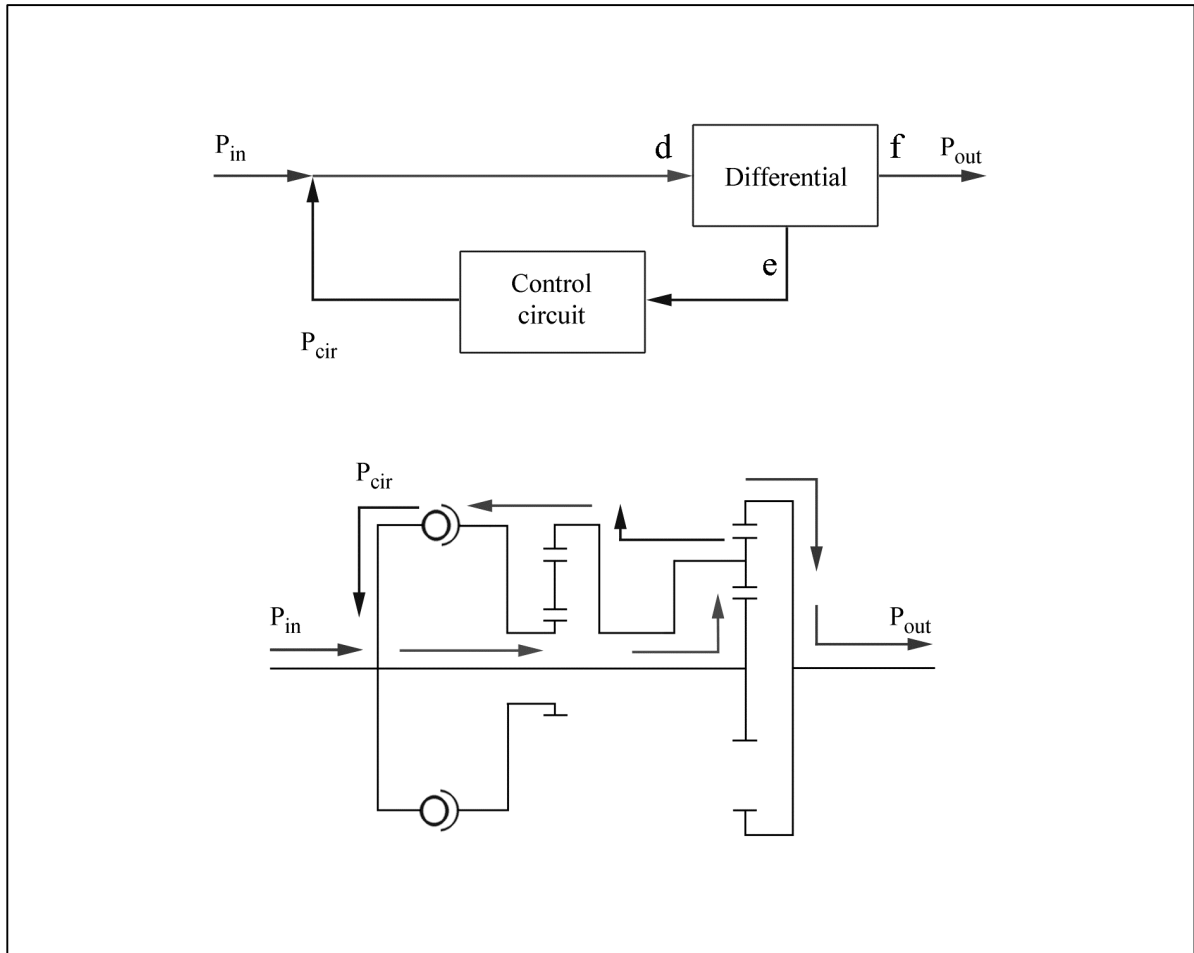


Figure 3.6: Ideal power flow through configuration 3.2.

circuit but does not transmit power directly to or from a point outside of the system. Element f is always the member projecting from the differential directly to a point outside the system but having no connection to the branch control circuit. Inspection of Figure 3.6 shows that for configuration 3.2, element d is the sun gear connected to the input shaft, element e is the planetary carrier, and element f is the ring gear connected to the output shaft. Once the elements are defined, it can be seen that element e is the member that circulates the power

through the variable element. Calculating the power flowing through element e gives the amount of power flowing through the variable element.

An expression for the amount of power circulating through the branch circuit can easily be developed from the general torque and energy relationships between the three elements. Γ is defined by equation 3.20

$$\Gamma = \frac{P_{cir}}{P_f} = \frac{T_e \mathbf{v}_e}{T_f \mathbf{v}_f} \quad (3.20)$$

where circulating power (P_{cir}) is the power flowing through the branch control circuit, the power flowing through element f is P_f , T_e and T_f are the torques transferred by elements e and f respectively and ω_e and ω_f are the angular velocities of those elements. By considering the differential as an isolated unit, the summation of torque's and powers can be equated to zero. This is shown in the following equations:

$$\sum T = T_d + T_e + T_f = 0 \quad (3.21)$$

$$\sum P = T_d \mathbf{v}_d + T_e \mathbf{v}_e + T_f \mathbf{v}_f = 0 \quad (3.22)$$

Solving the preceding equations simultaneously gives

$$\Gamma = \frac{\mathbf{v}_e (\mathbf{v}_f - \mathbf{v}_d)}{\mathbf{v}_f (\mathbf{v}_d - \mathbf{v}_e)} \quad (3.23)$$

or

$$\Gamma = \frac{r(1-R)}{1-r} \quad (3.24)$$

where

$$r = \frac{\mathbf{v}_e}{\mathbf{v}_d} \quad (3.25)$$

$$R = \frac{\mathbf{V}_d}{\mathbf{V}_f} \quad (3.26)$$

Γ can now be derived for configuration 3.2 by finding r and R by inspection. Since ω_d and ω_f by definition are equal to the input and output angular velocities respectively, R is equivalent to ω_{in}/ω_{out} . For configuration 3.2, ω_{in}/ω_{out} was derived and shown in Equation 3.7. By inspection of Figure 3.2 ω_e is equivalent to ω_c . Solving Equation 3.4 for ω_c/ω_{in} results in Equation 3.27 which is equivalent to r for configuration 3.2.

$$r = -\frac{\mathbf{g}_c}{\mathbf{g}_h} \quad (3.27)$$

Substituting Equation 3.7 and 3.27 into Equation 3.24, allows Γ to be calculated for this configuration. Using arbitrary values of $\gamma_g=1.5$, $\gamma_c=.32$, and varying γ_h between 1 and 5 an example curve for Γ vs. γ_h can be plotted, as illustrated in Figure 3.7.

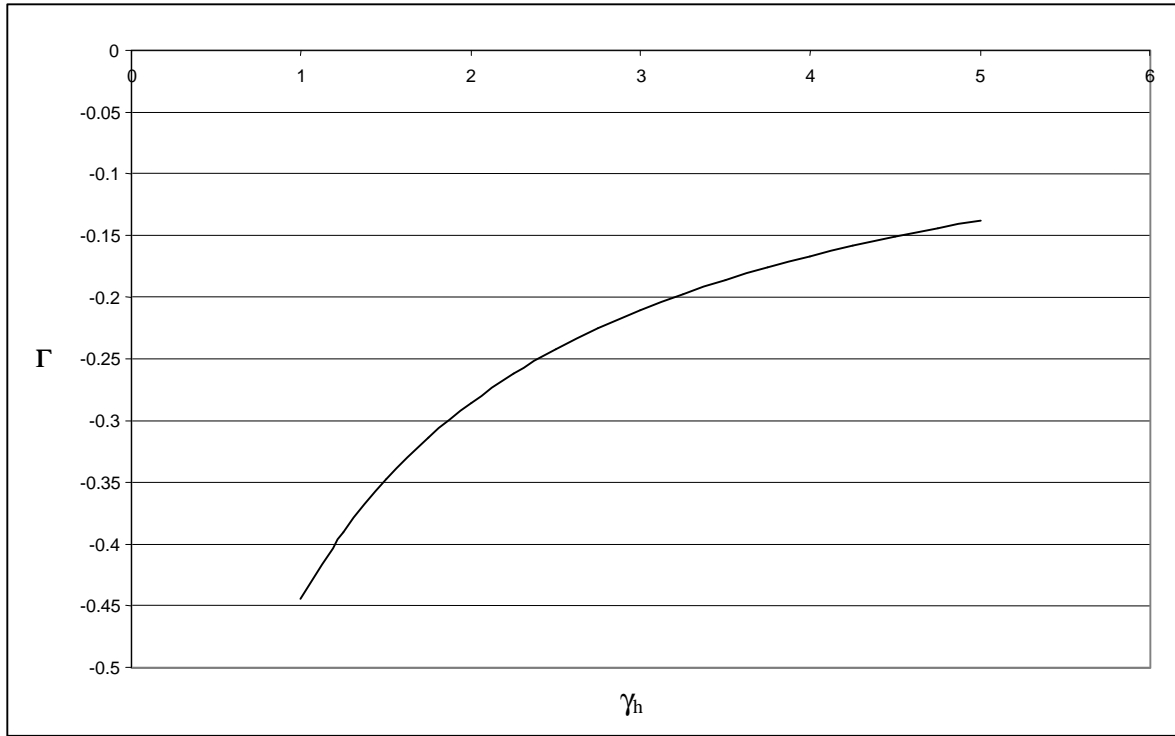


Figure 3.7: Γ vs. γ_h for configuration 3.2 using arbitrary values.

Several issues can be addressed once Γ is calculated for a configuration. First, if Γ is positive, then power flows into or out of the differential through both elements e and f. This is a power re-circulation case, where power is actually flowing backwards through the variable element. If Γ is negative, power flows into the differential through one of the elements (e or f) and out through the other. This is a power split case, since element f is always the output in an input coupled variable shunt transmission. Since Γ is the percentage of power that flows through the branch element, Γ greater than one or less than negative one means that there is a parasitic power flow. This is unsuitable because it means a variable element would have to be large enough to handle power greater than the input power. By using arbitrary values, plots of Γ vs. γ_h were created for each configuration and can be found

in Appendix A. The curves illustrate the trend of Γ for each configuration, and allow the designer to compare Γ of different configurations without actually using specific numbers.

3.5.2 Calculation of Gamma with Power Loss

Calculating Γ for the ideal case gives a good estimate of how the power flows in the system but does not give an adequate calculation of the power that is supplied to the variable element. The power supplied to the variable element is higher than the power circulated in the planetary gear train, due to the efficiency of the variable element. This case is illustrated below in Figure 3.8. This power (P_{O_i}) affects the sizing of the variable element and the overall efficiency of the transmission. To determine P_{O_i} the power balance of the system is determined. The method of deriving the power balance is presented here using Configuration 3.2 as an example.

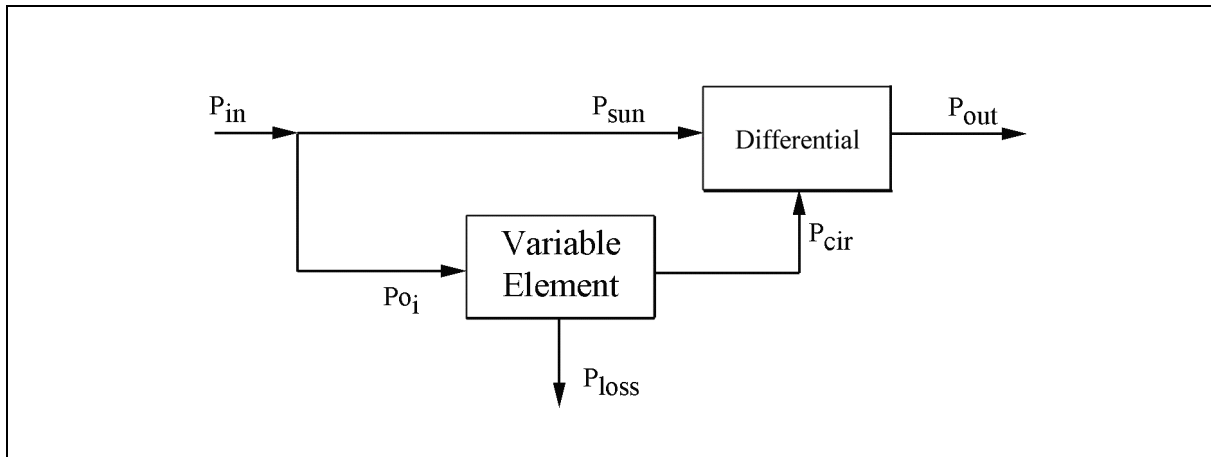


Figure 3.8: Power balance block diagram.

To develop the power balance for this system, the equations that govern the powers are determined from inspection of Figure 3.8. They are:

$$P_{sun} = P_{in} + P_{O_i} \quad (3.28)$$

$$P_{cir} = P_{O_i} - P_{loss} \quad (3.29)$$

$$P_{out} = P_{sun} + P_{cir} \quad (3.30)$$

The equation for P_{loss} using the definition of efficiency is

$$P_{loss} = P_{o_i} (1 - \eta) \quad (3.31)$$

where η is the efficiency of the variable element. P_{in} is known from the specification of the applications engine. If P_{o_i} is found, then all the other powers can be solved. P_{o_i} can be expressed as

$$P_{o_i} = T_{o_i} \omega_{in} \quad (3.32)$$

In this equation ω_{in} is known and T_{o_i} can be found by manipulating equations from section 1.3.2. The first step in deriving an expression for T_{o_i} is to substitute γ_g and γ_c into equation 3.13 and solve for T_{o_r} . This yields:

$$T_{out} = \frac{T_{o_r}}{g_c \left(\frac{1}{g_g} + 1 \right)} \quad (3.33)$$

Substituting this equation into 3.14 produces:

$$T_{o_i} = T_{in} - \frac{T_{o_r}}{g_c (g_g + 1)} \quad (3.34)$$

Rewriting equation 3.15 taking into account the efficiency of variable element results in the following equation:

$$hg_h T_{o_i} = T_{o_r} \quad (3.35)$$

Substituting equation 3.35 into 3.34 and solving for T_{o_i} results in an expression which can be used to solve for T_{o_i} . This expression is:

$$T_{o_i} = \frac{T_{in}}{1 + \frac{hg_h}{g_c (g_g + 1)}} \quad (3.36)$$

Now that there is an equation for To_i all of the powers in the system can be determined and a plot of the power balance can be created. Using the equations presented, a power balance curve for configuration 3.2 is illustrated in Figure 3.9. Arbitrary values of $\gamma_g = 1.5$, $\gamma_c = .32$, $P_{in} = 425 \text{ hp @ 2100 rpm}$ and varying a γ_h between 1 and 5 were used. For this example, an efficiency for the variable element of 80% was arbitrarily used.

The power balance illustrates several important design considerations. First, it allows the designer to size the variable element based on the maximum Po_i . Second, it illustrates that the efficiency of the variable element directly affects the size of the variable element and its effect should not be neglected. Comparing the ideal Γ from the last section to the power balance presented here, further supports this since there is a 5% difference in the input

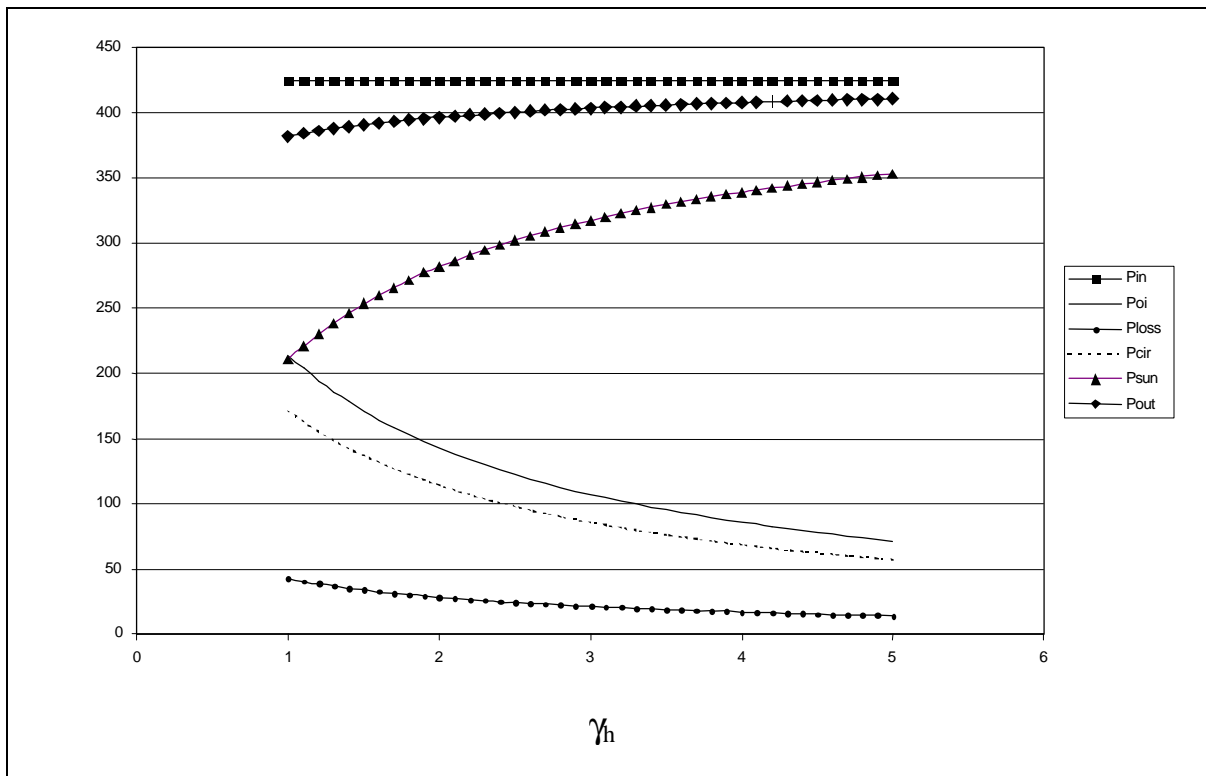


Figure 3.9: Example power balance curves for configuration 3.2.

