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Automatic hydraulic fracturing design for low permeability reservoirs using artificial intelligence

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Automatic Hydraulic Fracturing Design for Low Permeability Reservoirs Using Artificial Intelligence

Andrei Sergiu Popa

**Dissertation submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of**

**Doctor of Philosophy
in
Petroleum and Natural Gas Engineering**

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Fuzzy Logic, Vector Quantization, Lithology Identification
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ABSTRACT

Automatic Hydraulic Fracturing Design for Low Permeability Reservoirs Using Artificial Intelligence

Andrei Sergiu Popa

The hydraulic fracturing technique is one of the major developments in petroleum engineering in the last two decades. Today, nearly all the wells completed in low permeability gas reservoirs require a hydraulic fracturing treatment in order to produce at an economical level. This study presents a new methodology, applicable to tight gas reservoirs, for designing hydraulic fractures.

This study is intended to develop an automatic hydraulic fracture design tool to help users design fracture jobs without being an expert in the art and science of hydraulic fracturing. This process is composed entirely of an integration of several artificial intelligence techniques.

The methodology consists of three modules: formation stress determination, optimum treatment design and net treatment pressure prediction. The first module combines the classic approach of stress calculations with a fuzzy lithology identification system to better characterize the reservoir and estimate the stress profile. The result of this module is essential for the fracture treatment design. The second module incorporates an optimization system composed of neural networks and a genetic algorithm to search for the optimum treatment design. The third, and final, module is designed to predict the net treating pressure expected during fracturing. A one-dimensional vector quantization technique samples and extracts the main characteristic of the pressure profile. The net treatment pressure neural network generates the main features of the pressure profile and then reconstructs the entire signal.

The methodology was integrated in a computer program aimed to help petroleum engineers design optimum treatment schedules and predict net treatment pressure for hydraulic fracturing. This tool is designed to reduce the engineering time for designing optimum treatment schedules.

DEDICATION

To my wife Carrie, the best thing that ever happened in my life.

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CHAPTER 1: INTRODUCTION

1.1. Problem Statement

Hydraulic fracturing is a widely used technology to economically increase the productivity of oil and gas reservoirs. Over the last two decades, this technology has been heavily studied, and many sophisticated models have been created in an attempt to simulate the propagation and geometry of induced fractures. However, there are still shortcomings in that the actual results of the job rarely reflect the designed intentions. This is due to the fact that there are many external influences that can affect the results, but are unknown, not incorporated into the model, or not accurately modeled. Some of these influences include (but are not limited to) localized stresses around the wellbore, tortuosity of the reservoir, and the existence of natural minifrac that can alter the direction of propagation.

The current study presents an alternative to classical methods used in fracture design simulators. This work is intended to develop a fracture design simulator that helps users to design hydraulic frac jobs without being an expert in the art and science of hydraulic fracturing. This process is composed of an integration of several artificial intelligence techniques. The advantage of artificial intelligence is that it does not require a mathematical modeling of the problem.

The first step consists of a fuzzy logic decision system that determines the lithology that would assist in the determination of the stress profile. The results of this step are used in the second step, which involves a search and optimization using an integrated system of neural networks and genetic algorithms. The output of this system is the optimum fracturing treatment design. The final step uses vector quantization and neural networks to predict the net treating pressure.

The study verifies the capabilities of this approach in tight gas reservoirs (natural gas reservoirs with an average permeability of less than 0.1 mD). The outputs of this system (the fracturing treatment design and the net treating pressure) are verified against a commonly used 3-D fracture simulator, FRACPRO. The final outcome of this work is an integrated program able to design optimum fracturing treatment schedules in a matter of seconds once the stress profile has been completed.

1.2. Motivation for the Study

Over the years, many hydraulic fracture simulators have been developed using analytical and empirical equations to solve fracture modeling. However, no attempt has been made thus far to build an intelligent fracture simulator that uses only knowledge and intelligent techniques to design and implement optimum fracture treatments. This study intends to develop a tool, which learns from data, and designs optimum fracture treatments based on historical practices and results.

Due to the unavailability of actual fracturing data (geometry and net pressures), the data used in this study has been generated using a commercial fracture simulator. The intent is that this study should be repeated using the actual results from hydraulic fractures performed in a specific field or basin, which may or may not reflect the analytical fracture simulator.

The goal of the system is to learn from real fracturing data and optimize future treatments based on best practices. Accordingly, this intelligent tool will not only be able to design the optimum frac treatment, but will also be capable of reproducing the net pressure signature for that specific formation or reservoir. If successful, this methodology can be implemented in any field where significant hydraulic fracturing is performed.

The originality of the work is the development of a methodology that will design and monitor hydraulic fracture treatments based on historical results from previous jobs for a specific reservoir, rather than generalized analytical approximations.

1.3. Hydraulic Fracturing Background

Well stimulation methods have become very popular in the petroleum industry for maintaining or increasing well productivity. All stimulation practices adjust the skin, which is not due exclusively to damage, but is a multi-component variable for which stimulation may not affect all of its components. Therefore, identification of the individual components of the skin is important in choosing and designing the proper type of stimulation treatment. There are two main types of well stimulation methods employed presently: matrix acidizing and hydraulic fracturing. This study focuses on the practices of hydraulic fracturing.

Hydraulic fracturing is the process of pumping fluid into a wellbore at an injection rate that is high enough to cause the formation to break down. As the resistance to flow in the formation increases, the pressure in the wellbore increases until the breakdown pressure of the formation is exceeded. Once this occurs, a fracture is formed, and the injected fluid begins to move down the fracture. In most formations, a single, vertical fracture is created that propagates in two directions from the wellbore, usually 180 degrees apart and assumed to be identical in shape and size. In naturally fractured reservoirs, it is possible that multiple fractures may occur and each may propagate during a hydraulic fracture treatment.

1.3.1. History of Hydraulic Fracturing

According to a report by the Department of Energy, the first hydraulic fracturing treatment was pumped in 1947 on a Pan American Petroleum Corporation gas well in the Hugoton field¹. Located in Grant County, Kansas, Kelper Well No. 1 was a low productivity well, even though it had been acidized. Kelper Well No. 1 was selected for the first hydraulic fracture stimulation in order to directly compare the results to those of acidizing treatment¹.

Since that first treatment in 1947, hydraulic fracturing has become a standard treatment for stimulating the productivity of oil and gas wells.

1.3.2. Objective of Hydraulic Fracturing

Generally, hydraulic fracture treatments are used to increase the productivity or injectivity index of a producing or injection well, respectively. The productivity index is the volume of oil or gas that can be produced at a given pressure differential between the reservoir and the wellbore, while the injectivity index refers to how much fluid can be injected into the well at a given pressure differential.

There are many applications and uses of hydraulic fracturing; however, the majority of treatments are pumped for the following most common reasons:

- ▶ Increase the flow rate of oil and/or gas from low permeability reservoirs.
- ▶ Increase the area of drainage or the amount of formation in contact with the wellbore.
- ▶ Increase the flow rate of oil and/or gas from wells that have been damaged.
- ▶ Connect the natural fractures in a formation to the wellbore.

This study is mainly directed to low permeability reservoirs, which are typically excellent candidates for stimulation by hydraulic fracturing. A low permeability reservoir is one that has a high resistance to fluid flow. In many formations, chemical and/or physical processes alter a reservoir rock over geologic time. Sometimes, these diagenetic processes restrict the openings in the rock and reduce the ability of fluids to flow through the rock.

1.3.3. Data Requirements for Hydraulic Fracturing

The data required to run a fracture design simulator can be divided into two groups. The first group includes the data that can be controlled by the engineer such as well completion details, treatment volume, pad volume, injection rate, fracture fluid viscosity, fracture fluid density, fluid loss additives, propping agent type, and propping agent volume. The second group reflects data that must be measured or estimated, but cannot be controlled such as formation depth, formation permeability, *in-situ* stresses in the pay zone, *in-situ* stresses in the surrounding layers,

formation modulus, reservoir pressure, formation porosity, formation compressibility, and the thickness of the reservoir. The most critical data for the design of a fracture treatment are, roughly in order of importance:

- ▶ *In-situ* stress profile
- ▶ Formation permeability
- ▶ Fluid loss characteristics
- ▶ Total fluid volume pumped
- ▶ Propping agent type and amount
- ▶ Pad volume
- ▶ Fracture fluid viscosity
- ▶ Injection rate
- ▶ Formation modulus

Given a limited amount of time, attention should be focused on the most important parameters. In hydraulic fracture treatment design, the two most important parameters, by far, are the *in-situ* stress profile and the permeability profile of the zone to be stimulated along with that above and below the target zones.

Typically in new fields, most operating companies are normally willing to spend money to run logs, cut cores and run well tests to determine important factors such as the *in-situ* stress and the permeability of the major reservoir layers.

By using such data, along with fracture treatment records and production records, accurate data sets can be compiled. These data sets can be used on subsequent wells to optimize the fracture treatment designs. It is normally not practical to cut cores and run well tests on every well. Thus, the data obtained from cores and well tests must be correlated to log parameters so the logs on subsequent wells can be used to compile accurate data sets.

Data necessary for fracture treatment design can be obtained from a number of sources, including drilling records, completion records, well files, open hole geophysical logs, cores and core analyses, well tests, production data, geologic records, and other public records, such as publications. In addition, service companies provide data on their fluids, additives, and propping agents. Table 1-1 illustrates typical data needed to design a fracture treatment and possible sources for the data.

Data	Units	Sources
Formation Permeability	md	Cores, Well Tests, Correlations, Production Data
Formation Porosity	%	Cores, Logs
Reservoir Pressure	psi	Well Tests, Well Files, Regional Data
Formation Modulus	psi	Cores, Logs, Correlations
Formation Compressibility	psi	Cores, Logs, Correlations
Poisson's Ratio		Cores, Logs, Correlations
Formation Depth	ft	Logs, Drilling Records
In-situ Stress	psi	Well Tests, Logs, Correlations
Formation Temperature	°F	Logs, Well Tests, Correlations
Fracture Toughness	psi - $\sqrt{\text{in}}$	Cores, Correlations
Water Saturation	%	Logs, Cores
Net Pay Thickness	Ft	Logs, Cores
Gross Pay Thickness	Ft	Logs, Cores, Drilling Records
Formation Lithology		Cores, Drilling Records, Logs, Geologic Records
Wellbore Completion		Well Files, Completion Prognosis
Fracture Fluids		Service Company Information
Fracture Proppants		Service Company Information

Table 1-1: Necessary Data for Fracture Design

1.4. Research Tools – Intelligent Techniques

Three main artificial intelligence techniques are employed in the methodology of the Artificial Intelligence Hydraulic Fracturing Simulator: neural networks, genetic algorithms, and fuzzy logic.

Neural Networks are extremely powerful systems for pattern recognition and nonlinear multidimensional interpolation. They use distributive parallel processing to capture existing relationships between input parameters and the output of the system²⁻⁴. To achieve accurate predictions, neural networks must be trained and exposed to adequate and representative data.

Evolutionary computation it has its root in the biological evolution of the nature. Evolutionary computation is nothing other than searching or optimization algorithms inspired by the biological evolution of nature⁵. Evolutionary computation search and optimization is conducted: (1) based on multiple searching points or solutions candidates (population based search), (2) using operations inspired by biological evolution, such as crossover and mutation, (3) using little information of the searching space⁶. Genetic algorithms are a paradigm of

evolutionary computation. Genetic algorithms usually represent solutions for chromosomes with bit coding (genotype) and search for the better solution candidates using operations like selection, crossover and mutation.

Fuzzy logic theory provides a means for representing uncertainty⁷. Uncertainty is usually either due to the random nature of events or due to imprecision and ambiguity of information we have about the problem we are trying to solve. Fuzzy logic can use natural language to process information, and it provides efficient use of imprecise information. Fuzzy logic achieves this important task through fuzzy sets. In crisp sets, an object either belongs to a set or it does not. Fuzzy sets are used to determine the degree to which an object belongs to a set (an object may belong to multiple sets in varying degrees). Fuzzy set theory is a capable tool for modeling uncertainty associated with vagueness, imprecision, and/or a lack of information regarding a particular element of the problem at hand^{7,8}.

CHAPTER 2: LITERATURE REVIEW

2.1. Pre-Fracturing Formation Evaluation

Formation parameters used in designing a fracturing treatment can be obtained from wireline logging through direct or indirect measurements. These parameters include stress, shale volume, pressure, and permeability. Usually, the necessary data can be obtained from tools run in open hole on new wells. However, some of the data cannot be acquired in pre-existing cased wells.

2.1.1. Log Analysis Techniques

The wireline measurements used in fracturing treatment design are obtained primarily from sonic, gamma ray, density, and formation tester devices⁹. Sonic and density tools provide the measurements needed to calculate formation stresses, while gamma tools are the main instruments for determining the shale volume. Formation testers provide information about the pressure and permeability of the selected formations. However, these last two parameters can be also obtained through other procedures.

The records of sonic, gamma, and density tools are made continuously as the tools move through the wellbore. Typically the measurements are recorded as a function of depth, every 0.5 ft or 0.1 m. In logging terms, because of this dense sampling of formation properties, these measurements are often considered as being continuous. Continuous data such as this, used for designing fracturing jobs, provides an improved vertical resolution of stress variations.

Rock mechanical properties such as Poisson's ration, Young's modulus, shear modulus, and bulk compressibility are calculated from wireline measurements as an intermediate step in determining formation stress. When rock properties are derived from acoustic measurements, they are referred to as dynamic measurements.

2.1.1.1. Sonic Measurements

Acoustic energy propagates through matter as waves, the most commonly known being compressional and shear waves. By definition, acoustic slowness is the time required for an acoustic wave to travel a specific distance through a material, usually 1 ft or 1 m^{9,10}. Slowness is usually expressed in microseconds per foot or meter ($\mu\text{s}/\text{m}$). The wireline sonic tools use acoustic sources and sensors to measure the formation compressional and shear slowness, Δt_c and Δt_s , respectively. These are the two acoustic parameters needed in determining formation stresses. The acoustic velocity is the inverse of the acoustic slowness and is usually expressed in foot or meter per microseconds ($\text{m}/\mu\text{s}$).

Two types of sonic tools can be used to measure Δt_c and Δt_s , depending on whether the formation is slow⁹. A formation is slow when $\Delta t_s > \Delta t_f$, where Δt_f is borehole fluid slowness. The normal value of Δt_f for water is 189 $\mu\text{s}/\text{ft}$. Full waveform sonic tools use monopole acoustic transmitters and receivers to generate and sense compressional and shear waves that travel along the borehole wall. The tools are thus able to measure Δt_c and Δt_s . When the formation is slow, monopole tools cannot give rise to shear waves along the borehole wall and thus cannot measure Δt_s . For this reason, in slow formations, a dipole sonic tool is run. A dipole acoustic transmitter generates flexural waves that travel along the borehole wall and are sensed by dipoles receivers to measure flexural slowness as shear waves. Dipole tools also contain monopole acoustic transmitters and receivers to enable compressional slowness to be measured.

2.1.1.2. Density Measurements

Formation density (also known as formation bulk density, ρ_b) is the combined mass per unit volume of all materials in the formation whether solid, liquid, or gas. Wireline density tools contain a chemical source of gamma rays and two gamma ray detectors⁹⁻¹³. The measuring process uses the emission of gamma rays by the source into the formation. Some of the emitted gamma rays are scattered back to the tool and sensed by the detectors. The density measurement is based on the assumption that as formation density increases, the number of gamma rays scattered back to the tool decreases. The measured bulk density is used to determine the overburden pressure. Bulk density and overburden pressure are used with Δt_c and Δt_s in computing formation stresses.

Either traditional or spectral density tools can be used in open holes to measure ρ_b . Traditional density tools measure gamma radiation returning to the tool in a single broad energy range. By contrast, spectral density tools measure the amounts of gamma radiation returning to the tool in several specific energy ranges⁹. The spectral tools can provide a more accurate ρ_b measurement and also furnish a measurement of formation photoelectric factor used in identifying formation lithology.

2.1.1.3. Gamma Ray Measurements

The shale volume of the formation is determined through the use of gamma ray measurements. Wireline gamma ray tools contain a gamma ray detector but no gamma ray source^{9,12,13}. They measure the amount of gamma radiation present in the subsurface environment. The amount of such radiations emanating from a geologic formation is usually a good indicator of the shale volume. Shale volume is used in determining the formation's sanding potential.

Gamma ray tools are often run in combination with sonic and density tools, when either a conventional or spectral gamma ray tool can be used. Conventional tools measure gamma radiation in a single broad energy range while spectral tools measure the amount of gamma radiation in a larger number of energy bands. With spectral data, shale volume can be calculated more accurately, and clay types can be identified⁹.

Others tools are sometimes used to determine shale volume. These include spontaneous potential, neutron, neutron-density combination, and resistivity devices.

2.1.2. Closure Stress Calculations

In-situ stress has been widely accepted as the most important controlling factor for the vertical growth of hydraulic fractures¹⁴⁻¹⁶. The ability to determine stress profiles and therefore predict the vertical growth of hydraulic fractures is an essential part of a successful stimulation treatment in tight gas reservoirs.

2.1.2.1. Use of Hook's Law in Calculating the Log-Derived Stress

There are published techniques for computing the in-situ stress profile from log measurements and from core analysis^{10-13,17}. Several different methodologies for stress calculations follow.

- ▶ Hook's Law¹⁰⁻¹³ modified for stress calculation using log data is given by:

$$e_x = \frac{1}{E}(S_x - S_p) - \frac{\nu}{E}(S_y - S_p) - \frac{\nu}{E}(S_z - S_p) \quad \text{Equation 2-1}$$

where, e_x = elastic strain in the x direction

E = Young's modulus

S_x, S_y, S_z = stress in direction x, y, z

S_p = stress produced by internal pore pressure

ν = Poisson's ratio

When e_x is assumed to be zero, $S_x = S_y = S_z$ (closure stress), and $S_z = S_v$ (overburden stress), the above equation reduces to the classical equation:

$$S_h = \frac{\nu}{1-\nu} (S_v - S_p) + S_p \quad \text{Equation 2-2}$$

However, different studies have contributed to the equation in an attempt to match the log derived stress profile with the in-situ stress calculations.

- ▶ In 1986, Whitehead et al¹¹, modified the classical equation and proposed a new correlation given by:

$$Sh = \frac{\nu}{1-\nu} (1-P'_p) + P'_p \quad \text{Equation 2-3}$$

where, P'_p = equivalent formation pore pressure

$$P'_p = 1.15 (2.8P_f^2 + 1)^{1/2} - 1$$

P_f - formation pore pressure

In this equation it can be noticed that the S_v (overburden stress gradient) is equal to 1.

- ▶ In 1991, S.A. Holditch and Associates¹², using the data from a well analyzed in Texas, introduced a tectonic factor to the equation in an attempt to match the log derived stress profile with the measured in-situ stress of the formations. The new form of the correlation is:

$$Sh = \frac{\nu}{1-\nu} (S_v - S_p) + S_p + X \quad \text{Equation 2-4}$$

where, X = empirical stress factor

$$X = 0.794 \phi_{nls} - 0.075$$

ϕ_{nls} = neutron porosity log

The empirical stress factor, X , is obtained from the plot of delta stress versus neutron porosity log¹².

- ▶ In the same year, S.A. Holditch and Associates¹³, using the data from a well analyzed in Wyoming, determined a new form of the correlation:

$$Sh = \frac{\nu}{1-\nu} (S_v - S_p) + S_p(1 + \beta) \quad \text{Equation 2-5}$$

where, β = a unitless, shaliness or "softness" factor

$$\beta = (C_b - C_{bmin}) / (C_{bmax} - C_{bmin})$$

C_b = bulk compressibility computed from sonic and density log

C_{bmax} = max bulk compressibility expected for porous sandstone

C_{bmin} = min bulk compressibility expected for solid quartz

- ▶ Bragan & Associates¹⁷, in a report prepared for Gas Research Institute, evaluated the ABC Stress Derivations Methodology. The ABC method reduces the classical stress equation by using A, B and C coefficients:

$$Sh = A S_v + B S_p + C \quad \text{Equation 2-6}$$

A coefficient is assigned based on formation lithology. In practice, B is assumed to be 0.5 and C zero. A different version of the ABC Methodology calculates A and B coefficients using Poisson's ratio.

$$A = \frac{\nu}{1-\nu} \quad \text{Equation 2-7}$$

$$B = \frac{1-2\nu}{1-\nu} \quad \text{Equation 2-8}$$

2.1.2.2. Determination of Elastic Properties with Acoustic Logs

The elastic properties needed in stress calculations are routinely determined from acoustic logs¹⁰⁻¹³. The main parameters used include the following:

- ▶ Ratio of shear and compressional slowness, R:

$$R = \frac{\Delta ts}{\Delta tc} \quad \text{Equation 2-9}$$

where, Δts = shear slowness

Δtc = compressional slowness

- ▶ Compressional velocity, V_c :

$$V_c = \frac{10^6}{\Delta tc} \quad \text{Equation 2-10}$$

- ▶ Shear velocity, V_s :

$$V_s = \frac{10^6}{\Delta ts} \quad \text{Equation 2-11}$$

- ▶ Poisson's Ratio ν :

By definition, Poisson's ratio is the ratio of the transverse expansion to the longitudinal stress when a body is compressed^{10,11}. It can be determined from static loading tests or from acoustic (dynamic) measurements. The calculation of Poisson's ratio from acoustic log measurement tends to give more consistent results. For the purpose of this analysis, Poisson's ratio is related to the ratio, R, of compressional velocity, V_c , to shear velocity, V_s , through the following expression^{10,11}:

$$\nu = \frac{1}{2} \left[\frac{\left[\frac{V_c}{V_s} \right]^2 - 2}{\left[\frac{V_c}{V_s} \right]^2 - 1} \right] \quad \text{Equation 2-12}$$

$$\text{or } \nu = \frac{1}{2} \left[\frac{R^2 - 2}{R^2 - 1} \right] \quad \text{Equation 2-13}$$

- ▶ Shear modulus, G:

$$G = \frac{(1.36 * 10^4) * \rho_b}{\Delta t s^2} \quad \text{Equation 2-14}$$

$$\text{or } G = c * \rho_b * V_s^2 \quad \text{Equation 2-15}$$

where, c = unit constant
 ρ_b = bulk density

- ▶ Young's modulus, E:

$$E = 2 G * (1 + \nu) \quad \text{Equation 2-16}$$

$$\text{or } E = \frac{9K_b * G}{(3K_b + G)} \quad \text{Equation 2-17}$$

- ▶ Bulk modulus, K_b :

$$K_b = (1.36 * 10^{-8}) * \rho_b * (V_c^2 - \frac{4}{3} V_s^2) \quad \text{Equation 2-18}$$

$$\text{or } K_b = \frac{GE}{9G - 3E} \quad \text{Equation 2-19}$$

- ▶ Bulk compressibility, C_b :

$$C_b = \frac{1}{K_b} \quad \text{Equation 2-20}$$

2.1.2.3. Determination of Formation Pressure

Formation pressure is a major component in the stress profile calculation expression. It is known that in normally pressured zones the formation pressure contributes well over one half of the closure stress. For accurate calculations of the stress profile, the formation pressure must be known or closely estimated. In permeable zones, pressure can be determined from wireline testing; the most common procedure is the Repeat Formation Tester⁹⁻¹³.

In the case when well tests are not run, the formation pressure can be estimated using industry accepted correlations. The most common correlation consists of a linear dependency between formation pressure and depth.

2.1.3. Formation Tester Measurements

Formation testers provide stationary measurements of formation pressure^{10,11}. The tool is positioned at a depth at which pressure is to be measured and is then held stationary while pressure is recorded as a function of time. This is in contrast to most other logging tools, where measurements are usually recorded as a function of depth. Analysis of pressure measurements yields reservoir pressure and permeability. These parameters are used directly in the simulation program.

Pore pressure, p_p , is one of the most crucial parameters in evaluating the stress field around the borehole. Although the assumption of a 0.46 psia/ft pore pressure gradient is accurate enough for most analyses, the exceptions will cause erroneous stress profile calculations. For example, over-pressured zones and depleted zones have different pore pressure gradients and therefore drastically affect the final stress profile. Although reservoir engineers generally know the pore pressure gradients in such zones, the precise boundaries at which pressure gradients change must be identified. This can be accomplished using wireline formation tester or microfrac data. If such data are not available, over-pressured or depleted zones may be detected using density, resistivity, or sonic logs⁹.

Pore pressure gradients can vary on a smaller scale within a reservoir or zone of interest, dependent upon the density of the fluid present. Although a slight gradient shift may not be critical in the final stress evaluation, data from a wireline formation tester should be used when available. Density, resistivity, and sonic logs along with reservoir engineering data are usually not able to sufficiently resolve variations in pore pressure gradients within an interval.

2.1.4. In-Situ Stress Test Analysis

The objective of an in-situ stress test is to determine when a fracture opens (or closes) by monitoring pressure data¹⁰⁻¹³. With this information, the minimum principal stress can be estimated.

Several methods are used to estimate the minimum principal stress (or in-situ stress) for an interval. These methods include injection falloff or injection flowback tests, mini-fracs, and micro-fracs¹⁰⁻¹². Injection falloff tests are generally more accurate⁹. In these tests, very small fluid volumes (less than 500 gallons) are pumped at very low injection rates (usually less than 20 gpm). A downhole shut-off tool is used to record the pressure. In this way, the effects of after-flow or wellbore storage during shut-in or pressure falloff stages are minimized. Two types of analyses are used for determining the fracture closure pressure: the log-log plot of pressure versus time and the plot of square root of time versus pressure. The half-slope region on the log-log plot represents the linear flow and the end of the half slope line should agree with the straight line on the square root of time plot¹¹. The deviation of the data from the straight line represents the fracture closure pressure.

In-situ stress tests are performed in both open-hole and cased-hole. Even though open-hole tests are preferable, it is very difficult to get the packer to seal on the formation making the measurements untrustworthy¹¹. For this reason, cased-hole tests are run to determine the fracture closure pressure with a high rate of success. A good set of the packer in the cased hole allows for better control of the point of injection. However, a poor sealing of the packer in the casing allows communication with the annulus and the measurements become unclear.

2.2. Hydraulic Fracture Models

From the early years of oil and gas production, hydraulic fracturing seemed to be a very promising technology to increase the well flow rate and thus produce the well at higher economic levels. However, not much was known about the propagation and geometry of the fracture.

In 1955, Khristianovic and Zlethov¹⁸ proposed the first model to simulate fracture propagation. It was a simple two-dimensional model that considered the formation as a single layer bounded by two impermeable layers. Since this first model was introduced, many authors have proposed different hydraulic fracturing models. Throughout the years, the models have become more and more complex in an attempt to integrate the mathematical equations describing the fundamental and constitutive laws^{19,20}.

In 1969, Geertsma and de Klerk²¹ improved Khristianovic's¹⁸ model and proposed a new 2D-model known as the KGD Model. This model was a breakthrough at the time and is still incorporated in many commercial simulators. The particularities of this model are: uniform height of the fracture, rectangular cross section of the fracture, and the crack opening is solved in the

horizontal plane. A schematic description of the fracture geometry considered in the KGD model is presented in Figure 2-1.

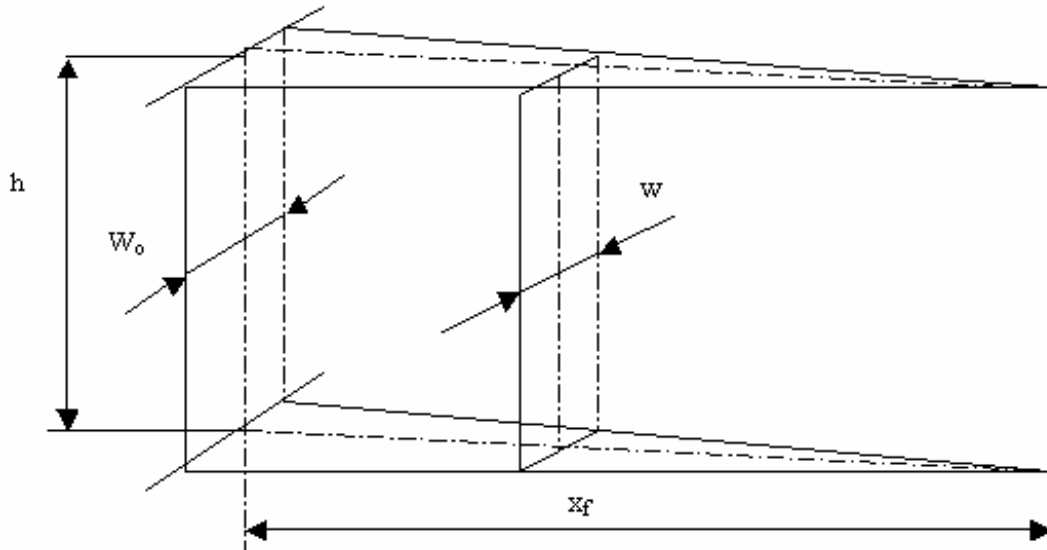


Figure 2-1: KGD Model Fracture Geometry

In 1961 Perkins and Kern²², and Nordgen¹⁵ in 1972, proposed a new 2D-model known as the PKN Model. They considered a new approach in solving the crack opening equations based on the assumption that the shape of the fracture in cross section, is not rectangular, but elliptical. The model considered the following assumptions: uniform height of the fracture, elliptical cross section of the fracture, and the crack opening is solved in the vertical plane. A schematic description of the fracture geometry considered by the PKN model is presented in Figure 2-2.

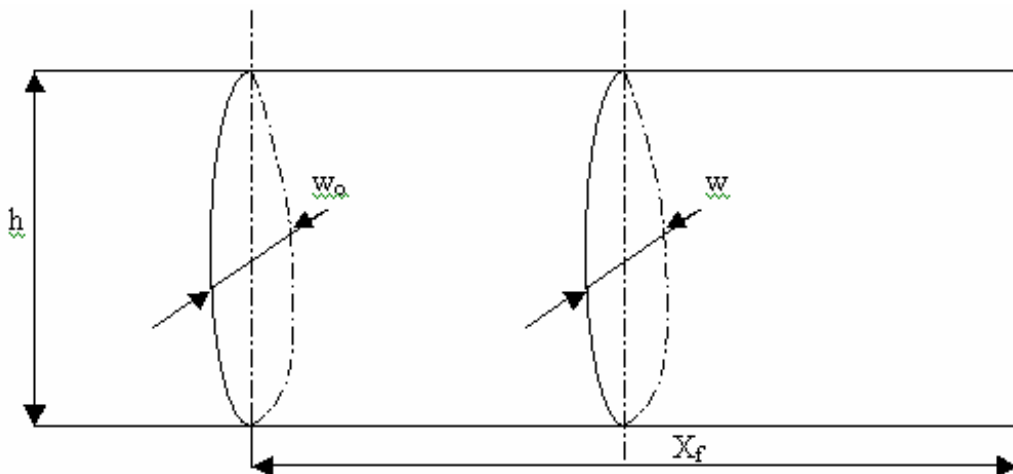


Figure 2-2: PKN Model Fracture Geometry

In 1982, Nolte and Smith¹⁴ developed a pseudo-three-dimensional model (p-3D), which was based mainly on the PKN system. The primary difference from the classical 2D-model is the estimation of the vertical fluid-flow term through the fracture, which provides a non-uniform height of the fracture.

In 1991, Mike Cleary²³⁻²⁵ developed one of the most complex 3-D fracturing models in existence today. The model was implemented in complex software called FRACPRO²⁶, by Resource Engineering Systems, Inc. (RES) with support from the Gas Research Institute, (GRI).

2.3. Hydraulic Fracturing Design

Hydraulic fracturing removes the skin around the wellbore, and also improves the well productivity by creating a large contact surface between the well and the reservoir¹⁹. Experience has demonstrated that hydraulic fracturing, in appropriate reservoirs, has increased and maintained well productivity to values by far greater than achieved by matrix acidizing.

Hydraulic fracturing is a complex treatment that deals with four main domains: formation characteristics, treatment fluid characteristics, proppant characteristics, and treatment type. Hydraulic fracture modeling uses fundamental laws (mass, momentum, and energy conservation), constitutive laws, and the previously mentioned domains to simulate the propagation and geometry of the fracture¹⁹.

2.3.1. Treatment Fluid Characterization

The fracturing fluid is a critical component of the hydraulic fracturing treatment due to its important functions: to open the fracture and transport the proppant into the fracture^{9,19,26}. For this reason, the fracturing fluid must not only have good viscous properties, but also many other properties that can be summarized as follows:

- ▶ Low friction pressure in tubing/casing and in the fracture
- ▶ Good fluid-loss control and low damage on the productive formation
- ▶ Rapidly break at the end of the treatment
- ▶ Low cost

Three types of fracturing fluids are currently available in the industry: oil-based fluids, water-based fluids, and multiphase fluids.

Oil-based fluids were the first fluids used in fracturing treatments because they were perceived to be less damaging to the production formation than water-based fluids. Also, their viscosity made them more attractive. However, oil-based fluids are expensive and operationally difficult to handle. Therefore, this type of fluid is now only used for extremely water-sensitive formations¹⁹.

Water-based fluids are the most widely used fracturing fluids in the industry¹⁹. After the discovery of polymers and crosslinkers, water-based fluids replaced most oil-based fluids due to their high performance and low cost. By adding polymers to a water-based fluid, the viscosity of the solution increases considerably as well as the proppant transport property of the fluid. All polymers (basically high molecular weight molecules) increase the viscosity of the solution at ambient temperatures. Because reservoir temperature is much higher, some solutions thin significantly. In these conditions, the polymer concentration can be increased to maintain the solution viscosity and offset the thermal conditions, but this approach is expensive. For these cases, the use of crosslinker agents has proved to be more efficient. Crosslinker agents are used to increase the effective molecular weight of the polymer and, as a result, increase the viscosity of the solution. The most used ions to crosslink water-soluble polymers are borate, Ti (IV), and Zr (IV).

Multiphase fluids are basically foams and emulsions. Foam is a stable mixture of liquid and gas. In the case of hydraulic fracturing, foams are created by adding gas to water-based or oil-based fluids. Oftentimes, surface-active agents are added to increase the stability of the foams. These agents, also known as surfactants, act at the gas-liquid interface to reduce the surface tension. The most common foams used in the industry are nitrogen (N₂) and carbon dioxide (CO₂) foam fracturing fluids.

Foams are not only good fluid-loss control fluids in low permeability formations, but are also non-damaging¹⁹. Foam fracturing fluids are described by a specific parameter called “foam quality” which is defined by the following relationship:

$$\text{Foam Quality} = \frac{\text{GasVolume}}{\text{FoamVolume}} \times 100 \quad \text{Equation 2-21}$$

It should be mentioned that foam fracturing fluids are becoming more commonly used in the industry.

The other type of multiphase fluids is the emulsion. An emulsion is a dispersion of two immiscible phases, usually stabilized with a surfactant. In the case of fracturing fluids, emulsions are created by mixing oil in water or water in oil and stabilized with a surfactant. The primary advantage of the emulsion-based fracturing fluids is its high viscosity, which confers a very good transport property. Disadvantages include high friction pressure and the high cost of the fluid (hydrocarbon phase).

There are a series of additives that are added to fracturing fluids to control their properties and increase their functionality and usability. Among these additives are buffers (to adjust the pH of the fluid), bactericides (to prevent viscosity loss due to bacterial degradation of the polymer), stabilizers (to prevent degradation of the polymer at high temperatures), breakers (to break the gel polymer at the end of the treatment), surfactants (to reduce the interface tension

between two immiscible substances), and fluid-loss additives (to control fluid-loss in the productive formation).

2.3.2. Proppant Characterization

Proppant is the material transported with the fracturing fluid and deposited in the fracture to keep the fracture open at the end of the treatment^{9,19,26}. Since the beginning of hydraulic fracturing, several materials have been used as proppants. Some proppants used in the industry today include sands, resin-coated sands, intermediate-strength proppants (ISP), and high-stress proppants.

Sand is by far the most widely used proppant due to its good performance and low cost. There are different types of sand, named for the region where they are found such as “Northern sand”, “Ottawa sand”, “Brady sand”, “Jordan sand”, etc.

Resin-coated sand is sand treated with resin coatings to improve the proppant strength. It has been proven that resin-coated sand provides higher conductivity and withstands higher confining pressure than conventional sands¹⁹. The cost of this proppant is higher than conventional sands.

Intermediate-strength proppants are basically fused ceramics. These proppants are used only in cases where the formation yields higher closure stress.

High-strength proppants are sintered bauxite and zirconium oxide. They yield a very high strength and are very seldom used due to high cost.

The main characteristics of propping agents that have an impact on fracture conductivity are:

- ▶ Proppant strength
- ▶ Grain size and grain size distribution
- ▶ Quality of the proppant
- ▶ Roundness and sphericity
- ▶ Proppant density

Proppant strength is the property of the proppant to resist the closure stress of the fracture without breaking or crushing. If the proppant is not strong enough and crushes, the permeability of the fracture is significantly reduced.

The conductivity of the fracture is related to the grain size and grain size distribution – the higher the grain size, the higher the conductivity. However, larger grain sizes cannot be used in deep wells due to the low strength to crushing.

The quality of the proppant is related to the purity (the amount of fines and impurities) of the proppant. Fines or impurities can drastically reduce the proppant-packed permeability.

Roundness and sphericity of the grains have a dramatic effect on the fracture conductivity. Proppant grain roundness is a measure of the relative sharpness of the grain corners, while grain sphericity is a measure of how close the proppant particle approaches the shape of the sphere. Higher roundness and sphericity of grains provide higher conductivity of the fracture and higher load to support before crushing.

2.3.3. Treatment Design

A classical hydraulic fracture treatment is designed to provide optimum length and conductivity to effectively stimulate the well. The treatment design consists of the total volume of fracturing fluid, total quantity of proppant used, and injection rate. The total volume of fracturing fluid is divided into pad volume and slurry volume. The pad is the clean volume of the fluid pumped at the beginning of the treatment to fracture the formation, while the slurry is the mixture of fluid and proppant (pumped after the pad) that transports and deposits the proppant in the fracture. Treatment schedules can be designed using different number of stages, proppant concentration, and pumping rate. In order to design an effective treatment job, the appropriate quantities of fluid and proppant must be pumped at a specific flow rate in order to achieve a desired propped length.

2.4. Fracture Design Simulators

The purpose of a fracture design simulator is to use a computer to simulate, as closely as possible, the actual downhole events that occur while performing a fracturing treatment. A number of reliable fracture design simulators are currently available on the market²⁰. The following discussion focuses on some of major programs that perform fracturing design using 2D, pseudo-3D, and 3D modeling.

2.4.1. 2D Models

2D models simulate the length and width of the fracture, assuming constant height. The most popular 2D models used the PKN or GDK fracture geometries.

2.4.1.1. PROP

PROP is Halliburton's 2D fracture model²⁷. It is based on Daneshy's numerical solution and is thus more flexible than other analytical models. It also uses the power-law model for gel fluids and Herschel-Bulkley for nitrogen or carbon dioxide phase fluids. PROP has capabilities to provide both GDK and PKN fracture geometries.

2.4.1.2. Chevron

Chevron's 2D fracture simulator is used for constant-height, hydraulically induced, vertical fractures using a power-law fluid²⁷. It can predict both PKN and GDK fracture geometries. It is most applicable for fracture design where geological conditions prevent height growth. It also has a proppant transport model that uses proppant settling and production, which calculates the final propped concentration, width, and bank height, given the settling velocity. Chevron's 2D simulator can predict possible problems caused by proppant bridging or screenout.

2.4.2. Pseudo-3D Models

Pseudo-3D models were developed based on the PKN model by removing the requirement of constant fracture height. However, they assume one-dimensional flow along the fracture length.

2.4.2.1. MFRAC-II

MFRAC-II is a pseudo-3D hydraulic fracturing simulator that models penny, GDK and PKN geometries and accounts for the parameters affecting fracture propagation and proppant transport²⁷. It also includes multilayer, asymmetrical confining stress contrast; fracture toughness and tip/overpressure effects; rock deformation; variable injection rate and time-dependent fluid rheology properties; multilayer leakoff with spurt loss; and 2D proppant transport. The fracture propagation model calculates fracture length, upper and lower heights, net pressure, efficiency, geometry parameters as function of time, and width variation as a function of height and confining stress. MFRAC-II also presents other options to allow customization and applicability over a broad range of conditions.

2.4.2.2. STIMPLAN

The STIMPLAN program, developed by NSI Inc., is a fracture design simulator with special modifications that allows for tip-screenout designs²⁷. At tip-screenout initiation, fracture extension is stopped and the program calculates a width increase based on the increase in the net treating pressure. This program will analyze complex formations composed of multiple productive layers with varying fluid-loss coefficients to calculate height growth including proppant settling and simulate fracture closure. It includes a history-matching module for net treating pressure to provide fracture geometry and behavior. STIMPLAN is well designed, easy to use, and is a popular choice among professionals who need an effective tool for designing fracturing treatments for high permeability formations²⁷.

2.4.2.3. ENERFRAC

ENERFRAC is Shell's pseudo-3D hydraulic fracture model that predicts fracture dimensions for uncontained and contained fractures²⁷. It accounts for fracture-tip effects through direct input of apparent fracture toughness or the instantaneous shut-in pressure minus the closure pressure, which can be determined in the field from mini-frac tests. ENERFRAC also incorporates other interacting processes such as viscous fluid flow, elastic rock deformations, and fluid loss²⁷.

2.4.2.4. TRIFRAC

S. A. Holditch and Associates uses TRIFRAC²⁷, a pseudo-3D model, which utilizes a finite-difference numerical approach to determine created and propped fracture dimensions from fracture propagation and proppant transport. It can use up to 22 multiple nonsymmetrical stress layers, with each layer having unique values for Young's modulus, Poisson's coefficient, and fracture toughness, permeability, porosity, and fluid-leak-off coefficients. The apparent viscosity of the fracturing fluid is calculated from the shear rate inside the fracture and owes its variations of temperature (a temperature calculation model is provided) and time to changes in flow-behavior index and consistency index. Initiating the hydraulic fracture from up to 10 different layers is possible. Special options can be used for nitrogen foam²⁷.

Proppant distribution, transport, and settling, along with fracture growth are simultaneously solved using the geometry computations module coupled with the finite-difference proppant transport simulator. It can compute the proppant profile for each time step for certain fluid velocities and proppant settling rates. TRIFRAC also has the ability to model PKN and GDK (including horizontal fracture) geometries. All models use the proppant-transport calculations module.

2.4.3. 3D Models

Fully 3D fracture models calculate the fracture geometry with no restrictions on fracture height. They consider 3D fluid flow into the fracture.

2.4.3.1. FRACPRO

Resource Engineering Systems, Inc. (RES) developed the FRACPRO^{26,27} program with support from the Gas Research Institute (GRI). Currently, FRACPRO belongs to Pinnacle Technologies.

The FRACPRO²⁶ system is designed to provide engineers with comprehensive tools for hydraulic fracturing design and analysis. The fracture design model has the capability of

acquiring real-time fracturing data during treatment. The program can be used to design fracturing treatments and can acquire downhole data during field operations from a treatment database to confirm the design estimates or perform detailed post-treatment analysis. Changes to the design and treatment can be made to better match the job data to design criteria. The capabilities of designing, monitoring, and analyzing the fracturing treatment make FRACPRO²⁶ a versatile model for both minifrac and fracture design.

The FRACPRO model can analyze several layers of formation with varying rock properties and fluid-loss coefficients. Also, the model allows the user to select either wall-building or non-wall-building fracturing fluids to be used during the treatment. The fluid selection feature is important, since both of these fluid types (HEC and borate-crosslinked fluids) are used in high permeability formations. HEC does not build a filter cake and is controlled by viscous invasion of the formation, whereas borate-crosslinked fluids simulate a wall-building fluid with high spurt volumes. Multiple fluid designs can be handled with changing fluid-loss coefficients and different fluid-loss characteristics of fracturing fluids^{26,27}.

2.4.3.2. GOHFER

Grid Oriented Hydraulic Fracture Extension Replicator (GOHFER)²⁷ is a planar 3D model provided by Marathon Oil Company. For elastic rock displacement, it uses a regular grid structure, and a planar 2D finite-difference grid for fluid flow. The fluid flow equations provide the areal pressure distribution and proppant transport, which are used iteratively with the solution for elastic deformation. This allows for the representation of perforations at various locations. Each grid node can be assigned the following values: net stress, pore pressure, permeability, porosity, wall-building coefficient, rock strength, Young's modulus, Poisson's ration, and other variable associated with fracture-wall roughness and tortuosity. Boussinesq is used as the fracture width equation. GOHFER can model multiple fracture initiation and can be used for any planar 3D geometry.

2.4.3.3. FRACPAC

The FRACPAC program, developed by Halliburton, is aimed to assist in design of tip-screenout fracturing treatments^{9,27}. The 3-D fracture geometry predictions from the XTENT program are incorporated with modifications to FRACPAC to allow pumping to continue after the tip-screenout initiates. Upon tip-screenout initiation, fracture length extension and fracture height extension stop and injection of additional slurry causes fracture width to grow. Calculations for fluid-loss, fracture width, proppant concentration, and net treating pressure during pumping continue after tip-screenout initiation. Treatment modeling ends when the user-specified increase in net treating pressure is reached.

The FRACPAC²⁷ design simulator offers very good input options such as capability of reading dynamic in-situ rock stress measurements directly from wireline logging files. The program also has limitations, especially in complex reservoirs. FRACPAC allows only one pay zone to be analyzed, and stress values across the zone are averaged. Values for fluid-loss are also limited; only two values are allowed for fluid-loss, one value for the pay zone and the other for outside the pay zone. In wells where several high permeability intervals are separated by small layers of shale, FRACPAC requires the user to make assumptions, which can cause the software to be difficult to use.

2.4.3.4. TerraFrac

TerraFrac is a commercially available, true 3D hydraulic fracture simulator provided by Terra Tek²⁷. TerraFrac uses discrete elements to partition the fracture in order to solve the equations. It uses 3D elasticity, 2D fluid flow, and fracture criteria for intensity of fracture growth²⁵. Proppant and temperature distribution are provided from 2D fluid flow. Multiple stages and multiple layers are available each having different fluids, proppants, injection rates; and in-situ stresses, Young's moduli, fracture toughness, Poisson's ratio, and leakoffs, respectively. TerraFrac also has special features for waterfloods, a quasi-Newton method for fluid pressures, and post-shut-in capabilities.

2.4.3.5. HYFRAC3D

HYFRAC3D²⁷ is a finite-element based code developed by S. H. Advani of Lehigh University that uses a set of coupled mass conservation, fluid momentum, constitutive elasticity, and fracture mechanics equations governing planar hydraulic fracture propagation in a multilayer reservoir. HYFRAC3D tracks the moving fracture front in a numerical scheme using a mapping technique of the baseline mesh in a unit circle to arbitrarily shaped fracture geometrics. It can also provide PKN results based on a 2D finite-element model simulator with standard PKN model equations, including vertical stiffness and one-dimensional fluid flow.

2.4.4. Comparison of Fracturing Simulators

In 1994, the Gas Research Institute sponsored the Stage Field Experiment No. 3 fracture experiment¹² in which fracture design results were submitted from several different simulators and compared; these results are documented in SPE paper 25890²⁷. Input data, including rock and reservoir data (depth, zone thickness, in-situ stress, Poisson's ratio, Young's modulus, and fracture toughness) along with treatment data (temperature, reservoir pressure, spurt loss, fluid-leakoff, viscosity, fluid volume, injection rate, and proppant), was provided.

Fracture design results were submitted for GOHFER, PROP, TRIFRAC, FRACPRO, ENERFRAC, MFRAC-II, Chevron, HYFRAC3D, STIMPLAN, and TerraFrac. The results from these models, given the same input parameters, had widely different fracture geometries. For example, in the five-layer case: FRACPRO calculated very short fractures, high net pressures, and large heights; GOHFER predicted short fractures and high net pressures; TRIFRAC, STIMPLAN, TERRAFRAC, and MFRAC-II all had longer fractures, less height, and somewhat lower net pressures; HYFRAC3D predicted somewhere in the middle.

The differences in results between simulators are partly attributed to the differences in the assumptions for each model. Furthermore, these simulators have many options available to provide better simulation of actual fracture behavior. The use of these options is arbitrary, depending on the reservoir experience of the engineer running the simulation. The authors of SPE paper 25890²⁷ also recognize that other formations, with different stress and lithology data could provide a considerably different comparison.

It is extremely difficult, if not impossible, to know exactly how the fracture propagates in reality. As such, it cannot be determined which fracture simulator is the best. Each has its advantages and disadvantages. Therefore, due to wide use throughout the industry and availability, the 3D model FRACPRO²⁶ was used for this study. However, any one of these fracture simulators can be used.

CHAPTER 3: BACKGROUND

Three main artificial intelligence techniques are employed in the methodology of the Artificial Intelligence Hydraulic Fracturing Simulator: neural networks, genetic algorithms, and fuzzy logic. Each is described in detail throughout the following sections.

3.1. Artificial Neural Networks

Artificial neural networks are information-processing systems developed as generalizations of mathematical models of human cognition and neural biology²⁸. They were born from the desire to produce systems capable of intelligent and sophisticated computations similar to those the human brain performs.

3.1.1. Overview

Biological neural networks are composed of neurons and dendritic connections that form the human brain². The basic processing element in this system is the neuron, while nerve fiber called dendrites form a tree-like network of connections to the cell body. The axon is a single fiber that extends from the cell body and branches into strands and substrands, which are then connected to other neurons through synapses. Figure 3-1 represents a diagram of a simple neuron.

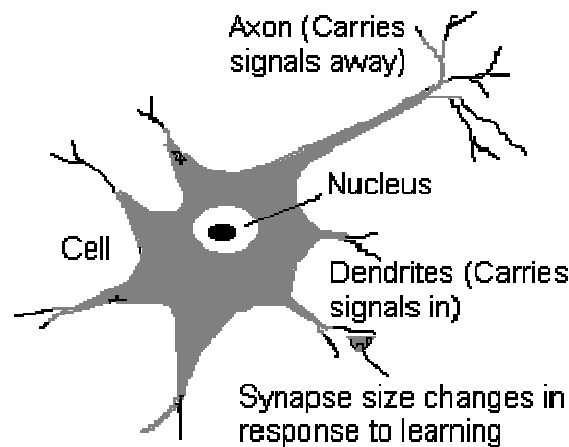


Figure 3-1: A Simple Neuron

Signals are transmitted from one neuron to another at the synapses and involve a complex chemical process in which the sender neuron releases specific transmitter substances to be absorbed by the receiver neuron. The effect of this reaction is the increase of electrical

potential of the receiver neuron. If the electrical potential reaches a certain threshold, the neuron will fire, sending a pulse down its axon. It is through this process and architecture that complex human learning, reaction, and behavior takes place.

Neural networks are designed to process information in a similar, but simplified, manner as the human brain. Resemblances to the human brain include the following:

- ▶ Knowledge acquired through a learning process
- ▶ Local processing in artificial neurons (known as nodes or processing elements)
- ▶ Storage of experiential knowledge available for future use in interneuron connections (synaptic weights)
- ▶ Massive parallel processing implemented by profuse connection pattern

Neural networks present some additional fundamental features providing them with the capability of solving a variety of complex scientific and engineering problems. These features include:

- ▶ Adaptability – the ability to deal with small and large data sets with minimal memory requirements; allows for the possibility of developing real-time online solutions where all data is not available at the beginning.
- ▶ Massive parallelism and high connectivity – large number of nodes share the burden of computation and often can operate independent of information made available by other nodes; the speed of computation is significantly increased (the output is generated instantaneously).
- ▶ Universal function estimation – ability to perform pattern recognition, detect trends, and solve highly non-linear problems through deriving meaning from complicated or imprecise data.

Since the birth of artificial neural networks in the 1980s, neural systems were built in many different forms. However, there are certain characteristics that are common between all paradigms.

- ▶ Weight parameters, adjusted to produce learning.
- ▶ Interconnected units (nodes).
- ▶ Learning rules that dictate information flow in the network.
- ▶ Optimization by aiming to minimize a cost function, energy function, or a complex combination of functions.

In neural computing, the artificial neuron is called the processing element or node. Although modeled after biological neurons, their artificial counterparts bear only a modest resemblance. Whereas neurons in the brain can perform at least 150 different processes, processing elements can only perform three:

- ▶ Evaluate input signals and determine the strength of each.

- ▶ Calculate the total for the combined input signals and compare this to a certain threshold level.
- ▶ Determine the output: to fire or not fire.

As biological neurons can be affected by mechanisms other than inputs, most neural networks attempt to simulate this feature through the use of synaptic weights. Much like the varying of synaptic strengths in biological neurons, weights are adaptive coefficients within the network that determine the intensity of the input signal, thus allowing for some inputs to be more important than others in their combination to form an output or impulse. As mentioned above, neural networks acquire information through the process of learning, which is completed through showing the network examples of input data and the expected output. The synaptic weights adjust themselves until the error between the actual expected output and the output generated by the neural network reaches a desired level. The weights are changed by an amount proportional to the error at that unit times the output of the unit feeding into the weight. This, in effect, allows the processing elements to modify their behavior in response to its inputs^{3,28}.

Several processing elements are combined to form a layer of nodes. Inputs are connected to many nodes using various weights, resulting in a series of outputs. The layer receiving the inputs is termed the input layer and generally only buffers the input signal, while the network outputs are generated from the output layer. All other layers are called hidden layers. The network is fully connected if every output from one layer is passed along to every node in the next layer.

3.1.2. Backpropagation Neural Networks

Back propagation networks (the most common in use today) allow for weight adjustments to be made in preceding layers of feed forward networks by “backing up” from outputs. Figure 3-2 presents a back-propagation neural network with two hidden layers. The first layer (input layer) reads the inputs and passes them to the next layer, (the first hidden layer). Next, the processed information is passed to the next layer (the second hidden layer) and so on.

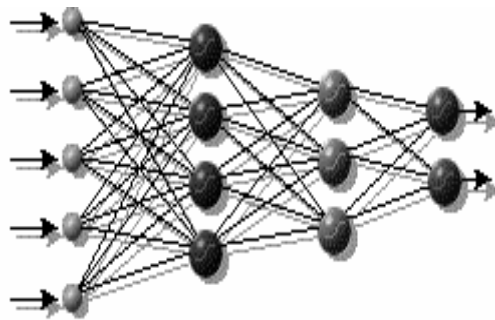


Figure 3-2: Typical Back Propagation Network

The weights associated to each interneuron connection affect the information so that the network can control the flow of information. The processing units in the hidden and output layers are characterized by the following activation function²:

$$Y = 1 / (1 + e^{-k(\sum W_{in} * X_{in})}) \quad \text{Equation 3-1}$$

The graphical representation of this function can be seen in Figure 3-3.

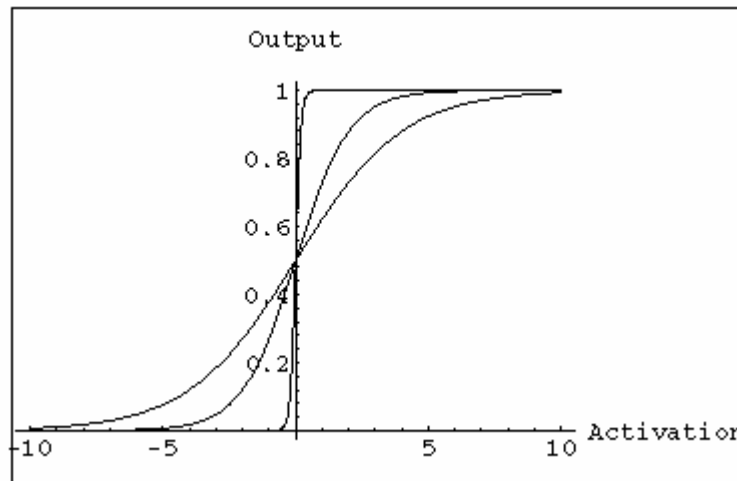


Figure 3-3: Activation Function for Back Propagation Networks

In a typical neural network data processing procedure, the data is divided into three separate sets:

- ▶ Training – set of examples used for learning to fit the parameters (weights) of the classifier.
- ▶ Testing (or calibration) – set of examples used to tune the parameters of a classifier, such as choosing the number of hidden units in a neural network.
- ▶ Production (or verification) – set of examples (not previously seen by the neural network to ensure its integrity and robustness) used only to assess the performance or generalization of a fully specified classifier.

3.1.3. Neural Nets versus Conventional Computing

Artificial neural networks are an alternative to conventional computing, but will not replace it. Neural networks are simply a different way of solving problems that typically cannot be done by conventional computing. Conventional computing (which must follow a specific set of

instructions) is restricted to problems that can already be solved, while neural networks are able to handle non-linearity, noise in the data, and a large number of parameters. Because neural networks learn by themselves using the examples provided to it, their operation can be unpredictable if not tested thoroughly. A summary of the differences between conventional computing and neural networking follows in Table 3-1:

Conventional Computing	Neural Networks
Deductive reasoning – known rules are applied to input data to produce an output.	Inductive reasoning – rules are constructed given training and testing data.
Computation is centralized, synchronous, and serial.	Computation is collective, asynchronous, and parallel.
Memory is packeted, literally stored, and location addressable.	Memory is distributed, internalized, and content addressable.
Not fault tolerant.	Fault tolerant, redundant and shared responsibilities.
Fast, measured in millionths of a second.	Slow, measured in thousandths of a second.
Exact.	Inexact.
Static connectivity.	Dynamic connectivity.
Applicable if rules are well defined with precise input data.	Applicable if rules are unknown or complicated, or if data is noisy or partial.

Table 3-1: Differences between Conventional Computing and Neural Networks

Thus, neural networking and conventional algorithmic computing are not in competition, but rather are complementary of each other. Many tasks and engineering problems require systems that use a combination of the two approaches in order for the best solution to be found.

3.1.4. The NeuroShell Software

The intelligent fracturing model proposed in this research relies heavily on the prediction capabilities of neural networks. Artificial neural networks are used in the fracturing treatment design (either alone or coupled with genetic algorithms) and for the formation pressure prediction response.

There are numerous artificial neural networks commercial software packages available off the shelf. Examples include: Neurogon, NeuroForcaster/Genetica, Matlab Neural Network Toolbox, NeuroShell, VBBackProp, N-Net, STATISTICA Neural Networks, NeuralWorks, NeuroClassifier, NeuroWindows, MacBrain, PathFinder, etc.

For this research, NeuroShell 2 software, a product of the Ward Systems Group, was used. NeuroShell 2 combines robust neural network architectures, a Microsoft® Windows user interface, and options to provide users with a powerful neural network environment. Previous

experience with the software together with the preliminary results from training neural network systems concluded that the Ward Architecture Net proved to be the best suited for the types of problems addressed in this research.

NeuroShell 2 includes twelve neural networks architectures for backpropagation training including the following: standard backpropagation type, recurrent networks; Wards nets; TurboProp; Kohonen Self Organizing Map networks; Probabilistic Neural Networks (PNN); General Regression Neural Networks (GRNN); and Group Method of Data Handling polynomial nets (GHMDH). The Ward nets type of backpropagation is able to detect different features in the data through the use of multiple slabs of neurons in the hidden layer, each with a different activation function. Activation functions are functions used internally in neurons to “fire” the neurons. A different activation function is applied to each slab in the hidden layer, which gives the network the possibility to discover novel features in a single pattern processed through the network.

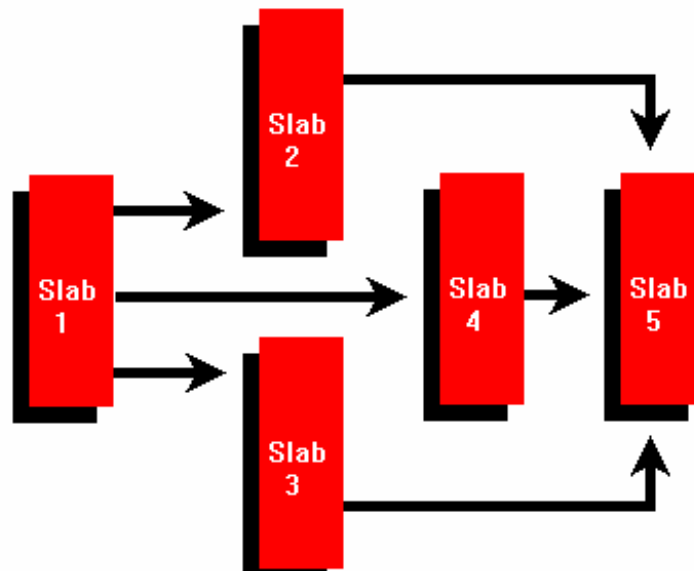


Figure 3-4: Multiple Slabs in an Artificial Neural Network

All backpropagation networks include calibration features, which prevent overtraining (thereby greatly reducing training time), and increase the network’s ability to generalize new data well.

The NeuroShell program offers a set of statistical tools to assist in model building and analysis. They include linear and multiple correlation tools.

The performance of the neural network is measured by two main parameters: the linear correlation coefficient (r) and the coefficient of multiple determination (R^2).

The linear correlation coefficient (r) is the statistical measure of the strength of the relationship between the actual versus the predicted outputs. The correlation coefficient ranges from -1 to +1, with values approaching 1 denoting a stronger positive linear relationship, values approaching -1 denoting a stronger negative linear relationship, and 0 meaning there is no linear relationship.

The coefficient of multiple determination (R^2) is a much better predictor for the performance of neural networks. It is a statistical indicator usually applied to multiple regression analysis and is a measure of how well the multiple regression equation fits the data. Values approaching +1 indicate very good fit, while values approaching 0 indicate very poor fit. The following is the equation used by NeuroShell 2 for the coefficient of multiple determination:

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2} \quad \text{Equation 3-2}$$

3.1.5. Role of Neural Networks in This Research

Neural networks have been used to develop a data driven model that, once trained, can mimic a classic fracturing simulator software. Due to their high response speed, neural networks provide the ideal platform for a genetic algorithm search to find the optimum solution for fracturing treatments.

The model consists of several neural networks, specifically trained to produce certain outputs of a fracture simulation. Five neural networks, representing different types of fracturing treatments, have been trained to mimic the fracture simulation while providing the fracture geometry as output. In addition, two more neural networks have been trained to predict the pressure profile developed during fracturing. The development of each neural network is presented in detailed in the methodology.

3.2. Genetic Algorithms

A genetic algorithm is a model of Machine Learning, which derives its behavior from a metaphor of the process of evolution in the nature. Genetic algorithms are a type of evolutionary computation, which attempts to apply the biological principle of natural evolution to artificial systems^{5,6}. Evolutionary computation is used to describe all computer-based problem solving systems that use computational models based on the known mechanisms of evolution as key elements in their design and implementation. Evolutionary computing includes genetic algorithms, evolutionary programming, evolution strategies, classifier systems, and genetic programming²⁸. All types of evolutionary computation are conceptually similar – they simulate evolution through selection, mutation, and reproduction.

3.2.1. Overview

A genetic algorithm is an iterative procedure that provides a powerful and adaptive search method, practical in complex optimization problems and valuable for application to spaces that are too large to be exhaustively searched. It can cover very large search spaces in a small amount of time without requiring an exact mathematical model of the problem. Genetic algorithms have the ability to simultaneously search several solutions and combine the best of these solutions to come up with the optimum combination from all these possibilities. The genetic algorithm maintains a constant-size population of individuals that evolve according to the rules of selection and other search operators. Each individual in the population, which is a possible solution in the given problem space, is represented by a chromosome (a finite string of symbols, often binary), and has a measure of fitness according to the particular environment. Using this information, reproduction occurs with attention focused on high fitness individuals (those that are better are more likely to survive and propagate their genetic materials). Thus, the offspring have a greater probability of a high measure of fitness.

3.2.2. Representation

A chromosome is a finite string of symbols that represents an individual in the population. The chromosome may be digitized (a binary string), analog (a series of real numbers), character-based, or a tree representation. Digitized (binary) chromosomes are most often used and will be used for explanatory purposes here. Figure 3-5 shows an example of a digitized chromosome.



Figure 3-5: Example of a Digitized Chromosome

3.2.3. Initial Population

Many random solutions to the given problem are generated in order to create the initial population. The fitness of each member of the initial population is evaluated according to a determined fitness function. The fitness is a numerical value, which is representative of the ability of the individual. The fitness value $eval(v_j)$ is calculated for each chromosome v_j ($j=1 \dots pop_size$), then the total population fitness is determined:

$$F = \sum_{j=1}^{\text{Pop_size}} \text{eval}(v_j) \quad \text{Equation 3-3}$$

3.2.4. Selection

In order to generate new and better populations, chromosome selection is extremely important so that the best chromosomes survive to produce the next generation. The most apt parents are selected from the population using the fitness values. More fit individuals will likely be selected several times in a generation, while less fit ones may not be selected at all.

The most widely used method is the classical selection (or roulette wheel) method, which is based on a probability distribution with respect to fitness evaluation. After the total population fitness is determined, the following steps are completed:

- ▶ Calculate the probability of selection P_j for each chromosome v_j ($j = 1 \dots \text{pop_size}$):

$$P_j = \frac{\text{eval}(v_j)}{F} \quad \text{Equation 3-4}$$

- ▶ Calculate the cumulative probability q_j for each chromosome v_j ($j = 1 \dots \text{pop_size}$):

$$F = \sum_{k=1}^j (P_k) \quad \text{Equation 3-5}$$

This selection process is based on spinning the roulette wheel pop_size times, where a single chromosome is selected each time for the new population.

3.2.5. Genetic Operators

After the parent chromosomes have been selected, the offspring must be created through implementation of genetic operations. There are several types of genetic operators such as crossover, fitness-proportionate reproduction, and mutation. In implementation, simple bit manipulation operations allow for the creation of new populations. The most common operators are crossover and mutation.

3.2.5.1. Crossover

Crossover, or recombination, involves the exchange of chunks of genetic information. It is reflective of sexual reproduction in nature, which allows for the creation of genetically different offspring of the same species. Crossover is quite important in genetic algorithms for it allows portions of chromosomes to be combined with portions of other chromosomes. A corresponding

point (or points) along the parent chromosomes is selected at random. The parent chromosomes are swapped at each crossover point to produce two offspring chromosome. Each offspring inherits genes from each parent. Three types of crossover are most common: classical crossover, double crossover, and uniform crossover.

In classical crossover, one random number (between one and the total number of bits in the chromosome) is generated to determine where the two chromosomes will exchange parts, as seen in Figure 3-6 below:

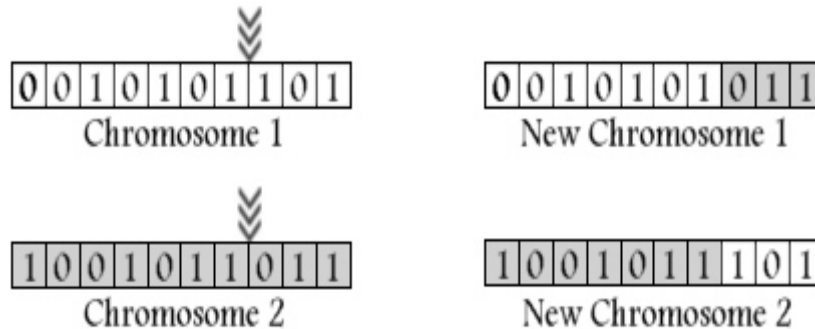


Figure 3-6: Classical Crossover

In double crossover, two random numbers (between one and the total number of bits in the chromosome) are generated to determine where the two chromosomes will be broken into three parts and the middle exchanged, as seen in Figure 3-7:

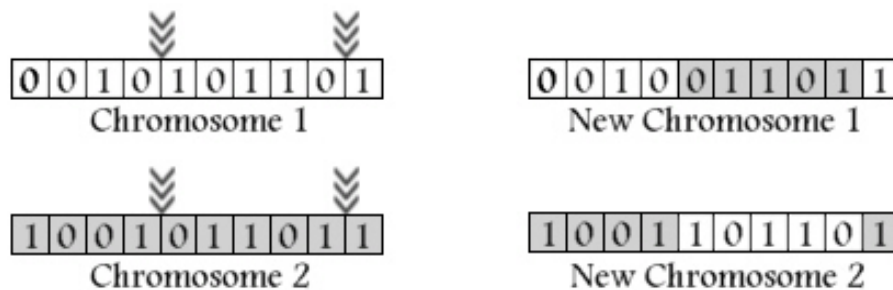


Figure 3-7: Double Crossover

In uniform crossover, an indefinite number of random numbers (between one and the total number of bits in the chromosome) are generated that will represent the bits exchanged between the two chromosomes, as seen in Figure 3-8:

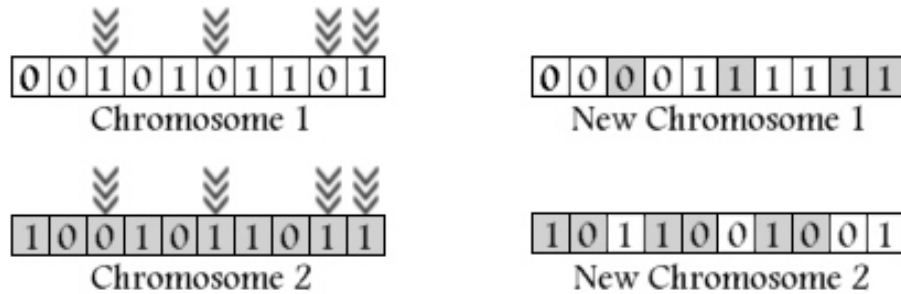


Figure 3-8: Uniform Crossover

3.2.5.2. Mutation

Mutation is another major operator in genetic algorithms, although its relative importance continues to be a matter of debate. In mutation, a bit is randomly selected and its value is altered to produce the new individual. For example as seen below in Figure 3-9, the fourth bit in the chromosome is randomly selected and its value is flipped from zero to one to form the offspring. In other types of chromosomes, such as analog, the new value is selected from amongst a parameter range. Mutation typically occurs with a small frequency.

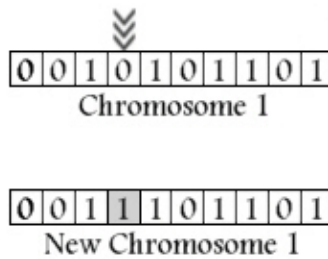


Figure 3-9: Mutation

3.2.6. How They Work

The following steps represent the implementation of the genetic algorithm process:

- ▶ Generate the initial population.
- ▶ Evaluate the fitness (a numerical value, representative of the ability of the individual) for all individuals in the population.
- ▶ Select the best individuals from those whose fitness has just been measured.
- ▶ Perform operations such as crossover, fitness-proportionate reproduction, and mutation on the best individuals.
- ▶ Generate a new population (offspring) via the operations just performed.
- ▶ Discard the old population and iterate using the new population.

Each iteration of this loop is referred to as a generation. During each iteration, the genetic operations (using the fitness measure) work to improve the population. This procedure is repeated until no improvement in the fitness value is observed. Figure 3-10 represents the genetic algorithm process:

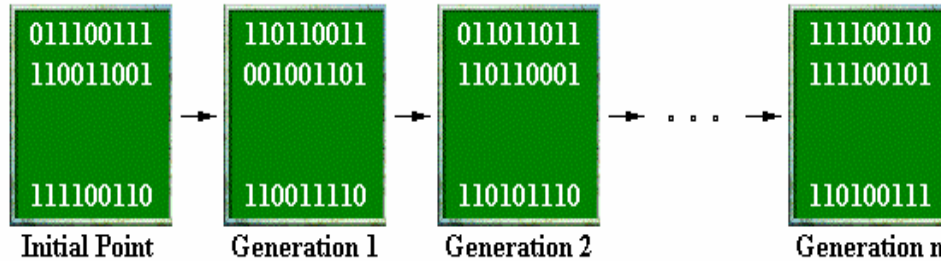


Figure 3-10: Genetic Algorithm Process Depiction

3.2.7. Genetic Algorithms versus Conventional Searching Techniques

It cannot be stressed too strongly that the genetic algorithm is not a random search for a solution to a problem. The genetic algorithm uses stochastic processes, but the result is distinctly non-random (better than random). Genetic algorithms are used for a number of different applications, including multidimensional optimization problems in which each bit in the chromosome represents the value for a different optimization parameter.

3.2.8. Role of Genetic Algorithms in This Research

In this study, genetic algorithms were combined with neural networks in a hybrid system, aimed at designing optimum fracturing treatments. The complexity of the problem is given by the number of parameters to be optimized. The problem calls for simultaneously finding the best combination of four parameters to match the desired fracture length and conductivity. Due to their unmatched capabilities of searching large spaces and providing accurate solutions in relative short time, genetic algorithms were the best fit technique for this research. The development of the genetic algorithm for designing optimum fracturing treatments is presented in detail in the methodology.

3.3. Fuzzy Theory

Fuzzy theory is a broad field that allows for the use of intermediate values between standard values (such as true/false, yes/no, 0/1, etc.) in an attempt to mimic the human thought process. In other words, it provides an array of grays to be defined between black and white and permits vague answers such as “OK” when asked, “How are you?” Fuzzy logic values are

continuous ranging from 0 to 1 (where 0 is false and 1 is true), allowing statements to be either false by various degrees or true by various degrees.

3.3.1. Fuzzy Sets

In traditional set theory, an element either belongs to the set (completely inside the set) or does not belong to the set (completely outside the set). Fuzzy set theory allows the element to belong partially to the set (both inside and outside the set) or belong to a certain amount, called the degree of membership⁷. Furthermore, the element can belong to multiple sets with varying degrees of membership. In addition, fuzzy sets correspond more closely to human language than tradition sets.

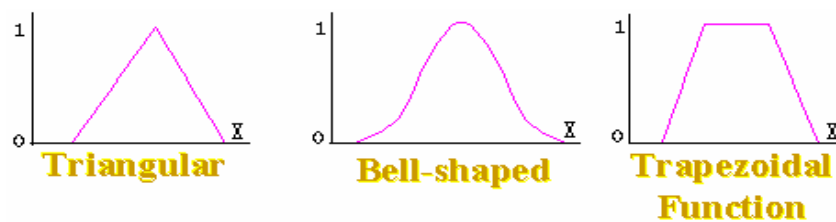


Figure 3-11: Common Fuzzy Membership Functions

All fuzzy sets are specific by their membership functions, provided by human experts. Some commonly used membership functions can be seen above in Figure 3-11. These are used with fuzzy rules that represent associations between input and output fuzzy sets.

3.3.2. Fuzzy Inference System

A collection of these rules is called a fuzzy inference system, which provides exceptional representation of expert knowledge and is highly interpretable. In a practical fuzzy inference system, two added steps (fuzzification and defuzzification) are required to combine crisp inputs with expert knowledge to produce a crisp output, as seen below in Figure 3-12.

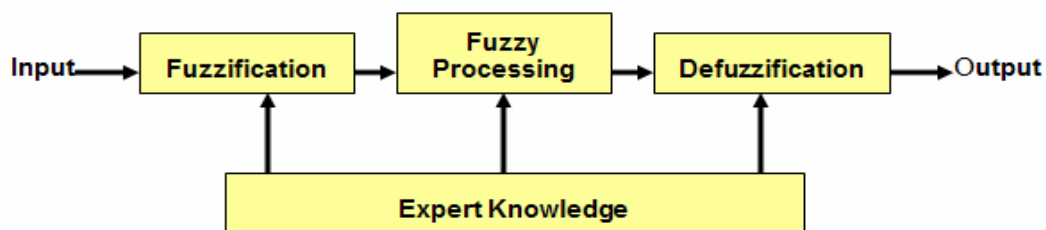


Figure 3-12: Fuzzy Inference System

Fuzzification, the first step in the inference system, is the process by which a fuzzy representation of a crisp input value is obtained. In other words, during the fuzzification process, the membership function is calculated for each crisp input for each fuzzy rule. The second step is rule output evaluation, which allows for the modification of the output fuzzy set for each rule based on the rule strength. The final step, defuzzification, is the process by which fuzzy output values are converted into the crisp equivalent.

The main benefit of fuzzy logic lies in the fact that it simplifies the process of tapping the knowledge of domain experts⁷. Fuzzy methodologies are most useful for human inference, particularly for applications that involve the following:

- ▶ Encompassing expert knowledge about unknown dependencies.
- ▶ Using post-processing of outputs for decision-making.
- ▶ Interpreting models produced by non-fuzzy methods.

3.3.3. Role of Fuzzy Logic in This Research

Fuzzy Logic has been used to develop a lithology identification expert system. This study introduces a new and automatic approach for lithology identification. The lithology is necessary for stress calculations when sonic logs are not recorded.

The methodology uses a combination of three different logs such as spontaneous potential, resistivity, gamma ray, or other available logs. Fuzzy sets are defined for each three selected logs and the expected output. The output of the system is the type of lithology. Expert knowledge is incorporated in the fuzzy rules. The final outcome of the system is the continuous lithology identification for every foot or half-foot of log data. This continuous data lithology is used later in an empiric stress equation to determine the stress profile required for fracturing simulation. The development of each fuzzy lithology system is presented in detail in the methodology.

3.4. Vector Quantization

Vector quantization is one of the most powerful technologies for data compression and dimension reduction²⁹. The dictionary definition of quantization is the division of a quantity into a discrete number of small parts, often assumed to be integral multiples of a common quantity. The objective of vector quantization is to minimize a well-defined approximation error when the number of prototypes is fixed beforehand³⁰. This is in contrast to clustering methods that implicitly define groupings by use of a computational procedure. In practice, the objective of vector quantization is to compress a particular sample by a pre-specified number of prototypes.

The original purpose of vector quantization was to encode continuous or discrete data vectors so that they could be transmitted over a digital communications channel. It is often used in speech and image processing where compression is necessary and some loss in quality can be endured.

A vector quantizer is a system for mapping a sequence of continuous or discrete vectors into a digital sequence suitable for communication or storage in a digital channel. Vector quantization is nothing more than an approximator. The idea is similar to that of “rounding-off” to the nearest integer. An example of a 1-dimensional VQ is shown in Figure 3-13:

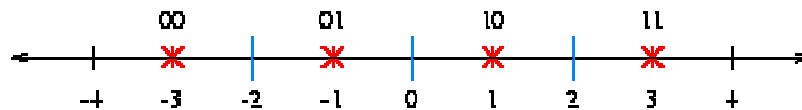


Figure 3-13: One-Dimensional Vector Quantizer

Every number less than -2 is approximated by -3. Every number between -2 and 0 is approximated by -1. Every number between 0 and 2 is approximated by +1. Every number greater than 2 is approximated by +3. Note that 2 bits uniquely represent the approximate values. This is a 1-dimensional, 2-bit VQ with the rate of 2 bits/dimension.

A vector quantizer system includes both an encoder and a decoder, as seen in Figure 3-14. First, the input vectors are divided into a constant number of separate regions, and an output vector for each region is found. Once this is complete, input vectors can be given to the encoder. The encoder will provide the index (or channel symbol) of the region to which the input vector belongs. The index is then transmitted over the digital channel. Once it reaches the decoder, it is mapped to its corresponding output vector^{29,30}.

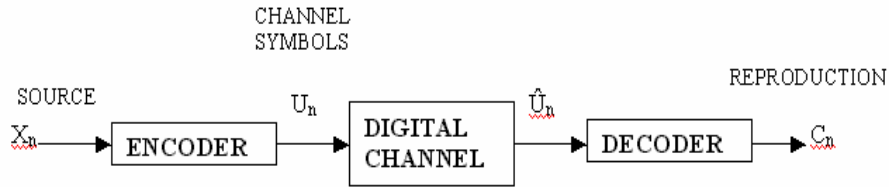


Figure 3-14: Vector Quantizer System

The data or information source $\{x_n, n=0,1,\dots\}$ is a sequence of random vectors. The encoder produces a sequence of channel symbols $\{u_n, n=0,1,\dots\}$. The sequence $\{\hat{u}_n, n=0,1,\dots\}$ is delivered to the receiver by the digital channel. The decoder maps this sequence into the final representation of $\{c_n, n=0,1,\dots\}$.

3.4.1. Role of Vector Quantization in This Research

In this study, vector quantization has been used to compress the pressure profile signal into a smaller number of prototypes (called pressure points or pressure quantizers) in order to train a neural network. The data available for a pressure profile can reach up to 1000 pressure points when small time steps are used in recording. However, using generated data with time steps of one minute, the range of data points is between 40 and 100. Since the number of pressure points is very large to be used as output in the neural network, a simple linear vector quantization algorithm was used to reduce the number of points to 8. Two end points were added to the existing, so that 10 points can represent every pressure profile, no matter its length. The development and implementation of the vector quantization system is presented in detail in the methodology.

CHAPTER 4: METHODOLOGY

4.1. Overview

The goal of the project is to develop an intelligent application to design optimum fracture treatments and predict the expected net treating pressure. The application is designed for tight gas reservoirs where, due to very low values of permeability, the wells have to be stimulated by hydraulic fracture treatments in order to achieve an economically adequate production rate.

A classical hydraulic fracture treatment is designed to provide optimum length and conductivity to effectively stimulate the well. Similarly, this application uses an intelligent system composed of neural networks and genetic algorithms to search for the optimum fracture treatment that creates the desired fracture length. This is an optimization problem that requires the optimum combination of four treatment parameters: total fluid volume, pad volume, proppant concentration, and injection rate. The methodology consists of three steps: lithology determination and stress profile calculation, optimum fracture treatment design, and net treatment pressure prediction. These steps are outlined in detailed in the following sections.

4.2. Model Description

The model described in Figure 4-1 presents the big picture methodology flow chart. Three main modules form the methodology, (1) lithology identification & stress profile, (2) fracture design treatment, and (3) pressure profile prediction. Each of these modules is described in detail in the following sections.