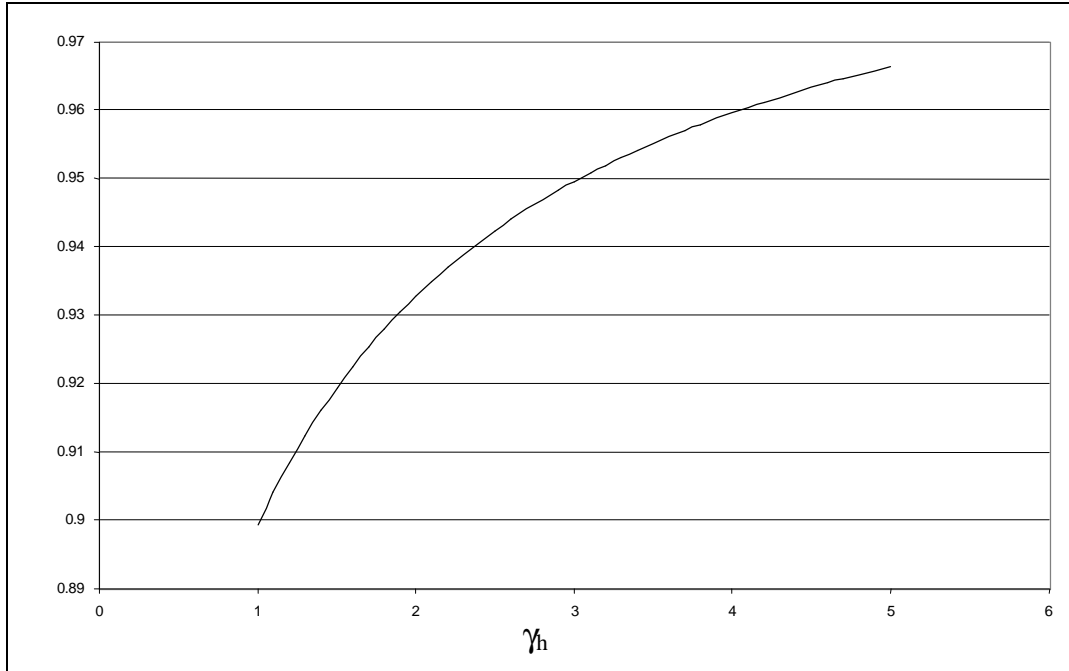


Figure 3.10: Estimated overall efficiency for example configuration 3.2.

power to the variable element. Therefore, the most efficient variable element not only increases the overall efficiency of the transmission but also decreases the size of the variable element and thus the overall size of the transmission. The last concept the power balance allows the designer to investigate is the overall efficiency of the transmission. The main power loss in the transmission is due to the efficiency of the variable element. Therefore, the overall efficiency can be estimated by dividing the power out by the power in across the entire operating range of the transmission.

3.6 Gamma – Span Relationship

The methods of calculating Γ and the span for a given configuration have been presented, but before any design can begin the relationship between the two must be understood. Changing either of these has an effect on the other. The general relationship

between the two is that the larger the span of a configuration, the larger Γ . This basic relationship is described using graphical analysis of an example.

Since equations for the span and Γ were derived for Configuration 3.2, this configuration will be used to examine the relationship between Γ and the span. Inspection of the equations (Equations 3.19, 3.7 and 3.27) reveal five variables: γ_g , γ_c , γ_h , γ_{hL} , and γ_{hH} . Two of these are determined by the designer, these are γ_g and γ_c . The others are defined by the variable element. There is an infinite combination of these two variables, that result in the same span for this configuration. This is illustrated in Figure 3.11, where the combinations of γ_g and γ_c were found for a span of 1.5. This was done, by solving Equation 3.19 for γ_c yielding:

$$g_c = \frac{g_{hL}g_{hH}(1 - span)}{(g_{hH}span - g_{hL})(g_g + 1)} \quad (3.37)$$

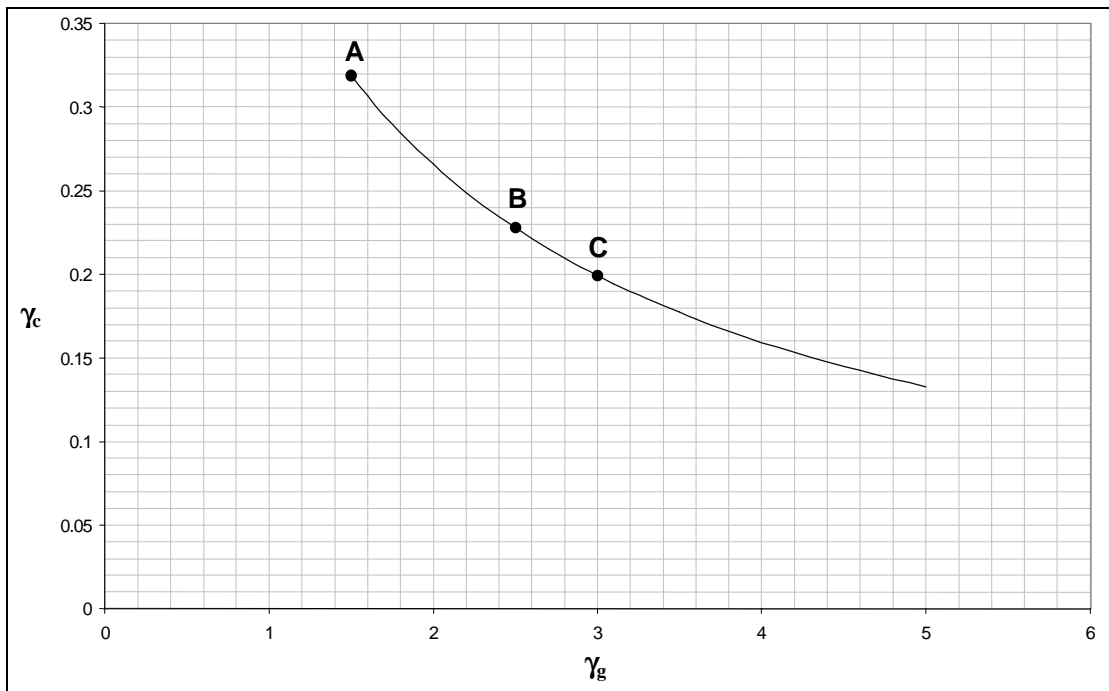


Figure 3.11: Possible combinations of γ_g and γ_c for configuration 3.2 with a span of 1.5.

Then by setting the span = 1.5, $\gamma_{hH} = 1$, $\gamma_{hL} = 5$, and varying of γ_g and solving for γ_c , several sets of data were calculated and used for the plot.

Since there are several combinations of γ_g and γ_c that yield the same span, their affects on Γ need to be understood to determine which of the combinations to use for a given application. To determine this, Equation 3.37 is substituted into Equations 3.7 and 3.27. The results of these substitutions are shown below.

$$\frac{w_{in}}{w_{out}} = - \frac{g_g g_h}{g_h + \frac{g_{hL} g_{hH} (1 - span)}{(g_{hH} span - g_{hL})(g_g + 1)}} \quad (3.38)$$

$$r = \frac{g_{hL} g_{hH} (1 - span)}{(g_{hH} span - g_{hL})(g_g + 1) g_h} \quad (3.39)$$

Γ is now in terms of γ_g and span. The effect of γ_g can be illustrated by calculating Γ while varying γ_g and fixing span and γ_h . This was done for several values of span and the data are illustrated in Figure 3.12. This figure shows that the combination of γ_g and γ_c results in the same Γ for any span. A curve can be plotted (Figure 3.13) which illustrates the relationship between the span and Γ of configuration 3.2. This figure is in terms of the absolute value of Γ and percentage of ratio coverage of the variable element. For example, if the variable element has a span of 5 and a span of 5 is required by the application then 100% of the power will flow through the variable element. The method of creating this plot can be used to create similar plots for each configuration. The plots of Γ vs. the ratio coverage are very useful and will be utilized in the design procedure.

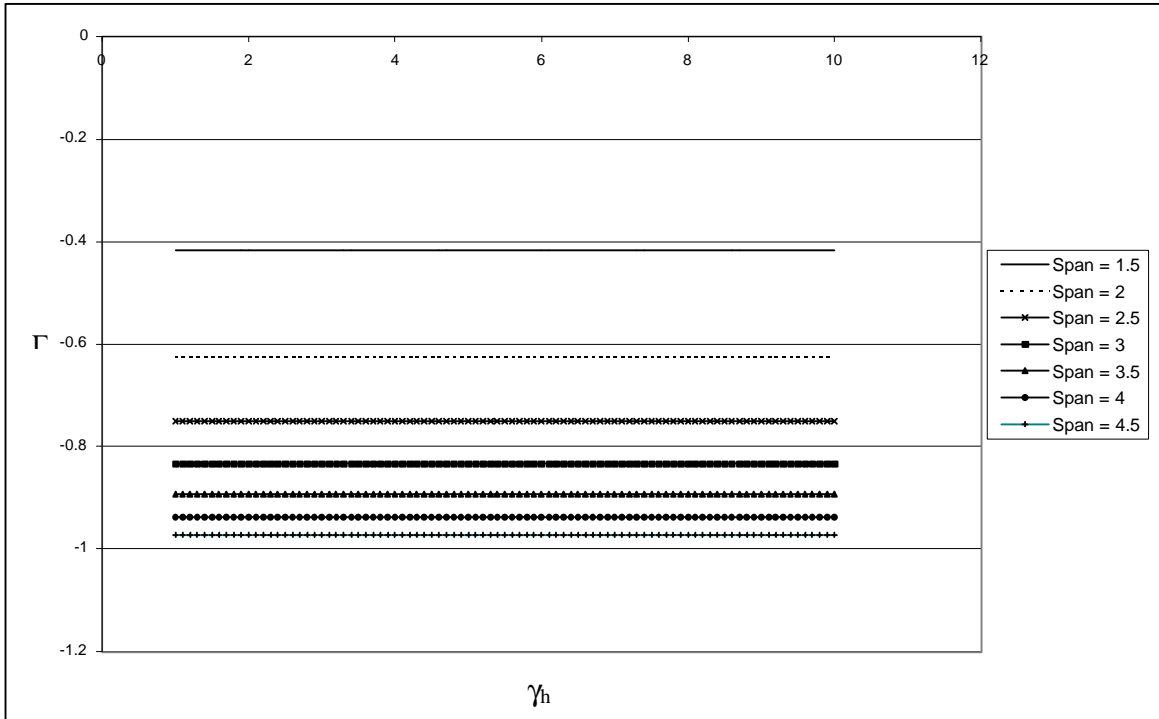


Figure 3.12: Γ vs. γ_g for configuration 3.2 with various spans.

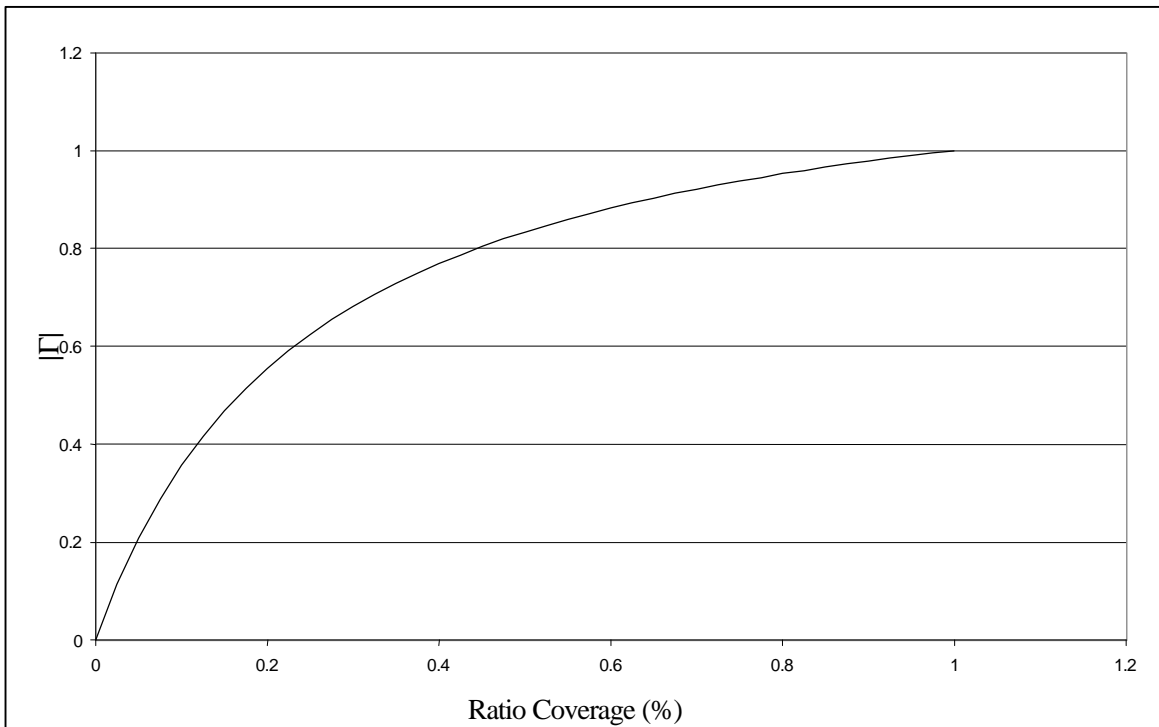


Figure 3.13: Absolute Γ vs. the ratio coverage of the variable element for configuration 3.2

Though the combinations of γ_g and γ_c which result in a certain span also equate to the same value of Γ , the different combinations have a large affect on the input to output angular velocity ratios of a configuration. Three combinations of γ_g and γ_c from Figure 3.11 (points A, B, and C) where used to plot the input to output velocity ratios for these points. This plot, Figure 3.14, shows that even though Γ is the same, the velocity ratios covered are different. This effect can be considered in the design procedure.

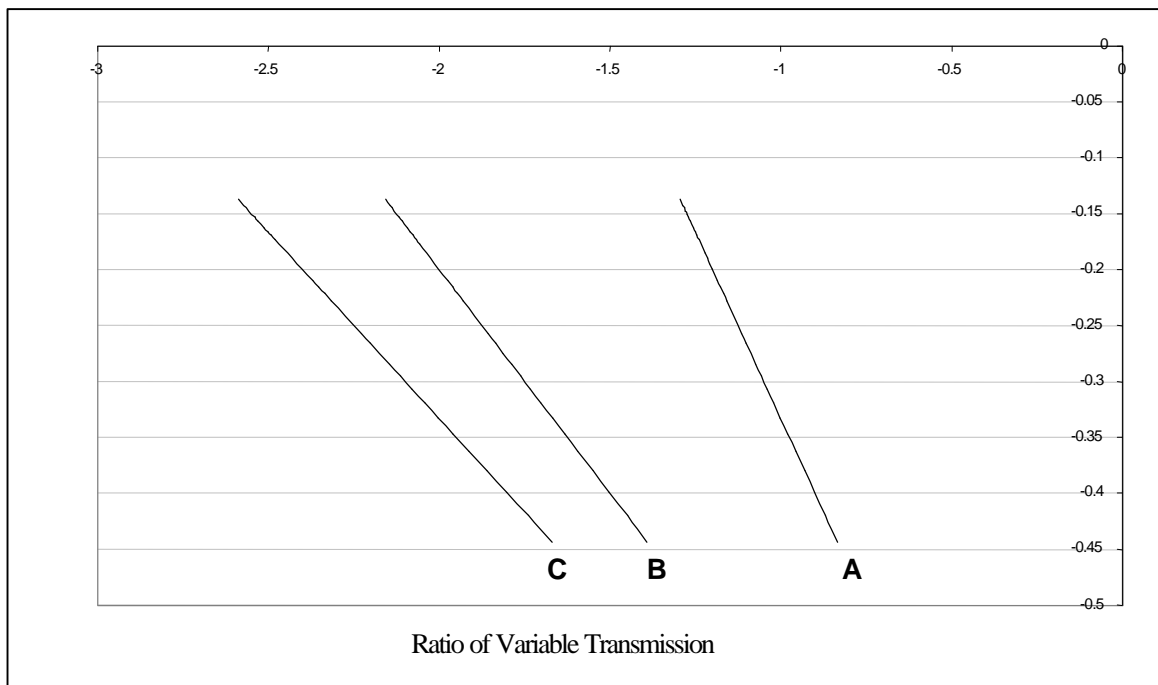


Figure 3.14: Comparison of the three combinations of γ_g and γ_c from Figure 3.10.

3.7 Variable Element

Any shaft to shaft continuously variable drive could be used as the variable element in a PS-CVT. For this study, two types of hydraulic elements are considered. Both are used in examples of the design procedure. These are hydrodynamic couplings and hydrostatic drives. Information was acquired on both types, and used below to describe them in detail.