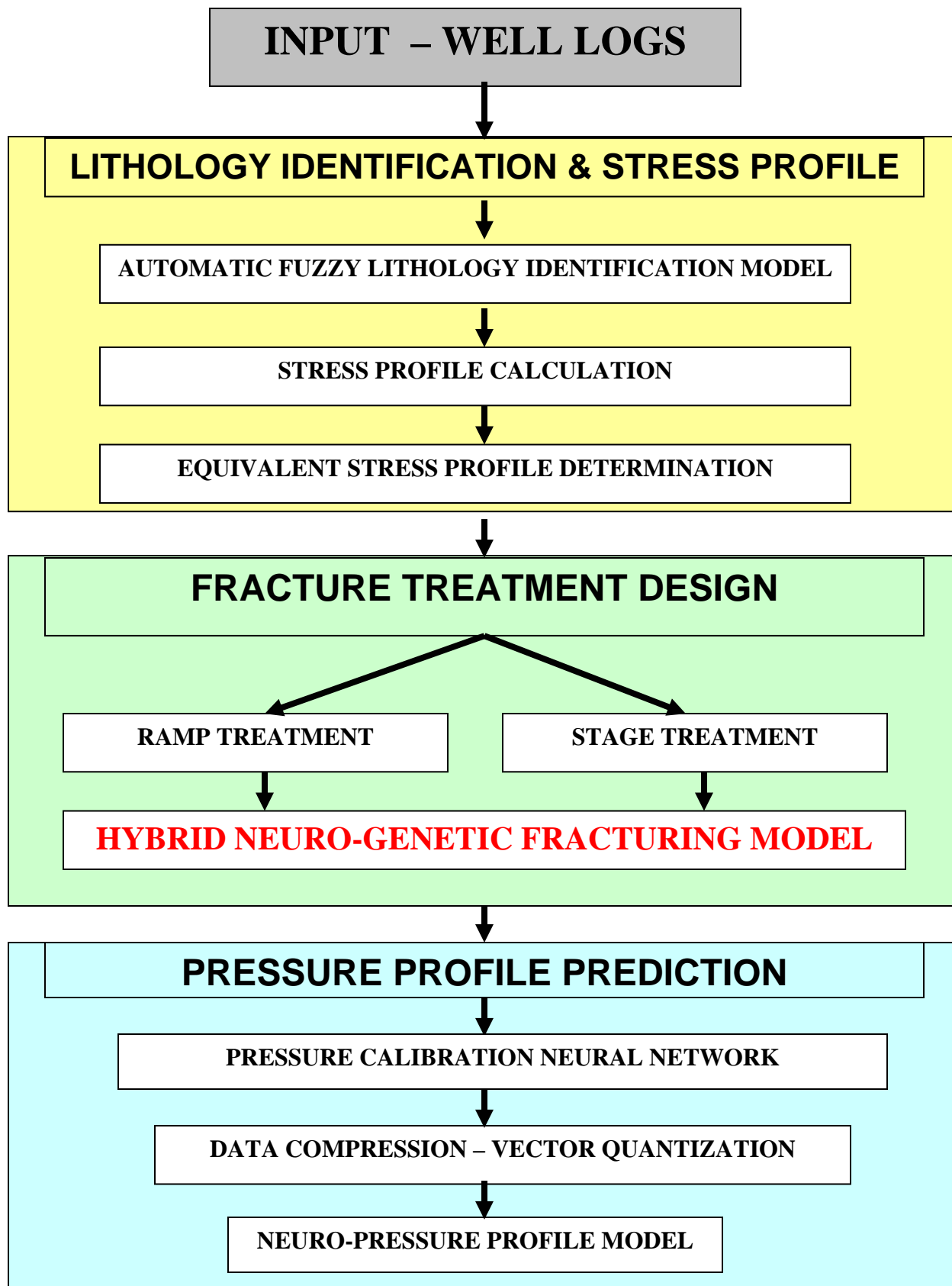


Figure 4-1: Methodology Flow Chart

4.3. Data Preparation Considerations

The methodology proposed in this study is intended to design optimum fracturing treatments and predict the formation pressure response during fracturing. If this methodology proves successful it is intended to be used for actual fracturing data in a reservoir or basin.

In order to test the feasibility of this methodology, sample data was generated using a commercial fracture simulator. FRACPRO software was used for this purpose due to its powerful 3D capabilities and widespread use in the industry.

As presented in a previous chapter, fracture simulation requires quite a significant amount of information. This information can be divided into four categories:

- ▶ Fracturing Fluids Characteristics - fracture fluid viscosity, fracture fluid density, fluid loss additives.
- ▶ Proppant Characteristics - propping agent type, and propping agent volume.
- ▶ Treatment Schedule - number of stages, treatment volume, pad volume, injection rate.
- ▶ Formation characteristics - formation depth, formation permeability, in-situ stresses in the pay zone, in-situ stresses in the surrounding layers, formation modulus, reservoir pressure, formation porosity, formation compressibility, and the thickness of the reservoir.

4.4. Fracturing Fluids

The main types of the treatment fluids and their characteristics and properties were described in section 2.3.1 Treatment Fluid Characterization.

Typically each service company develops its own fracturing fluids, using proprietary chemicals and compositions. Depending on the base chemical used and treatment scope, the fracturing fluids are grouped in different classes, such as: Boragel, Fracgel, Hybor, Thermagel, Versagel, etc.

FRACPRO's Fluid Library was used to generate the data needed for building the model. FRACPRO includes a Fluid Library, which contains several types of fluids developed by major service companies such as Halliburton, Dowell Schlumberger, BJ Services. As a result, the fluids bear their names. Examples of the Halliburton naming conventions for different types of fluids are presented below.

- ▶ HL_BOR_G25_1 class Boragel
- ▶ HL_FG_25_1 class Fracgel
- ▶ HL_HYB_25_1 class Hybor
- ▶ HL_THG_30 class Thermagel
- ▶ HL_VER_30 class Versagel

Since fluids from class Boragel are most often used in tight gas sands stimulation, HL_BOR_G30_1 was used for data generation. Once the methodology is proven, other fluids will be added to the final model.

4.5. Proppant Agents

The detail description and role of the propping agents were presented in section 2.3.2 Proppant Characterization.

In the industry there are several well-known types of sands, which typically carry the name of the region where they are produced. Examples are Brady, Jordan, Ottawa, Colorado sand, etc. Moreover, each type of sand comes in different mesh size such as: 20/40, 30/50, 16/30, 14/20, 12/20 etc.

Proppants already available in the FRACPRO's Proppant Library were used in generating the data needed to build the model. FRACPRO included a Proppant Library that contains the most popular types of proppants used in fracturing treatments.

Examples of the most common types of proppants presented below.

- ▶ Brady 20/40
- ▶ Jordan 16/30
- ▶ Ottawa 20/40
- ▶ Col_It 12/20 (Colorado Sand)

Since 20/40 mesh size is one of the most popular sands used in fracturing tight gas sands, Brady 20/40 sand was used in generating data. Again, once the methodology is proven other mesh sizes and sand brands can be added to the model.

4.6. Treatment Schedule Definition

Designing an optimum fracturing treatment is a complex process, which requires both art and science. Typically, the treatment is designed to provide an optimum fracture length and conductivity. To achieve these desired requirements, engineers are able to control a few main parameters such as fluid (pad and slurry), proppant concentration, and injection rate.

In the early times, due to mechanical limitations the fracturing treatments were executed in stages, each stage consisting of different amount of fluid (slurry) pumped at a specific proppant

concentration. These types of treatment are called “Stage Treatment Schedule” and are discussed in detail in this manuscript.

In this study two examples of Stage Treatment Schedules were considered in order to demonstrate the capability of this methodology in comprehensively addressing all possible variations:

- ▶ The Six Stage Treatment
- ▶ The Eight Stage Treatment

Table 4-1 presents in detail the structure of the Six Stage Treatment while Table 4-2 displays the structure of the Eight Stage Treatment.

The treatment schedule can be composed of as many stages as the user desires. In practice the total number of stages rarely exceeds ten. The example in Table 4-1 presents a stage treatment schedule containing five slurry stages, each one being characterized by specific fluid volume and proppant concentration. In this case the proppant is pumped in steps being constant during every stage. After the last slurry stage, one more stage is pumped. This last stage is called flush and is equal with the wellbore volume. Usually, during flush, Slickwater is pumped to transport the remaining slurry from the last stage from the tubing or casing into the formation, thus cleaning all the sand from the tubing/casing.

No	Stage	Volume (gal)	Rate (bbl/min)	Starting Proppant Conc (ppg)	Final Proppant Conc (ppg)	Fluid Type
1	Pad	21598	20	-	-	HL_HYB_30_1
2	Slurry	4773	20	1.5	1.5	HL_HYB_30_1
3	Slurry	4773	20	3	3	HL_HYB_30_1
4	Slurry	9546	20	4.5	4.5	HL_HYB_30_1
5	Slurry	11932	20	6	6	HL_HYB_30_1
6	Slurry	14318	20	7.5	7.5	HL_HYB_30_1
7	Flush	2150	20	-	-	SLICKWATER

Table 4-1: Six-Stage Treatment Schedule

Table 4-2 presents a stage treatment containing 7 slurry stages, one pad stage, and the flush (Slickwater). Depending upon engineering practices the Stage Treatments may have a smaller or larger number of stages.

No	Stage	Volume (gal)	Rate (bbl/min)	Starting Proppant Conc (ppg)	Final Proppant Conc (ppg)	Fluid Type
1	Pad	21598	20	-	-	HL_HYB_30_1
2	Slurry	3573	20	1	1	HL_HYB_30_1
3	Slurry	4793	20	2	2	HL_HYB_30_1
4	Slurry	6546	20	3	3	HL_HYB_30_1
5	Slurry	8732	20	4	4	HL_HYB_30_1
6	Slurry	11318	20	5	5	HL_HYB_30_1
7	Slurry	13434	20	6	6	HL_HYB_30_1
8	Slurry	16890	20	7	7	HL_HYB_30_1
9	Flush	2150	20	-	-	SLICKWATER

Table 4-2: Eight-Stage Treatment Schedule

With the advance of technology the new blenders allow the entire job to be pumped continuously ramping up the proppant concentration during the job. In this case the treatment consists of one pad stage and one slurry stage. This type of treatment is called “Ramp Treatment Stage”. The structure of the Ramp Treatment Schedule is described in Table 4-3.

No	Stage	Volume (gal)	Rate (bbl/min)	Starting Proppant Conc (ppg)	Final Proppant Conc (ppg)	Fluid Type
1	Pad	21598	20	-	-	HL_HYB_30_1
2	Slurry	39458	20	1	7.5	HL_HYB_30_1
3	Flush	2150	20	-	-	SLICKWATER

Table 4-3: Ramp Treatment Schedule

In comparison with the Stage Treatment Schedule the Ramp Treatment Schedule consists of two main stages, pad and slurry, and ends with the flush. As in the case of Stage Treatment Schedule the flush is equal with the wellbore volume. All the proppant is pumped as ramp typically starting with a concentration of 1 ppg to the final desired concentration.

FRACPRO is a user-friendly flexible tool that allows the possibility of using different types of treatment schedules, including both Stage Treatment Schedules and Ramp Treatment Schedules, each one being unique in its way. In this study the three types of treatments described above are considered.

4.7. Formation Characteristics Consideration

An accurate fracture simulation requires a significant amount of data not only related to the formation to be fractured but also considerable information describing the formations above and below. In a previous chapter the amount of information needed, its role in fracturing

simulation and sources of collection were described in detail. The most important information that controls the fracture geometry is the stress profile, which reflects in-situ stress contrast between the production formation and the barrier formations.

In practice not all desired information is available or easy to collect. Due to high cost some of the required information is uneconomic to collect. However, in most cases well logs are already available at no extra cost. The main information that can be obtained from logs is the depth and thickness of each formation. Moreover, logs provide significant information regarding the lithology of the productive formation and the surrounding layers.

Sonic logs, together with density logs and gamma ray logs are required for closure stress gradient calculation. If sonic logs are not run in the well, the Poisson coefficient cannot be computed and as a result nor the stress gradient.

This study proposes a new methodology to calculate the closure stress gradient without sonic logs. Instead, it uses basic logs such as spontaneous potential (SP), gamma ray (GR) and deep induction logs (or resistivity). The process employs a fuzzy logic approach to automatically determine formation lithology. Once lithology is identified, a model such as the ABC stress model can be used to determine the stress profile. This methodology is presented in detail in the following chapter.

The other information defining the reservoir is related to formation pressure and temperature, porosity, permeability and viscosity of the existing fluid. Some of this information can be either measured or estimated from correlations.

In this study most of these parameters will be calculated using well-accepted correlations.

4.8. Fuzzy Lithology Identification Approach

Reservoir lithology is vital, not only in formation characterization^{31,32}, but also in providing information used in fracture design and calculations. Several techniques use lithology description to calculate the stress profile. For example, the ABC default model¹⁷ developed by RES (section 2.1.2.1) involves the use of defined coefficients for different types of lithology. Prior knowledge of the lithology is required in order to assign the appropriate coefficient to the corresponding formation.

The technique proposed in this study presents a flexible approach for automatic lithology identification through the use of a fuzzy logic system. The intelligent procedure will determine the type of lithology and automatically assign the corresponding coefficient, necessary for stress calculation in Equation 2.6. The system allows for flexible, changeable rules to match different reservoirs in different zones. It is designed to use basic logs for lithology identification such as spontaneous potential, gamma ray and deep induction log (or resistivity). These three logs have been selected due to their high availability in most wells. However, other logs may be used with the system, if available.

Lithology identification usually is done visually using logs. Geologists, geophysicists, and engineers visually inspect the logs and identify the possible lithology based on specific deflections and values. Lithology identification is more accurate when using three or more logs, since at same depths different logs exhibit similar, yet specific characteristics.

In this study, it was identified that such qualitative information reflecting expert knowledge, can contribute to the determination of the in-situ stress. The best-known analytical technique for incorporation of expert knowledge into calculations is fuzzy logic.

Since formations transition from sandstone to shale, or from shale to carbonate, using crisp values in defining the lithology may induce significant errors. These zones of transition are defined by uncertainty, and this is where fuzzy sets are able to provide the best answer.

A three-input, one-output fuzzy expert system to determine the formation lithology from well logs was developed. The inputs to this system are Spontaneous Potential, Gamma Ray, and Deep Induction logs. As previously mentioned, other logs can be used in the system as well.

Three fuzzy sets (low, medium, and high) were defined for each input log. The range interval of the log is defined first and then the type of fuzzy set is considered. In Figure 4-2, the range interval was selected between 0 and 150. For example, a value of 40 mv of spontaneous potential has membership value of 0.67 in fuzzy set “Low” and a membership value of 0.33 in fuzzy set “Medium”. Similar fuzzy sets are defined for gamma ray log and induction log.

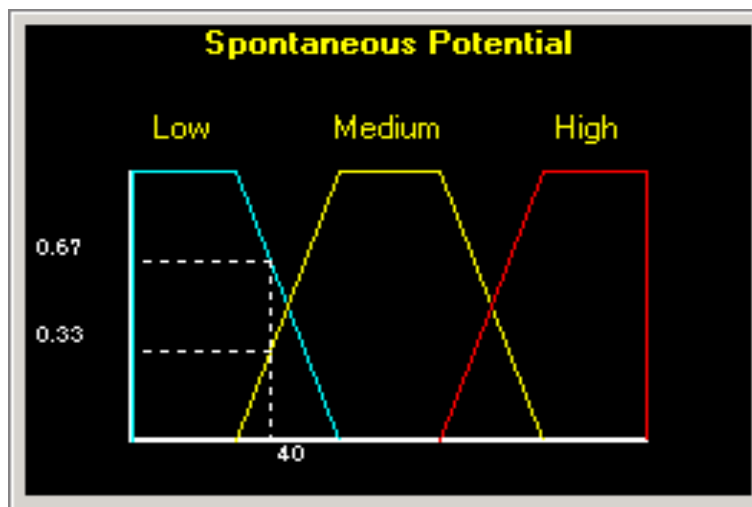


Figure 4-2: Three Fuzzy Sets for Spontaneous Potential

The fuzzy rules were defined based on the fuzzy sets. The three input logs, each one with three fuzzy sets require 27 fuzzy rules as shown in Table 4-4. Each rule was defined using expert knowledge such that the system can function for any reservoir or basin. The output of the system can be sandstone, shaly-sandstone (dirty sandstone), or shale. An example rule follows:

*If Deep Induction Log is “Low” and Gamma Ray is “High”
and Spontaneous Potential is “Medium”, then the formation is “Shale”*

		Deep Induction Log					Deep Induction Log					Deep Induction Log				
		Low	Medium	High			Low	Medium	High			Low	Medium	High		
Gamma Ray	Low	Shale	SS-Sh	SandStone	Low	SS-Sh	SandStone	SandStone	Low	SS-Sh	SandStone	SandStone	Low	SS-Sh	SandStone	SandStone
		T	FT	VT		FT	T	VT		FT	T	VT		FT	T	VT
	Medium	Shale	Shale	SS-Sh	Medium	SS-Sh	SS-Sh	SandStone	Medium	SS-Sh	SS-Sh	SandStone	Medium	SS-Sh	SS-Sh	SandStone
		T	FT	T		FT	T	FT		T	VT	T		T	VT	T
	High	Shale	Shale	Shale	High	Shale	Shale	Shale	High	Shale	Shale	Shale	High	Shale	Shale	Shale
		VT	NNT	NNT		VT	T	NNT		VT	NNT	NNT		VT	NNT	NNT
		Spontaneous Potential - LOW					Spontaneous Potential - MEDIUM					Spontaneous Potential - HIGH				

Table 4-4: Fuzzy Rules for the Fuzzy Lithology Identification System

The previous statement defines a rule of the fuzzy set. In addition, each rule is qualified using an approximate reasoning methodology as being true (T), very true (VT), fairly true (FT), and not necessarily true (NNT). In this context, not necessarily true refers to cases that are nearly impossible to be found in nature.

Using a fuzzy set theory-based approach for defining formation lithology provides the necessary means to identify rock types that cannot distinctly be defined as sandstone, shale or carbonate. Fuzzy terms, used for years in the oil industry such as sandy-shale or shaly-sand, are easily accommodated in this approach and contribute to success of the methodology presented in this study.

As seen in Table 4-4, the rules of log interpretation can be described by fuzzy terms used in fuzzy rules, which are incorporated in the fuzzy decision support system. During a fuzzy inference process, several fuzzy rules will be fired in parallel. The parallel rule firing, unlike sequential evaluation, is much closer to the human reasoning process. The output of the fuzzy inference process is the type of lithology. An example of the lithology determination by the fuzzy system is presented in Figure 4-3. In this figure, both sandstone and shale are represented in the fuzzy sets and the lithology estimated is dirty sandstone or shaly-sand.

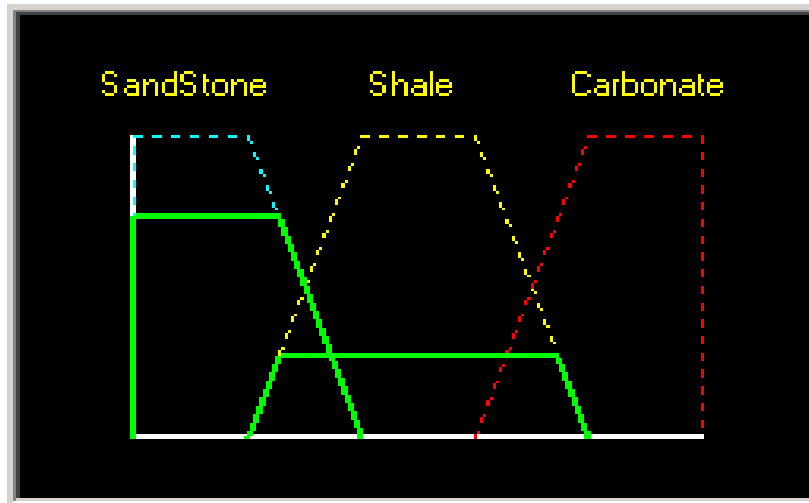


Figure 4-3: Fuzzy Decision System

The estimation is done for each recorded value of spontaneous potential, gamma ray and resistivity. Since logs are recorded each 0.5 or 1 ft, the fuzzy system assigns the type of lithology for every recording. With the lithology defined for every 0.5 or 1 ft, the system automatically assigns the corresponding lithology coefficients, needed in Equation 2-6 for stress calculation.

4.9. Stress Profile Calculations

The stress profile is known to be the most important input in designing a successful fracture treatment. Oftentimes, the formation intended for fracturing is not neatly bound by top and bottom layers. Occasionally, the productive formation itself contains many thin layers of nonproductive rock. It is important to understand the stresses in the surrounding zones as well as the productive zone. As mentioned in previous chapters, the stress profile can be estimated from log measurements. Since these calculations are crucial in the design of a fracture treatment, the calculated stress profile is usually calibrated with the stress values obtained from in-situ stress test analysis, when these are available.

The stress profile can be calculated using one of the models presented in section 2.1.2.1 Use of Hook's Law in Calculating the Log-Derived Stress. However, some of the models can be used only when sonic logs are available. In the absence of the sonic logs, the lithology model (ABC model) presented in section 2.1.2.1, can be used³³. The result is a continuous stress profile curve for the considered interval as shown in Figure 4-4. Next, the stress profile is processed to illustrate a clear definition of the stress zones.

Stress profile processing is an early step that engineers take in order to compress data and better define different layers. Similar consecutive stress values are grouped together to define possible shale and sandstone formations by visually analyzing the stress profile curve and

logs. For example, in Figure 4-4, there is a different stress value for every foot between 11,040 and 11,100 ft. However, these values are very similar and all within a close range. This interval would be considered one formation and represented by one stress value, which is calculated as the arithmetic average of all stress profile values between the two depths. This is shown in Figure 4-5, where the continuous jagged red curve represents the original stress profile and the thick blue step line shows the processed stress profile. In addition, Table 4-5 presents the lithology of the formations resulted after geology and engineering interpretation. In section 4.10 Equivalent Stress Profile, an explicit example of stress profile processing is provided.

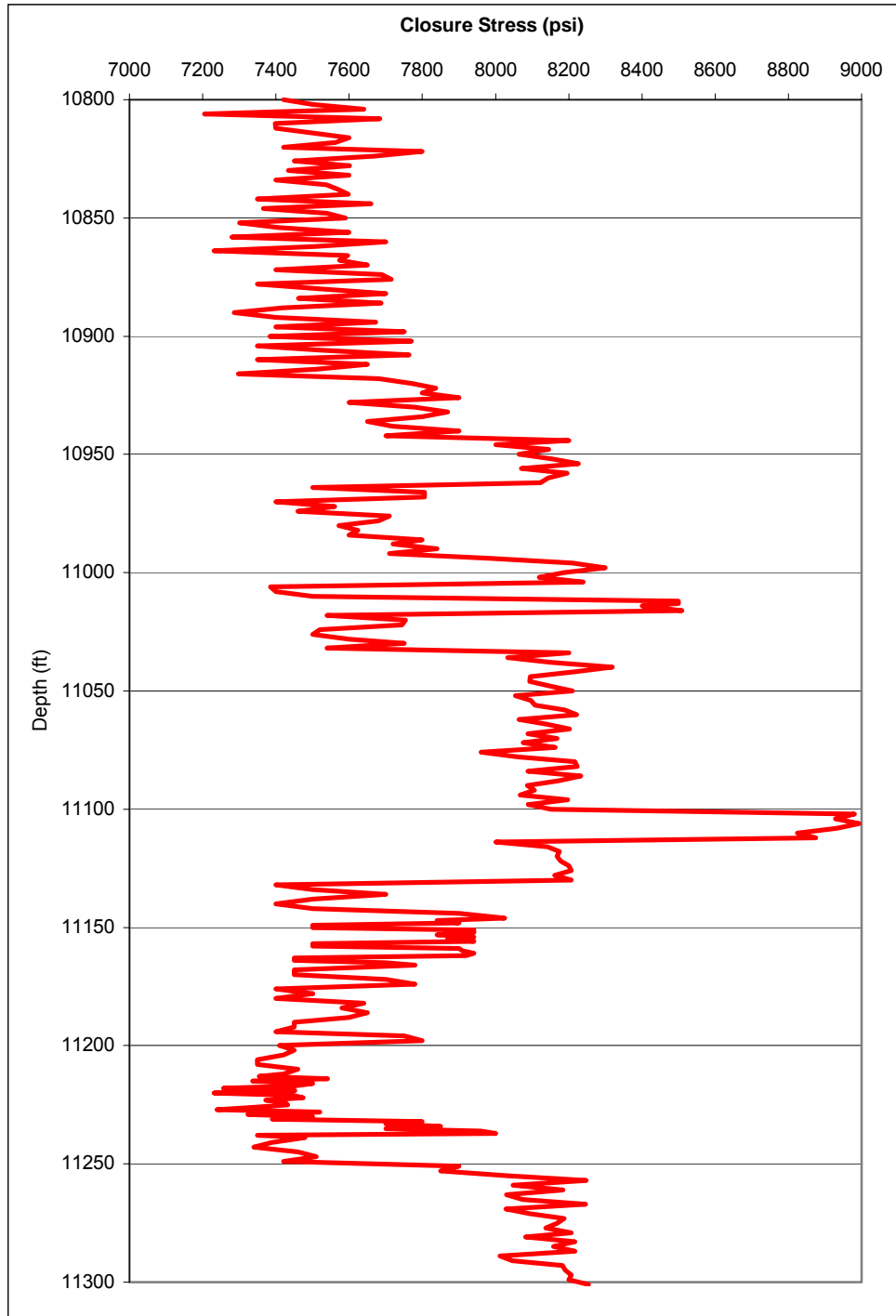


Figure 4-4: Log-Derived Stress Profile

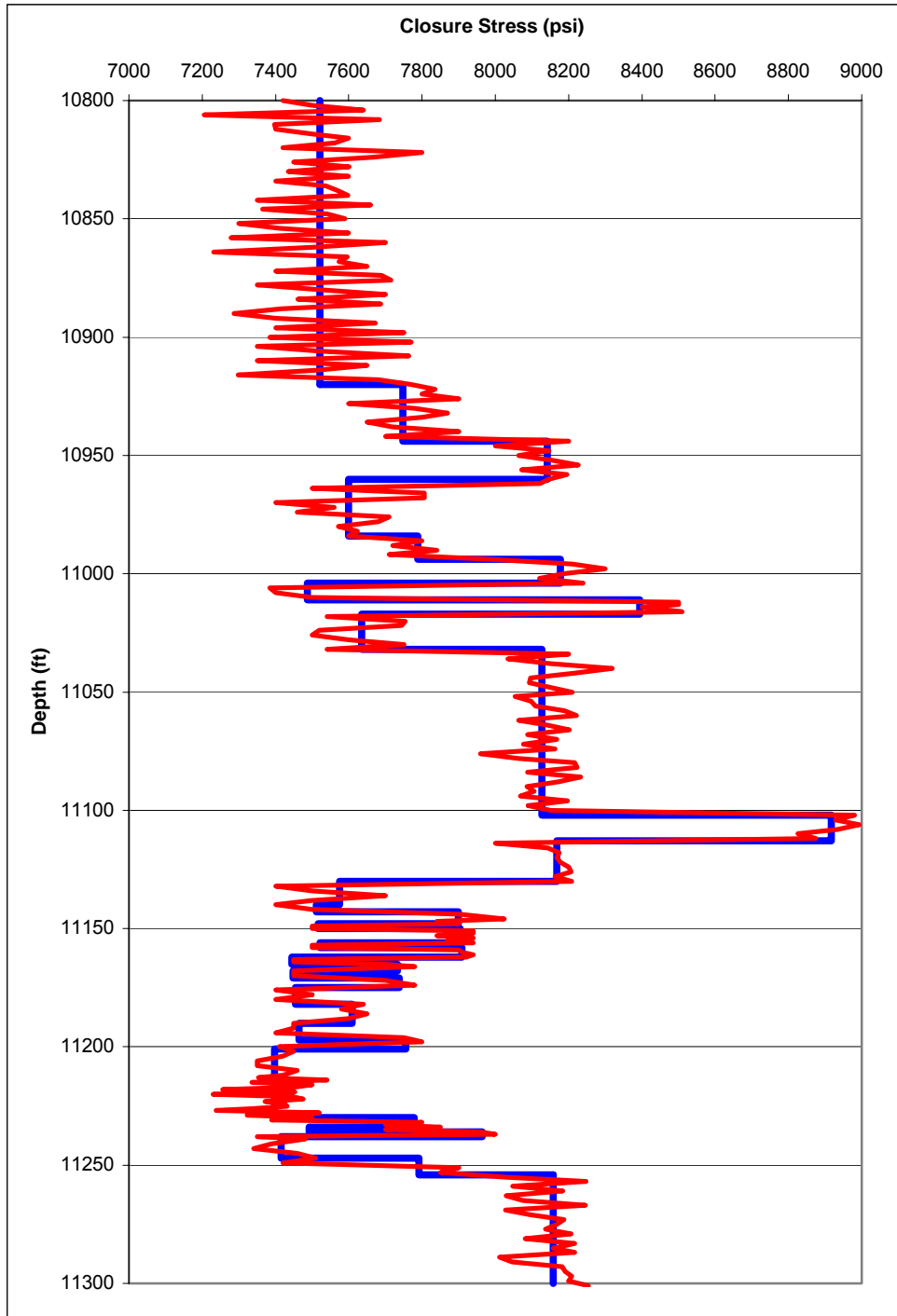


Figure 4-5: Original and Processed Stress Profile

Layer Description	Upper Depth	Lower Depth	Layer Thickness	Layer Permeability	Poisson's Ratio	Closure Str. Gradient	Closure Stress
Shale 1	10800	10920	120	0	0.23	0.69	7521.07
Shale 2	10920	10944	24	0	0.25	0.71	7747.24
Shale 3	10944	10960	16	0	0.29	0.74	8141.81
Shale 1	10960	10984	24	0	0.23	0.69	7598.63
Shale 2	10984	10994	10	0	0.25	0.71	7787.64
Shale 3	10994	11004	10	0	0.29	0.74	8176.75
Shale 4	11004	11011	7	0	0.22	0.68	7487.15
Shale 5	11011	11017	6	0	0.30	0.76	8394.24
Shale 1	11017	11032	15	0	0.23	0.69	7634.99
Shale 6	11032	11102	70	0	0.28	0.73	8127.56
Shale 7	11102	11113	11	0	0.34	0.80	8916.29
Shale 6	11113	11132	19	0	0.28	0.73	8168.32
Shale 4	11132	11140	8	0	0.22	0.68	7574.56
Redfork 1	11140	11143	3	0.01182	0.21	0.67	7511.35
Shale 2	11143	11148	5	0	0.25	0.71	7898.55
Redfork 1	11148	11150	2	0.01182	0.21	0.67	7516.40
Shale 2	11150	11156	6	0	0.25	0.71	7903.86
Redfork 1	11156	11158	2	0.01182	0.21	0.67	7521.80
Shale 2	11158	11162	4	0	0.25	0.71	7908.82
Redfork 2	11162	11165	3	0.01182	0.20	0.67	7444.05
Shale 1	11165	11168	3	0	0.23	0.69	7733.33
Redfork 2	11168	11171	3	0.01182	0.20	0.67	7448.05
Shale 1	11171	11175	4	0	0.23	0.69	7737.83
Redfork 2	11175	11182	7	0.01182	0.20	0.67	7454.05
Shale 4	11182	11190	8	0	0.22	0.68	7608.57
Redfork 2	11190	11197	7	0.01182	0.20	0.67	7464.05
Shale 1	11197	11201	4	0	0.23	0.69	7755.84
Redfork 3	11201	11230	29	0.01182	0.19	0.66	7398.08
Shale 1	11230	11234	4	0	0.23	0.69	7778.69
Redfork 2	11234	11236	2	0.01182	0.20	0.67	7491.73
Shale 2	11236	11238	2	0	0.25	0.71	7963.39
Redfork 3	11238	11247	9	0.01182	0.19	0.66	7415.89
Shale 1	11247	11254	7	0	0.23	0.69	7791.51
Shale 8	11254	11500	246	0	0.26	0.72	8157.74

Table 4-5: Reservoir Lithology

4.10. Equivalent Stress Profile

In Figure 4-5 the continuous scatter line represents the stress profile calculated every 0.5 or 1ft, while the bold step line illustrates a clear definition of the stress zones, which defines each potential layer. Table 4-5 clearly points to the large amount of input data (number of layers and their properties) needed for a proper fracture simulation. Moreover, no two reservoirs are the

same; reservoirs exist at different depths and have different lithologies. Therefore, based on the number of the inputs, it can be concluded that:

- ▶ The number of the layers may vary from 3 to 50 or more.
- ▶ Each layer is situated at a specific depth.
- ▶ Each layer has a unique thickness.
- ▶ Each layer has a unique closure gradient stress (and closure stress).
- ▶ Each layer has a unique permeability, if productive.

In the conditions outlined above, it is impractical to train a neural network using all the layers from the lithology. Therefore, it becomes necessary to develop a methodology that reduces the total number of layers to a few representative layers. Meanwhile, the methodology should have a universal applicability for different numbers of layers, different lithology, and different layer thicknesses. In addition, this equivalent lithology should provide not only the same output as the original profile, but also the same shape of the fracture when used in simulation.

This study proposes an algorithm to reduce the stress profile, and consequently the reservoir representation, to a representative model of a few layers.

4.10.1. Methodology for Generating Equivalent Stress Profile

A first attempt to develop an equivalent stress profile considered reducing the total number of layers to three. These layers were named as follows:

- ▶ Top layer – The formations above the productive formation merged together.
- ▶ Pay zone – The production layers and the intercalation merged together.
- ▶ Bottom layer – The formations below the pay zone merged together.

A real case of a well in Texas was considered to analyze the robustness of the three-layer profile compared to the original profile. The well has a complete data set and was hydraulic fractured for increased productivity. To determine the characteristics of the proposed three-layer model, an averaging technique was used. This methodology merges the layers into one layer with a thickness equal to the sum of the thicknesses of all layers. The methodology also determines the closure stress gradient of the equivalent layer, defined by the following relationship:

$$\sigma_{eq} = \frac{\sum \sigma_i * h_i}{\sum h_i} \quad \text{Equation 4-1}$$

where, σ_{eq} = closure stress gradient of the equivalent layer, psi/ft

σ_i = closure stress of each layer, psi

h_i = thickness of each layer, ft

A similar relationship was used to calculate the equivalent permeability of the productive formation.

$$K_{eq} = \frac{\sum k_i * h_i}{\sum h_i} \quad \text{Equation 4-2}$$

where, K_{eq} = equivalent permeability of the productive layer, mD

k_i = permeability of each layer, mD

h_i = thickness of each layer, ft

Using Equation 4-1, top layer is defined by an average stress value and thickness equal to sum of all layers thickness above. Similarly, the production layers were merged into one layer, as were the formations below the target formation. The permeability of the pay zone was calculated using Equation 4-2. Figure 4-6 shows the processed stress profile and the three layers equivalent profile.

FRACPRO was run in both cases, using the three layers stress profile and the original profile. Analyzing the results, it was concluded that the major difference between two cases was the geometry of the fracture.

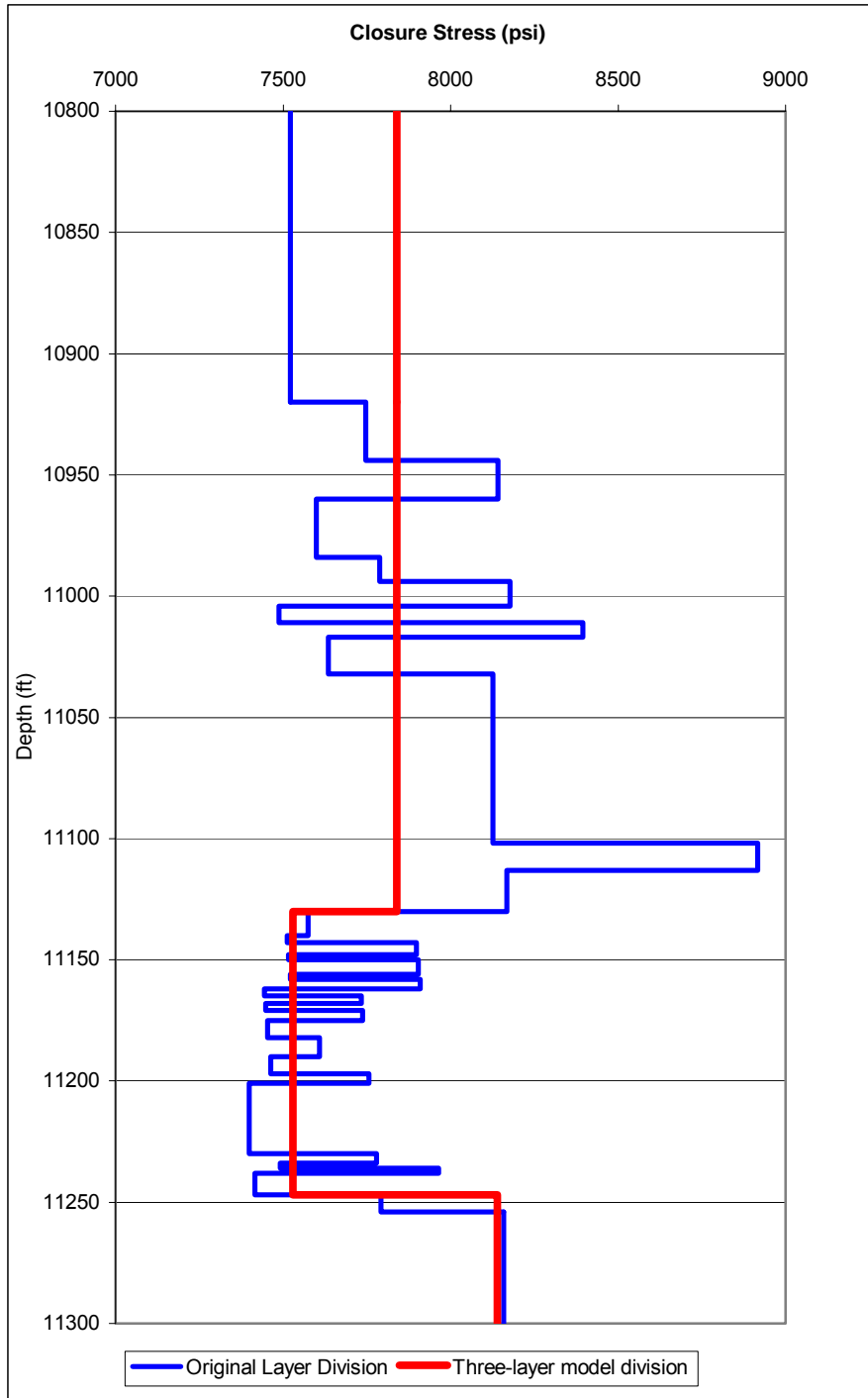


Figure 4-6: Three-Layer Equivalent Stress Profile

To study the impact of these three layers on the shape of the fracture and the typical outputs (fracture length, total height, width and conductivity), 72 random generated cases were considered. These cases were generated using the following assumptions:

- ▶ The number of layers above the productive formation was considered between 6 and 14 layers.
- ▶ The number of layers for the productive formation was considered between 6 and 12 layers.
- ▶ Poisson coefficient for each layer was randomly generated between 0.215 and 0.34. This was considered a good range for the coefficient.
- ▶ Each formation thickness was considered between 5 and 70 ft.
- ▶ The sum of layer thicknesses above the productive formation should not be greater than 270 ft.

The closure stress gradient was calculated using the relationship:

$$\text{GradPc} = \left(\frac{\nu}{1-\nu} \right) \text{Pob} + \left(1 - \left(\frac{\nu}{1-\nu} \right) \right) \text{Pp} \quad \text{Equation 4-3}$$

where, Pob = overburden pressure (psi/ft)

Pp = pore pressure (psi/ft)

ν = Poisson coefficient

The value for the overburden pressure was assumed as 1.1 psi/ft, which represents a typical value for this type of calculation. The pore pressure is usually defined between 0.43 and 0.65 psi/ft. A default average value of 0.54 psi/ft was used. The closure stress of each layer was calculated based on the gradient closure stress and depth to the middle of the layer.

$$\sigma_j = (H_j + H_{j+1}) / 2 * \text{GradPc} \quad \text{Equation 4-4}$$

where, σ_j = closure stress of the layer j, psi

Hj = depth to the top of the layer j, ft

H_{j+1} = depth to the bottom of the layer j, ft

hj = thickness of the layer j (hj = H_{j+1} – Hj), ft

A simple Visual Basic code was developed to generate the cases used in the study. The program generates a random number of layers, random values for Poisson Ratio and thickness, all within the specified ranges. Using these values, the gradient closure stress and formation closure stress is calculated for each generated layer.

4.10.2. Model Accuracy

Using the methodology presented above, all 72 generated cases were reduced to a three-layer model. For each of the 72 cases FRACPRO was run using both methodologies; first using all the layers (original profile) and then the three equivalent layers. The error value between the outputs of the original stress profile and the equivalent stress profile was calculated in order to measure the fit between the fracture geometries. For most of the 72 cases, the error for the fracture propped length between the two models was greater than 12%. This was a significantly high value that was not accepted for the equivalent model.

Analyzing the shape of the fracture for the two scenarios, it was concluded that the highest closure stress layer plays a very important role in the geometry of the fracture. This layer was named the significant overburden layer. Consequently, the significant overburden layer was placed at the bottom of the top layer, right above the target zone. Figure 4-7 shows the new stress profile arrangement with the overburden layer placed above pay zone. With this assumption FRACPRO was run again for the 72 cases. In this scenario, the error for the fracture propped length was diminished to a range between 0 and 8%, with an average of less than 4%. These results were considered satisfactory and this new four-layer model was adopted. Table 4-6 shows the error for the fracture propped length between the original lithology and the equivalent profile.

In addition, the influence of the position of the significant overburden layer within the sequence of layers on fracture propped length was studied. The entire top layer was divided in four equally regions as follows:

- ▶ Region A - topmost region of the top layer,
- ▶ Region B - right below region A,
- ▶ Region C - right below region B,
- ▶ Region D - the region right above the productive formation.

The study showed that higher error values are present when the overburden layer belongs to regions A and B. As the position of the overburden layer gets closer to the top of the productive formation, the error values diminish as seen in Table 4-6.

The newly added layer, placed right on top of the target formation, was named the immediate overburden layer in the model. It is defined as the highest stress layer in the top sequence of the lithology and proved to be the main component in controlling the fracture geometry. As a result, the equivalent stress profile adopted in this study consists of four-layer equivalent model. In this new representation, the number of inputs required for neural network training decreases to eight, as listed below:

1. Thickness of the top layer.
2. Closure gradient stress of the top layer.
3. Thickness of the overburden layer.
4. Closure gradient stress of the overburden layer.
5. Thickness of the pay zone.
6. Closure gradient stress of the pay zone.
7. Permeability of the pay zone.
8. Closure gradient stress of the bottom layer.

No	Region	Overburden Layer Thickness	Fracture Propped Length		Error
			All Layers	4 Layers Model	
		ft	ft	ft	
1	C	13.72	290	299	3.10
2	C	19	302.57	289.3	4.39
3	A	22.58	292.08	290.57	0.52
5	A	27	361.37	340.47	5.78
7	B	16.5	363.94	349.25	4.04
1	D	42.5	351.45	331.22	5.76
2	C	12.54	338.96	353.89	-4.40
5	B	12.8	356.41	337.22	5.38
6	D	7.77	347.41	337.73	2.79
7	B	19.5	350.63	340.75	2.82
9	C	15.7	347.49	337.88	2.77
11	C	28.3	326.85	330.09	-0.99
14	D	10.5	343.26	342.23	0.30
15	D	19.5	335.8	344.36	-2.55
16	C	19.63	329.25	317.23	3.65
18	B	8	355.8	334.34	6.03
19	D	10.45	360.54	337.54	6.38
20	D	24.54	346.52	329.86	4.81
19	D	11.49	353.33	333.93	5.49
20	B	11.68	332.37	335.63	-0.98
21	AB	24	358.36	338.76	5.47
22	C	8.93	344.42	332.08	3.58
23	B	10.68	301.51	329.95	-9.43
24	B	15.62	352.68	335.66	4.83
25	B	43.61	350.39	346.23	1.19
26	C	44.4	334.35	347.78	-4.02
27	B	21.54	351.93	334.74	4.88
28	B	24.8	340.23	341.06	-0.24
29	D	13.77	354.33	335.53	5.31
30	C	8.78	324.3	290.72	10.35
31	D	21.45	359.65	342.48	4.77
32	D	23.7	361.88	342.83	5.26
33	C	49.88	335.17	348.24	-3.90
34	A	36.18	311.59	346.13	-11.09
35	B	59.67	348.71	350.37	-0.48
36	D	30.3	357.04	350.55	1.82
37	D	25.95	358.4	343.56	4.14
38	D	20.22	355.3	341.61	3.85
39	D	11.76	355.66	338.94	4.70
40	A	40.05	346.47	347.55	-0.31
41	B	25.27	338.78	337.49	0.38
42	C	31.48	325.02	341.19	-4.98

Table 4-6: Accuracy of the Four Layer Equivalent Profile

A graphical representation of the original stress profile and the representative four-layer profile are seen in Figure 4-7.

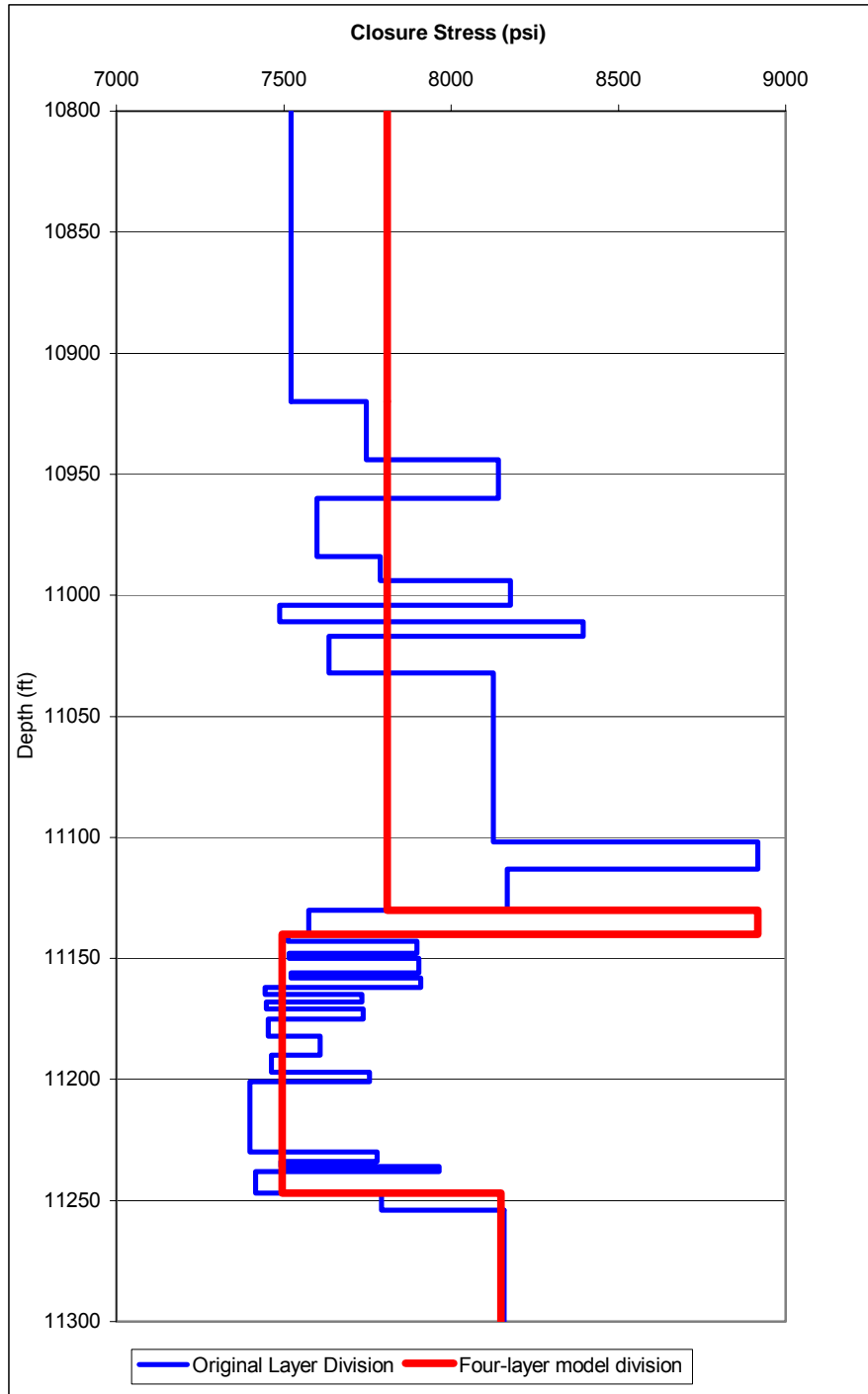


Figure 4-7: Four-Layer Equivalent Stress Profile

4.11. Database Generation for Fracture Design

In this study a combination of neural networks and genetic algorithms, both regarded as artificial intelligence tools, are used to search for the optimum fracture treatment to create the desired fracture length. Typically, the neural network is trained on historic data and captures the relationship between a certain data set of input parameters and the corresponding output parameters.

In order to train a neural network model with reasonable accuracy, one must have access to a significant amount of data. This study uses simulated data to prove its intended methodology.

4.11.1. Overview

Since fracture treatments are performed in fields or basins that typically contain hundreds of wells, large databases containing fracturing information are continuously generated. Moreover, these databases grow with every new treatment executed. The information recorded in these databases can be of a great value to engineers attempting to optimize fracturing treatments.

Since the information used in this work is simulated, the data must contain all required parameters needed for a complex fracturing treatment. Furthermore, it is required that the data would show significant variation in order to capture the relationships between the input and output³⁴. FRACPRO was used to generate the data for training the neural networks. The model was designed for tight gas sands where long fractures are desired.

4.11.2. Input/Output Classification and Data Statistics

Input data required for modeling a fracture simulation typically consists of treatment details and formation data as was stated in detail in section 1.3.3 Data Requirements for Hydraulic Fracturing. The output of the simulation contains several parameters. Since most fractures are designed to achieve a specific fracture length and conductivity, these two parameters are the most important outcomes of the system. In addition to fracture length and conductivity, a simulation run also calculates total fracture height and width. These two parameters, along with the fracture length, define the geometry of the fracture. FRACPRO provides a final important parameter: fracture efficiency, which is related to the effectiveness of fluid use during fracturing. If no leak-off occurs, the value of the fracture efficiency is 1.

The following sections will focus on the input and output parameters as defined for each of the treatment designs considered in the study. Statistics for each of the input/output data sets are included for each type of treatment.

4.11.2.1. Ramp Treatment Schedules

As described in section 4.6 Treatment Schedule Definition, ramp treatment schedules are pumped while continuously ramping up the proppant concentration during the job. The input parameters listed below are used for running FRACPRO to generate the output. A total number of 675 cases (listed in the Appendix Table A - 1) were generated for training and testing the neural network. Each case is unique and contains 14 different parameters as follows:

1. Total fluid volume – the range of values considered for this parameter was based on recommendations provided by Halliburton engineers. Typical treatment jobs performed by Halliburton use between 10,000 to 200,000 gallons.
2. Pad volume – typically is considered between 35–45% of total fluid volume.
3. Final proppant concentration – the range for this parameter was considered between 6 and 16 pounds per gallon (ppg). Typical ramp jobs are 6 to 10 ppg. The proppant is pumped as a ramp function, from a starting value of 1 ppg (default) to the final concentration.
4. Average injection rate – controls the time of the treatment. In the case of jobs where large amounts of fluid are pumped, the injection rate is higher, thus reducing the job time. The range for the injection rate was considered between 15 and 40 bbl/min.
5. Depth to the top of the “productive formation” – the range for the depth was considered to be 5000–15,000 ft. The depth controls the stress profile and pore pressure.
6. Reservoir pressure – considered as an input although calculated as a function of depth. In the case of tight gas sands, fracture jobs are executed in new wells or when the formation pressure has not dropped more than 2000 psia. Therefore, the reservoir pressure was calculated as the initial reservoir pressure (depth * pressure gradient) minus a value of 2000 multiplied by a random number. This assumption provides a good range in reservoir pressure values that can occur in reservoirs.
7. Permeability – controls the leakoff of the fluid in the formation. Since the study addresses the case of the tight gas sands, the values for this parameter were considered within a range of 0.001–0.1 mD. This permeability is actually the permeability of the “equivalent pay zone” which in a real case will be calculated using an averaging technique.
8. Top layer thickness – was considered in a range of 170–250 ft. Two facts contribute to this range selection:
 - ▶ It is time consuming to define lithology above the productive formation for more than 300 ft, assuming that logs are available
 - ▶ The upper height of the fracture does not develop higher than 200–250 ft.
9. Closure stress gradient for the equivalent top layer – considered between 0.72–0.85 psi/ft. This was concluded from the previous analysis when the layers above the productive formation were merged to form the equivalent top layer.

10. Overburden layer thickness – restricted to values between 10 and 50 ft. It has been observed from logs that a high stress layer does not usually present a high value of thickness.
11. Closure stress gradient of the overburden layer – considered within a range of 0.74–0.85 psi/ft. This parameter is designed to always be greater than the closure stress of the equivalent top layer.
12. Pay zone thickness – values between 50 and 130 ft. The pay zone thickness is an average of the productive sands, and the thin shale intercalations, when the later exists.
13. Closure stress gradient for the pay zone – considered within a range of 0.64 to 0.72 psi/ft. This value is, in a real case, obtained from the merging algorithm developed for the production layer.
14. Closure stress gradient of the bottom layer – considered within a range of 0.715 and 0.810 psi/ft. Most of the time, little information is known about the layers below the production formation or little lithology change exists. For these reasons, simulations consider only one layer below the production formation and take into account only its closure stress and no thickness. This is also the case of FRACPRO when the thickness of the bottom layer is not required as input.

Since porosity is an important reservoir characteristic, its influence on the simulation outputs was studied. Keeping all the inputs for the FRACPRO simulator constant, while altering porosity values from 7 to 13%, it was concluded that porosity does not induce significant difference in the geometry (height, width, length) of the fracture. Accordingly, the average value of 9% was considered satisfactory for all cases.

The leakoff fluid permeability ratio, K_p/K_l (ratio of formation pore-fluid permeability to formation leakoff-fluid permeability), was determined by expert engineers to have a range of 25–30.

The average pressure along the fracture was calculated as a function of the depth and the fracture closure gradient using the following formula:

$$\text{Average pressure in the fracture} = \text{Depth} * \text{Grad}_{\text{closure_stress}} \quad \text{Equation 4-5}$$

Since the slurry has a linear relationship with total volume and pad volume, it was not considered as an input in training the neural network.

All 14 inputs presented above were used in the FRACPRO simulator for generating the data. A simple Visual Basic computer program, using a random number generation function and the constraints defined above, generated 660 cases for the parameters within the specified ranges.

According to Halliburton experts, most ramp treatments in tight rock country ($k < 1 \text{ md}$) start with 1 lb/gal and are increased to around 6 lb/gal. In highly permeable sands, the ramping may grow in 2 lb/gal increments. It is intended to get 60% or better of the total sand volume in the last one or two stages.

4.11.2.2. Six Stage Treatment Schedules

The main difference between a stage treatment and a ramp treatment consists of the number of stages when pumping the slurry. Also, in the case of stage treatment the proppant concentration remains constant during each stage, which mathematically can be described as a step function. The six stages treatment as defined in this study consists of five steps and the pad.

All 14 inputs described in the ramp treatment section remain the same. In addition, three more parameters are added to describe volume and proppant concentration by stage for this type of treatment. These three new inputs are:

1. Initial proppant concentration in the first slurry stage. For this parameter three values were considered: 1, 1.5, and 2 ppg. Halliburton engineers suggested these values, which are typically used when designing treatments.
2. Proppant concentration increment – used to define the proppant concentration for the intermediate stages, which are the stages between the first and the last. The range values for the final proppant concentration were defined in the previous section. The proppant concentration increment is calculated using the following relationship:

$$PCI = \frac{FPC - IPC}{S - 1} \quad \text{Equation 4-6}$$

where, PCI = proppant concentration increment. ppg

FPC = final proppant concentration, ppg

IPC = initial proppant concentration, ppg

S = number of stages excluding pad

3. Fluid volume increment – defines the fluid volume for the intermediate stages. The fluid volumes by stage are not generated randomly, as each stage must respect a specified range. The fluid volumes for each stage are defined as a function of the total fluid volume. Table 4-7 shows the stage volume as percentage of the total treatment volume.

Stage Type	Percentage from Total Fluid Volume
Pad (Stage 1)	35 - 45 %
Stage 2	5 - 10 %
Stage 3	5 - 10%
Stage 4	10 -15%
Stage 5	15 - 20%
Stage 6	15 - 25%

Table 4-7: Percentage of Total Fluid Volume per Stage for Six-Stage Treatment Schedule

In order to generate stage treatments, which respects the intervals defined in Table 4-7, the value of the fluid increment was thoroughly studied. Different types of combinations were considered and analyzed. However the one meeting the most requirements is structured as shown in Table 4-8.

Stage Type	Stage Volume Structure
Pad (Stage 1)	(35 - 45) % from Total Fluid Volume
Stage 2	2 * Increment
Stage 3	2 * Increment
Stage 4	3 * Increment
Stage 5	4 * Increment
Stage 6	5 * Increment

Table 4-8: Stage Fluid Volume Structure of Six-Stage Treatment Schedule

To achieve this structure, a new relationship was introduced to calculate the fluid increment. This equation defines the fluid increment volume as:

$$FI = \frac{TFV - PFV}{N} \quad \text{Equation 4-7}$$

where, FI = fluid volume increment, gal

TFV = total fluid volume, gal

PFV = pad fluid volume, gal

N = parameter calculated function of number of stages in the treatment

(In the present case of 5 slurry stages, N is equal to 16.)

The graphical representation of the six-stage treatment is shown in Figure 4-8.

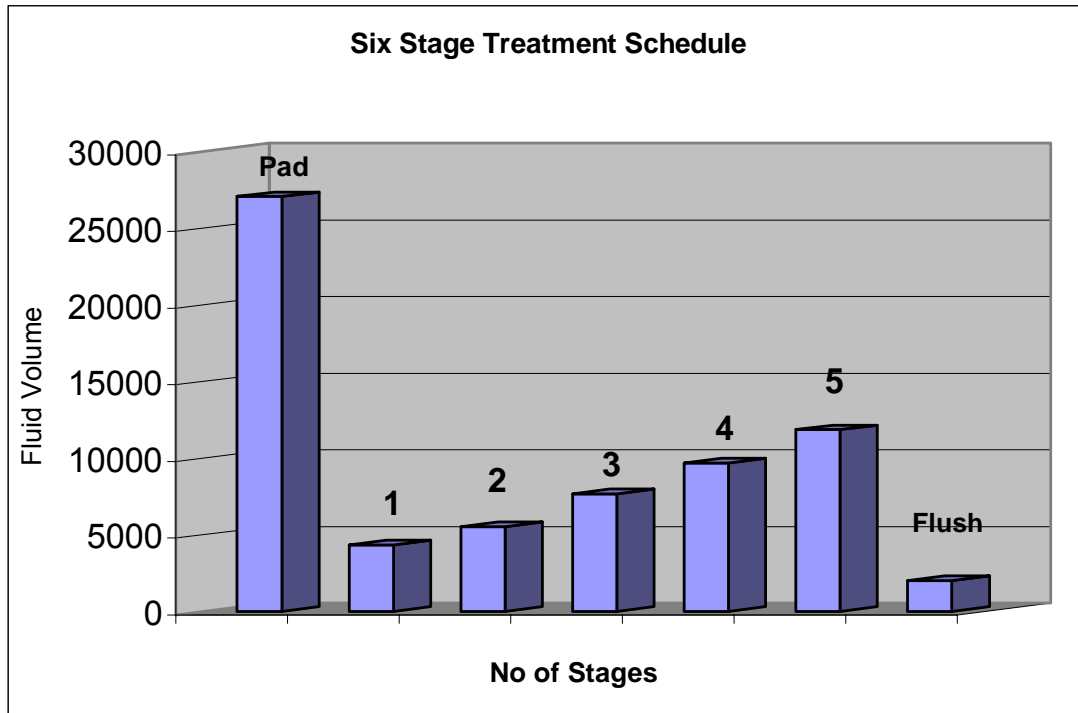


Figure 4-8: Graphical Representation of the Six Stage Design.

The 17 inputs defined above were used in FRACPRO simulator for generating the data, which can be viewed in the Appendix Table A - 2. The final system consists of a total number of 17 inputs and 5 outputs.

4.11.2.3. Eight Stage Treatment Schedules

A treatment with large numbers of stages (eight or more) is designed when job placement problems are anticipated. For example, if the desired conductivity would be achieved from 5 and 6 lb/gal stages, but there might be problems placing 5 and 6 lb/gal into the formation, then smaller incremental steps of 1, 2, and 3 lb/gal stages are designed so they can be analyzed as each stage enters the formation. In this case, if problems occur, the job design can be altered to overcome any formation problems that might exist (i.e. increase rate, increase volume etc.). As a rule of thumb, if the reservoir is prone to job placement problems a higher number of stages are recommended.

The design of the eight-stage treatment runs basically on the same platform as the six-stage treatment. The only difference consists in the percentage of fluid volume injected for each stage. As in the case of six stages treatment, the system consists of 17 parameters as defined

above. The proppant concentration increment and the fluid volume increment are calculated with equations 4-6 and 4-7.

The initial proppant concentration in the first slurry stage was considered as having three values: 1, 1.1, 1.2 ppg. These values were suggested by Halliburton engineers, which are typically used when designing treatments. The proppant concentration increment, used to define the proppant concentration for the intermediate stages, is calculated using the same equation (4.6). Table 4-9 shows how proppant increment concentration is applied in defining each of the treatment stages.

Stage	Proppant Conc
Pad (Stage 1)	-
Stage 2	X_i
Stage 3	x_i+1*p_inc
Stage 4	x_i+2*p_inc
Stage 5	x_i+3*p_inc
Stage 6	x_i+4*p_inc
Stage 7	x_i+5*p_inc
Stage 8	x_f

Table 4-9: Proppant Concentration for Eight-Stage Treatment Schedule

The fluid volume increment is used to define the fluid volume for the intermediate stages. Again the fluid volumes by stage are not generated randomly; each stage must respect a specified range. The fluid volumes for each stage are defined as a function of the total fluid volume. Table 4-10 shows the stage volume as percentage of the total treatment volume.

Stage Type	Percentage from Total Fluid Volume
Pad (Stage 1)	25 - 43 %
Stage 2	5 - 8 %
Stage 3	5 - 8%
Stage 4	8 -10%
Stage 5	8 -10%
Stage 6	12 - 20%
Stage 7	15 - 20%
Stage 8	12 - 15%

Table 4-10: Percentage of Total Fluid Volume per Stage for Eight-Stage Treatment Schedule

The ranges provided in Table 4-10 are typical ranges for an eight-stage treatment design. In order to generate stage treatments that respect the ranges defined in Table 4-10, a thorough

analysis of the fluid increment value was carried out. One equation that respects the structure of stages is a derivation of equation (4.7), where N is equal to 23. The number N is the sum of all fluid increments as can be seen in Table 4-11.

Stage	Type I Fluid Volume
Pad (Stage 1)	30-40%
Stage 2	2*inc
Stage 3	2*inc
Stage 4	3*inc
Stage 5	3*inc
Stage 6	4*inc
Stage 7	4*inc
Stage 8	5*inc

Table 4-11: Type I Stage Fluid Volume Structure of Eight-Stage Treatment Schedule

The treatment with the structure presented in Table 4-11 column two, was named “8 Stages Design Type I”. The graphical representation of this type of treatment is shown in Figure 4-9.

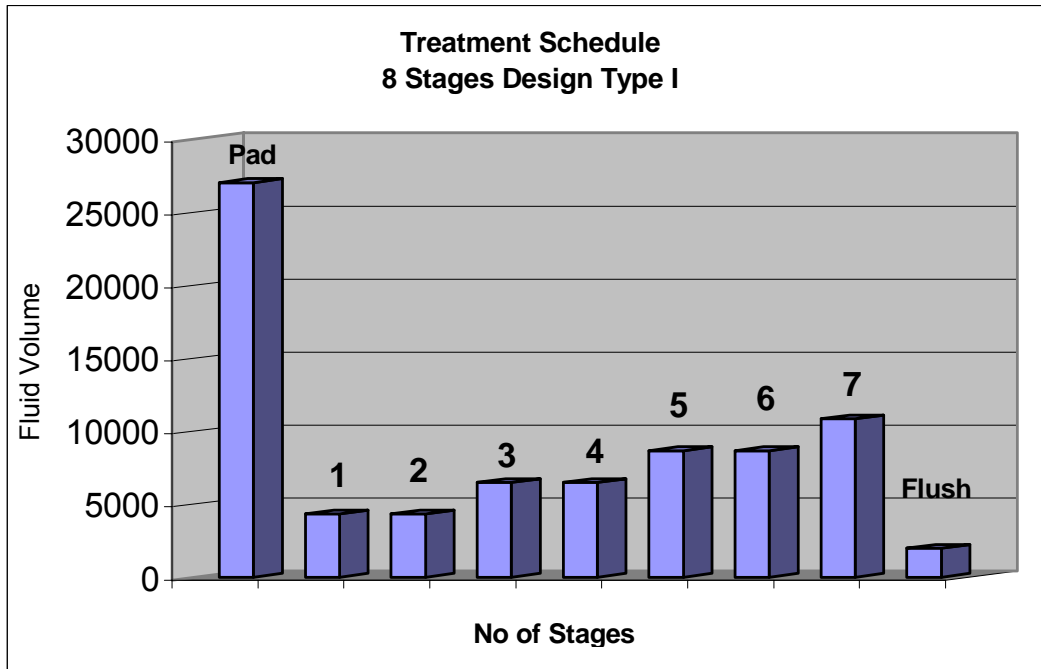


Figure 4-9: Type I Eight Stage Design

In designing fracture treatments, the first proppant stages are basically an extension of the pad portion of the job. Increasing proppant concentrations build fracture width and serve as an indicator for any near wellbore or fracture growth problems. The last proppant stage is

designed to provide maximum conductivity near well bore, and therefore is not a significant percentage of the job.

With these considerations, the volume for each stage was altered to accommodate for designs that provide higher conductivity near the well bore. Two more structures for the eight-stage treatment were defined as shown in Table 4-12.

	Type II	Type III
Stage	Fluid Volume	Fluid Volume
Pad	30-40%	30-40%
Stage 1	2*inc	2*inc
Stage 2	2.5*inc	2.5*inc
Stage 3	3*inc	3*inc
Stage 4	4.5*inc	4*inc
Stage 5	4*inc	4.5*inc
Stage 6	5*inc	5*inc
Stage 7	3*inc	2*inc

Table 4-12: Types II and III Stage Fluid Volume Structure of Eight-Stage Treatment Schedule

The treatment with the structure presented in Table 4-12 column two was named “8 Stages Design Type II”. The graphical representation of this type of treatment is shown in Figure 4-10.

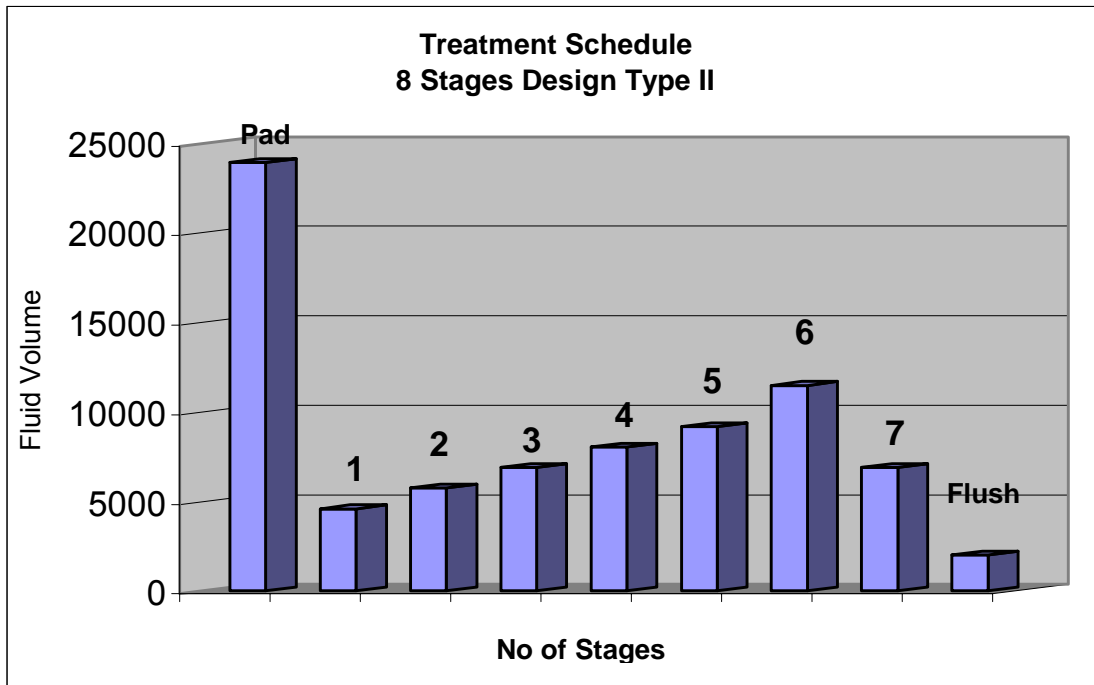


Figure 4-10: Type II Eight Stage Design

The treatment with the structure presented in Table 4-12 column three, was named “8 Stages Design Type III”. The graphical representation of this type of treatment is shown in Figure 4-11.

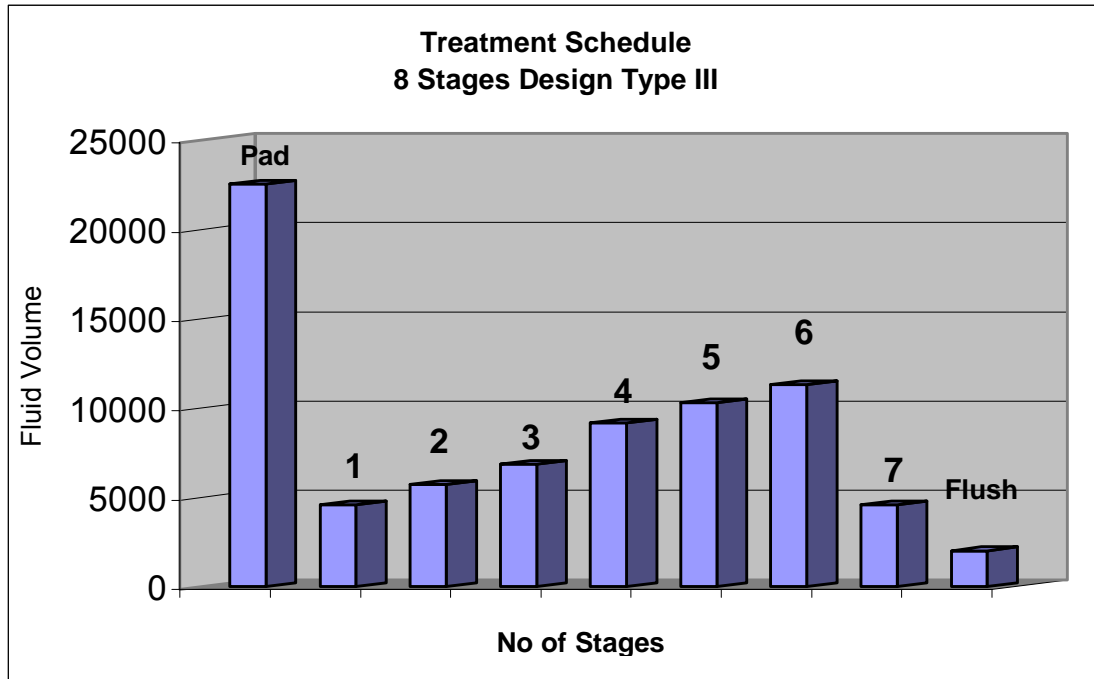


Figure 4-11: Type III Eight Stage Design

The comparison between the three types of treatments is shown in Figure 4-12. As can be seen from the graph, Types II and III are designed so that higher proppant concentrations are carried with low amount of fluid, thus creating high conductivity around the well bore.

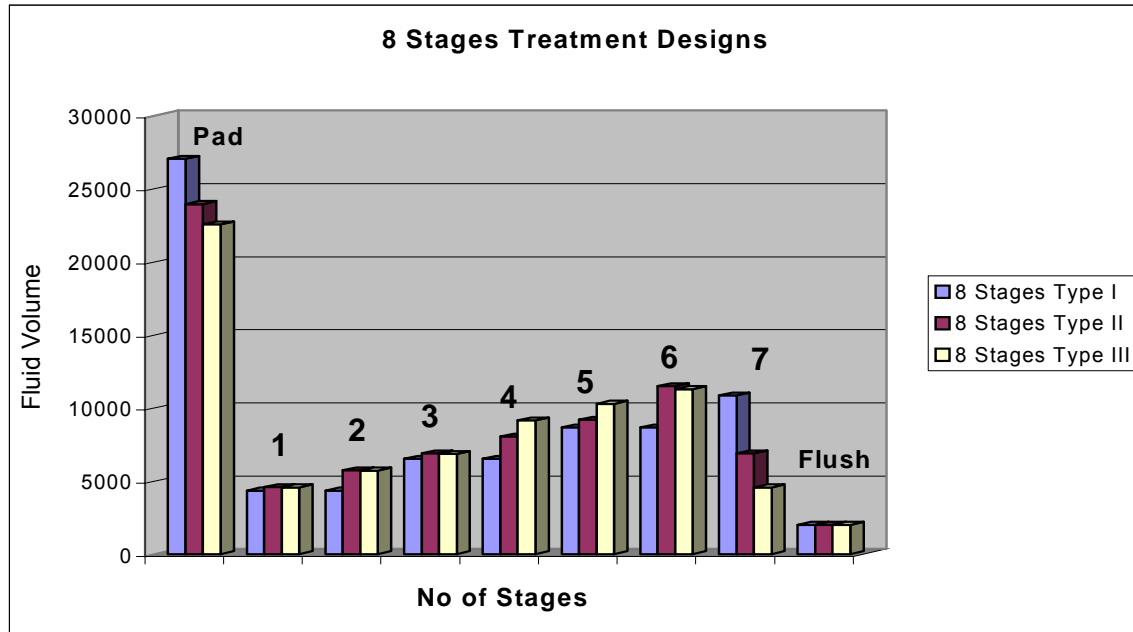


Figure 4-12: Comparison of Eight Stage Treatment Schedules

All 17 inputs were used in the FRACPRO simulator for data generation. For each of the three types of treatments, 200 cases were generated and can be viewed in the Appendix Table A - 3, Table A - 4, and Table A - 5.

4.12. Fracture Design Neural Networks

The next step in the development of an intelligent tool that designs optimum fracture treatments is to build a module that replaces the hydraulic fracture simulator. Since neural networks have a natural propensity to store knowledge and make it available for later use, they can be trained to mimic the fracture simulation and provide the same output as the computer model.

It is impractical, if not impossible, to couple vendor software such as FRACPRO with a search engine, like genetic algorithms, particularly because FRACPRO requires human data entry for each new run. Furthermore, a simulation run may take up to a minute depending upon the complexity of the reservoir lithology and treatment design. In addition, no source code of the software was available or easy to acquire.

Since neural networks can learn, they can be trained to perform a particular task, such as the output of a fracturing simulation treatment. Neural networks learn from presented data and can generalize to a certain extent, but they cannot predict something that is beyond the range of the training data. Therefore, it is important for the training data to be representative and cover a wide range of possibilities. The module that replaces the fracture simulator is composed of a few

neural networks with specific functionality. These networks are the heart of the optimization methodology presented in this study.

The geometry of the fracture is defined by fracture length, fracture height, and fracture width. Three additional parameters, fracture efficiency and proppant concentration and dimensionless fracture conductivity, are also carefully considered when placing a treatment job. All six parameters represent the output of this system.

In training the neural networks that will replace FRACPRO, a commercial product named “NeuroShell 3.0” developed by Ward System Groups was used. “NeuroShell 3.0” provides a relatively wide variety of net architectures and training algorithms, with a friendly user interface.

4.12.1. Developing Neural Networks for Ramp Treatments

The first neural network developed was the ramp treatment network. It was designed to mimic a hydraulic fracture simulator for ramp treatments schedules. Table 4-13 summarizes the inputs and outputs used for training the neural network. The final system consisted of a total number of 14 inputs and 6 outputs. Column two defines the names of each parameter, column three identifies whether it is an input or output, and column four provides the range of each parameter.

Index	Parameter Name	Type	Range Interval	Units
1	Total fluid volume	input	10,000 - 200,000	gal
2	Pad volume	input	3,500 - 90,000	gal
3	Final proppant Concentration	input	6 - 15	ppg
4	Flow pumping rate	input	15 - 40	bbf/min
5	Depth to the "pay zone"	input	5,000 - 15,000	ft
6	Reservoir pressure	input	1,500 - 7,000	psia
7	Permeability	input	0.001 - 0.1	mD
8	Top layer thickness	input	170 - 250	ft
9	Top layer closure stress gradient	input	0.72 - 0.85	psi/ft
10	Overburden layer thickness	input	Oct-50	ft
11	Overburden layer closure stress gradient	input	0.74 - 0.85	psi/ft
12	Pay zone thickness	input	50 - 130	ft
13	Pay zone closure stress gradient	input	0.64 - 0.72	psi/ft
14	Bottom layer closure stress gradient	input	0.71 - 0.81	psi/ft
1	Fracture Total Height	output	220 - 600	ft
2	Fracture Propped Length	output	170 - 900	ft
3	Fracture Maximum Width	output	0.14 - 1.5	in
4	Dimensionless Conductivity Ratio	output	1 - 500	
5	Fracture Proppant Concentration	output	0.4 - 2.0	lb/ft ²
6	Fracture Efficiency	output	0.2 - 0.97	

Table 4-13: Inputs/Outputs for Ramp Treatment Neural Network

FRACPRO was used for generating a representative set of examples, namely pairs of input and output samples, necessary for training the neural network as described in section 4.11.2.1. The results are displayed in the Appendix Table A - 1.

Due to the complexity of the problem and the wide range of parameters, several neural network types and corresponding architectures were considered. Because of its outstanding performance, a back-propagation neural network with an architecture consisting of three hidden layers with different activation functions was used in training. The graphical representation of the selected neural network architecture is shown in Figure 4-13.

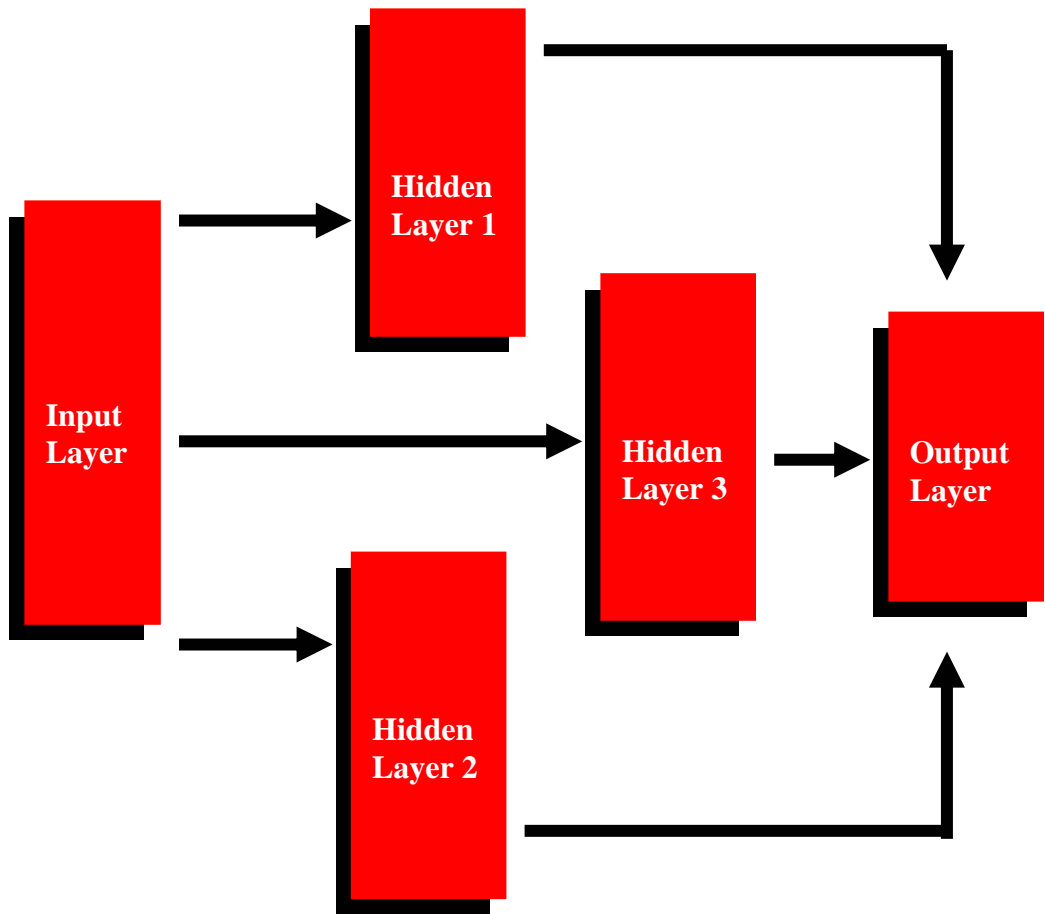


Figure 4-13: Back-Propagation Neural Network Architecture

A summary architecture description of the Neural Networks presented in Figure 4-13 is given in Table 4-14.

Optimization Network	Input Layer	Hidden Layer 1	Hidden Layer 2	Hidden Layer 3	Output	Learning Rate	Momentum
Number of Neurons	14	12	12	12	6	0.1	0.1
Type of activation Function	Linear	Gaussian	Tanh	Gaussian	Logistic		

Table 4-14: Optimization Network for Ramp Treatment Schedule

The ramp treatment optimization network has one input layer with 14 input neurons, three hidden layers with 36 hidden neurons (12 in each hidden layer) and 6 output neurons. The learning rate was 0.1 and the momentum 0.1.

4.12.2. Developing Neural Networks for Stage Treatments

Two scenarios were considered in order to demonstrate the capability of the methodology by addressing multiple variations of the stage treatment design schedules. Once the model proved that these two types of stage treatment schedules provide accurate results, the model can be expanded to any number of stages that can be reasonably implemented in the field.

The next two sections provide a detailed explanation of the implementation for the six and eight stage treatments schedules design.

4.12.2.1. The Six Stage Treatment

The second neural network developed was the six-stage treatment network. This network models cases where fracturing treatments are placed in stages with a step in proppant concentration for each stage. Previously, an algorithm was introduced to represent different stages while keeping the lowest possible number of inputs. An example of the structure of a six-stage treatment was defined in Table 4-1.

Table 4-15 summarizes the inputs/outputs neural network system. Column two defines the names of each parameter, column three identifies the type of parameter, and column four provides the range of each parameter.

Index	Parameter Name	Type	Range Interval	Units
1	Total fluid volume	Input	10,000 - 200,000	gal
2	Pad volume	Input	3,500 - 90,000	gal
3	Increment volume	Input	Calculated	gal
4	Initial proppant concentration	input	1, 1.5, 2	ppg
5	Proppant concentration increment	input	Calculated	ppg
6	Final proppant Concentration	input	10.8	ppg
7	Flow pumping rate	input	15 - 40	bbbl/min
8	Depth to the "pay zone"	input	5,000 - 15,000	ft
9	Reservoir pressure	input	1,500 - 7,000	psia
10	Permeability	input	0.001 - 0.1	mD
11	Top layer thickness	input	170 - 250	ft
12	Top layer closure stress gradient	input	0.72 - 0.85	psi/ft
13	Overburden layer thickness	input	Oct-50	ft
14	Overburden layer closure stress gradient	input	0.74 - 0.85	psi/ft
15	Pay zone thickness	input	50 - 130	ft
16	Pay zone closure stress gradient	input	0.64 - 0.72	psi/ft
17	Bottom layer closure stress gradient	input	0.71 - 0.81	psi/ft
1	Fracture Total Height	output	220 – 600	ft
2	Fracture Propped Length	output	170 – 900	ft
3	Fracture Maximum Width	output	0.14 - 1.5	in
4	Dimensionless Conductivity Ratio	output	1 - 500	
5	Fracture Proppant Concentration	output	0.4 - 2.0	lb/ft ²
6	Fracture Efficiency	output	0.2 - 0.97	

Table 4-15: Inputs/Outputs for Six-Stage Treatment Schedule Neural Network

As in the case of the ramp treatment schedule, FRACPRO was used for generating data necessary for training the neural network. The stage treatment neural network used has an identical architecture with the one used for ramp treatment (Figure 4-13). A summary description of the neural networks architecture is presented in Table 4-16.

Optimization Network	Input Layer	Hidden Layer 1	Hidden Layer 2	Hidden Layer 3	Output	Learning Rate	Momentum
Number of Neurons	17	11	11	11	6	0.1	0.1
Type of activation Function	Linear	Gaussian	Tanh	Gaussian	Logistic		

Table 4-16: Optimization Network for Six-Stage Treatment Schedule

The six-stage optimization network has one input layer with 17 input neurons, three hidden layers with 33 hidden neurons (11 in each hidden layer) and 6 output neurons. The learning rate was 0.1 and the momentum 0.1.

4.12.2.2. The Eight Stage Treatment

The third type of neural network was the eight-stage treatment network. A small study was performed to determine the minimum number of cases needed to be generated in order to train an acceptable accurate neural network to mimic the type of treatment. 660 cases were generated for the ramp treatment and 675 cases for the six-stage treatment. This was based on the runs already available from the six-stage treatment schedule.

The procedure consisted of the following steps:

1. 100 random cases were selected from the 675 total available.
2. The cases were divided into the training set and the testing set, representing 90% and 10% respectively of the 100 random cases.
3. A neural network was trained using the training and testing cases. Once optimized the correlation coefficient for the training set is recorded.
4. The process is repeated each time increasing the number of random cases selected using a step of 25 or 50 cases. The correlation coefficient for each step is recorded for five of the outputs.

Table 4-17 presents each step of the process, the structure of the training and testing sets as well as the recorded correlation coefficients for each step. The first column represents the step number, while column two shows the total number of cases selected for the current step. Columns three and four show the structure of the training and testing sets. The last five columns are the values for the correlation coefficients for each of the five outputs considered in the neural network. The last row in the table corresponds to the correlation coefficients for the optimized six-stage treatment neural network.

	Number of Cases				Correlation Coefficient Values				
	Total # of cases	Training Set, # cases	Clibration Set, # of cases	Verification Set, # cases	Fracture Efficiency	Propped Length, (ft)	Dimmensionless Conductivity Ratio	Maximum Propped Width (in)	Fracture Height, (ft)
Step 1	100	90	10	578	0.801	0.88	0.848	0.889	0.87
Step 2	150	135	15	528	0.846	0.921	0.834	0.904	0.904
Step 3	200	180	20	478	0.838	0.929	0.892	0.902	0.918
Step 4	250	225	25	428	0.838	0.94	0.88	0.899	0.912
Step 5	300	270	30	378	0.828	0.946	0.898	0.913	0.918
Step 6	350	315	35	328	0.864	0.931	0.905	0.919	0.93
Step 7	400	360	40	278	0.887	0.939	0.901	0.919	0.927
Step 8	425	382	43	253	0.858	0.929	0.908	0.935	0.918
Step 9	450	405	45	228	0.883	0.943	0.935	0.945	0.942
Step 10	475	427	48	203	0.884	0.955	0.929	0.945	0.938
Step 11	500	450	50	177	0.882	0.958	0.925	0.939	0.933
Step 12	525	472	53	153	0.907	0.954	0.925	0.94	0.924
Step 13	550	495	55	127	0.927	0.954	0.941	0.951	0.924
Step 14	575	517	58	103	0.943	0.943	0.938	0.942	0.916
Step 15	600	540	60	75	0.937	0.946	0.933	0.945	0.939
All	675	480	120	75	0.917	0.947	0.907	0.914	0.929

Table 4-17: Correlation Coefficients for Eight Stage Treatment Schedule Neural Network

The graphical representation of the correlation coefficients values versus the number of cases considered in training and testing the neural network is seen in Figure 4-14.

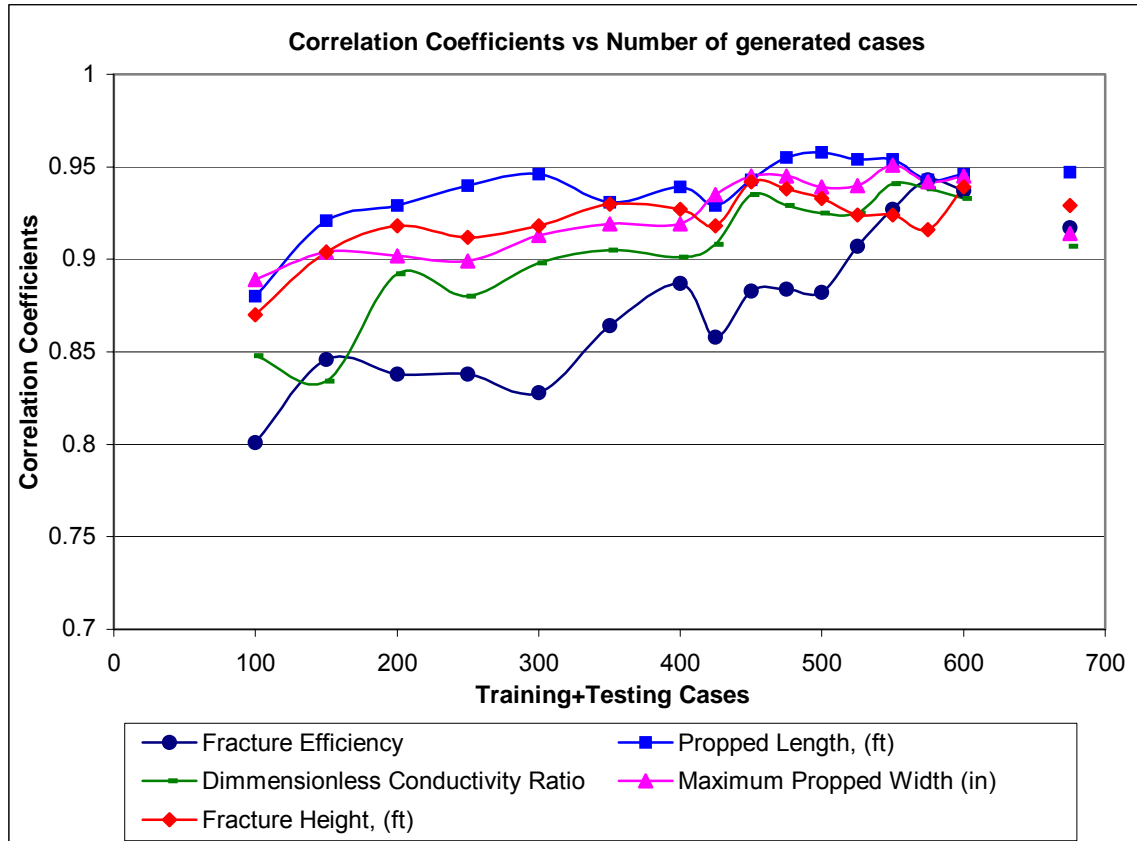


Figure 4-14: Correlation Coefficient Variation with Number of Cases

Table 4-17 and Figure 4-14 show that values for the correlation coefficients are improving with the increase of number of cases. With the exception of fracture efficiency, the values for correlation coefficients are near or above 0.9 for 200 or more cases. Since values above 0.9 are acceptable, 200 cases or more were generated for training the eight stage treatment schedule neural networks.

As with the six stage neural network, three types of treatment structures were also considered for the eight stage neural network. The structures of these three types are defined in Table 4-11 and Table 4-12.

The inputs and outputs for the eight stage neural network are identical to the six stages treatment and are summarized in Table 4-15.

Again, FRACPRO was used to generate data for each type of the three treatments schedules (seen in Appendix Table A - 3, Table A - 4, and Table A - 5). With this data, three neural networks were trained to mimic FRACPRO. All three types neural networks have an identical architecture to that used for ramp and six stage treatments (Figure 4-13).

A summary description of the neural network architectures is presented in Table 4-18. The eight stage optimization networks have one input layer with 17 input neurons, three hidden

layers with 36 hidden neurons (12 in each hidden layer), and 6 output neurons. The learning rate was 0.1 and the momentum 0.1.

Optimization Network	Input Layer	Hidden Layer 1	Hidden Layer 2	Hidden Layer 3	Output	Learning Rate	Momentum
Number of Neurons	17	12	12	12	5	0.1	0.1
Type of activation Function	Linear	Gaussian	Tanh	Gaussian	Logistic		

Table 4-18: Optimization Network for Eight Stage Treatment Schedule

4.13. Neuro-Genetic Optimization Module for Fracture Treatment Design

The objective of this study is to create a front-end optimization routine for hydraulic fracturing models to design the optimum treatment schedule that creates a desired fracture propped length. This is optimization problem requires the optimum combination of four treatment parameters: total fluid volumes, pad volume, proppant concentration and injection rate³⁵. The optimum fracture treatment design is found through the use of a two-step method that integrates neural networks with a search engine, in this case a genetic algorithm^{8,36-38}.

The first step involves building the neural networks modules to mimic a fracture simulator, as detailed in sections 4.12.1 and 4.12.2. The second step of the methodology uses the genetic algorithm to search for the optimum fracture design treatment. Genetic algorithms complete the optimization process, which consists of the initialization of a population, fitness evaluation, selection, crossover and mutation to produce the next generation³⁹. This process is repeated for each generation until converged to the final solution.

4.13.1. Treatment Optimization Design Overview

In designing the optimum hydraulic fracturing treatment an important variable is the dimensionless fracture conductivity. This variable incorporates both fracture conductivity and length. The dimensionless fracture conductivity C_{FD} , introduced by Cinco-Ley⁴⁰ is defined by the equation:

$$C_{FD} = \frac{k_f w}{k x_f} \quad \text{Equation 4-8}$$

where, k_f = proppant pack fracture permeability, md
 w = average propped fracture width, in
 k = formation permeability, md
 x_f = fracture half-length, ft

In high permeability reservoirs, increasing the fracture conductivity is the primary demand, while fracture length is of secondary importance. In low permeability reservoirs, fracture conductivity does not play the primary role, since long fracture lengths are required for the success of the treatment. Accordingly, since this study addresses low permeability reservoirs, fracture length dictates the main optimization followed by the conductivity.

The main outputs of FRACPRO, which were modeled with the neural network, are fracture geometry (height, length, and width), fracture efficiency, proppant concentration, and dimensionless conductivity ratio. FRACPRO does not directly provide the fracture conductivity, but instead the dimensionless conductivity ratio.

According to the FRACPRO manual, the fracture conductivity is derived as the harmonic means of cell conductivities in each region. During simulation, a conductivity value is calculated for each grid block by interpolating the proppant permeability versus laboratory-determined stress. Harmonic mean is the natural way to average conductivities in series and is consistent with the way simulators use permeability to calculate flow between grid blocks. Under harmonic mean averaging, the lowest conductivity tends to dominate.

Since the implementation of genetic algorithms used in this study can only search for one parameter at time, finding the optimum combination of fracture length and conductivity requires a two-step process. The methodology consists of the following steps:

1. Define the desired fracture length.
2. Utilize a genetic algorithm to find the desired optimum fracture length. The result of the genetic optimization is an entire population of solutions evolving around the optimal solution.
3. Select the best 10% to 20% of the final population. For each potential solution, calculate the fracture conductivity using equation 4-8.
4. Find the solution with the conductivity value closest to the desired one.
5. If not ideal, minor adjustments in the injection rate or proppant concentrations can achieve a desired solution.

Two genetic algorithms were developed, one for the ramp treatment schedule, the other for the stage treatment schedule. Each of these algorithms uses two neural networks, one as fitness function and the other for optimization.

The steps for this process, in further detail, are as follows:

1. Generate the initial population of potential treatment designs solutions.
2. Run each one of the potential solutions through the fitness function neural network.
3. Compare the outcome (fracture propped length) to the desired propped length, and rank the treatments based on the error between them.
4. Select the top ranked treatment designs to create the new generation and discard the rest.
5. Generate a new population of treatment designs by performing crossover and mutation processes on the selected treatment designs selected in Step 4.
6. Repeat process from Step 2 until desired error convergence is reached.

4.13.2. Rapid Screening Solutions

Rapid screening solutions provide a fast way to estimate potential treatment designs that may fit the desired criteria (fracture length, conductivity, volume of fluid and proppant). It can quickly provide an indication of the size of the treatment (volumes of fluid and proppant) needed to create a fracture with the specified length. The process represents the first generation of genetic optimization.

In this study, a rapid screening solution was developed for each treatment schedule. First, a desired fracture length was set, and then one hundred random treatment designs were generated. These treatments, together with the reservoir data, are sent to the neural network where the corresponding output (the fracture propped length) is created. The treatments are ranked descending according to the fitness function represented by desired propped length so that the top ranked treatment design has the closest calculated fracture length to the desired fracture length. The top five to ten treatments designs were deemed acceptable solutions and were considered as a starting point for the genetic algorithm initial population.

4.13.3. Ramp Treatment Optimization Design

Neural networks provide the fitness function for genetic algorithms to solve the highly non-linear optimization problem of finding the optimum treatment schedule in order to create the desired fracture length and conductivity. The data involved in this process is divided in two categories:

- ▶ Constant data represented by the formation characteristics
- ▶ Treatment design parameters that can be optimized
 - Total fluid volume
 - Pad fluid volume
 - Slurry volume

- Initial proppant concentration
- Final proppant concentration
- Injection flow rate

Since slurry volume represents the total fluid volume minus the pad volume, it was excluded from the optimization. Furthermore, initial proppant concentration was set to 1 ppg for each case generated, and since it was constant it was not considered for optimization either. With these assumptions, the optimum treatment design represents a combination of the following four parameters:

- ▶ Total fluid volume
- ▶ Pad fluid volume
- ▶ Final proppant concentration
- ▶ Injection flow rate

In addition, some constraints were set for the injection rate, such as for small treatment jobs the injection flow rate should not converge towards the high values.

The first step in developing the genetic algorithm module is problem encoding. The binary representation of each parameter is determined. Concatenating the binary representation of the four parameters to be optimized generates one chromosome. The total number of chromosomes forms a population. The number of the bits used for each variable is presented in Table 4-19.

Type	Variable	Range	Number of Bits
Ramp	Total Fluid Volume	10,000 - 200,000 gal	18
	Pad Volume	3,500 – 90,000 gal	6
Treatment	Final Proppant Concentration	16	8
	Injection Rate	10 – 20 bpm	6

Table 4-19: Ramp Treatment Design Encoding

The algorithm creates an initial population of chromosomes by randomly populating the bits, corresponding to each parameter in the chromosomes. This is followed by the selection process, crossover, mutation, and fitness evaluation to form a new population.

4.13.4. Stage Treatments Optimization Design

Since both the six stage and the eight stage treatments are designed on the same platform, they can be defined by the same variables as the ramp treatment (total fluid volume, pad volume, slurry volume, initial proppant concentration, final proppant concentration and injection rate). Using the same considerations, the slurry will not be considered in the

optimization. In the case of stage treatments the initial proppant concentration was set to be equal to the proppant increment for each stage. The same constraints were applied to the injection rate as related to the size of the treatments. Table 4-20 contains the encoding for the stage treatment design, which applies to both six and eight stage treatments.

Type	Variable	Range	Number of Bits
Stage Treatment	Total fluid volume	10,000 - 200,000 gal	18
	Pad volume	3,500 - 90,000 gal	6
	Incremental proppant concentration	1- 2 ppg	5
	Final proppant concentration	11 ppg	7
	Injection Rate	10 – 40 bpm	7

Table 4-20: Stage Treatment Design Encoding

4.14. Pressure Profile Prediction

Monitoring the pressure in real time is the most precious information that a fracturing engineer can acquire. This provides the real and most accurate response of the formation as it reacts to the fracturing process. The net-pressure matching during fracturing is aimed to support and assist the engineer in two ways. First, a good pressure match while fracturing assures engineer that the treatment is being placed as scheduled. Secondly, good matching between the observed pressure (which is the formation response) and the calculated pressure predicted by the model supports the results of the fracture geometry and fracture conductivity.

This study includes a model to predict the expected net treating pressure for the designed fracture. This model consists of a set of neural networks that mimic the expected net pressure profile during fracturing. This pressure profile can later be used for real time matching.

4.14.1. Overview

During a hydraulic fracturing job, the pressure signal can be recorded for any desired time interval. Generally this produces more data points than the neural network can handle as output data. Vector quantization, a powerful technique for data compression, was utilized to resolve this situation.

A simple linear vector quantization technique was used to sample and extract the main characteristics (called quantizers) of the pressure profile. The net treating pressure neural network was trained using these quantizers and, when applied to a new case, reconstructed the entire pressure signal. This technique was used in the development of the pressure profile neural network and was trained in the case of ramp treatments.

4.14.2. Defining Pressure Components Used For Pressure Matching

There are several pressure components that are either calculated or measured during a fracture simulation. These components are defined below:

1. Net Pressure: the pressure in the fracture minus the closure stress that is calculated by the model.
2. Observed Net Pressure: the pressure in the fracture minus the closure stress that is obtained by using one of the three possible measured pressures (bottomhole, surface, or dead string). This pressure is calculated only if a database is available.
3. Tophole Pressure: the pressure at the top of the wellbore, also called treating pressure.
4. Bottomhole Pressure: the calculated bottomhole pressure. This is calculated when it is not an input channel for the simulation.
5. Measured Bottomhole Pressure: the pressure in the wellbore at the fracture initiation depth that is derived from the pressure selected as input channel.
6. Hydrostatic Pressure: the hydrostatic pressure of the fluid and proppant in the wellbore.
7. Wellbore Friction: the calculated wellbore friction pressure. This calculation takes into account different fluids, different proppant concentrations, and different velocities in the wellbore.
8. Perf Friction: the calculated perforation friction pressure.
9. Near Wellbore Friction: the calculated near-wellbore friction.

In pressure matching, FRACPRO uses two specific pressure variables to compare: the observed net pressure and the net pressure. The fracture simulator predicts the net pressure, but there are three choices to calculate the observed net pressure based on the pressure measured at the well site in the database. The observed net pressure calculation options are outlined below:

1. Calculate Observed Net Pressure from Bottomhole Pressure

The recorded bottomhole pressure data is read directly into the simulator as measured bottomhole pressure. Perforation pressure, near wellbore friction, and closure stress in the pay zone are subtracted from the measured bottomhole pressure to give the observed net pressure.

$$P_{\text{observed net}} = P_{\text{bottomhole}} - P_{\text{perf}} - P_{\text{near wellbore}} - P_{\text{closure}} \quad \text{Equation 4-9}$$

1. Calculate Observed Net Pressure from Dead String Pressure

The pressure measured in the dead string is read into the simulator. The hydrostatic pressure in the dead string, plus the hydrostatic pressure from the end of the dead string down to the center of the fracture, is added to the measured dead string pressure. The wellbore friction present from the bottom of the dead string down to the perforations is subtracted to give the measured bottomhole pressure. The perforation friction, near

wellbore friction, and closure stress in the pay zone are subtracted from the measured bottomhole pressure to give the observed net pressure.

$$P_{\text{observed net}} = P_{\text{dead string}} + P_{\text{dead string hydrostatic}} - P_{\text{perf}} - P_{\text{near wellbore}} - P_{\text{closure}} \quad \text{Equation 4-10}$$

2. Calculate Observed Net Pressure from Surface Treating Pressure

The recorded surface treating pressure is read into the simulator directly as tophole pressure. The hydrostatic pressure is added and the wellbore friction is subtracted from the tophole pressure to give the measured bottomhole pressure. The perforation friction, near wellbore friction, and closure stress in the pay zone are subtracted from the measured bottomhole pressure to give the observed net pressure.

$$P_{\text{observed net}} = P_{\text{treating}} + P_{\text{hydrostatic}} - P_{\text{perf}} - P_{\text{near wellbore}} - P_{\text{closure}} \quad \text{Equation 4-11}$$

4.14.3. Pressure Matching Discussion

The pressure profile module is intended to reproduce the net pressure developed during fracturing as predicted by the fracture simulator. This pressure is the observed net pressure during the pressure matching process.

Figure 4-15 shows an example of the pressure matching process, where the jagged red line represents the observed net pressure while the smooth blue line represents the net pressure predicted by the fracture simulator. According to Halliburton engineers this example is considered to be a good match between the two pressures.

The net pressure profile predicted in Figure 4-15 was generated using the original lithology of the reservoir, consisting of a significant number of layers, and the designed treatment schedule. One of the problems with using complex lithologies with a high number of layers is the exponential increase of the number of inputs in designing the neural network. Generating complex lithologies to simulate them using FRACPRO becomes an impractical and time-consuming task.

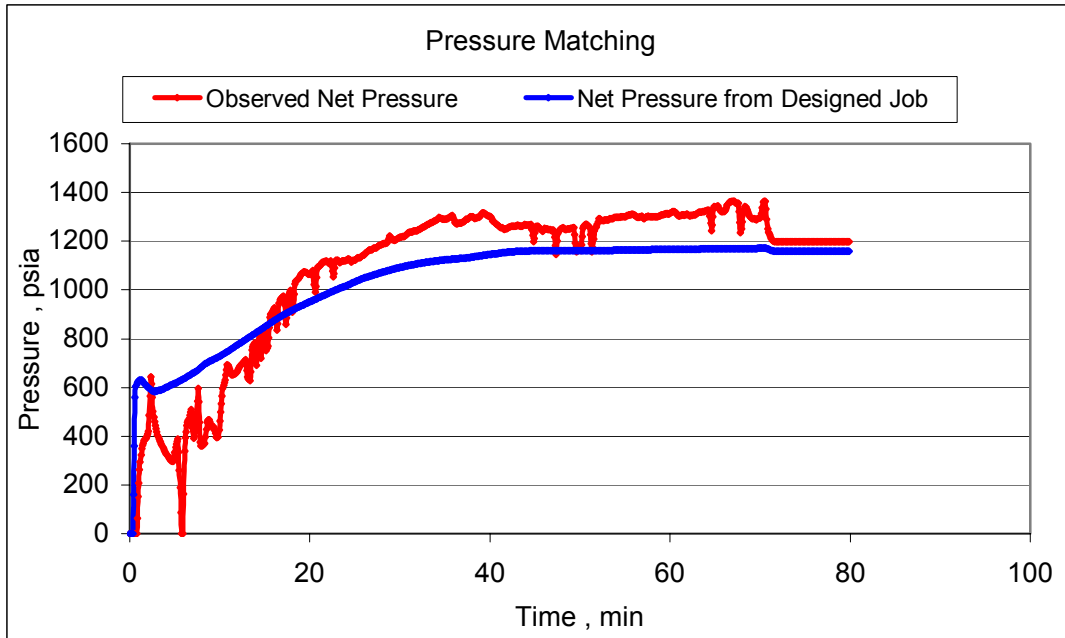


Figure 4-15: Pressure Matching Example between Observed Net Pressure and Predicted Net Pressure

In section 4.10 a new methodology was introduced in order to reduce the number of layers to a four-layer equivalent profile. This equivalent four-layer profile proved to be consistent in providing accurate fracture geometry results compared to the original profile when simulated with FRACPRO.

Since the net pressure profile data is needed to train the neural networks, a first attempt was made to use the four-layer representative profile in the process of data generation. However, before running hundreds of cases, the net pressure profile calculated using original reservoir lithology and the net pressure profile using the equivalent four layers were compared. In order to achieve this, ten random reservoir lithologies were generated, and the corresponding equivalent four-layer profile was calculated. Each case was run through FRACPRO using both scenarios: the entire lithology and the equivalent four-layer profile. The same ramp treatment design was used in all simulations.

The results posed a new problem. In all ten cases, the net pressure profile calculated using the equivalent four-layer algorithm was significantly lower than the net pressure profile calculated using original lithology. Three examples that clearly illustrate this behavior are shown below in Figure 4-16, Figure 4-17, and Figure 4-18. The blue diamond line represents the net pressure profile calculated from the original lithology, and the pink triangular line is the net pressure profile for the four-layer equivalent model.

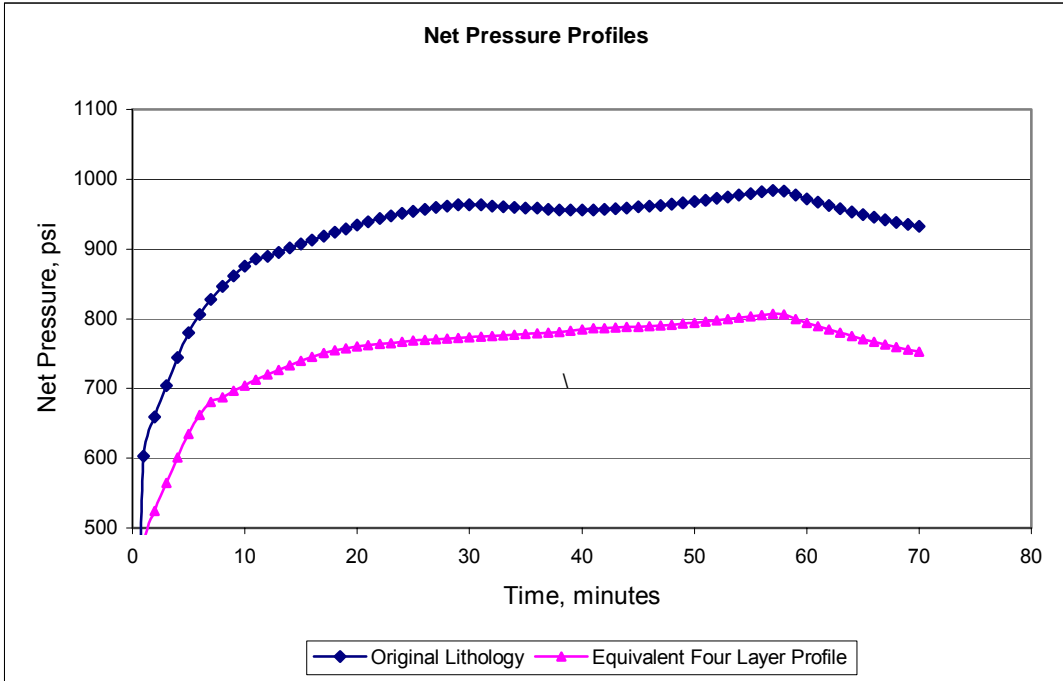


Figure 4-16: Net Pressures for Original Lithology and Equivalent Profile from Case 1

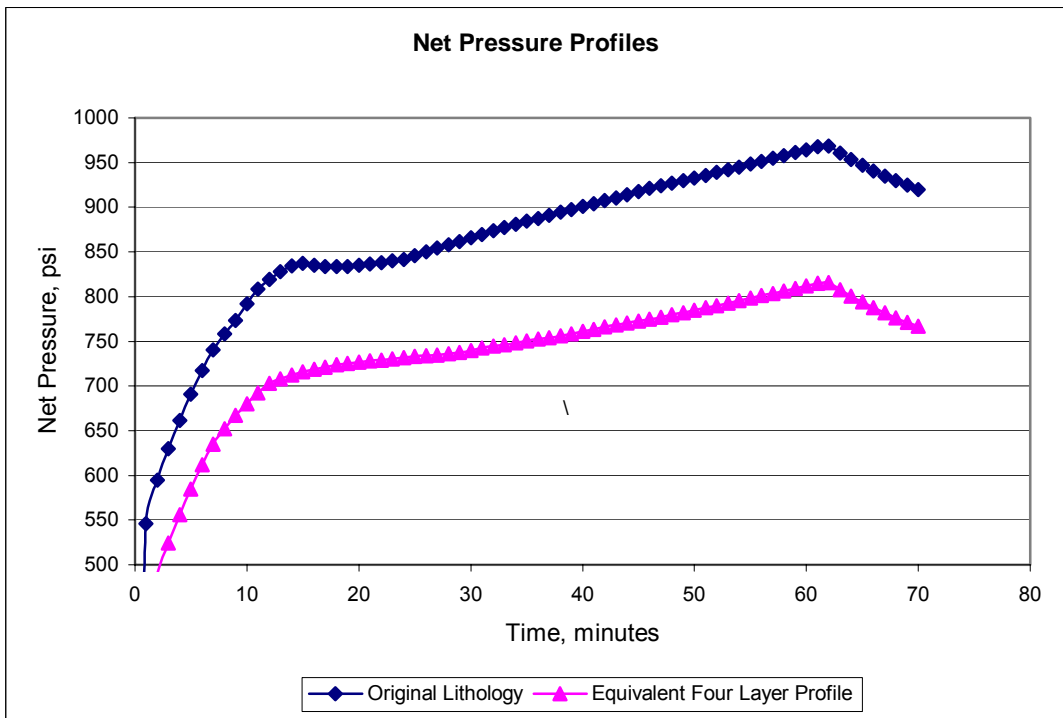


Figure 4-17: Net Pressures for Original Lithology and Equivalent Profile from Case 3

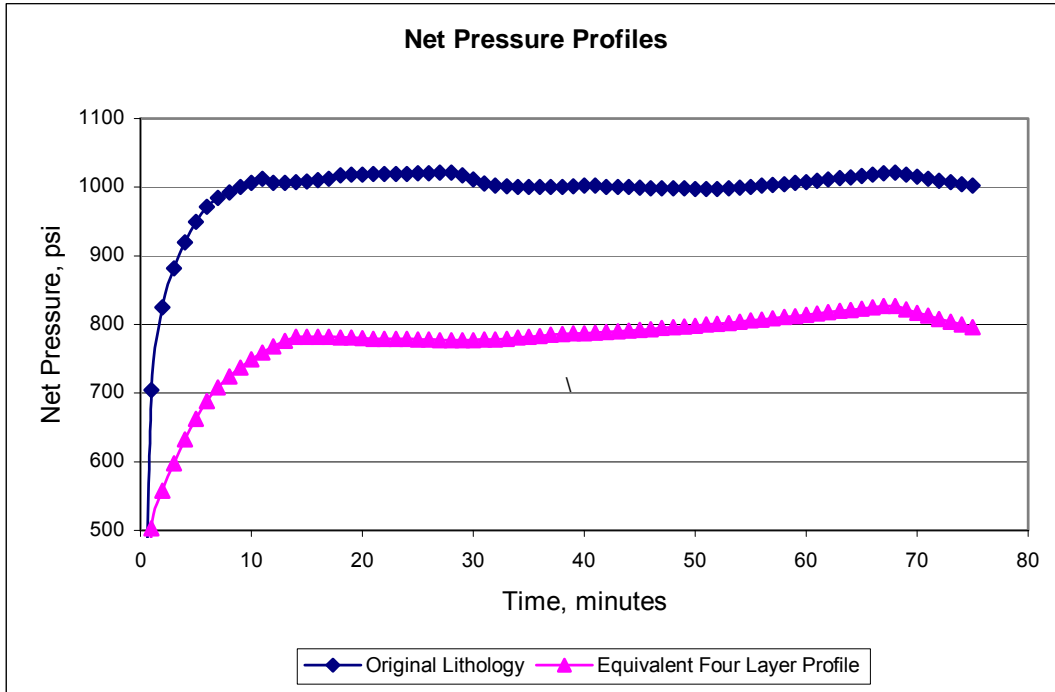


Figure 4-18: Net Pressures for Original Lithology and Equivalent Profile from Case 8

The difference between the original and equivalent profiles was more severe in some cases than others. A credible reason for the net pressure profile to exhibit much lower pressure values for the entire treatment may be related to the stress contrast between the four layer equivalent profiles. This difference and inconsistency was not acceptable for modeling, so a new approach was needed. The goal of this new model was to determine a similar representative stress profile able to provide the same pressure profile as the original lithology stress profile.

4.14.3.1. Closure Stress Coefficient Alteration

The first approach in finding a new representative four-layer stress profile was to alter the closure stress gradients of the four layers in a controlled way. This was done such that the new pressure profile calculated from FRACPRO would provide a closer match with the pressure profile generated by the original stress profile. Only four parameters out of the total eight for the representative four-layer stress model were considered for alteration, namely the closure stress gradients for each of the four layers.

1. Top layer closure stress coefficient
2. Limiting layer closure stress coefficient
3. Pay zone closure stress coefficient
4. Bottom layer closure stress coefficient

Due to the relatively high number of parameters that can be modified from one run to another the study was conducted in two scenarios. The first scenario considered alterations of the closure stress gradients for top layer, overburden layer and bottom layer. The second scenario altered the closure stress coefficient only for the pay zone layer maintaining the same equivalent permeability.

4.14.3.1.1. Top, Overburden, and Bottom Layers

The first scenario (altering the closure stress gradients for top, overburden, and bottom layers) was completed in two sub-steps:

- A. Closure stress gradients for the top layer, limiting layer and bottom layer are increased with a percentage between 2-5%.
- B. Closure stress gradients for only the top layer and bottom layer are increased with a percentage between 2-5%.

The same ten cases analyzed previously were used in this analysis. FRACPRO was run using the original stress profile and the net pressure profile was recorded. Next, the simulator was run using the altered representative four-layer stress profile. The net pressure profile was again recorded and then compared with the one generated by the original stress profile. The representative four-layer stress profile was then altered four times for scenario A and four times for scenario B. This generated eight additional net pressure profiles when run through FRACPRO. The stress data corresponding to the altered four-layer equivalent profiles for scenario A and scenario B are shown in

Table 4-21 and Table 4-22 respectively.

Layer Name	Equivalent Profile	Equivalent TOB 2%	Equivalent TOB 3%	Equivalent TOB 4%	Equivalent TOB 5%
		2%	3%	4%	5%
Top Layer	0.729	0.751	0.758	0.765	0.773
Overburden Layer	0.791	0.815	0.823	0.831	0.611
Pay Zone	0.646	0.665	0.672	0.678	0.395
Bottom Layer	0.715	0.736	0.744	0.751	0.282

Table 4-21: Closure Stress Gradient Values for Equivalent Altered Profiles from Scenario A Case 1

Layer Name	Equivalent Profile	Equivalent TB 2%	Equivalent TB 3%	Equivalent TB 4%	Equivalent TB 5%
		2%	3%	4%	5%
Top Layer	0.729	0.751	0.758	0.765	0.773
Overburden Layer	0.791	0.791	0.791	0.791	0.791
Pay Zone	0.646	0.646	0.646	0.646	0.646
Bottom Layer	0.715	0.736	0.744	0.751	0.758

Table 4-22: Closure Stress Gradient Values for Equivalent Altered Profiles from Scenario B Case 1

Figure 4-19 shows eight net pressure profiles, the original stress profile, the equivalent four-layer stress profile, and six altered equivalent four-layer profiles (three from scenario A and three from scenario B). The following descriptions explain the details for each stress profile:

- ▶ Original Lithology: represents the pressure profile for the original stress profile.
- ▶ Equivalent Profile: represents the pressure profile corresponding to the representative four layers as defined in section 4.10.
- ▶ Equivalent TOB 3%: represents the case when top layer, overburden layer and bottom layer closure stress coefficients are increased by 3%.
- ▶ Equivalent TOB 4%: represents the case when top layer, overburden layer and bottom layer closure stress coefficients are increased by 4%.
- ▶ Equivalent TOB 5%: represents the case when top layer, overburden layer and bottom layer closure stress coefficients are increased by 5%.
- ▶ Equivalent TB 3%: represents the case when only top layer and bottom layer closure stress coefficients are increased by 3%.
- ▶ Equivalent TB 4%: represents the case when only top layer and bottom layer closure stress coefficients are increased by 3%.
- ▶ Equivalent TB 5%: represents the case when only Top layer and Bottom layer closure stress coefficients are increased by 3%.

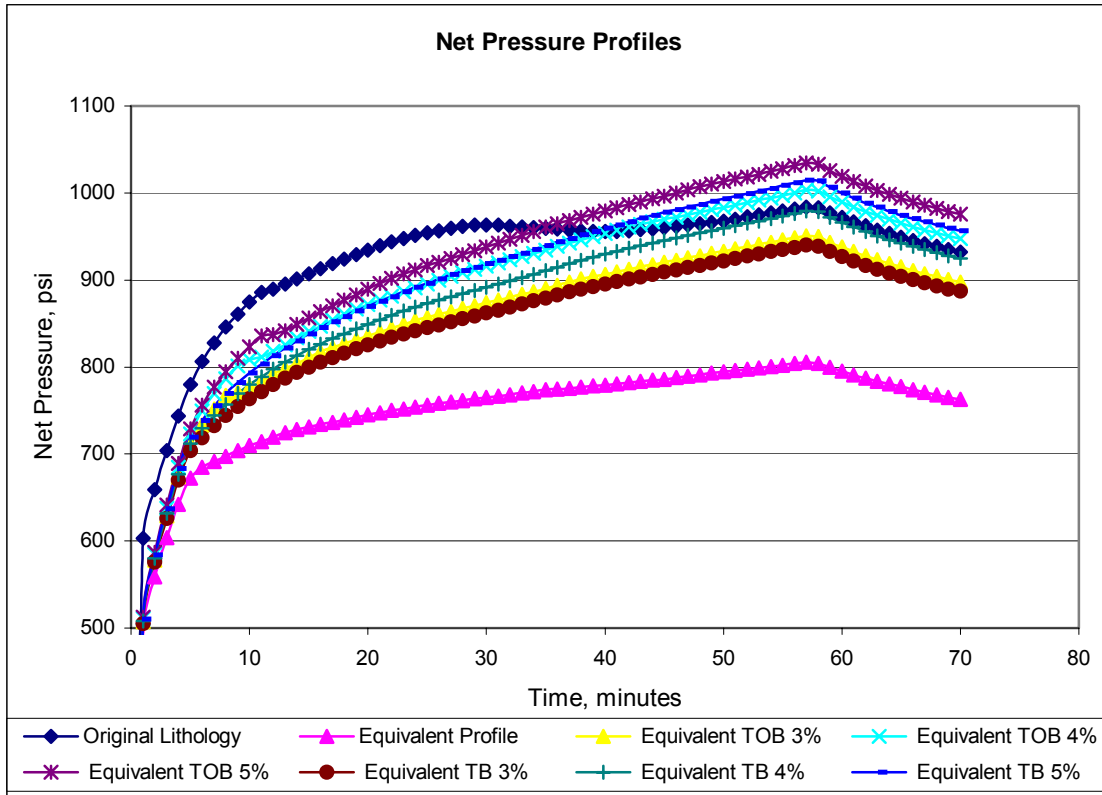


Figure 4-19: Net Pressure Profile Comparisons for Equivalent Altered Profiles from Case 1

The net pressure profiles for Equivalent TOB 2% and Equivalent TB 2% are not displayed on the figure since their results were very close to the equivalent four-layer profile, which clutters the graph and makes it difficult to see.

Another example is Case 3 presented below. The stress values corresponding to the altered four-layer equivalent profiles for scenario A and scenario B are shown in

Layer Name	Equivalent Profile	Equivalent TOB 2%	Equivalent TOB 3%	Equivalent TOB 4%	Equivalent TOB 5%
		2%	3%	4%	5%
Top Layer	0.750	0.765	0.773	0.780	0.788
Overburden Layer	0.809	0.825	0.833	0.841	0.849
Pay Zone	0.665	0.665	0.665	0.665	0.665
Bottom Layer	0.730	0.745	0.752	0.759	0.767

Table 4-23 and

Table 4-24.

Layer Name	Equivalent Profile	Equivalent TOB 2%	Equivalent TOB 3%	Equivalent TOB 4%	Equivalent TOB 5%
		2%	3%	4%	5%
Top Layer	0.750	0.765	0.773	0.780	0.788
Overburden Layer	0.809	0.825	0.833	0.841	0.849
Pay Zone	0.665	0.665	0.665	0.665	0.665
Bottom Layer	0.730	0.745	0.752	0.759	0.767

Table 4-23: Closure Stress Gradient Values for Equivalent Altered Profiles from Scenario A Case 3

Layer Name	Equivalent Profile	Equivalent TB 2%	Equivalent TB 3%	Equivalent TB 4%	Equivalent TB 5%
		2%	3%	4%	5%
Top Layer	0.75	0.765	0.7725	0.78	0.7875
Overburden Layer	0.809	0.809	0.809	0.809	0.809
Pay Zone	0.665	0.665	0.665	0.665	0.665
Bottom Layer	0.73	0.7446	0.7519	0.7592	0.7665

Table 4-24: Closure Stress Gradient Values for Equivalent Altered Profiles from Scenario B Case 3

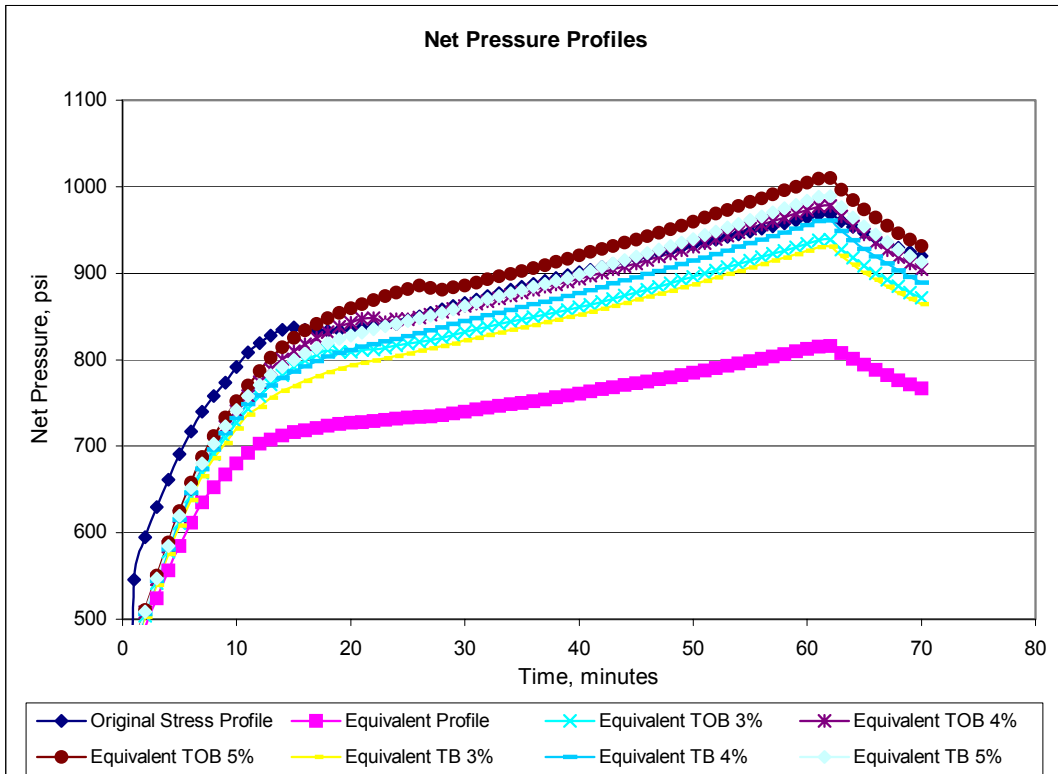


Figure 4-20: Net Pressure Profile Comparisons for Equivalent Altered Profiles from Case 3

This analysis confirmed that increasing the stress contrast between the non-productive layers and the pay zone layer leads to an increase in the net pressure profile values. Analyzing the shapes of the ten cases showed that some of the altered four-layer profiles almost overlap with original net pressure profile, while others come close in pressure values but not in shape. Case 3 (Figure 4-20) is an example where the net pressure profile generated using Equivalent TOB 4% provided a similar match to the net pressure profile calculated from original lithology. Furthermore, Case 1 (Figure 4-19) shows an example where pressure values are close using Equivalent TOB 5% or TOB 4% but they are not able to reproduce the shape of the pressure profile. The most acceptable pressure match is obtained when the closure stress coefficients of the three non-producing layers of the profile were increased by around 4%.

4.14.3.1.2. Pay Zone Layer

The second scenario altered the closure stress coefficient only for the pay zone layer while maintaining the same equivalent permeability. The closure stress gradient for the pay zone layer was decreased by a percentage of 3 – 7% psi/ft. Table 4-25 shows an example of this approach, which was performed each of the previous ten cases in order to provide consistent results.

Table 4-25 shows the altered values of the pay zone closure stress gradient, while Figure 4-21 shows the net pressure profile from Case 1.

- ▶ Original Lithology: represents the pressure profile for the original stress profile.
- ▶ Equivalent Profile: represents the pressure profile corresponding to the equivalent four-layer profile.
- ▶ Equivalent PZ 3%: represents the case when closure stress gradient of the pay zone layer, is decreased by 3%.
- ▶ Equivalent PZ 4.5%: represents the case when closure stress gradient of the pay zone layer, is decreased 4.5%.
- ▶ Equivalent PZ 6%: represents the case when closure stress gradient of the pay zone layer, is decreased by 6%.

Layer Name	Equivalent Profile	Equivalent PZ-3%	Equivalent PZ-4.5%	Equivalent PZ-6%
Top Layer	0.729	0.729	0.729	0.729
Overburden Layer	0.791	0.791	0.791	0.791
Pay Zone	0.646	0.626	0.616	0.607
Bottom Layer	0.715	0.715	0.715	0.715

Table 4-25: Closure Stress Gradient Values for Pay Zone Equivalent Altered Profiles from Case 1

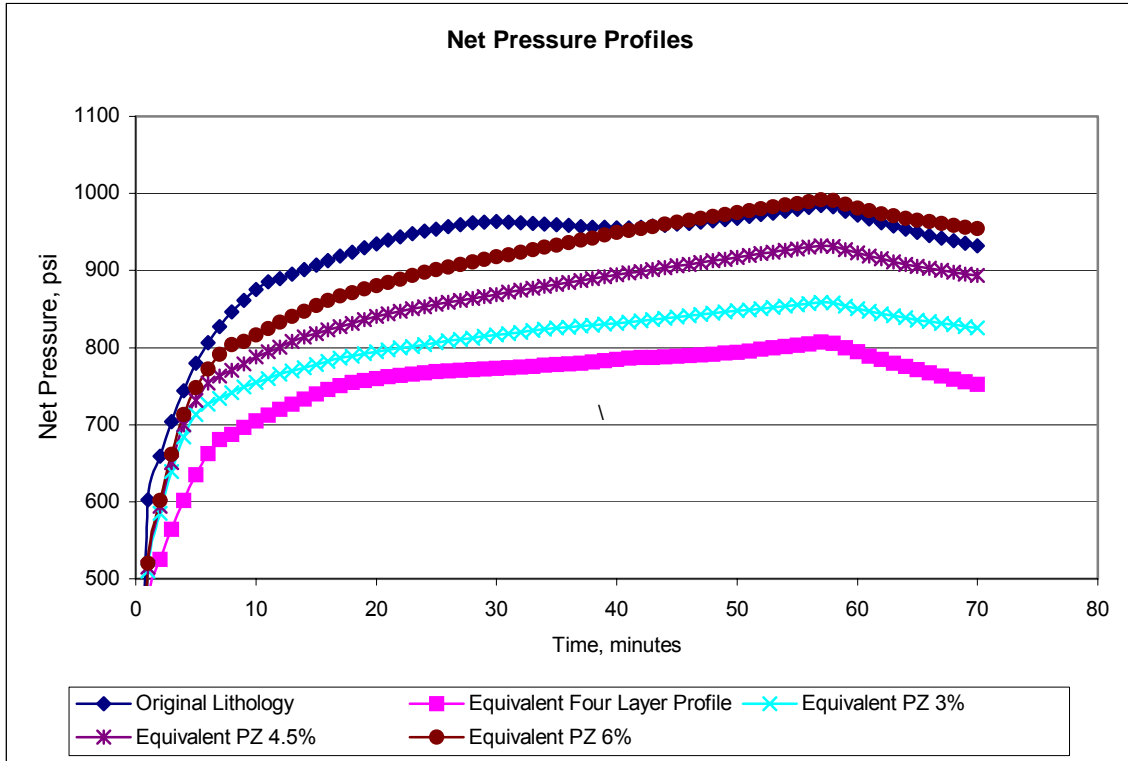


Figure 4-21: Net Pressure Profile Comparisons for Pay Zone Equivalent Altered Profiles from Case 1

Table 4-26 and Figure 4-22 display the analysis from Case 3.

Layer Name	Equivalent Profile	Equivalent PZ-3%	Equivalent PZ-4.5%	Equivalent PZ-6%
Top Layer	0.750	0.765	0.773	0.780
Overburden Layer	0.809	0.825	0.833	0.841
Pay Zone	0.665	0.645	0.635	0.625
Bottom Layer	0.730	0.745	0.752	0.759

Table 4-26: Closure Stress Gradient Values for Pay Zone Equivalent Altered Profiles from Case 1

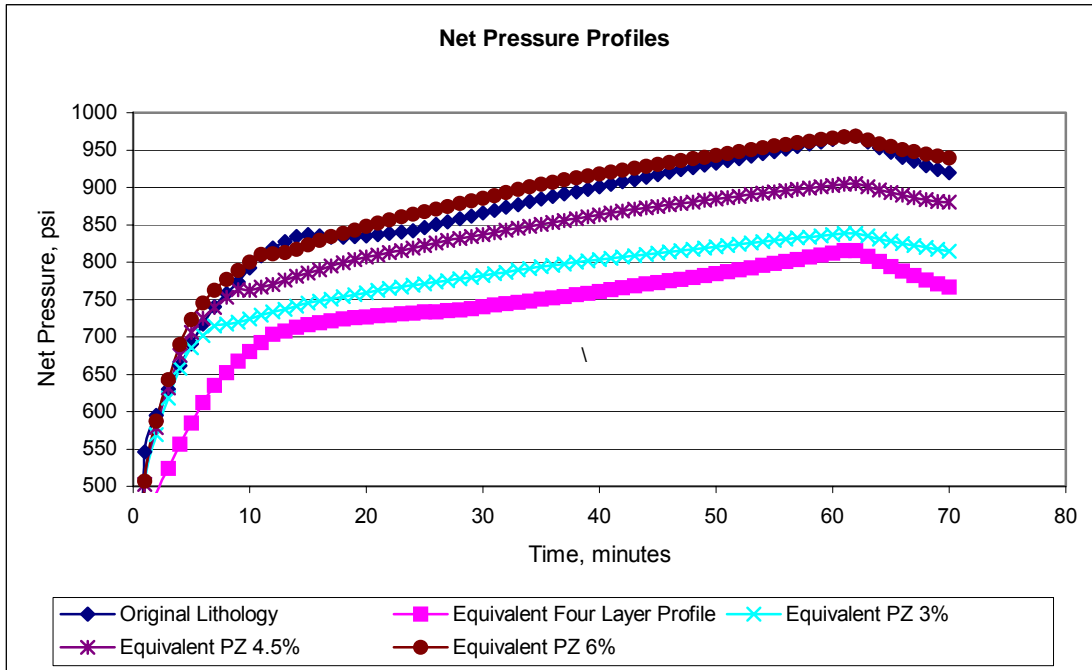


Figure 4-22: Net Pressure Profile Comparisons for Pay Zone Equivalent Altered Profiles from Case 3

Again, much like the altered profiles from top, overburden, and bottom layers previously, altering the closure stress gradient of the pay zone produces a stronger contrast with the non-productive zones. This leads to increase in the net pressure profile values. In this case, the shapes of the ten cases show that some of the altered four-layer profiles provide a much better overlap, both in values and shapes, with the net pressure profile calculated from the original stress profile. For example, the net pressure profile generated using an Altered Equivalent PZ 6% provided an excellent match with the original net pressure profile in Case 1 (Figure 4-21) and Case 3 (Figure 4-22). Similar matches were observed for the remaining cases. The most acceptable pressure matches were obtained when the closure stress gradient of the pay zone was decreased by a percentage between 4.5 and 6%.

4.14.3.1.3. Closure Stress Coefficient Alteration Summary

The closure stress coefficient alteration process was conducted in two parts until the best pressure match was reached:

- ▶ Top, Overburden, and Bottom Layer

This part consisted of two scenarios, the first where the top, overburden, and bottom layers are altered and the second where only the top and bottom layers were altered. Based on the analysis of these two scenarios, it was noted that the increase of the

closure stress coefficient for the overburden layer does not significantly influence the pressure profile.

► Pay Zone Layer

The closure stress coefficient was decreased by a percentage of 3 – 7% psi/ft from the equivalent four-layer stress profile. The alteration of the representative layer profile was done empirically based on the rock mechanic and hydraulic fracture behavior.

In conclusion, the alteration of the pay zone closure stress gradient was preferred between the two scenarios presented above. This is for two reasons: first, it provided a better pressure match, both in pressure values and shape; and secondly because of the minimum number of parameters required to be altered (only the closure stress gradient of the pay zone). However, it was still unable to be determined what the critical percentage of the pay zone closure stress gradient should be decreased in order to provide the best net pressure profile match.

4.14.3.2. Correction Percentage Value Determination

The ten previously analyzed cases were not considered sufficient to define a set percentage value that, when applied to the pay zone closure stress gradient, would provide an acceptable match of the net pressure profiles. Further analysis proceeded of which the steps involved are as follows:

1. Generate 50 random cases of reservoir lithology profile where each case contains the following components:
 - a. Random number of layers above the first permeable productive formation called top layers.
 - b. Each top layer has a random thickness and a random closures stress gradient.
 - c. Random number of productive permeable formations called pay zones.
 - d. Each productive formation has a random thickness and a random closure stress gradient.
 - e. Random number of thin intercalations between the pay zones.
 - f. Random number of layers below the deepest permeable productive formation called bottom layers.
 - g. Each bottom layer has a random thickness and a random closures stress gradient.

This data was generated using a simple computer program. Logic and certain constraints were embedded in the code so that unrealistic cases could not be generated.

2. Apply the equivalent four-layer profile methodology to each of the fifty random cases and calculate the eight parameters defining the four-layer profile.
3. Select first randomly generated case. Design a unique treatment schedule for this case. Run FRACPRO using the reservoir lithology profile, and record the net pressure profile. This net pressure profile was named the original net pressure profile.

4. Using the same treatment schedule, run FRACPRO using the equivalent four-layer stress profile. Record the net pressure profile. This net pressure profile was named the equivalent net pressure profile.
5. Alter the pay zone closure stress gradient of the equivalent four-layer profile using percentage values between 2 and 8%. Using the same treatment design, run FRACPRO for each altered equivalent four-layer profile. Record net pressures for each run. Typically, start with a value of 3%. Calculate the net pressure profile and plot it on the same graph with the original and equivalent net pressure profiles.
6. Increase the percentage value until a good match is achieved. Record each percentage used, the altered equivalent four-layer profile and the calculated net pressure profile.
7. Repeat the process from step 3.

An example of a generated reservoir lithology is seen in Table 4-27, while Table 4-28 shows the processed equivalent four-layer profile for the same case. Figure 4-23 shows both profiles, original and equivalent, together with two altered equivalent profiles.

Layer Index	Top Depth, ft	Thickness, ft	Closure Stress Gradient, psi/ft	Permeability, md
Layer 1	7080	34	0.751	0
Layer 2	7114	22	0.772	0
Layer 3	7136	12	0.731	0
Layer 4	7148	17	0.756	0
Layer 5	7165	12	0.755	0
Layer 6	7177	27	0.787	0
Layer 7	7204	33	0.738	0
Layer 8	7237	8	0.706	0
Layer 9	7245	11	0.737	0
Layer 10	7256	12	0.787	0
Layer 11	7268	29	0.8	0
Layer 12	7297	18	0.712	0
Pay Zone1	7315	12	0.664	0.006
Pay Zone2	7327	6	0.664	0.006
Layer 13	7333	10	0.719	0
Pay Zone 3	7343	22	0.664	0.006
Pay Zone 4	7365	13	0.687	0.057
Pay Zone 5	7378	19	0.664	0.006
Layer 14	7397	10	0.729	0
Pay Zone 6	7407	9	0.687	0.057
Layer 14	7416	84	0.722	0
Layer 15	7500	-	0.722	0

Table 4-27: Generated Reservoir Lithology Case

Layer Index	Top Depth, ft	Thickness, ft	Closure Stress Gradient, psi/ft	Permeability, md
Top Layer	7080	206	0.758	
Overburden Layer	7286	29	0.8	
Pay Zone	7315	101	0.6813	0.015
Bottom Layer	7416		0.722	

Table 4-28: Processed Equivalent Four-Layer Profile

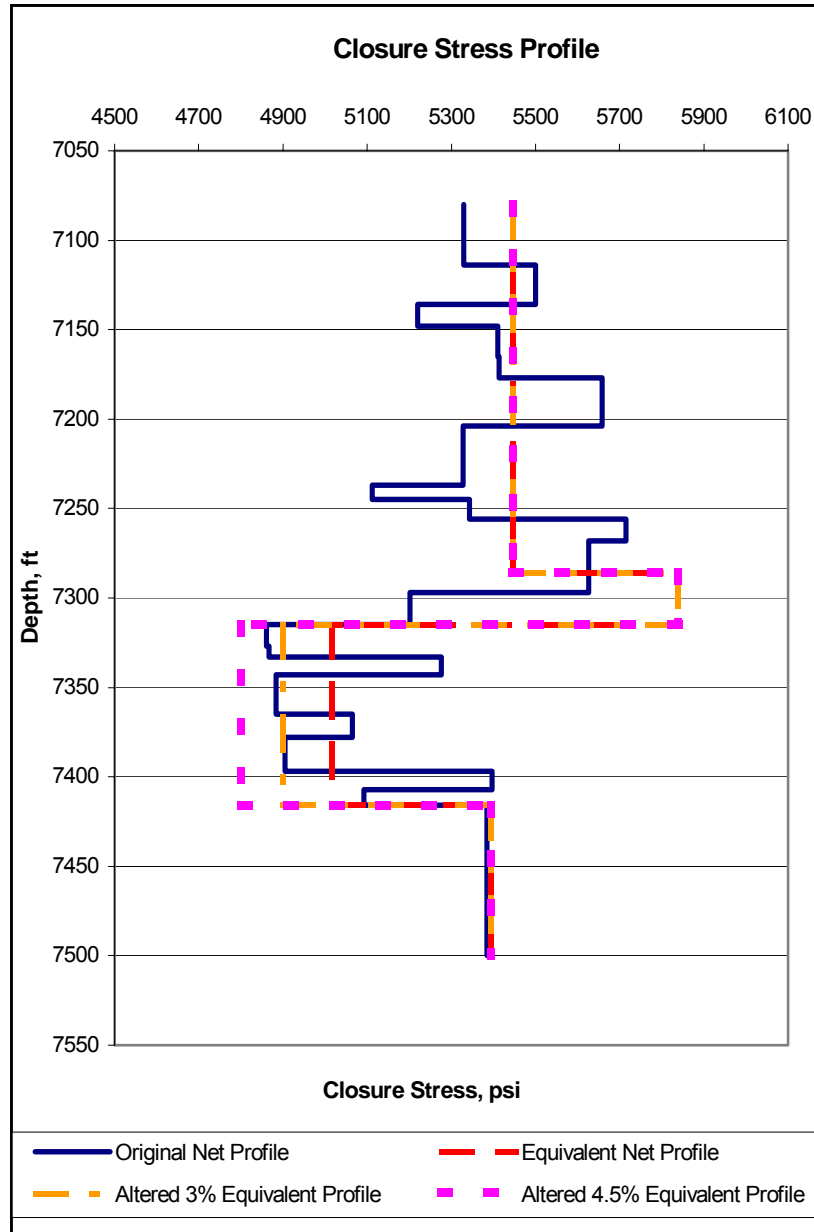


Figure 4-23: Original, Equivalent, and Altered Equivalent Stress Profiles

The process described above was applied to all fifty generated cases. Several cases of the matching process follow. The remaining examples exhibited similar patterns, the only difference being the value of the net pressure profiles.

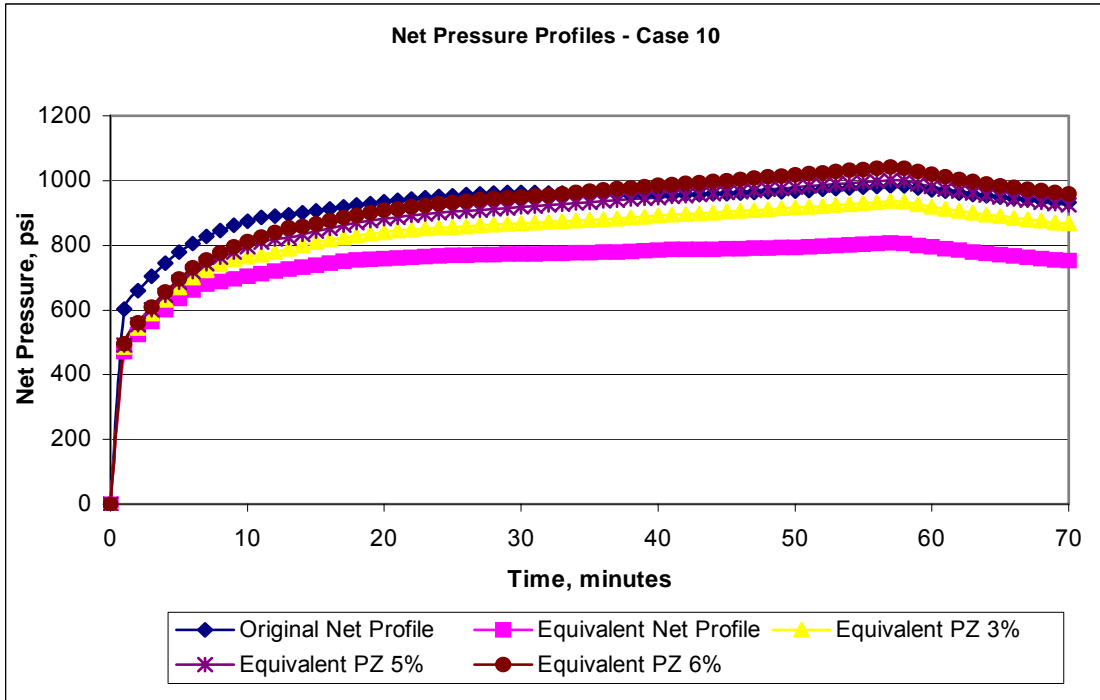


Figure 4-24: Net Pressure Profile Matching Process for Case 10

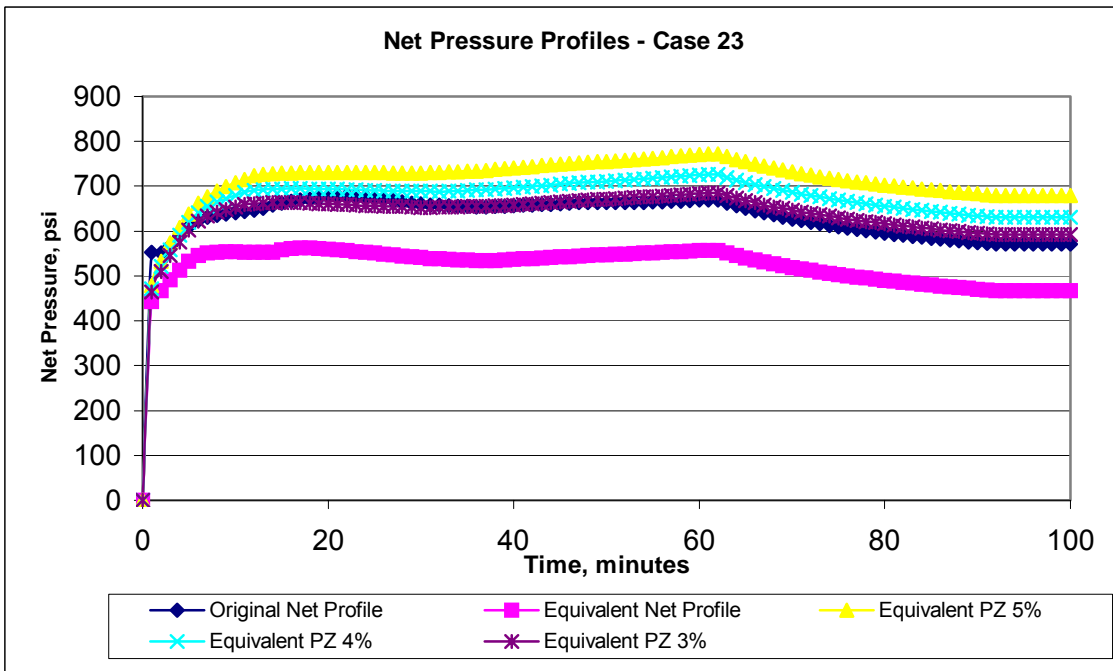


Figure 4-25: Net Pressure Profile Matching Process for Case 23

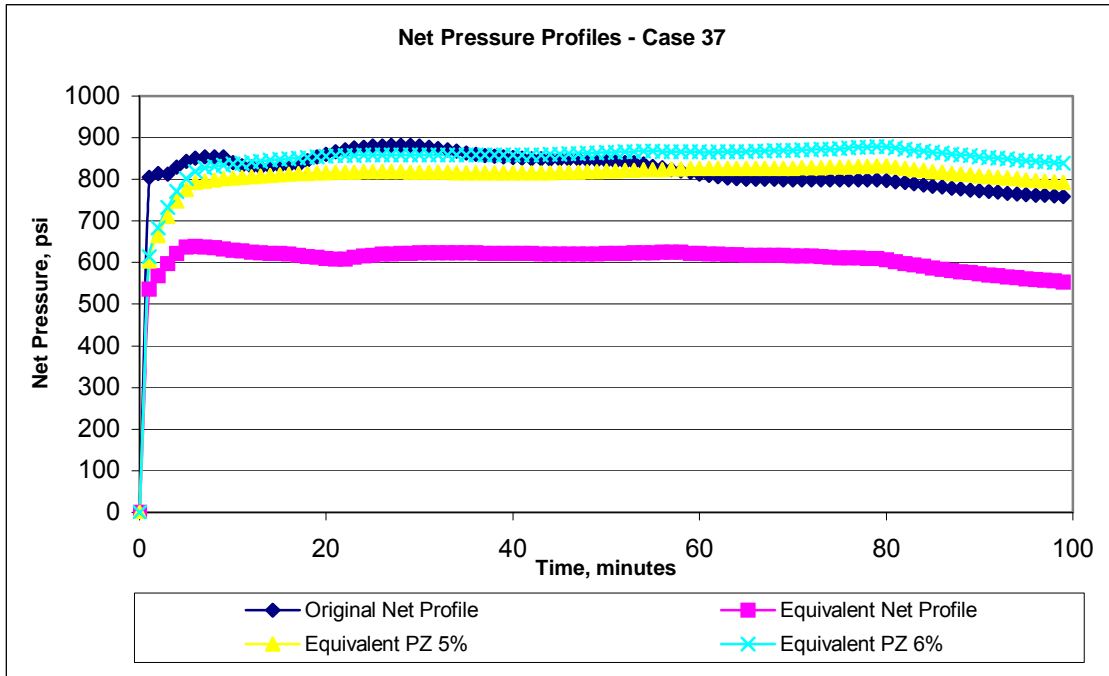


Figure 4-26: Net Pressure Profile Matching Process for Case 37

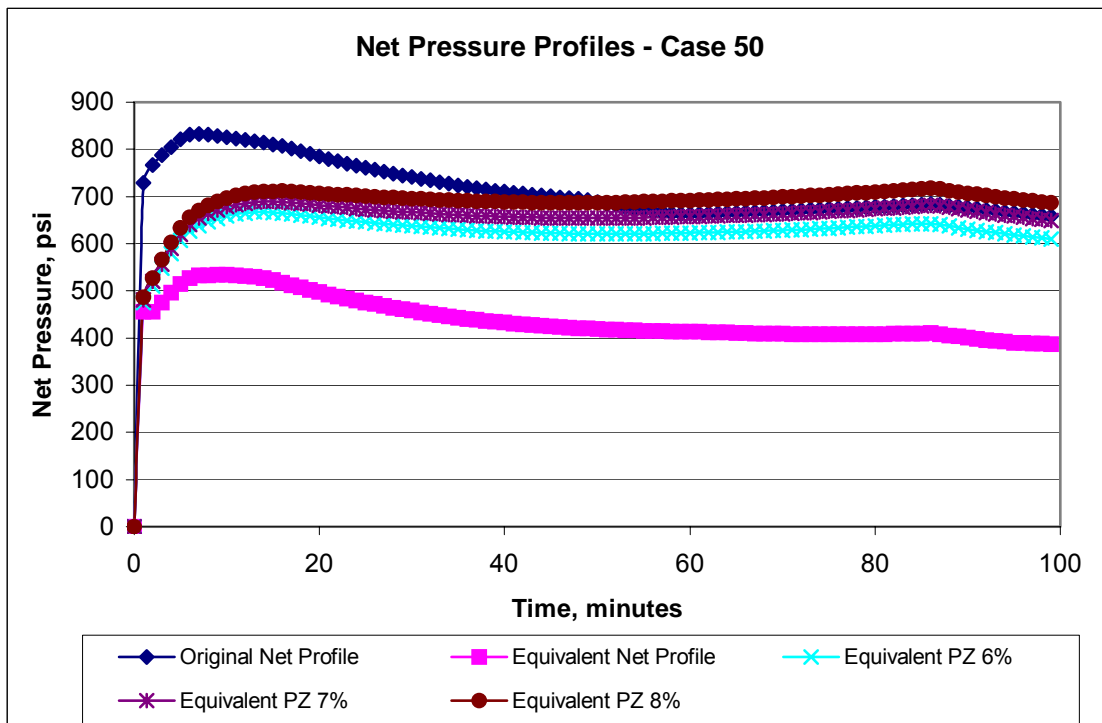


Figure 4-27: Net Pressure Profile Matching Process for Case 50

Due to the fact that the process of entering data and preparing each simulation is very time consuming, only fifty cases were generated and analyzed. An in-depth visual analysis of all fifty cases allowed better interpretation of each case and matching. It should be noted that some correction percentage coefficients were slightly modified to allow for better matching. Still yet, in some of the cases the matching is not perfect and some pressure domains still do not overlap.

The four examples randomly selected and previously presented show that the correction percentage coefficient appears to have sporadic values. For example, the correction percentage coefficient in Case 10 (Figure 4-24) is 6%, Case 23 (Figure 4-25) is 3%, Case 37 (Figure 4-26) is 5%, and Case 50 (Figure 4-27) is around 8%. Table 4-29 shows the correction percentage coefficient for all fifty cases.

Case Index	Correction percentage value, %		Case Index	Correction percentage value, %
Case 1	6		Case 26	9
Case 2	9		Case 27	3
Case 3	4		Case 28	9
Case 4	4.5		Case 29	5
Case 5	4.5		Case 30	1.5
Case 6	2.5		Case 31	0
Case 7	3.5		Case 32	1
Case 8	3		Case 33	2.5
Case 9	2.5		Case 34	3.5
Case 10	6.5		Case 35	6
Case 11	3.5		Case 36	1
Case 12	9.5		Case 37	5.5
Case 13	9.5		Case 38	3
Case 14	9.5		Case 39	3.5
Case 15	9.5		Case 40	6
Case 16	5.5		Case 41	7
Case 17	6.5		Case 42	4
Case 18	2		Case 43	1.5
Case 19	10		Case 44	5.5
Case 20	10		Case 45	3
Case 21	10		Case 46	9
Case 22	3		Case 47	4
Case 23	3		Case 48	2.5
Case 24	8		Case 49	4.5
Case 25	5.5		Case 50	8

Table 4-29: Correction Percentage Coefficients for 50 Cases

The frequency distribution of the correction percentage coefficient was plotted. Figure 4-28 shows that the distribution is fairly uniform and no clear answer was concluded.

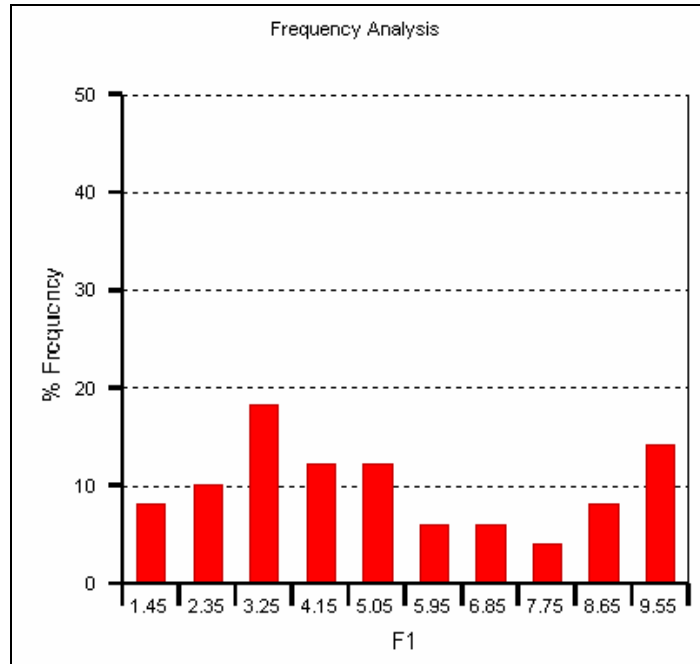


Figure 4-28: Frequency Distribution Analysis for Percentage Coefficient

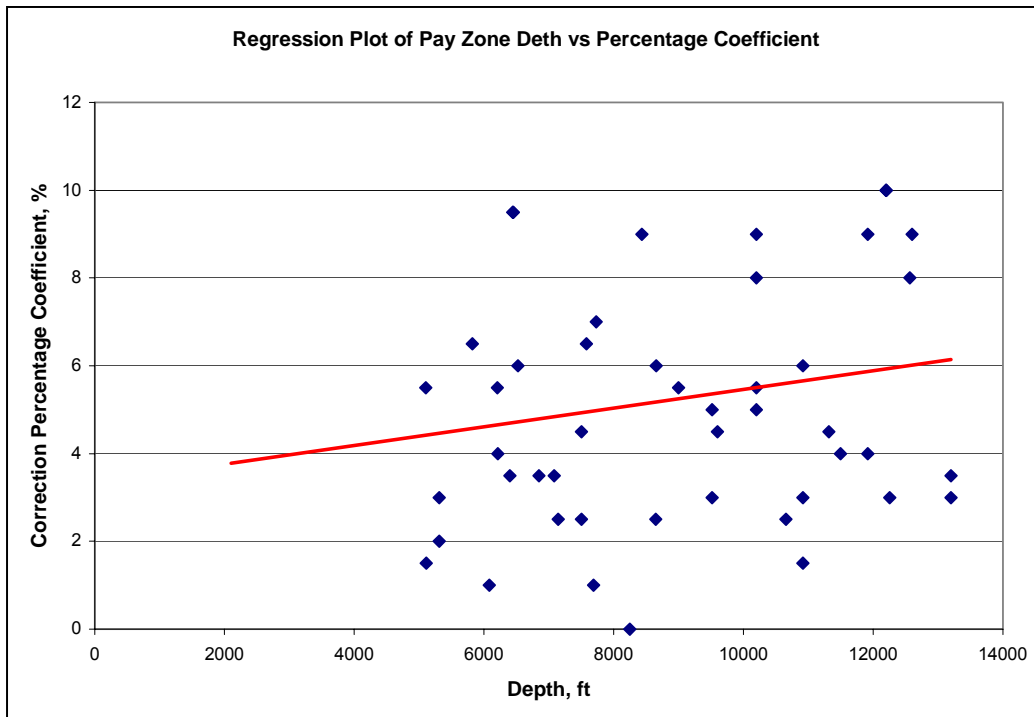


Figure 4-29: Regression Plot of Pay Zone Depth vs. Correction Percentage Coefficient

Since, the stress profile is directly proportional with the depth, a regression was run comparing the depth of the pay zone with the reservoir lithology versus the correction percentage coefficient. Figure 4-29 clearly shows that the percentage coefficient does not depend on the reservoir depth. Since no default value or significant relationship was found to help estimate the percentage coefficient, a new process was initiated to examine the data.

4.14.3.3. Equivalent Shift Delta Pressure Profile

Careful analysis of the shapes of the pressure profiles indicated that the equivalent net pressure profile shape was much closer to the shape of the original net pressure profile than most of the altered equivalent pressure profiles. In a few cases, the entire equivalent net pressure profile was shifted up, by adding a constant pressure value, until a very close match with the original net profile was achieved. This constant pressure value was called the delta pressure shift (D_p), and the new matching profile was called equivalent shift D_p profile. Each case was processed separately. The delta pressure shift was determined for each of the fifty cases such that the new equivalent D_p shift profile would match as closely as possible the original net pressure profile. Figure 4-30, Figure 4-31, Figure 4-32, and Figure 4-33 show the same cases presented above, with the difference that the matching net pressure is the equivalent D_p shift. It is clearly visible that equivalent D_p shift represents a much better match to the original net pressure. However, a different delta pressure shift was determined for each case in order to achieve this match. Table 4-30 summarizes the D_p shift for each of the fifty cases.

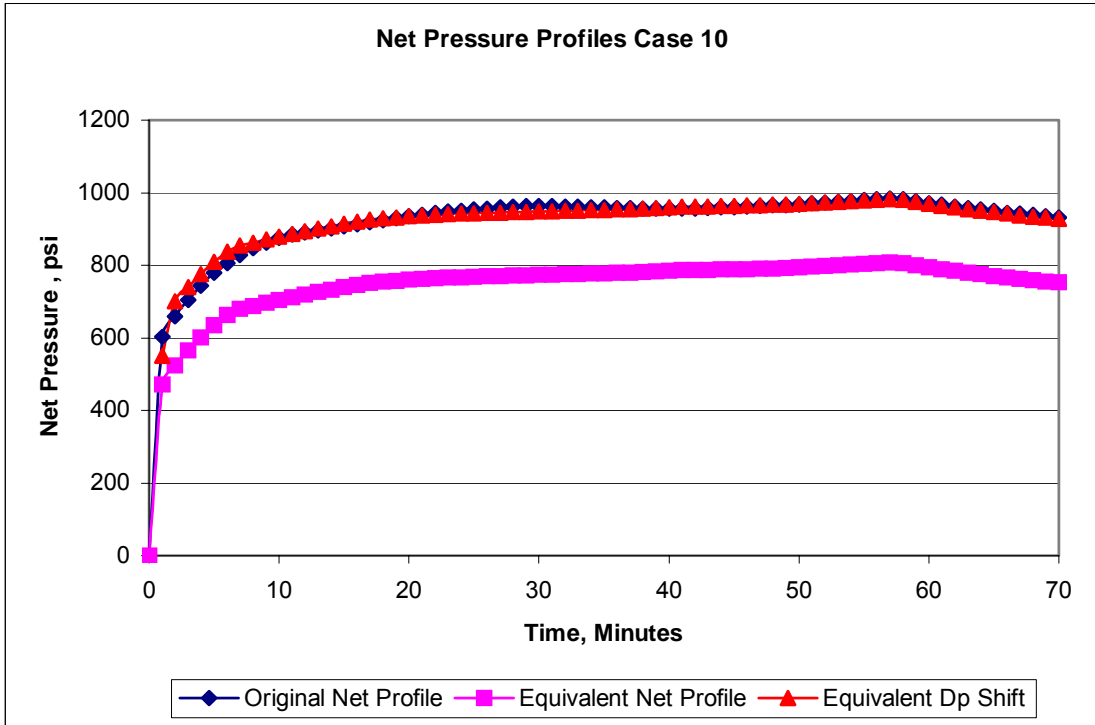


Figure 4-30: Equivalent Dp Shift Matching Process for Case 10

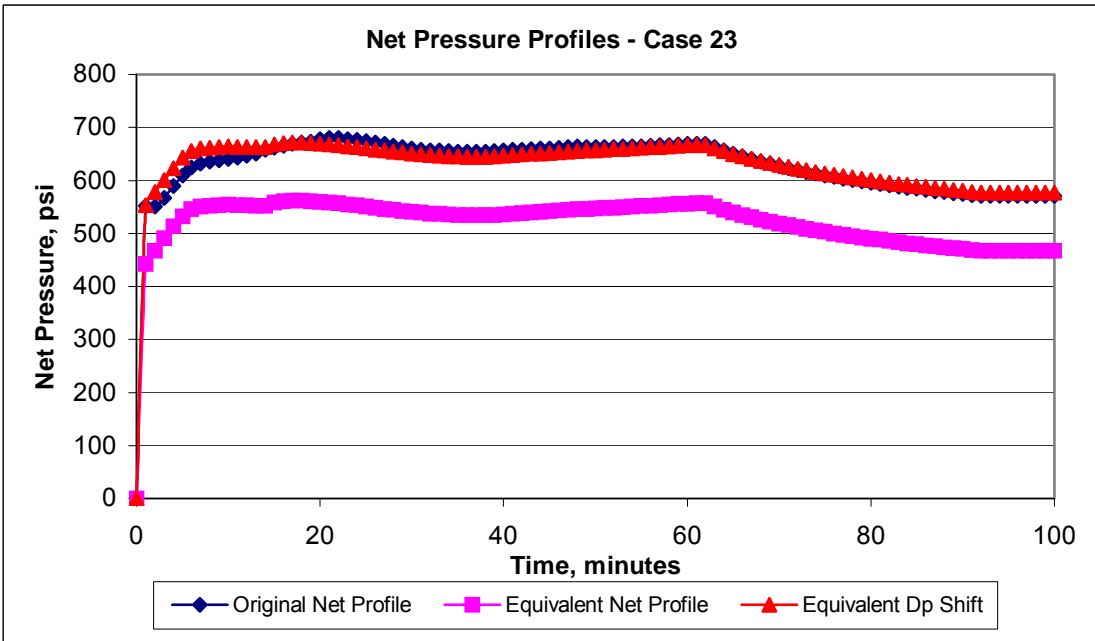


Figure 4-31: Equivalent Dp Shift Matching Process for Case 23

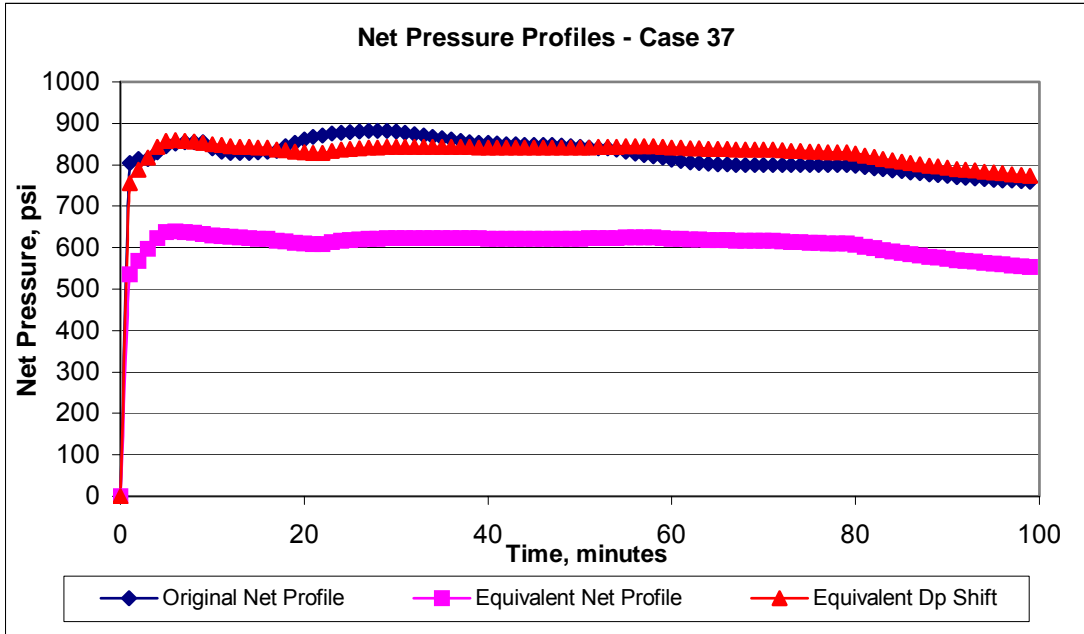


Figure 4-32: Equivalent Dp Shift Matching Process for Case 37

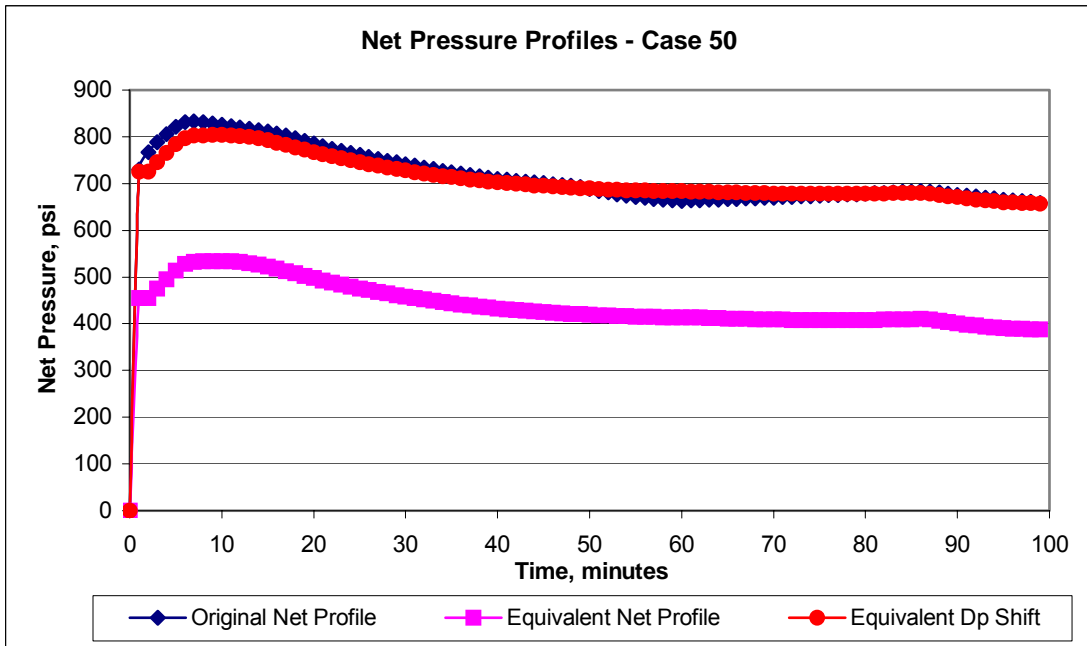


Figure 4-33: Equivalent Dp Shift Matching Process for Case 50

Case Index	Delta Pressure Shift, psi		Case Index	Delta Pressure Shift, psi
Case 1	175		Case 26	230
Case 2	300		Case 27	50
Case 3	125		Case 28	230
Case 4	220		Case 29	200
Case 5	130		Case 30	40
Case 6	40		Case 31	0
Case 7	110		Case 32	40
Case 8	110		Case 33	80
Case 9	55		Case 34	90
Case 10	160		Case 35	150
Case 11	95		Case 36	20
Case 12	260		Case 37	220
Case 13	260		Case 38	240
Case 14	260		Case 39	175
Case 15	260		Case 40	180
Case 16	120		Case 41	270
Case 17	190		Case 42	120
Case 18	50		Case 43	40
Case 19	235		Case 44	120
Case 20	235		Case 45	140
Case 21	235		Case 46	195
Case 22	80		Case 47	140
Case 23	145		Case 48	280
Case 24	160		Case 49	240
Case 25	200		Case 50	260

Table 4-30: Dp Shift for 50 Cases

The delta pressure shift has minimum value of 0 (meaning that the equivalent net pressure profile matches the original net pressure profile) and a maximum of 300 psi. It has been proven earlier that there is no direct correlation between the percentage coefficient and any other parameter. The same can be said regarding the delta pressure shift. It is still undetermined as to the exact delta pressure shift required in each case to acceptably and accurately match the original pressure profile.