3.7.1 Hydrodynamic Couplings

The first variable element used in an example of the design procedure is a hydrodynamic coupling. In this report, information from catalogs provided by the company Voith Transmissions Inc. were used. The type of variable speed turbo coupling selected is illustrated in Figure 3.15 (Voith 1997).

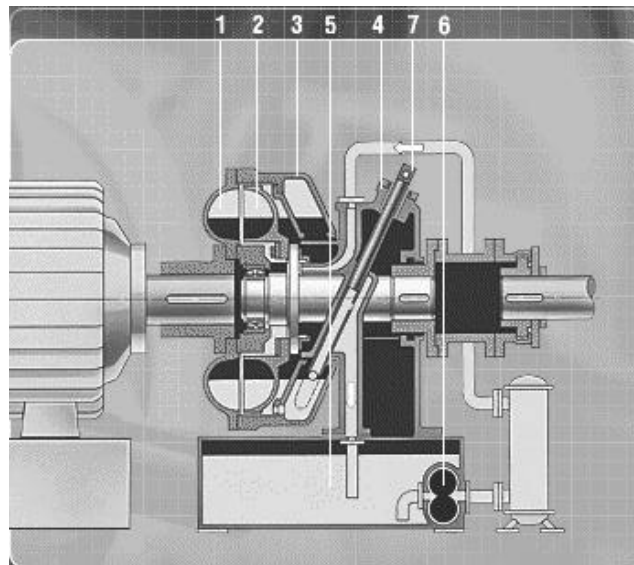


Figure 3.15: Voith SVN hydraulic variable speed turbo coupling.

The hydraulic coupling is a fluid coupling which transmits the power input by the mass forces of a fluid which is circulated in a closed system between an impeller on the driving shaft and a runner on the driven shaft. The impeller and runner are labeled 1 and 2 respectively in Figure 3.15. The oil fill in the working compartment can be varied between

completely filled and drained. The amount of oil in the working compartment determines the speed at the output side of the coupling and is dependent upon the position of the scoop tube. The scoop tube is labeled 7 in the figure. The Voith SVN hydraulic coupling is available in sizes from 100 to 1000 kW (Voith 1997). The actual selection of the coupling is associated with operational ranges required for the transmission.

The performance characteristics of these type of hydraulic couplings are illustrated in Figure 3.16 (Voith 1997), in which a "decreasing" torque curve for the output shaft is illustrated. The speed ratio refers to the output-to-input velocity ratio (ω_{out}/ω_{in}) as does the relative torque (T_{out}/T_{in}). The family of curves indicates the scoop tube position changing from minimum (almost horizontal) to maximum (almost vertical).

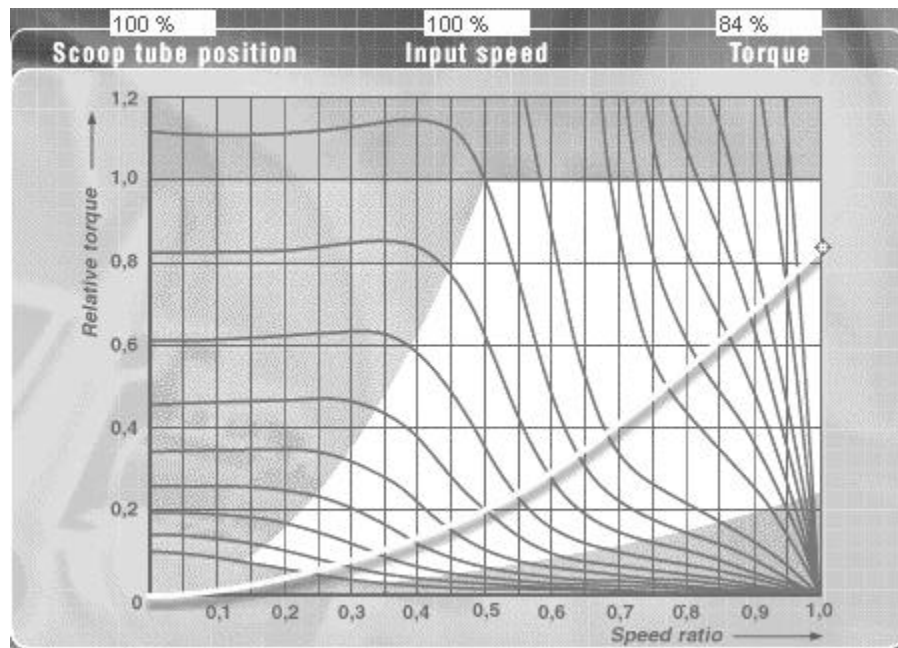


Figure 3.16: Relative torque versus speed ratio (ω_{out}/ω_{in}) for hydrodynamic coupling from the Voith Catalog.

3.7.2 Hydrostatic Drives

There were several companies that manufacture hydrostatic drives, and in general they have similar features. For this design study information provided by Sauer-Sundstrand was used. A typical hydrostatic drive is illustrated in Figure 3.17 (Sauer-Sundstrand 1997).

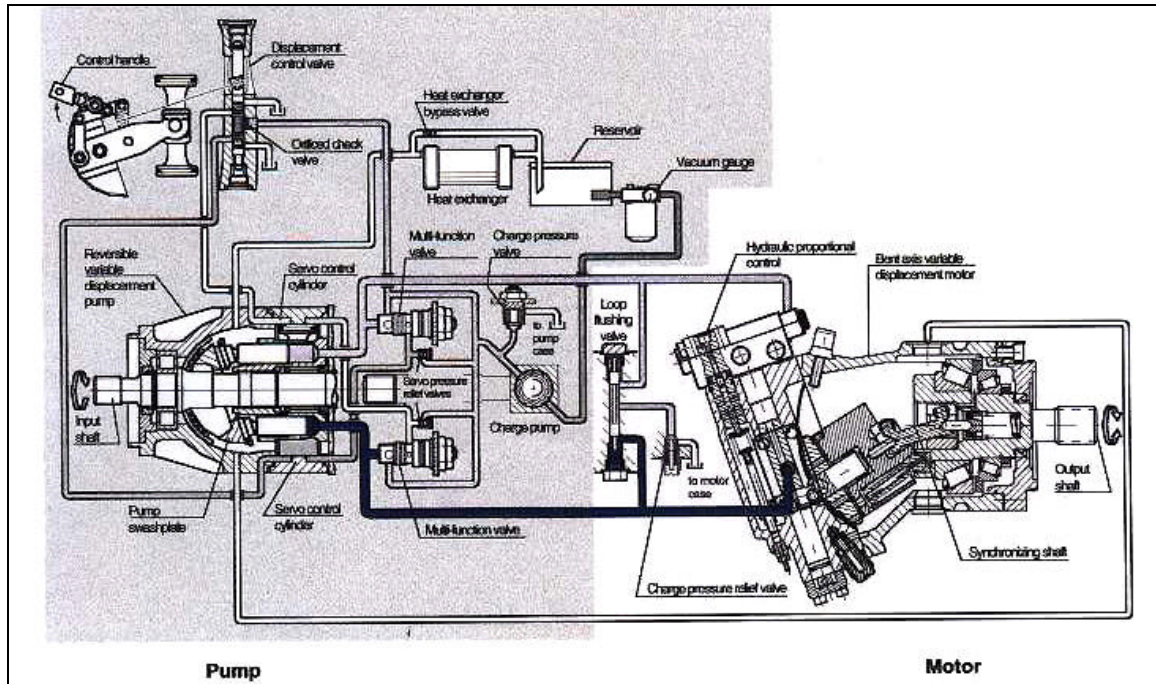


Figure 3.17: Hydrostatic transmission configuration with a variable pump and motor.

A hydrostatic drive consists of 2 main components a hydraulic pump and a hydraulic motor. Each of these can have either a fixed or variable displacement. These 4 components can be combined to create 4 different circuits, which exhibit different performance characteristics. It was determined that a variable displacement pump driving a variable displacement motor was the best circuit for this application (Bretz 1998).

In general these types of elements are more efficient overall and more compact than torque converters and hydrodynamic couplings. The efficiency of a hydrostatic drive is based on three factors. These factors are the percentage of maximum displacement, percentage of rated continuous speed, and system pressure. Figure 3.18 shows a typical

efficiency plot for a Series 51 Sauer-Sundstrand motor. Note that this is only for maximum displacement.

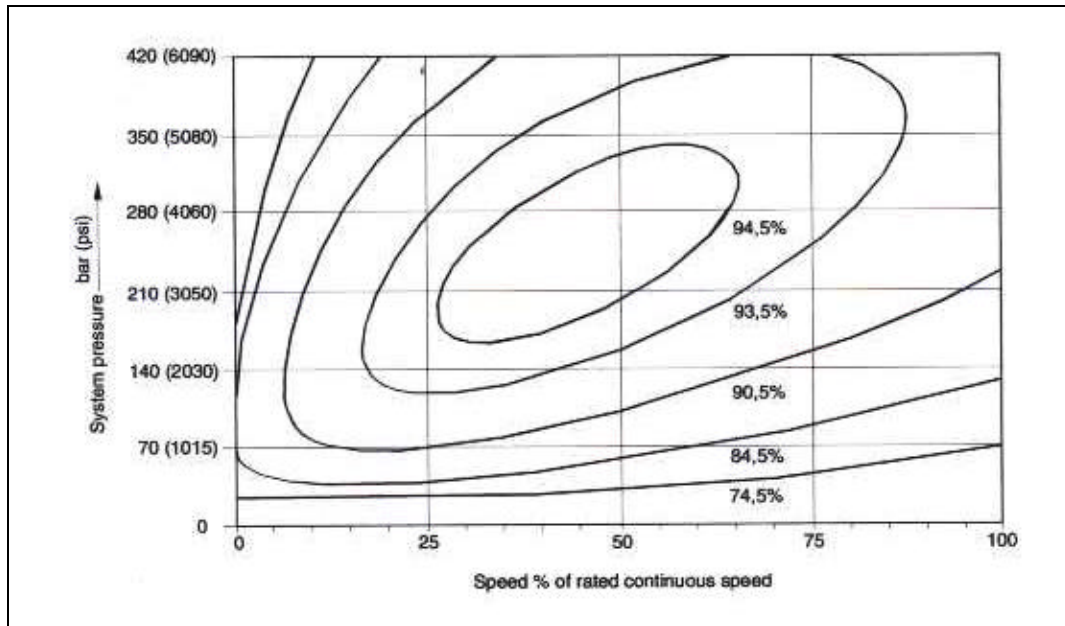


Figure 3.18: Overall efficiency at maximum displacement for Sauer-Sundstrand Series 51 motor.

To determine the pump and motor combination that would best suit the two applications, Sauer-Sundstrand was contacted. Due to the large number of variables in selecting a pump and motor combination and obtaining efficiency data for the configurations Sauer-Sundstrand used computer modeling. Using input and output information provide by the theoretical calculations Sauer-Sundstrand was able to match a hydrostatic drive to meet the needs of the example application. The specifics are presented later in the example of the conceptual design procedure.

Chapter 4 Conceptual Design Procedure

4.1 Introduction

Given the engine characteristics, output requirements, and eligible configurations, how does one design the gear train? This involves determining the gear ratios, sizing the gears and selecting a variable element, and is usually the result of a conceptual design procedure. This chapter outlines the steps required to complete a conceptual design for a multistage power split continuously variable transmission for an industrial application.

4.2 Conceptual Design Procedure

The complete design procedure has been outlined in flow chart form, Figure 4.1. The flow chart contains all the necessary steps required to develop a conceptual design of a multistage PS-CVT. This process starts with defining the input variables, which are the input power, input speed range, and output speed requirements, and ends with selected gear ratios and sizes, specification of variable element and theoretical efficiency of the design. The design procedure contains 14 steps, which are numbered in Figure 4.1. To help the reader follow the steps of the flow chart in the description below, step numbers are included in parentheses at the beginning of each step description.

To begin the conceptual design procedure, the specifications of the application must be defined (1). The values that are required are: input power, the operating speed range of the input device, and the transmission output speed range. Once these specifications are determined, the actual process can begin. The first step in the procedure is to determine the maximum allowable Γ (2). The larger the allowable Γ , the larger the variable

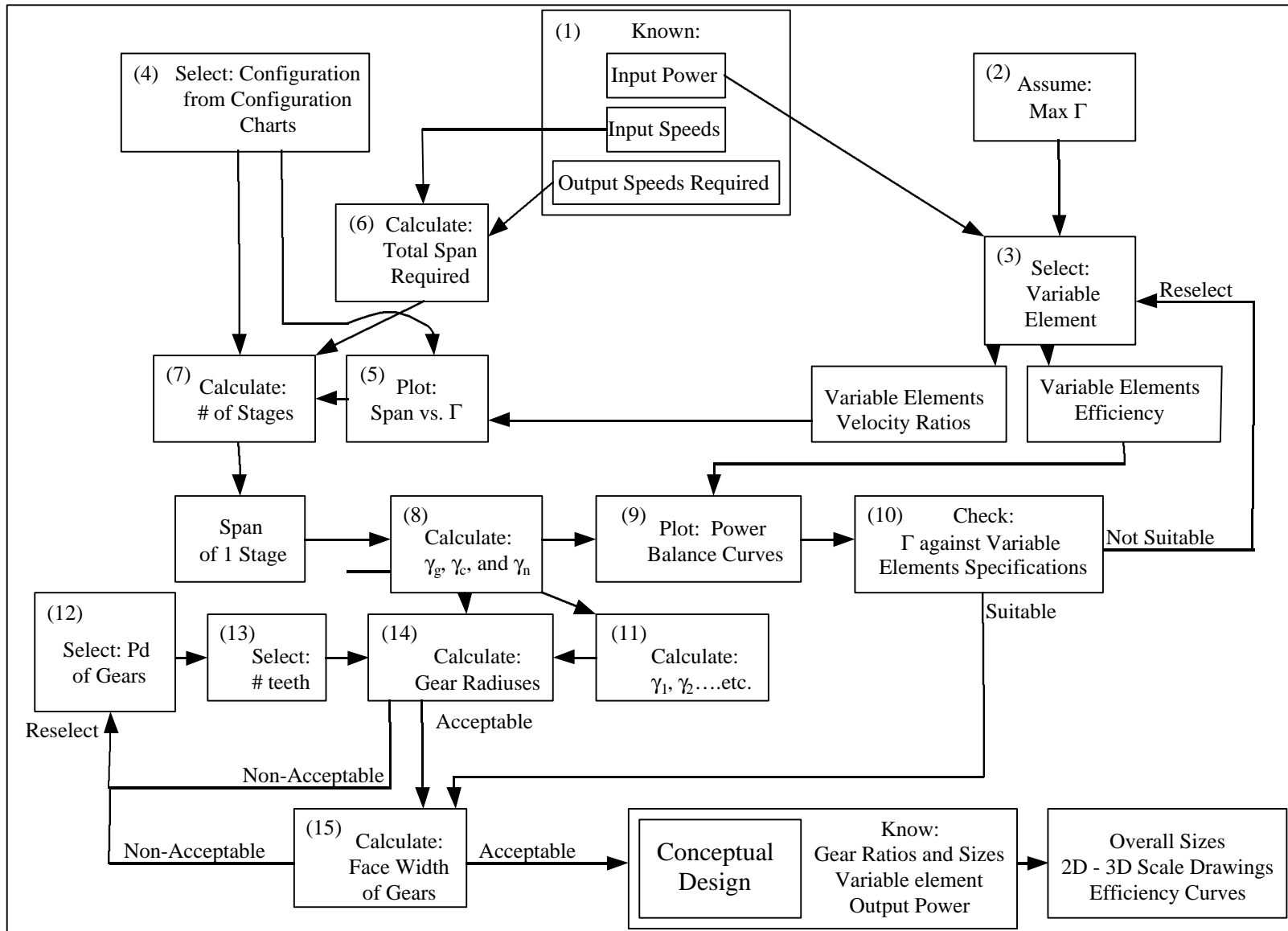


Figure 4.1: Conceptual design procedure flow diagram.

element. Smaller Γ result in the less power flow through the variable element and therefore, a higher overall efficiency for the transmission. Based on the sizes of the power capacities of the variable elements available and the total input power the designer can estimate a maximum Γ . After the maximum Γ has been determined, it can be used in combination with the input power to select a variable element (3). An initial variable element is selected from available literature (this selection will be checked later in the procedure). Selection of the variable element results in specification of the velocity ratio range and the efficiency of the variable element.

The next selection made by the designer, is the configuration to use for the conceptual design (4). The designer uses the charts, found in Appendix A, and selects one of the sixty combinations based on the Γ and velocity ratios curves which meet the requirements of the application. Once this step is completed, the equations for the properties of that configuration can be used with the variable element's velocity ratio range to plot a span vs. Γ curve (5). From this curve, the allowable span for a stage due to the maximum Γ is determined. Next, using the input speed range and the transmission output speed range, required the overall span of the transmission can be determined (6). Using the allowable span for a stage, and the overall span required, the number of stages that is needed can be determined (7). Starting with one stage, the number of stages is increased until the span required for a stage is less then the allowable span. If the span required for a given number of stages is slightly larger then the allowable span and does not warrant another stage, the maximum Γ can be slightly increased. This can be done without returning to step 2 to repeat all the steps as it will be checked later in the design procedure.