4.14.4. Calibration Neural Network

Neural networks have a superior capability of extracting hidden information from complex non-linear data. They are able to learn from patterns in data and predict outcomes when presented with new data. It was believed that a complex relationship or correlation exists

between the reservoir data (depth, stress, and thickness) and the delta pressure shift; therefore, neural networks were employed to solve this problem.

4.14.4.1. Data Considerations

A neural network was built and trained to predict the delta pressure shift using the available reservoir data. Table 4-31 summarizes the inputs and outputs, while the data used in training the neural network system is shown in Table 4-32.

Index	Parameter Name, Unit	Type	Range
1	Depth To Top Layer, ft	Input	5102 -13200
2	Top Layer Thickness, ft	Input	92 -249
3	Top Layer Closure Stress, psi/ft	Input	0.72 -0.773
4	Depth to Overburden Layer, ft	Input	5276 -13363
5	Overburden Layer Thickness, ft	Input	7 -81
6	Overburden Layer Stress Gradient, psi/ft	Input	0.73 -0.83
7	Depth to Pay Zone Layer, ft	Input	5329 -13415
8	Pay Zone Thickness, ft	Input	63 -148
9	Pay Zone Closure Stress Gradient, psi/ft	Input	0.646 -0.69
10	Pay Zone Permeability, md	Input	0.002 -0.554
11	Depth to Bottom Layer, ft	Input	5419 -13497
12	Bottom Layer Closure Stress Gradient, psi/ft	Input	0.702 -0.741
1	Pressure Shift, psi	Output	0 -300

Table 4-31: Parameters for Delta Pressure Shift Neural Network

Index	Top Layer			Overburden Layer			Pay Zone				Bottom Layer		Output
	Depth To Top Layer, ft	Top Layer Thickness, ft	Top Layer Closure Stress, psi/ft	Depth to Overburden Layer, ft	Overburden Layer Thickness, ft	Overburden Layer Stress Gradient, psi/ft	Depth to Pay Zone Layer, ft	Pay Zone Thickness, ft	Pay Zone Closure Stress Gradient, psi/ft	Pay Zone Permeability, md	Depth to Bottom Layer, ft	Bottom Layer Closure Stress Gradient, psi/ft	Pressure Shift, psi
Case 1	10920	205	0.729	11125	15	0.791	11140	100	0.646	0.027	11240	0.73	175
Case 2	12600	224	0.745	12824	16	0.774	12840	100	0.656	0.0159	12940	0.71	300
Case 3	11500	215	0.75	11715	19	0.809	11734	90	0.665	0.023	11824	0.73	125
Case 4	9600	200	0.772	9800	30	0.82	9830	80	0.665	0.039	9910	0.74	220
Case 5	7500	238	0.751	7738	16	0.799	7754	78	0.658	0.035	7832	0.724	130
Case 6	7500	240	0.753	7740	14	0.808	7754	78	0.673	0.013	7832	0.715	40
Case 7	6400	232	0.745	6632	8	0.8	6640	76	0.67	0.0254	6716	0.718	110
Case 8	10920	197	0.734	11117	23	0.787	11240	100	0.677	0.05	11340	0.718	110
Case 9	8650	203	0.741	8853	7	0.757	8860	107	0.676	0.03	8967	0.726	55
Case 10	7580	237	0.728	7817	14	0.768	7831	109	0.674	0.036	7940	0.71	160
Case 11	7080	206	0.758	7286	29	0.8	7315	101	0.6813	0.015	7416	0.722	95
Case 12	6450	198	0.741	6648	29	0.803	6677	103	0.672	0.0057	6780	0.728	260
Case 13	6450	198	0.741	6648	29	0.803	6677	103	0.672	0.0057	6780	0.728	260
Case 14	6450	198	0.741	6648	29	0.803	6677	103	0.672	0.0057	6780	0.728	260
Case 15	6450	198	0.741	6648	29	0.803	6677	103	0.672	0.0057	6780	0.728	260
Case 16	6210	195	0.743	6405	33	0.777	6438	101	0.674	0.023	6539	0.73	120
Case 17	5820	192	0.726	6012	13	0.771	6025	92	0.675	0.022	6117	0.709	190
Case 18	5310	216	0.769	5526	12	0.809	5538	74	0.664	0.0072	5612	0.717	50
Case 19	12200	198	0.75	12398	40	0.798	12438	91	0.678	0.016	12529	0.721	235
Case 20	12200	198	0.75	12398	40	0.798	12438	91	0.678	0.016	12529	0.721	235
Case 21	12200	198	0.75	12398	40	0.798	12438	91	0.678	0.016	12529	0.721	235
Case 22	9520	199	0.757	9719	46	0.798	9765	115	0.673	0.042	9880	0.728	80
Case 23	9520	219	0.755	9739	26	0.8	9765	115	0.689	0.018	9880	0.72	145
Case 24	10200	236	0.739	10436	16	0.783	10452	148	0.667	0.01	10600	0.73	160
Case 25	10200	236	0.739	10436	16	0.783	10452	148	0.667	0.01	10600	0.73	200
Case 26	8435	213	0.734	8648	9	0.786	8657	143	0.677	0.009	8800	0.717	230
Case 27	5310	216	0.767	5526	12	0.795	5538	74	0.663	0.007	5612	0.719	50
Case 28	10200	249	0.747	10449	16	0.789	10465	135	0.672	0.011	10600	0.73	230
Case 29	10200	249	0.747	10449	16	0.789	10465	135	0.672	0.011	10600	0.73	200
Case 30	10920	193	0.744	11113	54	0.763	11167	71	0.671	0.0033	11238	0.71	40
Case 31	8245	181	0.746	8426	37	0.778	8463	67	0.663	0.254	8530	0.732	0
Case 32	7687	194	0.748	7881	27	0.762	7908	124	0.687	0.0098	8032	0.718	40
Case 33	7145	183	0.738	7328	24	0.783	7352	109	0.665	0.0153	7461	0.702	80
Case 34	6851	207	0.726	7058	37	0.745	7095	96	0.683	0.0813	7191	0.719	90
Case 35	6521	184	0.742	6705	40	0.774	6745	128	0.679	0.027	6873	0.712	150
Case 36	6080	161	0.74	6241	9	0.754	6250	107	0.689	0.0425	6357	0.72	20
Case 37	8998	185	0.75	9183	33	0.787	9216	67	0.69	0.002	9283	0.722	220
Case 38	13200	160	0.773	13360	55	0.795	13415	63	0.6721	0.0385	13478	0.721	240
Case 39	13200	163	0.761	13363	52	0.796	13415	82	0.674	0.0135	13497	0.74	175
Case 40	8657	189	0.752	8846	22	0.809	8868	132	0.681	0.027	9000	0.722	180
Case 41	7732	217	0.72	7949	17	0.742	7966	94	0.689	0.039	8060	0.715	270
Case 42	6214	198	0.731	6412	23	0.779	6435	81	0.685	0.11	6516	0.73	120
Case 43	5112	187	0.738	5299	37	0.767	5336	103	0.684	0.049	5439	0.741	40
Case 44	5102	174	0.753	5276	53	0.774	5329	90	0.68	0.024	5419	0.721	120
Case 45	12254	217	0.754	12471	17	0.83	12488	90	0.676	0.0287	12578	0.715	140
Case 46	11920	139	0.724	12059	81	0.73	12140	100	0.682	0.007	12240	0.718	195
Case 47	11920	92	0.736	12012	28	0.748	12040	100	0.687	0.009	12140	0.712	140
Case 48	10658	244	0.732	10902	24	0.777	10926	108	0.67	0.024	11034	0.726	280
Case 49	11318	244	0.727	11562	24	0.75	11586	108	0.668	0.03	11694	0.726	240
Case 50	12568	214	0.73	12782	17	0.762	12799	112	0.681	0.0132	12911	0.731	260

Table 4-32: Data for Training the Delta Pressure Shift Neural Network

The first attempt to train a neural network using the data presented above resulted in very poor performance with correlation coefficients around 0.8 and R^2 values around 0.65 for testing and verification set. Multiple architectures and types of neural networks were tried in the attempt to build the model. It was concluded that the existing data was either insufficient or too sparse since it was randomly generated. Generating additional data would be very time consuming and would not necessarily guarantee the solution of the problem.

From previous experience and literature review^{41,42}, utilizing white noise has helped the training process of the neural network while being able to recognize and predict the original data with higher performance. Therefore, white noise was introduced into the system to attempt to overcome the previously described problem.

4.14.4.2. White Noise Data Generation

The process of white noise generation consists of a series of steps. The first of which is data selection. Since the existing dataset has only fifty cases, seven were extracted to form the verification set, representing 14% of the entire data set. These cases were set aside and were not utilized in noise generation. The remaining 43 cases together with the yet to be generated white noise cases will form the training and calibration data sets.

In this study, a five step process was used for white noise generation:

1. Using a random number generator, one case was extracted from the 43 remaining.
2. For the selected case, a new value for each parameter was generated such that the new value was within 5% higher or lower than the original value.
3. The result is a white noise case.
4. The process was repeated from step 1 to 3, 10 times. The result is one set of 10 white noise cases.

The process outlined above was repeated five times. Table 4-33 shows the five sets of white noise data.

Index	Top Layer			Overburden Layer			Pay Zone				Bottom Layer		Output
	Depth To Top Layer, ft	Top Layer Thickness, ft	Top Layer Closure Stress, psi/ft	Depth to Overburden Layer, ft	Overburden Layer Thickness, ft	Overburden Layer Stress Gradient, psi/ft	Depth to Pay Zone Layer, ft	Pay Zone Thickness, ft	Pay Zone Closure Stress Gradient, psi/ft	Pay Zone Permeability, md	Depth to Bottom Layer, ft	Bottom Layer Closure Stress Gradient, psi/ft	Pressure Shift, psi
Noise Set 1													
Noise 5	7446	231	0.743	7553	16	0.796	7610	77	0.648	0.0344	7695	0.708	127
Noise 28	9908	248	0.725	10237	16	0.772	10418	132	0.665	0.0107	10537	0.719	229
Noise 12	6360	195	0.736	6457	28	0.801	6629	102	0.662	0.0055	6697	0.719	258
Noise 26	8191	210	0.720	8544	9	0.773	8467	139	0.675	0.0087	8593	0.716	228
Noise 10	7556	236	0.722	7642	14	0.764	7606	106	0.670	0.0356	7886	0.707	156
Noise 1	10756	201	0.718	10869	15	0.770	10918	99	0.632	0.0266	11072	0.715	175
Noise 15	6269	193	0.733	6554	29	0.786	6550	101	0.664	0.0056	6739	0.710	258
Noise 40	8454	184	0.735	8648	22	0.793	8782	128	0.668	0.0267	8971	0.708	180
Noise 34	6838	202	0.725	6890	36	0.733	6947	94	0.672	0.0799	7026	0.710	88
Noise 21	12151	196	0.749	12224	39	0.786	12273	91	0.666	0.0157	12418	0.700	232
Noise Set 2													
Noise 24	10084	229	0.724	10355	16	0.761	10255	144	0.654	0.0100	10325	0.717	155
Noise 18	5212	210	0.761	5397	12	0.805	5422	73	0.659	0.0072	5510	0.715	50
Noise 35	6366	182	0.720	6571	40	0.758	6634	125	0.662	0.0269	6684	0.704	146
Noise 30	10746	192	0.743	11110	54	0.742	10954	69	0.654	0.0032	10970	0.691	39
Noise 2	12421	224	0.735	12804	16	0.769	12555	100	0.642	0.0157	12597	0.701	297
Noise 20	11903	194	0.732	12189	39	0.791	12347	91	0.668	0.0159	12194	0.712	233
Noise 38	13023	158	0.751	13307	53	0.795	13134	63	0.660	0.0379	13170	0.717	238
Noise 44	5015	174	0.745	5234	52	0.755	5323	90	0.662	0.0240	5367	0.715	119
Noise 49	11024	243	0.718	11524	24	0.735	11323	105	0.657	0.0294	11616	0.708	237
Noise 4	9372	194	0.763	9591	30	0.819	9580	79	0.651	0.0384	9874	0.739	216
Noise Set 3													
Noise 32	7685	191	0.738	7682	27	0.743	7816	121	0.669	0.0098	7850	0.710	39
Noise 22	9398	198	0.754	9537	45	0.788	9643	112	0.671	0.0410	9741	0.719	78
Noise 48	10440	238	0.722	10852	23	0.754	10635	105	0.655	0.0240	10824	0.724	273
Noise 7	6304	226	0.741	6462	8	0.794	6458	75	0.668	0.0253	6534	0.702	109
Noise 46	11695	136	0.715	11998	80	0.715	11988	100	0.673	0.0068	11891	0.717	195
Noise 25	10020	234	0.724	10127	16	0.765	10371	144	0.656	0.0100	10520	0.711	200
Noise 9	8434	197	0.730	8779	7	0.745	8812	105	0.674	0.0298	8714	0.704	55
Noise 42	6134	196	0.721	6351	23	0.773	6320	79	0.673	0.1083	6328	0.709	117
Noise 14	6443	197	0.731	6577	29	0.790	6483	101	0.668	0.0057	6632	0.708	260
Noise 3	11271	209	0.738	11594	19	0.787	11533	88	0.654	0.0225	11675	0.709	125
Noise Set 4													
Noise 11	7004	205	0.745	7250	28	0.777	7149	101	0.673	0.0148	7315	0.718	93
Noise 19	12171	197	0.735	12043	39	0.783	12147	90	0.665	0.0157	12404	0.704	230
Noise 8	10903	196	0.720	11016	23	0.775	11019	99	0.665	0.0489	11334	0.705	110
Noise 41	7697	215	0.710	7878	17	0.723	7766	93	0.673	0.0387	7933	0.705	268
Noise 39	12938	162	0.739	12989	51	0.788	13237	80	0.667	0.0132	13144	0.732	170
Noise 50	12469	210	0.717	12542	17	0.745	12483	109	0.667	0.0131	12885	0.710	257
Noise 31	8206	181	0.741	8183	37	0.767	8413	66	0.646	0.2519	8285	0.721	0
Noise 13	6282	193	0.735	6569	28	0.800	6507	102	0.661	0.0056	6714	0.728	259
Noise 36	6078	161	0.737	6215	9	0.733	6104	105	0.672	0.0416	6245	0.715	20
Noise 16	6173	194	0.732	6384	32	0.769	6329	100	0.663	0.0223	6371	0.716	118
Noise Set 5													
Noise 6	7490	234	0.732	7542	14	0.797	7661	77	0.658	0.0128	7805	0.715	40
Noise 17	5725	187	0.708	5973	13	0.749	5873	91	0.666	0.0216	6048	0.690	186
Noise 23	9400	213	0.745	9636	26	0.787	9682	114	0.672	0.0177	9789	0.715	142
Noise 27	5186	213	0.756	5426	12	0.782	5450	74	0.662	0.0069	5571	0.719	49
Noise 29	10015	242	0.727	10250	16	0.774	10344	132	0.657	0.0108	10521	0.729	199
Noise 33	7008	182	0.736	7264	24	0.779	7187	107	0.662	0.0149	7284	0.696	79
Noise 37	8811	181	0.737	9143	33	0.780	8951	66	0.682	0.0020	9053	0.715	218
Noise 43	5058	186	0.734	5296	36	0.754	5198	102	0.676	0.0482	5425	0.727	39
Noise 45	12232	213	0.733	12352	17	0.811	12369	90	0.667	0.0284	12245	0.711	139
Noise 47	11674	91	0.718	11995	28	0.737	11793	98	0.685	0.0089	12125	0.711	140

Table 4-33: White Noise Data for Training the Calibration Neural Network

4.14.4.3. Calibration Neural Network Performance

Five neural networks were trained using the original data set and five sets of white noise data in the following order.

1. The first neural network used the first noise set, which was added to the 43 cases and loaded into NeuroShell. The software defined the training and calibration sets randomly using all 53 cases, and trained the neural network. The performance of the neural network is given in the Table 4-34. The final verification was done using the seven cases set left aside.
2. The second neural network used the first and the second noise data sets together with the original 43 cases. This time the final set consisted of 63 cases. NeuroShell defined the training and calibration sets and trained the second neural network.
3. The process was repeated; each time adding a new white noise data set which increased the total number of cases by ten. The third, fourth, and fifth neural networks were trained this way. The same seven cases, previously extracted from the original fifty cases, were used to measure the performance improvement for each of the neural networks.

	Training Set	Calibration Set	Verification Set
Number of Cases	43	10	7
R^2	0.9991	0.4345	0.8749
r^2	0.9992	0.5351	0.9207
Correlation Coefficient	0.9996	0.7315	0.9595

Table 4-34: Calibration Neural Network, 1st Noise Data Set Included

	Training Set	Calibration Set	Verification Set
Number of Cases	53	10	7
R^2	0.9970	0.6319	0.8535
r^2	0.9977	0.7296	0.9080
Correlation Coefficient	0.9988	0.8542	0.9529

Table 4-35: Calibration Neural Network, 1st and 2nd Noise Data Sets Included

	Training Set	Calibration Set	Verification Set
Number of Cases	63	10	7
R^2	0.6380	0.6494	0.6802
r^2	0.6671	0.6786	0.8217
Correlation Coefficient	0.8167	0.8238	0.9065

Table 4-36: Calibration Neural Network, 1st, 2nd, and 3rd Noise Data Sets Included

	Training Set	Calibration Set	Verification Set
Number of Cases	73	10	7
R ²	0.9886	0.9178	0.9194
r ²	0.9912	0.9369	0.9199
Correlation Coefficient	0.9956	0.9679	0.9591

Table 4-37: Calibration Neural Network, 1st, 2nd, 3rd, and 4th Noise Data Sets Included

	Training Set	Calibration Set	Verification Set
Number of Cases	83	10	7
R ²	0.9838	0.9123	0.8804
r ²	0.9856	0.9335	0.9077
Correlation Coefficient	0.9927	0.9662	0.9527

Table 4-38: Calibration Neural Network, All 5 Noise Data Sets Included

The first neural network (Table 4-34) shows very poor performance in the calibration set; while the second neural network (Table 4-35) shows a slight improvement in the calibration set but the same performance for the verification set. The third neural network (Table 4-36), where three noise data sets were added, shows very poor performance in both calibration and verification sets. The fourth and fifth neural networks (Table 4-37 and Table 4-38, respectively) show very good performance for all three sets – training, calibration, and verification. However, the fourth neural network shows modestly better performance for the verification set (R2 = 0.919). The graphical representations of the accuracy of the fourth neural network training, calibration, and verifications sets are shown in Figure 4-34, Figure 4-35, and Figure 4-36 respectively.

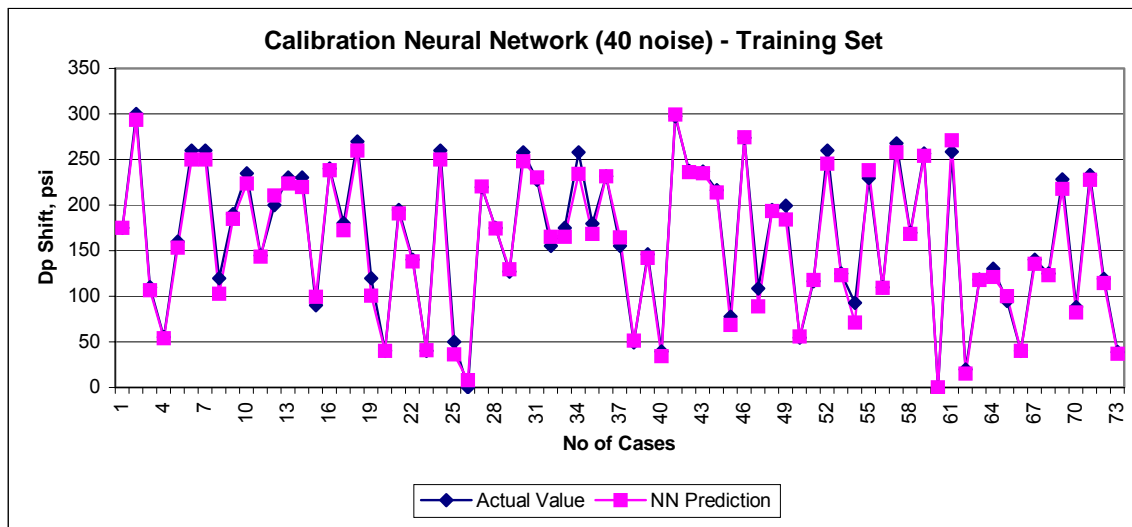


Figure 4-34: Calibration Neural Network Training Set Accuracy

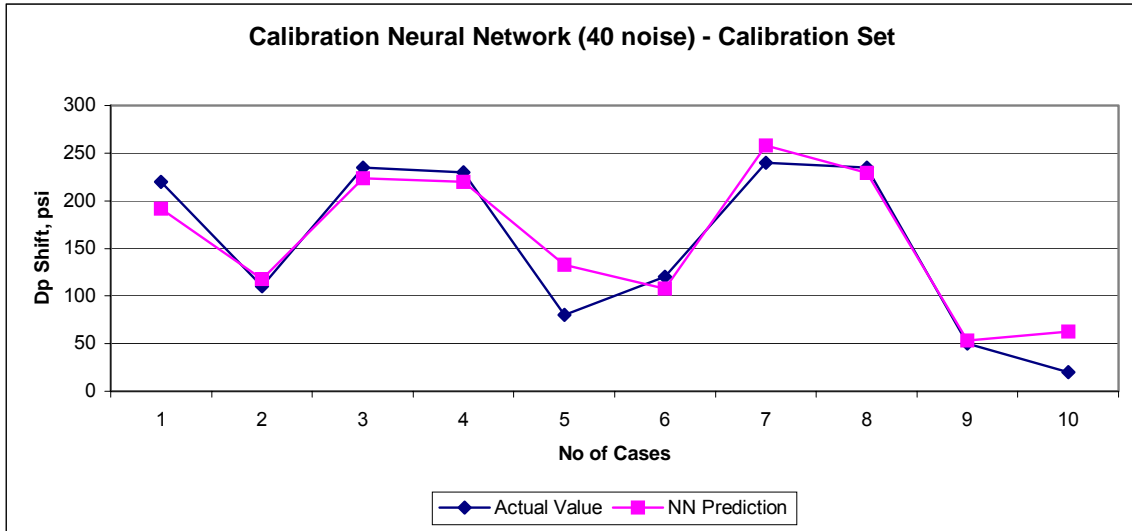


Figure 4-35: Calibration Neural Network Calibration Set Accuracy

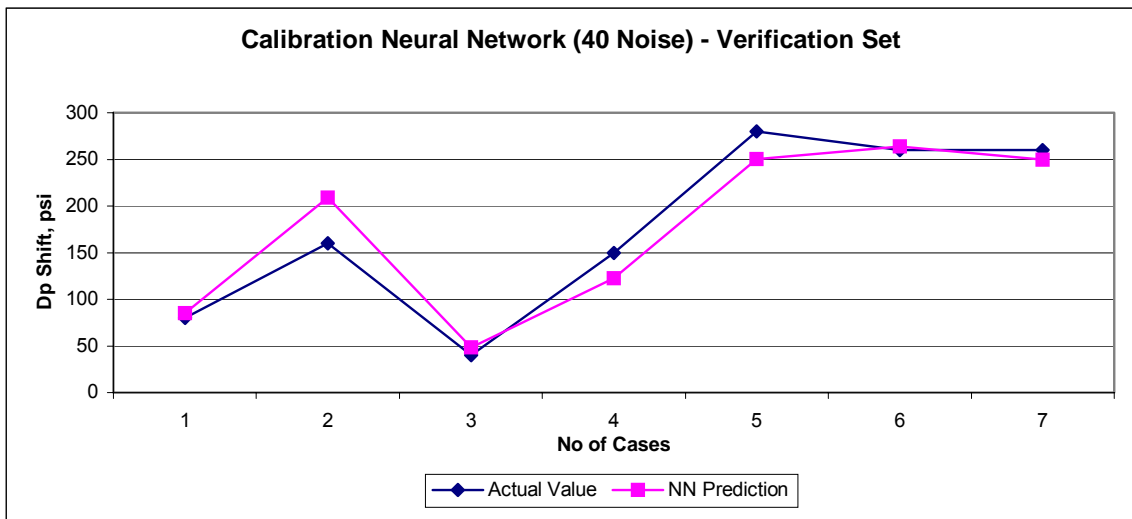


Figure 4-36: Calibration Neural Network Verification Set Accuracy

Due to its improved performance, the fourth neural network was selected to be the calibration neural network. The calibration neural network allows continuing the use of the equivalent four-layer profile for net pressure prediction in the methodology.

4.14.5. Neural Network for Pressure Profile Prediction

As previously described, significant work was done to find a methodology able to generate the same net pressure profile when using either the equivalent four-layer profile or original lithology profile with FRACPRO. The result was a calibration neural network trained to add the shift of delta pressure to the net pressure profile generated using the equivalent four-

layer profile in order to match the net pressure profile from the original lithology. A neural network model was proposed in this study to mimic the fracture simulator and provide the net pressure profile. The development of this system is covered in the following sections.

4.14.5.1. Data Considerations

Since data was needed to train a neural network model, 100 cases were randomly generated using the same algorithm used for ramp treatment schedules as stated in section 4.11.2.1. The inputs are presented in Table 4-3, while the output is the net pressure profile. FRACPRO was run for all 100 cases, and the net pressure profile was recorded for each case.

Each one of the 100 net pressure profiles was generated using a unique treatment design (total fluid, pad, and proppant amount) and specific reservoir conditions (depth, layer stress values, permeability, etc). Since the duration of the job is equal to the ratio of total fluid plus proppant to the injection rate, the treatments have various duration times. The data for each treatment is recorded every minute, resulting in net pressure profile vectors having up to 100 pressure points. In addition, each net pressure profile has a different number of points, based on its duration. Examples of pressure profiles with different duration time for different treatments are presented in Figure 4-37.

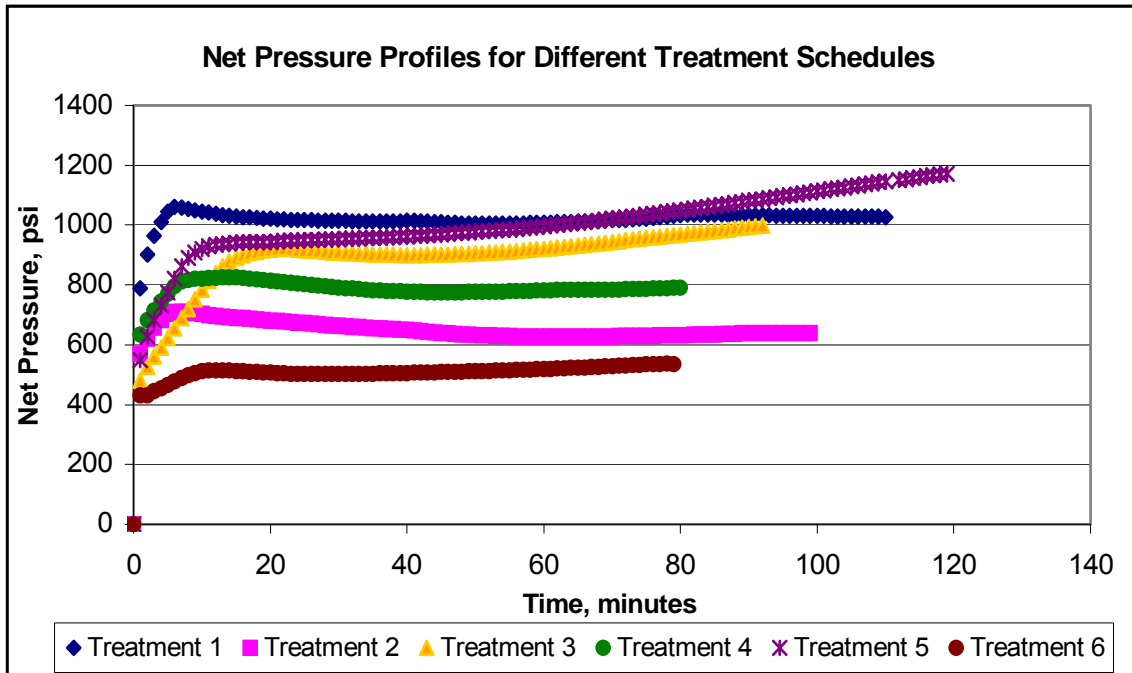


Figure 4-37: Net Pressure Profiles with Various Duration Times

It is impractical, if not impossible, to use all these points to train a neural network. In these conditions, a methodology to reduce the number of pressure points while maintaining a representative profile is needed. Since vector quantization is one of the most powerful technologies for data compression and dimension reduction, a modified version of the technique was used.

4.14.5.2. Data Sampling using Vector Quantization

The objective of vector quantization is to compress a particular sample by a pre-specified number of prototypes. In this case, the prototypes are the number of pressure points able to reproduce the original pressure profile.

Typically vector quantization is used in communication and data processing, or other high dimensional domains. For this, both an encoder and decoder are needed. However, in this study, the pressure profile is a one-dimensional vector, so a simple extraction of prototypes (pressure points) is sufficient. The net pressure profile will be reproduced using polynomial interpolation.

Vector quantization is simply an approximator. The classic LBG VQ^{30,43} design algorithm is an iterative algorithm:

1. Define feature vectors and final number of quantizers. For a number N , of feature vectors X_1, X_2, \dots, X_n , a number of k quantizers can be found such that $2^k < N$.
2. Initialize the first two quantizers, q_1 and q_2 , out of K (usually by assigning a random number to them).
3. Calculate the Euclidian distance from each of the quantizers to all feature vectors. The minimum distance determines X_i 's membership to one of the quantizers.
4. Recalculate q_1 and q_2 .
5. Repeat steps 3 and 4 until there is no change in q_1 and q_2 .
6. Split each quantizer such that the new number of quantizers is $K = 2K$.
7. Repeat step 3 through 6 until the final number of quantizers is reached.

In this study, each feature vector is one-dimensional. The time vector consists of values for every minute from 0 to final injection time. Similar net pressure vectors consist of values for pressure recorded for each time step, one minute in this case.

The neural network must have a constant number of outputs for each case; therefore, the number of pressure points for all pressure profiles must be consistent in order for the neural network to function. Since not all the pressure profiles have the same duration, sampling at the same time intervals is not possible. Hence, vector quantization was used to extract a fixed number of prototypes (pressure points) from each pressure profile. The algorithm extracts the time and the net pressure for each prototype. By analyzing the shape of each pressure profile, it

was concluded that 10 points are sufficient to reproduce the pressure profile with acceptable accuracy.

Since vector quantization methodology grows in steps of 2^K where K is the iteration step, the maximum number of prototypes that can be calculated is 8 (<10 points considered). A computer program using the algorithm described below was developed to sample the pressure profile:

1. Use the vector quantization methodology described above to determine the 8 time prototypes (or time quantizers) for the time vector.
2. For this time quantizer, calculate the pressure quantizer (or pressure point) as the average of the pressure values corresponding to the time points represented by the time quantizer.
3. Add two more points to the set of time/pressure quantizers. These points are the time at 1 minute and time of the final injection minute with the corresponding pressures. This ensures that the entire pressure profile is captured, including the start and end.
4. The 10 pressure points (8 quantizers and 2 end points) are the output of the net pressure neural network.

Figure 4-38 and Figure 4-39 show the continuous original net pressure profile generated by FRACPRO (blue diamond points) and the compressed 10-point pressure profile (red square points).

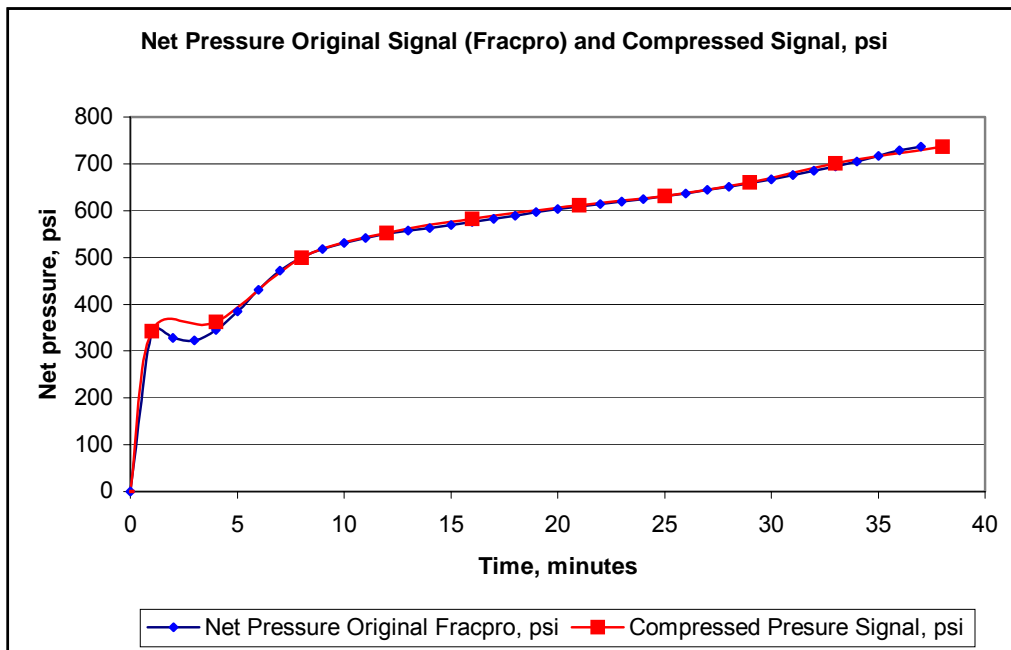


Figure 4-38: Net Pressure Profiles for Original Signal and Compressed Signal Example 2

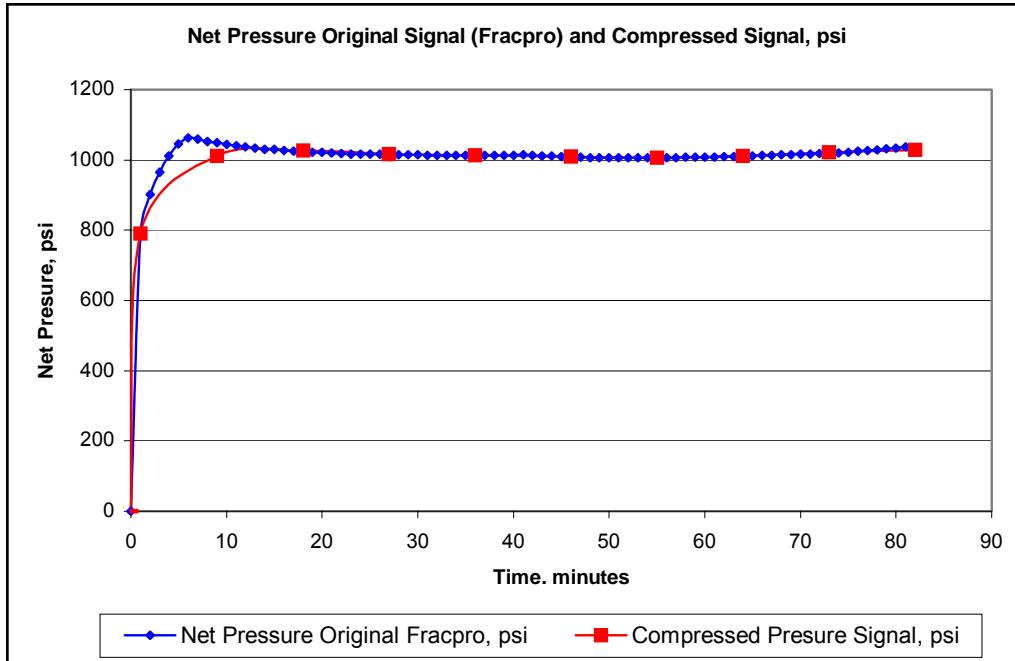


Figure 4-39: Net Pressure Profiles for Original Signal and Compressed Signal Example 2

The algorithm presented above is very convenient when the pressure signal is reproduced back (decoding). Once the size of the treatment and rate are known, the injection time can be easily calculated. Since the time vector is linear, quantizers for any job duration can be calculated by applying the back vector quantization technique.

4.14.5.3. Neural Network Design

The neural network design consists of the inputs and outputs presented in Table 4-39. The net pressure prediction network has one input layer with 14 input neurons, three hidden layers with 21 hidden neurons (7 in each hidden layer) and 10 output neurons. The learning rate was 0.1 and the momentum 0.1. A summary architecture description of the neural network is given in Table 4-40.

Index	Parameter Name	Type	Units
1	Total fluid volume	input	gal
2	Pad volume	input	gal
3	Final proppant Concentration	input	ppg
4	Flow pumping rate	input	bbbl/min
5	Depth to the "pay zone"	input	ft
6	Reservoir pressure	input	psia
7	Permeability	input	mD
8	Top layer thickness	input	ft
9	Top layer closure stress gradient	input	psi/ft
10	Overburden layer thickness	input	ft
11	Overburden layer closure stress gradient	input	psi/ft
12	Pay zone thickness	input	ft
13	Pay zone closure stress gradient	input	psi/ft
14	Bottom layer closure stress gradient	input	psi/ft
1	Pressure Point 1 (at 1 minute)	output	psi
2	Pressure Point 2	output	psi
3	Pressure Point 3	output	psi
4	Pressure Point 4	output	psi
5	Pressure Point 5	output	psi
6	Pressure Point 6	output	psi
7	Pressure Point 7	output	psi
8	Pressure Point 8	output	psi
9	Pressure Point 9	output	psi
10	Pressure Point 10 (final minute)	output	psi

Table 4-39: Inputs and Outputs for the Net Pressure Neural Network

Net Pressure Network	Input Layer	Hidden Layer 1	Hidden Layer 2	Hidden Layer 3	Output	Learning Rate	Momentum
Number of Neurons	14	7	7	7	10	0.1	0.1
Type of activation Function	Linear	Gaussian	Tanh	Gaussian	Logistic		

Table 4-40: Net Pressure Prediction Neural Network

4.15. Summary

The methodology used in this study combines a series of artificial intelligence tools in a unique manner to build an intelligent tool that designs optimum treatment schedules and predicts net treatment pressure for hydraulic fracturing. Three main modules form the methodology.

In the first module, a new and innovative fuzzy expert system is used for lithology identification. This approach proved to be very effective and provided accurate results. The expert knowledge incorporated in the system makes this tool a great companion for any need of lithology identification. The lithology is then used with a classic approach to calculate the stress profile. Next, the stress profile is processed to illustrate a clear definition of the stress zones. The final part of the module proposes a simple approach to reduce the complex multi-zone lithology stress profile to a four-layer equivalent profile. The equivalent stress profile represents the input to the second module.

The second module incorporates an optimization system composed of neural networks and a genetic algorithm to search for the optimum treatment design. Several types of neural networks were developed to design different types of treatments in this module. This combination of neural networks and genetic algorithms demonstrated fast convergence to optimum solution.

The third, and final, module was designed to predict the fracturing net pressure profile. First, a neural network was developed that calibrates the net pressure profile resulted from the equivalent layer profile with the net pressure profile resulted from original multi-layer profile. Next, a one-dimensional vector quantization technique was used to extract the main characteristic of the pressure profile. A net treatment pressure neural network that uses the pressure quantizers was built to predict the net pressure profile. In the end, the calibration neural network and net pressure neural network predict the pressure profile, and the entire signal is reconstructed.

CHAPTER 5: RESULTS & DISCUSSION

5.1. Fuzzy Logic Lithology Technique for Stress Profile Calculations

Over time, several models and equations have been developed to determine the stress profile. The ABC stress profile relationship, developed by Bragan & Associates¹⁷, modifies the original Hook's Law equation and introduces the concept of lithology-driven stress calculations. The authors defined a default coefficient for each type of lithology for use in Equation 2-6. However, this approach requires visual and manual coefficient assignment for each formation or zone in addition to the presence of an expert in log interpretation and lithology identification. Moreover, it does not provide any values for transition lithology such as dirty sandstones (shaly sandstone) or shaly carbonates. The fuzzy logic expert system developed in this study identifies lithology without any visual or manual effort and also compensates for transition zones. The system automatically determines the lithology and feeds the corresponding coefficient into Equation 2-6 for stress calculation.

The methodology presented in section 4.8 Fuzzy Lithology Identification Approach was tested on an example well in North Appleby Field, Nacogdoches County, Texas. Three basic logs (spontaneous potential, gamma ray, and deep induction) were obtained from the well. Table 5-1 shows the raw log data for a 30-foot interval (7700-7730 ft) with recordings made every half-foot.

Depth, ft	Spontaneous Potential, mV	Gamma Ray, API	Deep Induction, ohm-m
7700	-2.6004	66.9102	5.6821
7700.5	-2.7207	53.8349	5.2905
7701	-2.8409	41.6089	4.5072
7701.5	-2.8362	39.2341	4.2387
7702	-2.7065	45.2734	5.4921
7702.5	-2.7017	51.4805	8.5872
7703	-5.1892	62.0075	11.0407
7703.5	-5.1845	61.9653	12.3901
7704	-5.4297	52.178	10.8485
7704.5	-5.425	42.1078	6.8412
7705	-5.2875	42.636	6.0067
7705.5	-5.1577	46.3846	6.1237
7706	-5.028	54.2014	5.7805
7706.5	-5.1404	58.5403	5.2827
7707	-5.167	62.2593	4.886
7707.5	-5.0372	64.3599	4.3709
7708	-4.9075	72.039	4.0023
7708.5	-4.9106	80.6483	3.7591
7709	-5.1558	79.1327	3.5644
7709.5	-5.2761	76.7478	3.454
7710	-5.6464	74.3729	3.4072
7710.5	-5.7745	73.2683	3.4293
7711	-6.2697	72.4146	3.4614
7711.5	-7.015	72.3839	3.4739
7712	-7.3931	72.3226	3.4716
7712.5	-7.8493	64.3412	6.0416
7713	-8.7196	54.9708	18.8887
7713.5	-9.4648	48.0022	19.487
7714	-10.3351	42.7926	19.8692
7714.5	-11.0882	40.3232	20.3156
7715	-11.6928	42.9956	20.495
7715.5	-12.1881	46.9197	20.6529
7716	-12.9646	48.5773	20.5413
7716.5	-13.7255	48.6449	20.4508
7717	-14.2208	51.779	20.596
7717.5	-14.341	57.4551	21.3248
7718	-14.2113	57.6	22.2165
7718.5	-14.2066	55.8496	31.7005
7719	-16.0456	49.3581	34.0012
7719.5	-20.1659	47.1824	30.3164
7720	-21.7861	43.1226	30.2078
7720.5	-22.0314	45.2378	31.8776
7721	-22.0267	47.0289	34.3436
7721.5	-22.0219	48.8217	37.9456
7722	-21.8922	43.5953	40.7235
7722.5	-22.1062	41.4603	42.3581
7723	-21.9609	43.2454	44.1007
7723.5	-22.2061	46.3126	45.3438
7724	-22.5608	48.1888	46.4139
7724.5	-22.7123	44.0543	46.6483
7725	-23.3638	45.9305	45.1318
7725.5	-24.1091	43.012	46.823
7726	-24.6043	46.2128	44.0916
7726.5	-24.7246	44.2214	43.891
7727	-24.8449	43.9759	42.3834
7727.5	-24.8401	42.2625	36.3123
7728	-25.3354	37.8586	32.1787
7728.5	-25.5651	35.7222	31.4269
7729	-25.8103	36.2846	31.1947
7729.5	-26.0556	38.8916	29.387
7730	-26.4415	40.8915	27.2507

Table 5-1: Log Data from Example Well in Texas

First, the fuzzy sets were defined for each of the three input parameters. Figure 5-1 shows the data defining the fuzzy sets and the three fuzzy sets for deep induction. Similar sets were defined for spontaneous potential and gamma ray.

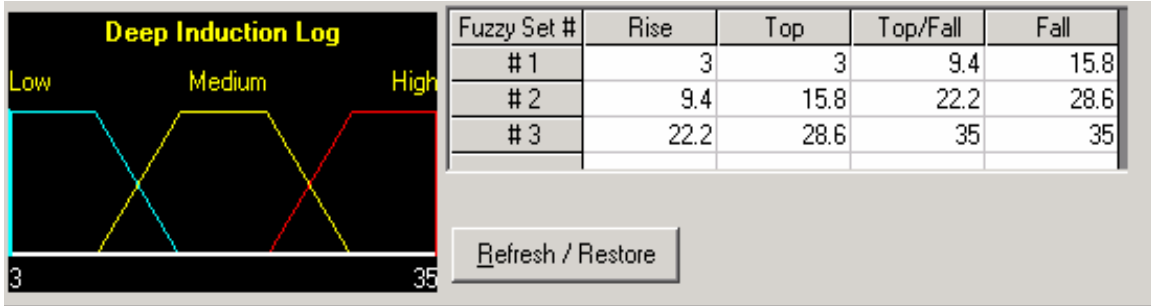


Figure 5-1: Fuzzy Sets for Deep Induction Log

Figure 5-2 shows the parallel processing and decision-making using the fuzzy logic lithology system. In this figure, the system is processing the log values at a depth of 7,845 feet (22.45mV for spontaneous potential, 38.2API for gamma ray, and 24.38 ohm-m for deep induction).

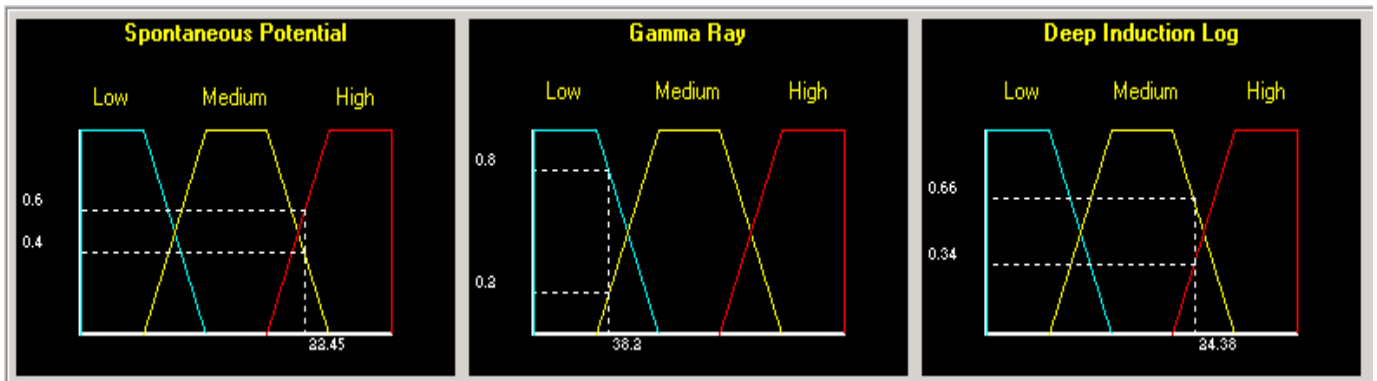


Figure 5-2: Fired Rules in the Process of Lithology Identification

Figure 5-2 shows the fired rules during the decision process for the values mentioned above. Eight rules were fired simultaneously to provide the output of the system. The last column shows the strength of each rule.

Rule #		Spontaneous Potential		Gamma Ray		Deep Induction Log		Status	Truth Qualification
Rule # 11	If	Medium	And	Low	And	Medium	Then	SandStone	T
Rule # 12	If	Medium	And	Low	And	High	Then	SandStone	UT
Rule # 14	If	Medium	And	Medium	And	Medium	Then	SS-Sh	T
Rule # 15	If	Medium	And	Medium	And	High	Then	SandStone	FT
Rule # 20	If	High	And	Low	And	Medium	Then	SandStone	T
Rule # 21	If	High	And	Low	And	High	Then	SandStone	UT
Rule # 23	If	High	And	Medium	And	Medium	Then	SS-Sh	UT
Rule # 24	If	High	And	Medium	And	High	Then	SandStone	T

Table 5-2: Fuzzy Rules for the Fuzzy Lithology Identification System

The outcome of the fuzzy lithology system for the example given above is shown in Figure 5-3. The case belongs to fuzzy set sandstone with a degree of membership equal to 0.6 and to fuzzy set shaly-sandstone with a degree of membership equal to 0.3.

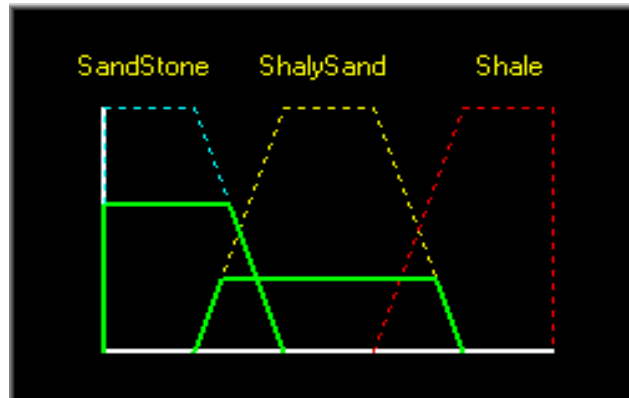


Figure 5-3: Outcome of the Fuzzy Lithology System

Table 5-3 shows the degree of membership value for every half-foot of recorded log data for each of the three lithology fuzzy sets between 7,700 and 7,730 feet. The system identified a shale formation from 7,700-7,714 ft, followed by a transition zone (7,714-7,718 ft) identified as shaly-sandstone (dirty sandstone), followed by a cleaner sandstone formation. The logs and lithology output for the interval analyzed in this study is presented in Figure 5-4. The column plotted at the right side of the figure represents the lithology identified using fuzzy lithology system. This lithology was verified by an expert in the domain and proved to be very accurate.

Depth, ft	Lithology	Degree of Membership		
		Sandstone	Shaly-Sandstone	Shale
7700	Shale	0	0	0.85
7700.5	Shale	0	0	1
7701	Shale	0	0	0.51
7701.5	Shale	0	0	0.71
7702	Shale	0	0	0.84
7702.5	Shale	0	0	1
7703	Shale	0	0	0.64
7703.5	Shale	0	0	0.53
7704	Shale	0	0	0.77
7704.5	Shale	0	0	0.56
7705	Shale	0	0	0.6
7705.5	Shale	0	0	0.94
7706	Shale	0	0	1
7706.5	Shale	0	0	0.95
7707	Shale	0	0	0.61
7707.5	Shale	0	0	0.66
7708	Shale	0	0	1
7708.5	Shale	0	0	1
7709	Shale	0	0	1
7709.5	Shale	0	0	1
7710	Shale	0	0	1
7710.5	Shale	0	0	1
7711	Shale	0	0	1
7711.5	Shale	0	0	1
7712	Shale	0	0	1
7712.5	Shale	0	0	0.66
7713	Shale	0	0.06	0.91
7713.5	Shale	0	0.2	0.72
7714	Shale	0.36	0.36	0.49
7714.5	Sandstone	0.5	0.39	0.25
7715	SS-SH	0.36	0.61	0.24
7715.5	SS-SH	0.01	0.7	0.16
7716	SS-SH	0	0.85	0.06
7716.5	SS-SH	0	0.99	0
7717	SS-SH	0	1	0
7717.5	SS-SH	0	1	0
7718	SS-SH	0	1	0
7718.5	Sandstone	1	0	0
7719	Sandstone	1	0	0
7719.5	Sandstone	0.74	0	0
7720	Sandstone	0.48	0	0
7720.5	Sandstone	0.52	0	0
7721	Sandstone	0.52	0	0
7721.5	Sandstone	0.52	0	0
7722	Sandstone	0.5	0	0
7722.5	Sandstone	0.6	0	0
7723	Sandstone	0.51	0	0
7723.5	Sandstone	0.56	0	0
7724	Sandstone	0.62	0	0
7724.5	Sandstone	0.65	0	0
7725	Sandstone	0.77	0	0
7725.5	Sandstone	0.64	0	0
7726	Sandstone	0.93	0	0
7726.5	Sandstone	0.75	0	0
7727	Sandstone	0.73	0	0
7727.5	Sandstone	0.57	0	0
7728	Sandstone	0.87	0	0
7728.5	Sandstone	1	0	0
7729	Sandstone	0.98	0	0
7729.5	Sandstone	0.8	0	0
7730	Sandstone	0.64	0.31	0

Table 5-3: Lithology Identification using Fuzzy Lithology System

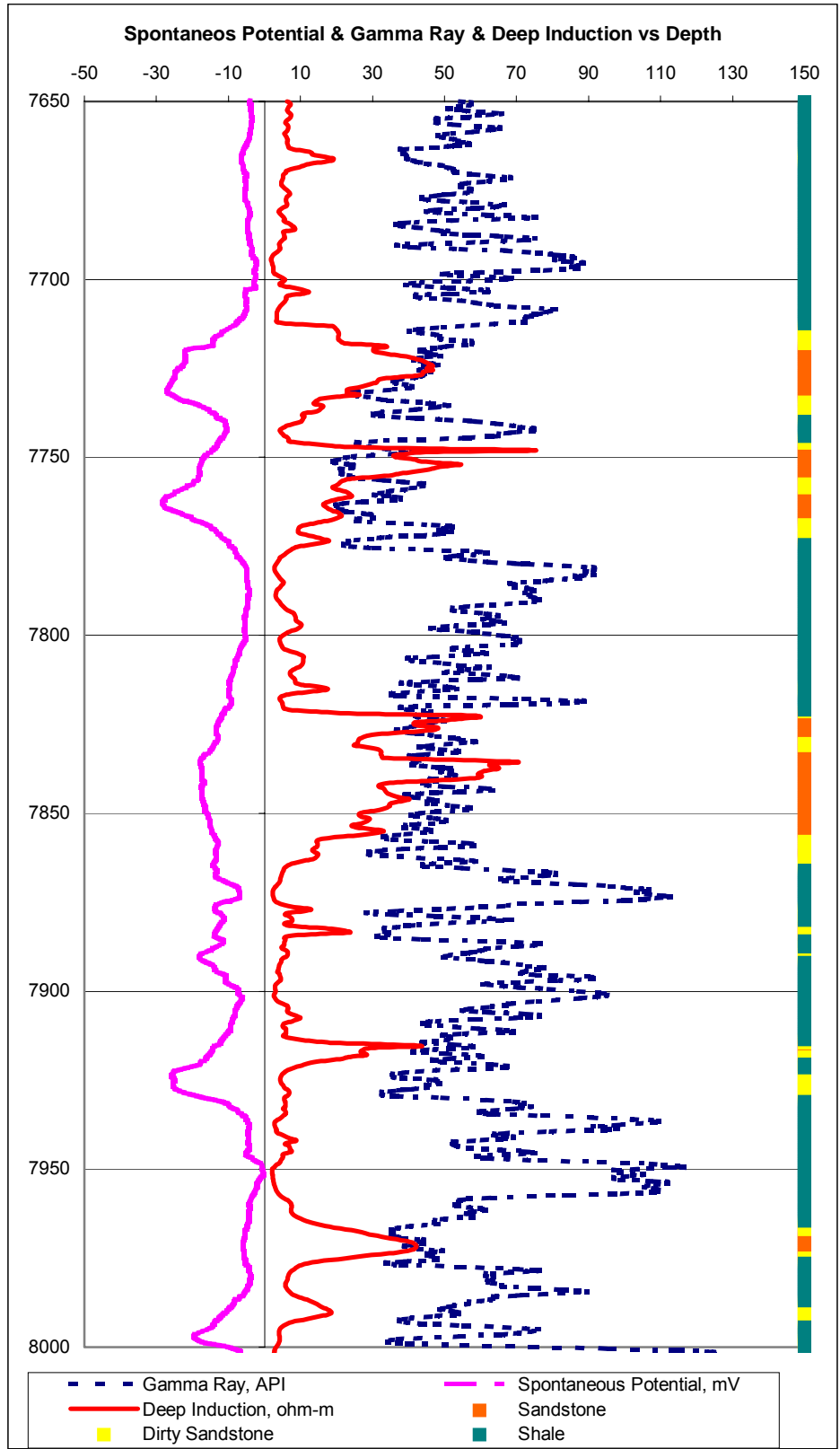


Figure 5-4: Spontaneous Potential, Gamma Ray, Deep Induction, and Lithology

It is clearly visible that sandstone formations (red in the lithology column) are present where resistivity (deep induction) and spontaneous potential have high values and gamma ray values are low.

After lithology has been established, Lithology Coefficient A was automatically assigned, and Equation 2-6 was used to calculate the stress profile. Table 5-4 shows closure stress calculation for every half-foot of log data, while Figure 5-5 shows the stress profile of the interval.

Next, the stress profile underwent a manual-processing step to clearly illustrate different existing stress layers. This process was done visually, using the stress profile curve and logs, by grouping together similar consecutive stress values to define possible shale and sandstone formations. Figure 5-5 illustrates the separation between different layers with similar characteristics, such as values of closure stress. The horizontal lines and boxes on the right side of the graph show a definition of the four layers between 7,777 and 7,850 feet.

- ▶ Layer 1 (7,777 – 7,793.5 ft) represents clean shale.
- ▶ Layer 2 (7,794 – 7,821 ft) is shale with small intercalations of sandstone. These intercalations indicate the presence of sandstone nearby.
- ▶ Layer 3 (7,821.5 – 7,832 ft) is clean sandstone as indicated from Layer 2.
- ▶ Layer 4 (7,832 – 7,850 ft) is poor sandstone (dirty sandstone).

Although every 0.5 ft in these four layers may have a different closure stress value, the final outcome is one closure stress value for each of the four layers. These values are calculated as the arithmetic average of all the stress values between the two depths. Table 5-5 shows the stress profile processing, which is plotted in Figure 5-6.

Depth, ft	Lithology	Degree of Membership			A Coefficien	B Coefficien	C Coefficien	Closure Stress, psi
		Sandstone	Shaly- Sandstone	Shale				
7700	Shale	0	0	0.85	0.524	0.5	0	6162
7700.5	Shale	0	0	1	0.53	0.5	0	6210
7701	Shale	0	0	0.51	0.5104	0.5	0	6052
7701.5	Shale	0	0	0.71	0.5184	0.5	0	6117
7702	Shale	0	0	0.84	0.5236	0.5	0	6160
7702.5	Shale	0	0	1	0.53	0.5	0	6212
7703	Shale	0	0	0.64	0.5156	0.5	0	6096
7703.5	Shale	0	0	0.53	0.5112	0.5	0	6061
7704	Shale	0	0	0.77	0.5208	0.5	0	6139
7704.5	Shale	0	0	0.56	0.5124	0.5	0	6071
7705	Shale	0	0	0.6	0.514	0.5	0	6085
7705.5	Shale	0	0	0.94	0.5276	0.5	0	6195
7706	Shale	0	0	1	0.53	0.5	0	6215
7706.5	Shale	0	0	0.95	0.528	0.5	0	6199
7707	Shale	0	0	0.61	0.5144	0.5	0	6089
7707.5	Shale	0	0	0.66	0.5164	0.5	0	6106
7708	Shale	0	0	1	0.53	0.5	0	6217
7708.5	Shale	0	0	1	0.53	0.5	0	6217
7709	Shale	0	0	1	0.53	0.5	0	6217
7709.5	Shale	0	0	1	0.53	0.5	0	6218
7710	Shale	0	0	1	0.53	0.5	0	6218
7710.5	Shale	0	0	1	0.53	0.5	0	6219
7711	Shale	0	0	1	0.53	0.5	0	6219
7711.5	Shale	0	0	1	0.53	0.5	0	6219
7712	Shale	0	0	1	0.53	0.5	0	6220
7712.5	Shale	0	0	0.66	0.5164	0.5	0	6110
7713	Shale	0	0.06	0.91	0.5264	0.5	0	6191
7713.5	Shale	0	0.2	0.72	0.5188	0.5	0	6130
7714	Shale	0.36	0.36	0.49	0.5044	0.5	0	6014
7714.5	Sandstone	0.5	0.39	0.25	0.445	0.5	0	5533
7715	SS-SH	0.36	0.61	0.24	0.4576	0.5	0	5636
7715.5	SS-SH	0.01	0.7	0.16	0.4964	0.5	0	5950
7716	SS-SH	0	0.85	0.06	0.4924	0.5	0	5918
7716.5	SS-SH	0	0.99	0	0.49	0.5	0	5899
7717	SS-SH	0	1	0	0.49	0.5	0	5900
7717.5	SS-SH	0	1	0	0.49	0.5	0	5900
7718	SS-SH	0	1	0	0.49	0.5	0	5900
7718.5	Sandstone	1	0	0	0.4	0.5	0	5171
7719	Sandstone	1	0	0	0.4	0.5	0	5172
7719.5	Sandstone	0.74	0	0	0.413	0.5	0	5277
7720	Sandstone	0.48	0	0	0.426	0.5	0	5383
7720.5	Sandstone	0.52	0	0	0.424	0.5	0	5367
7721	Sandstone	0.52	0	0	0.424	0.5	0	5368
7721.5	Sandstone	0.52	0	0	0.424	0.5	0	5368
7722	Sandstone	0.5	0	0	0.425	0.5	0	5376
7722.5	Sandstone	0.6	0	0	0.42	0.5	0	5336
7723	Sandstone	0.51	0	0	0.4245	0.5	0	5373
7723.5	Sandstone	0.56	0	0	0.422	0.5	0	5353
7724	Sandstone	0.62	0	0	0.419	0.5	0	5329
7724.5	Sandstone	0.65	0	0	0.4175	0.5	0	5317
7725	Sandstone	0.77	0	0	0.4115	0.5	0	5269
7725.5	Sandstone	0.64	0	0	0.418	0.5	0	5322
7726	Sandstone	0.93	0	0	0.4035	0.5	0	5205
7726.5	Sandstone	0.75	0	0	0.4125	0.5	0	5278
7727	Sandstone	0.73	0	0	0.4135	0.5	0	5287
7727.5	Sandstone	0.57	0	0	0.4215	0.5	0	5352
7728	Sandstone	0.87	0	0	0.4065	0.5	0	5231
7728.5	Sandstone	1	0	0	0.4	0.5	0	5178
7729	Sandstone	0.98	0	0	0.401	0.5	0	5187
7729.5	Sandstone	0.8	0	0	0.41	0.5	0	5260
7730	Sandstone	0.64	0.31	0	0.4324	0.5	0	5442

Table 5-4: Example of Stress Calculation using ABC Methodology

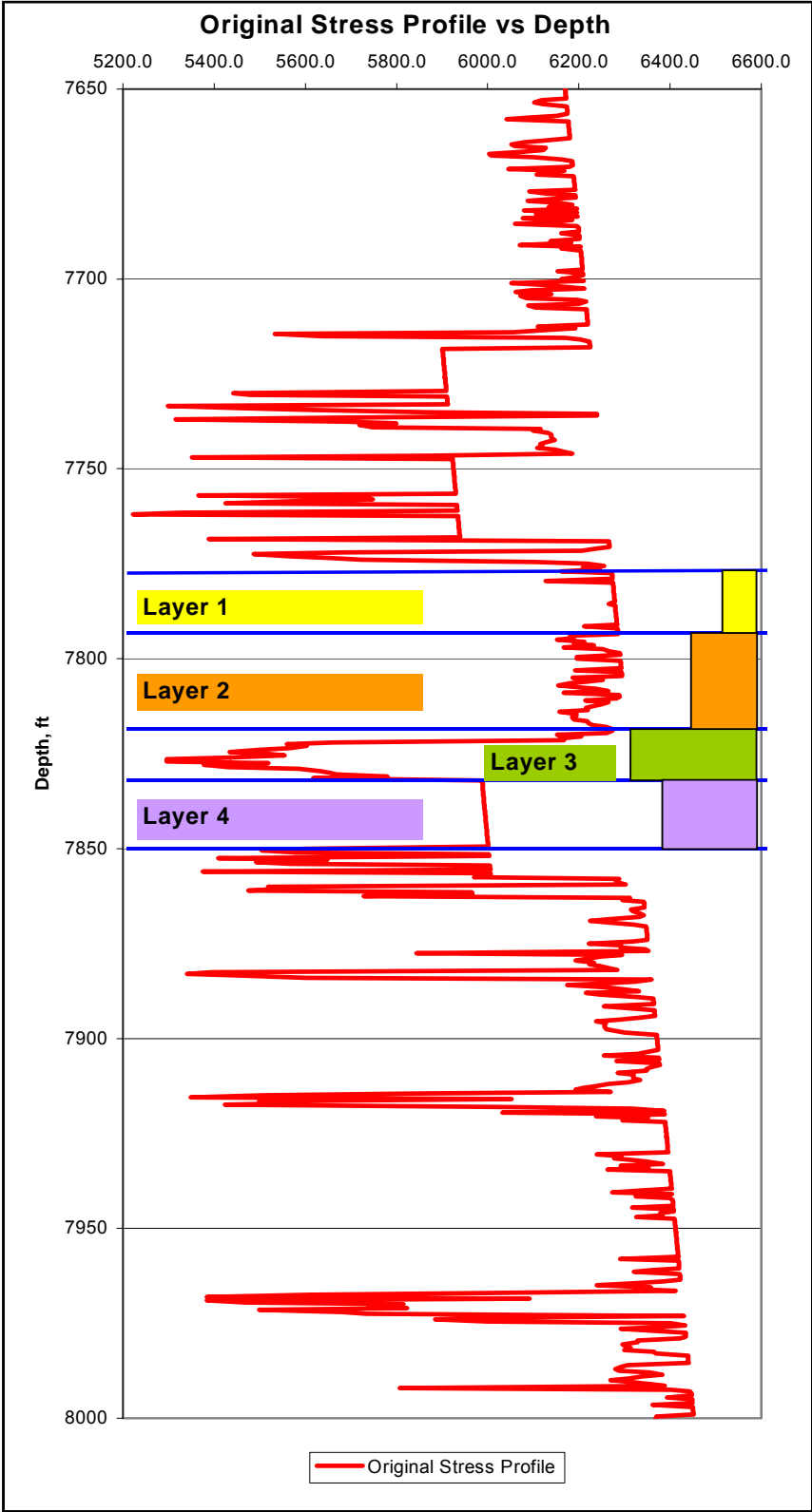


Figure 5-5: Processing Methodology for Original Stress Profile

Layer 1		Layer 2		Layer 3		Layer 4	
Depth, ft	Closure Stress, psi	Depth, ft	Closure Stress, psi	Depth, ft	Closure Stress, psi	Depth, ft	Closure Stress, psi
7777	6161	7794	6181	7821.5	6167	7832.5	5988
7777.5	6273	7794.5	6188	7822	5662	7833	5988
7778	6273	7795	6152	7822.5	5559	7833.5	5989
7778.5	6273	7795.5	6212	7823	5604	7834	5989
7779	6274	7796	6189	7823.5	5567	7834.5	5989
7779.5	6127	7796.5	6232	7824	5501	7835	5990
7780	6275	7797	6167	7824.5	5435	7835.5	5990
7780.5	6275	7797.5	6253	7825	5516	7836	5991
7781	6275	7798	6266	7825.5	5554	7836.5	5991
7781.5	6276	7798.5	6289	7826	5436	7837	5991
7782	6276	7799	6290	7826.5	5296	7837.5	5992
7782.5	6277	7799.5	6195	7827	5296	7838	5992
7783	6277	7800	6196	7827.5	5518	7838.5	5993
7783.5	6277	7800.5	6291	7828	5378	7839	5993
7784	6278	7801	6292	7828.5	5430	7839.5	5993
7784.5	6278	7801.5	6292	7829	5586	7840	5994
7785	6279	7802	6292	7829.5	5631	7840.5	5994
7785.5	6266	7802.5	6293	7830	5653	7841	5994
7786	6279	7803	6192	7830.5	5668	7841.5	5995
7786.5	6280	7803.5	6294	7831	5780	7842	5995
7787	6280	7804	6294	7831.5	5617	7842.5	5996
7787.5	6281	7804.5	6294	7832	5988	7843	5996
7788	6281	7805	6187			7843.5	5996
7788.5	6281	7805.5	6253			7844	5997
7789	6282	7806	6220			7844.5	5997
7789.5	6282	7806.5	6181			7845	5998
7790	6283	7807	6155			7845.5	5998
7790.5	6283	7807.5	6195			7846	5998
7791	6283	7808	6238			7846.5	5999
7791.5	6212	7808.5	6265			7847	5999
7792	6284	7809	6167			7847.5	5999
7792.5	6285	7809.5	6289			7848	6000
7793	6285	7810	6289			7848.5	6000
7793.5	6285	7810.5	6283			7849	6001
		7811	6214			7849.5	6001
		7811.5	6264			7850	5601
		7812	6248				
		7812.5	6232				
		7813	6216				
		7813.5	6220				
		7814	6158				
		7814.5	6194				
		7815	6194				
		7815.5	6185				
		7816	6189				
		7816.5	6219				
		7817	6222				
		7817.5	6229				
		7818	6259				
		7818.5	6273				
		7819	6273				
		7819.5	6260				
		7820	6152				
		7820.5	6205				
		7821	6160				
Interval, ft	Average Closure Stress, psi	Interval, ft	Average Closure Stress, psi	Interval, ft	Average Closure Stress, psi	Interval, ft	Average Closure Stress, psi
7777 - 7793.5	6269	7794 - 7821	6230	7821.5 - 7832	5584	7832.5 - 7850	5984

Table 5-5: Example of Stress Profile Processing for Four Layers

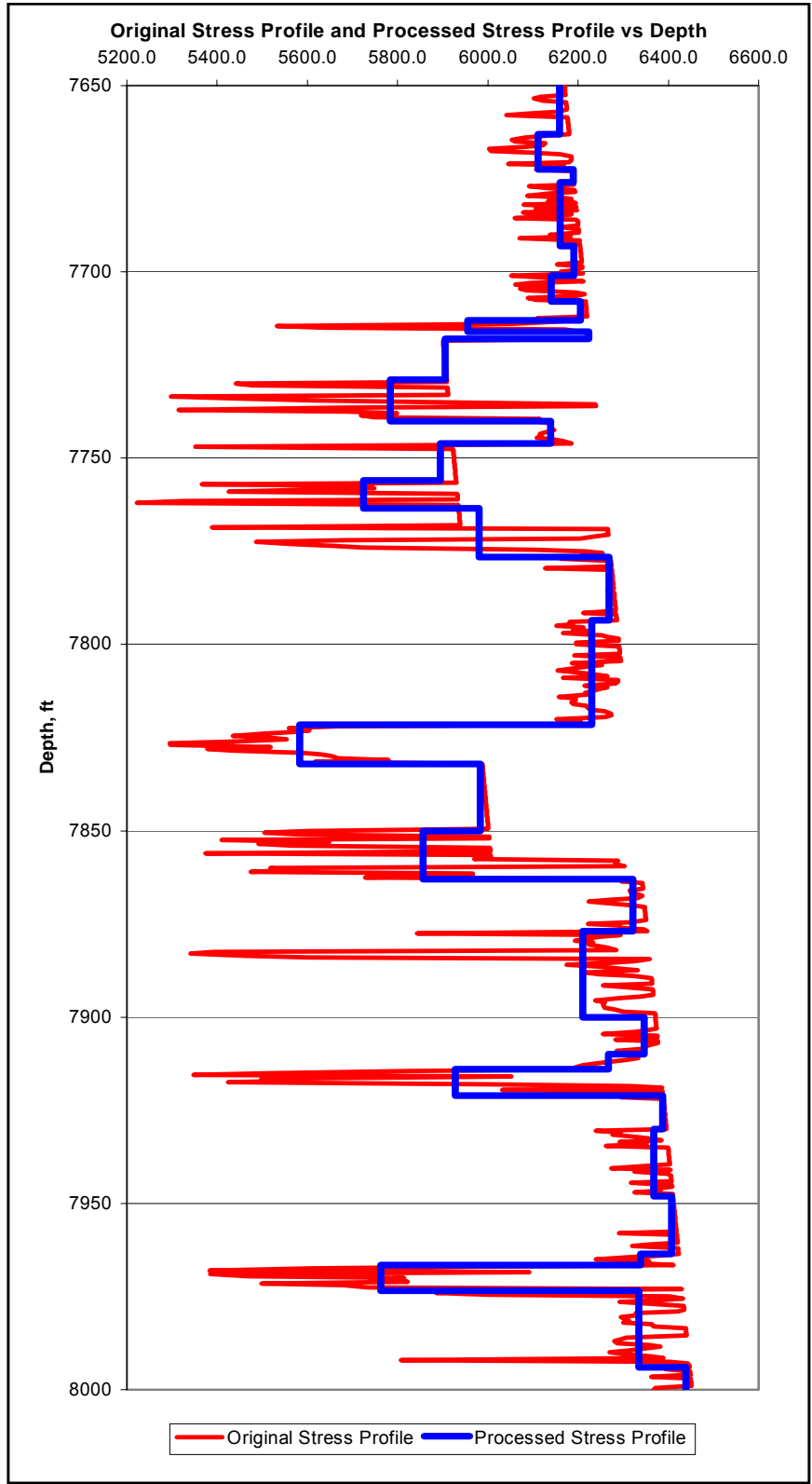


Figure 5-6: Original Stress Profile and Processed Stress Profile

The final processed stress profile consists of 32 layers and is displayed in Table 5-6. Column two shows the depth interval of each layer while column three gives the average closure stress values of each layer.

Layer Index	Interval Depth, ft	Closure Stress, psi
Layer 1	7650 - 7663	6159
Layer 2	7663 - 7672.5	6111
Layer 3	7672.5 - 7676	6189
Layer 4	7676 - 7693	6161
Layer 5	7693 - 7701	6190
Layer 6	7701 - 7708	6140
Layer 7	7708 - 7713	6205
Layer 8	7713 - 7716	5955
Layer 9	7716 - 7718	6224
Layer 10	7718 - 7729	5905
Layer 11	7729 - 7740	5784
Layer 12	7740 - 7746	6139
Layer 13	7746 - 7756	5895
Layer 14	7756 - 7763.5	5724
Layer 15	7763.5 - 7776.5	5980
Layer 16	7776.5 - 7793.5	6269
Layer 17	7793.5 - 7821.5	6230
Layer 18	7821.5 - 7832	5584
Layer 19	7832 - 7850	5984
Layer 20	7850 - 7863	5856
Layer 21	7863 - 7877	6321
Layer 22	7877 - 7900	6211
Layer 23	7900 - 7910	6347
Layer 24	7910 - 7914	6267
Layer 25	7914 - 7921	5928
Layer 26	7921 - 7930	6387
Layer 27	7930 - 7948	6367
Layer 28	7948 - 7963.5	6407
Layer 29	7963.5 - 7966.5	6339
Layer 30	7966.5 - 7973.5	5763
Layer 31	7973.5 - 7994	6334
Layer 32	7994 - 8000	6438

Table 5-6: Example of Stress Profile Processing for Four Layers

In Figure 5-7, the equivalent four-layer stress profile concludes the first module covering reservoir characterization for fracturing simulation. The equivalent four-layer profile is the main input to the optimization module and the net pressure prediction module.

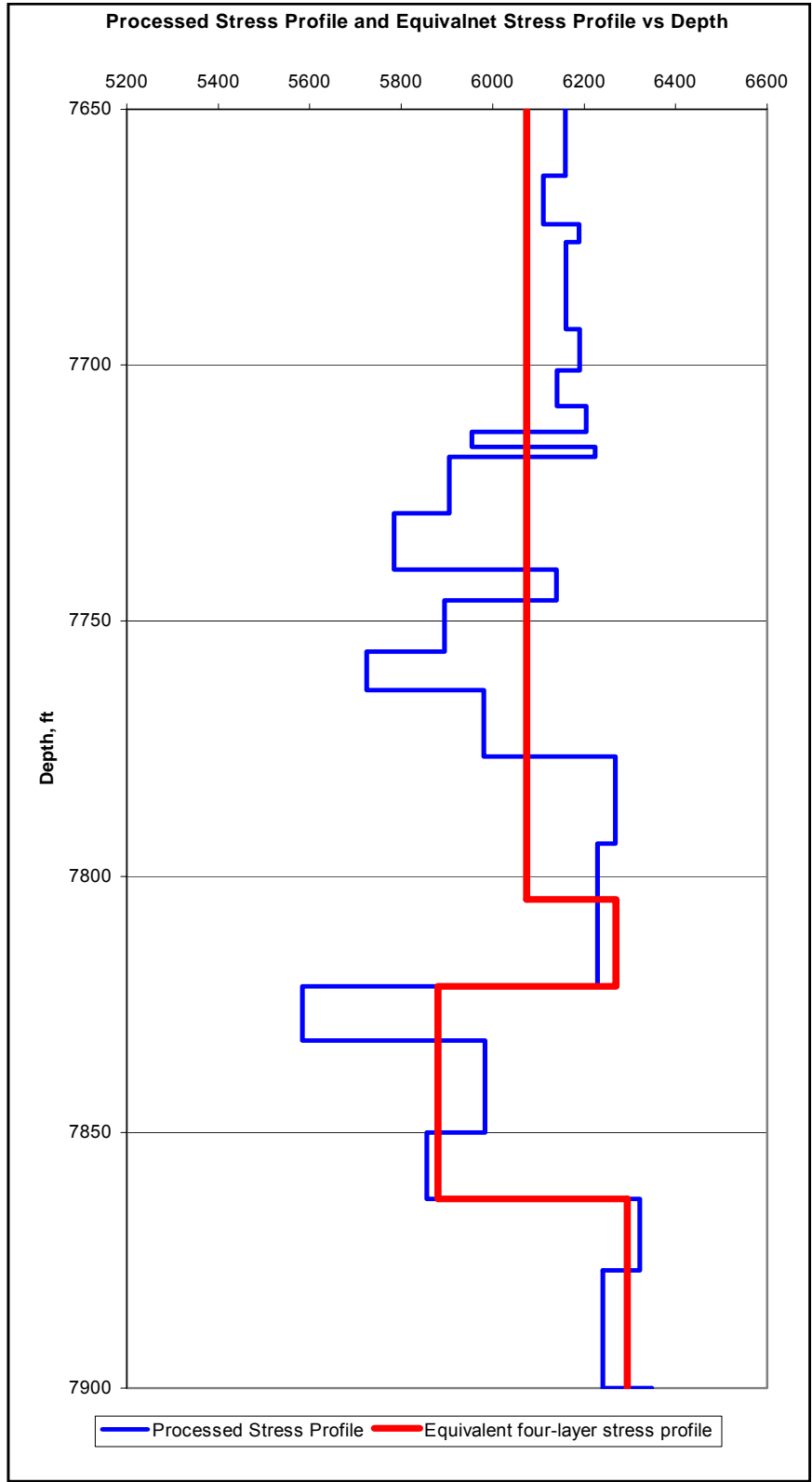


Figure 5-7: Processed Stress Profile and Equivalent Four-Layer Stress Profile

5.2. Neural Network Fracturing Model Accuracy

Two types of fracture treatment design were developed in this study in order to demonstrate the capability of the tool while comprehensively addressing all possible variations. These are the ramp treatment design and stage treatment design. The total number of five neural networks was trained as follows:

- ▶ One neural net for ramp treatment design
- ▶ One neural network for six stage treatment design
- ▶ Three neural networks for three types of eight stage treatment design

The criteria for an acceptable accurate neural network were measured using R^2 and the correlation coefficient. Values higher than 0.8 for R^2 are considered acceptable, while values greater than 0.9 for correlation coefficient show a good agreement in trend between actual values and predicted. In an ideal case, R^2 and correlation coefficient are equal to 1, where the actual values and the predicted values would fall on the 45-degree line. This study illustrates the performance of the neural network, by the closeness of points to the 45° line.