The span required for one stage has been calculated and the process of determining gear ratios and sizes can begin. The first ratios that are calculated are the ratios of the control unit (γ_c and γ_n) and the planetary gear train (γ_g) (8). These can be determined using the relationships developed in the previous chapter. The next step is to plot the power balance curves. This is done using the gear ratios from step 8 and the efficiency of the variable element which was a result of step 3. The main product of this plot is a value of the actual power that flows through the variable element (Γ). This actual Γ is used as a check to determine if the specified variable element can handle the maximum calculated power from the power balance curves (10). If the specified variable element is not suitable then a new one must be selected and the process repeated to this point until a suitable one is found.

The next step is to determine the ratios for the external gearbox, γ_1 , γ_2 , etc. (11). These are the ratios which are used to vary the stages, and thus the number of stages determines the number of ratios which need to be calculated. The first ratio is determined by making the low speed ratio of stage one equivalent to the lowest speed ratio required. Then the second stage low speed ratio is made equivalent to the high speed ratio of stage two. This process is repeated until all the ratios have been calculated. If this process is done correctly the high speed ratio of the last stage should match the high speed requirement of the transmission. When these values are determined, all of the transmission gear ratios are known.

The next two steps, determining the diametral pitch (P_d) and the number of teeth for each gear, determine the sizes (diameter and face width) of the gears. Changing either of these values effects the sizes of the gears. The process to determine the diametral pitch and number of teeth for each gear is iterative. The acceptability of the sizes is left to the discretion of the designer.

To start the iterative process of sizing the gears, a diametral pitch is assumed (12). The diametral pitch may be different for each gear ratio. Generally the larger the diametral pitch the narrower the face width for the same diameter gear. Larger diametral pitches should be used for the gear box ratios because they transfer all the force through one set of teeth. The control gears and the planetary gear train may have smaller diametral pitch because they have three pairs of teeth in contact. The initial assumption will be checked. However, a good initial assumption will result in less iterations to complete the procedure. Regardless of the accuracy of the initial assumption, an acceptable diametral pitch will be found by following the process. After a diametral pitch is assumed, the number of teeth for each gear set is determined to match the desired ratio (13). The only constraint here is that gears sets on parallel shafts must have the same center distances. This can be checked with standard gear equations.

Once the diametral pitch and the number of teeth for each gear has been assumed, the radii for each gear can be calculated using standard gear equations (14). The acceptability of the values calculated for each gear is up to the discretion of the designer. The criteria used for the gears radii can be related to a number of factors, which are determined by the application. If the gear radii are not acceptable the diametral pitch or the number of teeth per gear must be adjusted. Once acceptable radii have been found for all gears, the gears face

widths can be calculated using AGMA standards (Avallone and Baumeister 1996, Shigley and Mischke 1989). Power balance curves are used to determine the loads being transferred by each gear set. The acceptability of the gear face width is again, at the designer's discretion. When acceptable gear sizes have been determined the conceptual design procedure has been completed.

Several things have been determined in the conceptual design procedure. These are the gear ratios and sizes, the variable element, and the output power. Using this information the designer can: estimate the overall size of the transmission, develop 2D and 3D drawings, and produce efficiency curves. This information is useful in determining the feasibility of a multistage power split continuously variable transmission to fulfill the requirements of an application.

Chapter 5 Example Conceptual Design

5.1 Application Description

In this chapter, the conceptual design procedure is applied to the development of a power split continuously variable transmission for an industrial application. In general, this process, may be used as the initial step in determining whether a PS-CVT is a viable alternative. To determine the feasibility for an application, the procedure addresses the following issues.

1. Can a power split configuration be found which meets the input/output speed and power requirements of the application?
2. Is the overall size of the transmission within the design requirement?
3. What is the estimated efficiency of the resulting PS-CVT design?

Once favorable results are obtained from the design procedure described here, transition to the refined design stage is made.

An industrial pump has been chosen for use as example of an industrial application. In the current pump system an electronically controlled automatic transmission is used to transmit power from the power source to the pump. The conceptual design procedure is utilized to determine if a PS-CVT is a feasible replacement option for this application. Two conceptual designs, utilizing different variable elements, are considered in the study. One uses a hydrodynamic coupling and a second one uses a hydrostatic drive.

5.2 Conceptual design using a hydraulic coupling

The first step in the procedure was to define the application. The power source for the industrial pump is a 425 hp diesel engine that operates between 1300 and 2100 rpm. Speed reduction in this system is achieved with a transmission and 8.6:1 worm gearing on the pump. This combination results in pump operation speeds of 47 to 244 rpm and transmission output speeds of between 408 and 2100 rpm. The PS-CVT will be replacing a 4 speed automatic transmission with ratios varying from 3.19:1 to 1:1. The block diagram, Figure 5.1, illustrates the application requirements.

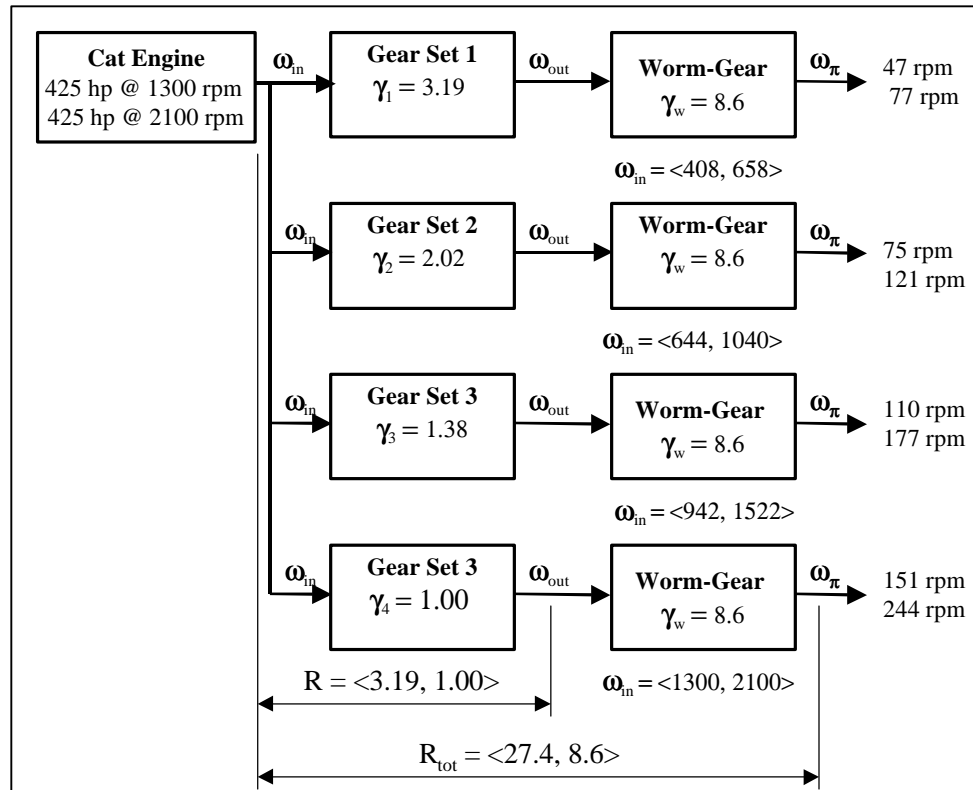


Figure 5.1: Block diagram of automatic transmission to be replaced in example application.

For step 2 of the procedure, a maximum power fraction of .5 was assumed implying that half of the input power will flow through the variable element. With this information, a coupling was selected from the Voith (1997) catalog, completing step three. A Γ of .5 and a

maximum input power of 425 hp, requires that the selected coupling have a capacity above 212 hp. The table in catalog (Voith 1997) indicates that the smallest unit that meets this requirement is a Voith SVN 316, which has a 316 hp capacity. The coupling velocity ratio range (range of γ_h) varies between 5:1 to 1:1, and with an efficiency of approximately $1/\gamma_h$.

Step 4 consists of PS-CVT input-output configuration selection from the configuration charts found in Appendix A. Since a hydrodynamic coupling is utilized in this example, the Γ range must fall between 0 and -1 , implying that the power is never regenerative. The inclusion of a reduction gearbox in the PS-CVT requires that the velocity ratio be negative such that the input and output of the entire transmission will rotate in the same direction. For this application, configuration 3.2 was selected. This configuration offers the advantages of a large, negative velocity ratio span while fulfilling the requirements for Γ , as stipulated above.

Step 5 utilizes the range of velocity ratios resulting from step three to construct a plot of the span- Γ relationship. For this application, this relationship was the same as described in the example plot (Figure 3.13) presented in chapter 3. Calculation of the total span required for the application was accomplished by dividing the low speed ratio by the high speed ratio of the automatic transmission being replaced. This results in an overall span for this application of 3.19, completing step 6.

Step 7, involves the calculation of the number of stages required to cover the overall span for the application. This was accomplished by an iteration process using the following equation.

$$span_i = \sqrt[N_s]{span} \quad (5.1)$$

In the equation, span_i is the span required for each stage, N_s is the number of stages, and span refers to the overall span required for the application. Each iteration begins by assuming a number of stages and computing the span required for each stage using Equation 5.1. An additional 5% is added to the calculated span in order to account for discrepancies that occur downstream in the process when gears are selected to match theoretical values. The resulting span is then divided by the total ratio range of the variable element which produces a value referred to as the “ratio coverage” of the variable element. Once the ratio coverage is determined, the span- Γ plot constructed in step 5, is utilized to ascertain the value of Γ that corresponds to the span for the number of stages used. This value is then compared to the maximum allowable Γ selected in step 2. If it is below the allowable value, then the number of stage is acceptable. If not, the iteration must be repeated using a larger number of stages. This iteration process is completed for the example configuration starting with a single stage and adding one stage with each iteration until a Γ of less than .5 is obtained. For this application use of a single stage results in an overall span of 3.35 (after 5% is added), which corresponds to a coupling ratio coverage of 58%. The Γ value obtained from the span- Γ curve, Figure 3.13, was over 80%, which is above the maximum allowable. Repeating the process with two stages, results in a value for Γ of 59%. With three stages, the span of each stage becomes 1.52 and the resulting value of Γ was calculated to be 42% which is below the maximum allowable. The iteration process was then complete with three stages proving to be sufficient for the system requirements.

In step 8, the values for γ_g and γ_c were determined. This involves finding combinations of the two unknown variables that produce a span of 1.52 and was accomplished using Equation 3.37 which is:

$$g_c = \frac{g_{hL}g_{hH}(1 - span)}{(g_{hH}span - g_{hL})(g_g + 1)}$$

The results were plotted in Figure 5.2. The PS-CVT contains two reductions.

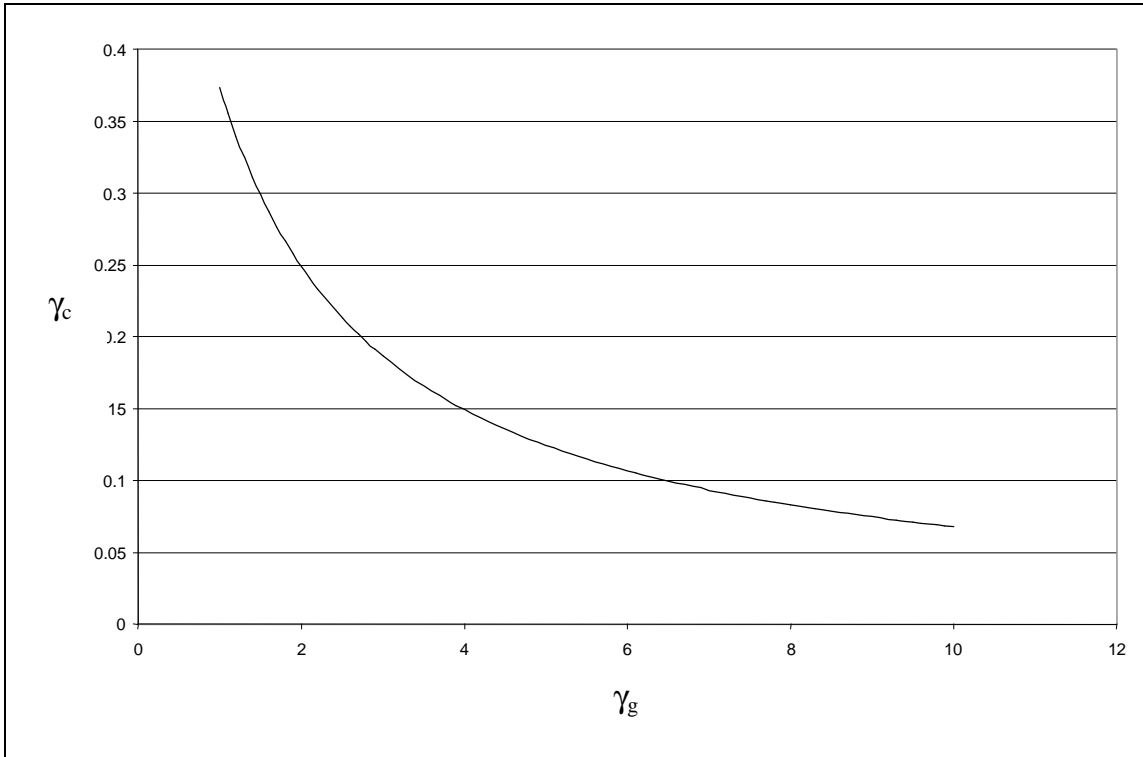


Figure 5.2: Possible combinations of γ_g and γ_c which yield a span of 1.52 for example application.

These include the power split unit, which is the input/output configuration and the gearbox, which contains the reductions for each stage. It is desirable that the resulting values of γ_g and γ_c produce the smallest gear sizes for the two planetary gear trains of the power split unit. In Appendix A, γ_g and γ_c are defined as N_R/N_S and N_o/N_c respectively. Where N_S and N_o are the number of teeth on the sun gears and N_R and N_c are the number of teeth on the ring gears of these planetary gear trains. Since it is desirable to minimize gear size tooth counts for the sun gear is held at a fixed assumed, minimum value while N_c and N_R are allowed to vary. Figure 5.2 illustrates the inverse relationship between γ_g and γ_c . Since N_o and N_S are held

fixed. A decrease in γ_c must be accomplished by an increase in N_c . Similarly, as γ_g increases, N_R must be increased. Consequently moving left to right along the γ_g - γ_c curve results in increasing ring gear sizes. To reduce the size of these gear trains a γ_g and γ_c are selected from the left portion of the curve. The minimum value that can be used for γ_g , considering geometric limitations, has been determined to be 1.5. The corresponding γ_c is .3. These are the values used for the example application.

Step 9 of this procedure involves plotting a set of power balance curves using the input power and the efficiency of the coupling. The equations for the powers of configuration 3.2 were derived in chapter 3. The following Equation 3.36 was used to solve for T_{O_i}

$$T_{O_i} = \frac{T_{in}}{1 + \frac{hg_h}{g_c(g_g + 1)}}$$

The value obtained was then substituted into the following equations (Equations 3.28 through 3.32) to obtain the powers of the system.

$$P_{sun} = P_{in} + P_{O_i}$$

$$P_{cir} = P_{O_i} - P_{loss}$$

$$P_{out} = P_{sun} + P_{cir}$$

$$P_{loss} = P_{O_i} (1 - \eta)$$

$$P_{O_i} = T_{O_i} \omega_{in}$$

The resulting powers are plotted against γ_h and is shown below in Figure 5.3. P_{O_i} is the amount of power that is transmitted through the coupling and has a maximum value of 180 hp as shown by the power balance curves. This is below the capacity of the selected coupling.

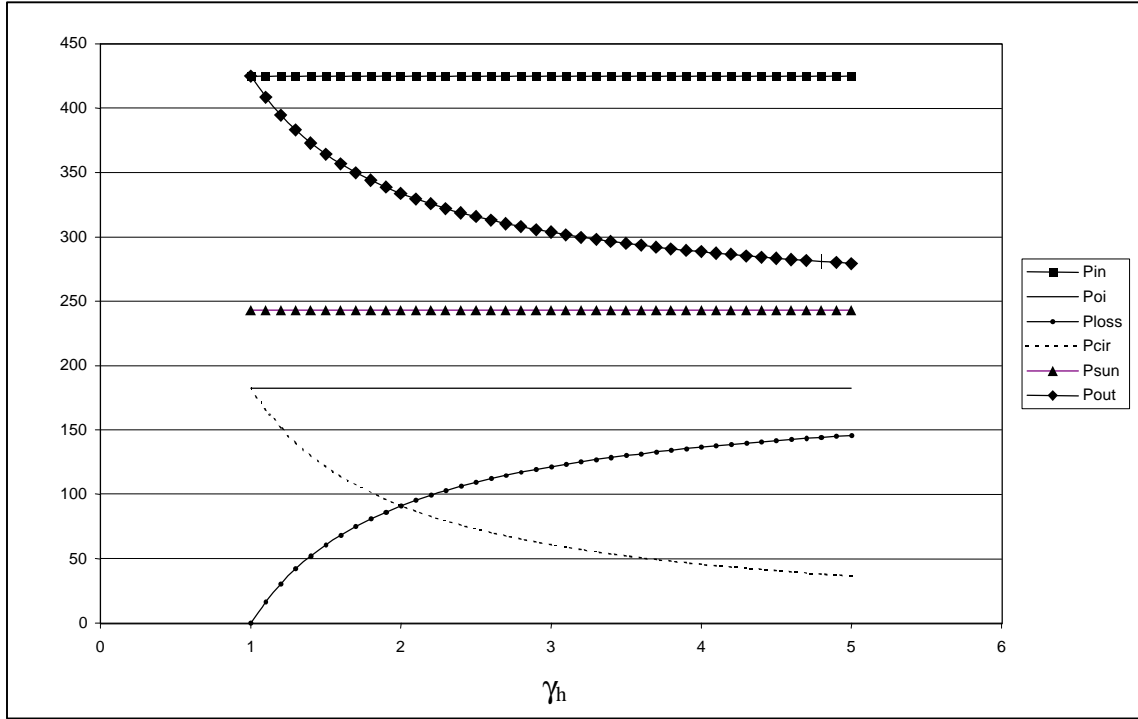


Figure 5.3: Power balance curves for example application.