5.2.1. Neural Network for Ramp Treatment

The architecture of the neural network for ramp treatment has three hidden layers with different activation functions, 14 inputs and 6 outputs as described in detail in 4.12.1 Developing Neural Networks for Ramp Treatments. Figure 5-8 through Figure 5-25 show the correlation between the actual FRACPRO values and the neural network predicted values of the training set, calibration set, and verification set for each of the 6 output parameters (fracture efficiency, propped fracture length, proppant concentration, dimensionless conductivity ratio, max fracture width, and fracture height).

Figure 5-8 shows the training set results for fracture efficiency. R^2 is 0.89, r^2 is 0.89, and the correlation coefficient is 0.94.

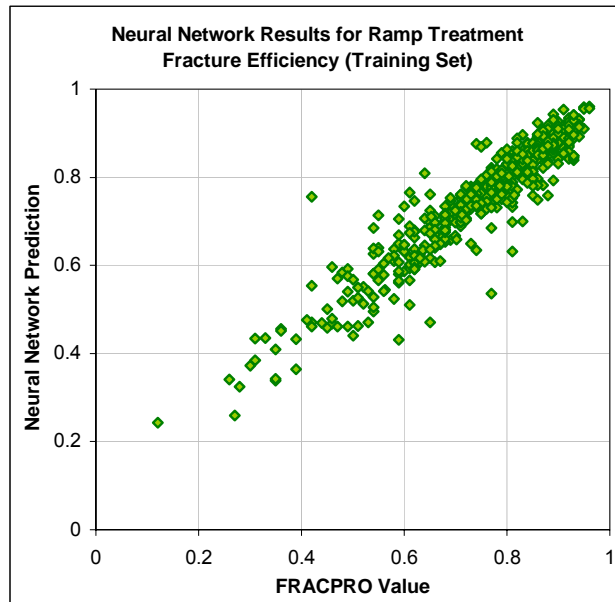


Figure 5-8: Ramp Treatment Training Set Results for Fracture Efficiency

Figure 5-9 shows the calibration set results for fracture efficiency. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.91.

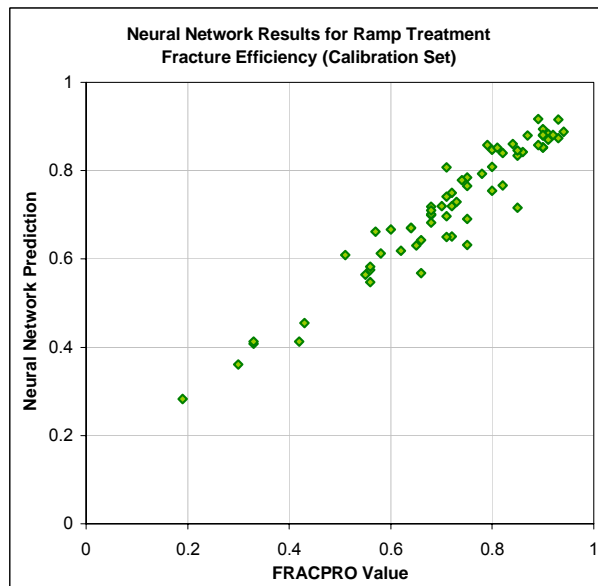


Figure 5-9: Ramp Treatment Calibration Set Results for Fracture Efficiency

Figure 5-10 shows the verification set results for fracture efficiency. R^2 is 0.91, r^2 is 0.92, and the correlation coefficient is 0.96.

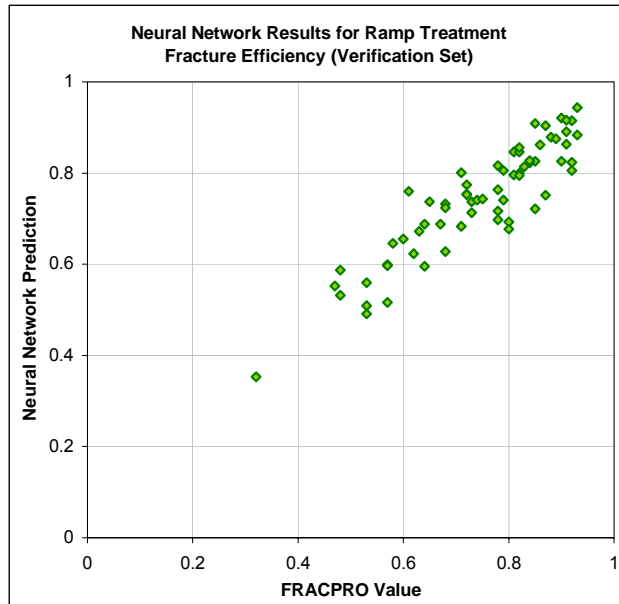


Figure 5-10: Ramp Treatment Verification Set Results for Fracture Efficiency

Figure 5-11 shows the training set results for propped fracture length. R^2 is 0.92, r^2 is 0.92, and the correlation coefficient is 0.96.

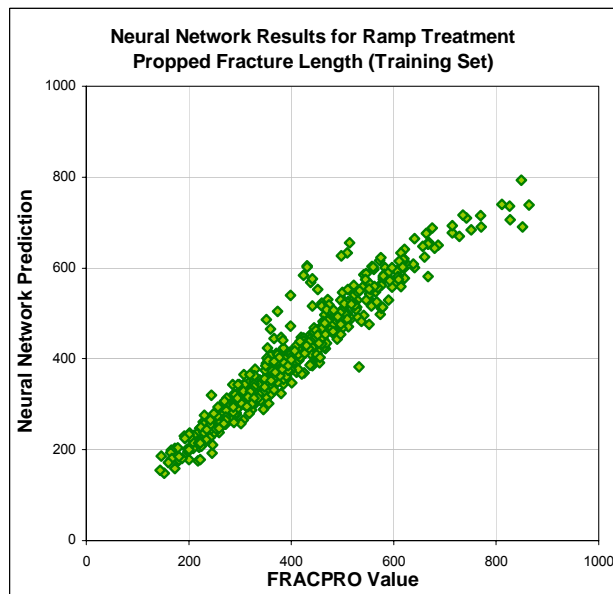


Figure 5-11: Ramp Treatment Training Set Results for Propped Fracture Length

Figure 5-12 shows the calibration set results for propped fracture length. R^2 is 0.90, r^2 is 0.91, and the correlation coefficient is 0.95.

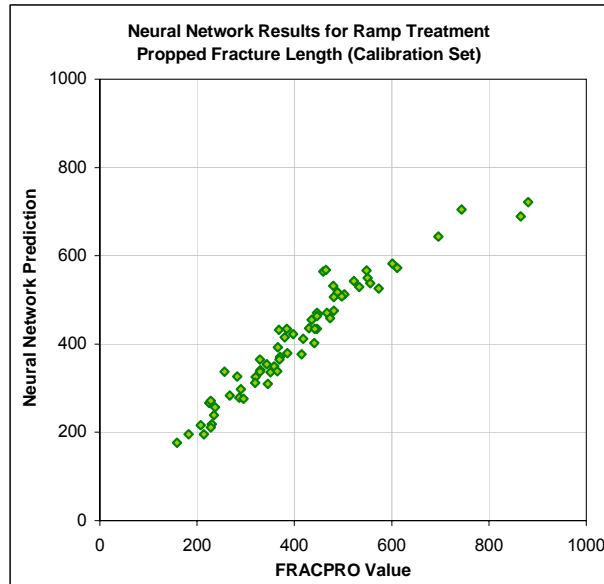


Figure 5-12: Ramp Treatment Calibration Set Results for Propped Fracture Length

Figure 5-13 shows the verification set results for propped fracture length. R^2 is 0.90, r^2 is 0.89, and the correlation coefficient is 0.94.

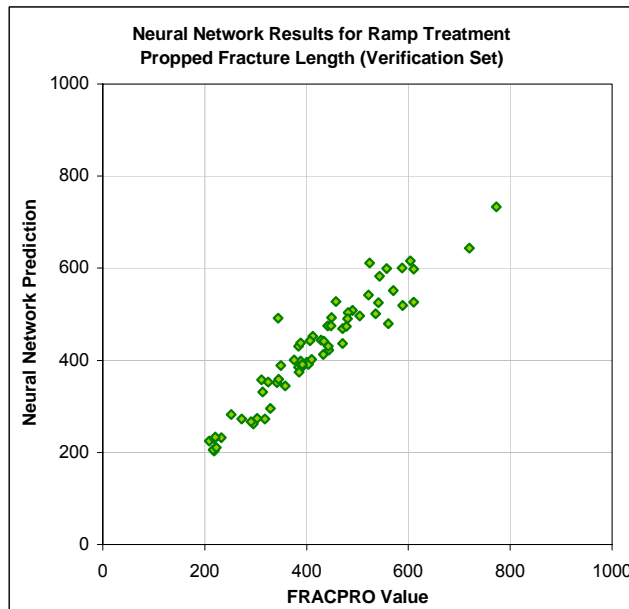


Figure 5-13: Ramp Treatment Verification Set Results for Propped Fracture Length

Figure 5-14 shows the training set results for proppant concentration. R^2 is 0.85, r^2 is 0.85, and the correlation coefficient is 0.92.

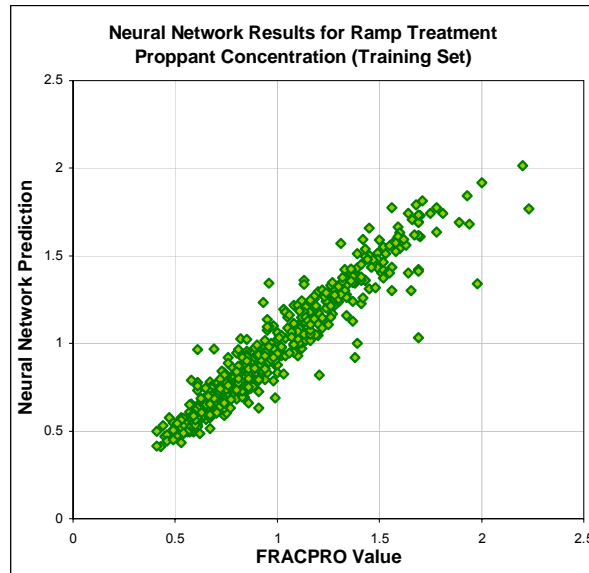


Figure 5-14: Ramp Treatment Training Set Results for Proppant Concentration

Figure 5-15 shows the calibration set results for proppant concentration. R^2 is 0.78, r^2 is 0.79, and the correlation coefficient is 0.89.

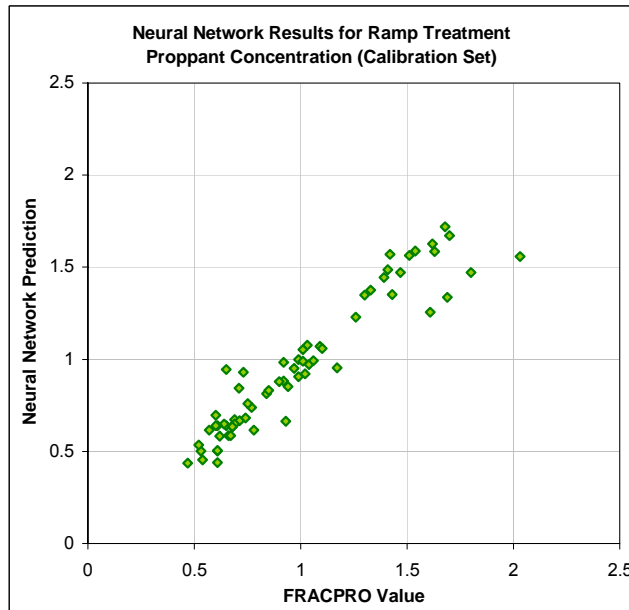


Figure 5-15: Ramp Treatment Calibration Set Results for Proppant Concentration

Figure 5-16 shows the verification set results for proppant concentration. R^2 is 0.83, r^2 is 0.84, and the correlation coefficient is 0.92.

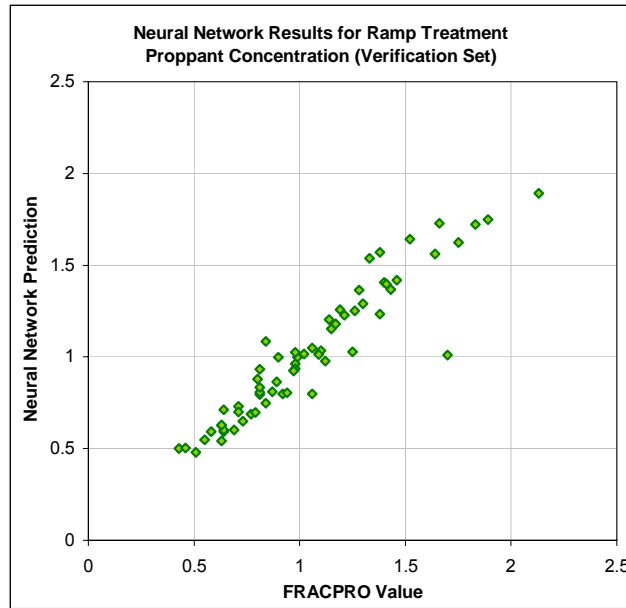


Figure 5-16: Ramp Treatment Verification Set Results for Proppant Concentration

Figure 5-17 shows the training set results for dimensionless conductivity ratio. R^2 is 0.87, r^2 is 0.88, and the correlation coefficient is 0.94.

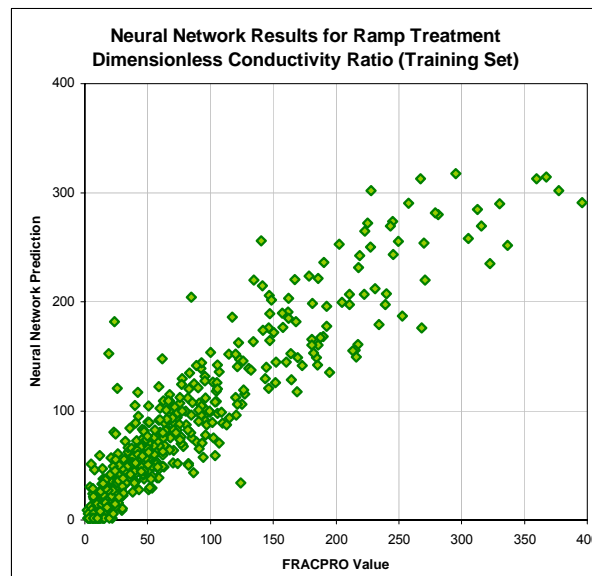


Figure 5-17: Ramp Treatment Training Set Results for Dimensionless Conductivity Ratio

Figure 5-18 shows the calibration set results for dimensionless conductivity ratio. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.91.

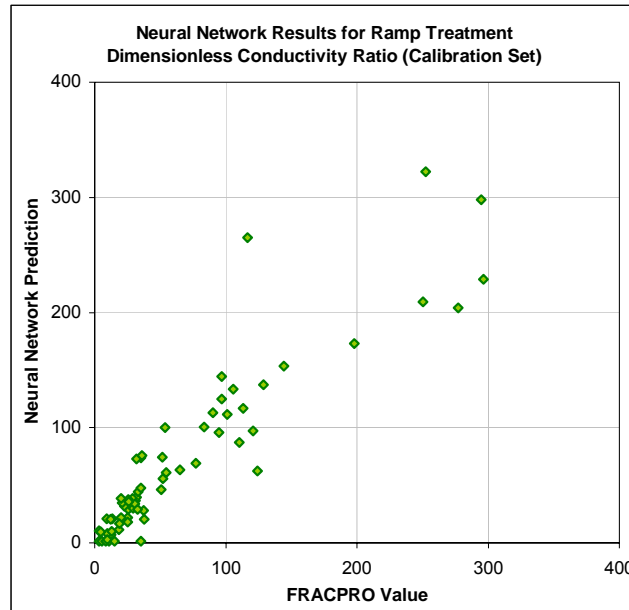


Figure 5-18: Ramp Treatment Calibration Set Results for Dimensionless Conductivity Ratio

Figure 5-19 shows the verification set results for dimensionless conductivity ratio. R^2 is 0.87, r^2 is 0.87, and the correlation coefficient is 0.93.

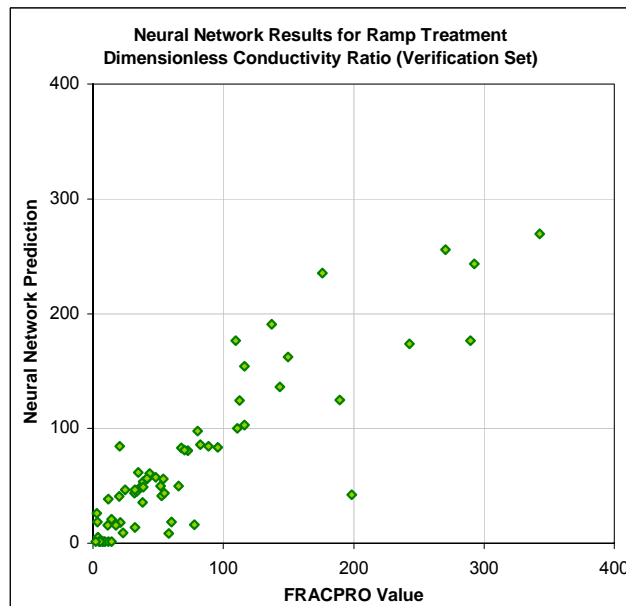


Figure 5-19: Ramp Treatment Verification Set Results for Dimensionless Conductivity Ratio

Figure 5-20 shows the training set results for maximum fracture width. R^2 is 0.95, r^2 is 0.95, and the correlation coefficient is 0.97.

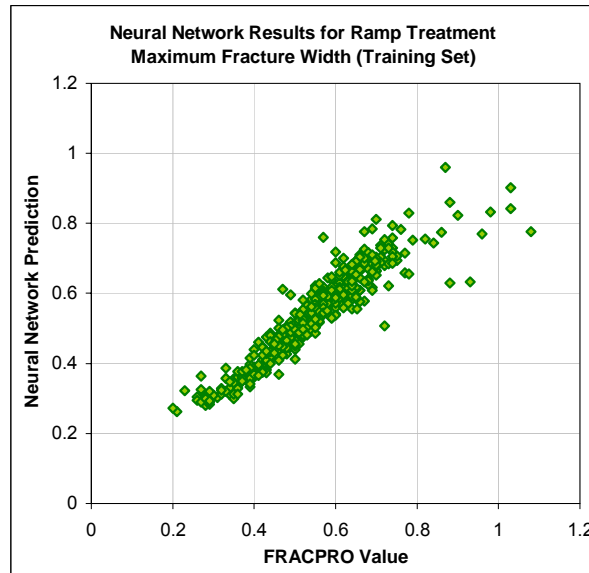


Figure 5-20: Ramp Treatment Training Set Results for Maximum Fracture Width

Figure 5-21 shows the calibration set results for maximum fracture width. R^2 is 0.95, r^2 is 0.95, and the correlation coefficient is 0.97.

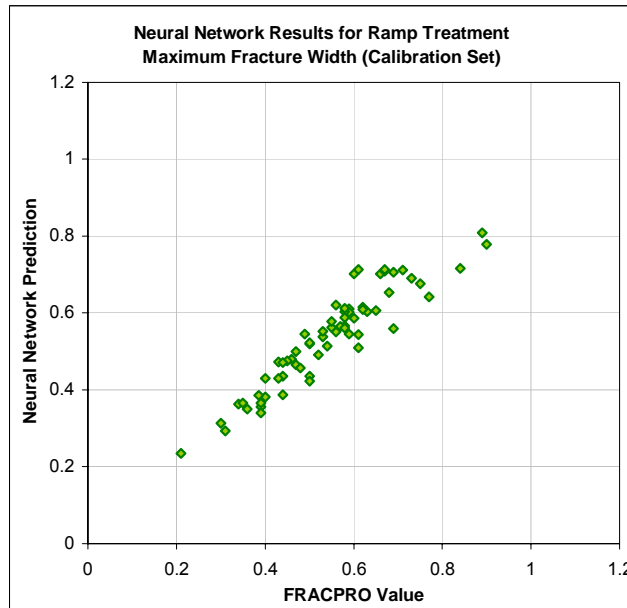


Figure 5-21: Ramp Treatment Calibration Set Results for Maximum Fracture Width

Figure 5-22 shows the verification set results for maximum fracture length. R^2 is 0.95, r^2 is 0.95, and the correlation coefficient is 0.98.

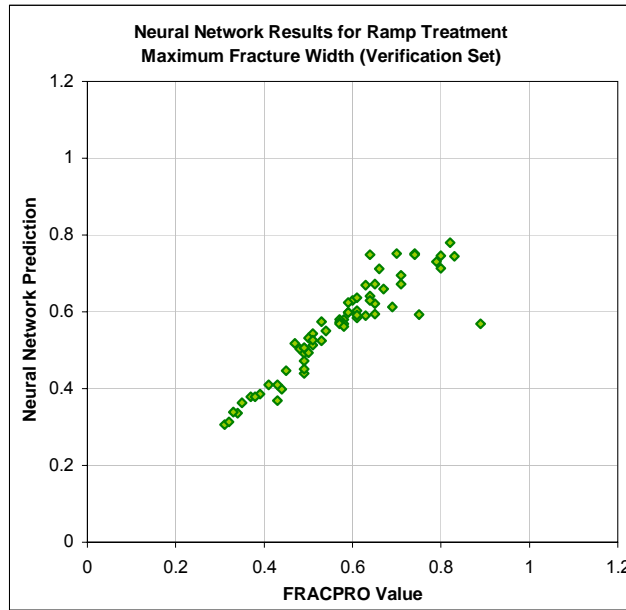


Figure 5-22: Ramp Treatment Verification Set Results for Maximum Fracture Width

Figure 5-23 shows the training set results for fracture height. R^2 is 0.91, r^2 is 0.91, and the correlation coefficient is 0.95.

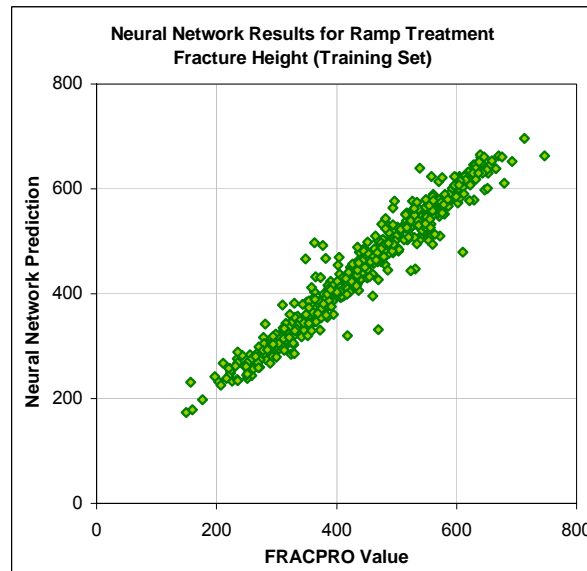


Figure 5-23: Ramp Treatment Training Set Results for Fracture Height

Figure 5-24 shows the calibration set results for fracture height. R^2 is 0.88, r^2 is 0.89, and the correlation coefficient is 0.94.

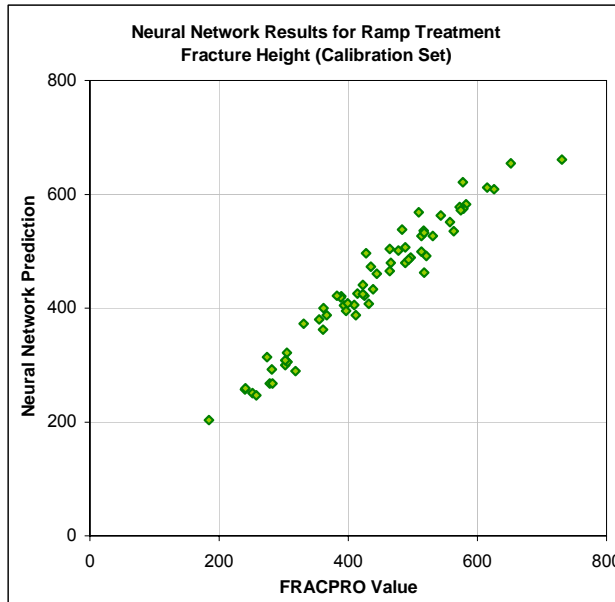


Figure 5-24: Ramp Treatment Calibration Set Results for Fracture Height

Figure 5-25 shows the verification set results for fracture height. R^2 is 0.89, r^2 is 0.90, and the correlation coefficient is 0.95.

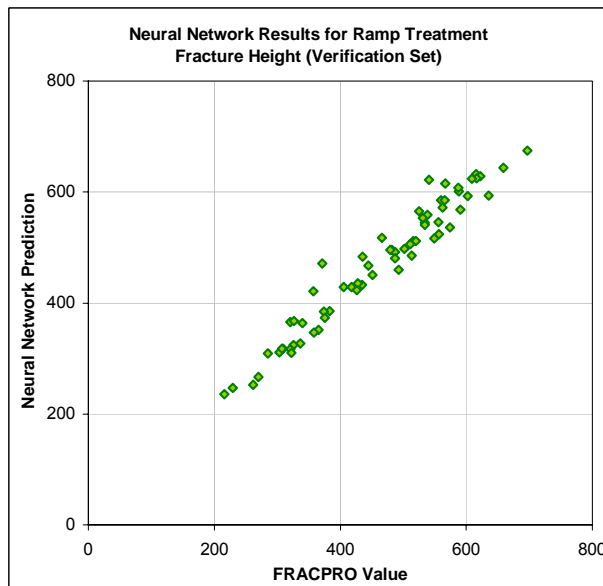


Figure 5-25: Ramp Treatment Verification Set Results for Fracture Height

The details of the results for each of the six output parameters in the training, calibration, and verification sets can be seen in Table 5-7.

	Fracture Efficiency	Propped Fracture Length	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width	Fracture Height
Training Set (528 cases)						
R ²	0.8866	0.9236	0.8463	0.8737	0.9454	0.9063
r ²	0.8888	0.9237	0.8543	0.8758	0.9475	0.9109
Correlation Coefficient, r	0.9428	0.9611	0.9243	0.9358	0.9734	0.9544
Calibration Set (66 cases)						
R ²	0.8319	0.9047	0.7841	0.8264	0.9450	0.8814
r ²	0.8326	0.9062	0.7866	0.8332	0.9500	0.8883
Correlation Coefficient, r	0.9125	0.9519	0.8869	0.9128	0.9747	0.9425
Verification Set (66 cases)						
R ²	0.9143	0.9028	0.8268	0.8686	0.9504	0.8914
r ²	0.9213	0.8869	0.8391	0.8731	0.9544	0.9006
Correlation Coefficient, r	0.9598	0.9418	0.9160	0.9344	0.9769	0.9490

Table 5-7: Ramp Treatment Neural Network Results

Fracture length and fracture width exhibit the best performance in prediction. This is excellent, particularly since this study addresses tight gas formations where the design optimization criteria is fracture length. Fracture height and fracture efficiency closely follow with regards to performance. Proppant concentration and dimensionless conductivity ratio show a little lower performance than fracture length; however, both have strong correlation coefficients and values above 0.8 for R². As seen in the previous figures for fracture length, fracture height, and fracture width, there is good agreement between the points and the 45° line.

The training and calibration sets for dimensionless conductivity ratio, Figure 5-17 and Figure 5-18, show sparser values around the 45° line, compared to other parameters, while the verification set shows poor prediction performance at high values (Figure 5-19). However, current performance was the best achieved and was used in the optimization module.

5.2.2. Neural Network for Six Stage Treatment

The architecture of the neural network for six stage treatments has three hidden layers with different activation functions, 17 inputs and 6 outputs as described in detail in 4.12.2.1 The Six Stage Treatment. Figure 5-26 through Figure 5-43 show the correlation between the actual FRACPRO values and the neural network predicted values of the training set, calibration set, and verification set for each of the 6 output parameters (fracture efficiency, propped fracture length, proppant concentration, dimensionless conductivity ratio, max fracture width, and fracture height).

Figure 5-26 shows the training set results for fracture efficiency. R^2 is 0.84, r^2 is 0.86, and the correlation coefficient is 0.93.

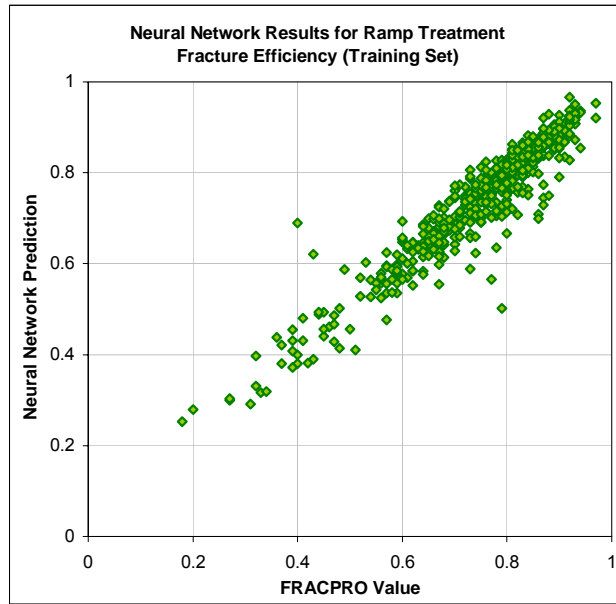


Figure 5-26: Six Stage Treatment Training Set Results for Fracture Efficiency

Figure 5-27 shows the calibration set results for fracture efficiency. R^2 is 0.81, r^2 is 0.82, and the correlation coefficient is 0.90.

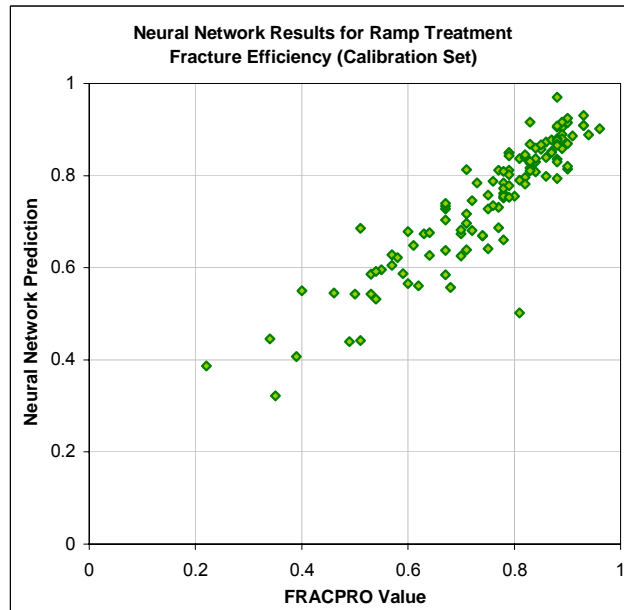


Figure 5-27: Six Stage Treatment Calibration Set Results for Fracture Efficiency

Figure 5-28 shows the verification set results for fracture efficiency. R^2 is 0.89, r^2 is 0.90, and the correlation coefficient is 0.95.

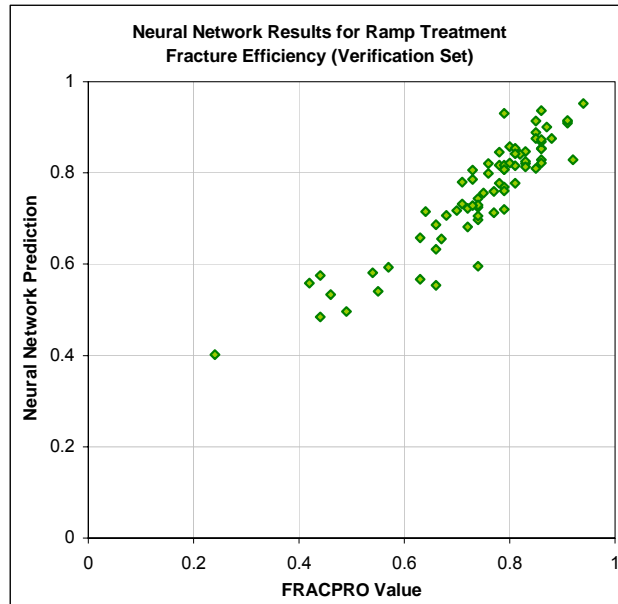


Figure 5-28: Six Stage Treatment Verification Set Results for Fracture Efficiency

Figure 5-29 shows the training set results for propped fracture length. R^2 is 0.90, r^2 is 0.89, and the correlation coefficient is 0.95.

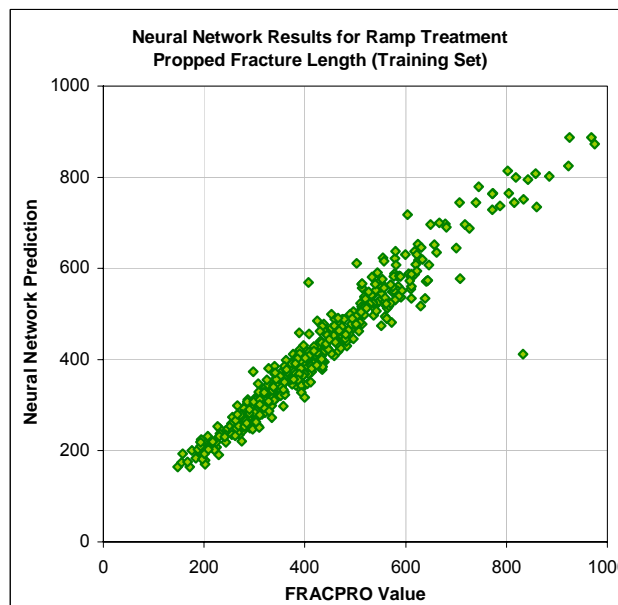


Figure 5-29: Six Stage Treatment Training Set Results for Propped Fracture Length

Figure 5-30 shows the calibration set results for propped fracture length. R^2 is 0.87, r^2 is 0.87, and the correlation coefficient is 0.93.

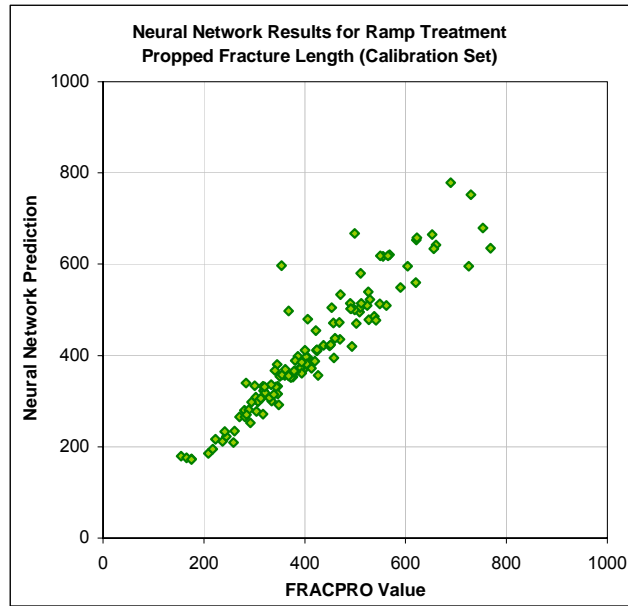


Figure 5-30: Six Stage Treatment Calibration Set Results for Propped Fracture Length

Figure 5-31 shows the verification set results for propped fracture length. R^2 is 0.93, r^2 is 0.93, and the correlation coefficient is 0.97.

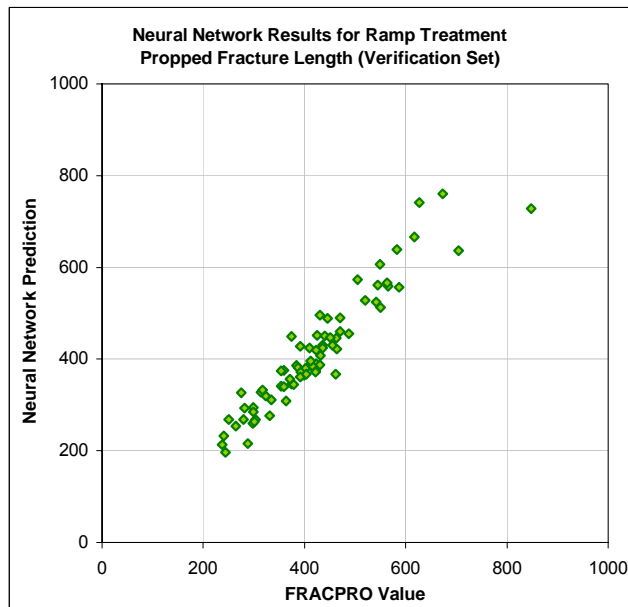


Figure 5-31: Six Stage Treatment Verification Set Results for Propped Fracture Length

Figure 5-32 shows the training set results for proppant concentration. R^2 is 0.88, r^2 is 0.89, and the correlation coefficient is 0.94.

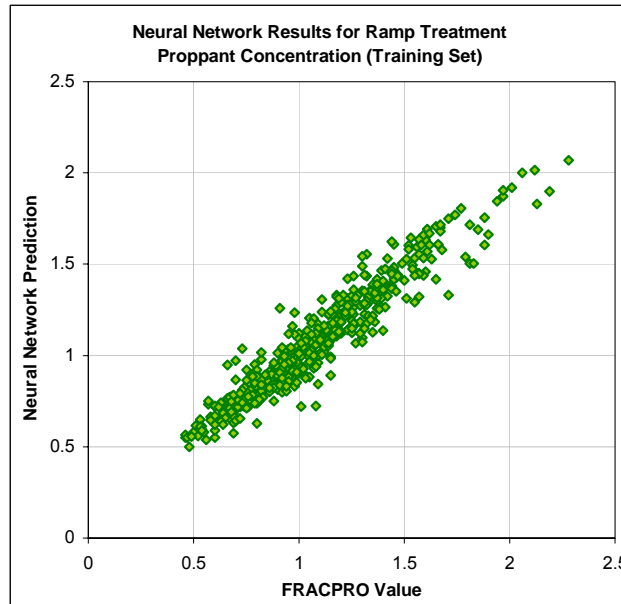


Figure 5-32: Six Stage Treatment Training Set Results for Proppant Concentration

Figure 5-33 shows the calibration set results for proppant concentration. R^2 is 0.83, r^2 is 0.84, and the correlation coefficient is 0.92.

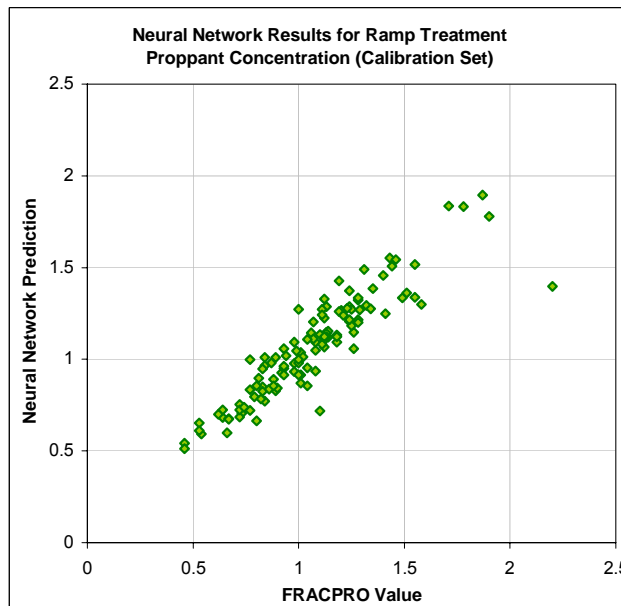


Figure 5-33: Six Stage Treatment Calibration Set Results for Proppant Concentration

Figure 5-34 shows the verification set results for proppant concentration. R^2 is 0.89, r^2 is 0.91, and the correlation coefficient is 0.95.

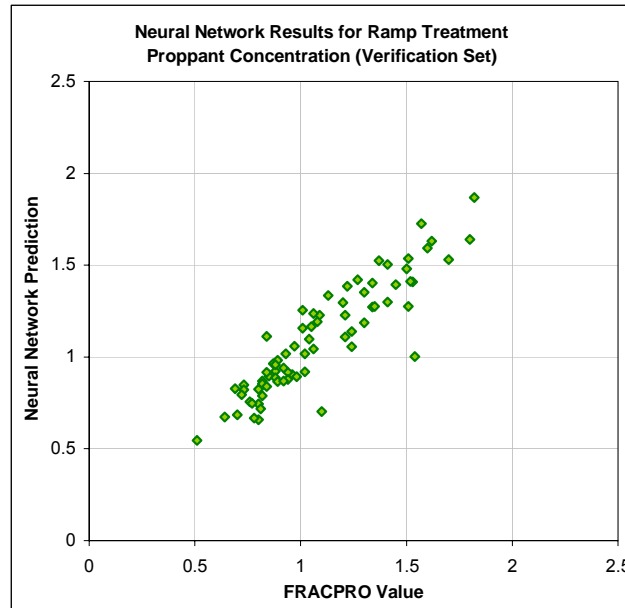


Figure 5-34: Six Stage Treatment Verification Set Results for Proppant Concentration

Figure 5-35 shows the training set results for dimensionless conductivity ratio. R^2 is 0.80, r^2 is 0.81, and the correlation coefficient is 0.90.

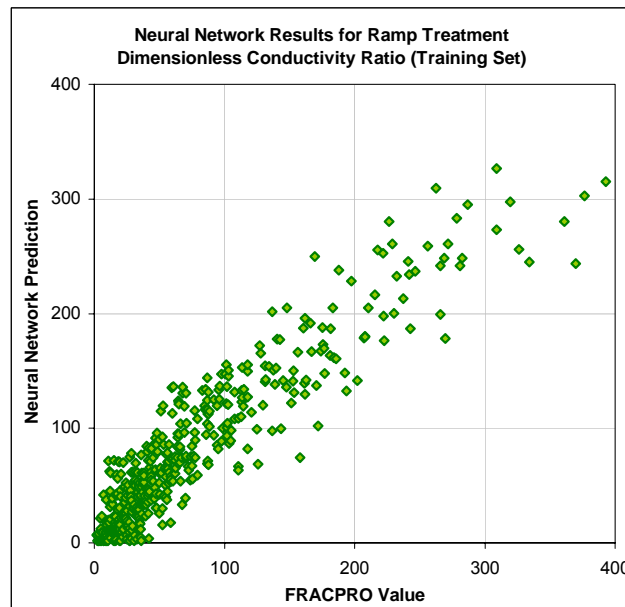


Figure 5-35: Six Stage Treatment Training Set Results for Dimensionless Conductivity Ratio

Figure 5-36 shows the calibration set results for dimensionless conductivity ratio. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.85.

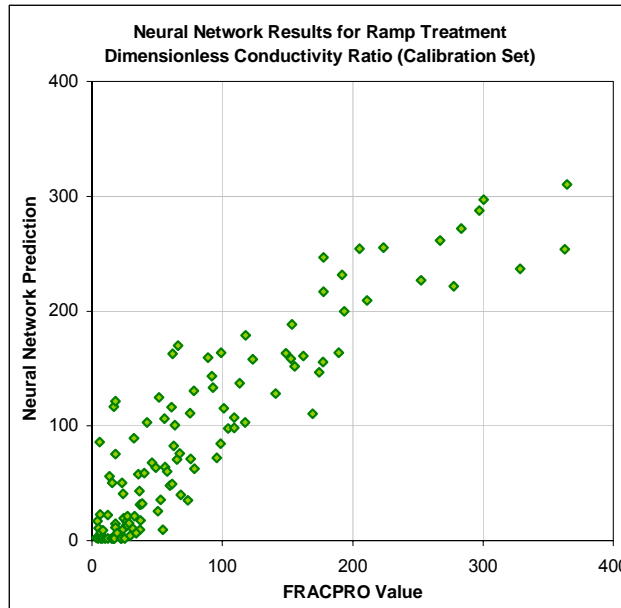


Figure 5-36: Six Stage Treatment Calibration Set Results for Dimensionless Conductivity Ratio

Figure 5-37 shows the verification set results for dimensionless conductivity ratio. R^2 is 0.80, r^2 is 0.80, and the correlation coefficient is 0.86.

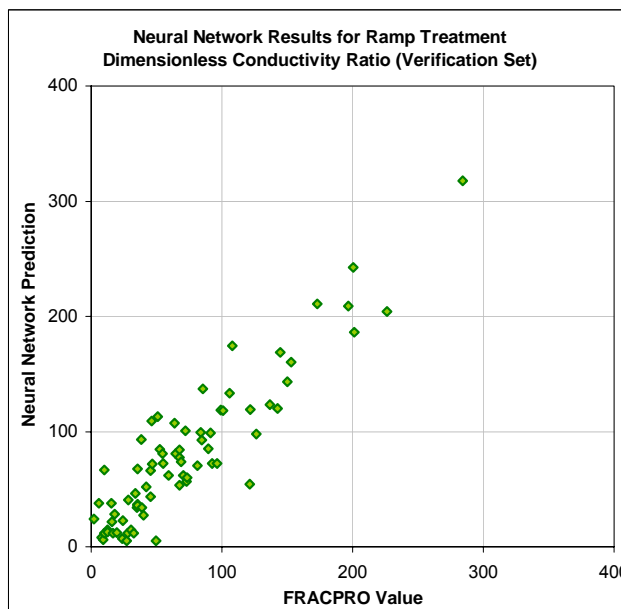


Figure 5-37: Six Stage Treatment Verification Set Results for Dimensionless Conductivity Ratio

Figure 5-38 shows the training set results for maximum fracture length. R^2 is 0.84, r^2 is 0.84, and the correlation coefficient is 0.92.

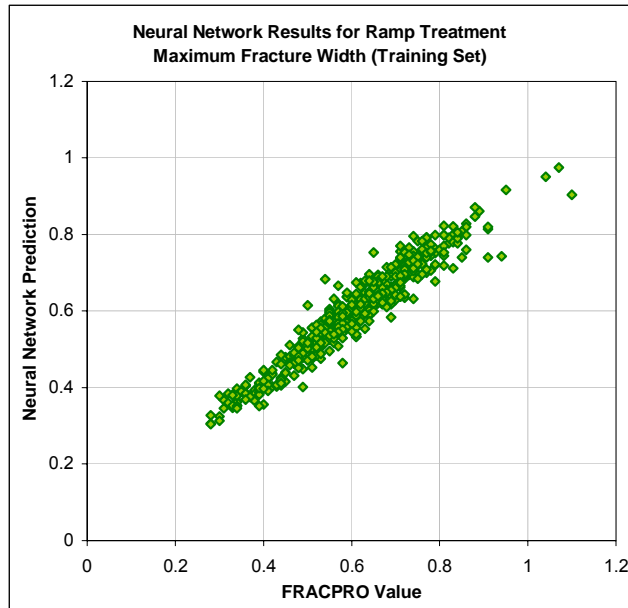


Figure 5-38: Six Stage Treatment Training Set Results for Maximum Fracture Width

Figure 5-39 shows the calibration set results for maximum fracture width. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.91.

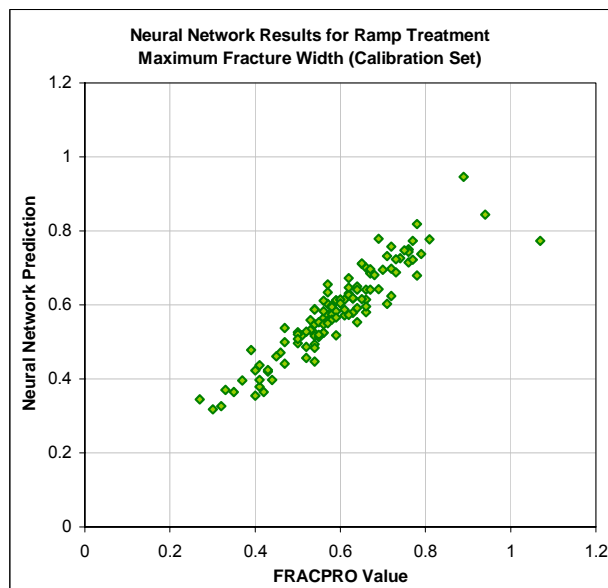


Figure 5-39: Six Stage Treatment Calibration Set Results for Maximum Fracture Width

Figure 5-40 shows the verification set results for maximum fracture width. R^2 is 0.85, r^2 is 0.86, and the correlation coefficient is 0.93.

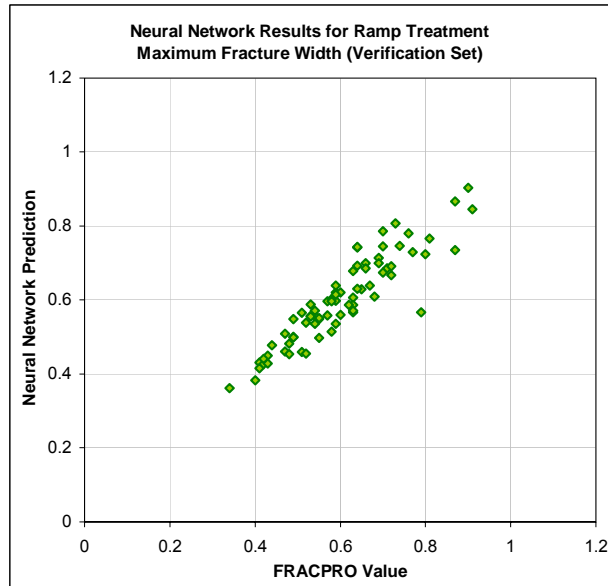


Figure 5-40: Six Stage Treatment Verification Set Results for Maximum Fracture Width

Figure 5-41 shows the training set results for fracture height. R^2 is 0.94, r^2 is 0.94, and the correlation coefficient is 0.97.

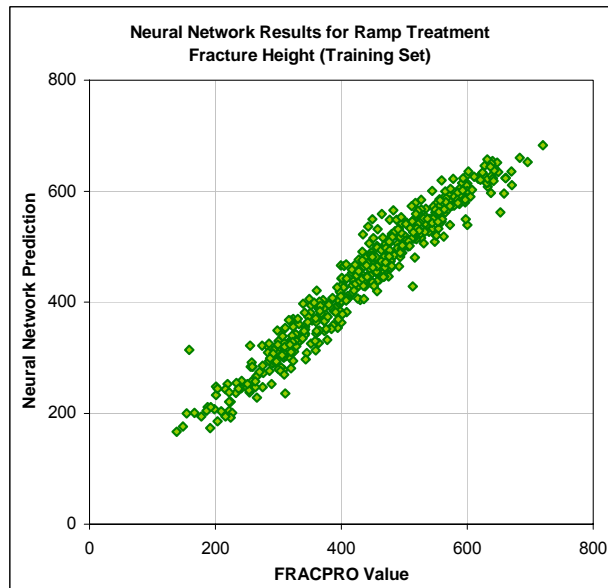


Figure 5-41: Six Stage Treatment Training Set Results for Fracture Height

Figure 5-42 shows the calibration set results for fracture height. R^2 is 0.91, r^2 is 0.91, and the correlation coefficient is 0.95.

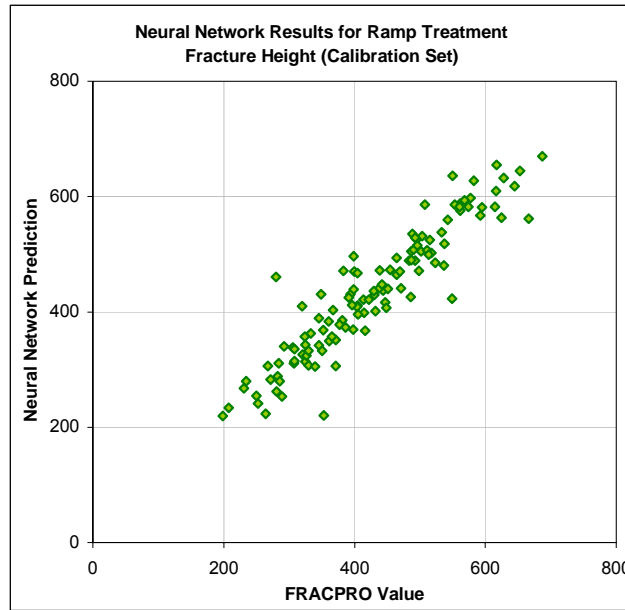


Figure 5-42: Six Stage Treatment Calibration Set Results for Fracture Height

Figure 5-43 shows the verification set results for fracture height. R^2 is 0.92, r^2 is 0.92, and the correlation coefficient is 0.96.

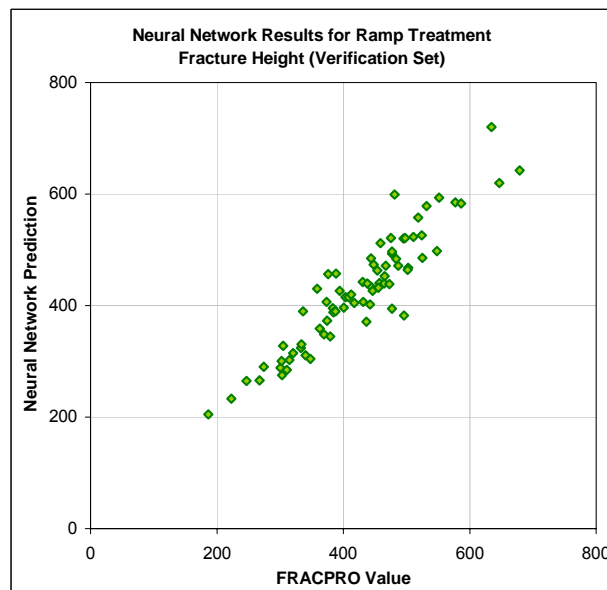


Figure 5-43: Six Stage Treatment Verification Set Results for Fracture Height

The details of the results for each of the six output parameters in the training, calibration, and verification sets can be seen in Table 5-8.

	Fracture Efficiency	Propped Fracture Length	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width	Fracture Height
Training Set (480 cases)						
R ²	0.8440	0.8980	0.8770	0.8030	0.8390	0.9370
r ²	0.8630	0.8900	0.8860	0.8100	0.8410	0.9370
Correlation Coefficient, r	0.9290	0.9480	0.9410	0.9000	0.9170	0.9680
Calibration Set (120 cases)						
R ²	0.8060	0.8650	0.8290	0.8260	0.8270	0.9080
r ²	0.8150	0.8660	0.8410	0.8270	0.8280	0.9090
Correlation Coefficient, r	0.9020	0.9300	0.9190	0.8520	0.9100	0.9520
Verification Set (75 cases)						
R ²	0.8850	0.9310	0.8880	0.8020	0.8520	0.9200
r ²	0.8960	0.9330	0.9080	0.8010	0.8630	0.9210
Correlation Coefficient, r	0.9480	0.9660	0.9530	0.8590	0.9290	0.9590

Table 5-8: Six Stage Treatment Neural Network Results

Fracture length and fracture height exhibit the best performance in prediction for the six stage neural network. The performance for fracture efficiency and proppant concentration follow closely behind. Fracture width and dimensionless conductivity ratio show lower performance than fracture length; however, both have strong correlation coefficients and values above 0.8 for R². The figures for the training, calibration, and verification sets of fracture length and fracture height show values that fall close the 45° line.

The training and calibration sets for dimensionless conductivity ratio, Figure 5-35 and Figure 5-36, show poorer alignment with the 45° line compared to the other parameters. The R² value is lowest for dimensionless conductivity; however, this performance was the best achieved and was used in the optimization module.

5.2.3. Neural Networks for Eight Stage Treatment

The architecture of the neural networks for eight stage treatments has three hidden layers with different activation functions, 17 inputs and 5 outputs as described in detail in 4.12.2.2 The Eight Stage Treatment. Three neural networks were trained for eight stages treatments representing Type I, Type II, and Type III.

5.2.3.1. Type I

Figure 5-44 through Figure 5-58 show the correlation between the actual FRACPRO values and the neural network predicted values of the training set, calibration set, and verification set for each of the 5 output parameters (fracture efficiency, propped fracture length, proppant concentration, max fracture width, and fracture height).

Figure 5-44 shows the training set results for fracture efficiency. R^2 is 0.96, r^2 is 0.96, and the correlation coefficient is 0.98.

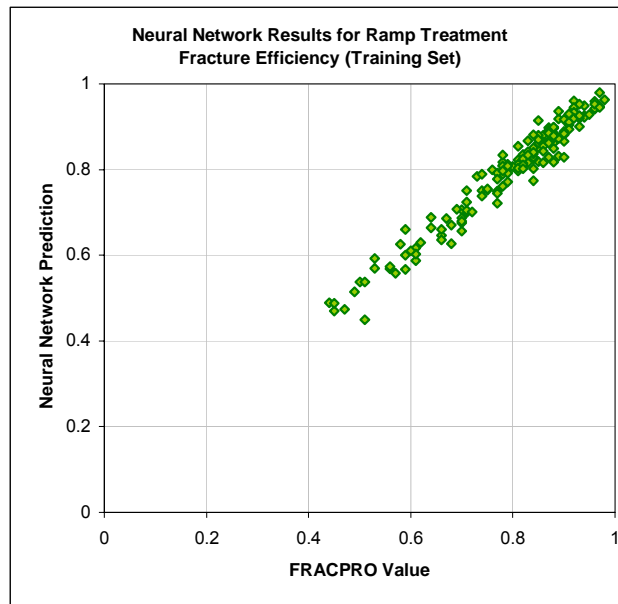


Figure 5-44: Eight Stage Type I Treatment Training Set Results for Fracture Efficiency

Figure 5-45 shows the calibration set results for fracture efficiency. R^2 is 0.76, r^2 is 0.79, and the correlation coefficient is 0.89.

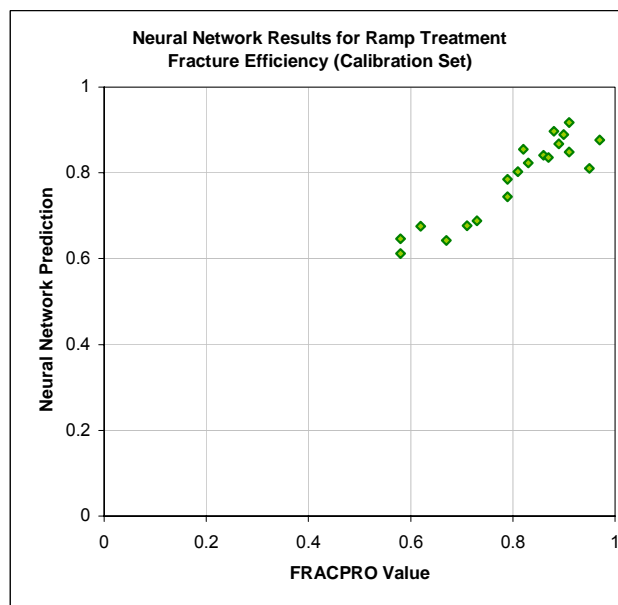


Figure 5-45: Eight Stage Type I Treatment Calibration Set Results for Fracture Efficiency

Figure 5-46 shows the verification set results for fracture efficiency. R^2 is 0.72, r^2 is 0.76, and the correlation coefficient is 0.87.

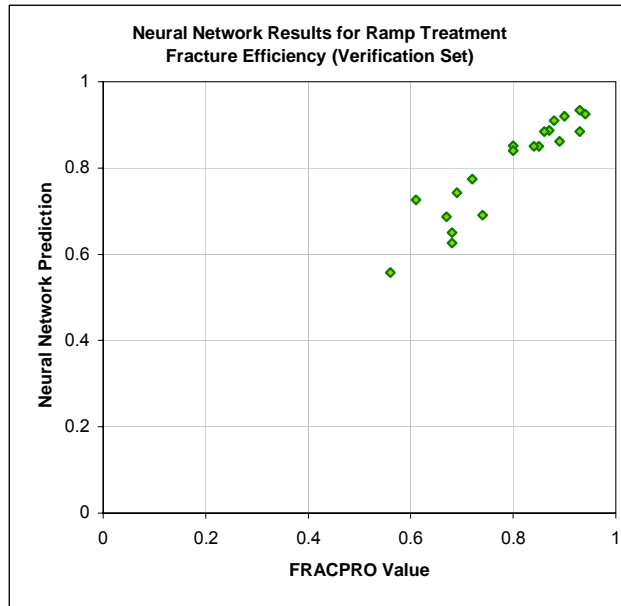


Figure 5-46: Eight Stage Type I Treatment Verification Set Results for Fracture Efficiency

Figure 5-47 shows the training set results for propped fracture length. R^2 is 0.92, r^2 is 0.93, and the correlation coefficient is 0.96.

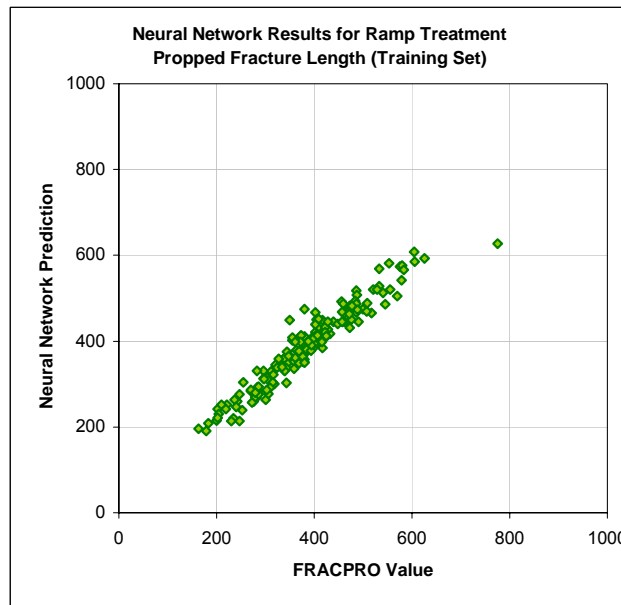


Figure 5-47: Eight Stage Type I Treatment Training Set Results for Propped Fracture Length

Figure 5-48 shows the calibration set results for propped fracture length. R^2 is 0.88, r^2 is 0.89, and the correlation coefficient is 0.95.

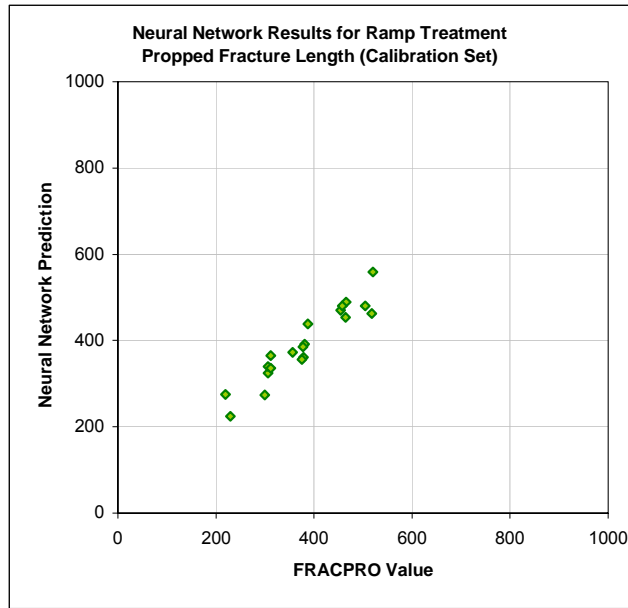


Figure 5-48: Eight Stage Type I Treatment Calibration Set Results for Propped Fracture Length

Figure 5-49 shows the verification set results for propped fracture length. R^2 is 0.91, r^2 is 0.92, and the correlation coefficient is 0.96.

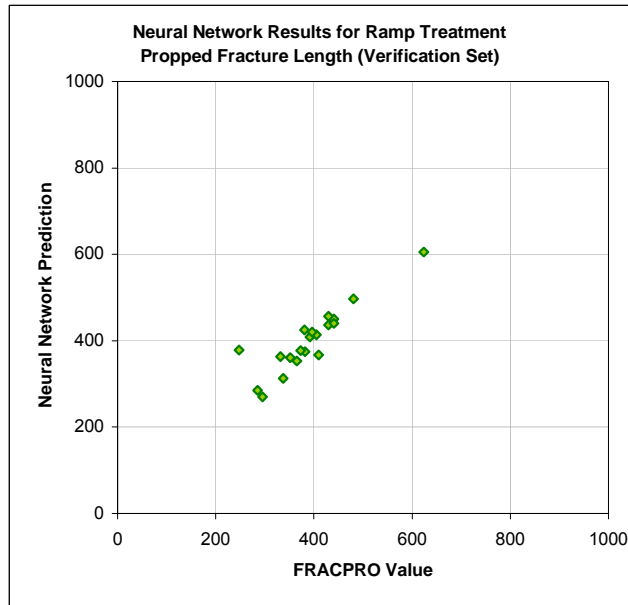


Figure 5-49: Eight Stage Type I Treatment Verification Set Results for Propped Fracture Length

Figure 5-50 shows the training set results for proppant concentration. R^2 is 0.80, r^2 is 0.84, and the correlation coefficient is 0.94.

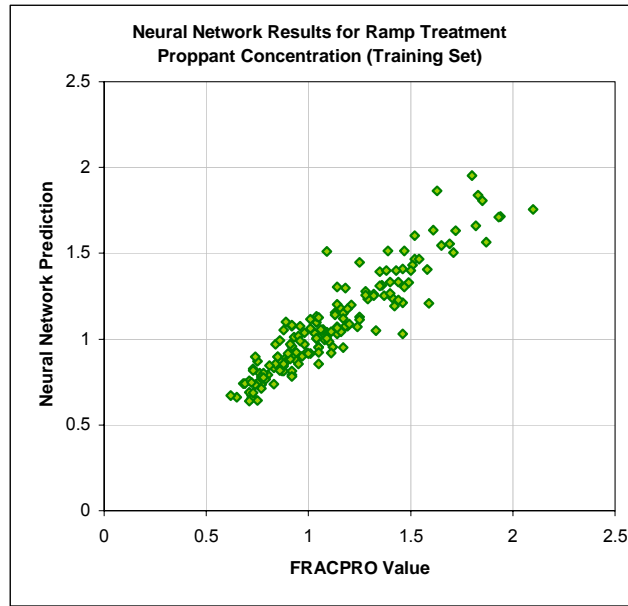


Figure 5-50: Eight Stage Type I Treatment Training Set Results for Proppant Concentration

Figure 5-51 shows the calibration set results for proppant concentration. R^2 is 0.86, r^2 is 0.82, and the correlation coefficient is 0.94.

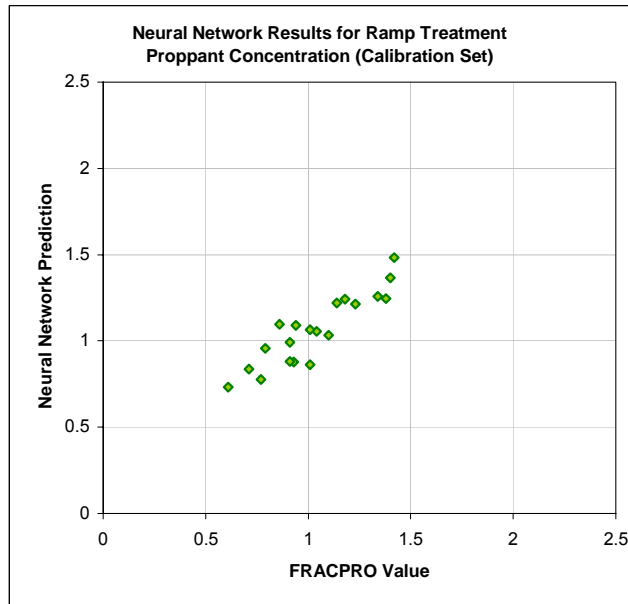


Figure 5-51: Eight Stage Type I Treatment Calibration Set Results for Proppant Concentration

Figure 5-52 shows the verification set results for proppant concentration. R^2 is 0.90, r^2 is 0.82, and the correlation coefficient is 0.90.

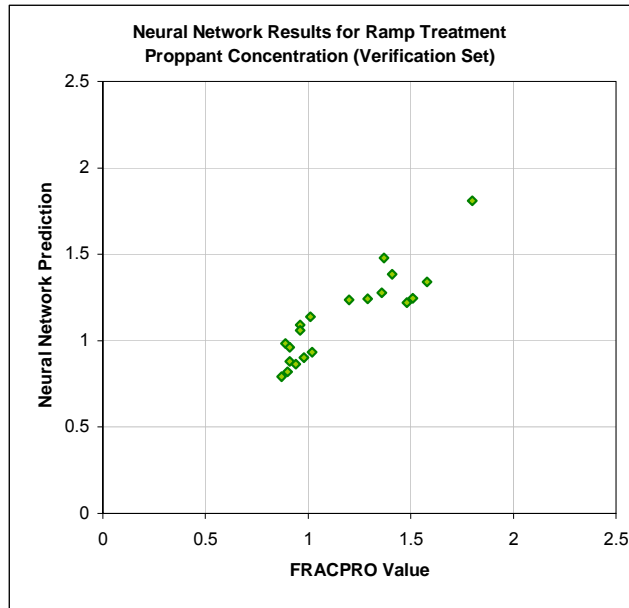


Figure 5-52: Eight Stage Type I Treatment Verification Set Results for Proppant Concentration

Figure 5-53 shows the training set results for maximum fracture width. R^2 is 0.82, r^2 is 0.83, and the correlation coefficient is 0.91.

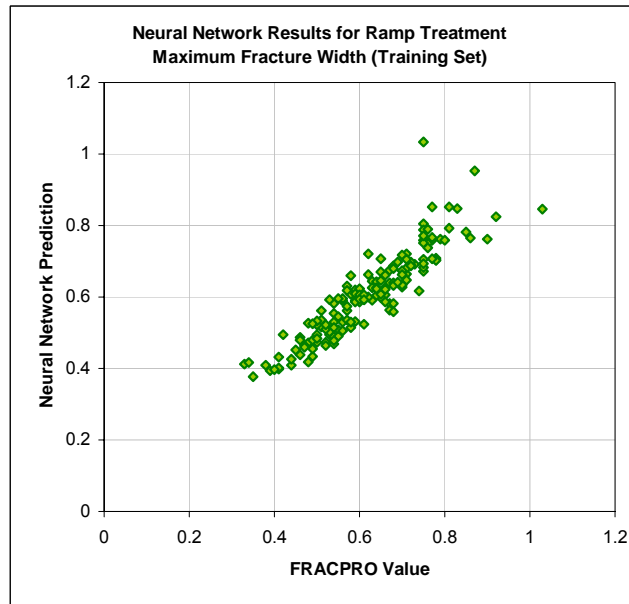


Figure 5-53: Eight Stage Type I Treatment Training Set Results for Maximum Fracture Width

Figure 5-54 shows the calibration set results for maximum fracture width. R^2 is 0.87, r^2 is 0.87, and the correlation coefficient is 0.92.

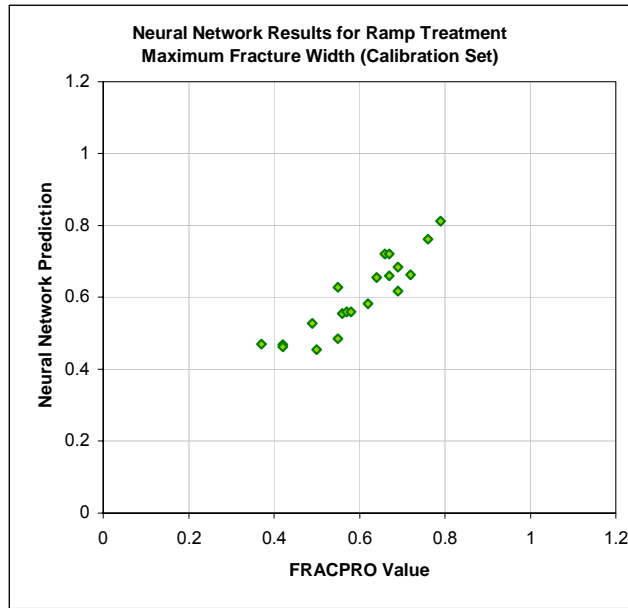


Figure 5-54: Eight Stage Type I Treatment Calibration Set Results for Maximum Fracture Width

Figure 5-55 shows the verification set results for maximum fracture width. R^2 is 0.84, r^2 is 0.86, and the correlation coefficient is 0.95.

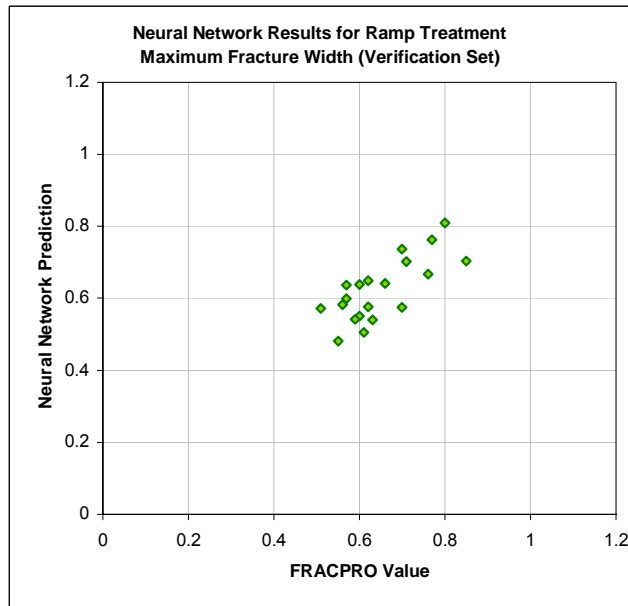


Figure 5-55: Eight Stage Type I Treatment Verification Set Results for Maximum Fracture Width

Figure 5-56 shows the training set results for fracture height. R^2 is 0.90, r^2 is 0.90, and the correlation coefficient is 0.95.

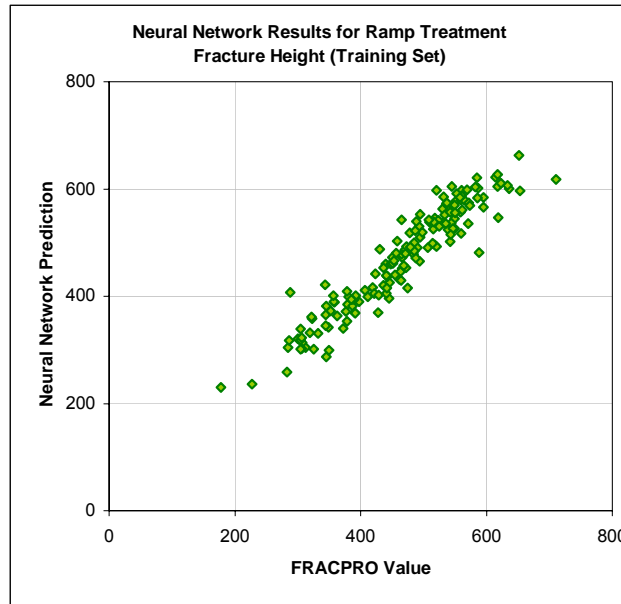


Figure 5-56: Eight Stage Type I Treatment Training Set Results for Fracture Height

Figure 5-57 shows the calibration set results for fracture height. R^2 is 0.88, r^2 is 0.90, and the correlation coefficient is 0.95.

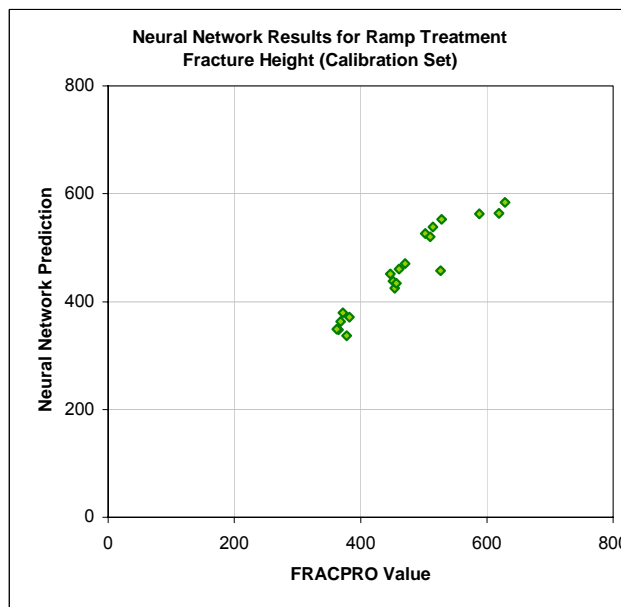


Figure 5-57: Eight Stage Type I Treatment Calibration Set Results for Fracture Height

Figure 5-58 shows the verification set results for fracture height. R^2 is 0.90, r^2 is 0.90, and the correlation coefficient is 0.95.

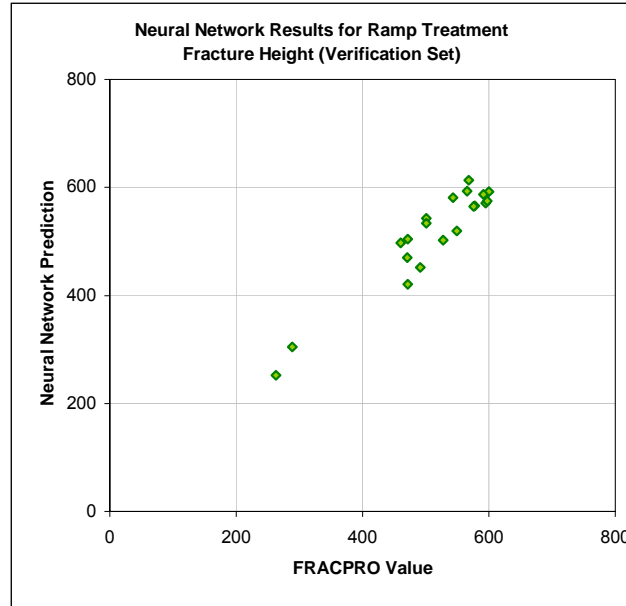


Figure 5-58: Eight Stage Type I Treatment Verification Set Results for Fracture Height

The details of the results for each of the five output parameters in the training, calibration, and verification sets can be seen in Table 5-9.

	Fracture Efficiency	Propped Fracture Length	Proppant Concentration	Max Fracture Width	Fracture Height
Training Set (162 cases)					
R^2	0.9570	0.9241	0.8034	0.8163	0.8953
r^2	0.9575	0.9263	0.8443	0.8287	0.8987
Correlation Coefficient, r	0.9785	0.9624	0.9413	0.9103	0.9480
Calibration Set (20 cases)					
R^2	0.7587	0.8785	0.8611	0.8679	0.8779
r^2	0.7900	0.8931	0.8151	0.8710	0.9030
Correlation Coefficient, r	0.8900	0.9451	0.9443	0.9192	0.9503
Verification Set (20 cases)					
R^2	0.7163	0.9125	0.8998	0.8387	0.8975
r^2	0.7618	0.9204	0.8153	0.8594	0.9030
Correlation Coefficient, r	0.8728	0.9594	0.9829	0.9479	0.9503

Table 5-9: Eight Stage Type I Treatment Neural Network Results

Note that in training this system only 202 cases were used compared to the two previous neural networks where more than 600 cases were available.

Much like the ramp treatment and six stage neural networks, fracture length and fracture height show the best performance, followed by fracture efficiency and fracture width. Fracture proppant concentration shows lower performance in this neural network. Dimensionless fracture conductivity was not collected for the eight-stage treatment and is not available for prediction.

5.2.3.2. Type II

Figure 5-59 through Figure 5-73 show the correlation between the actual FRACPRO values and the neural network predicted values of the training set, calibration set, and verification set for each of the 5 output parameters (fracture efficiency, propped fracture length, proppant concentration, max fracture width, and fracture height).

Figure 5-59 shows the training set results for fracture efficiency. R^2 is 0.81, r^2 is 0.81, and the correlation coefficient is 0.90.

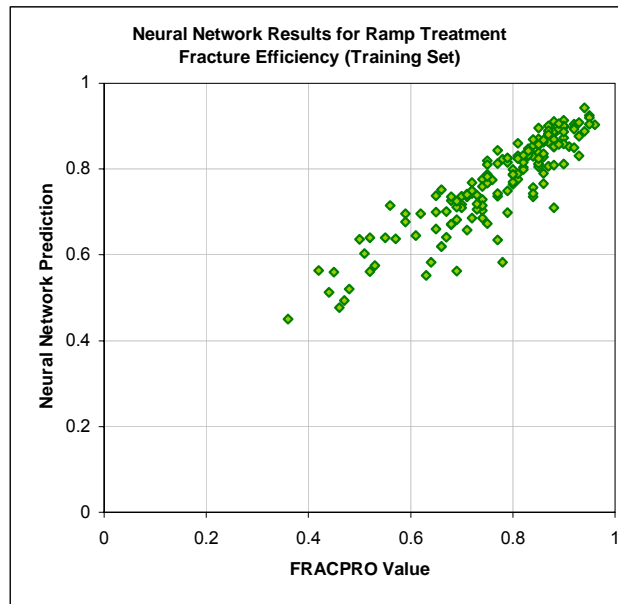


Figure 5-59: Eight Stage Type II Treatment Training Set Results for Fracture Efficiency

Figure 5-60 shows the calibration set results for fracture efficiency. R^2 is 0.91, r^2 is 0.91, and the correlation coefficient is 0.95.

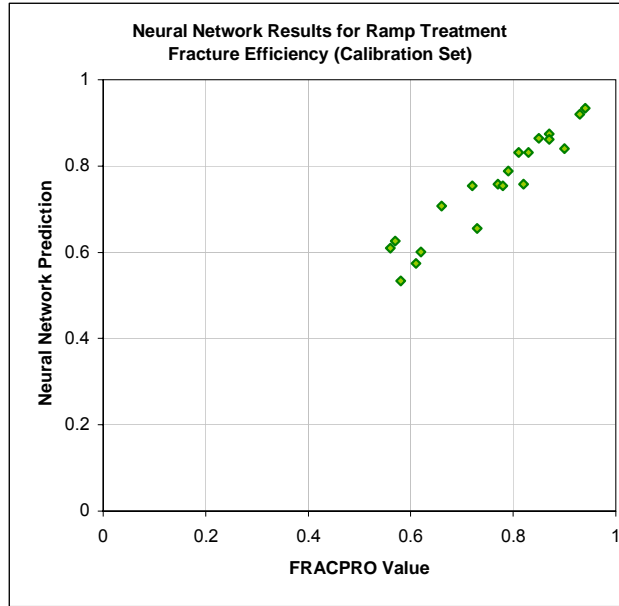


Figure 5-60: Eight Stage Type II Treatment Calibration Set Results for Fracture Efficiency

Figure 5-61 shows the verification set results for fracture efficiency. R^2 is 0.83, r^2 is 0.89, and the correlation coefficient is 0.95.

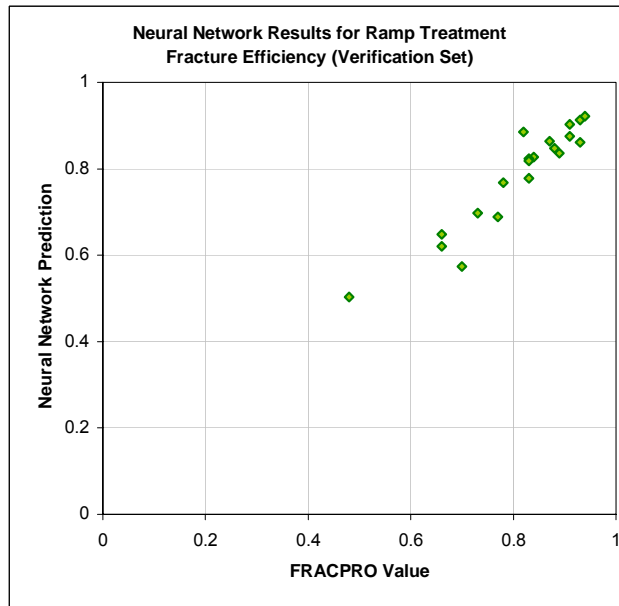


Figure 5-61: Eight Stage Type II Treatment Verification Set Results for Fracture Efficiency

Figure 5-62 shows the training set results for propped fracture length. R^2 is 0.89, r^2 is 0.89, and the correlation coefficient is 0.95.

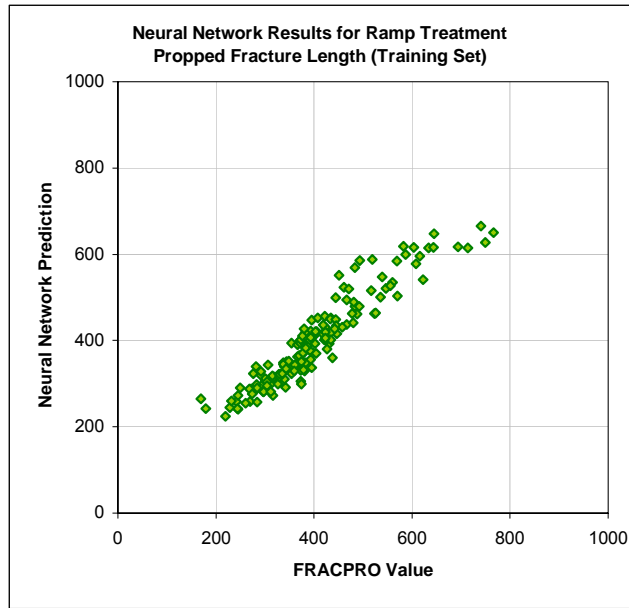


Figure 5-62: Eight Stage Type II Treatment Training Set Results for Propped Fracture Length

Figure 5-63 shows the calibration set results for propped fracture length. R^2 is 0.84, r^2 is 0.86, and the correlation coefficient is 0.93.

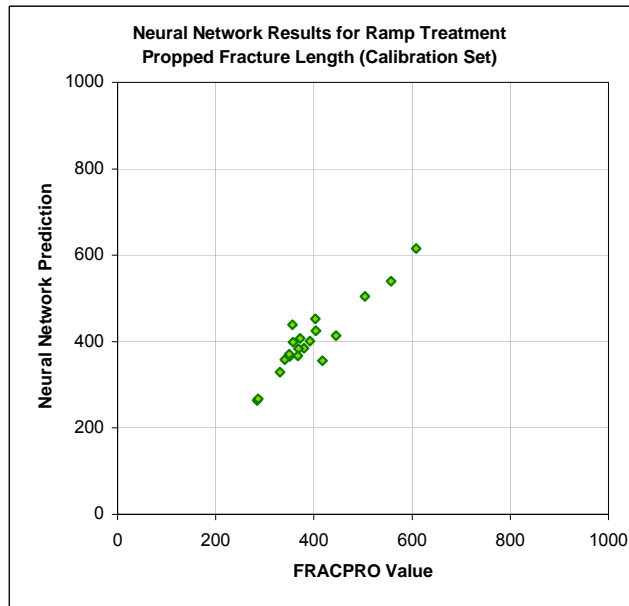


Figure 5-63: Eight Stage Type II Treatment Calibration Set Results for Propped Fracture Length

Figure 5-64 shows the verification set results for propped fracture length. R^2 is 0.95, r^2 is 0.95, and the correlation coefficient is 0.98.

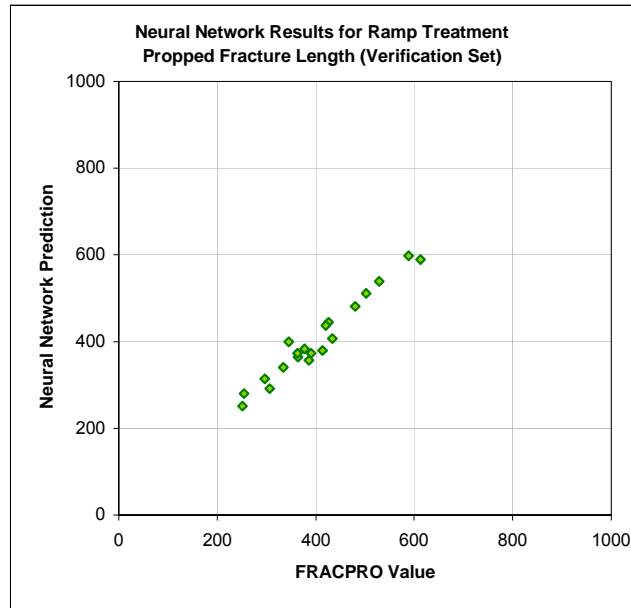


Figure 5-64: Eight Stage Type II Treatment Verification Set Results for Propped Fracture Length

Figure 5-65 shows the training set results for proppant concentration. R^2 is 0.74, r^2 is 0.78, and the correlation coefficient is 0.88.

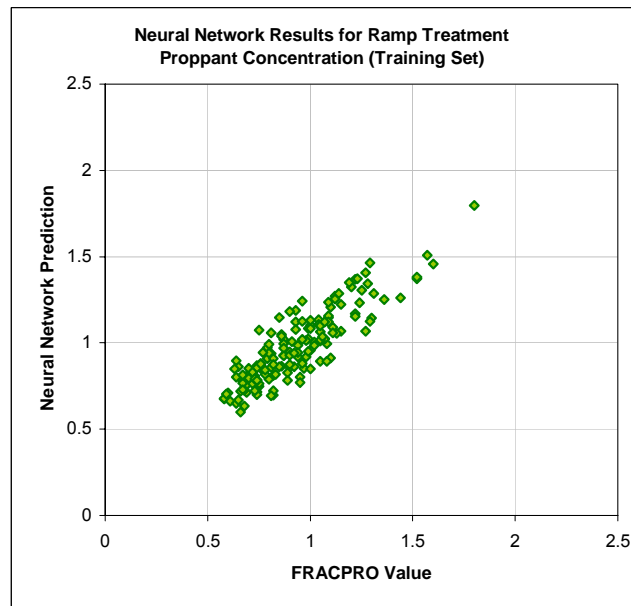


Figure 5-65: Eight Stage Type II Treatment Training Set Results for Proppant Concentration

Figure 5-66 shows the calibration set results for proppant concentration. R^2 is 0.84, r^2 is 0.85, and the correlation coefficient is 0.92.

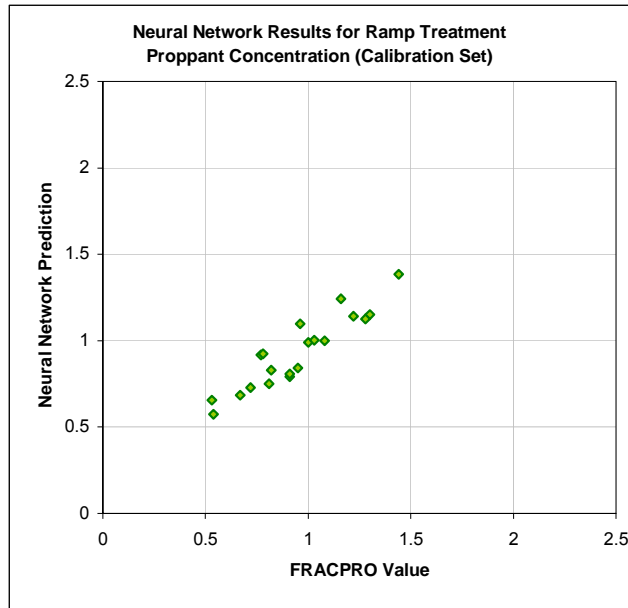


Figure 5-66: Eight Stage Type II Treatment Calibration Set Results for Proppant Concentration

Figure 5-67 shows the verification set results for proppant concentration. R^2 is 0.91, r^2 is 0.91, and the correlation coefficient is 0.95.

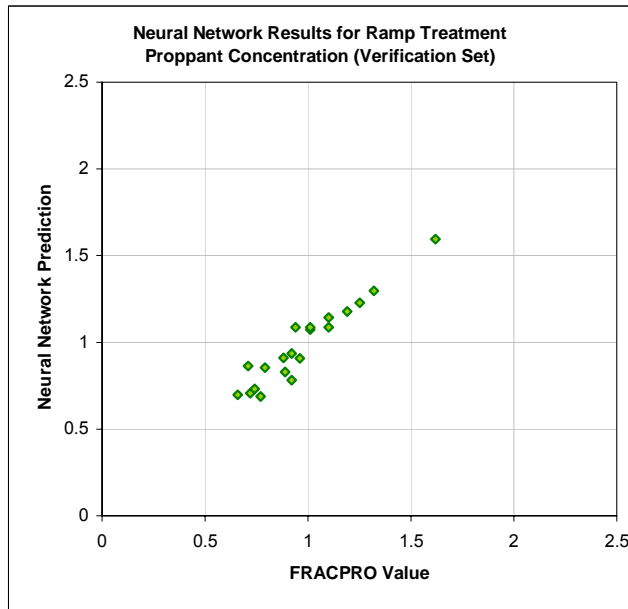


Figure 5-67: Eight Stage Type II Treatment Verification Set Results for Proppant Concentration

Figure 5-68 shows the training set results for maximum fracture width. R^2 is 0.81, r^2 is 0.81, and the correlation coefficient is 0.90.

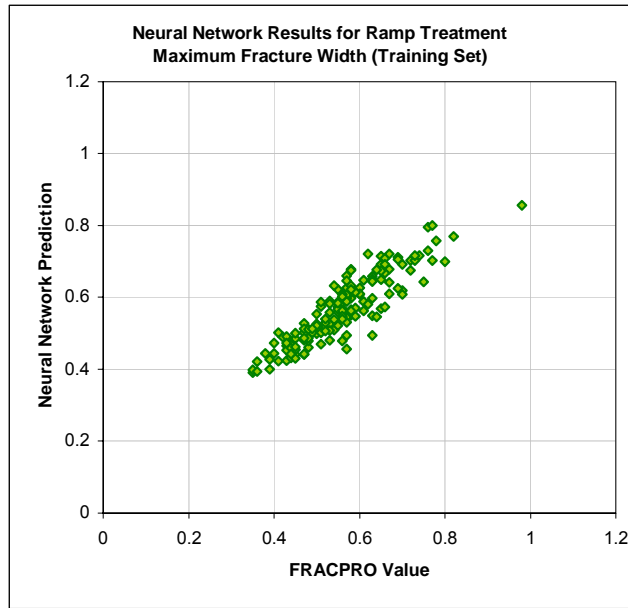


Figure 5-68: Eight Stage Type II Treatment Training Set Results for Maximum Fracture Width

Figure 5-69 shows the calibration set results for maximum fracture width. R^2 is 0.90, r^2 is 0.90, and the correlation coefficient is 0.95.

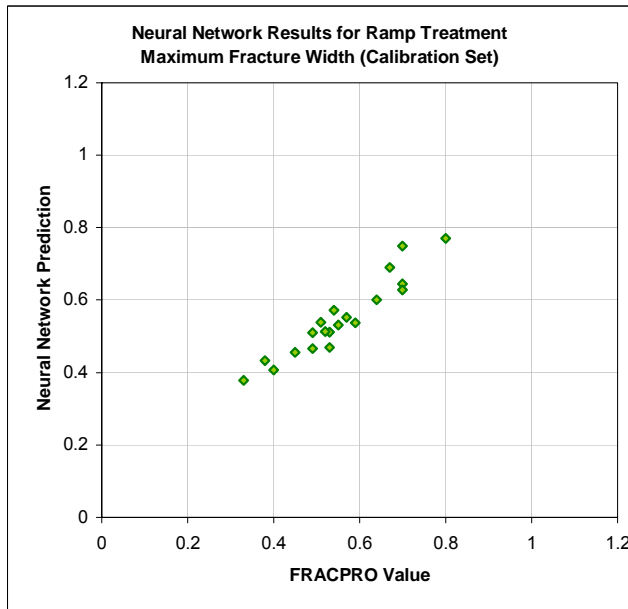


Figure 5-69: Eight Stage Type II Treatment Calibration Set Results for Maximum Fracture Width

Figure 5-70 shows the verification set results for maximum fracture width. R^2 is 0.81, r^2 is 0.81, and the correlation coefficient is 0.89.

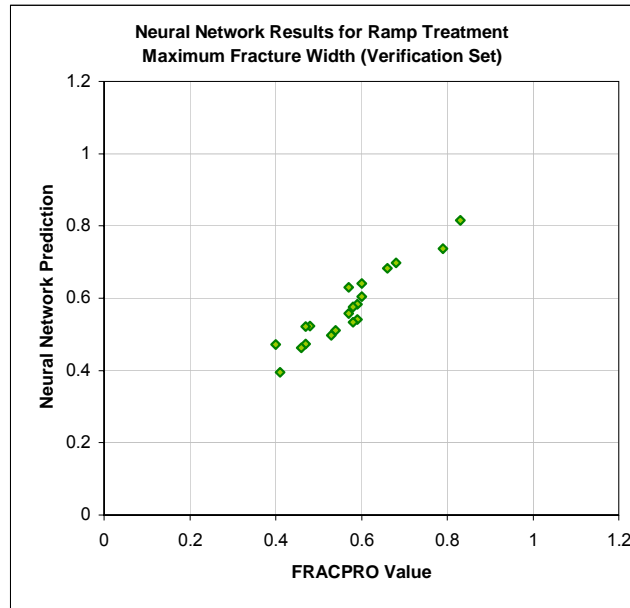


Figure 5-70: Eight Stage Type II Treatment Verification Set Results for Maximum Fracture Width

Figure 5-71 shows the training set results for fracture height. R^2 is 0.84, r^2 is 0.86, and the correlation coefficient is 0.93.

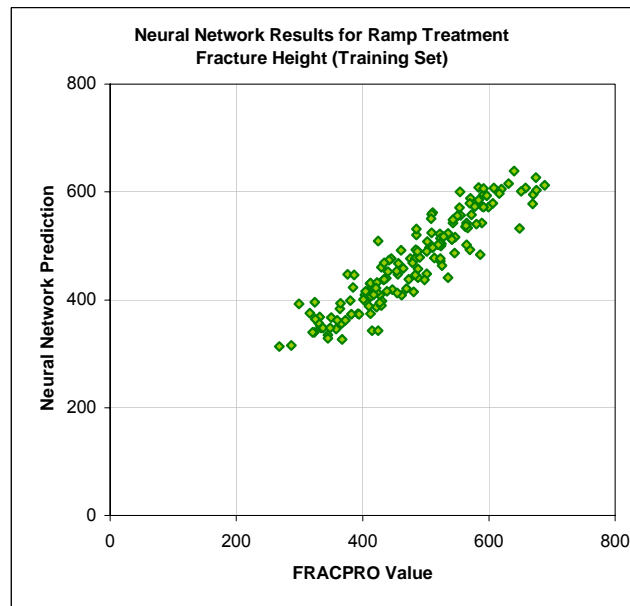


Figure 5-71: Eight Stage Type II Treatment Training Set Results for Fracture Height

Figure 5-72 shows the calibration set results for fracture height. R^2 is 0.88, r^2 is 0.88, and the correlation coefficient is 0.94.

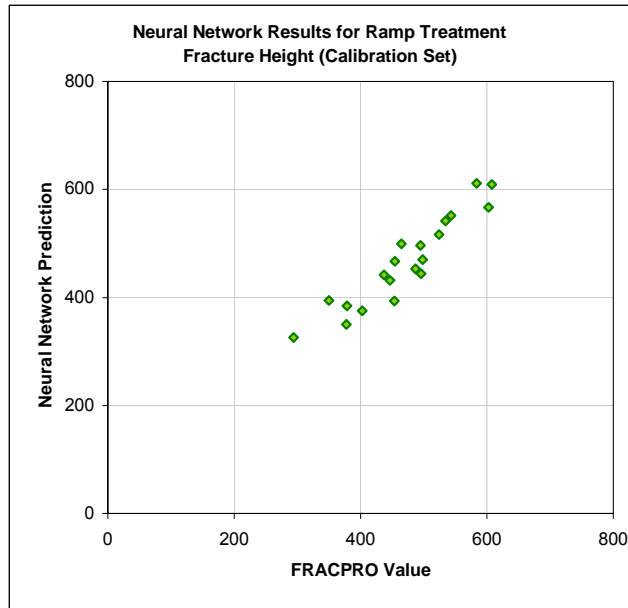


Figure 5-72: Eight Stage Type II Treatment Calibration Set Results for Fracture Height

Figure 5-73 shows the verification set results for fracture height. R^2 is 0.91, r^2 is 0.92, and the correlation coefficient is 0.96.

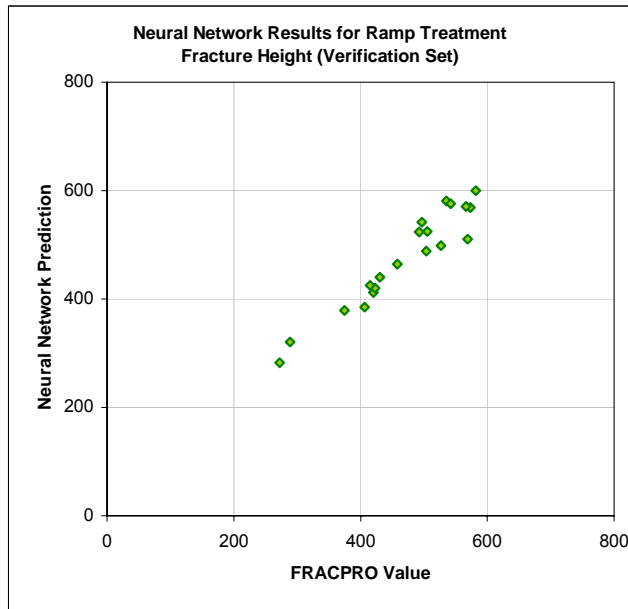


Figure 5-73: Eight Stage Type II Treatment Verification Set Results for Fracture Height

The details of the results for each of the five output parameters in the training, calibration, and verification sets can be seen in Table 5-10.

	Fracture Efficiency	Propped Fracture Length	Proppant Concentration	Max Fracture Width	Fracture Height
Training Set (160 cases)					
R ²	0.8100	0.8913	0.7375	0.8056	0.8448
r ²	0.8136	0.8935	0.7754	0.8056	0.8567
Correlation Coefficient, r	0.9020	0.9452	0.8806	0.8976	0.9256
Calibration Set (20 cases)					
R ²	0.9054	0.8437	0.8426	0.9021	0.8755
r ²	0.9088	0.8618	0.8498	0.9021	0.8823
Correlation Coefficient, r	0.9533	0.9283	0.9219	0.9498	0.9393
Verification Set (20 cases)					
R ²	0.8298	0.9543	0.9091	0.8114	0.9127
r ²	0.8945	0.9547	0.9118	0.8140	0.9207
Correlation Coefficient, r	0.9458	0.9771	0.9549	0.8934	0.9595

Table 5-10: Eight Stage Type II Treatment Neural Network Results

The results of the Type II neural network are much like the Type I neural network for eight stage treatments. Fracture length and fracture height show the best performance, followed by fracture efficiency and fracture width. Fracture proppant concentration shows lower performance. Dimensionless fracture conductivity was not collected for eight-stage treatment and is not available for prediction. Again, only 200 cases were used in developing this neural network.

5.2.3.3. Type III

Figure 5-74 through Figure 5-88 show the correlation between the actual FRACPRO values and the neural network predicted values of the training set, calibration set, and verification set for each of the 5 output parameters (fracture efficiency, propped fracture length, proppant concentration, max fracture width, and fracture height).

Figure 5-74 shows the training set results for fracture efficiency. R^2 is 0.86, r^2 is 0.87, and the correlation coefficient is 0.93.

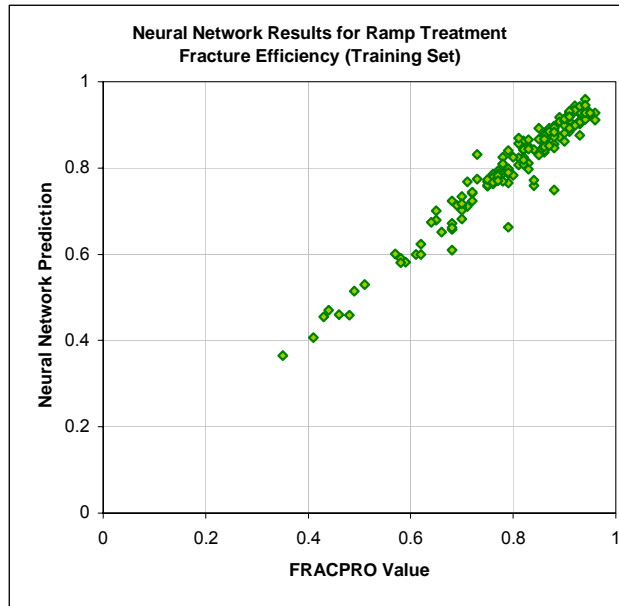


Figure 5-74: Eight Stage Type III Treatment Training Set Results for Fracture Efficiency

Figure 5-75 shows the calibration set results for fracture efficiency. R^2 is 0.94, r^2 is 0.95, and the correlation coefficient is 0.97.

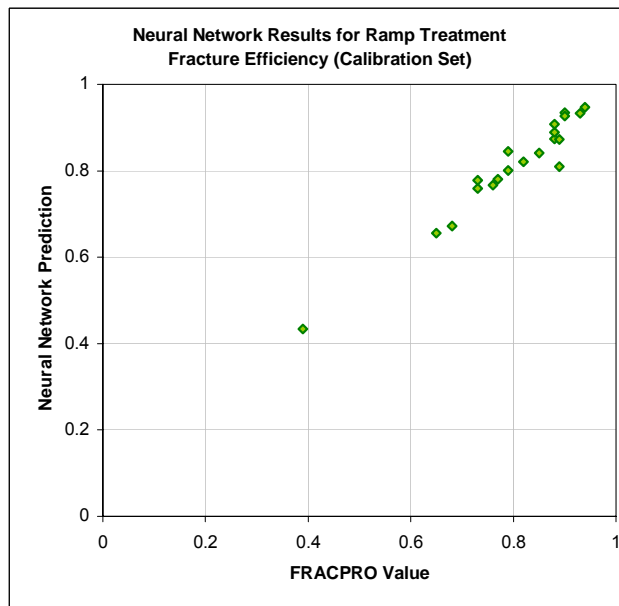


Figure 5-75: Eight Stage Type III Treatment Calibration Set Results for Fracture Efficiency

Figure 5-76 shows the verification set results for fracture efficiency. R^2 is 0.91, r^2 is 0.92, and the correlation coefficient is 0.96.

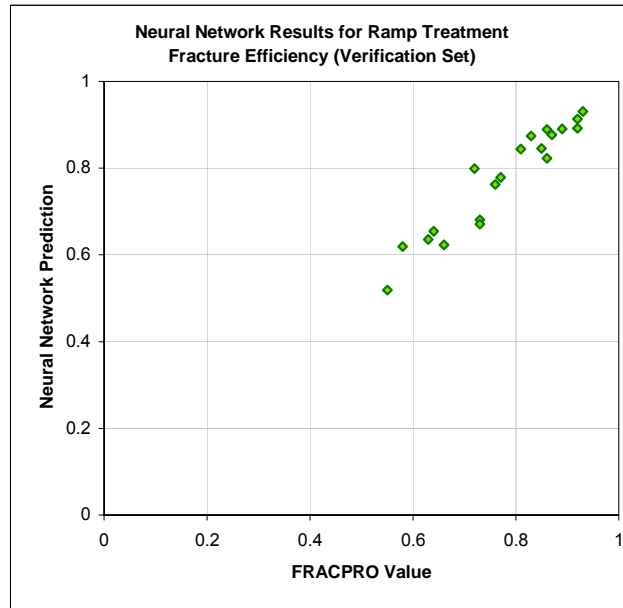


Figure 5-76: Eight Stage Type III Treatment Verification Set Results for Fracture Efficiency

Figure 5-77 shows the training set results for propped fracture length. R^2 is 0.92, r^2 is 0.92, and the correlation coefficient is 0.96.

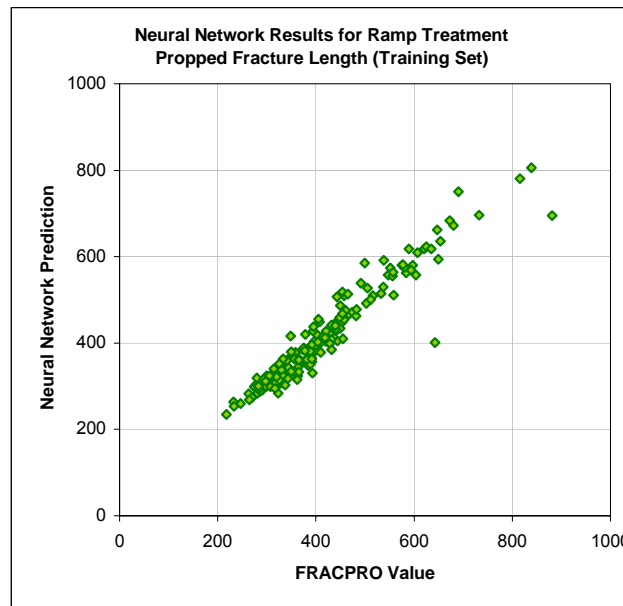


Figure 5-77: Eight Stage Type III Treatment Training Set Results for Propped Fracture Length

Figure 5-78 shows the calibration set results for propped fracture length. R^2 is 0.94, r^2 is 0.96, and the correlation coefficient is 0.98.

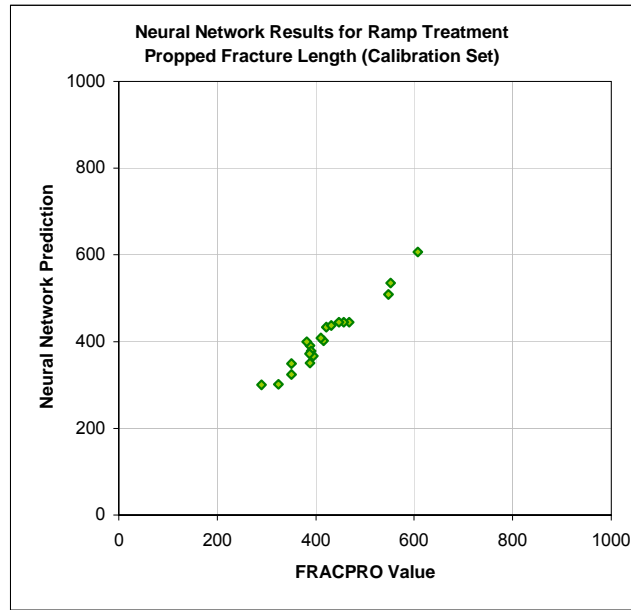


Figure 5-78: Eight Stage Type III Treatment Calibration Set Results for Propped Fracture Length

Figure 5-79 shows the verification set results for propped fracture length. R^2 is 0.94, r^2 is 0.96, and the correlation coefficient is 0.98.

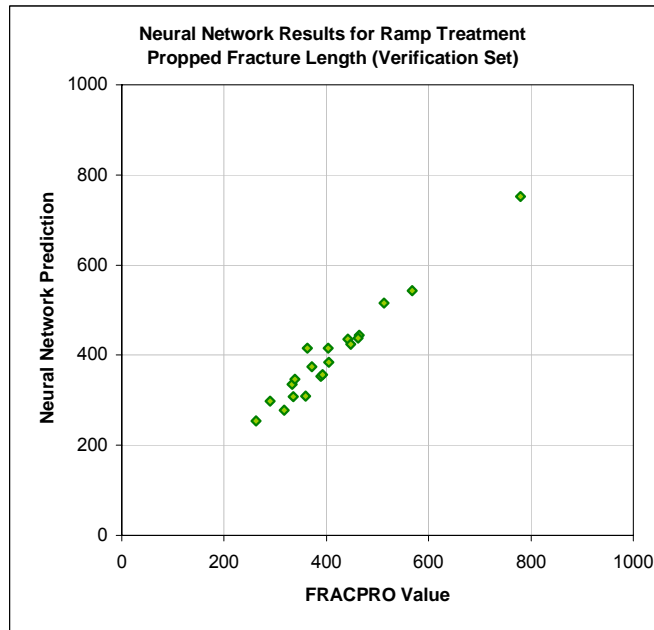


Figure 5-79: Eight Stage Type III Treatment Verification Set Results for Propped Fracture Length

Figure 5-80 shows the training set results for proppant concentration. R^2 is 0.92, r^2 is 0.92, and the correlation coefficient is 0.96.

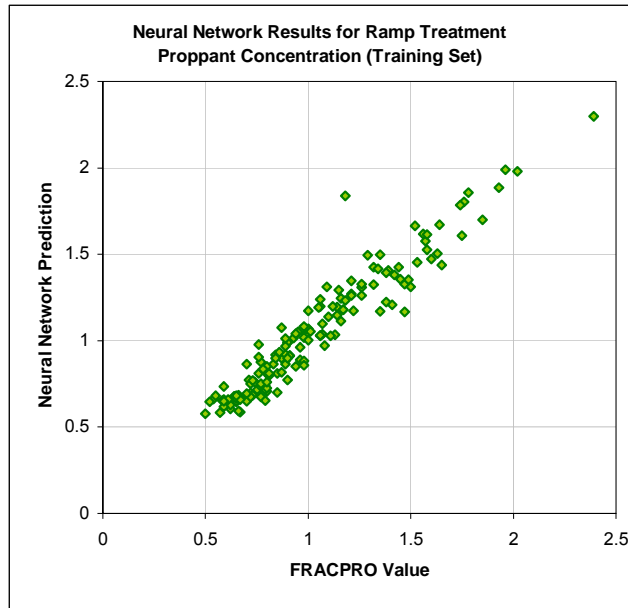


Figure 5-80: Eight Stage Type III Treatment Training Set Results for Proppant Concentration

Figure 5-81 shows the calibration set results for proppant concentration. R^2 is 0.94, r^2 is 0.94, and the correlation coefficient is 0.97.

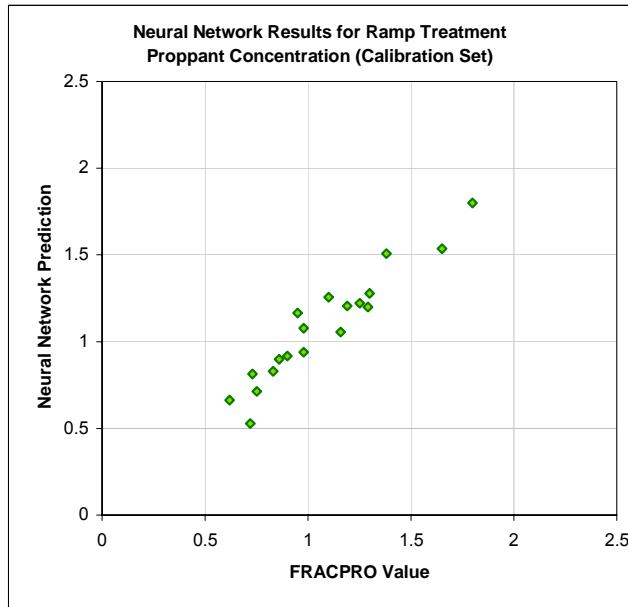


Figure 5-81: Eight Stage Type III Treatment Calibration Set Results for Proppant Concentration

Figure 5-82 shows the verification set results for proppant concentration. R^2 is 0.89, r^2 is 0.91, and the correlation coefficient is 0.95.

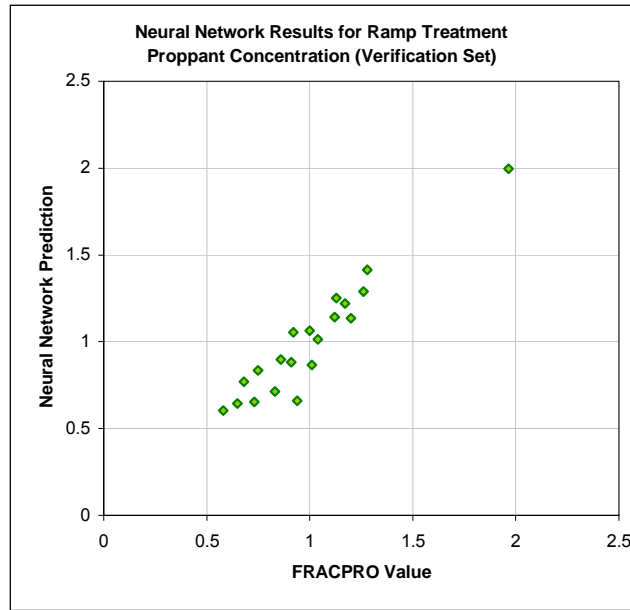


Figure 5-82: Eight Stage Type III Treatment Verification Set Results for Proppant Concentration

Figure 5-83 shows the training set results for maximum fracture width. R^2 is 0.86, r^2 is 0.87, and the correlation coefficient is 0.94.

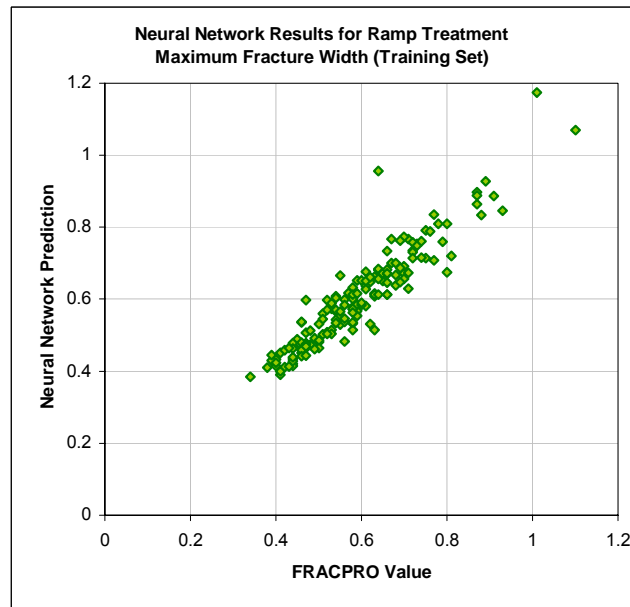


Figure 5-83: Eight Stage Type III Treatment Training Set Results for Maximum Fracture Width

Figure 5-84 shows the calibration set results for maximum fracture width. R^2 is 0.92, r^2 is 0.93, and the correlation coefficient is 0.96.

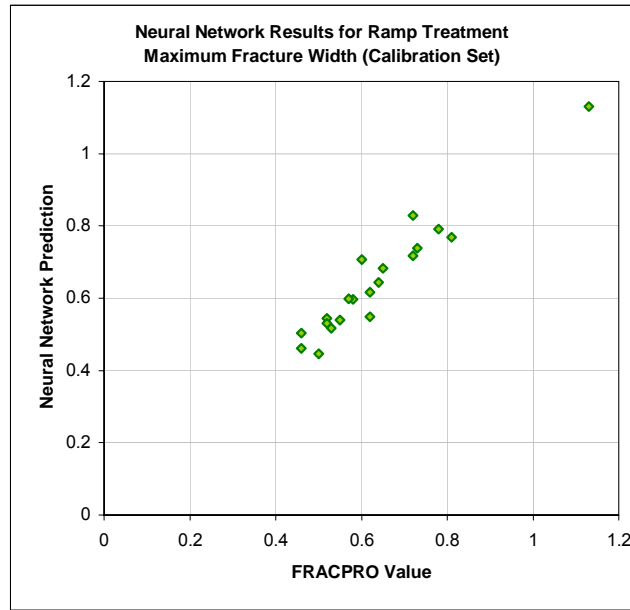


Figure 5-84: Eight Stage Type III Treatment Calibration Set Results for Maximum Fracture Width

Figure 5-85 shows the verification set results for maximum fracture width. R^2 is 0.93, r^2 is 0.94, and the correlation coefficient is 0.97.

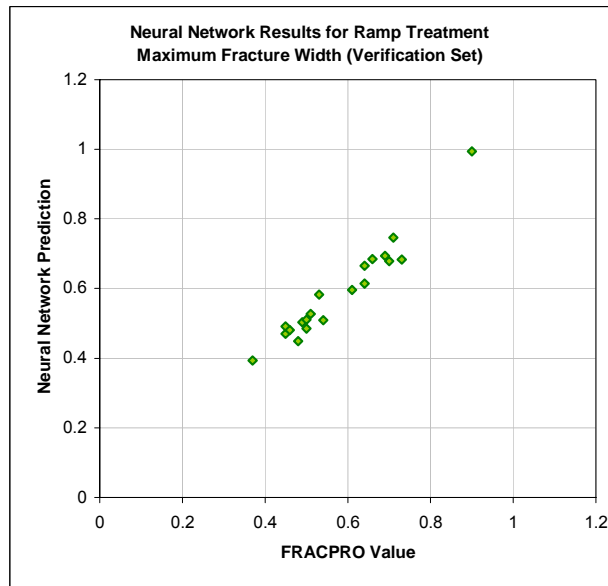


Figure 5-85: Eight Stage Type III Treatment Verification Set Results for Maximum Fracture Width

Figure 5-86 shows the training set results for fracture height. R^2 is 0.84, r^2 is 0.85, and the correlation coefficient is 0.92.

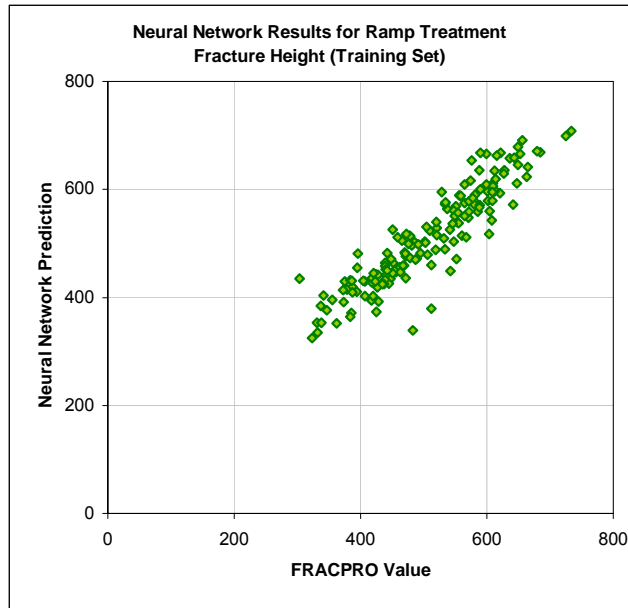


Figure 5-86: Eight Stage Type III Treatment Training Set Results for Fracture Height

Figure 5-87 shows the calibration set results for fracture height. R^2 is 0.88, r^2 is 0.88, and the correlation coefficient is 0.94.

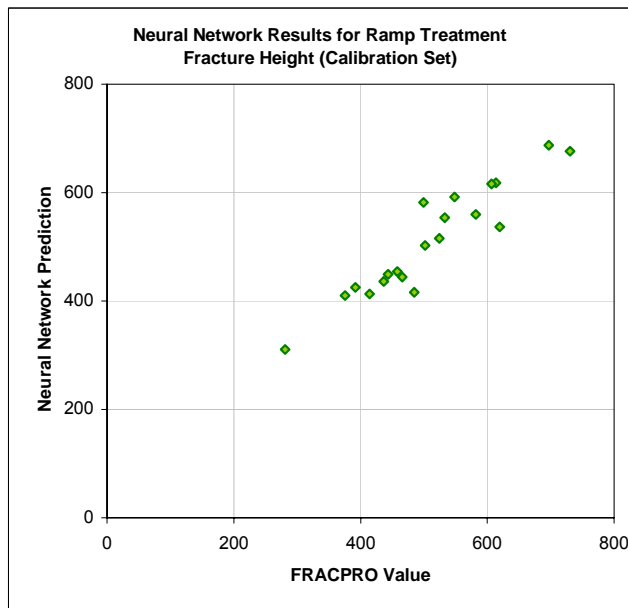


Figure 5-87: Eight Stage Type III Treatment Calibration Set Results for Fracture Height

Figure 5-88 shows the verification set results for fracture height. R^2 is 0.90, r^2 is 0.92, and the correlation coefficient is 0.96.

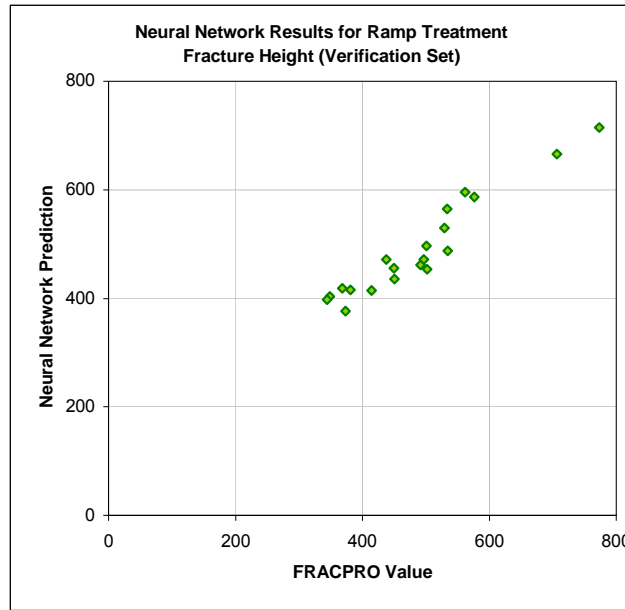


Figure 5-88: Eight Stage Type III Treatment Verification Set Results for Fracture Height

The details of the results for each of the five output parameters in the training, calibration, and verification sets can be seen in Table 5-11.

	Fracture Efficiency	Propped Fracture Length	Proppant Concentration	Max Fracture Width	Fracture Height
Training Set (162 cases)					
R^2	0.8601	0.9184	0.9163	0.8605	0.8426
r^2	0.8653	0.9187	0.9198	0.8742	0.8460
Correlation Coefficient, r	0.9302	0.9585	0.9591	0.9350	0.9198
Calibration Set (20 cases)					
R^2	0.9424	0.9377	0.9386	0.9157	0.8787
r^2	0.9493	0.9567	0.9432	0.9255	0.8796
Correlation Coefficient, r	0.9743	0.9781	0.9712	0.9620	0.9379
Verification Set (20 cases)					
R^2	0.9129	0.9436	0.8866	0.9251	0.8976
r^2	0.9198	0.9567	0.9068	0.9414	0.9162
Correlation Coefficient, r	0.9590	0.9781	0.9522	0.9702	0.9572

Table 5-11: Eight Stage Type III Treatment Neural Network Results

As with the other two eight-stage treatment neural networks, 202 cases were used in training. Fracture length and fracture height show the best performance, followed by fracture efficiency and fracture width. Fracture proppant concentration shows lower performance. Dimensionless fracture conductivity was not collected for the eight-stage treatment and is not available for prediction.

5.2.4. Summary

Figure 5-8 through Figure 5-88 show the training, calibration and verification sets for the five neural networks developed for treatment design.

The ramp and the five-stage neural networks exhibit better performance of prediction since more data was used in training the systems. In both cases, the fracture length, fracture height, and fracture width have high R^2 and correlation coefficient values. With few exceptions, the cases are very close to the 45° line in each set. The dimensionless conductivity ratio shows the lowest prediction performance in both neural networks. Even though R^2 and the correlation coefficient are good, the training, calibration, and verification plots show sparser distribution along the 45° line.

The three types of eight-stage neural networks were trained using only a third of the volume of data used for the ramp and six-stage networks. As a result, the expected performance of the neural networks was not as high. However, all three networks show strong performance capabilities for fracture length and fracture height prediction. Fracture width closely follows. The lowest performance was exhibited in predicting the fracture efficiency and fracture proppant concentration. Dimensionless fracture conductivity was not collected for the eight-stage treatment and it is not available for prediction.

Among the many neural networks that were tried for each type of treatment design, the five networks presented above proved to be the best selection for the optimization model.

5.3. Net Pressure Neural Network

The final neural network developed in this study is the net pressure prediction network. This neural network generates the pressure profile from the equivalent stress profile. This signal is corrected using the calibration neural network to match the net pressure profile developed when using the original stress profile.

The architecture of the neural network for net pressure has three hidden layers with different activation functions, 14 inputs and 10 outputs as described in detail in section 4.14.5.3 Neural Network Design. Figure 5-89 through Figure 5-118 show the correlation between the actual FRACPRO values and the neural network predicted values of the training set, calibration set, and verification set for each of the 10 pressure point output parameters.

Figure 5-89 shows the training set results for pressure point 1. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.91.

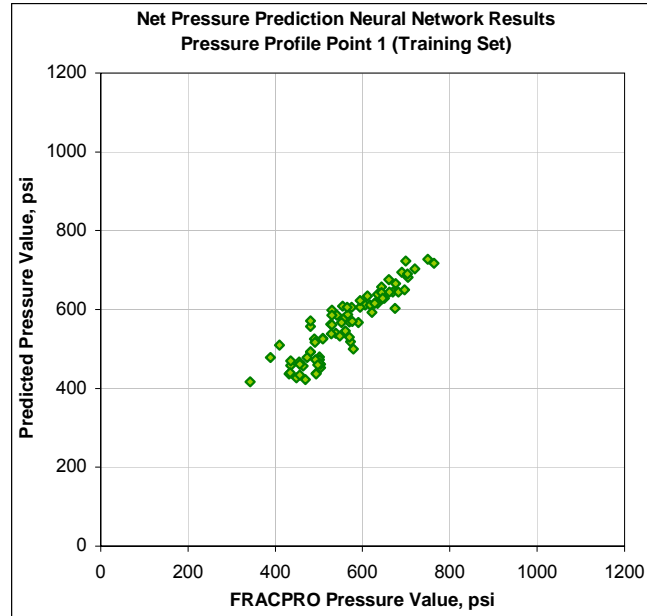


Figure 5-89: Net Pressure Neural Network Training Set Results for Pressure Point 1

Figure 5-90 shows the calibration set results for pressure point 1. R^2 is 0.73, r^2 is 0.75, and the correlation coefficient is 0.86.

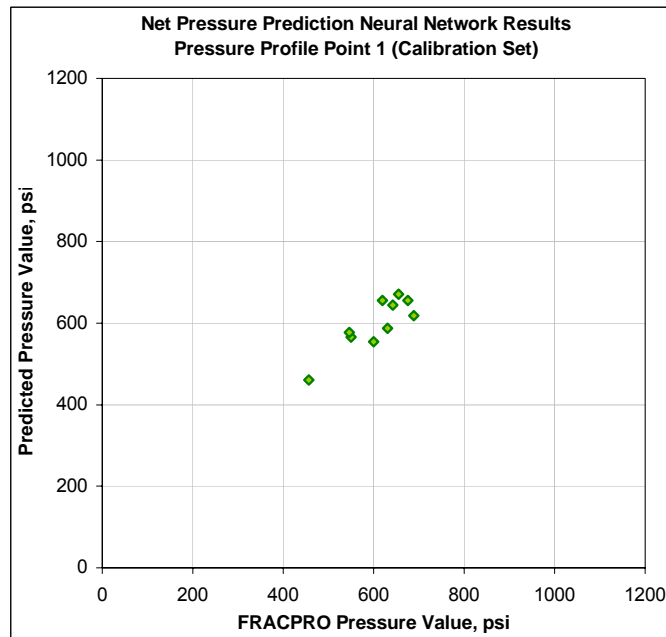


Figure 5-90: Net Pressure Neural Network Calibration Set Results for Pressure Point 1

Figure 5-91 shows the verification set results for pressure point 1. R^2 is 0.81, r^2 is 0.84, and the correlation coefficient is 0.92.

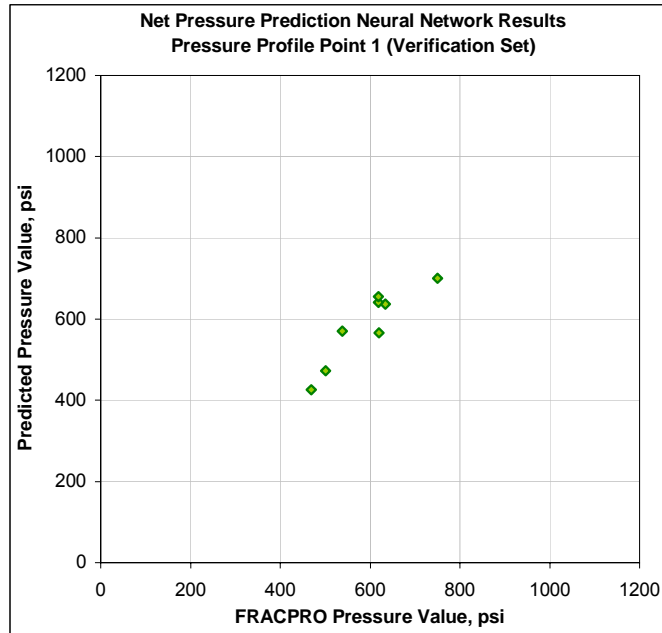


Figure 5-91: Net Pressure Neural Network Verification Set Results for Pressure Point 1

Figure 5-92 shows the training set results for pressure point 2. R^2 is 0.80, r^2 is 0.80, and the correlation coefficient is 0.89.

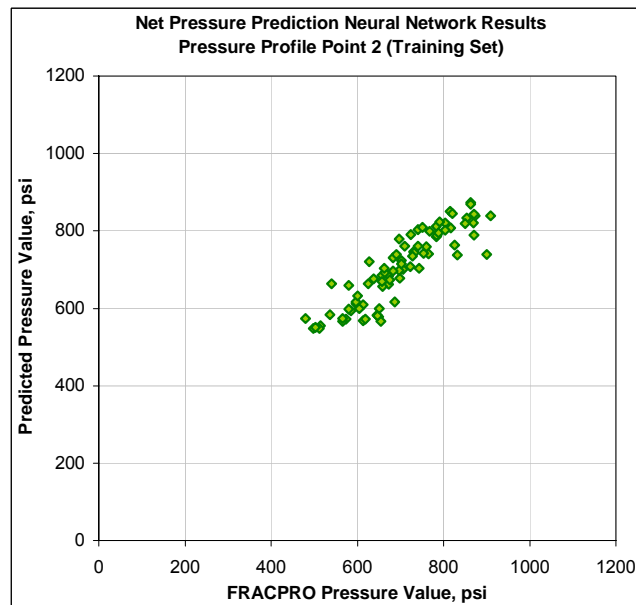


Figure 5-92: Net Pressure Neural Network Training Set Results for Pressure Point 2

Figure 5-93 shows the calibration set results for pressure point 2. R^2 is 0.80, r^2 is 0.84, and the correlation coefficient is 0.92.

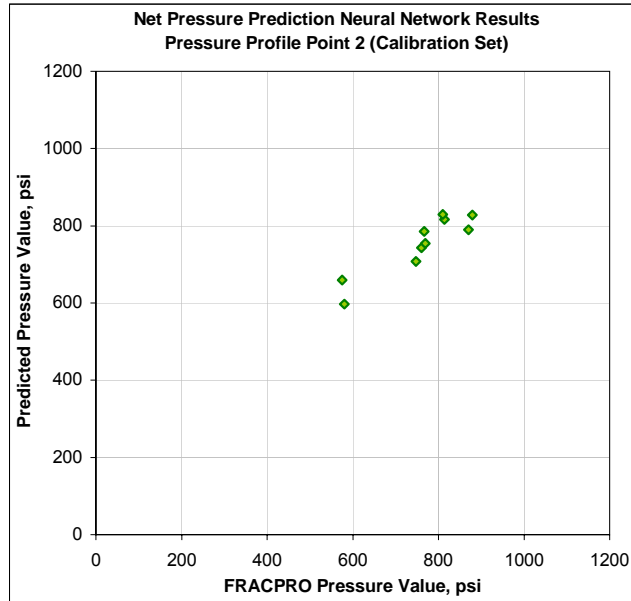


Figure 5-93: Net Pressure Neural Network Calibration Set Results for Pressure Point 2

Figure 5-94 shows the verification set results for pressure point 2. R^2 is 0.78, r^2 is 0.81, and the correlation coefficient is 0.90.

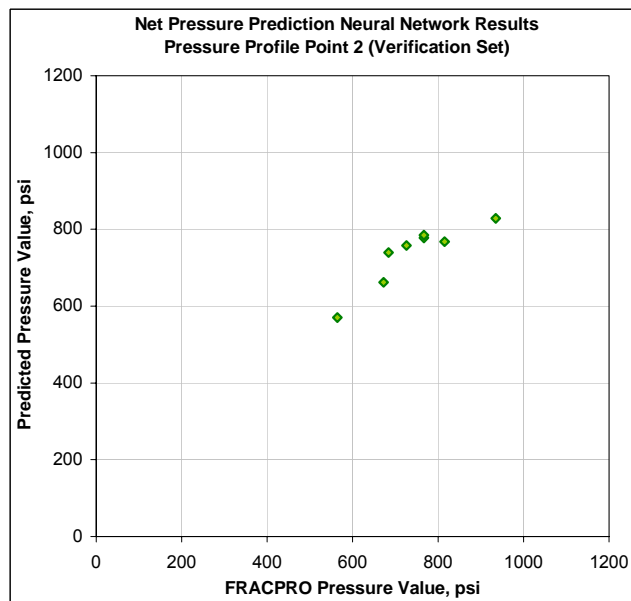


Figure 5-94: Net Pressure Neural Network Verification Set Results for Pressure Point 2

Figure 5-95 shows the training set results for pressure point 3. R^2 is 0.81, r^2 is 0.81, and the correlation coefficient is 0.90.

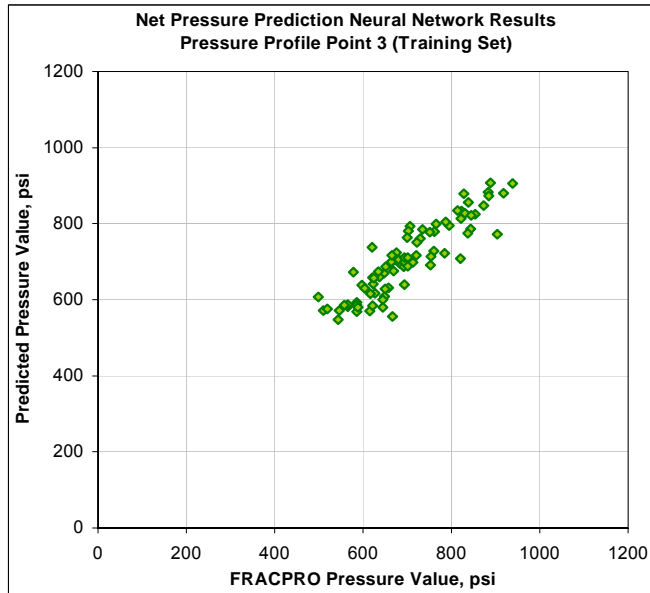


Figure 5-95: Net Pressure Neural Network Training Set Results for Pressure Point 3

Figure 5-96 shows the calibration set results for pressure point 3. R^2 is 0.82, r^2 is 0.85, and the correlation coefficient is 0.92.

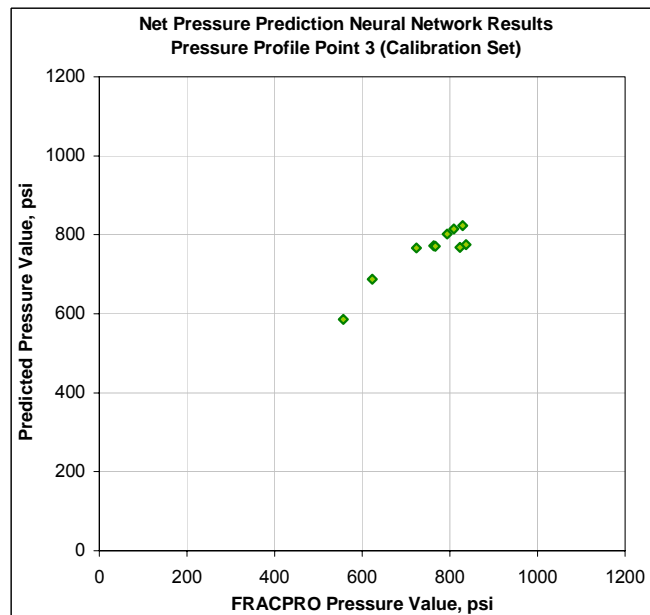


Figure 5-96: Net Pressure Neural Network Calibration Set Results for Pressure Point 3

Figure 5-97 shows the verification set results for pressure point 3. R^2 is 0.79, r^2 is 0.81, and the correlation coefficient is 0.90.

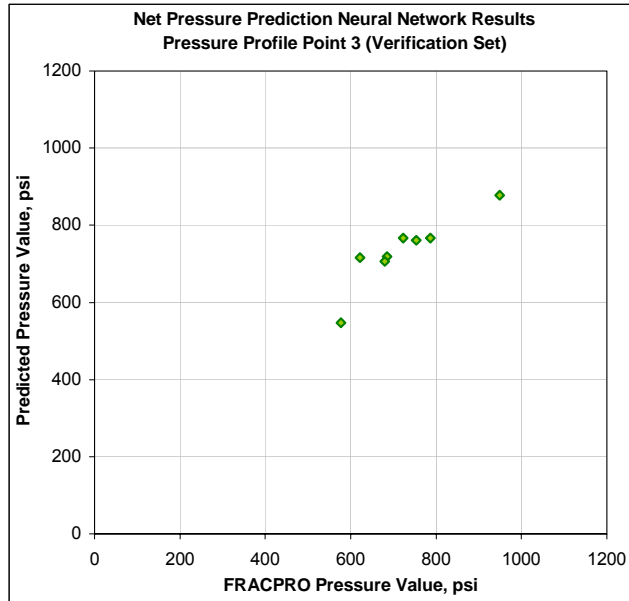


Figure 5-97: Net Pressure Neural Network Verification Set Results for Pressure Point 3

Figure 5-98 shows the training set results for pressure point 4. R^2 is 0.82, r^2 is 0.82, and the correlation coefficient is 0.91.

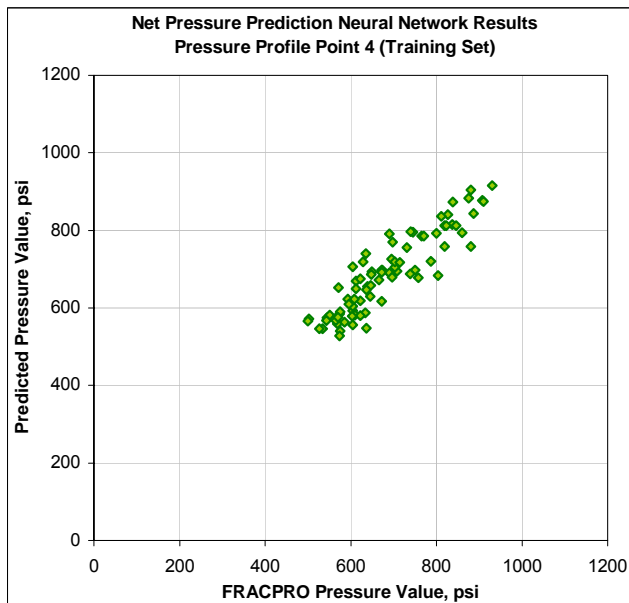


Figure 5-98: Net Pressure Neural Network Training Set Results for Pressure Point 4

Figure 5-99 shows the calibration set results for pressure point 4. R^2 is 0.85, r^2 is 0.87, and the correlation coefficient is 0.92.

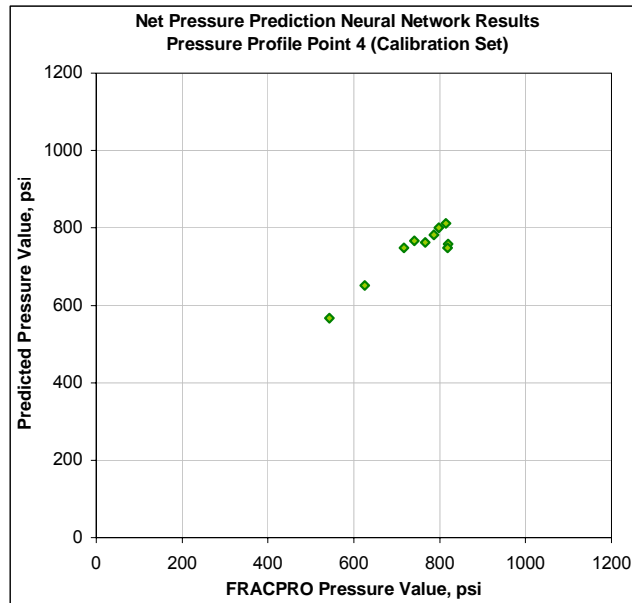


Figure 5-99: Net Pressure Neural Network Calibration Set Results for Pressure Point 4

Figure 5-100 shows the verification set results for pressure point 4. R^2 is 0.86, r^2 is 0.92, and the correlation coefficient is 0.96.

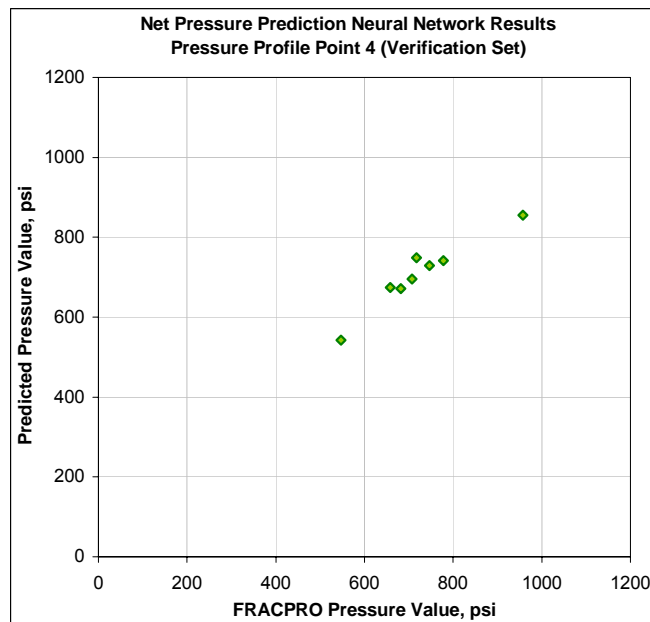


Figure 5-100: Net Pressure Neural Network Verification Set Results for Pressure Point 4

Figure 5-101 shows the training set results for pressure point 5. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.91.

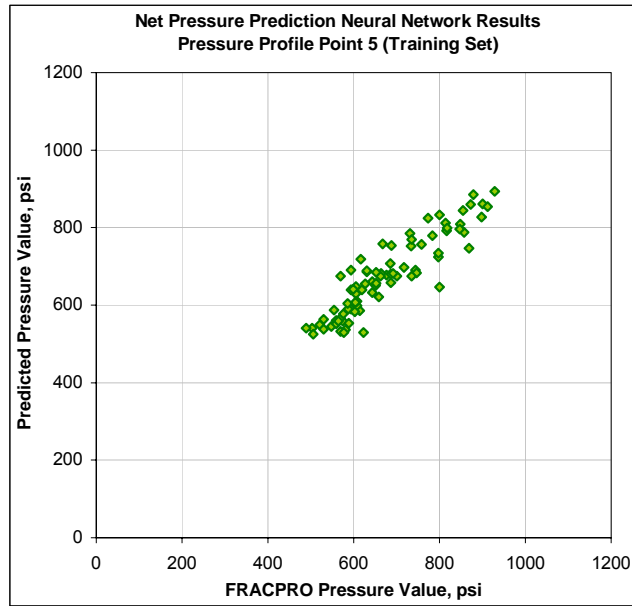


Figure 5-101: Net Pressure Neural Network Training Set Results for Pressure Point 5

Figure 5-102 shows the calibration set results for pressure point 5. R^2 is 0.84, r^2 is 0.86, and the correlation coefficient is 0.93.

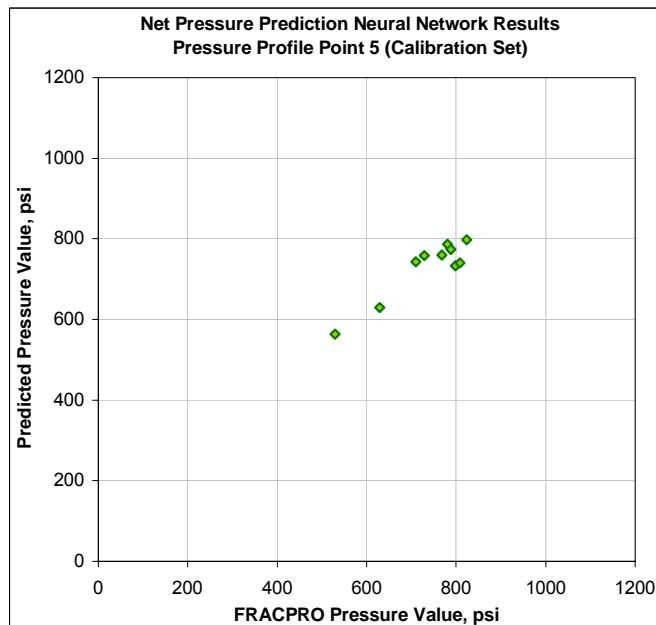


Figure 5-102: Net Pressure Neural Network Calibration Set Results for Pressure Point 5

Figure 5-103 shows the verification set results for pressure point 5. R^2 is 0.82, r^2 is 0.87, and the correlation coefficient is 0.93.

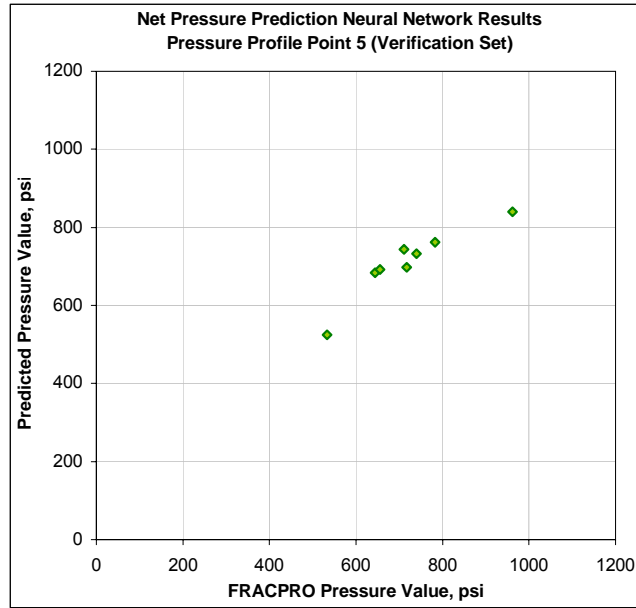


Figure 5-103: Net Pressure Neural Network Verification Set Results for Pressure Point 5

Figure 5-104 shows the training set results for pressure point 6. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.91.

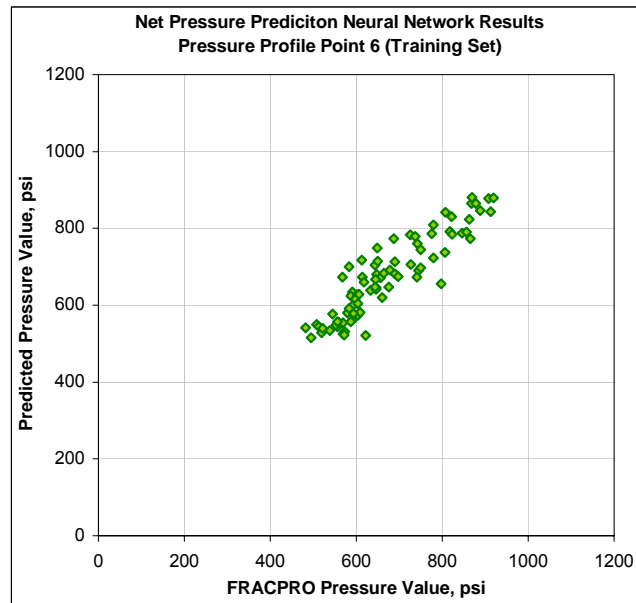


Figure 5-104: Net Pressure Neural Network Training Set Results for Pressure Point 6

Figure 5-105 shows the calibration set results for pressure point 6. R^2 is 0.85, r^2 is 0.86, and the correlation coefficient is 0.93.

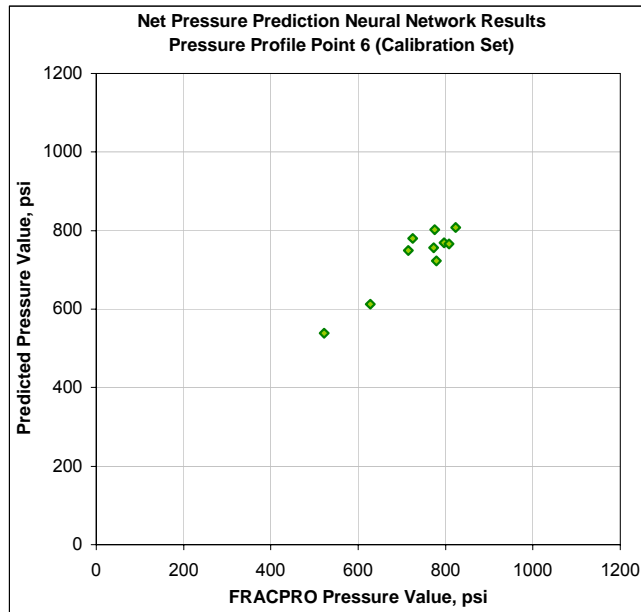


Figure 5-105: Net Pressure Neural Network Calibration Set Results for Pressure Point 6

Figure 5-106 shows the verification set results for pressure point 6. R^2 is 0.86, r^2 is 0.89, and the correlation coefficient is 0.94.

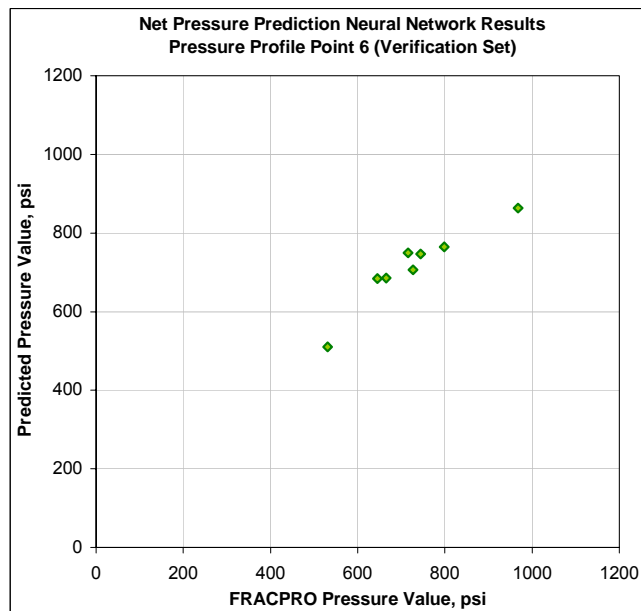


Figure 5-106: Net Pressure Neural Network Verification Set Results for Pressure Point 6

Figure 5-107 shows the training set results for pressure point 7. R^2 is 0.82, r^2 is 0.82, and the correlation coefficient is 0.91.

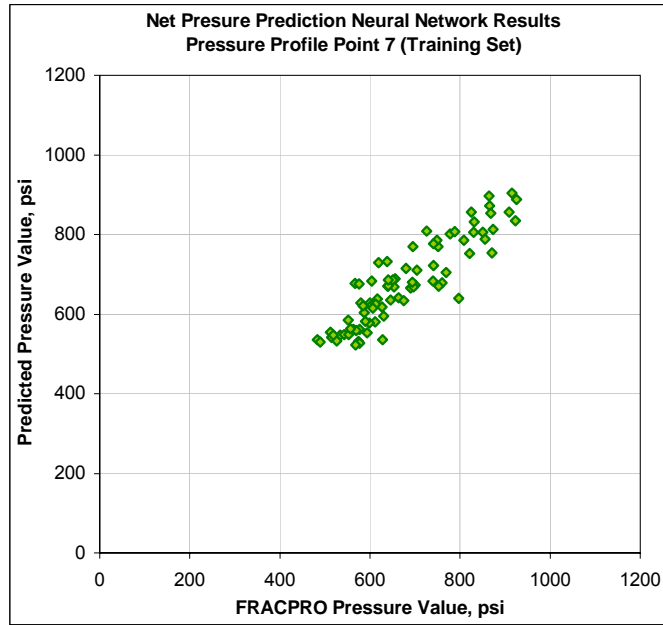


Figure 5-107: Net Pressure Neural Network Training Set Results for Pressure Point 7

Figure 5-108 shows the calibration set results for pressure point 7. R^2 is 0.83, r^2 is 0.83, and the correlation coefficient is 0.91.

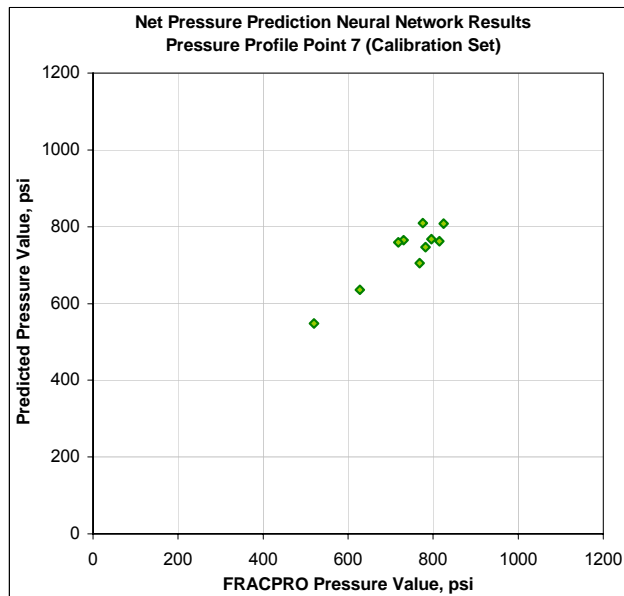


Figure 5-108: Net Pressure Neural Network Calibration Set Results for Pressure Point 7

Figure 5-109 shows the verification set results for pressure point 7. R^2 is 0.85, r^2 is 0.89, and the correlation coefficient is 0.94.

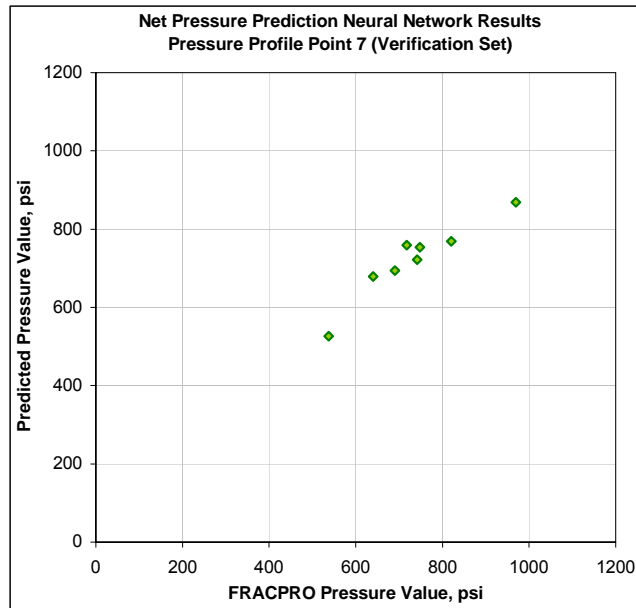


Figure 5-109: Net Pressure Neural Network Verification Set Results for Pressure Point 7

Figure 5-110 shows the training set results for pressure point 8. R^2 is 0.83, r^2 is 0.84, and the correlation coefficient is 0.91.

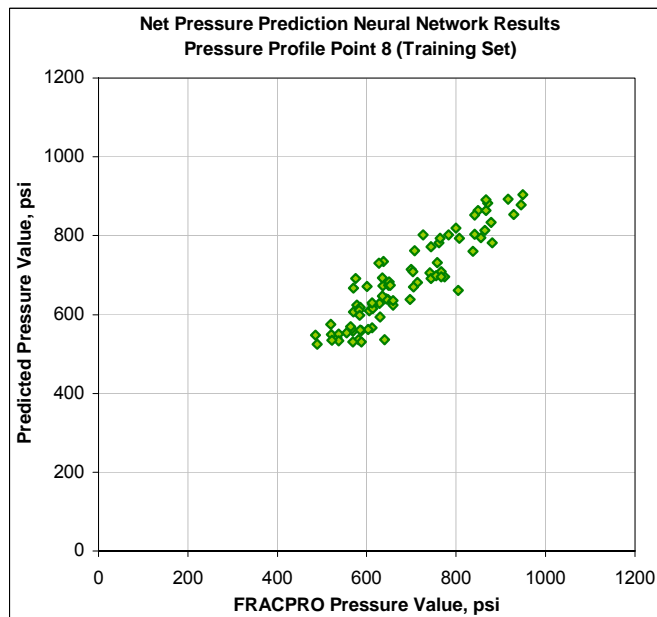


Figure 5-110: Net Pressure Neural Network Training Set Results for Pressure Point 8

Figure 5-111 shows the calibration set results for pressure point 8. R^2 is 0.79, r^2 is 0.80, and the correlation coefficient is 0.89.

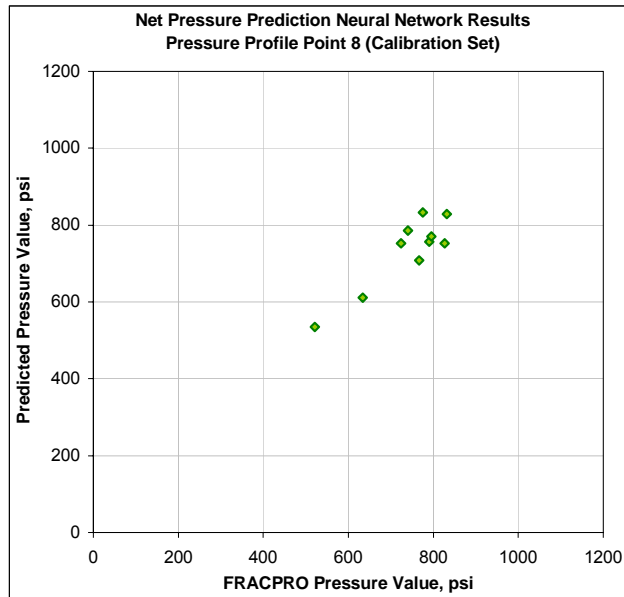


Figure 5-111: Net Pressure Neural Network Calibration Set Results for Pressure Point 8

Figure 5-112 shows the verification set results for pressure point 8. R^2 is 0.84, r^2 is 0.89, and the correlation coefficient is 0.94.

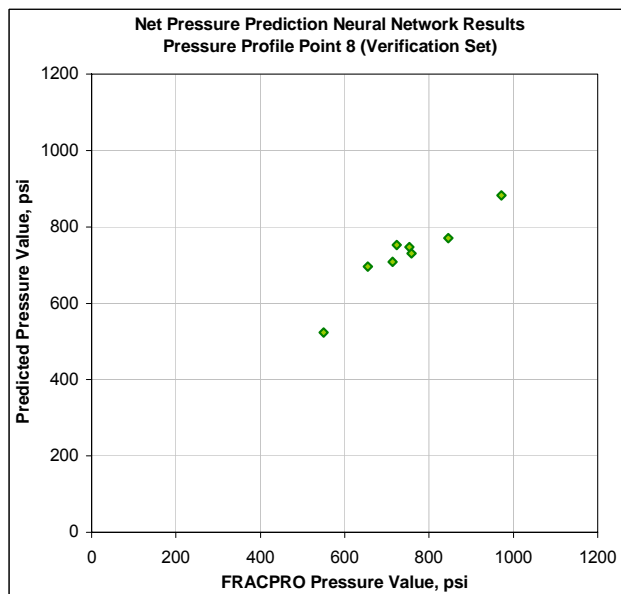


Figure 5-112: Net Pressure Neural Network Verification Set Results for Pressure Point 8

Figure 5-113 shows the training set results for pressure point 9. R^2 is 0.81, r^2 is 0.81, and the correlation coefficient is 0.90.

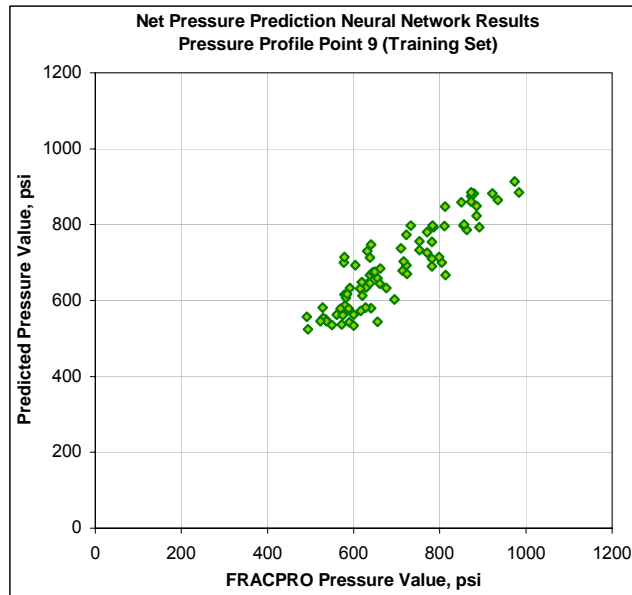


Figure 5-113: Net Pressure Neural Network Training Set Results for Pressure Point 9

Figure 5-114 shows the calibration set results for pressure point 9. R^2 is 0.81, r^2 is 0.81, and the correlation coefficient is 0.90.

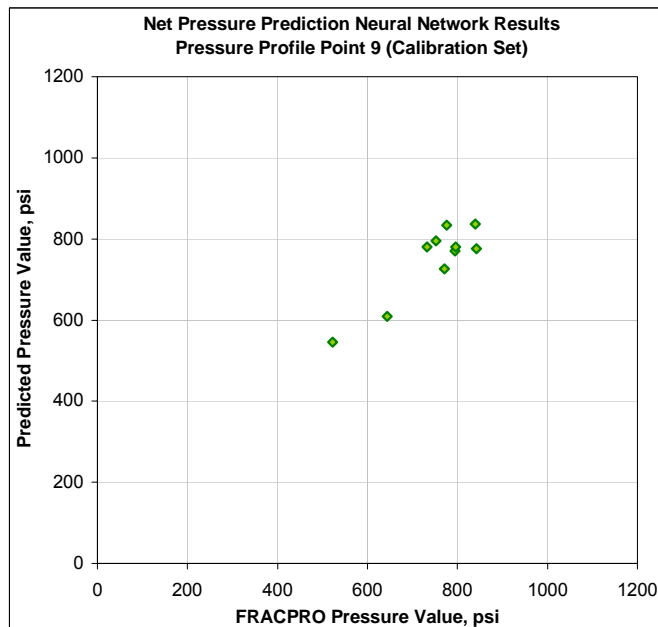


Figure 5-114: Net Pressure Neural Network Calibration Set Results for Pressure Point 9

Figure 5-115 shows the verification set results for pressure point 9. R^2 is 0.82, r^2 is 0.85, and the correlation coefficient is 0.92.