In step 10 of the procedure the gearbox ratios, γ_1 , γ_2 , and γ_3 are determined. The power split unit reduction is fixed by the selection of γ_g and γ_c . The low and high speed ratios of the power split unit were then calculated by substituting γ_{hL} and γ_{hH} for γ_h in the velocity ratio equation. For configuration 3.2, Equation 3.7,

$$\frac{w_{in}}{w_{out}} = -\frac{g_g g_h}{g_h + g_c (g_g + 1)}$$

was used to determine these values. In this application γ_{hL} and γ_{hH} are 5 and 1 respectively. Substitution of these values results in a low speed ratio of -1.3 and a high speed ratio -.85. The gearbox ratios for each stage are calculated to meet the required ratios of the application. To determine γ_1 , the lowest transmission ratio was divided by the low speed ratio of the power split unit. This yields a value for γ_1 of -1.55. The high speed ratio of this stage was obtained by multiplying the high speed ratio of the power split unit by γ_1 . The high speed

ratio of stage one is 2.17, for stepless shifts to occur, this must also be equal to the low speed ratio of the transmission for stage two. Calculation of γ_2 is accomplished by dividing this low speed ratio by the low speed ratio of the power split unit. This results in a value for γ_2 of -1.73. The high speed ratio of stage two was calculated using the same method as described for stage one. This process is repeated one more time in order to determine γ_3 , which was -1.15. As a final check the high speed velocity ratio should be computed and compared to the overall required high speed velocity ratio. If the high speed ratio for the final stage was less than or equal to the overall required high speed ratio then the gearbox ratios have been determined correctly. For the example, the high speed ratio was .97, which compares favorably to the target value of 1.0.

Steps 12 through 15 calculate the gear sizes. A spreadsheet was set up to make this part of the procedure more efficient, since it requires several iterations. The resulting gear sizes for the example application can be found in Appendix B. The first step in sizing the gears was to select a diametral pitch. The initial selection of a diametral pitch for the planetary and control gears was 8. For the gearbox gears a value of 5 was chosen. The initial number of teeth for each gear was selected to achieve the desired ratios calculated above. Gear selection was subject to the following two additional constraints. For the two planetary gear trains, the radius of the sun plus the diameter of the planet or idler should always equal to the radius of the ring gear. Since the gear ratios γ_1 , γ_2 , and γ_3 are on parallel shafts, the center distance between the gear sets should also be kept equal. The face width of each gear was calculated using AGMA bending strength standards (Avallone and Baumeister 1996, Shigley and Mischke 1989). The tangential load on each gear was calculated using the maximum powers transmitted by the gears, which was found from the power balance curves.

By selecting a gear material and setting all the constants to standard values, the face width was calculated. The requirements for this application are to keep the ring gears of the planetary and control gear trains smaller than the diameter of the coupling and to keep the face width of all the gears less than 1.5 inches. After several iterations using the spreadsheet, satisfactory values were obtained. This completes the conceptual design procedure. The conceptual design is illustrated with two diagrams. Figure 5.4 is a wire-frame schematic diagram, which relates to Figure 5.5, the block diagram that describes the ratios and velocities of the transmission.

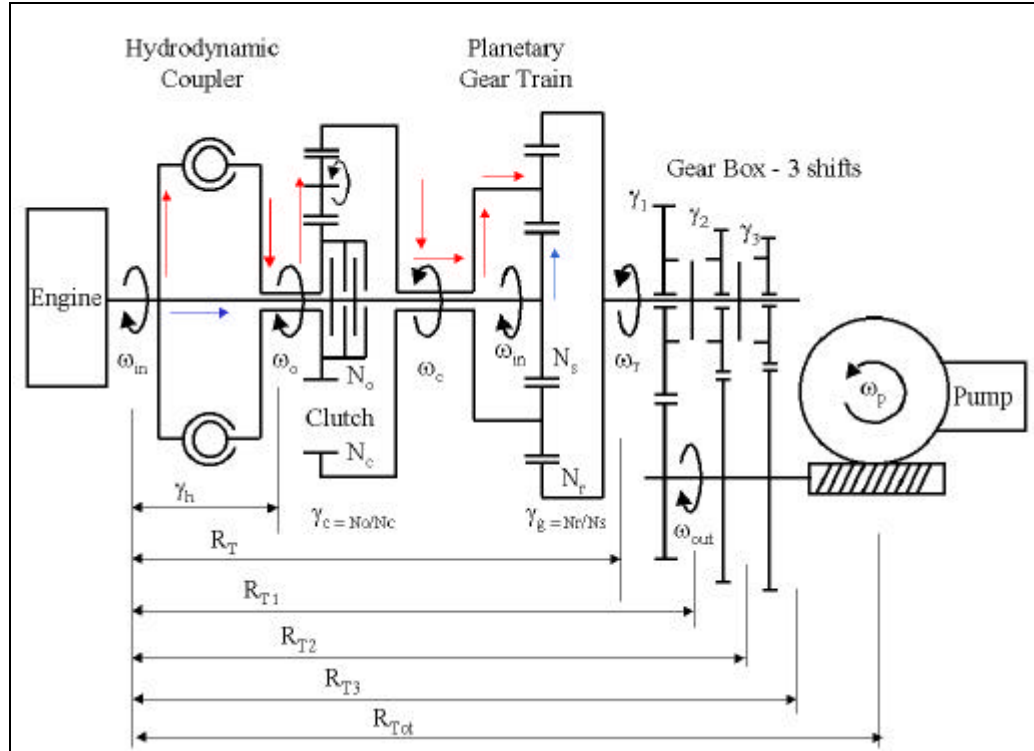


Figure 5.4: Description of example conceptual design system with coupling.

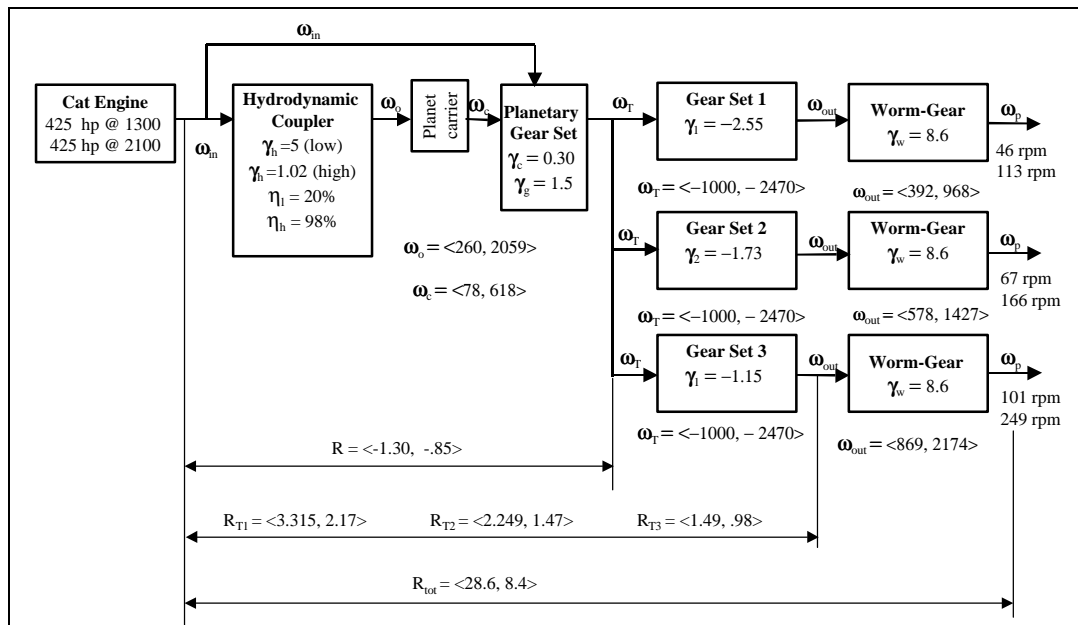


Figure 5.5: Block diagram of PS-CVT design using a Voith coupling for a well service pump.

Using the calculated values for gear ratios and sizes, and the selected coupling and output power, several things are determined about the design. First, the gear and coupling

dimensions are used to develop a 2D drawing of the conceptual design. A 2D rendering of the conceptual PS-CVT design is presented in Figure 5.6. From this illustration in combination with the coupling gear sizes, the overall size of the transmission may be estimated. The overall sizes are then compared to those of the automatic transmission currently in use. These are presented in Table 5.1.

Dimension	Concept PS-CVT w/ coupling	Automatic Transmission
Length	36"	41"
Width	31.5"	21.8"
Height	20"	26.6"

Table 5.1: Dimension comparison of developed PS-CVT concept with automatic transmissions.

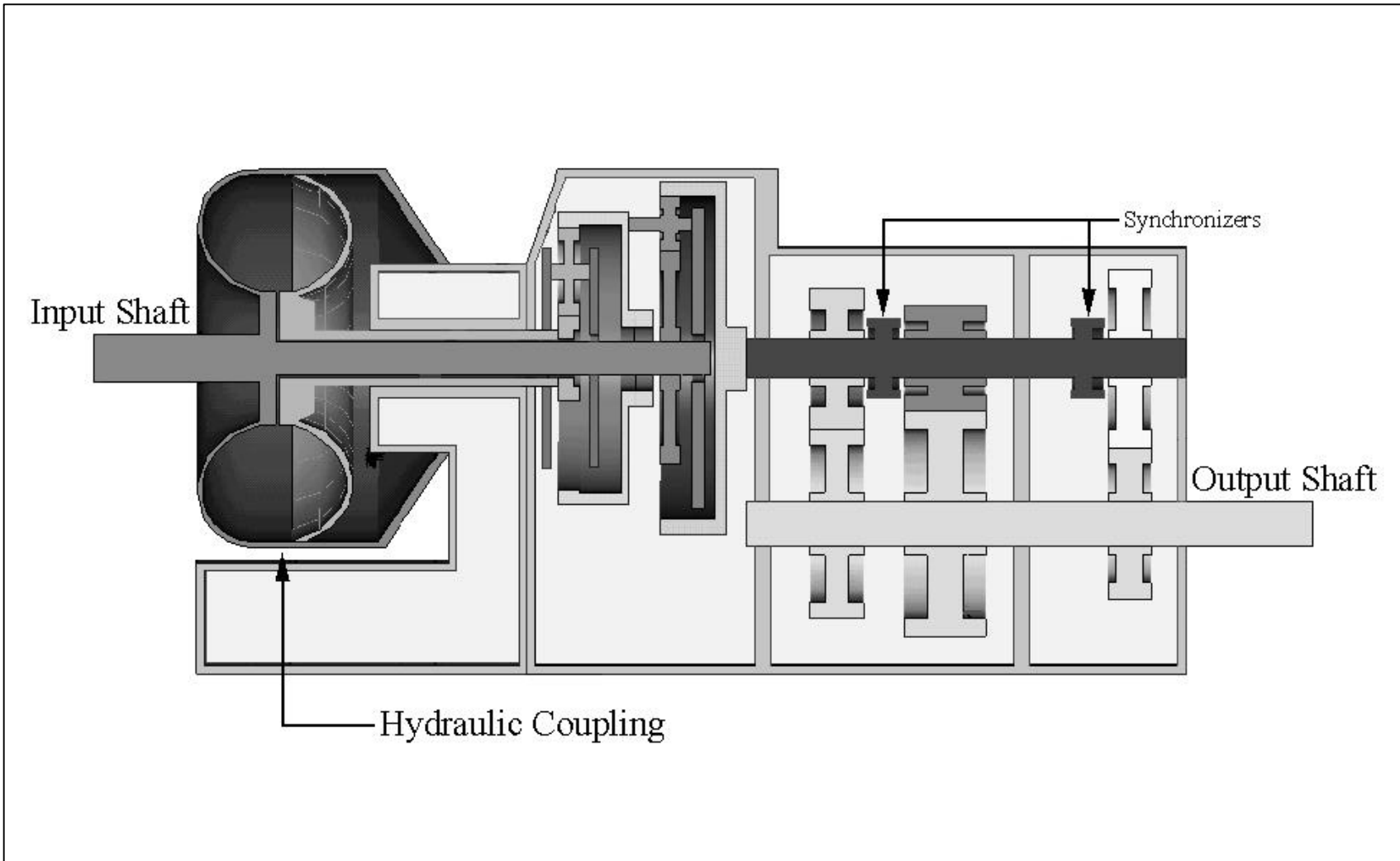


Figure 5.6: 2D rendering of the conceptual design using a coupling.

The efficiency of this conceptual design was estimated using the power balance curves. Dividing the output power by the input power created Figure 5.7, of the estimated efficiencies.

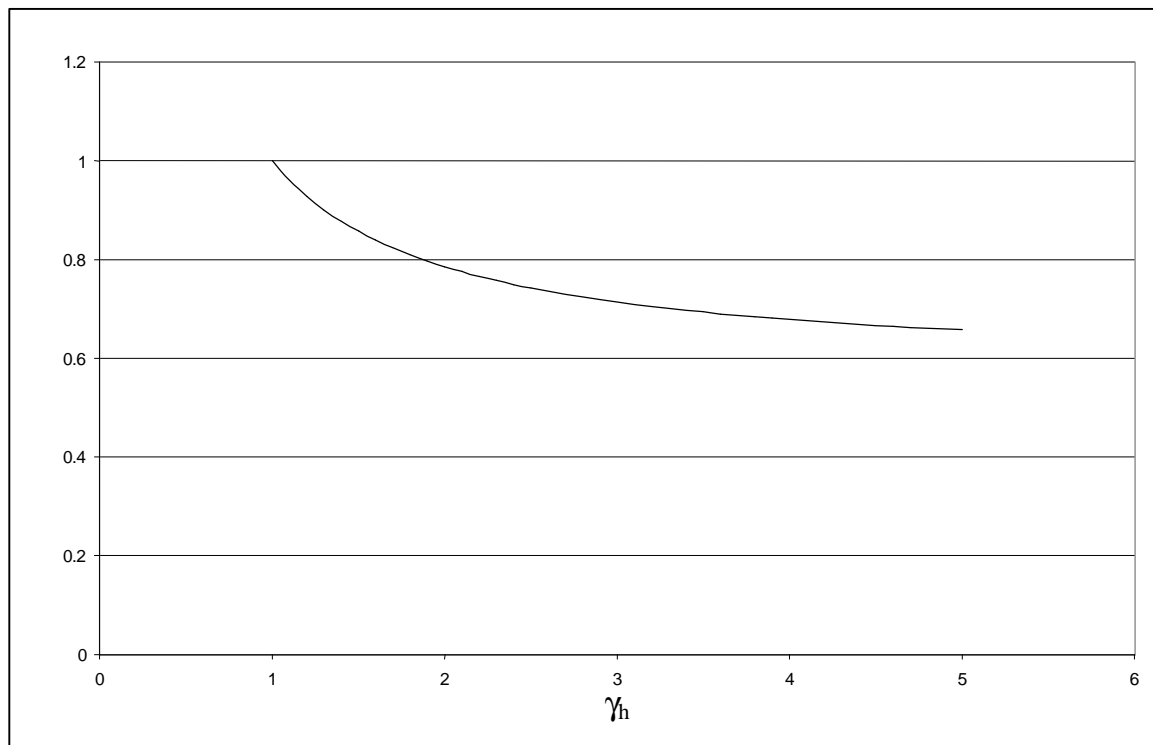


Figure 5.7: Estimated efficiency of the conceptual design using coupling.

5.3 Conceptual design using a hydrostatic drive

A second conceptual design for this application utilizes a hydrostatic drive for the variable element. Following the same steps outlined in the previous section the sizes and efficiencies of the alternative design are determined. The results of the process are presented in following figures. The overall design specifications are shown with a wire-frame diagram in Figure 5.8 and a block diagram shown in Figure 5.9.

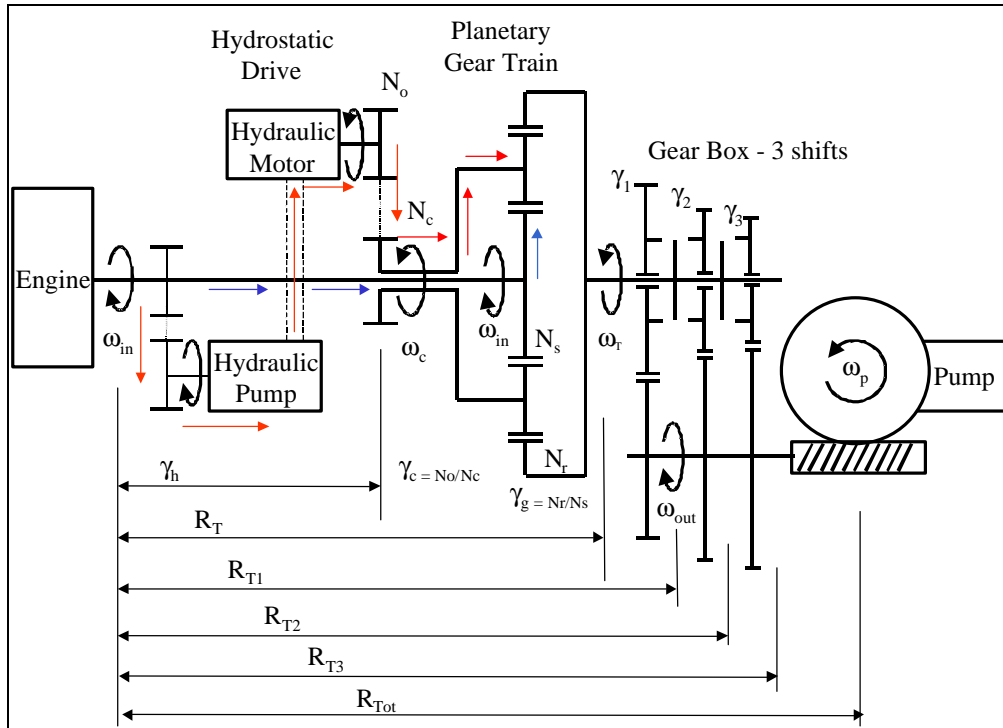


Figure 5.8: Description of example conceptual design system with hydrostatic drive.