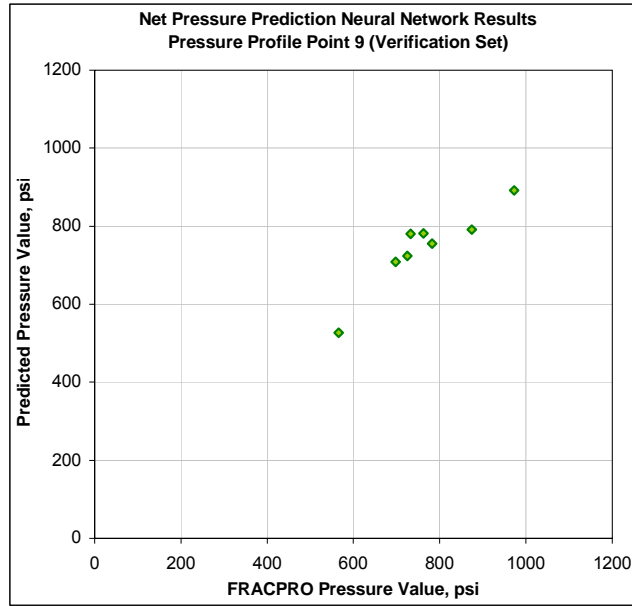


Figure 5-115: Net Pressure Neural Network Verification Set Results for Pressure Point 9

Figure 5-116 shows the training set results for pressure point 10. R^2 is 0.79, r^2 is 0.79, and the correlation coefficient is 0.89.

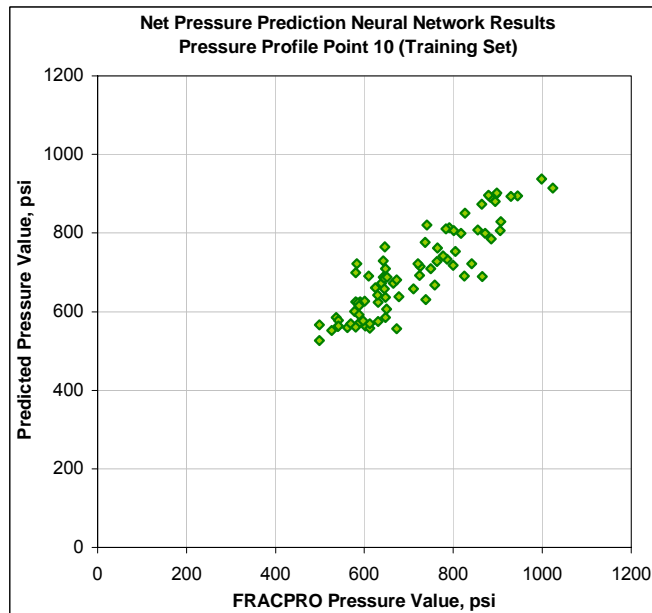


Figure 5-116: Net Pressure Neural Network Training Set Results for Pressure Point 10

Figure 5-117 shows the calibration set results for pressure point 10. R^2 is 0.79, r^2 is 0.80, and the correlation coefficient is 0.90.

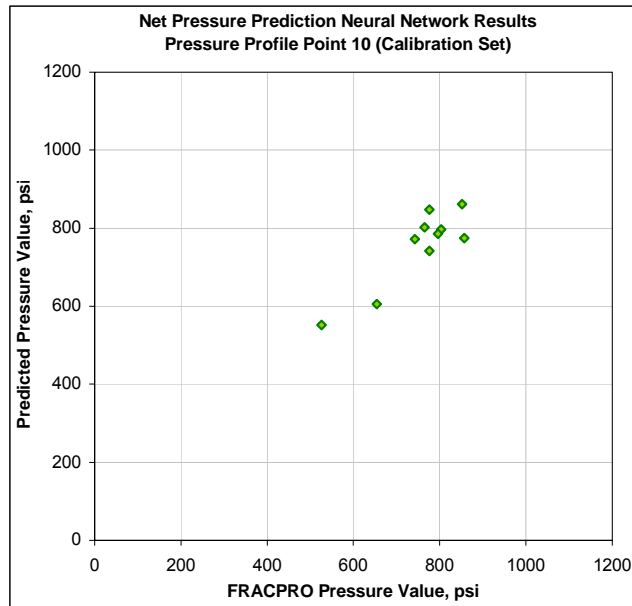


Figure 5-117: Net Pressure Neural Network Calibration Set Results for Pressure Point 10

Figure 5-118 shows the verification set results for pressure point 10. R^2 is 0.76, r^2 is 0.88, and the correlation coefficient is 0.94.

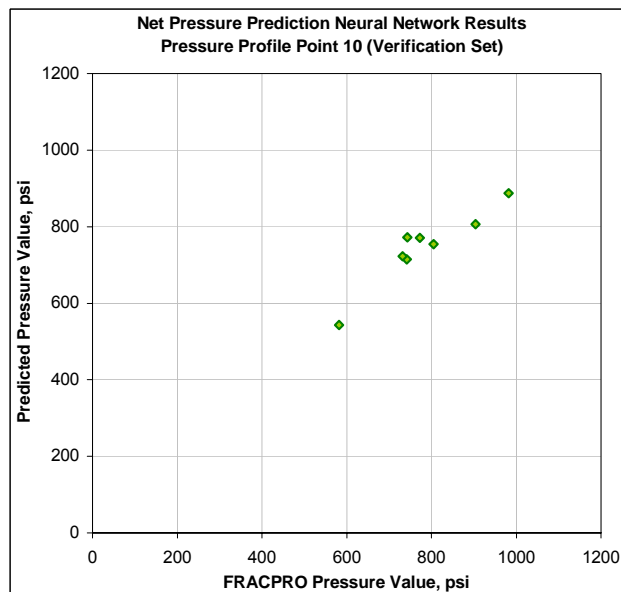


Figure 5-118: Net Pressure Neural Network Verification Set Results for Pressure Point 10

The details of the results for each of the 10 points output pressure in the training, calibration, and verification sets can be seen in Table 5-12.

	Pressure Profile Point 1	Pressure Profile Point 2	Pressure Profile Point 3	Pressure Profile Point 4	Pressure Profile Point 5	Pressure Profile Point 6	Pressure Profile Point 7	Pressure Profile Point 8	Pressure Profile Point 9	Pressure Profile Point 10
Training Set (82 cases)										
R ²	0.8317	0.7993	0.8120	0.8230	0.8314	0.8332	0.8188	0.8341	0.8051	0.7865
r ²	0.8320	0.8008	0.8139	0.8242	0.8324	0.8333	0.8189	0.8356	0.8097	0.7924
Correlation Coefficient, r	0.9121	0.8949	0.9022	0.9079	0.9123	0.9128	0.9050	0.9141	0.8998	0.8902
Calibration Set (10 cases)										
R ²	0.7331	0.8024	0.8220	0.8502	0.8346	0.8546	0.8285	0.7864	0.8068	0.7885
r ²	0.7469	0.8410	0.8533	0.8659	0.8578	0.8564	0.8349	0.8004	0.8131	0.8028
Correlation Coefficient, r	0.8642	0.9171	0.9238	0.9305	0.9262	0.9254	0.9138	0.8946	0.9017	0.8960
Verification Set (8 cases)										
R ²	0.8050	0.7830	0.7924	0.8552	0.8178	0.8576	0.8535	0.8394	0.8197	0.7577
r ²	0.8420	0.8074	0.8103	0.9222	0.8657	0.8885	0.8926	0.8901	0.8522	0.8775
Correlation Coefficient, r	0.9176	0.8986	0.9002	0.9603	0.9304	0.9426	0.9448	0.9434	0.9232	0.9368

Table 5-12: Net Pressure Prediction Neural Network Results

Table 5-12 gives the R² and correlation coefficients for each of the pressure points. It can be seen that the prediction performance of the neural network is weaker at the end points (first two and last two points). Some of these points even have R² values under 0.8 in the calibration and verification sets. However, correlation coefficients show higher values, in most cases above 0.9. The data indicates that the middle pressure points have a better correlation in regard to the neural network prediction capability. These pressure points have higher correlation coefficients and R² values around 0.8.

5.3.1. Summary

The net pressure neural network was trained with only 100 data cases. Even with the low number of cases, the network has proven good accuracy in predicting the pressure points and the resulting pressure profiles.

Different types of neural networks were tried. Since the performance of the network is weaker for the end points, neural networks were developed that predict one point at a time in an attempt to improve performance. However, even with only one output, the performance did not improve. Moreover, a better match was observed when reconstructing the pressure signal using the 10-point neural network compared to the match observed from the 1-point networks. Similar observations were made after attempts using two neural networks of five outputs each.

After additional examination, it was concluded that using all 10 pressure points simultaneously enables the neural network to achieve a better correlation between points and

more accurate shape of the signal. This characteristic was not achieved when using separate one-point prediction neural networks.

In Figure 5-89 through Figure 5-118, each pressure point is plotted versus the actual value separately. Unfortunately, this does not provide a clear picture of the 10-point pressure signal results. Eight cases used in the verification set (not in the training process) are presented to examine the neural network pressure prediction capability. In Figure 5-119 through Figure 5-126, the FRACPRO pressure profile obtained using the equivalent four-layer profile and neural network predicted pressure profile is plotted.

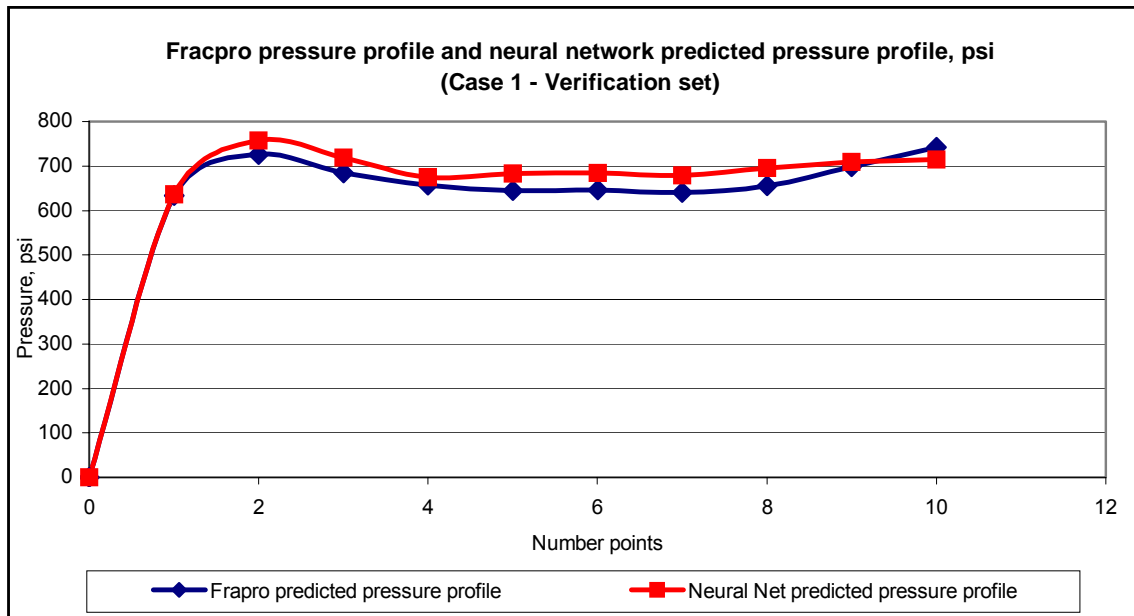


Figure 5-119: Case 1 FRACPRO and Neural Network Pressure Profiles

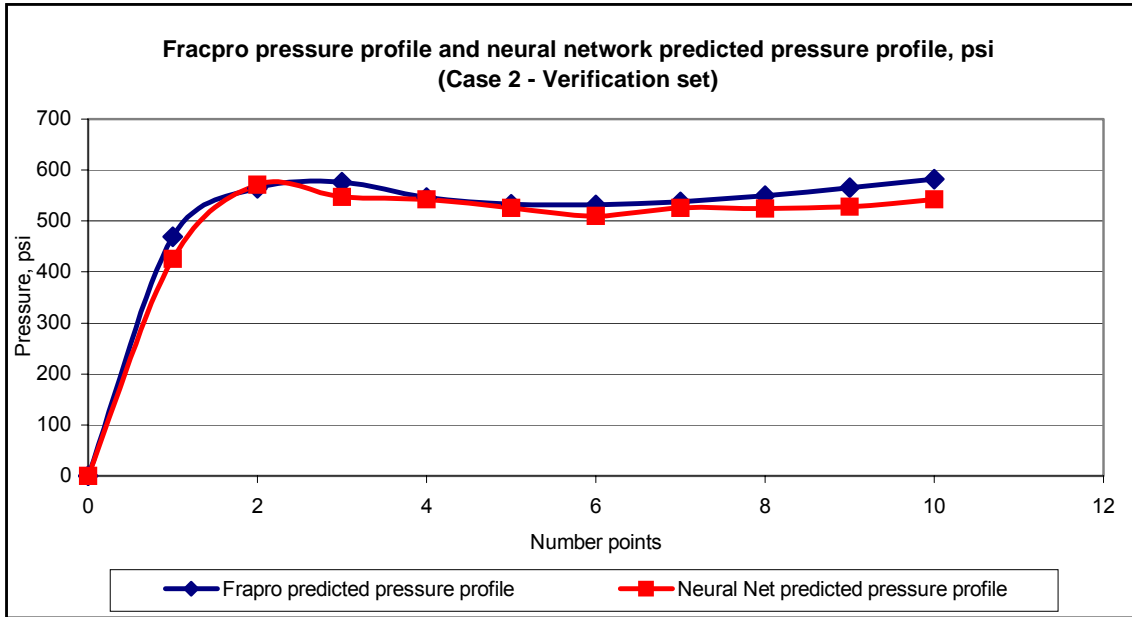


Figure 5-120: Case 2 FRACPRO and Neural Network Pressure Profiles

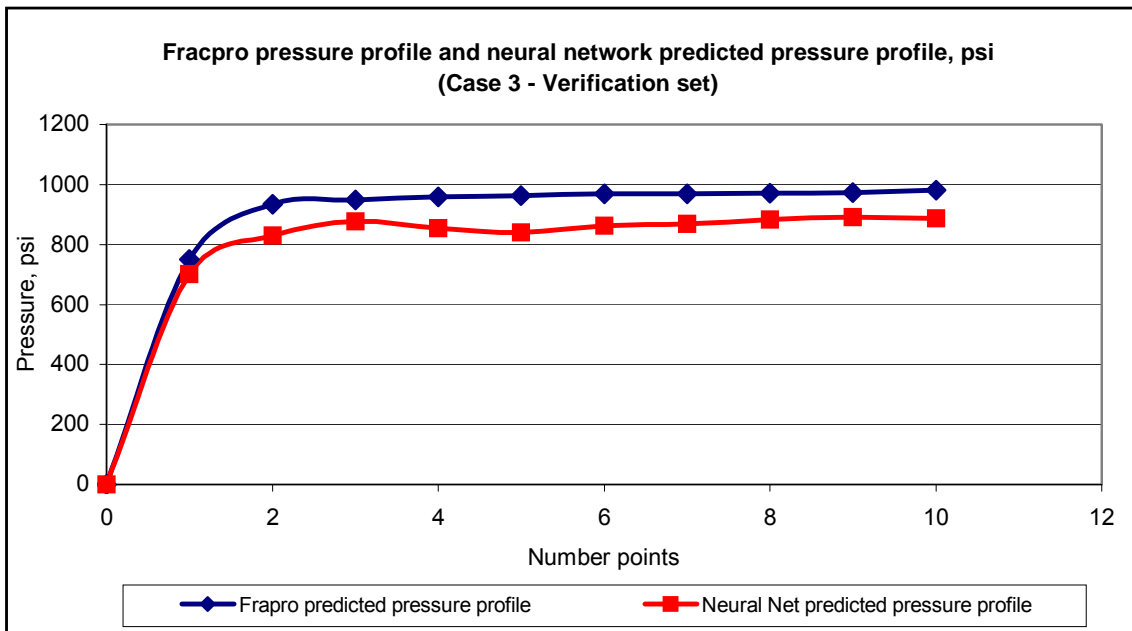


Figure 5-121: Case 3 FRACPRO and Neural Network Pressure Profiles

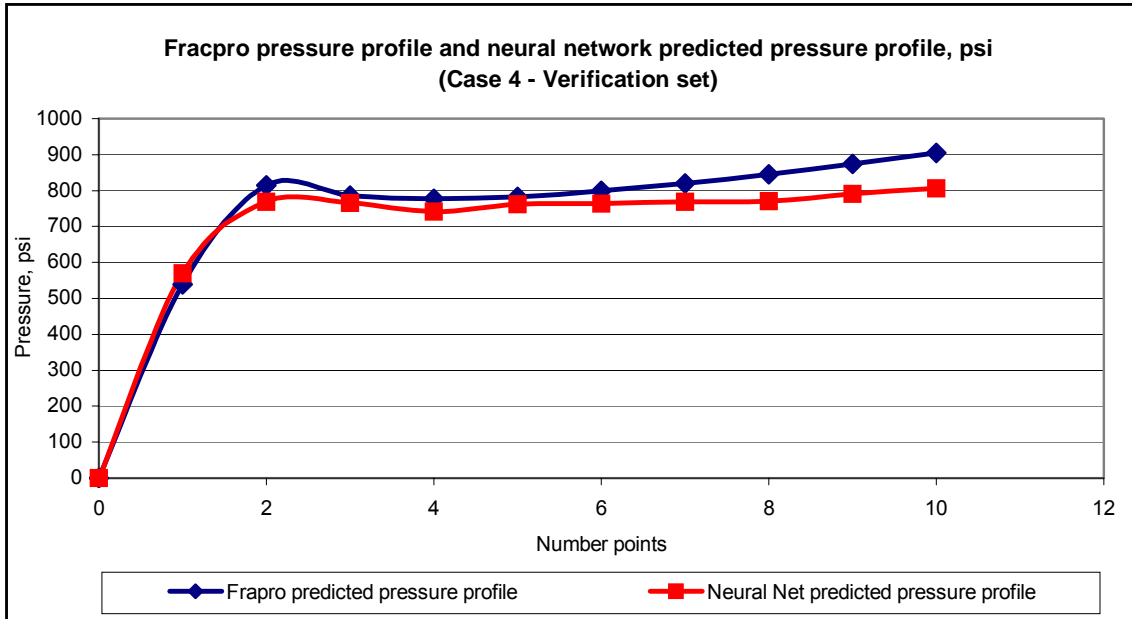


Figure 5-122: Case 4 FRACPRO and Neural Network Pressure Profiles

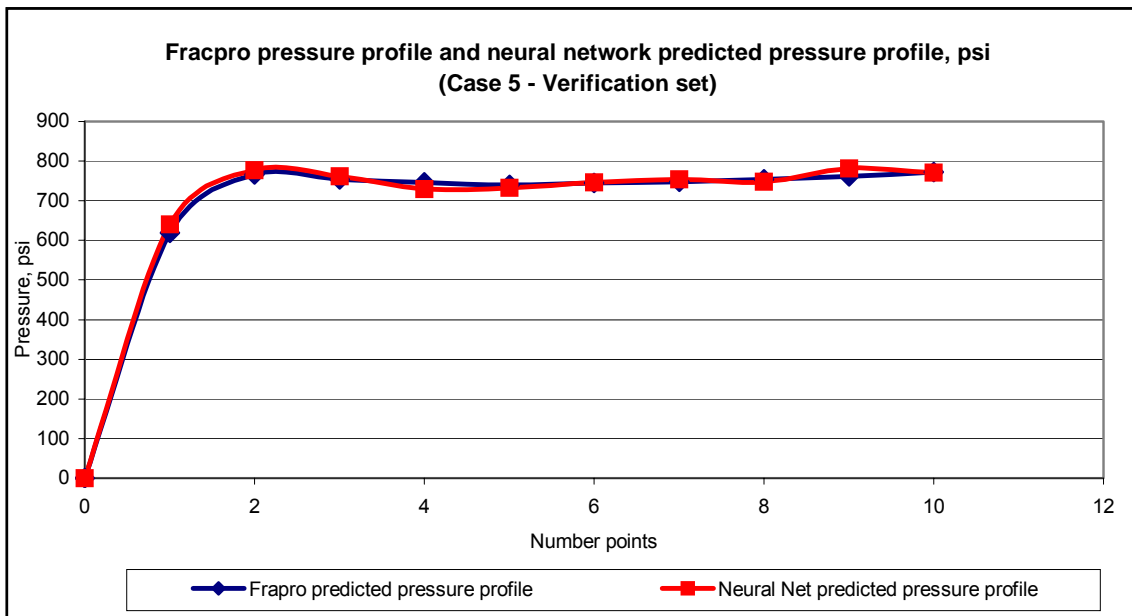


Figure 5-123: Case 5 FRACPRO and Neural Network Pressure Profiles

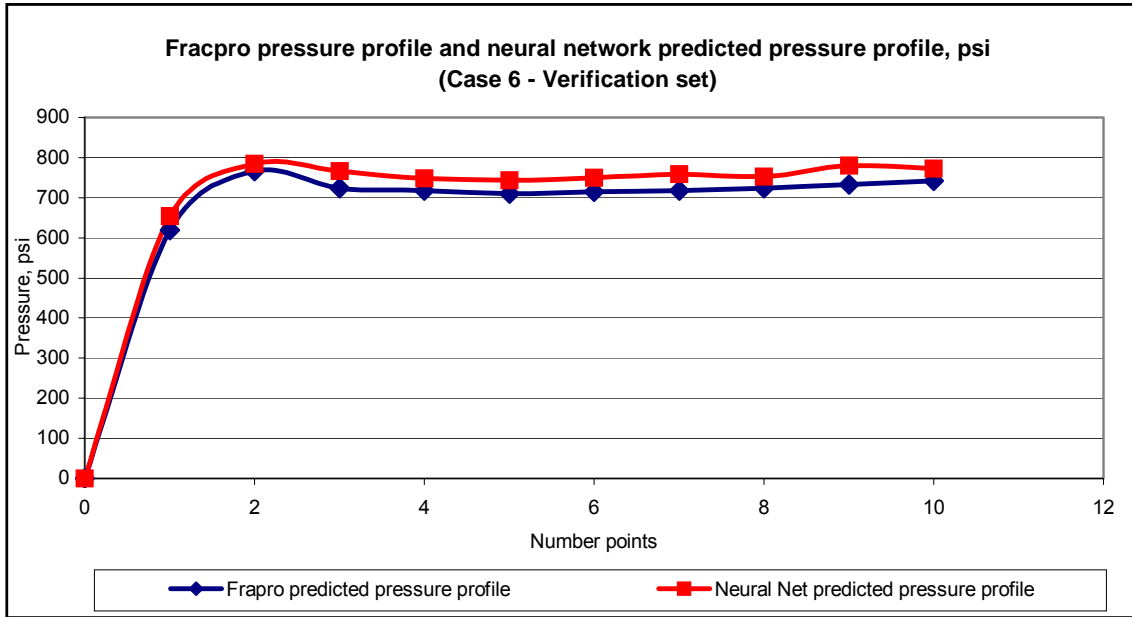


Figure 5-124: Case 6 FRACPRO and Neural Network Pressure Profiles

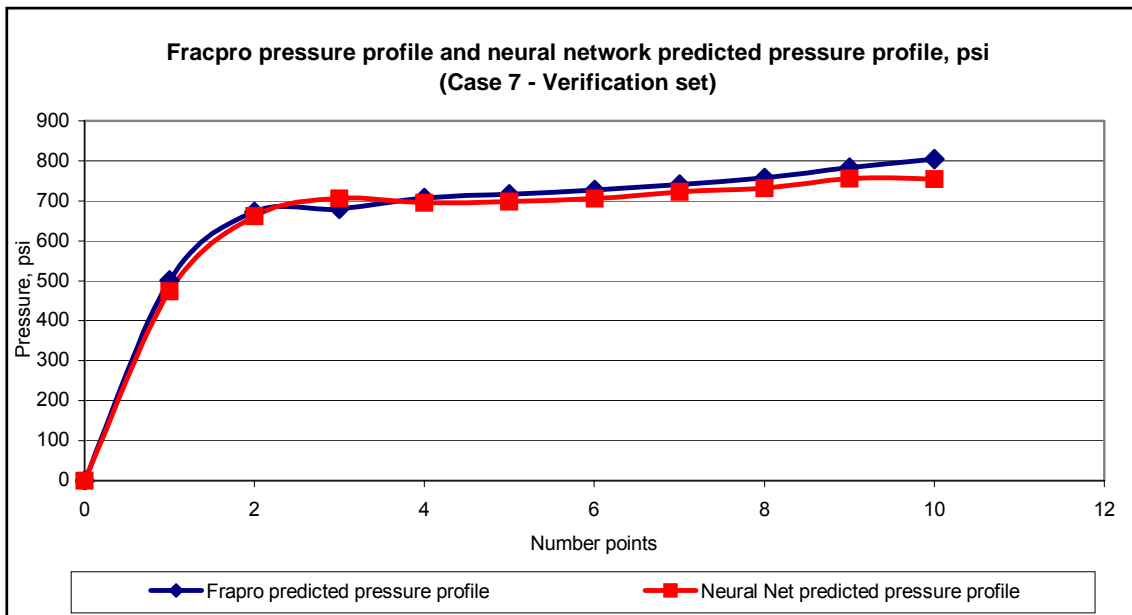


Figure 5-125: Case 7 FRACPRO and Neural Network Pressure Profiles

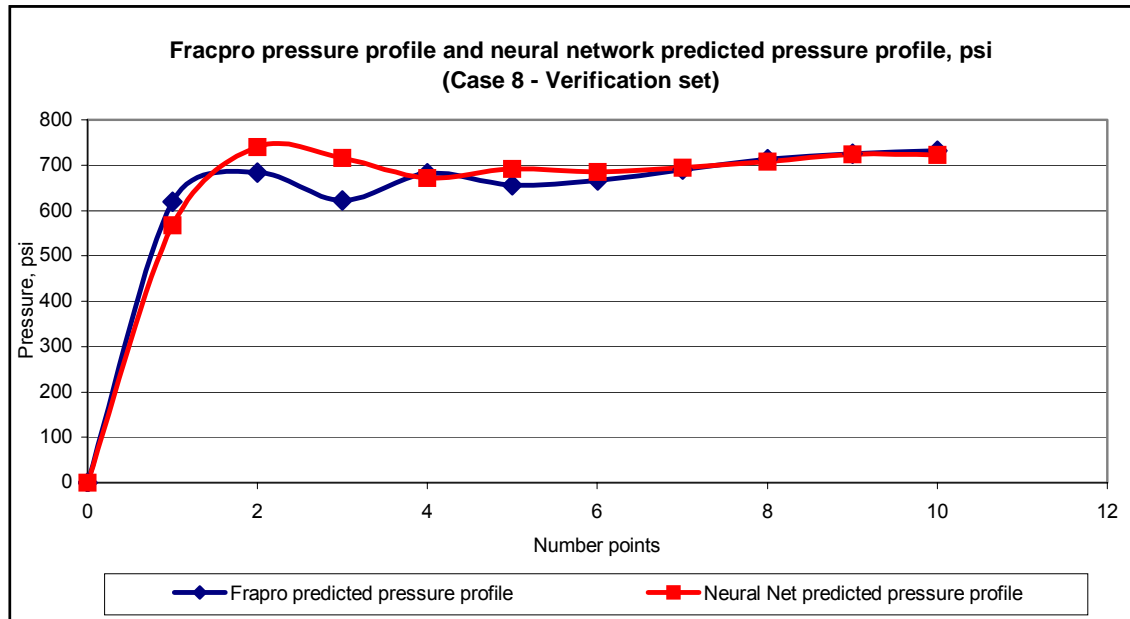


Figure 5-126: Case 8 FRACPRO and Neural Network Pressure Profiles

Figure 5-119 through Figure 5-126 graphically show the two pressure profiles, one from FRACPRO and the other generated by the neural network. The first point in each of the eight cases is called pressure point one. Similarly, for the remaining points to pressure point ten. In each pressure point set, there is at least one point where the neural network prediction significantly different than the actual value. For example, the neural network prediction of point seven is very close to the actual value in seven out of eight cases. However, in Case 3 (Figure 5-121), point seven falls below the actual by about 70 psi. This is more than enough to cause the R^2 for point seven to drop to 0.85. Point 10 falls short in three cases (2, 3 and 4), which drops the R^2 to 0.76. In other words, a few points in a set may tend to imply that the neural network is not accurate. However, the accuracy should not be judged by single point prediction since the output of the system is the entire signal. The closer the neural network pressure profile is to the shape and values of the FRACPRO pressure profile, the better the accuracy. Cases 1 through 8 show that, with exception of case 3, the pressure profiles generated by the neural network are very close to the FRACPRO generated profiles in values and shape. Even for Case 3, the shape of the neural network profile is very close to the FRACPRO profile.

The selected net pressure neural network provided good results when tested against FRACPRO generated data. Moreover, although the R^2 values are not exceptional, the correlation coefficients are above 0.9, with some up to 0.96. This proves the capability of reproducing the shape of the signal. Similar results were observed when working with logs, where neural networks were very capable of identifying layer signatures while falling short in predicting their

picks. As a result, the R^2 values were low, even though the neural networks were considered good performers.

5.4. Neuro-Genetic Model Verification

The optimum fracture treatment design is found through the use of a two-step method that integrates neural networks with genetic algorithms. The methodology presented in section 4.13 is implemented in a computer program that automatically runs different scenarios and provides optimum solutions. Since this study addresses tight sands, the tool was designed such that the optimization search is performed on fracture length. The fracture conductivity is calculated from the dimensionless coefficient.

5.4.1. Tool Design vs FRACPRO vs Actual for Treatment Schedule

To prove the accuracy of the proposed methodology, a comparison between the treatment schedule from a fracturing job performed by Halliburton and the treatment design generated by the computer program was initiated.

The original job is a ramp treatment schedule, performed at a depth of 11,440 ft in a tight gas reservoir. The job was designed to create a propped fracture length of 450 ft. The other characteristics of the treatment, fluid volumes, proppant concentration, and pumping rate are presented in Table 5-13. Since the fracture propped length of the original job was 450 feet, the same value was set as the target for the optimization criteria. The tool was run several times, and three examples of ramp treatment schedules were selected. The characteristics of these examples are included in Table 5-13. The FRACPRO simulator was run using the three selected treatments, and the results for each treatment were recorded. These results are presented in comparison with the output from the optimization tool.

	Case #	Treatment Characteristics					NN & GA		FRACPRO Results	
		Total Fluid Volume (gal)	Pad Fluid Volume (gal)	Starting Propp Conc (ppg)	Final Propp Conc (ppg)	Pump Rate (bpm)	Predictions		Propp Length (ft)	Frac Conductivity (mD-ft)
							Propp Length (ft)	Frac Conductivity (mD-ft)		
Original Job		59929	21504	1	7.5	20			451.6	315
NN & GA Suggested Optimum Treatments	Case 1	72041	28974	1	9	23	450.3	203.87	453.5	359
	Case 2	65843	20940	1	7.5	20	450.8	192.28	458.1	309
	Case 3	61674	39448	1	6	29	449.1	154.1	455.5	232

Table 5-13: Original Ramp Treatment with Optimum Suggested Treatments

The optimization routine suggested treatment designs with fracture propped length virtually equal to the desired value. The fracture propped length created by FRACPRO, when the suggested optimum treatment is run, is very close to the desired value. The largest difference between the original target and the suggested design propped length when run in FRACPRO is approximately 8 feet. This corresponds to a percentage error of 2%.

Note that each suggested optimum treatment has different characteristics:

- ▶ The treatments have various total fluid and pad volumes.
- ▶ The treatments have different final proppant concentrations.
- ▶ The treatments suggest different injection rates.

The second example tests the accuracy of the system for an eight-stage treatment design. The original job was pumped in the Teapot Formation in Kaye Field, Oklahoma. The treatment induced a fracture with a 403 ft length. Since it believed that the treatment was designed for a 400 ft fracture length, this value was set as the optimization criteria for the tool. The optimization module was run for each of the three eight-stage treatment types.

The optimum treatments suggested for each type of eight-stage job are presented in Table 5-14 along with the original Halliburton treatment. The table also shows the comparison between the fracture length suggested by the optimization routine, the fracture length generated by FRACPRO, and the fracture length of the original treatment. The three types of suggested eight stage treatments provided very close fracture length values compared FRACPRO. However the proppant concentration values are all higher than the original treatment.

	Original Treatment			NN&GA Optimum Suggested Treatment								
				Type I			Type II			Type III		
	Fluid Volume	Pump Rate	Propp Conc	Fluid Volume	Pump Rate	Propp Conc	Fluid Volume	Pump Rate	Propp Conc	Fluid Volume	Pump Rate	Propp Conc
	gal	bmp	ppg	gal	bmp	ppg	gal	bmp	ppg	gal	bmp	ppg
Stage 1	20,762	20	0	27,054	20	0	22,945	24	0	22,548	22	0
Stage 2	2,585	20	1	4,340	20	1.3	4,593	24	1	4,573	22	1.1
Stage 3	4,571	20	2	4,340	20	2.6	5,741	24	2	5,716	22	2.2
Stage 4	7,224	20	3	6,511	20	3.9	6,890	24	3	6,859	22	3.3
Stage 5	9,152	20	4	6,511	20	5.2	8,038	24	4	9,146	22	4.4
Stage 6	10,950	20	5	8,681	20	6.5	9,186	24	5	10,289	22	5.5
Stage 7	5,088	20	6	8,681	20	7.8	11,483	24	6	11,289	22	6.6
Stage 8	8,886	20	0	10,851	20	9.1	6,890	24	7	4,573	22	7.7
Flush				2,000	20	0	2,000	24	0	2,000	22	0
Total	69,218			78,969			77,766			76,993		
Fracture Propped Length												
NN&GA Prediction	-			400.5			399.5			400.2		
FRACPRO Results	403.71			400.19			394.04			404.25		
Proppant Concentration												
NN&GA Prediction	-			1.21			0.91			1.4		
FRACPRO Results	0.74			1.07			0.81			1.2		

Table 5-14: Original Halliburton Treatment with Eight Stage Optimum Suggested Treatments

Note that the provided Halliburton treatment actually consists of seven stages, and the fracturing fluid used was slightly different than the one used in training the neural network. Since the original treatment consisted of seven stages, a six-stage design was also generated for comparison. Table 5-15 shows the suggested six-stage optimum treatment along with the original Halliburton treatment.

	Original Treatment			NN&GA Optimum Suggested Treatment		
	Fluid Volume	Pump Rate	Propp Conc	Fluid Volume	Pump Rate	Propp Conc
	gal	bmp	ppg	gal	bmp	ppg
Stage 1	20,762	20	0	26,269	28	0
Stage 2	2,585	20	1	6,565	28	1
Stage 3	4,571	20	2	6,565	28	2.8
Stage 4	7,224	20	3	9,848	28	4.5
Stage 5	9,152	20	4	13,131	28	6.3
Stage 6	10,950	20	5	16,413	28	8
Stage 7	5,088	20	6	2,000	28	0
Stage 8	8,886	20	0			
Flush						
Total	69,218			80,791		
Fracture Propped Length						
NN&GA Prediction	-			400.8		
FRACPRO Results	403.71			396.7		
Proppant Concentration						
NN&GA Prediction	-			1.02		
FRACPRO Results	0.74			0.96		

Table 5-15: Original Halliburton Treatment with Six Stage Optimum Suggested Treatment

All treatments suggested by the optimization tool created a fracture propped length very close to the desired when run on FRACPRO. However, the system tends to provide slightly larger treatments, with higher total volumes and either more proppant or higher final proppant concentration.

Overall, the intent of a tool to design fracturing treatments was achieved. Using this tool, engineers can select the treatment that best fits their needs from a large variety of available treatment designs.

5.4.2. Design versus FRACPRO for Net Pressure Prediction

The accuracy of the pressure prediction module was tested using the three ramp treatment designs suggested in Table 5-13. The net pressure profile was recorded when each of the ramp treatments were run with FRACPRO. This profile was compared with the net pressure

profile created by the two neural networks (net pressure neural network and calibration neural network).

Figure 5-127 through Figure 5-129 shows the comparison of the FRACPRO pressure profile and the system predicted profile.

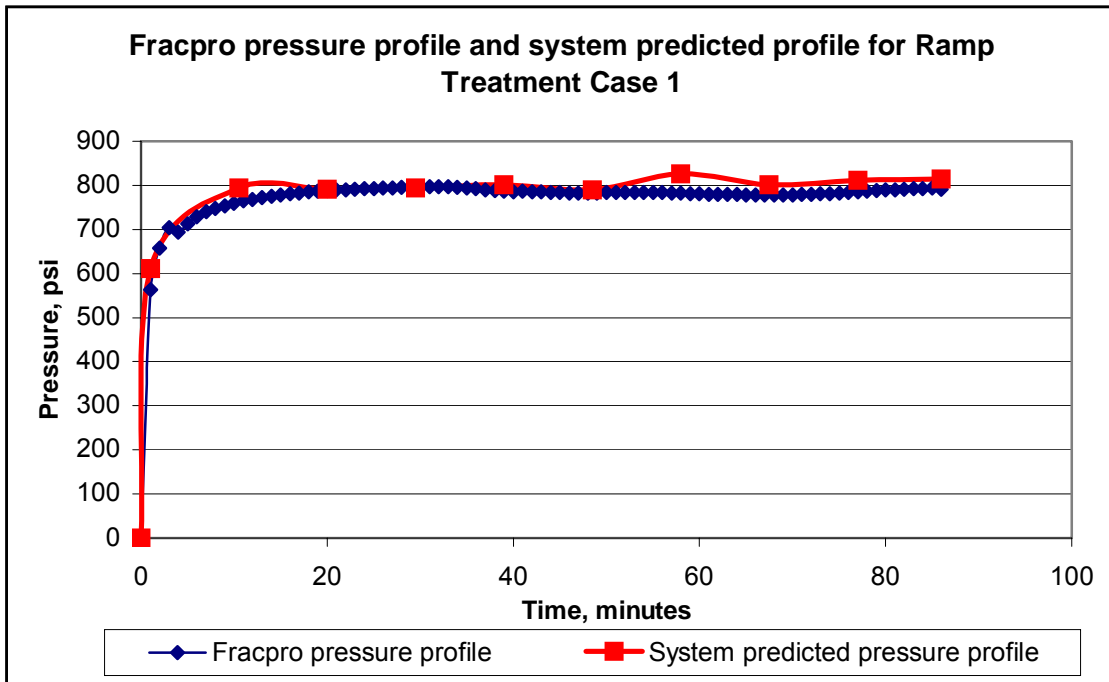


Figure 5-127: FRACPRO and System Generated Pressure Profile for Ramp Case 1

Figure 5-127 shows the best match among the three cases. It corresponds to the largest treatment suggested by the optimization tool.

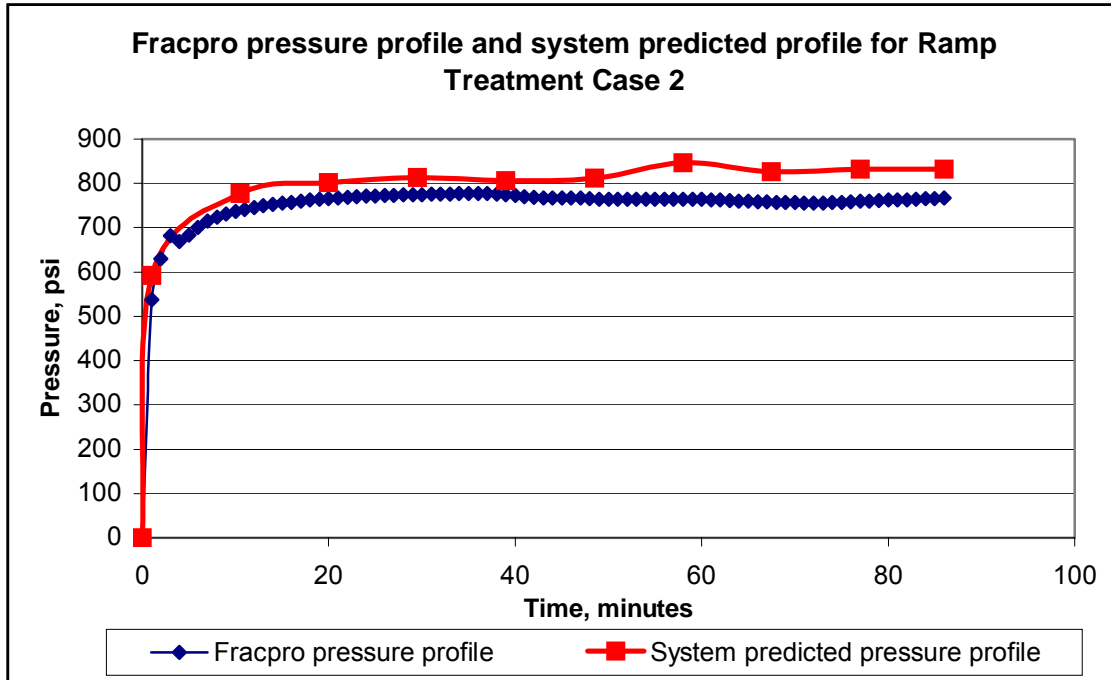


Figure 5-128: FRACPRO and System Generated Pressure Profile for Ramp Case 2

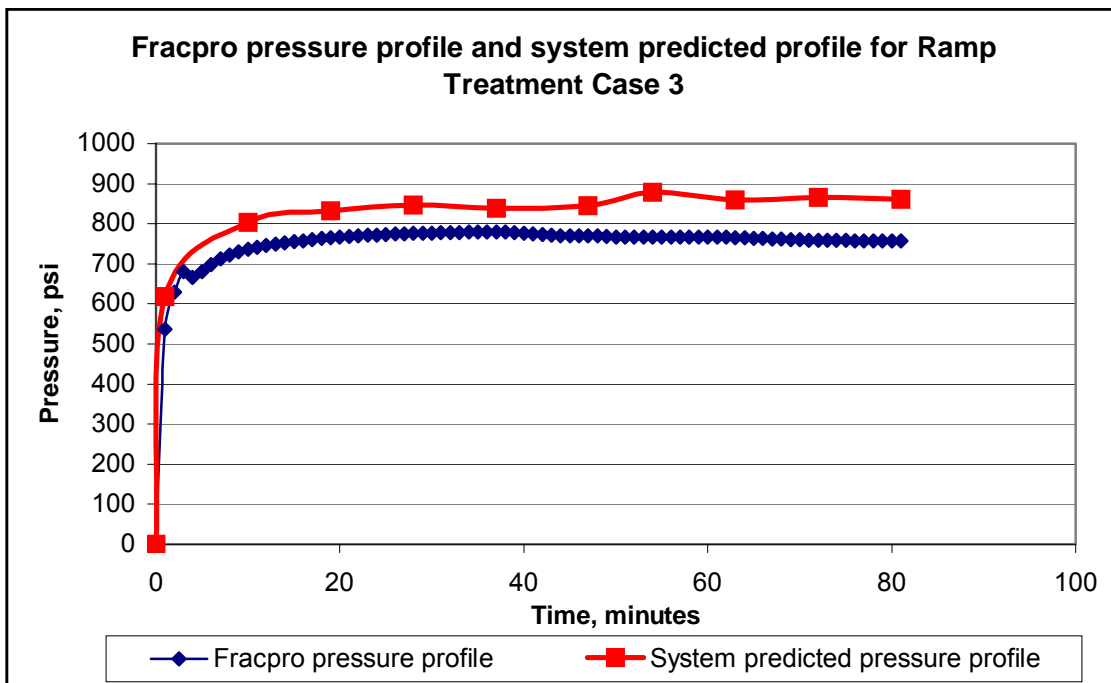


Figure 5-129: FRACPRO and System Generated Pressure Profile for Ramp Case 3

In Figure 5-128, the system predicted pressure profile is above the FRACPRO pressure profile. This case corresponds to a job very similar to the original one, except that a larger fluid volume was injected.

In the last case, Figure 5-129, the system overestimates the pressure profile. In this case, the fluid volume was closest to the original job, but the treatment had a much lower final proppant concentration.

CHAPTER 6: FUTURE RESEARCH

1. The methodology presented in this study proposed an equivalent four-layer stress profile to deal with the large amount of data necessary for complex reservoir characterization. This equivalent four-layer profile proved very effective in preserving the geometry of the fracture when used in the simulation. However, a significant gap was observed between the net pressure profile using the original lithology and the pressure profile using the resulting equivalent stress profile. In this study, the gap was corrected using a calibration neural network trained using only formation characteristics. Further research should be conducted into the cause of this gap. There is potential to develop a new equivalent layer profile model that may predict the correct net pressure profile without the use of a calibration neural network.
2. In section 4.12.2.2 The Eight Stage Treatment, Table 4-17 and Figure 4-14 proved that neural network performance improves with the number of cases. Since, only 200 cases were used to train each of the eight-stage treatment types, it is proposed that new types of neural networks use at least 500 generated cases. If needed, more cases should be generated for each type of eight-stage treatment and the neural network retrained. Additionally, the dimensionless conductivity ratio should be included in the data collected.
3. The neural network for predicting the net pressure profile was trained using only 100 cases. The intention was to research the possibility of predicting the net pressure profile using neural networks. The performance of the neural networks is very acceptable considering the low number of cases used in training. However, to improve its performance, more cases should be generated. Furthermore, it appears that the accuracy of the neural network lessens at the extremities of the stress profile (first and last few points). It is proposed to use more points, particularly to define the first 10 –15 minutes of the pressure signal.
4. Finally, future work should focus on reproducing this study using actual field data. Major companies possess databases with significant amounts of data required to reproduce this study. Fields where more than 80 wells are frac'ed every year, like Lost Hills, California, would provide a great source of data for both treatment schedules and pressure profiles. Not only could the application of this research facilitate the design of optimum fracturing treatments, it also has the potential of identifying other best practices that could lead to major savings for the company.

CHAPTER 7: SOFTWARE IMPLEMENTATION

The methodology presented in this study was incorporated into a computer application. This application is intended to be a companion tool for any commercial hydraulic fracture simulator to assist engineers in optimizing fracturing treatment schedules. Figure 7-1 shows the start-up form of the application. The user can select an already existing well or load data for a new well.

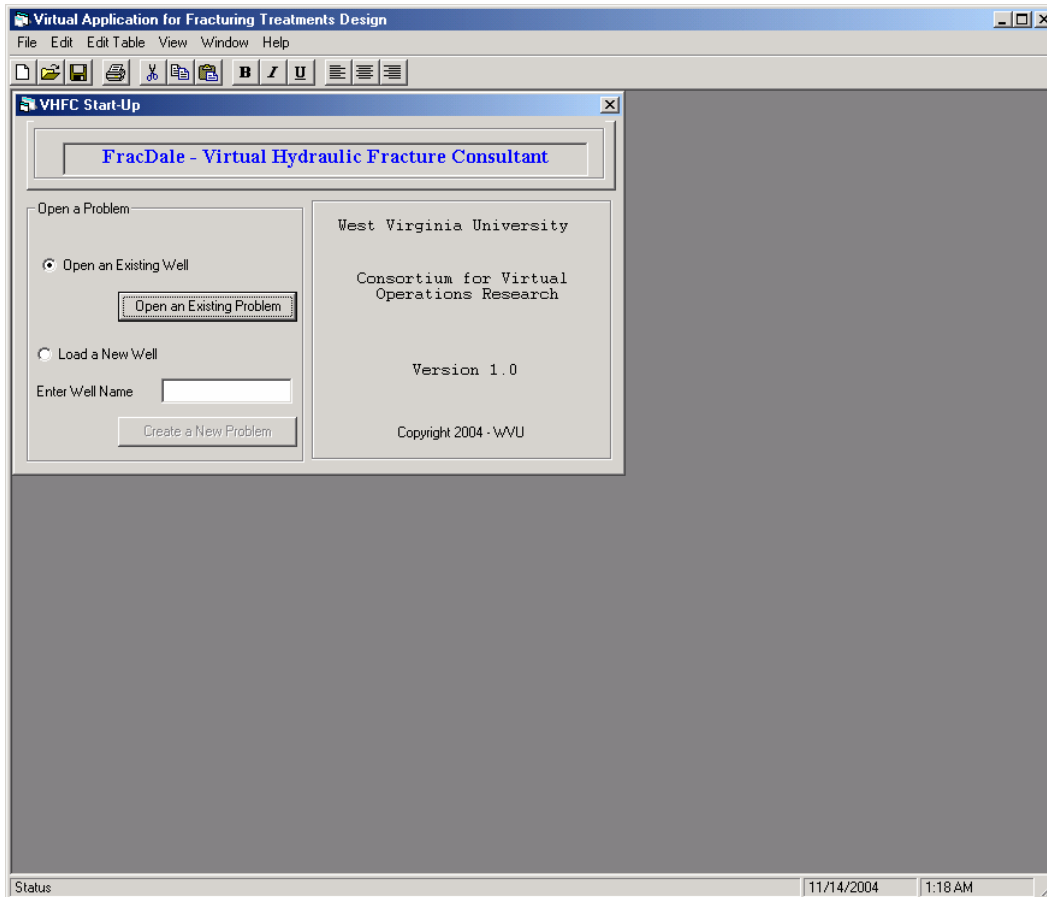


Figure 7-1: Start-Up Form

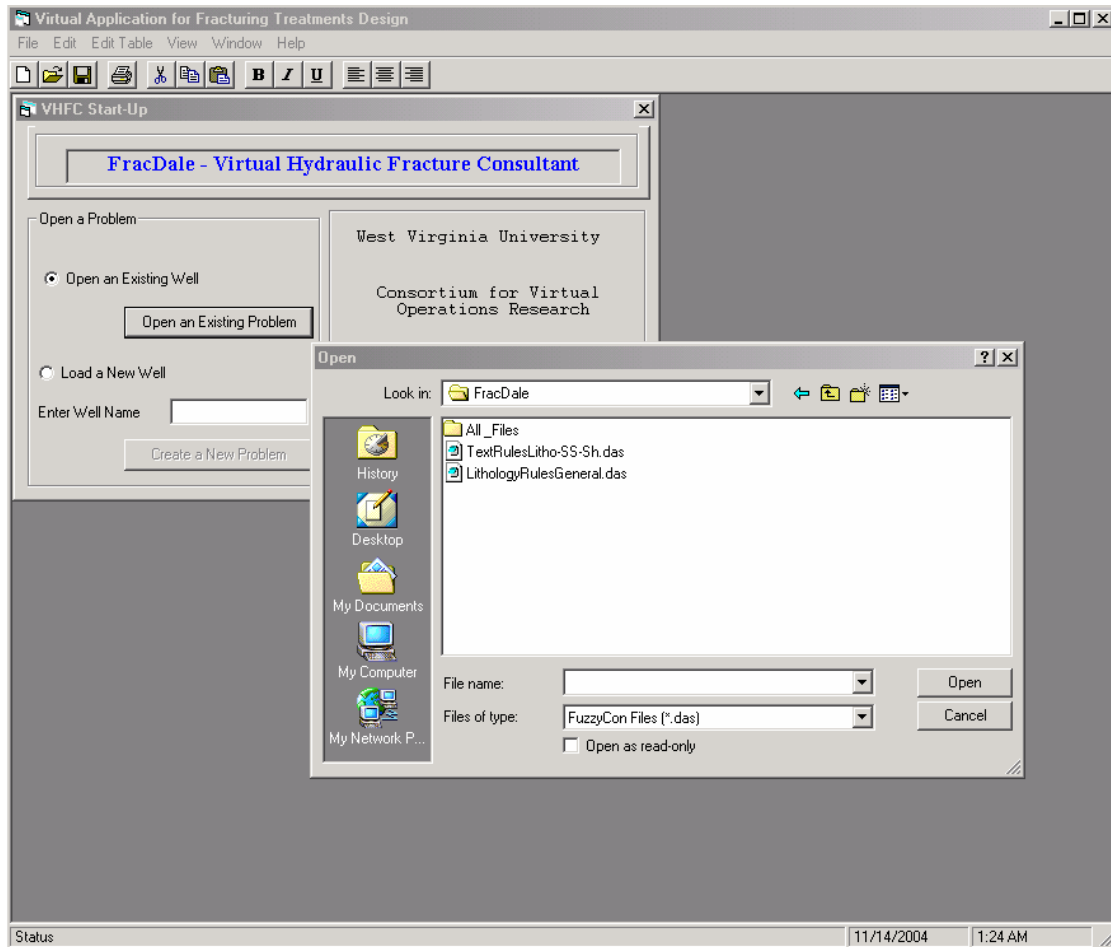


Figure 7-2: Loading a Well File

Figure 7-2 shows the process of opening a well file. This screen is the same whether the user is loading a pre-existing file or creating a new well and importing log data from an external file. The application allows the user to import data from text files, Microsoft Excel files, or databases.

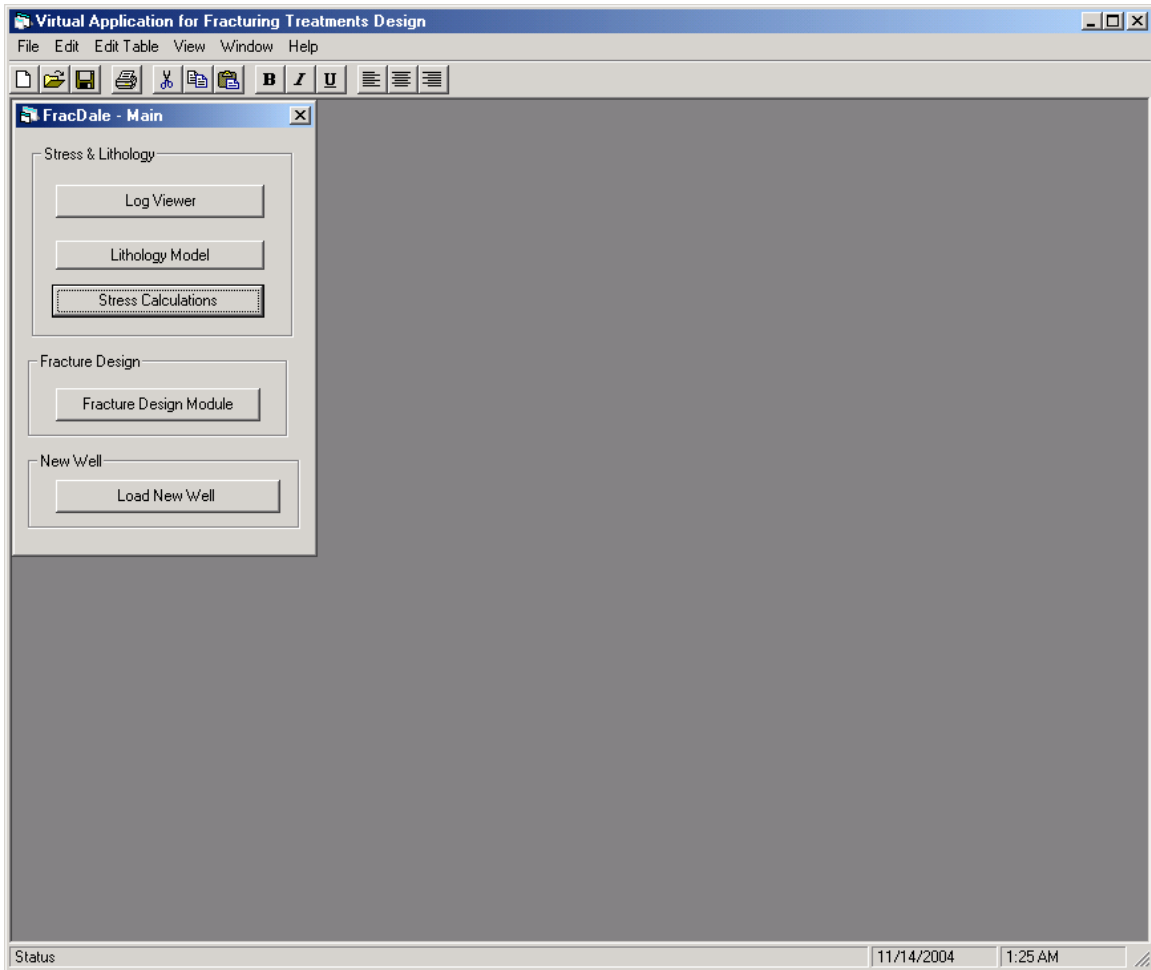


Figure 7-3: Main Console Form

Figure 7-3 shows the Main Console of the program. From this form, users have access to all modules of the program at any time. This form is active any time the program is running. The main modules are the Stress & Lithology Modules, which includes the Log Viewer, Lithology Model, Stress Calculations, and Fracture Design Module. Typically the normal flow process starts with Log Viewer model.

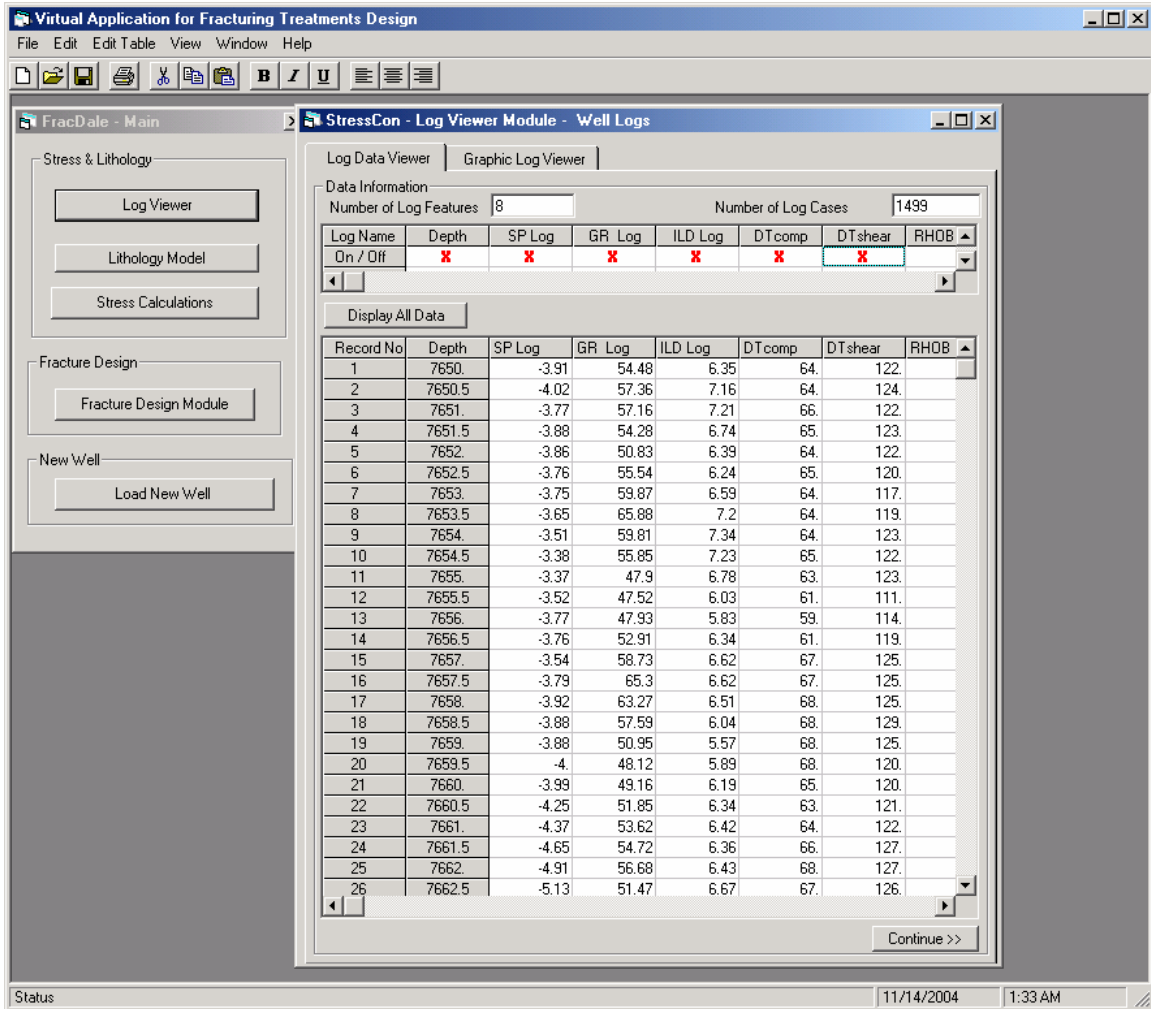


Figure 7-4: Log Viewer Model Form – Log Data Viewer Tab

Figure 7-4 shows the Log Viewer form, which has two tabs. In the first tab, log data is presented in table format. The log values are available for visual analysis. From the top table, the user can select the logs he or she wants to plot. Once the desired logs are selected they will be plotted on the second tab. One can return to this tab at any time to select a different combination of logs or to plot one log at a time.

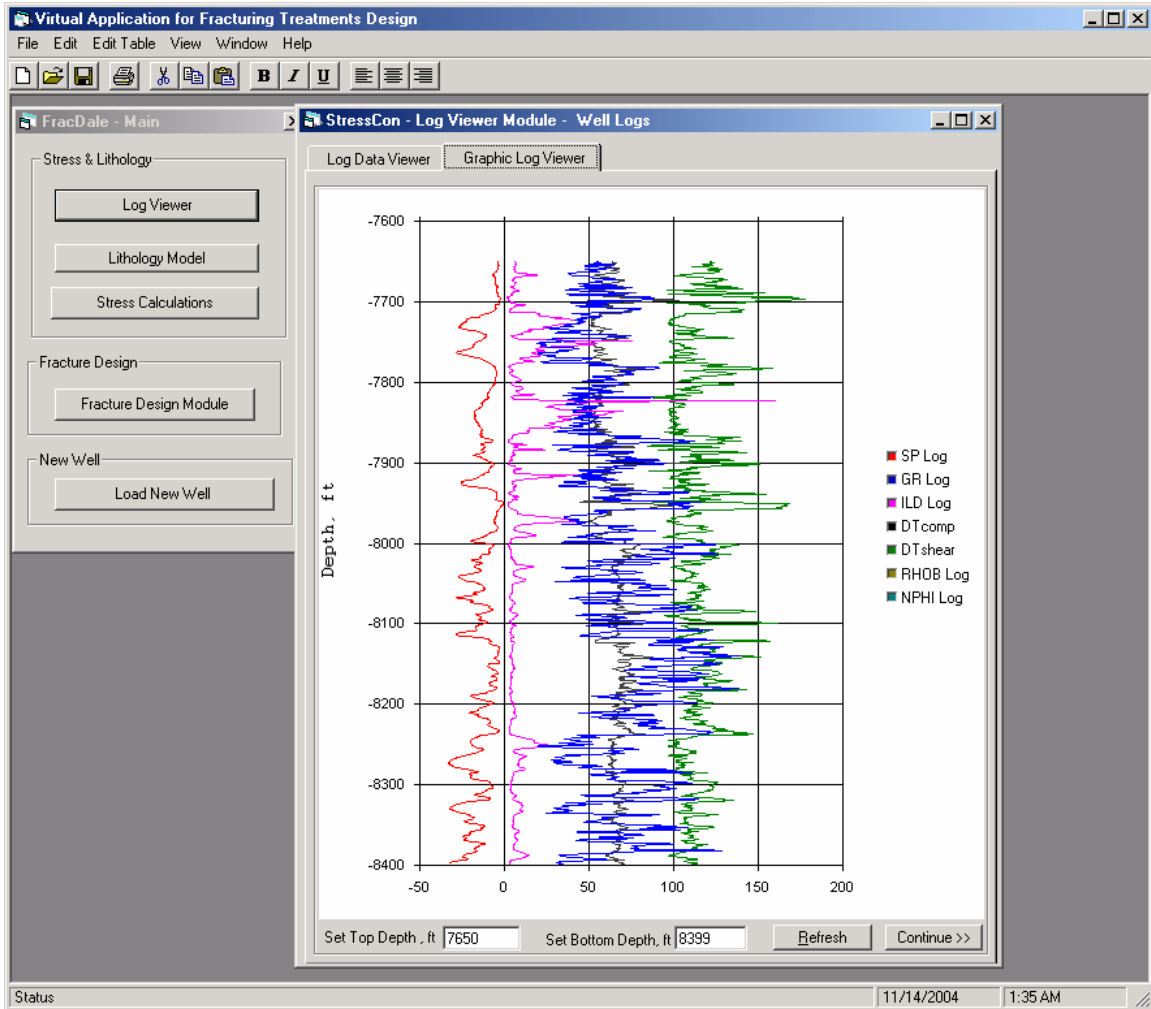


Figure 7-5: Log Viewer Model Form – Graph Log Viewer Tab

Figure 7-5 shows the plot of the selected logs from the first tab. The legend indicates which logs were selected for analysis. The scale of the graph can be changed to accommodate the desired intervals.

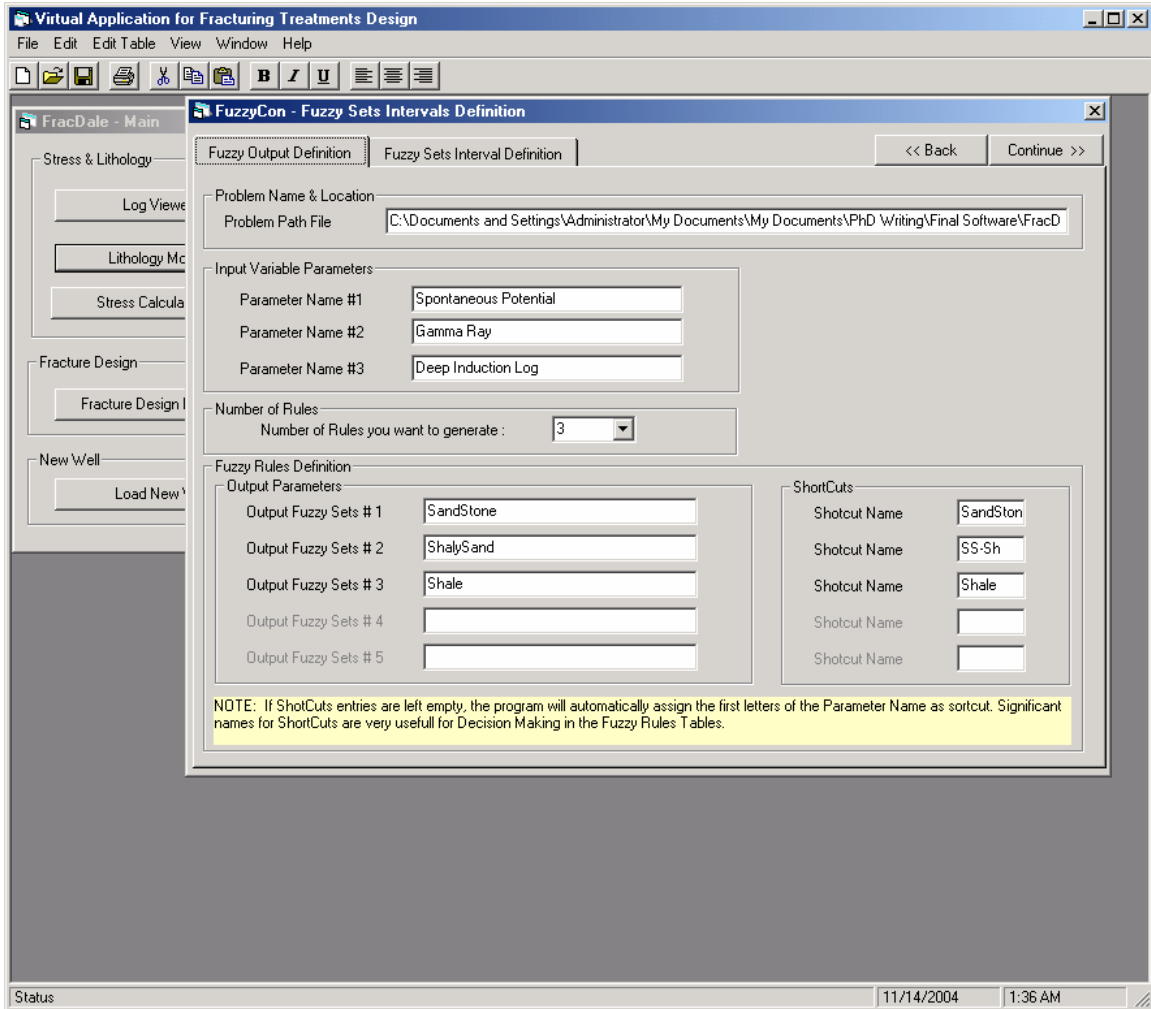


Figure 7-6: Lithology Model Design Form – Input/Output Definition Tab

Figure 7-6 shows the Design form of the Lithology Model. The analysis is done with defined fuzzy lithology rules loaded from an external file. However the user can design his/her own rules as well. The form includes two tabs, Fuzzy Input/Output Definition and Fuzzy Sets Interval Definition. Figure 7-6 shows the first tab where the user enters the names of the input parameters and the outcomes of the system.

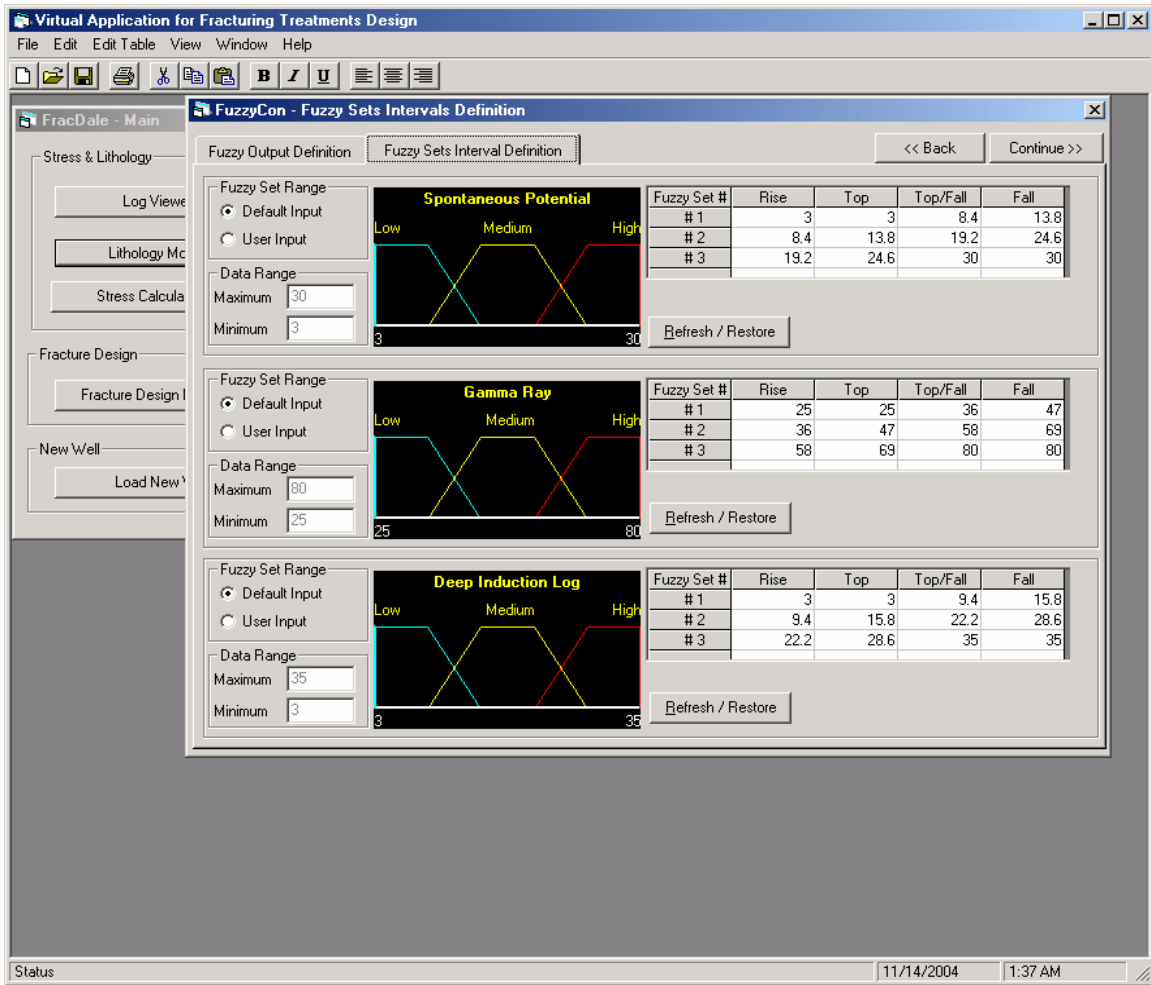


Figure 7-7: Lithology Model Design Form – Fuzzy Sets Interval Definition Tab

Figure 7-7 shows the second tab of Lithology Model Design form entitled Fuzzy Sets Interval Definition. As the name implies, the intervals for the fuzzy sets are entered in this section. The default is the trapezoidal fuzzy set. However, each one can be modified to reflect specific cases. The user can access this form at any time during fuzzy design.

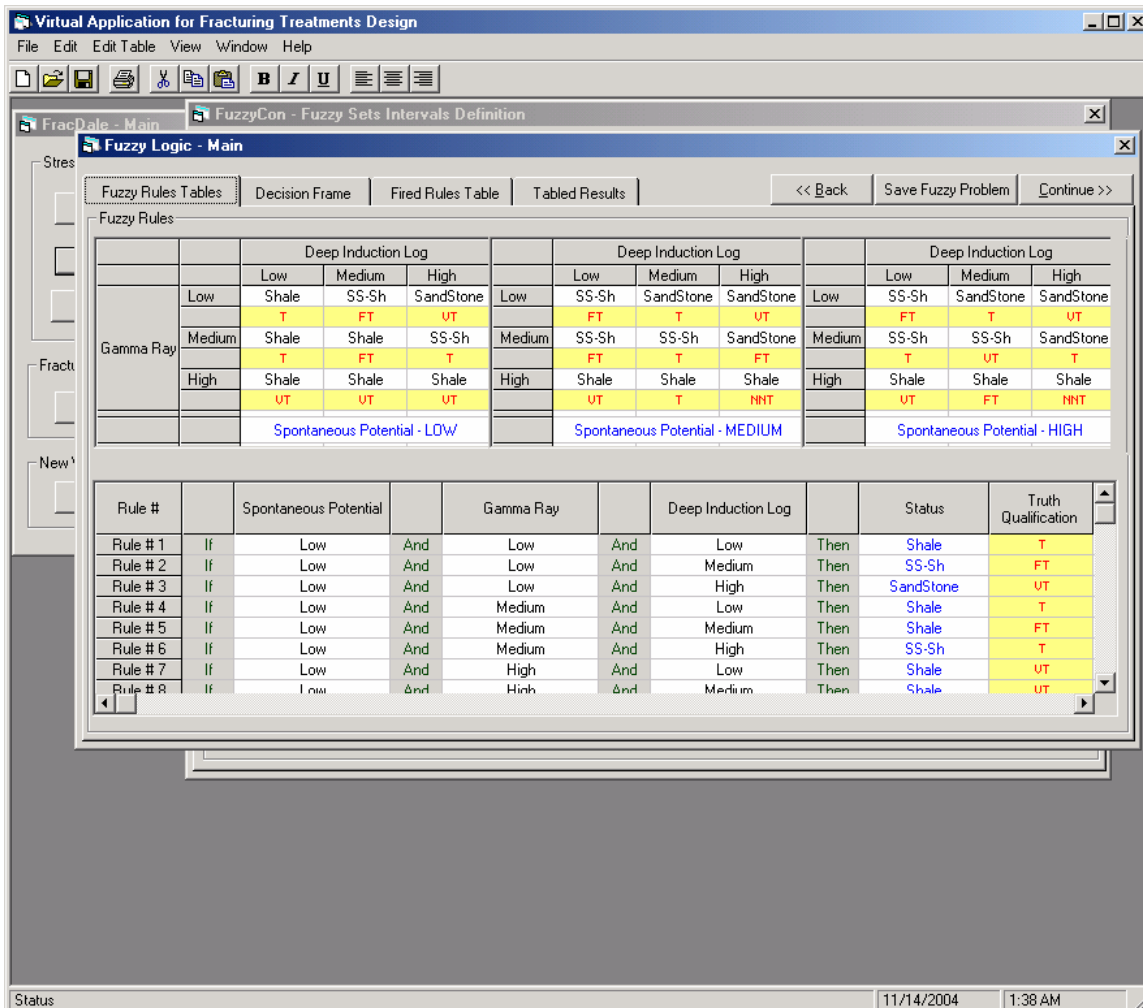


Figure 7-8: Lithology Model Main Form – Fuzzy Rules Table Tab

Figure 7-8 shows the main form of the Lithology Model. This form consists of four tabs: Fuzzy Rules Table, Decision Frame, Fired Rules Table, and Tabled Results. Figure 7-8 presents the first tab, Fuzzy Rules Table. The rules are set in the three-grid matrix using data previously entered in the Lithology Model Design form, Input/Output Definition Tab (Figure 7-6). If a well file is loaded the rules are already present in the grids; otherwise, the grids are empty and the user is required to build the rules. The next table shows the rules in line format.

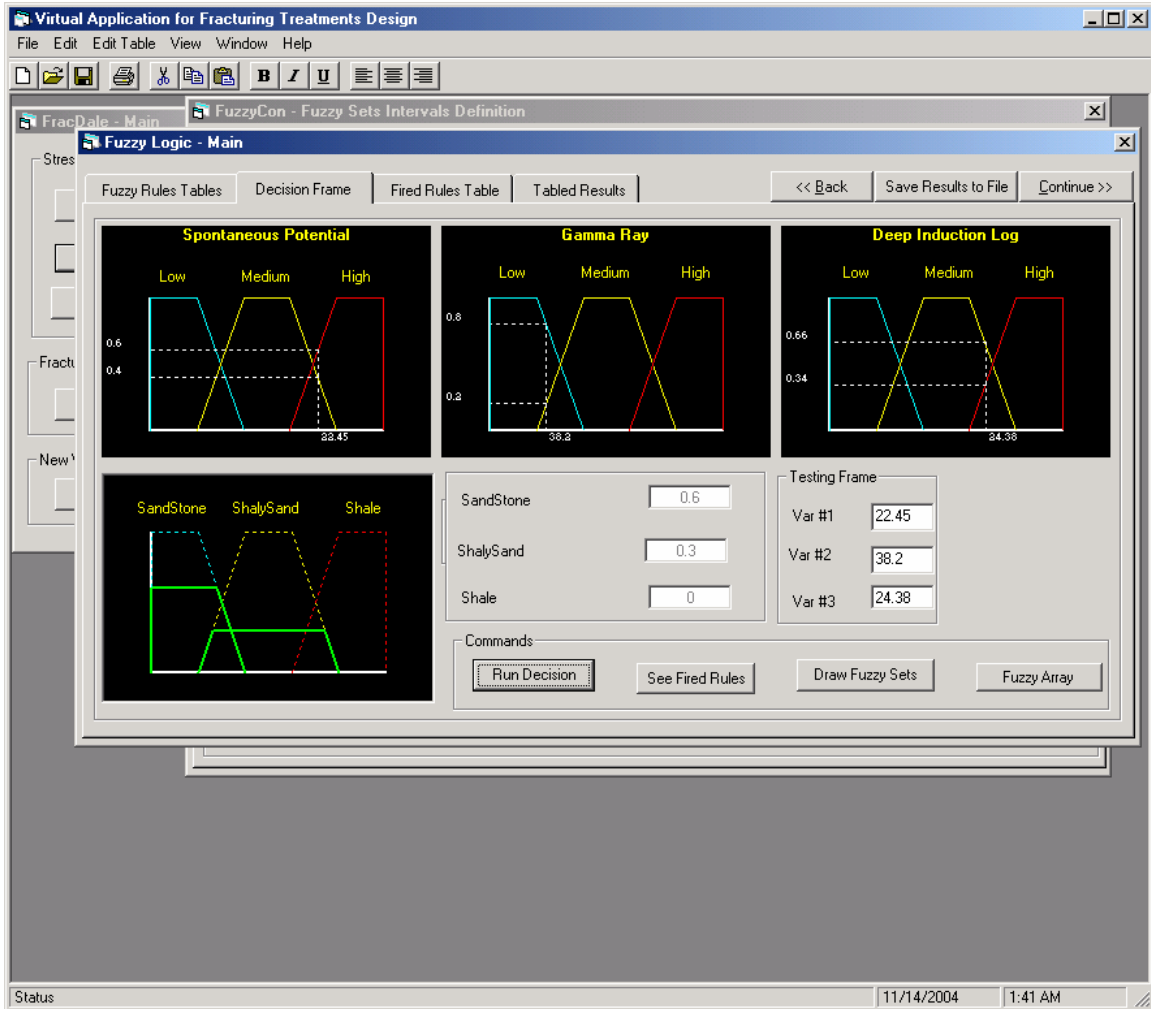


Figure 7-9: Lithology Model Main Form – Decision Frame Tab

Figure 7-9 shows the Decision Frame tab of the main form in the Lithology Model. This tab illustrates the decision process. Fuzzy logic decision is a parallel process where multiple rules are fired in the same time. Figure 7-9 shows an example of a decision where each input value belongs to more than one set. The outcome of the system is given in degrees of membership, graphically presented in the plot on the bottom of the screenshot. The user can make decisions firing cases one by one or the entire set of cases at once. To fire all cases, the Fuzzy Array command button is used.