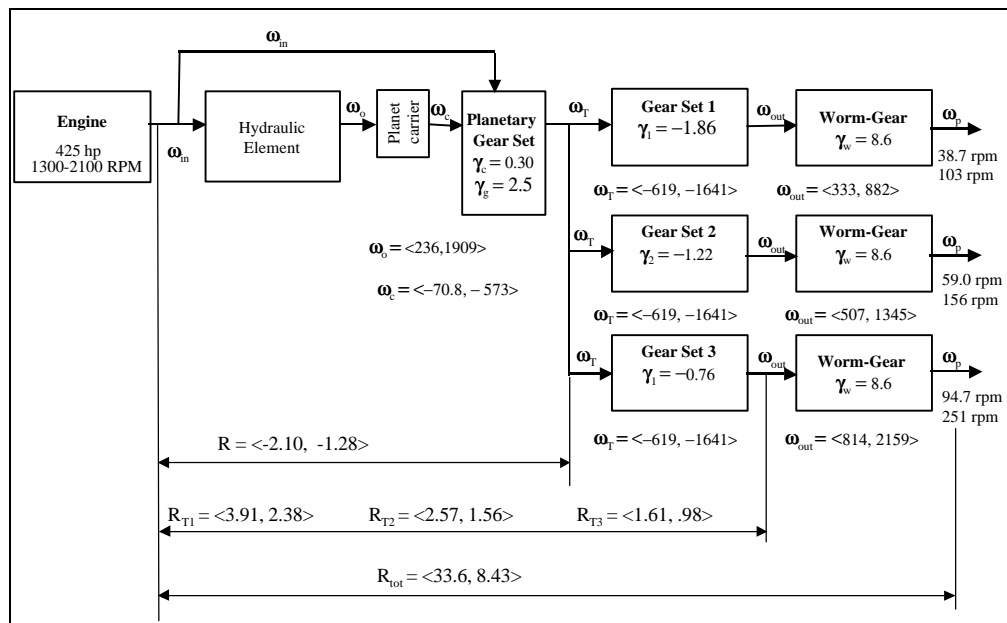


Figure 5.9: Block Diagram of PS-CVT design using a hydrostatic drive for a well service pump.

In the design procedure, the power balance diagram, Figure 5.10, was developed for the hydrostatic drive. The estimated efficiency was plotted in Figure 5.11 using the data from the power balance curve.

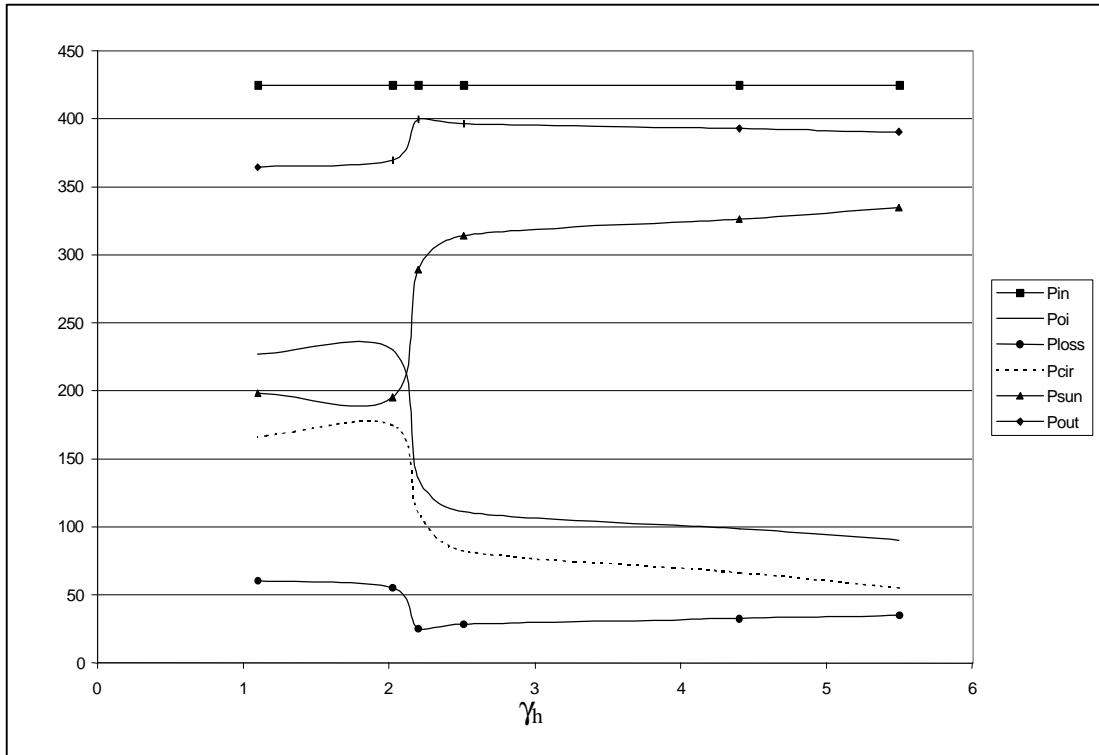


Figure 5.10: Power balance curve for conceptual PS-CVT using a hydrostatic drive.

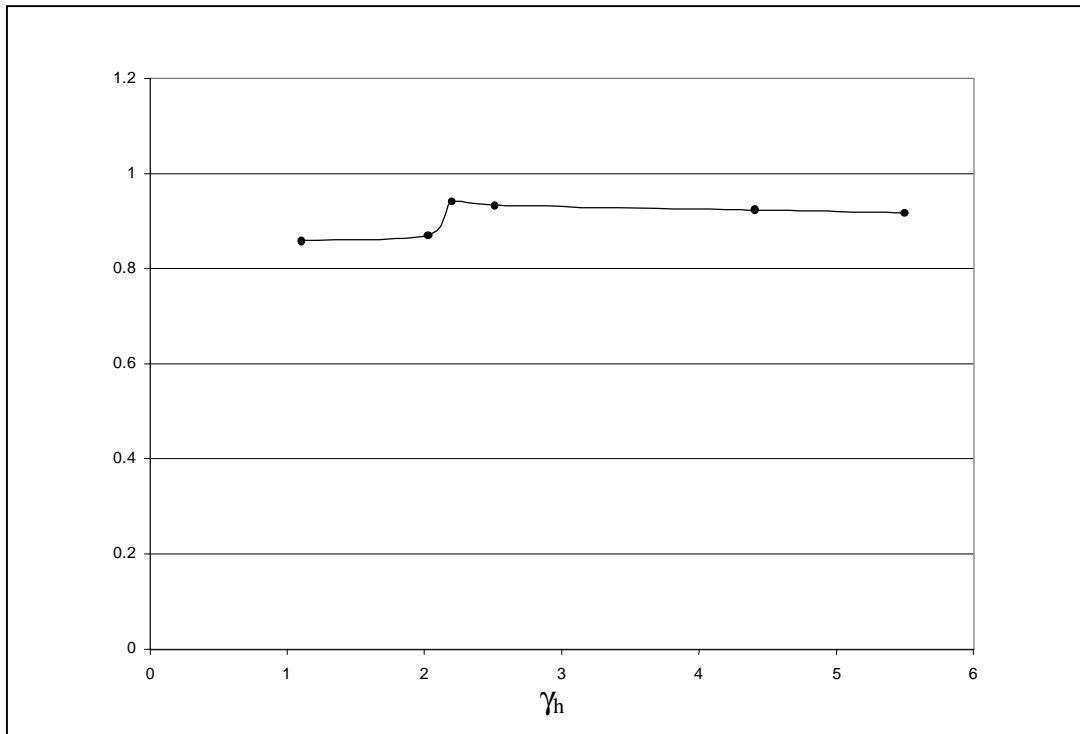


Figure 5.11: Estimated efficiency of the PS-CVT with hydrostatic drive.

With the results for the procedure a 2D drawing of the conceptual design is presented in Figure 5.12. Using this illustration, along with the gear sizes (found in Appendix C) the overall dimensions for this design were estimated. The length, width, and height is 25", 15", and 26" respectively.

Close examination of the results of the two conceptual designs reveals that the size and efficiency of the design utilizing a hydrostatic drive is more favorable than those of the hydrodynamic coupling configuration. The examples illustrated that the conceptual design procedure can be utilized to determine if a PS-CVT is a feasible replacement option for a given application. They also show that the information resulting from the conceptual design procedure facilitates the comparison of alternative PS-CVT designs. The use of a standardized design procedure allows for progression through the conceptual design phase of the application in an effective manner.

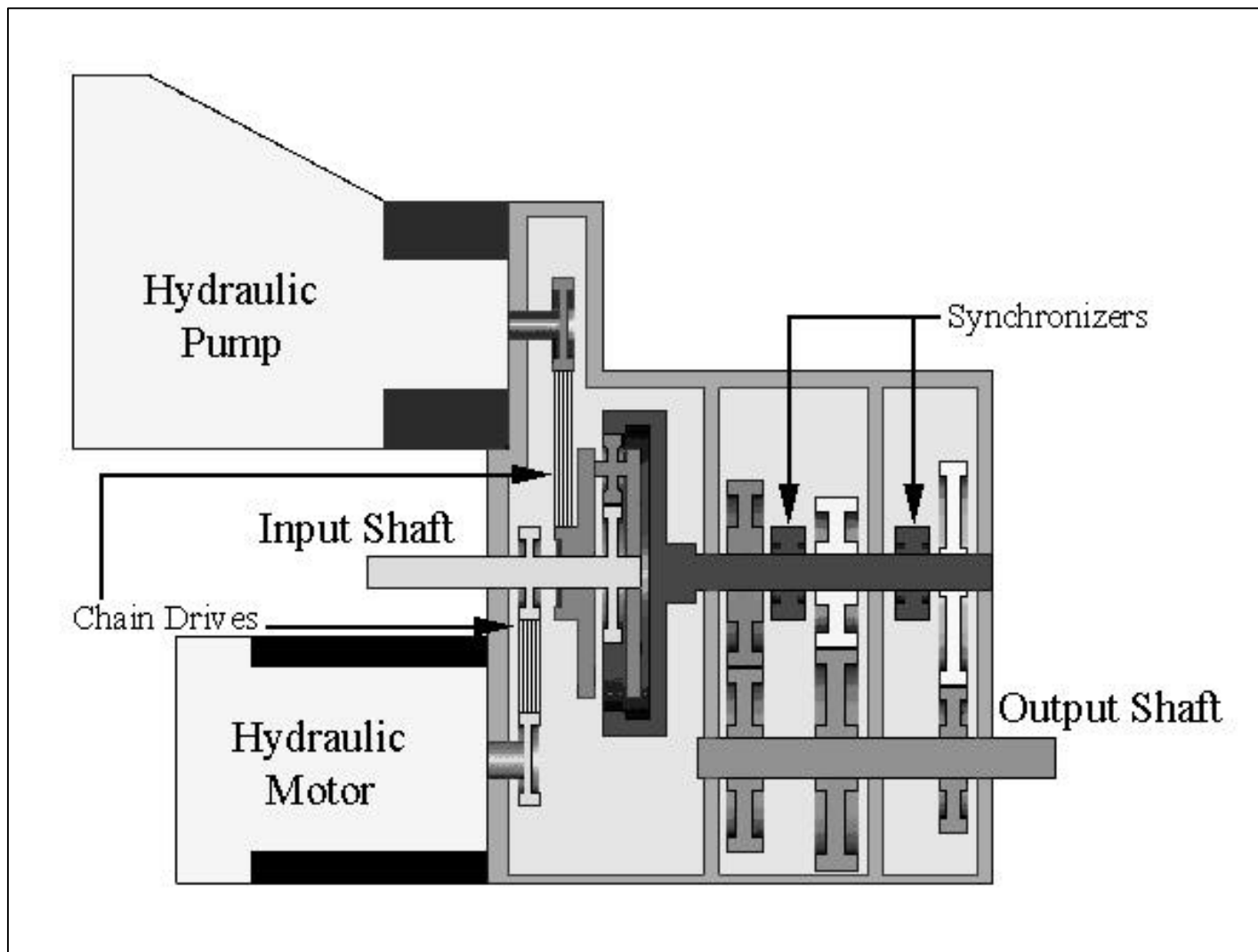


Figure 5.12: 2D illustration of the PS-CVT utilizing a hydrostatic drive.

Chapter 6 Conclusions

6.1 Conclusions

Although much research has been devoted to the development of continuously variable transmissions, a literature search returned limited information on conceptual design procedures for a PS-CVT. The majority of the research to date has dealt with specialized designs that consist of a configuration and a variable element, for a specific application.

The purpose of this work was to develop a conceptual design procedure for a power split continuously variable transmission for industrial applications. This 15 step procedure was outlined using a flow chart. The results of the procedure can be used to estimate overall size, produce 2D and 3D scale drawings and estimate efficiencies of the conceptual transmission. This procedure offers a means to improve the efficiency of the conceptual design stage of a PS-CVT. It also allows the designer to determine more effectively the feasibility of a PS-CVT for a specific application in the conceptual stages.

A industrial pump was used as an example industrial application to illustrate the conceptual design procedure. Two conceptual designs, using different variable elements where completed for this application. The two variable elements considered were a hydrodynamic coupling and a hydrostatic drive. The two examples also illustrated how the results of the procedure can be used to determine whether a PS-CVT is a viable alternative for the well service pump system.

6.2 Contributions

The work of this thesis has several contributions to the knowledge of power split continuously variable transmission design as well as contributions to Halliburton Energy Services. The configuration charts can be used to streamline the process of determining an input coupled variable shunt configuration for a given application. The conceptual design procedure can be used to: develop a PS-CVT, compare conceptual designs, and determine feasibility of PS-CVTs for specific applications. In the process of demonstrating the procedure with examples of an industrial pumping application, the feasibility of a PS-CVT for an industrial pump application was studied.

6.3 Recommendations

It is recommended that further refinement of the conceptual procedure be completed. This would include optimization of several steps, such as gear sizes, gearbox ratios, and the span. Several steps may also be added. These would include determining shaft sizes, bearing sizes and weights, and housings material and size for the PS-CVT. Considering the number of combinations and steps involved, it would be useful to develop a conceptual design program. This could make the conceptual design stage of a PS-CVT even more efficient.

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Appendix A: Configuration Charts

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
1.1	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$v_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_{in}$ $v_a = v_{out}$ $v_R = v_c$	$\frac{v_{in} + g_g v_c}{g_g + 1}$	$\frac{g_g g_h + g_h}{g_h + g_c g_n g_g}$	$R = \frac{g_g g_h + g_h}{g_h + g_c g_n g_g}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
1.2	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$v_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_{in}$ $v_a = v_c$ $v_R = v_{out}$	$\frac{v_c (g_g + 1) - v_{in}}{g_g}$	$\frac{g_g g_h}{g_h - g_c g_n (g_g + 1)}$	$R = \frac{g_g g_h}{g_h - g_c g_n (g_g + 1)}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
1.3	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$v_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_{out}$ $v_a = v_{in}$ $v_R = v_c$	$v_{in} (g_g + 1) - g_g v_c$	$\frac{g_h}{g_h (g_g + 1) - g_c g_n g_g}$	$R = \frac{g_h}{g_h (g_g + 1) - g_c g_n g_g}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
1.4	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$v_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_c$ $v_a = v_{in}$ $v_R = v_{out}$	$\frac{v_{in} (g_g + 1) - v_c}{g_g + 1}$	$\frac{g_h (g_g + 1)}{g_h (g_g + 1) - g_c g_n}$	$R = \frac{g_h (g_g + 1)}{g_h (g_g + 1) - g_c g_n}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
1.5	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$V_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_c$ $V_a = V_{out}$ $V_R = V_{in}$	$\frac{V_c + g_g V_{in}}{g_g + 1}$	$\frac{g_h (g_g + 1)}{g_h g_g + g_c g_n}$	$R = \frac{g_h (g_g + 1)}{g_h g_g + g_c g_n}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
1.6	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$V_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{out}$ $V_a = V_c$ $V_R = V_{in}$	$g_g (v_c - v_{in}) + v_c$	$\frac{g_h}{g_h g_g - g_c g_n (g_g + 1)}$	$R = -\frac{g_h}{g_h g_g - g_c g_n (g_g + 1)}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
2.1	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$-v_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{in}$ $V_a = V_{out}$ $V_R = V_c$	$\frac{V_{in} + g_g V_c}{g_g + 1}$	$\frac{g_g g_h + g_h}{g_h - g_c g_n g_g}$	$R = \frac{g_s g_h + g_h}{g_h - g_c g_n g_g}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
2.2	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$-v_{in} \frac{g_c g_n}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{in}$ $V_a = V_c$ $V_R = V_{out}$	$\frac{v_c (g_g + 1) - v_{in}}{g_g}$	$-\frac{g_g g_h}{g_h + g_c g_n (g_g + 1)}$	$R = -\frac{g_g g_h}{g_h + g_c g_n (g_g + 1)}$ $r = \frac{g_c g_n}{g_h}$ $g = \frac{r(1-R)}{1-r}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
2.3	$\mathbf{g}_i = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $\mathbf{g}_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$- \mathbf{v}_{in} \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_c$	$\mathbf{v}_{in} (\mathbf{g}_g + 1) - \mathbf{g}_g \mathbf{v}_c$	$\frac{\mathbf{g}_h}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_n \mathbf{g}_g}$	$R = \frac{\mathbf{g}_h}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_n \mathbf{g}_g}$ $r = \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
2.4	$\mathbf{g}_i = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $\mathbf{g}_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$- \mathbf{v}_{in} \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{v}_{in} (\mathbf{g}_g + 1) - \mathbf{v}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_n}$	$R = \frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_n}$ $r = \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
2.5	$\mathbf{g}_i = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $\mathbf{g}_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$- \mathbf{v}_{in} \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_{in}$	$\frac{\mathbf{v}_c + \mathbf{g}_g \mathbf{v}_{in}}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h \mathbf{g}_g - \mathbf{g}_c \mathbf{g}_n}$	$R = \frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h \mathbf{g}_g - \mathbf{g}_c \mathbf{g}_n}$ $r = \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
2.6	$\mathbf{g}_i = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $\mathbf{g}_n = \frac{N_2}{N_1}$ $N_o + N_1 = N_c + N_2$	$- \mathbf{v}_{in} \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_{in}$	$\mathbf{g}_g (\mathbf{v}_c - \mathbf{v}_{in}) + \mathbf{v}_c$	$-\frac{\mathbf{g}_h}{\mathbf{g}_g \mathbf{g}_h + \mathbf{g}_c \mathbf{g}_n (\mathbf{g}_g + 1)}$	$R = -\frac{\mathbf{g}_h}{\mathbf{g}_g \mathbf{g}_h + \mathbf{g}_c \mathbf{g}_n (\mathbf{g}_g + 1)}$ $r = \frac{\mathbf{g}_c \mathbf{g}_n}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_r/ω_{out}	Power split	Curves
3.1	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o < N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{in}$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_c$	$\frac{\mathbf{V}_{in} + \mathbf{g}_g \mathbf{V}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h - \mathbf{g}_c \mathbf{g}_g}$	$R = \frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h - \mathbf{g}_c \mathbf{g}_g}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
3.2	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o < N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{in}$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{V}_c (\mathbf{g}_g + 1) - \mathbf{V}_{in}}{\mathbf{g}_g}$	$-\frac{\mathbf{g}_g \mathbf{g}_h}{\mathbf{g}_h + \mathbf{g}_c (\mathbf{g}_g + 1)}$	$R = -\frac{\mathbf{g}_g \mathbf{g}_h}{\mathbf{g}_h + \mathbf{g}_c (\mathbf{g}_g + 1)}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
3.3	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o < N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_c$	$\mathbf{V}_{in} (\mathbf{g}_g + 1) - \mathbf{g}_g \mathbf{V}_c$	$\frac{\mathbf{g}_h}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g}$	$R = \frac{\mathbf{g}_h}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
3.4	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o < N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{V}_{in} (\mathbf{g}_g + 1) - \mathbf{V}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c}$	$R = \frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h (\mathbf{g}_g + 1) + \mathbf{g}_c}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
3.5	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $N_o < N_c$	$-V_{in} \frac{g_c}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_c$ $V_a = V_{out}$ $V_R = V_{in}$	$\frac{V_c + g_g V_{in}}{g_g + 1}$	$\frac{g_h (g_g + 1)}{g_h g_g - g_c}$	$R = \frac{g_h (g_g + 1)}{g_h g_g - g_c}$ $r = -\frac{g_c}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
3.6	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $N_o < N_c$	$-V_{in} \frac{g_c}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{out}$ $V_a = V_c$ $V_R = V_{in}$	$g_g (V_c - V_{in}) + V_c$	$-\frac{g_h}{g_g g_h + g_c (g_g + 1)}$	$R = -\frac{g_h}{g_g g_h + g_c (g_g + 1)}$ $r = -\frac{g_c}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
4.1	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $N_o > N_c$	$-V_{in} \frac{g_c}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{in}$ $V_a = V_{out}$ $V_R = V_c$	$\frac{V_{in} + g_g V_c}{g_g + 1}$	$\frac{g_h (g_g + 1)}{g_h - g_c g_g}$	$R = \frac{g_h (g_g + 1)}{g_h - g_c g_g}$ $r = -\frac{g_c}{g_h}$ $g = \frac{r(1-R)}{1-r}$	
4.2	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_o}{N_c}$ $N_o > N_c$	$-V_{in} \frac{g_c}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{in}$ $V_a = V_c$ $V_R = V_{out}$	$\frac{V_c (g_g + 1) - V_{in}}{g_g}$	$-\frac{g_g g_h}{g_h + g_c (g_g + 1)}$	$R = -\frac{g_g g_h}{g_h + g_c (g_g + 1)}$ $r = -\frac{g_c}{g_h}$ $g = \frac{r(1-R)}{1-r}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
4.3	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_c$	$\mathbf{V}_{in}(\mathbf{g}_g + 1) - \mathbf{g}_g \mathbf{V}_c$	$\frac{\mathbf{g}_h}{\mathbf{g}_h(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g}$	$R = \frac{\mathbf{g}_h}{\mathbf{g}_h(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
4.4	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{V}_{in}(\mathbf{g}_g + 1) - \mathbf{V}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{\mathbf{g}_h(\mathbf{g}_g + 1) + \mathbf{g}_c}$	$R = \frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{\mathbf{g}_h(\mathbf{g}_g + 1) + \mathbf{g}_c}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
4.5	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_{in}$	$\frac{\mathbf{V}_c + \mathbf{g}_g \mathbf{V}_{in}}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{\mathbf{g}_h \mathbf{g}_g - \mathbf{g}_c}$	$R = \frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{\mathbf{g}_h \mathbf{g}_g - \mathbf{g}_c}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
4.6	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$-\mathbf{V}_{in} \frac{\mathbf{g}_c}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_{in}$	$\mathbf{g}_g(\mathbf{V}_c - \mathbf{V}_{in}) + \mathbf{V}_c$	$\frac{\mathbf{g}_h}{\mathbf{g}_g \mathbf{g}_h + \mathbf{g}_c(\mathbf{g}_g + 1)}$	$R = \frac{\mathbf{g}_h}{\mathbf{g}_g \mathbf{g}_h + \mathbf{g}_c(\mathbf{g}_g + 1)}$ $r = -\frac{\mathbf{g}_c}{\mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
5.1	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$\mathbf{v}_{in} \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{in}$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_c$	$\frac{\mathbf{V}_{in} + \mathbf{g}_g \mathbf{V}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h + (\mathbf{g}_c + 1) \mathbf{g}_g}$	$R = \frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h + (\mathbf{g}_c + 1) \mathbf{g}_g}$ $r = \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$ $\mathbf{g} = \frac{r(1 - R)}{1 - r}$	
5.2	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$\mathbf{v}_{in} \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{in}$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{v}_c (\mathbf{g}_g + 1) - \mathbf{V}_{in}}{\mathbf{g}_g}$	$-\frac{\mathbf{g}_g \mathbf{g}_h}{\mathbf{g}_h - (\mathbf{g}_c + 1) (\mathbf{g}_g + 1)}$	$R = -\frac{\mathbf{g}_g \mathbf{g}_h}{\mathbf{g}_h - (\mathbf{g}_c + 1) (\mathbf{g}_g + 1)}$ $r = \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$ $\mathbf{g} = \frac{r(1 - R)}{1 - r}$	
5.3	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$\mathbf{v}_{in} \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_c$	$\mathbf{v}_{in} (\mathbf{g}_g + 1) - \mathbf{g}_g \mathbf{v}_c$	$\frac{\mathbf{g}_h}{\mathbf{g}_h (\mathbf{g}_g + 1) - (\mathbf{g}_c + 1) \mathbf{g}_g}$	$R = \frac{\mathbf{g}_h}{\mathbf{g}_h (\mathbf{g}_g + 1) - (\mathbf{g}_c + 1) \mathbf{g}_g}$ $r = \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$ $\mathbf{g} = \frac{r(1 - R)}{1 - r}$	
5.4	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$\mathbf{v}_{in} \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{in}$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{v}_{in} (\mathbf{g}_g + 1) - \mathbf{v}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h (\mathbf{g}_g + 1) - (\mathbf{g}_c + 1)}$	$R = \frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h (\mathbf{g}_g + 1) - (\mathbf{g}_c + 1)}$ $r = \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$ $\mathbf{g} = \frac{r(1 - R)}{1 - r}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
5.5	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$\mathbf{v}_{in} \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_{in}$	$\frac{\mathbf{V}_c + \mathbf{g}_g \mathbf{V}_{in}}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h \mathbf{g}_g + (\mathbf{g}_c + 1)}$	$R = \frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_h \mathbf{g}_g + (\mathbf{g}_c + 1)}$ $r = \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
5.6	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_o}{N_c}$ $N_o > N_c$	$\mathbf{v}_{in} \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_{in}$	$\mathbf{g}_g (\mathbf{v}_c - \mathbf{v}_{in}) + \mathbf{v}_c$	$-\frac{\mathbf{g}_h}{\mathbf{g}_g \mathbf{g}_h - (\mathbf{g}_c + 1)(\mathbf{g}_g + 1)}$	$R = -\frac{\mathbf{g}_h}{\mathbf{g}_g \mathbf{g}_h - (\mathbf{g}_c + 1)(\mathbf{g}_g + 1)}$ $r = \left(\frac{\mathbf{g}_c + 1}{\mathbf{g}_h} \right)$ $\mathbf{g} = \frac{r(1-R)}{1-r}$	
1'1	$\mathbf{g}_h = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $\mathbf{g}_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$\mathbf{v}_c = \mathbf{v}_{in} \mathbf{g}_c \mathbf{g}_n$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_o$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_c$	$\frac{\mathbf{V}_o + \mathbf{g}_g \mathbf{V}_c}{\mathbf{g}_g + 1}$	$\frac{(\mathbf{g}_g + 1) \mathbf{g}_h}{1 + \mathbf{g}_c \mathbf{g}_n \mathbf{g}_g \mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{1 + \mathbf{g}_c \mathbf{g}_n \mathbf{g}_g \mathbf{g}_h}$ $r = \mathbf{g}_c \mathbf{g}_n \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
1'2	$\mathbf{g}_h = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $\mathbf{g}_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$\mathbf{v}_c = \mathbf{v}_{in} \mathbf{g}_c \mathbf{g}_n$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_o$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{v}_c (\mathbf{g}_g + 1) - \mathbf{v}_o}{\mathbf{g}_g}$	$\frac{\mathbf{g}_g \mathbf{g}_h}{\mathbf{g}_h \mathbf{g}_c \mathbf{g}_n (\mathbf{g}_g + 1)}$	$R = \frac{\mathbf{g}_g}{\mathbf{g}_h \mathbf{g}_c \mathbf{g}_n (\mathbf{g}_g + 1)}$ $r = \mathbf{g}_c \mathbf{g}_n \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_n/ω_{out}	Power split	Curves
1'.3	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{out}$ $V_a = V_o$ $V_R = V_c$	$v_o (g_g + 1) - g_g v_c$	$\frac{g_h}{(g_g + 1) - g_c g_n g_s g_h}$	$R = \frac{1}{(g_g + 1) - g_c g_n g_s g_h}$ $r = g_c g_n g_s g_h$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
1'.4	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_c$ $V_a = V_o$ $V_R = V_{out}$	$\frac{v_o (g_g + 1) - v_c}{g_g + 1}$	$\frac{g_h (g_g + 1)}{(g_g + 1) - g_c g_n g_s g_h}$	$R = \frac{(g_g + 1)}{(g_g + 1) - g_c g_n g_s g_h}$ $r = g_c g_n g_s g_h$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
1'.5	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_c$ $V_a = V_{out}$ $V_R = V_o$	$\frac{v_c + g_g v_o}{g_g + 1}$	$\frac{g_h (g_g + 1)}{g_g + g_c g_n g_s g_h}$	$R = \frac{(g_g + 1)}{g_g + g_c g_n g_s g_h}$ $r = g_c g_n g_s g_h$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
1'.6	$g_h = \frac{V_{in}}{V_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $V_s = V_{out}$ $V_a = V_c$ $V_R = V_o$	$g_g (v_c - v_o) + v_c$	$\frac{g_h}{g_s (g_c g_n g_s - 1) + g_c g_n g_s g_h}$	$R = \frac{1}{g_s (g_c g_n g_s - 1) + g_c g_n g_s g_h}$ $r = g_c g_n g_s g_h$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
2'1	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = -v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_o$ $v_a = v_{out}$ $v_R = v_c$	$v_o + g_g v_c$ $g_g + 1$	$\frac{(g_g + 1)g_h}{1 - g_h g_c g_n g_g}$	$R = -\frac{g_g}{1 + g_g g_c (g_g + 1)}$ $r = -\frac{g_c g_n g_h}{g_g + 1}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
2'2	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = -v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_o$ $v_a = v_c$ $v_R = v_{out}$	$\frac{v_c (g_g + 1) - v_o}{g_g}$	$-\frac{g_g g_h}{1 + g_c g_n g_h (g_g + 1)}$	$R = \frac{g_g}{1 + g_g g_c (g_g + 1)}$ $r = -\frac{g_c g_n g_h}{g_g + 1}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
2'3	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = -v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_{out}$ $v_a = v_o$ $v_R = v_c$	$v_o (g_g + 1) - g_g v_c$	$\frac{g_h}{(g_g + 1) + g_c g_n g_g g_h}$	$R = \frac{1}{(g_g + 1) + g_c g_n g_g g_h}$ $r = -\frac{g_c g_n g_h}{g_g + 1}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
2'4	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = -v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_c$ $v_a = v_o$ $v_R = v_{out}$	$\frac{v_o (g_g + 1) - v_c}{g_g + 1}$	$\frac{g_h (g_g + 1)}{(g_g + 1) + g_c g_n g_h}$	$R = \frac{(g_g + 1)}{(g_g + 1) + g_c g_n g_h}$ $r = -\frac{g_c g_n g_h}{g_g + 1}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
2.5	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = -v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_c$ $v_a = v_{out}$ $v_R = v_o$	$\frac{v_c + g_g v_o}{g_g + 1}$	$\frac{g_h (g_g + 1)}{g_g - g_c g_n g_h}$	$R = \frac{(g_g + 1)}{g_g - g_c g_n g_h}$ $r = -\frac{g_c g_n g_h}{r(1-R)}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
2.6	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $g_n = \frac{N_2}{N_1}$ $N_i + N_1 = N_c + N_2$	$v_c = -v_{in} g_c g_n$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_{out}$ $v_a = v_c$ $v_R = v_o$	$g_g (v_c - v_o) + v_c$	$-\frac{g_h}{g_g (g_c g_n g_h + 1) + g_c g_n g_h}$	$R = \frac{1}{g_g (g_c g_n g_h + 1) + g_c g_n g_h}$ $r = -\frac{g_c g_n g_h}{r(1-R)}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
3.1	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $N_i < N_c$	$v_c = -v_{in} g_c$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_o$ $v_a = v_{out}$ $v_R = v_c$	$\frac{v_o + g_g v_c}{g_g + 1}$	$\frac{g_h (g_g + 1)}{1 - g_c g_n g_h}$	$R = \frac{(g_g + 1)}{1 - g_c g_n g_h}$ $r = -\frac{g_c g_n g_h}{r(1-R)}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	
3.2	$g_h = \frac{v_{in}}{v_o}$ $g_c = \frac{N_i}{N_c}$ $N_i < N_c$	$v_c = -v_{in} g_c$ $v_o = \frac{v_{in}}{g_h}$	$g_g = \frac{N_R}{N_s}$ $v_s = v_o$ $v_a = v_c$ $v_R = v_{out}$	$\frac{v_c (g_g + 1) - v_o}{g_g}$	$-\frac{g_g g_h}{1 + g_c g_n (g_g + 1)}$	$R = -\frac{g_g}{1 + g_c g_n (g_g + 1)}$ $r = -\frac{g_c g_n g_h}{r(1-R)}$ $g = \frac{r(1-R)}{1-r}$ $\%P = 1 + g$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
3.3	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_i < N_c$	$\mathbf{V}_c = -\mathbf{V}_{in} \mathbf{g}_c$ $\mathbf{V}_o = \frac{\mathbf{V}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_o$ $\mathbf{V}_R = \mathbf{V}_c$	$\mathbf{v}_o(\mathbf{g}_g + 1) - \mathbf{g}_g \mathbf{v}_c$	$\frac{\mathbf{g}_h}{(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g \mathbf{g}_h}$	$R = \frac{1}{\mathbf{g}_g + 1 + \mathbf{g}_c \mathbf{g}_h}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
3.4	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_i < N_c$	$\mathbf{V}_c = -\mathbf{V}_{in} \mathbf{g}_c$ $\mathbf{V}_o = \frac{\mathbf{V}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_o$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{v}_o(\mathbf{g}_g + 1) - \mathbf{v}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_h}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
3.5	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_i < N_c$	$\mathbf{V}_c = -\mathbf{V}_{in} \mathbf{g}_c$ $\mathbf{V}_o = \frac{\mathbf{V}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_o$	$\frac{\mathbf{v}_c + \mathbf{g}_g \mathbf{v}_o}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{\mathbf{g}_g - \mathbf{g}_c \mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{\mathbf{g}_g - \mathbf{g}_c \mathbf{g}_h}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
3.6	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_i < N_c$	$\mathbf{V}_c = -\mathbf{V}_{in} \mathbf{g}_c$ $\mathbf{V}_o = \frac{\mathbf{V}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_o$	$\mathbf{g}_g(\mathbf{v}_c - \mathbf{v}_o) + \mathbf{v}_c$	$-\frac{\mathbf{g}_h}{\mathbf{g}_g(\mathbf{g}_c \mathbf{g}_h + 1) + \mathbf{g}_c \mathbf{g}_h}$	$R = -\frac{1}{\mathbf{g}_g(\mathbf{g}_c \mathbf{g}_h + 1) + \mathbf{g}_c \mathbf{g}_h}$ $= -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
4.1	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = -\mathbf{v}_{in}\mathbf{g}_c$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_o$ $\mathbf{V}_a = \mathbf{V}_{out}$ $\mathbf{V}_R = \mathbf{V}_c$	$\frac{\mathbf{v}_o + \mathbf{g}_g \mathbf{v}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{1 - \mathbf{g}_c \mathbf{g}_g \mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{1 - \mathbf{g}_c \mathbf{g}_g \mathbf{g}_h}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	<p>Win/Wout & Power Split</p> <p>Hydraulic Ratio</p>
4.2	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = -\mathbf{v}_{in}\mathbf{g}_c$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_o$ $\mathbf{V}_a = \mathbf{V}_c$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{v}_c (\mathbf{g}_g + 1) - \mathbf{v}_o}{\mathbf{g}_g}$	$-\frac{\mathbf{g}_g \mathbf{g}_h}{1 + \mathbf{g}_c \mathbf{g}_h (\mathbf{g}_g + 1)}$	$R = -\frac{\mathbf{g}_g}{1 + \mathbf{g}_c \mathbf{g}_h (\mathbf{g}_g + 1)}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	<p>Win/Wout & Power Split</p> <p>Hydraulic Ratio</p>
4.3	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = -\mathbf{v}_{in}\mathbf{g}_c$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_{out}$ $\mathbf{V}_a = \mathbf{V}_o$ $\mathbf{V}_R = \mathbf{V}_c$	$\mathbf{v}_o (\mathbf{g}_g + 1) - \mathbf{g}_g \mathbf{v}_c$	$\frac{\mathbf{g}_h}{(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g \mathbf{g}_h}$	$R = \frac{1}{(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g \mathbf{g}_h}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	<p>Win/Wout & Power Split</p> <p>Hydraulic Ratio</p>
4.4	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = -\mathbf{v}_{in}\mathbf{g}_c$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{V}_s = \mathbf{V}_c$ $\mathbf{V}_a = \mathbf{V}_o$ $\mathbf{V}_R = \mathbf{V}_{out}$	$\frac{\mathbf{v}_o (\mathbf{g}_g + 1) - \mathbf{v}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g \mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{(\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_g \mathbf{g}_h}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	<p>Win/Wout & Power Split</p> <p>Hydraulic Ratio</p>

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
4.5	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = -\mathbf{v}_{in}\mathbf{g}_c$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_c$ $\mathbf{v}_a = \mathbf{v}_{out}$ $\mathbf{v}_R = \mathbf{v}_o$	$\frac{\mathbf{v}_c + \mathbf{g}_g \mathbf{v}_o}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{\mathbf{g}_g - \mathbf{g}_c \mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{\mathbf{g}_g - \mathbf{g}_c \mathbf{g}_h}$ $r = -\frac{\mathbf{g}_c \mathbf{g}_h}{\mathbf{g}_g - \mathbf{g}_c \mathbf{g}_h}$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
4.6	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = -\mathbf{v}_{in}\mathbf{g}_c$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_{out}$ $\mathbf{v}_a = \mathbf{v}_c$ $\mathbf{v}_R = \mathbf{v}_o$	$\mathbf{g}_g (\mathbf{v}_c - \mathbf{v}_o) + \mathbf{v}_c$	$-\frac{\mathbf{g}_h}{\mathbf{g}_g (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_h}$	$R = -\frac{1}{\mathbf{g}_g (\mathbf{g}_g + 1) + \mathbf{g}_c \mathbf{g}_h}$ $r = -\mathbf{g}_c \mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
5.1	$\mathbf{g}_h = \frac{\mathbf{V}_{in}}{\mathbf{V}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = \mathbf{v}_{in}(\mathbf{g}_c + 1)$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_o$ $\mathbf{v}_a = \mathbf{v}_{out}$ $\mathbf{v}_R = \mathbf{v}_c$	$\frac{\mathbf{v}_o + \mathbf{g}_g \mathbf{v}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h (\mathbf{g}_g + 1)}{1 + (\mathbf{g}_c + 1)\mathbf{g}_g \mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{1 + (\mathbf{g}_c + 1)\mathbf{g}_g \mathbf{g}_h}$ $r = (\mathbf{g}_c + 1)\mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
5.2	$\mathbf{g}_h = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = \mathbf{v}_{in}(\mathbf{g}_c + 1)$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_o$ $\mathbf{v}_a = \mathbf{v}_c$ $\mathbf{v}_R = \mathbf{v}_{out}$	$\frac{\mathbf{v}_c (\mathbf{g}_g + 1) - \mathbf{v}_o}{\mathbf{g}_g}$	$-\frac{\mathbf{g}_g \mathbf{g}_h}{1 - (\mathbf{g}_c + 1)(\mathbf{g}_g + 1)\mathbf{g}_h}$	$R = -\frac{\mathbf{g}_g}{1 - (\mathbf{g}_c + 1)(\mathbf{g}_g + 1)\mathbf{g}_h}$ $r = (\mathbf{g}_c + 1)\mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	

Config.	input variables	ω_c	output variables	ω_{out}	ω_{in}/ω_{out}	Power split	Curves
5.3	$\mathbf{g}_h = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = \mathbf{v}_{in}(\mathbf{g}_c + 1)$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_{out}$ $\mathbf{v}_a = \mathbf{v}_o$ $\mathbf{v}_R = \mathbf{v}_c$	$\mathbf{v}_o(\mathbf{g}_g + 1) - \mathbf{g}_g \mathbf{v}_c$	$\frac{\mathbf{g}_h}{(\mathbf{g}_g + 1) - (\mathbf{g}_c + 1)\mathbf{g}_s \mathbf{g}_h}$	$R = \frac{1}{(\mathbf{g}_g + 1) - (\mathbf{g}_c + 1)\mathbf{g}_s \mathbf{g}_h}$ $r = (\mathbf{g}_c + 1)\mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
5.4	$\mathbf{g}_h = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = \mathbf{v}_{in}(\mathbf{g}_c + 1)$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_c$ $\mathbf{v}_a = \mathbf{v}_o$ $\mathbf{v}_R = \mathbf{v}_{out}$	$\frac{\mathbf{v}_o(\mathbf{g}_g + 1) - \mathbf{v}_c}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{(\mathbf{g}_g + 1) - (\mathbf{g}_c + 1)\mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{(\mathbf{g}_g + 1) - (\mathbf{g}_c + 1)\mathbf{g}_h}$ $r = (\mathbf{g}_c + 1)\mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
5.5	$\mathbf{g}_h = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = \mathbf{v}_{in}(\mathbf{g}_c + 1)$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_c$ $\mathbf{v}_a = \mathbf{v}_{out}$ $\mathbf{v}_R = \mathbf{v}_o$	$\frac{\mathbf{v}_c + \mathbf{g}_g \mathbf{v}_o}{\mathbf{g}_g + 1}$	$\frac{\mathbf{g}_h(\mathbf{g}_g + 1)}{\mathbf{g}_g + (\mathbf{g}_c + 1)\mathbf{g}_h}$	$R = \frac{(\mathbf{g}_g + 1)}{\mathbf{g}_g + (\mathbf{g}_c + 1)\mathbf{g}_h}$ $r = (\mathbf{g}_c + 1)\mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	
5.6	$\mathbf{g}_h = \frac{\mathbf{v}_{in}}{\mathbf{v}_o}$ $\mathbf{g}_c = \frac{N_i}{N_c}$ $N_o > N_c$	$\mathbf{v}_c = \mathbf{v}_{in}(\mathbf{g}_c + 1)$ $\mathbf{v}_o = \frac{\mathbf{v}_{in}}{\mathbf{g}_h}$	$\mathbf{g}_g = \frac{N_R}{N_s}$ $\mathbf{v}_s = \mathbf{v}_{out}$ $\mathbf{v}_a = \mathbf{v}_c$ $\mathbf{v}_R = \mathbf{v}_o$	$\mathbf{g}_g(\mathbf{v}_c - \mathbf{v}_o) + \mathbf{v}_c$	$\frac{\mathbf{g}_h}{\mathbf{g}_s((\mathbf{g}_c + 1)\mathbf{g}_h - 1) + (\mathbf{g}_c + 1)\mathbf{g}_h}$	$R = \frac{1}{\mathbf{g}_s((\mathbf{g}_c + 1)\mathbf{g}_h - 1) + (\mathbf{g}_c + 1)\mathbf{g}_h}$ $r = (\mathbf{g}_c + 1)\mathbf{g}_h$ $\mathbf{g} = \frac{r(1-R)}{1-r}$ $\%P = 1 + \mathbf{g}$	

Appendix B: Gear Sizes of Example Hydrodynamic Coupling Conceptual Design

425 HP Application with Voith SVN 313 coupling							
Face Width for Planetary and Control Gears using AGMA Bending Strength Formula							
Gear		N_p	N_s	N_R	N_o	N_c	N_{idler}
Dynamic factor	$K_v = (A/(A+V^{1/2}))^B$	1.1207658	1.120766		1.092633		
AGMA transmission accuracy-level number	Q_v	11	11		11		
	$A = 50+56(1-B)$	92	92		92		
	$B = (12-Q_v)^{2/3}/4$	0.25	0.25		0.25		
Pitch line velocity (ft/min)	$V = \pi * n_p * d/12$	2826	2826		1530.75		
Speed of pinion (rev/min)	n_p	7200	1800		1800		
Pitch diameter of pinion (in)	$d = N/P_d$	1.5	6	9	3.25	10.5	3.625
Number of teeth	N	12	48	72	26	84	29
Diametral pitch (teeth per inch)	P_d	8	8	8	8	8	8
Transmitted pinion torque (in-lb)	T		8221		6659		
Transmitted tangential load (lb)	$W_t = 2*T/d$	913.44444	913.4444		1365.949		
Overload factor	K_o	1			1		
Size factor	K_s	1			1		
Load distribution factor	K_m	1.3			1.375		
Rim thickness factor	K_B	1			1		
Geometry factor for bending strength	J	0.32			0.35		
AGMA bending strength (lb/in ²)	s_{at}	75000			75000		
Stress cycle factor for bending strength	Y_N	0.95			1		
Temperature factor	K_T	1			1		
Reliability factor	K_R	1			1		
Safety factor for bending strength	S_F	1			1		
Allowable bending stress number (lb/in ²)	$s_t = s_{at} * Y_N / (S_F * K_T * K_R)$	71250			75000		
Face width (in)	F	0.466977	0.466977	0.466977	0.62542	0.62542	0.62542

425 HP Application with Voith SVN 313 coupling							
Face Width for Reduction Gears using AGMA Bending Strength Fundamental Formula							
Gear		N ₁	N ₃	N ₆	N ₂	N ₄	N ₅
Dynamic factor	$K_v = (A/(A+V^{1/2}))^B$	1.089771	1.100718	1.141647			
AGMA transmission accuracy-level number	Q _v	11	11	11			
	A = 50+56(1-B)	92	92	92			
	B = (12-Q _v) ^{2/3} /4	0.25	0.25	0.25			
Pitch line velocity (ft/min)	$V = \pi * n_p * d / 12$	1425.56	1853.228	4132.449			
Speed of pinion (rev/min)	n _p	1362	1362	2078			
Pitch diameter of pinion (in)	$d = N/P_d$	4	5.2	7.6	10.2	9	6.6
Number of teeth	N	20	26	38	51	45	33
Diametral pitch (teeth per inch)	P _d	5	5	5	5	5	5
Transmitted pinion torque (in-lb)	T	13326	13326	8730			
Transmitted tangential load (lb)	$W_t = 2*T/d$	6663	5125.385	2297.368			
Overload factor	K _o	1	1	1			
Size factor	K _s	1	1	1			
Load distribution factor	K _m	1.3	1.3	1.3			
Rim thickness factor	K _B	1	1	1			
Geometry factor for bending strength	J	0.36	0.38	0.36			
AGMA bending strength (lb/in ²)	s _{at}	75000	75000	75000			
Stress cycle factor for bending strength	Y _N	1	1	1			
Temperature factor	K _T	1	1	1			
Reliability factor	K _R	1	1	1			
Safety factor for bending strength	S _F	1	1	1			
Allowable bending stress number (lb/in ²)	$s_t = s_{at} * Y_N / (S_F * K_T * K_R)$	75000	75000	75000			
Face width (in)	F	1.748054	1.286681	0.631411	1.748054	1.286681	0.631411

Appendix C: Gear Sizes of Example Hydrodynamic Hydrostatic Drive Conceptual Design

425 HP Application with Hydrostatic Drive							
Face Width for Planetary and Control Gears using AGMA Bending Strength Formula							
Gear		N_P	N_S	N_R	N_O	N_C	N_{Idler}
Dynamic factor	$K_v = (A/(A+V^{1/2}))^B$	1.1067582	1.106758		1.089421		
AGMA transmission accuracy-level number	Q_v	11	11		11		
	$A = 50+56(1-B)$	92	92		92		
	$B = (12-Q_v)^{2/3}/4$	0.25	0.25		0.25		
Pitch line velocity (ft/min)	$V = \pi * n_p * d/12$	2119.5	2119.5		1413		
Speed of pinion (rev/min)	n_p	3600	1800		1800		
Pitch diameter of pinion (in)	$d = N/P_d$	2.25	4.5	9	3	11	4
Number of teeth	N	18	36	72	24	88	32
Diametral pitch (teeth per inch)	P_d	8	8	8	8	8	8
Transmitted pinion torque (in-lb)	T		8221		6659		
Transmitted tangential load (lb)	$W_t = 2*T/d$	1217.9259	1217.926		1479.778		
Overload factor	K_o	1			1		
Size factor	K_s	1			1		
Load distribution factor	K_m	1.3			1.375		
Rim thickness factor	K_B	1			1		
Geometry factor for bending strength	J	0.32			0.35		
AGMA bending strength (lb/in ²)	s_{at}	75000			75000		
Stress cycle factor for bending strength	Y_N	0.95			1		
Temperature factor	K_T	1			1		
Reliability factor	K_R	1			1		
Safety factor for bending strength	S_F	1			1		
Allowable bending stress number (lb/in ²)	$s_t = s_{at} * Y_N / (S_F * K_T * K_R)$	71250			75000		
Face width (in)	F	0.6148541	0.614854	0.614854	0.675547	0.675547	0.675547

425 HP Application with Hydrostatic Drive							
Face Width for Reduction Gears using AGMA Bending Strength Fundamental Formula							
Gear		N_1	N_3	N_6	N_2	N_4	N_5
Dynamic factor	$K_v = (A/(A+V^{1/2}))^B$	1.09726	1.107167	1.116744			
AGMA transmission accuracy-level number	Q_v	11	11	11			
	$A = 50+56(1-B)$	92	92	92			
	$B = (12-Q_v)^{2/3}/4$	0.25	0.25	0.25			
Pitch line velocity (ft/min)	$V = \pi \cdot n_p \cdot d/12$	1710.672	2138.34	2609.968			
Speed of pinion (rev/min)	n_p	1362	1362	2078			
Pitch diameter of pinion (in)	$d = N/P_d$	4.8	6	4.8	7.2	6	7.2
Number of teeth	N	24	30	24	36	30	36
Diametral pitch (teeth per inch)	P_d	5	5	5	5	5	5
Transmitted pinion torque (in-lb)	T	13326	13326	8730			
Transmitted tangential load (lb)	$W_t = 2 \cdot T/d$	5552.5	4442	3637.5			
Overload factor	K_o	1	1	1			
Size factor	K_s	1	1	1			
Load distribution factor	K_m	1.3	1.3	1.3			
Rim thickness factor	K_B	1	1	1			
Geometry factor for bending strength	J	0.36	0.38	0.36			
AGMA bending strength (lb/in ²)	s_{at}	75000	75000	75000			
Stress cycle factor for bending strength	Y_N	1	1	1			
Temperature factor	K_T	1	1	1			
Reliability factor	K_R	1	1	1			
Safety factor for bending strength	S_F	1	1	1			
Allowable bending stress number (lb/in ²)	$s_t = s_{at} \cdot Y_N / (S_F \cdot K_T \cdot K_R)$	75000	75000	75000			
Face width (in)	F	1.466722	1.121657	0.977927	1.466722	1.121657	0.977927