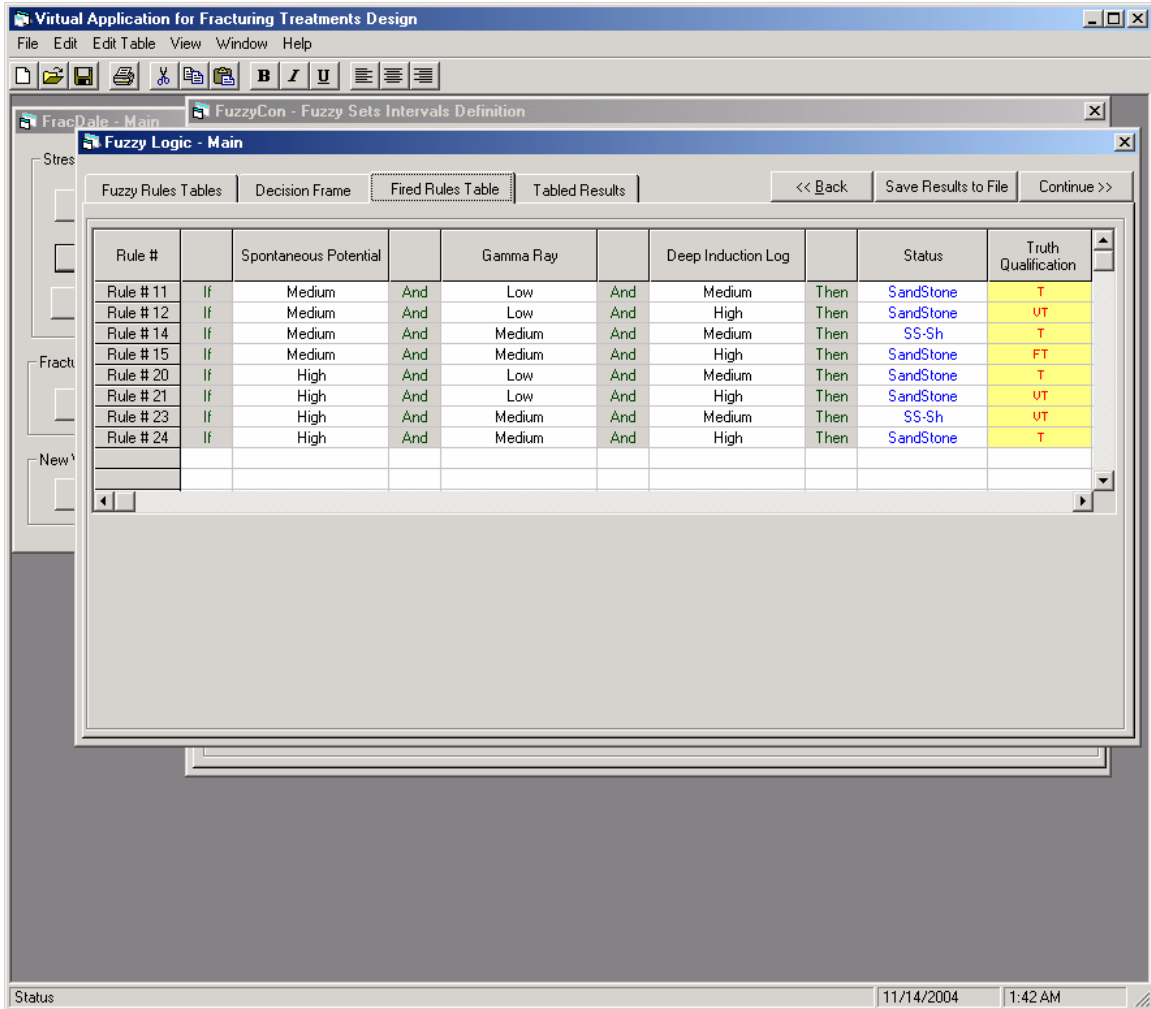


Figure 7-10: Lithology Model Main Form – Fired Rules Table Tab

Figure 7-10 shows the Fired Rules Table tab of the main form in the Lithology Model. This tab shows all the rules that were fired for the example presented in the previous tab. This information is typically used for a single case decision where in-depth analysis is required.

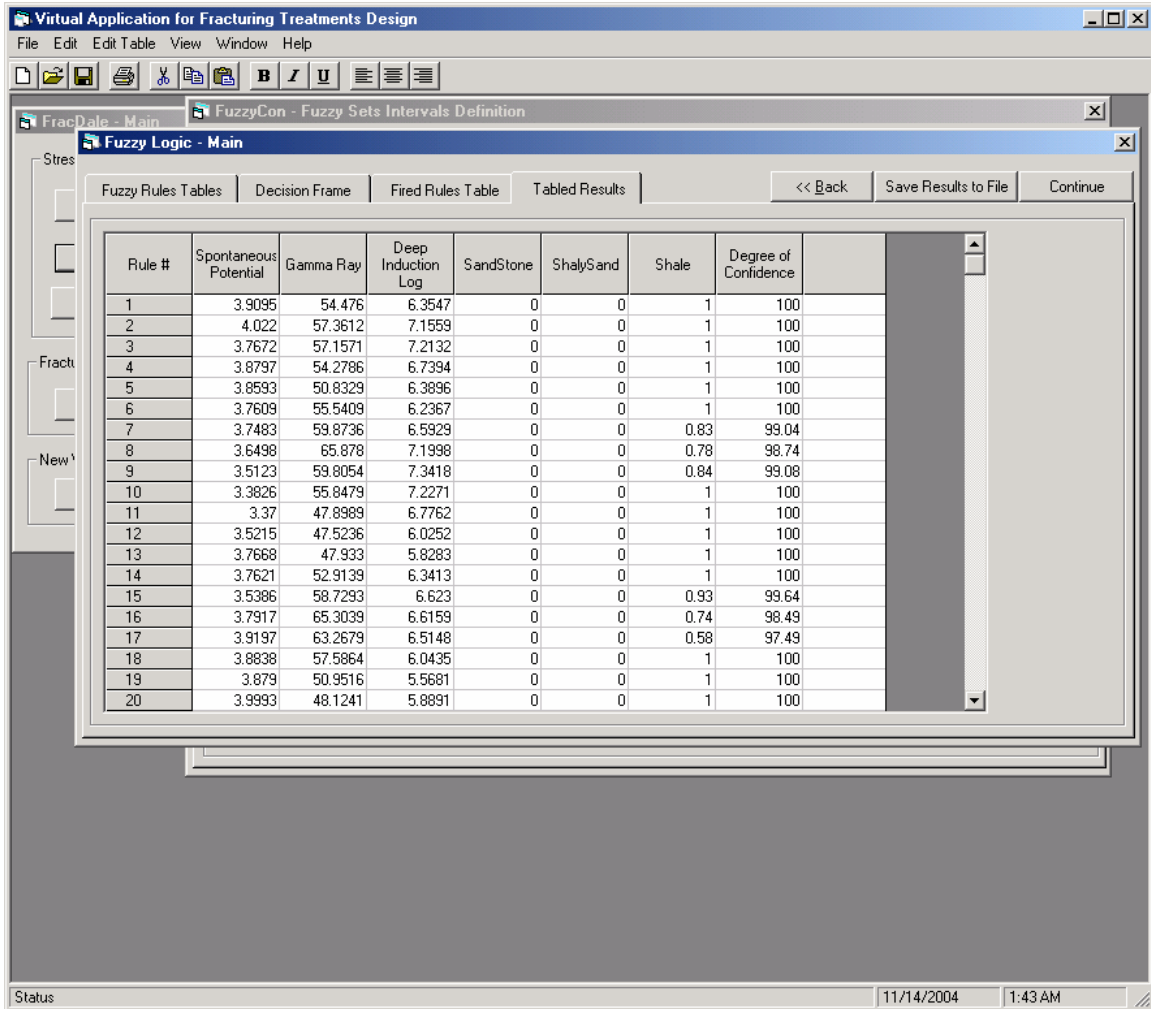


Figure 7-11: Lithology Model Main Form – Tabled Results Tab

Figure 7-11 shows the Tabled Results tab of the main form in the Lithology Model. The table includes all cases from the data set involved in the decision process. The outcome of the decision is presented in columns four, five, and six, while the degree of confidence for each decision is given in column seven. This information can be used as is or as the input for a different process. In this application, the outcome of the decision process is used with a classic stress equation to calculate the stress profile.

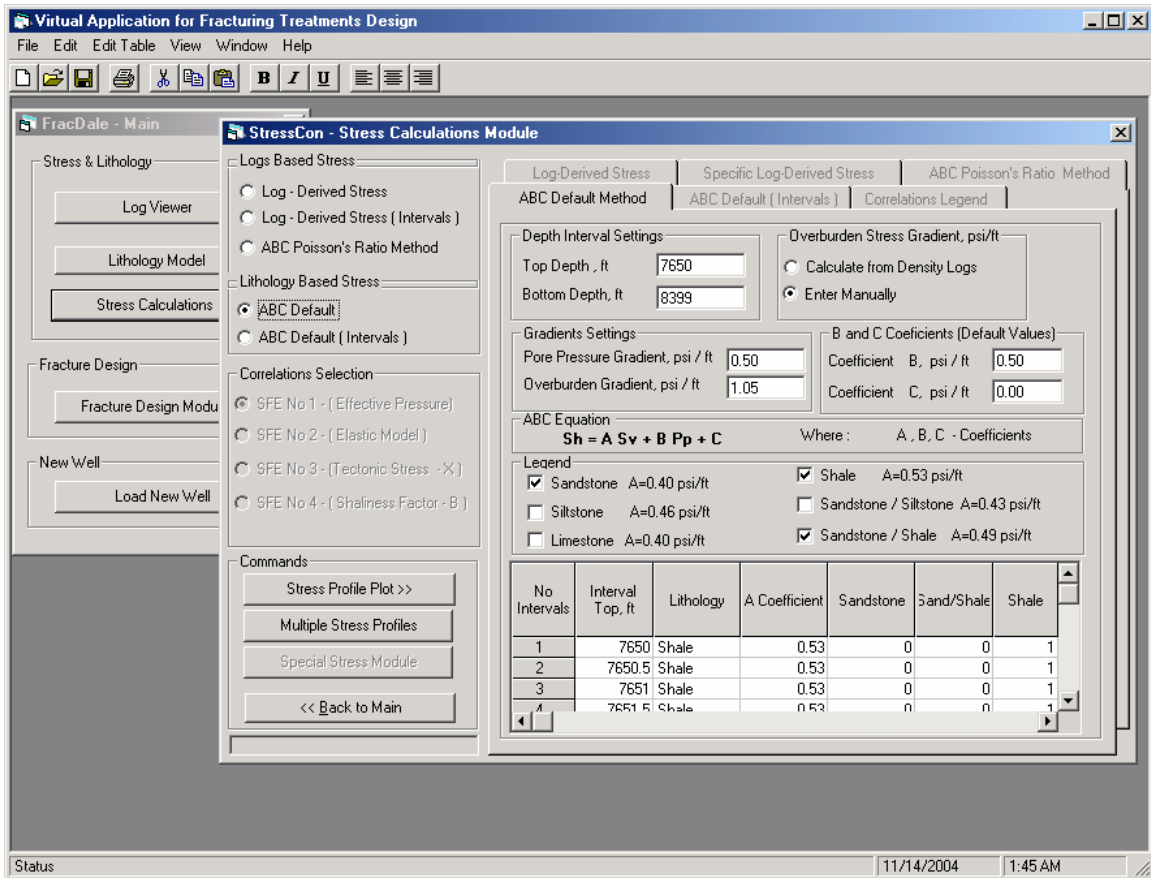


Figure 7-12: Stress Calculation Module Form – ABC Default Method Tab

Figure 7-12 shows the Stress Calculation Module Main form. This module includes all available techniques and equations for stress calculations. On the left side of the form, a radio button is provided for each model. By selecting a model, a tab is activated on the right part of the screen that has the details regarding the model along with the requested and default information. Figure 7-12 presents the ABC Default Model selection, since this model was used in the study. The inner ABC Default tab shows information regarding data and gradients used in calculations. The table from the bottom of the screen displays the lithology identification interpretation as an outcome of the fuzzy lithology system. A routine assigns the corresponding lithology coefficient and the calculated stress.

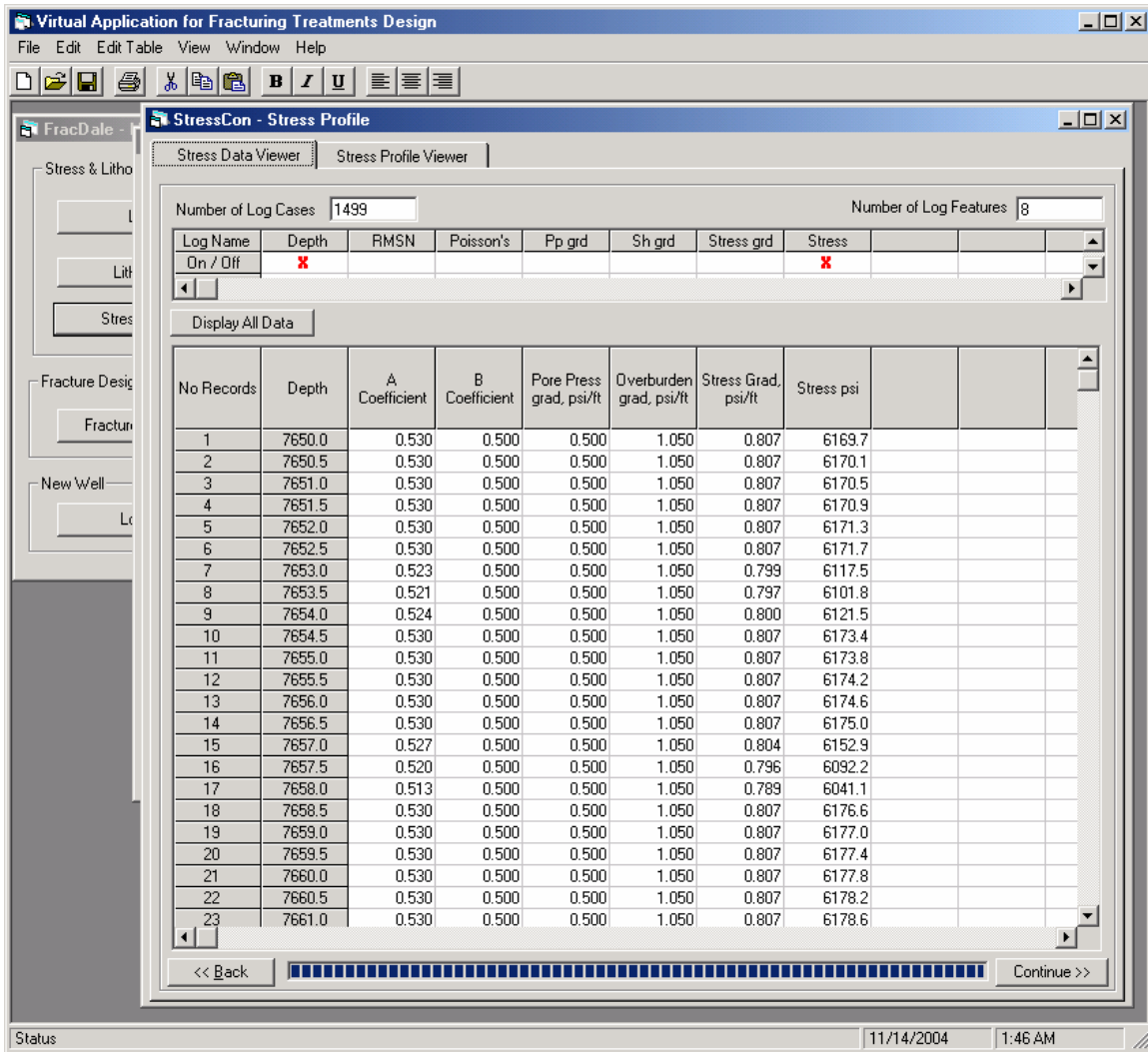


Figure 7-13: Stress Profile Form – Stress Data Viewer Tab

Figure 7-13 shows the Stress Profile form. This form presents two tabs: Stress Data Viewer and Stress Profile Viewer. The results of the stress calculation done in the Stress Calculation Module form are imported into the Stress Data Viewer tab of this form. Depths, Poisson's coefficients, overburden gradients, stress values, and others are displayed in table format. Similar to the Log Viewer form, the user can select which parameter to plot. The stress profile presents the most interest for graphic visualization since this processing will be a direct input into the fracture treatment optimization.

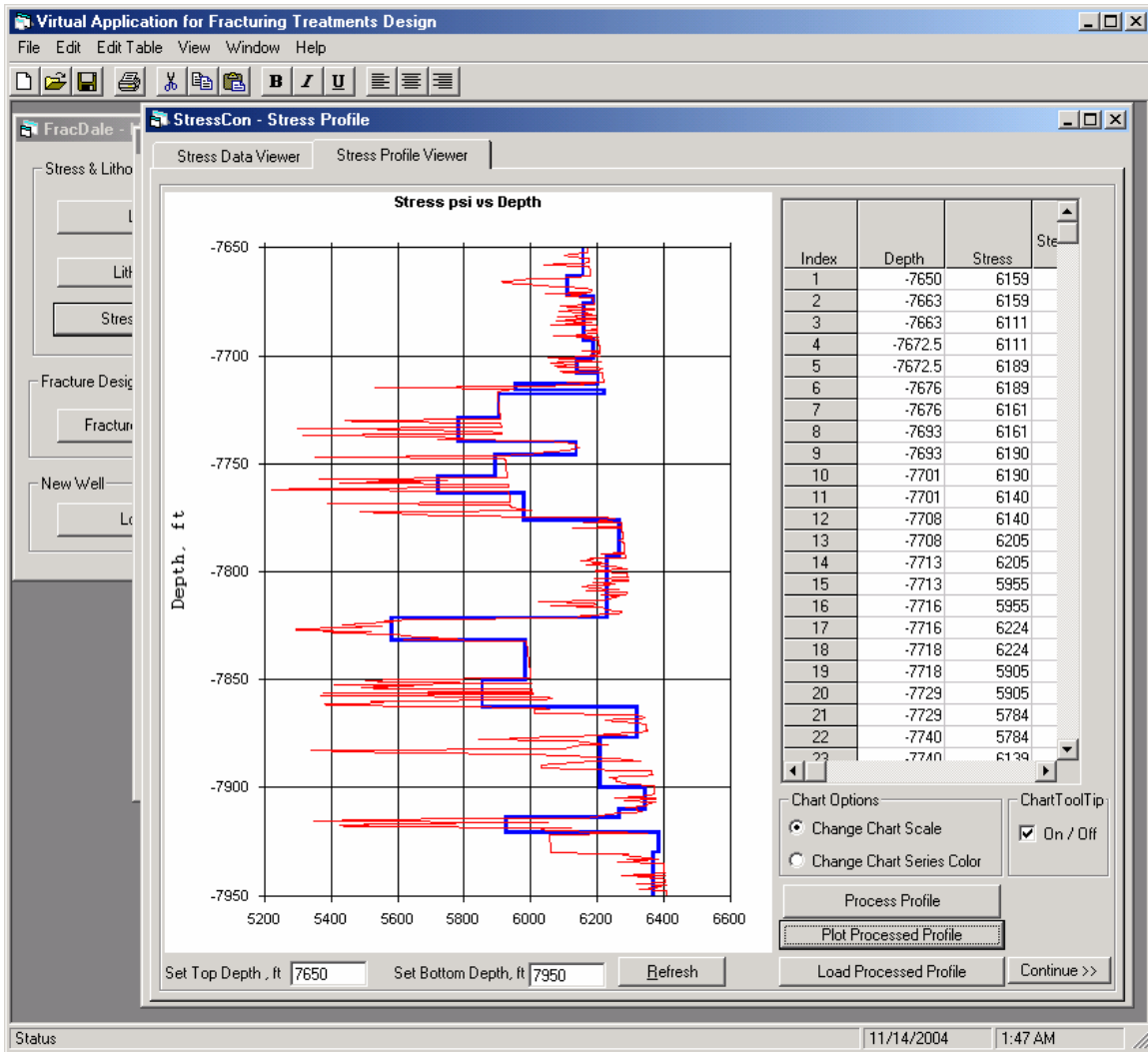


Figure 7-14: Stress Profile Form – Stress Data Viewer Tab

Figure 7-14 shows the Stress Profile Viewer under the Stress Profile form. This tab displays the graphical representation of the stress profile, or any other parameter selected in the previous tab. The stress profile processing is completed during this step. At the right side of the form, a table allows the user to visually define the formation with similar stress characteristics. If an existing well is loaded, this file already exists and can be loaded; otherwise, the user can enter the depths for each potential layer. The application will calculate an average stress for the selected interval and plot the processed profile on the chart. The processed profile is then loaded in the fracture design module where it will be used to derive the four-layer equivalent stress profile.

Virtual Application for Fracturing Treatments Design

File Edit View Window Help

Front End - Stress Profile

Import Well Data File

Import Data File D:\MDI_VFracPro\sample1.txt

100%

No Cr	Depth ft	Closure Stre	Stress	Permeability	Layer Thickr
1	10800	0.693	7526	0.00000	120
2	10920	0.709	7751	0.00000	24
3	10944	0.743	8137	0.00000	16
4	10960	0.693	7604	0.00000	24
5	10984	0.709	7791	0.00000	10
6	10994	0.743	8172	0.00000	10
7	11004	0.680	7485	0.00000	7
8	11011	0.762	8393	0.00000	6
9	11017	0.693	7640	0.00000	15
10	11032	0.734	8123	0.00000	70
11	11102	0.803	8919	0.00000	11
12	11113	0.734	8164	0.00000	19

Set margins for top layer

Set first formation 2

Set bottom formation layer 13

Set margins for middle layer

Set first formation 14

Set bottom formation layer 32

Set margins for bottom layer

Set first formation 33

Set bottom formation layer 34

Representative Four Layers - Properties

Thickness of the Top Layer , ft 209.00

Closure Stress Grad. of Top Layer , psi/ft 0.725

Thickness of the Overburden Layer , ft 11.00

Closure Stress Grad. of the Overburden Layer , psi/ft 0.802

Thickness of the Pay Zone Layer , ft 107.00

Closure Stress Grad. of the Pay Zone , psi/ft 0.6765

Equivalent Permeability of the Pay Zone, mD 0.007401

Closure Stress Grad. of the Bottom Layer, psi/ft 0.7170

Exit Calculate Equivalent Profile Display Profile Continue

Status 10/17/99 6:07 PM

Figure 7-15: Equivalent Stress Profile Form

Figure 7-15 shows the Equivalent Stress Profile form. This form allows for the user of multiple data source for stress profiling. If a stress profile was already processed with this application, it can be loaded and used for optimization. If a processed stress profile is available from an external source, such as Excel or text files, it can be loaded into the application at this point. The processed stress profile is displayed in the table at the left side of the form. The application identifies the production zones based on permeability contrasts and automatically calculates the equivalent four-layer stress profile.

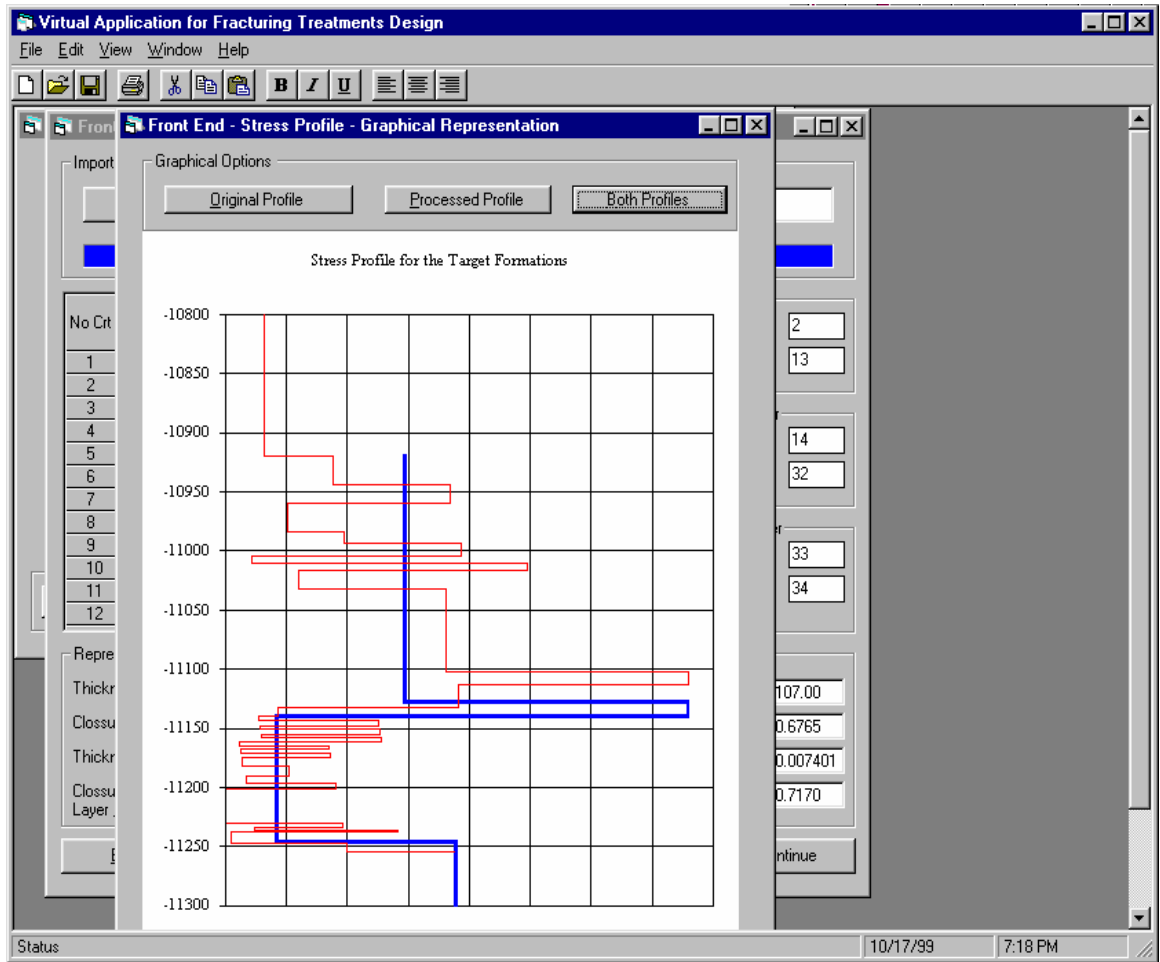


Figure 7-16: Equivalent Stress Profile Form

Figure 7-16 shows the graphical representation of the equivalent four-layer stress profile. The form allows for plotting the original profile and the equivalent profile separately. In addition, both profiles can be plotted in the same time. Next, the equivalent four-layer profile is sent to the main fracture design optimization treatment form, where it is used as direct input in the optimization process.

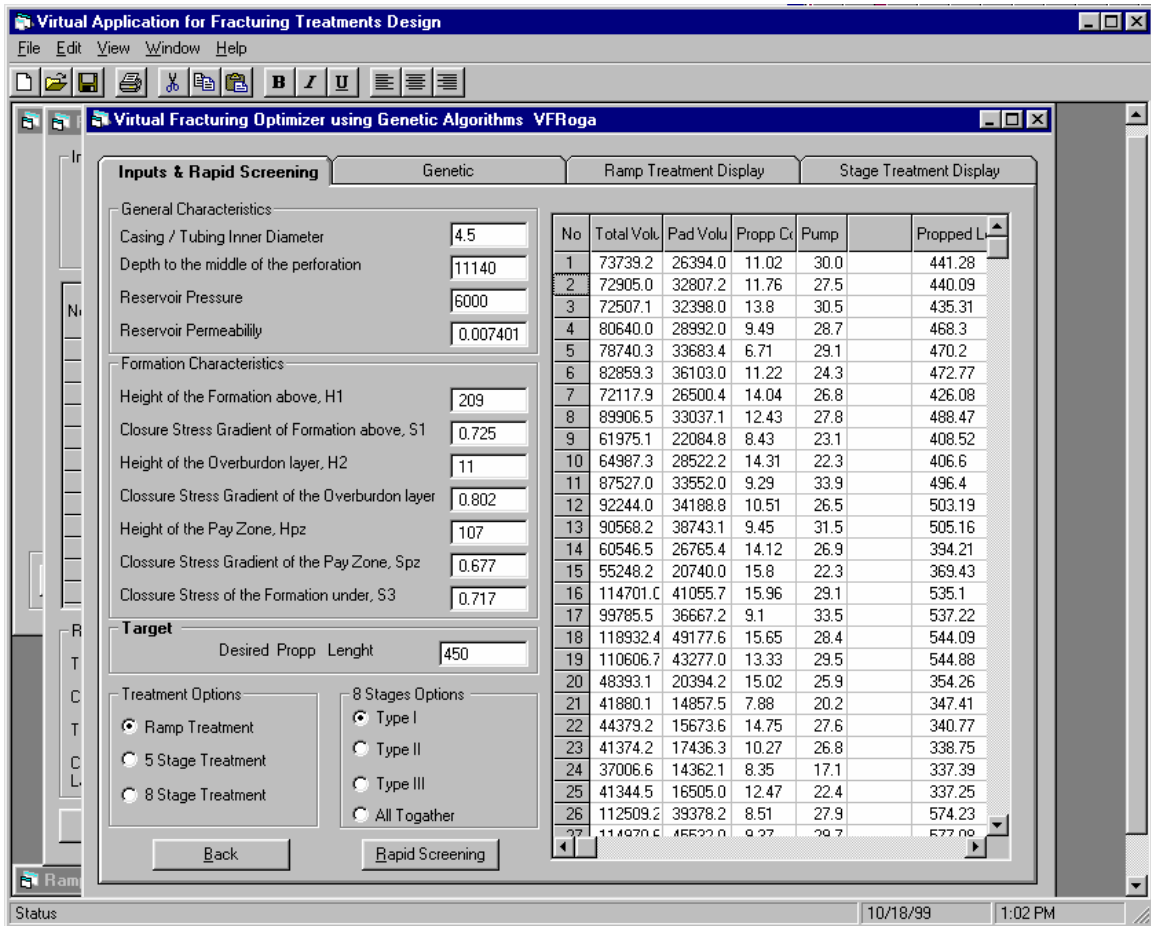


Figure 7-17: Main Optimization Form – Rapid Screening Tab

Figure 7-17 shows the Main Optimization form. This form consists of four tabs. The current figure presents the Rapid Screening tab. The left side of the tab shows the reservoir inputs loaded from the previous form. The types of treatments available for optimization are given at the bottom left of the form. The selection can be done using the radio buttons. A rapid screening represents the first generation of the genetic optimization. An initial population is generated and the output is ranked from the best to the weakest after each case is run through the fitness function. The ranked list is displayed in table format at the right side of the tab. The user can visually analyze each job and the predicted outcome. Double-clicking on each job index brings up the next form.

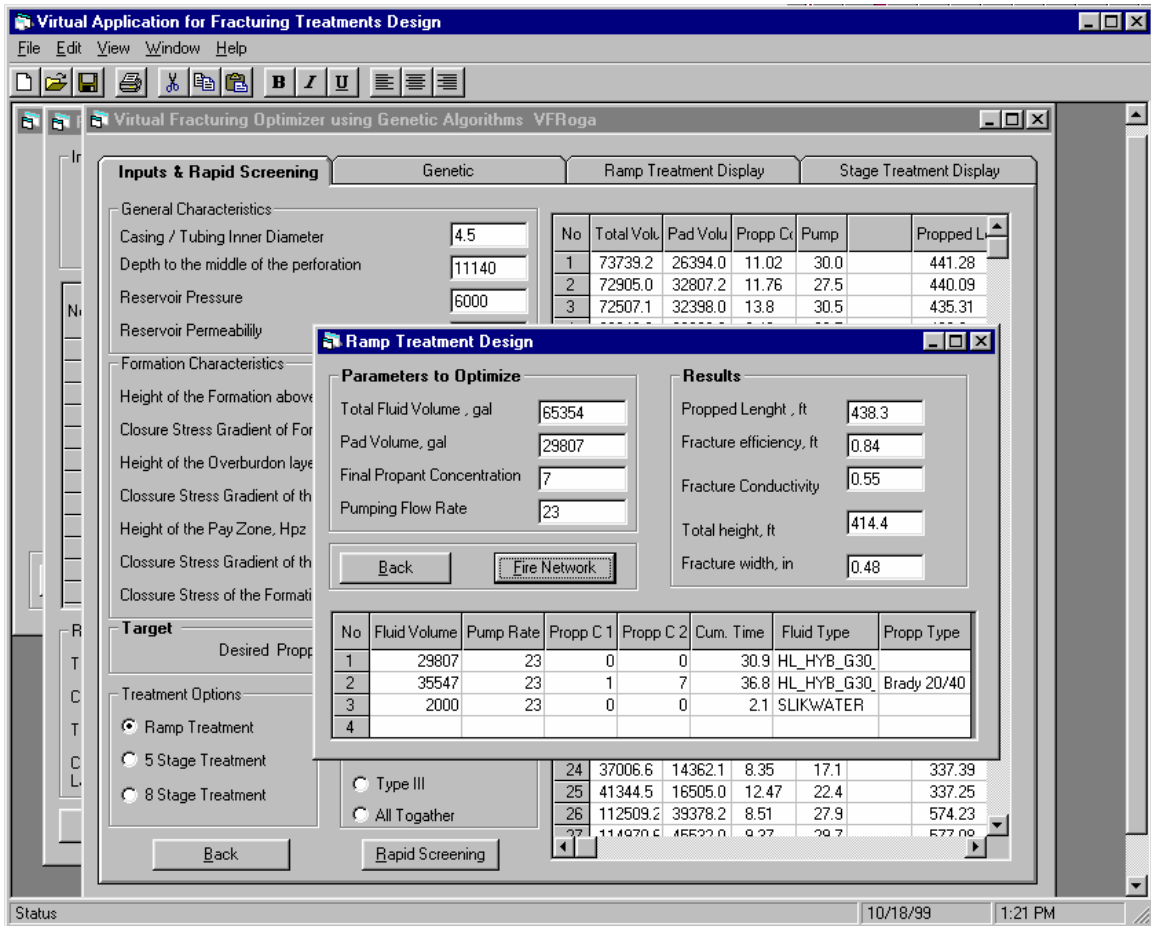


Figure 7-18: Ramp Treatment Form

Figure 7-18 shows the Ramp Treatment form. This form displays the description of the ramp treatment suggested by the application in detail. Each of the treatments displayed in the Main Optimization form, Rapid Screening Tab, can be further analyzed in this form. The treatment design can manually be modified, and the neural network can be fired again for the altered treatment design. The rapid screening provides a fast and easy way to manually design optimum treatments.

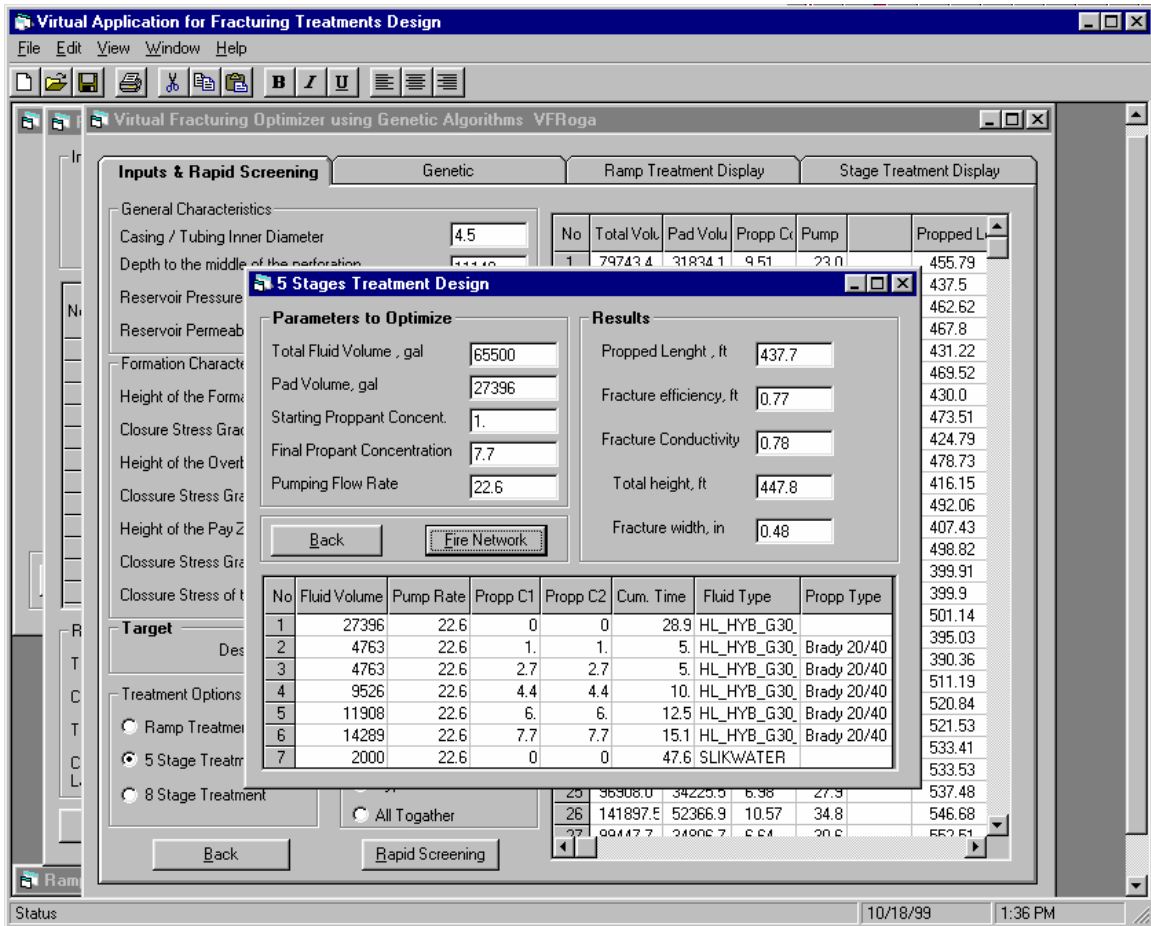


Figure 7-19: Five Stage Treatment Design Form

Figure 7-19 shows the Five Stage Treatment Design form. This form comes up when the Five Stage Treatment radio button is selected. Moreover, this form is identical to the ramp treatment form with the exception that it designs five stage treatments. Each of the treatments displayed in the Main Optimization form, Rapid Screening Tab, can be further analyzed in this form. The treatment design can manually be modified, and the neural network can be fired again for the altered treatment design.

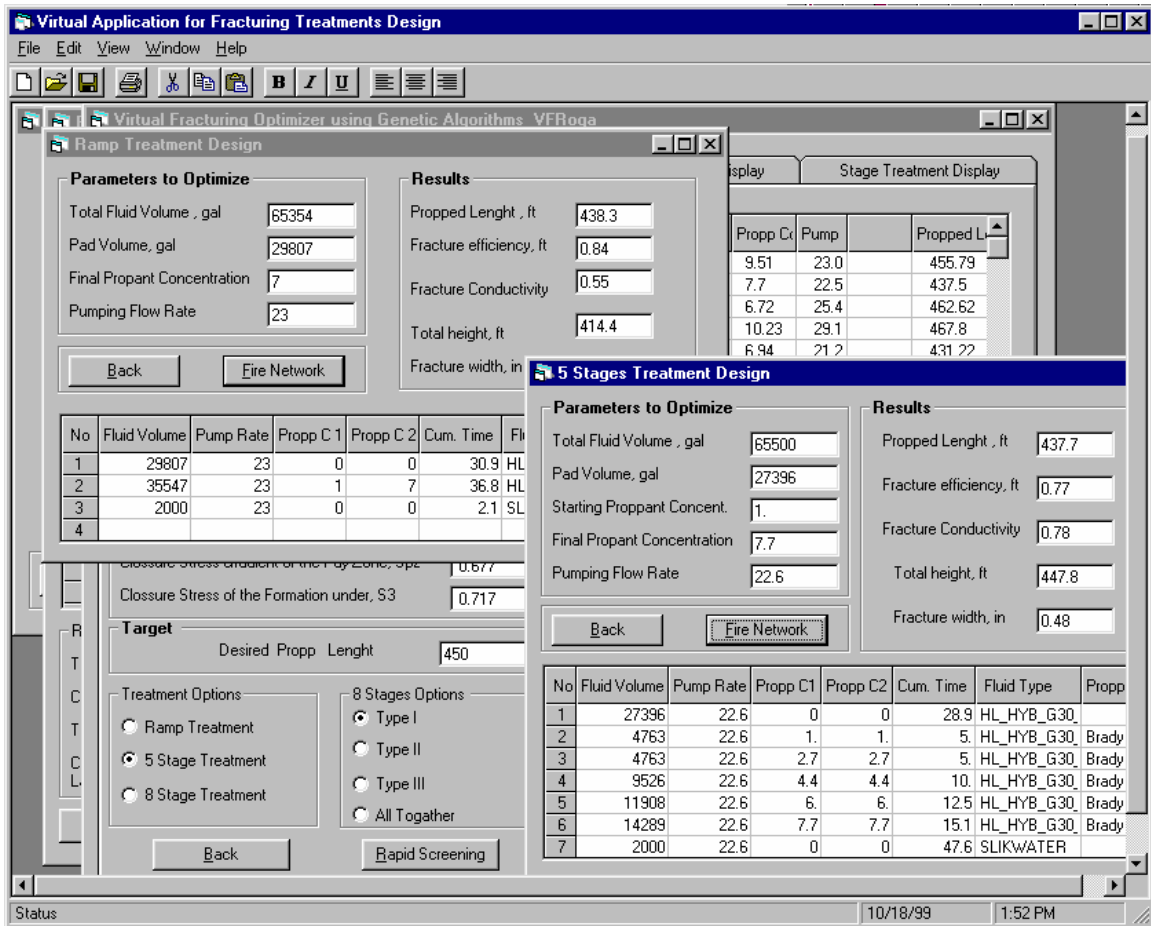


Figure 7-20: Multiple Screens Comparison – Ramp Treatment & Five Stage Treatment

Figure 7-20 shows two screens where the same fracture target lengths are compared. One screen, at the top left shows the optimum ramp treatment while the second screen at the bottom right shows the five-stage treatment. This capability of the application makes comparing different treatments very easy.

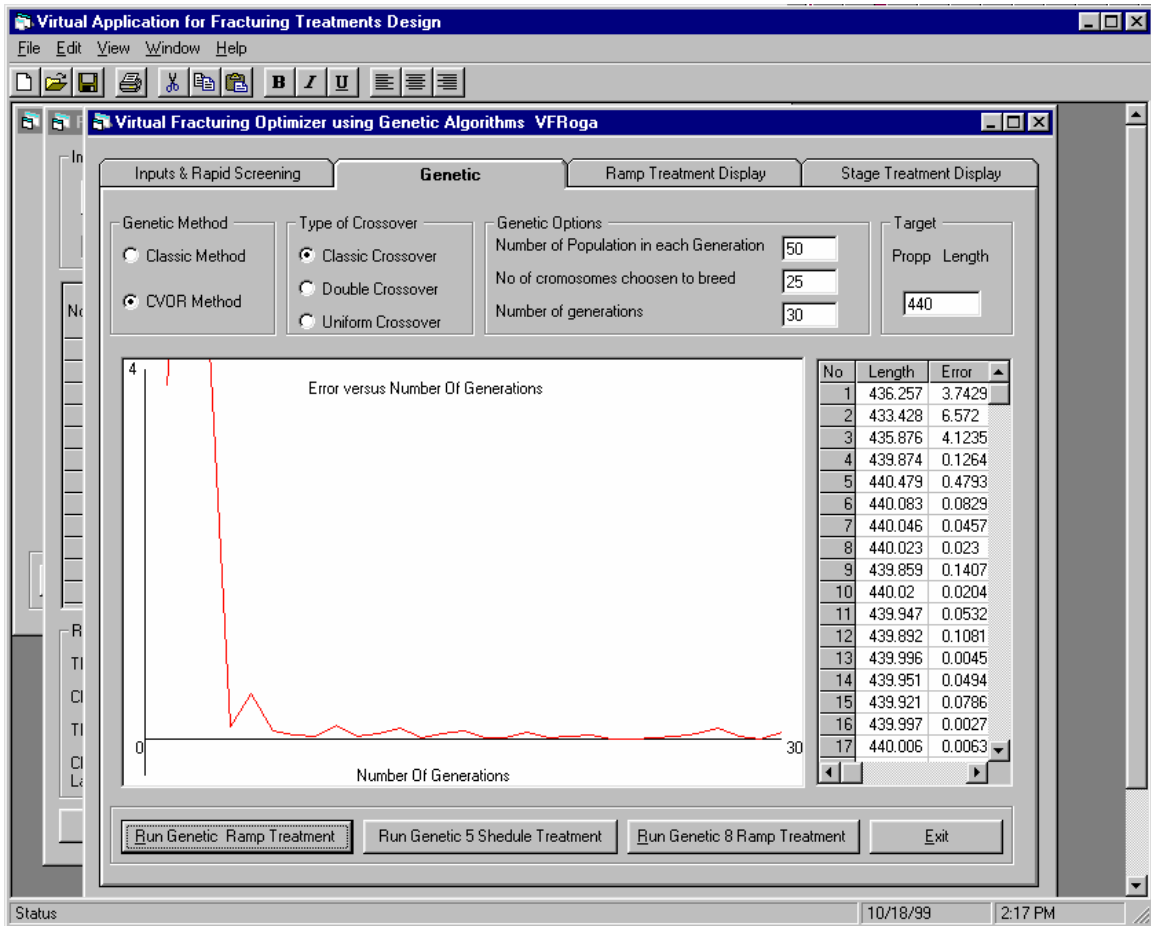


Figure 7-21: Main Optimization Form – Genetic Tab

Figure 7-21 shows the Main Optimization form, Genetic tab. This form was designed specifically for optimization using genetic algorithms. The typical genetic parameters are given at the top of the form. The user can modify these parameters as desired. The convergence speed is measured graphically using a plot. The plot shows the error between best solution and target versus the number of generations. The solutions of the final population are ranked ascending in the table at the right side of the tab.

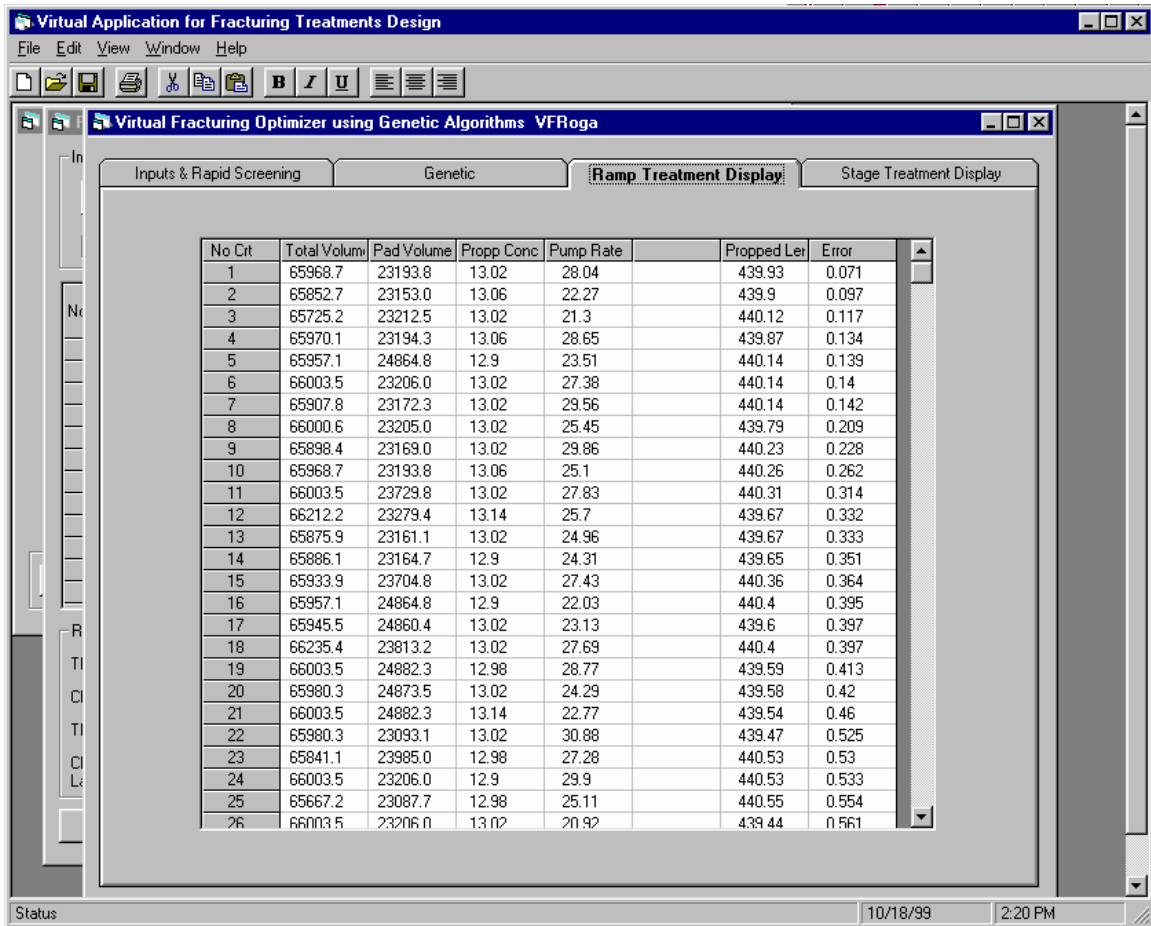


Figure 7-22: Main Optimization Form – Ramp Treatment Display Tab

Figure 7-22 shows the Main Optimization form, Ramp Treatment Display tab. This screen shows the best solutions suggested by the optimization routine. These solutions are displayed in table format. Double-clicking on the solution index number brings up the ramp treatment form where the user can visualize the detailed treatment schedule. A similar tab is available for stage treatments schedules.

CHAPTER 8: CONCLUSIONS

1. Optimum treatment design can be achieved using a combination of artificial intelligent tools without the use of a traditional hydraulic fracture simulator. The study proved that three basic logs, together with a series of artificial intelligence tools, are sufficient to provide robust initial optimum solution for a treatment design.
2. Stress profiles can be sufficiently determined using a fuzzy expert system for lithology identification together with a basic stress equation. The fuzzy expert system makes use of three basic logs (spontaneous potential, gamma ray, and resistivity) or any other available logs. The power of the fuzzy logic lithology identification system is particularly compelling when sonic logs are not available.
3. A new methodology to reduce the complex lithology profile to an equivalent four-layer stress profile was introduced. This reduced the large amount of data necessary for complex reservoir characterization to eight main parameters. The equivalent four-layer profile proved very efficient in preserving the fracture geometry during simulation.
4. A neuro-genetic optimization procedure was developed to design optimum fracture treatments. The procedure integrates power and speed of neural networks with the fast convergence of genetic algorithms to provide solution to a complex four-parameter optimization problem.
5. A one-dimensional modified version of vector quantization was used to compress the large number of data recordings from the pressure profile to ten pressure points. This compression allowed the use of neural networks for pressure profile prediction. Without this reduction, the large amount of data would have made impossible to train a predictive model.
6. Two neural networks were successfully developed to predict net pressure profile developed during a fracture treatment. One neural network compensates for the difference between the pressure profile obtained from the original lithology and the pressure profile generated using equivalent four-layer profile. The other neural network is used for net pressure profile prediction. Combined, they provide the expected net pressure profile equivalent to that resulting from the original lithology.
7. The methodology was tested for an example treatment that was designed by Halliburton experts. The treatment design suggested by applying this methodology was found to be in good agreement with the Halliburton-designed treatment.

8. A new computer application was developed to address the continuous effort towards fracture optimization. This application integrated the methodology presented in this study into a user-friendly computer application intended to be a companion tool to any fracture simulation software.
9. The modular implementation of the developed software permits altering the neural networks without modifying the source code. This allows the program to be used in any field or with data from simulators other than FRACPRO.
10. The methodology used in this study can be used with other simulators or reproduced using real field data. Since the development of this methodology was focused on tight gas sands, it is directly applicable to similar formations. However, the methodology is flexible enough to be applied to any kind of field where fracturing is utilized to increase performance.

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APPENDIX

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
188,201	76,582	9.8	30	8,236	3,769	0.0461	228	0.761	12	0.792	122	0.706	0.790	0.680	432.80	635.63	1.33	20.03	0.64
188,201	76,582	9.8	30	8,236	3,769	0.0086	228	0.790	12	0.823	122	0.695	0.764	0.800	610.92	582.81	1.03	51.62	0.69
120,214	47,534	13.3	28	5,438	1,982	0.0464	174	0.738	20	0.752	93	0.683	0.778	0.750	349.98	548.67	1.59	63.21	0.65
120,214	47,534	13.3	28	5,438	1,982	0.0066	174	0.752	20	0.789	93	0.683	0.738	0.890	370.45	543.68	1.51	359.88	0.66
16,821	6,927	9.9	23	7,500	2,918	0.0367	176	0.731	13	0.774	117	0.693	0.769	0.460	152.62	251.68	0.84	48.15	0.31
16,821	6,927	9.9	23	7,500	2,918	0.0033	176	0.731	13	0.774	117	0.693	0.769	0.810	172.44	267.87	0.70	227.91	0.35
39,351	15,540	14.5	22	7,202	3,068	0.0231	180	0.742	29	0.798	84	0.661	0.747	0.710	260.11	324.15	1.26	82.05	0.53
131,874	55,101	7.5	22	8,142	2,276	0.0430	187	0.764	27	0.799	134	0.653	0.769	0.540	615.29	409.62	0.78	27.18	0.63
131,874	55,101	7.5	25	8,142	2,276	0.0089	187	0.788	27	0.823	134	0.710	0.769	0.710	492.92	532.68	0.75	46.78	0.57
178,443	74,888	6.6	25	5,640	1,146	0.0172	212	0.780	20	0.801	85	0.697	0.733	0.880	419.53	669.99	0.85	57.73	0.60
149,220	60,097	14.0	21	5,854	2,208	0.0023	189	0.755	31	0.799	98	0.669	0.758	0.920	442.73	532.72	1.63	84.90	0.75
149,220	60,097	14.0	21	5,854	2,208	0.0023	189	0.764	31	0.786	98	0.650	0.720	0.640	463.69	550.29	1.51	25.86	0.67
23,341	9,409	9.8	16	6,250	2,553	0.0177	199	0.758	11	0.815	90	0.689	0.785	0.690	198.24	281.93	0.82	106.93	0.40
150,488	54,975	9.1	22	6,870	2,863	0.0165	181	0.741	34	0.803	62	0.673	0.731	0.880	471.05	615.24	0.99	52.85	0.58
150,317	53,030	13.5	26	7,790	2,509	0.0039	172	0.781	29	0.846	57	0.670	0.769	0.920	501.03	602.23	1.35	204.66	0.62
98,898	42,715	6.0	18	6,953	2,553	0.0071	170	0.762	30	0.782	105	0.700	0.736	0.810	384.67	527.63	0.60	70.82	0.47
61,055	26,072	7.6	17	5,921	2,223	0.0073	199	0.750	19	0.841	111	0.691	0.748	0.800	305.57	444.76	0.69	146.42	0.44
86,515	33,288	15.6	18	7,787	3,082	0.0402	170	0.777	22	0.783	106	0.701	0.767	0.540	371.34	447.50	1.49	36.40	0.58
152,750	58,126	13.7	28	5,388	1,557	0.0336	179	0.746	31	0.762	90	0.663	0.777	0.770	386.56	493.59	1.78	94.33	0.72
147,295	56,116	10.8	27	8,278	3,900	0.0182	208	0.774	11	0.799	121	0.690	0.742	0.720	503.93	559.76	1.15	36.49	0.64
147,295	56,116	10.8	27	8,278	3,900	0.0066	208	0.743	11	0.780	121	0.700	0.737	0.850	443.59	622.57	1.17	109.71	0.60
37,569	15,725	6.7	21	7,542	2,172	0.0300	234	0.786	14	0.816	106	0.686	0.772	0.480	272.13	320.59	0.57	24.80	0.39
31,461	12,495	6.4	19	8,387	2,960	0.0165	180	0.744	29	0.808	117	0.698	0.746	0.560	241.70	341.63	0.53	37.36	0.34
31,461	12,495	6.4	19	8,387	2,960	0.0052	180	0.744	29	0.766	117	0.660	0.720	0.720	267.20	305.33	0.53	100.95	0.40
188,307	74,838	14.2	21	5,998	1,772	0.0337	231	0.765	14	0.829	71	0.675	0.746	0.930	500.54	640.58	1.53	217.54	0.70
188,307	74,838	14.2	21	5,998	1,772	0.0500	231	0.765	14	0.790	100	0.698	0.740	0.690	443.39	652.04	1.70	42.71	0.68
161,689	65,428	11.5	26	6,360	2,618	0.0146	190	0.737	22	0.778	123	0.672	0.743	0.780	428.70	566.49	1.46	116.06	0.71
161,689	65,428	11.5	26	6,360	2,618	0.0023	190	0.780	22	0.830	123	0.702	0.765	0.900	487.69	603.91	1.20	245.62	0.65
136,794	51,211	11.1	25	6,686	2,015	0.0364	205	0.733	10	0.804	81	0.677	0.759	0.760	374.47	569.05	1.43	49.42	0.68
136,794	51,211	11.1	25	6,686	2,015	0.0042	205	0.765	10	0.824	81	0.707	0.767	0.900	370.21	526.94	1.56	367.52	0.77

Table A - 1: Data Generated for Ramp Treatment

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
24,567	9,152	10.4	18	6,266	2,228	0.0104	205	0.750	25	0.788	101	0.707	0.778	0.710	197.43	310.71	0.86	185.52	0.39
126,108	48,743	12.2	20	6,758	2,803	0.0041	174	0.767	23	0.819	132	0.704	0.734	0.860	400.81	565.73	1.30	292.48	0.61
119,135	51,634	15.5	27	6,176	2,181	0.0681	229	0.786	23	0.837	91	0.703	0.783	0.870	403.46	516.18	1.52	289.80	0.65
119,135	51,634	15.5	27	6,176	2,181	0.0681	229	0.749	23	0.799	91	0.670	0.745	0.640	380.26	512.93	1.62	35.07	0.62
149,673	56,005	15.9	30	7,715	2,925	0.0114	211	0.760	23	0.783	122	0.662	0.767	0.730	540.75	501.45	1.66	95.62	0.74
149,673	56,005	15.9	30	7,715	2,925	0.0114	211	0.800	23	0.833	92	0.710	0.840	0.830	404.57	556.16	2.00	134.35	0.74
95,141	36,899	15.6	29	6,718	2,071	0.0170	176	0.776	30	0.844	127	0.663	0.730	0.680	369.93	435.01	1.70	128.90	0.68
95,141	36,899	15.6	29	6,718	2,071	0.0033	176	0.763	30	0.804	127	0.697	0.737	0.860	343.51	487.94	1.63	252.43	0.65
101,619	35,902	11.7	20	5,047	2,390	0.0100	208	0.783	21	0.791	60	0.675	0.746	0.740	379.34	530.26	1.20	23.68	0.59
34,401	13,459	15.7	17	7,126	3,420	0.1215	197	0.767	24	0.776	126	0.658	0.752	0.270	200.64	314.97	1.56	31.65	0.42
73,631	29,186	15.1	28	8,196	3,128	0.0295	228	0.725	35	0.847	112	0.681	0.749	0.590	350.89	400.68	1.45	45.81	0.59
73,631	29,186	15.1	28	8,196	3,128	0.0069	228	0.742	35	0.790	112	0.690	0.720	0.810	308.72	442.68	1.49	192.47	0.59
136,740	47,946	8.5	29	7,317	3,635	0.0012	195	0.789	12	0.800	65	0.702	0.765	0.960	494.34	604.95	0.85	438.10	0.59
185,498	76,789	8.3	18	7,871	3,265	0.0042	193	0.764	31	0.816	119	0.685	0.737	0.860	561.29	619.19	0.90	103.89	0.62
20,186	8,467	6.3	23	7,364	3,576	0.0491	222	0.743	31	0.800	57	0.666	0.779	0.620	219.77	258.21	0.47	14.70	0.33
143,944	62,412	15.3	29	6,036	2,006	0.0386	193	0.748	12	0.811	113	0.701	0.785	0.800	345.85	491.31	2.20	215.16	0.86
137,148	59,667	11.7	26	8,111	3,188	0.0073	195	0.744	24	0.751	68	0.673	0.736	0.880	448.96	546.84	1.16	95.87	0.62
190,430	79,607	13.3	30	7,277	3,531	0.0235	185	0.754	19	0.783	104	0.678	0.783	0.800	432.88	628.84	1.71	57.93	0.70
190,430	79,607	13.3	30	7,277	3,531	0.0054	185	0.790	19	0.820	104	0.710	0.783	0.870	560.66	602.57	1.38	137.05	0.70
95,259	36,174	12.9	20	6,557	1,784	0.0073	202	0.779	34	0.829	54	0.686	0.786	0.880	382.48	446.01	1.41	222.65	0.67
31,651	12,212	8.5	28	5,744	1,982	0.0818	226	0.777	35	0.802	104	0.673	0.738	0.390	217.36	323.65	0.79	32.23	0.36
16,202	6,348	6.9	16	7,364	2,463	0.0154	233	0.762	27	0.788	89	0.667	0.779	0.530	208.45	229.02	0.51	53.95	0.34
62,537	22,554	14.0	17	6,227	2,550	0.0818	188	0.798	24	0.830	88	0.710	0.780	0.520	294.46	412.32	1.42	35.28	0.54
72,823	31,706	12.3	24	7,852	3,287	0.0157	203	0.757	34	0.761	53	0.662	0.741	0.840	370.19	415.61	1.04	50.65	0.54
81,375	30,527	8.4	22	8,259	2,981	0.0089	213	0.760	29	0.804	84	0.710	0.738	0.840	350.25	503.40	0.82	67.76	0.48
30,061	12,457	12.8	15	6,730	2,123	0.0112	180	0.735	23	0.810	56	0.683	0.745	0.800	227.63	322.35	0.97	146.75	0.43
175,697	69,827	7.1	27	5,123	2,080	0.0575	180	0.777	22	0.826	60	0.700	0.797	0.830	406.44	692.34	0.93	26.23	0.64
30,150	11,060	13.0	19	7,101	3,530	0.0818	210	0.781	22	0.834	63	0.692	0.767	0.600	213.45	323.61	1.12	23.17	0.43
86,222	32,587	8.1	23	7,941	2,086	0.0058	205	0.764	16	0.756	97	0.682	0.769	0.790	414.82	466.12	0.77	97.02	0.56
105,574	37,646	10.3	25	6,704	2,697	0.0124	229	0.755	18	0.807	61	0.672	0.777	0.890	351.94	539.44	1.20	122.42	0.62

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
105,574	37,646	10.3	25	6,704	2,697	0.0076	229	0.755	18	0.807	61	0.692	0.780	0.860	338.17	503.42	1.34	217.97	0.68
57,575	25,302	13.7	19	7,173	3,209	0.0076	227	0.768	12	0.809	107	0.699	0.765	0.790	291.34	390.10	1.20	167.05	0.53
60,595	26,596	14.4	25	8,011	3,088	0.0488	177	0.767	29	0.811	128	0.703	0.733	0.480	262.03	393.77	1.43	35.80	0.49
13,578	4,997	13.2	19	6,547	2,297	0.0075	190	0.775	14	0.838	112	0.707	0.786	0.640	158.12	239.93	0.92	294.69	0.34
56,398	23,307	13.0	21	6,597	2,645	0.0315	232	0.758	24	0.822	134	0.685	0.762	0.540	277.74	386.87	1.25	70.20	0.51
61,335	24,189	11.6	18	6,569	2,701	0.0012	208	0.740	23	0.819	131	0.665	0.735	0.900	329.57	422.36	0.99	116.70	0.53
106,559	46,581	9.7	26	5,178	1,814	0.0159	172	0.782	33	0.795	78	0.670	0.771	0.830	402.60	492.32	0.97	91.19	0.58
100,951	43,256	9.1	27	5,590	2,238	0.0062	237	0.769	21	0.819	73	0.700	0.765	0.910	353.97	517.38	0.95	218.77	0.56
94,702	35,525	12.5	28	7,882	2,594	0.0197	233	0.776	18	0.782	55	0.695	0.747	0.840	365.31	497.90	1.28	52.53	0.60
94,702	35,525	12.5	28	7,882	2,594	0.0900	233	0.776	18	0.782	95	0.706	0.790	0.490	363.24	446.63	1.43	16.28	0.59
26,126	9,364	8.7	21	6,660	2,500	0.0213	174	0.723	30	0.759	85	0.657	0.750	0.650	233.03	308.64	0.69	67.74	0.37
175,793	67,485	11.7	30	7,037	3,336	0.0737	211	0.739	27	0.753	83	0.689	0.724	0.740	445.63	656.13	1.39	16.67	0.62
184,517	70,474	10.9	30	5,707	2,393	0.0053	201	0.767	19	0.834	105	0.694	0.749	0.900	478.23	658.69	1.28	270.32	0.66
98,653	42,370	13.1	26	6,459	1,484	0.0018	194	0.760	28	0.781	96	0.675	0.743	0.920	382.93	480.07	1.26	244.92	0.61
36,753	15,639	6.4	20	7,022	3,508	0.0277	202	0.773	31	0.813	56	0.707	0.745	0.810	220.78	356.71	0.61	28.72	0.40
117,093	49,780	7.2	27	6,457	2,530	0.0874	220	0.789	32	0.770	99	0.675	0.771	0.600	359.59	518.53	0.90	15.19	0.56
117,093	49,780	7.2	27	6,457	2,530	0.0020	220	0.789	32	0.770	99	0.675	0.771	0.930	366.44	543.11	0.85	181.26	0.58
149,133	60,213	15.6	30	7,098	3,579	0.0465	230	0.751	15	0.809	89	0.687	0.746	0.750	429.54	578.49	1.68	31.77	0.66
30,961	13,082	7.3	16	5,938	2,731	0.0040	205	0.768	31	0.788	133	0.701	0.766	0.800	224.51	330.74	0.61	296.44	0.39
141,814	51,837	12.6	30	7,251	3,550	0.0330	225	0.736	14	0.763	110	0.673	0.724	0.740	433.98	588.38	1.40	38.14	0.64
191,842	70,965	12.6	30	7,793	3,547	0.0459	182	0.755	20	0.787	119	0.710	0.773	0.720	419.20	652.66	1.75	29.89	0.74
191,842	70,965	12.6	30	7,793	3,547	0.0010	182	0.755	20	0.787	119	0.680	0.720	0.950	511.23	628.24	1.49	234.38	0.74
82,366	30,446	9.7	25	6,023	2,847	0.0153	239	0.800	28	0.840	96	0.718	0.800	0.800	359.11	479.06	0.97	86.78	0.55
88,841	33,114	8.7	25	5,311	1,871	0.0382	197	0.782	19	0.775	61	0.677	0.750	0.810	355.52	503.58	0.91	43.02	0.53
39,182	17,138	12.8	18	7,733	3,747	0.0046	195	0.754	22	0.776	109	0.691	0.727	0.850	244.86	366.30	0.98	227.28	0.45
108,011	38,862	6.8	27	6,504	2,314	0.0135	207	0.730	19	0.752	94	0.685	0.723	0.860	376.51	580.56	0.76	67.39	0.52
140,287	57,933	10.6	28	6,027	2,501	0.0065	227	0.762	32	0.783	95	0.678	0.742	0.900	440.22	570.88	1.13	189.72	0.62
84,646	34,494	12.3	25	8,004	3,832	0.0322	204	0.774	18	0.838	128	0.705	0.788	0.650	376.98	440.18	1.18	27.70	0.56
84,646	34,494	12.3	25	8,004	3,832	0.0032	204	0.763	18	0.787	128	0.681	0.732	0.890	365.86	437.54	1.22	271.10	0.61
84,646	34,494	12.3	25	8,004	3,832	0.0010	204	0.764	18	0.787	128	0.674	0.748	0.930	409.66	451.12	1.06	175.77	0.61

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
120,306	43,572	10.2	29	7,732	2,548	0.0127	205	0.744	18	0.750	68	0.702	0.726	0.890	387.53	591.86	1.12	67.43	0.56
88,488	31,631	6.8	27	6,867	2,720	0.0272	200	0.764	11	0.785	52	0.684	0.745	0.870	385.67	516.86	0.69	25.55	0.50
116,859	43,581	6.8	27	7,763	1,990	0.0097	228	0.722	31	0.791	64	0.653	0.748	0.870	453.44	486.00	0.80	59.34	0.64
87,206	34,011	14.6	25	7,443	3,191	0.0032	179	0.763	23	0.784	128	0.674	0.774	0.870	395.77	440.87	1.36	336.48	0.64
143,203	58,662	15.1	30	5,105	2,112	0.0099	194	0.736	33	0.781	124	0.699	0.724	0.870	355.39	601.69	1.81	315.81	0.70
157,542	58,527	12.7	30	6,699	3,041	0.0439	233	0.747	20	0.793	70	0.704	0.737	0.850	388.57	616.64	1.64	38.44	0.71
77,706	33,359	11.7	28	7,209	3,550	0.0066	230	0.785	24	0.839	133	0.720	0.787	0.830	335.76	441.62	1.12	148.74	0.56
80,226	28,440	8.2	25	5,568	2,500	0.0213	174	0.763	27	0.838	98	0.658	0.789	0.780	375.92	434.75	0.89	72.81	0.59
138,120	57,846	6.9	28	6,810	3,224	0.0224	186	0.733	31	0.744	99	0.674	0.740	0.820	393.54	588.77	0.84	38.25	0.57
116,101	50,486	8.7	30	7,627	2,920	0.0311	227	0.773	16	0.799	101	0.704	0.758	0.730	409.57	531.81	0.88	20.60	0.54
136,922	48,387	14.8	35	6,681	1,950	0.0494	184	0.726	15	0.817	70	0.692	0.748	0.890	425.37	559.85	1.68	173.14	0.71
38,283	16,051	13.3	16	5,493	1,405	0.0161	230	0.792	11	0.831	83	0.720	0.788	0.780	227.89	352.16	1.15	162.09	0.48
12,688	5,545	15.0	19	7,494	2,510	0.0016	188	0.752	33	0.793	56	0.691	0.769	0.900	173.07	234.28	0.80	295.49	0.33
31,986	12,228	9.5	20	6,858	1,511	0.0372	172	0.768	21	0.794	94	0.685	0.728	0.590	224.31	328.82	0.85	42.44	0.40
21,852	8,054	10.8	18	7,193	3,586	0.0519	186	0.742	27	0.762	59	0.651	0.736	0.670	215.80	281.45	0.80	29.19	0.38
18,944	6,705	11.2	20	6,775	2,930	0.0094	222	0.753	32	0.781	131	0.704	0.764	0.680	178.10	284.91	0.87	190.45	0.36
31,808	13,850	7.6	17	6,265	2,838	0.0646	193	0.799	26	0.821	114	0.684	0.738	0.460	221.83	311.16	0.69	24.65	0.36
182,194	68,142	9.1	35	5,519	2,836	0.0045	226	0.773	17	0.807	82	0.700	0.766	0.950	418.09	639.72	1.30	377.49	0.75
184,981	67,117	14.7	35	7,912	3,528	0.0095	193	0.758	17	0.786	125	0.694	0.749	0.830	509.34	625.73	1.66	105.77	0.73
142,162	50,021	8.3	17	5,648	2,835	0.0183	171	0.745	12	0.787	54	0.681	0.737	0.930	384.33	577.56	1.17	94.78	0.69
127,344	50,742	6.5	26	6,142	2,638	0.0074	205	0.761	22	0.793	51	0.696	0.788	0.940	369.13	647.00	0.74	123.35	0.60
73,020	29,796	9.6	25	5,235	1,991	0.0310	175	0.793	19	0.837	83	0.694	0.759	0.780	315.61	445.55	0.98	59.78	0.53
116,066	44,766	8.0	28	8,171	3,989	0.0072	171	0.747	33	0.757	80	0.673	0.721	0.900	450.82	533.68	0.81	66.89	0.56
128,685	55,350	12.8	32	6,379	2,666	0.0047	238	0.768	33	0.823	110	0.698	0.781	0.890	392.38	512.13	1.47	330.31	0.69
101,034	35,409	7.5	26	7,549	3,743	0.0197	203	0.782	24	0.813	121	0.684	0.741	0.740	425.06	480.99	0.83	33.22	0.58
13,189	5,044	7.0	15	6,852	2,548	0.0054	233	0.737	21	0.785	95	0.685	0.725	0.750	165.98	255.58	0.45	161.79	0.27
138,962	56,770	12.2	30	7,788	3,600	0.0230	171	0.781	35	0.804	58	0.696	0.730	0.880	376.50	552.93	1.52	45.92	0.64
11,412	4,939	7.4	15	6,272	2,804	0.0251	228	0.749	31	0.770	61	0.677	0.741	0.540	145.75	211.24	0.54	26.98	0.26
73,657	27,986	10.9	24	5,493	2,492	0.0135	214	0.754	30	0.787	92	0.698	0.740	0.850	301.42	476.60	1.12	157.08	0.54
90,516	36,317	6.4	29	6,263	3,234	0.0045	225	0.756	27	0.782	102	0.688	0.741	0.920	357.80	507.58	0.68	178.70	0.52

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
167,771	69,779	7.5	36	7,695	3,613	0.0098	207	0.782	25	0.820	113	0.698	0.772	0.840	553.21	587.78	0.78	41.14	0.62
140,220	53,034	9.9	35	7,950	3,482	0.0110	233	0.771	17	0.774	94	0.674	0.731	0.860	467.94	537.05	1.13	62.37	0.66
59,263	25,194	15.0	21	5,986	1,593	0.0086	179	0.722	14	0.740	68	0.652	0.730	0.890	286.22	407.65	1.34	281.51	0.56
181,169	72,607	7.7	40	7,237	3,076	0.0271	174	0.768	16	0.792	115	0.704	0.785	0.790	433.81	632.14	1.05	29.30	0.65
181,169	72,607	7.7	40	7,237	3,076	0.0072	174	0.768	16	0.792	115	0.690	0.745	0.880	510.84	620.16	0.91	75.68	0.64
145,050	53,324	6.8	35	7,998	3,420	0.0020	171	0.766	15	0.801	114	0.720	0.772	0.920	384.67	540.60	1.06	242.68	0.75
72,451	29,798	7.5	28	8,309	3,814	0.0077	234	0.782	31	0.804	58	0.709	0.776	0.900	368.89	463.86	0.65	53.56	0.47
82,045	28,755	8.6	28	5,776	2,546	0.0055	178	0.767	29	0.803	99	0.693	0.763	0.890	356.27	489.52	0.89	243.08	0.55
79,855	33,033	9.1	26	8,284	2,915	0.0425	236	0.777	24	0.793	125	0.657	0.761	0.530	436.37	353.51	0.91	16.43	0.57
96,072	37,124	7.5	30	8,093	3,825	0.0236	220	0.753	34	0.793	100	0.697	0.744	0.920	384.14	519.64	0.71	198.62	0.51
67,889	29,547	7.9	25	8,264	3,620	0.0398	224	0.767	17	0.794	90	0.698	0.758	0.850	313.57	428.07	0.77	58.19	0.49
46,261	19,267	6.3	24	7,417	3,056	0.0245	211	0.729	23	0.821	99	0.683	0.734	0.790	251.97	383.11	0.63	65.76	0.41
47,798	19,218	6.9	26	5,457	2,321	0.0181	184	0.741	21	0.750	89	0.687	0.718	0.900	228.30	389.35	0.78	97.00	0.50
63,096	27,325	8.4	26	5,066	2,078	0.0471	193	0.722	18	0.767	59	0.684	0.725	0.900	247.42	428.87	0.92	44.56	0.55
60,607	22,873	6.7	17	7,817	2,615	0.0055	201	0.734	27	0.827	87	0.686	0.723	0.910	314.68	449.88	0.63	104.47	0.46
56,885	23,983	8.0	17	6,239	2,011	0.0290	173	0.739	26	0.794	81	0.684	0.738	0.870	286.51	418.81	0.75	40.47	0.48
45,557	19,542	7.0	20	5,898	1,917	0.0310	205	0.753	22	0.803	75	0.675	0.735	0.810	259.42	381.39	0.64	38.04	0.43
46,644	17,192	7.2	26	6,233	2,529	0.0497	171	0.752	30	0.802	76	0.667	0.743	0.740	286.99	394.69	0.65	24.36	0.43
64,505	26,504	7.8	22	5,568	2,741	0.0314	183	0.744	31	0.789	99	0.695	0.743	0.860	281.36	447.82	0.81	49.56	0.51
48,005	20,351	6.3	22	5,333	1,820	0.0303	174	0.728	18	0.755	89	0.683	0.720	0.820	249.26	413.88	0.60	43.67	0.40
56,279	24,458	7.0	24	6,429	1,984	0.0327	213	0.739	14	0.805	80	0.680	0.732	0.800	286.94	425.32	0.64	30.84	0.43
57,098	21,031	6.9	24	7,829	2,736	0.0222	240	0.734	30	0.812	83	0.672	0.743	0.810	331.72	423.26	0.62	26.59	0.46
64,367	27,231	7.5	19	6,734	2,572	0.0184	183	0.746	16	0.806	79	0.682	0.742	0.840	312.36	433.99	0.71	48.49	0.49
59,496	25,774	6.6	18	6,801	2,670	0.0394	213	0.731	30	0.768	106	0.668	0.746	0.550	295.31	408.45	0.65	28.26	0.45
43,341	18,802	7.6	22	5,035	2,448	0.0164	236	0.759	27	0.776	97	0.670	0.730	0.740	236.66	366.56	0.74	113.38	0.44
61,683	26,548	6.9	20	6,415	2,547	0.0204	226	0.744	23	0.807	66	0.685	0.734	0.900	282.03	430.02	0.70	48.80	0.49
53,458	19,938	7.2	15	5,006	2,309	0.0259	189	0.758	23	0.767	86	0.693	0.722	0.810	255.28	426.33	0.77	77.57	0.49
139,524	57,390	7.5	30	8,455	2,287	0.0063	177	0.758	16	0.814	91	0.698	0.746	0.930	480.08	588.97	0.75	66.75	0.56
40,681	16,186	13.5	18	10,435	4,921	0.0228	182	0.748	28	0.806	114	0.710	0.764	0.750	295.46	361.22	0.97	32.01	0.44
57,199	22,632	15.0	28	11,411	5,176	0.0110	188	0.751	28	0.769	63	0.687	0.745	0.910	342.18	357.39	0.84	20.47	0.47

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
171,255	62,924	15.7	38	8,649	3,942	0.0088	197	0.746	17	0.760	116	0.684	0.734	0.920	500.01	604.88	1.69	130.55	0.74
171,255	62,924	8.7	38	8,649	3,942	0.0345	197	0.788	17	0.835	116	0.723	0.753	0.850	489.05	626.15	1.04	18.46	0.63
38,750	16,670	7.7	19	9,951	3,340	0.0085	209	0.771	30	0.805	95	0.686	0.776	0.810	353.36	304.31	0.55	40.96	0.50
38,750	16,670	7.7	19	9,951	3,340	0.0645	209	0.771	30	0.805	95	0.714	0.776	0.610	285.14	334.44	0.61	8.20	0.39
196,759	69,742	11.8	24	10,666	4,663	0.0092	235	0.761	12	0.809	123	0.690	0.736	0.900	619.98	617.87	1.25	46.91	0.72
83,282	29,407	6.5	29	11,147	3,916	0.0021	175	0.743	26	0.784	110	0.679	0.744	0.920	517.03	481.02	0.50	89.00	0.52
83,282	29,407	6.5	29	11,147	3,916	0.0740	175	0.743	26	0.784	110	0.692	0.744	0.640	442.47	481.47	0.58	3.96	0.45
35,863	12,782	12.8	18	11,220	5,280	0.0086	187	0.806	21	0.841	51	0.730	0.772	0.930	241.67	384.68	0.99	67.71	0.43
43,136	16,869	8.7	16	10,876	4,693	0.0022	189	0.786	30	0.806	129	0.712	0.787	0.880	385.02	316.58	0.62	119.93	0.50
43,136	16,869	8.7	16	10,876	4,693	0.0090	189	0.786	30	0.806	129	0.730	0.787	0.800	329.22	365.17	0.63	34.79	0.44
40,865	15,714	13.9	22	9,868	4,920	0.0122	206	0.764	23	0.768	101	0.660	0.716	0.850	306.53	341.70	1.05	73.42	0.51
104,089	39,938	9.6	29	10,516	3,548	0.0862	202	0.753	13	0.785	130	0.690	0.739	0.590	443.16	483.58	0.95	6.58	0.55
147,768	64,406	15.5	35	10,510	3,717	0.0064	196	0.731	13	0.754	98	0.671	0.734	0.930	464.97	493.81	1.70	127.22	0.77
79,792	33,417	13.6	26	10,329	5,019	0.0255	224	0.799	28	0.849	104	0.731	0.782	0.800	391.86	417.81	1.19	23.09	0.58
55,586	22,762	8.2	21	10,642	4,037	0.0092	233	0.764	14	0.789	63	0.688	0.796	0.830	296.77	418.31	0.73	57.38	0.49
55,586	22,762	8.2	21	10,642	4,037	0.0754	233	0.764	14	0.763	63	0.688	0.731	0.830	308.99	404.93	0.73	50.93	0.47
165,073	60,059	6.7	35	10,144	4,217	0.0410	215	0.745	27	0.788	52	0.700	0.749	0.760	443.45	558.12	1.00	14.85	0.69
165,073	60,059	6.7	35	10,144	4,217	0.0041	215	0.745	27	0.788	52	0.691	0.738	0.910	573.97	650.54	0.67	70.06	0.55
95,475	35,074	10.1	27	8,739	3,775	0.0067	171	0.755	20	0.772	53	0.670	0.737	0.890	453.87	487.43	0.90	101.97	0.56
79,143	32,975	16.0	26	11,303	5,423	0.0076	210	0.736	19	0.793	68	0.687	0.755	0.850	435.14	435.50	1.16	63.33	0.56
128,638	53,480	13.3	24	10,796	4,406	0.0055	240	0.784	27	0.826	76	0.716	0.781	0.860	541.50	524.57	1.08	66.85	0.59
84,740	33,417	6.8	28	9,519	4,593	0.0074	212	0.754	35	0.806	100	0.675	0.761	0.880	494.63	451.18	0.55	40.35	0.52
117,993	47,268	11.4	26	9,644	4,102	0.0028	239	0.805	23	0.844	88	0.730	0.781	0.910	453.09	514.10	1.11	194.76	0.62
70,961	27,443	7.4	26	8,884	3,881	0.0315	230	0.752	25	0.829	61	0.673	0.748	0.780	394.48	444.82	0.64	17.12	0.48
88,264	35,276	6.6	30	10,423	4,215	0.0093	229	0.792	19	0.823	95	0.701	0.770	0.780	447.93	413.86	0.66	33.24	0.59
30,555	12,236	7.8	17	10,617	5,307	0.0034	219	0.738	21	0.758	131	0.674	0.727	0.780	275.96	312.89	0.56	124.85	0.40
58,529	22,207	13.8	24	11,184	4,873	0.0158	199	0.756	21	0.781	131	0.695	0.761	0.590	351.88	358.98	1.22	46.75	0.53
166,733	72,293	15.1	38	11,471	5,640	0.0061	233	0.745	31	0.772	87	0.685	0.755	0.890	459.18	463.93	2.03	124.03	0.90
133,751	57,853	9.5	30	9,198	4,020	0.0094	229	0.784	21	0.840	100	0.720	0.766	0.830	467.12	532.87	0.96	54.56	0.59
133,751	57,853	9.5	30	9,198	4,020	0.0182	229	0.766	21	0.780	100	0.685	0.744	0.760	500.08	520.36	0.92	28.96	0.60

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
72,811	26,280	12.7	25	11,666	5,503	0.0421	200	0.751	29	0.771	128	0.661	0.758	0.390	460.02	294.34	1.37	18.01	0.56
104,389	39,533	7.0	30	9,322	4,082	0.0948	196	0.763	32	0.784	55	0.682	0.744	0.920	470.93	525.17	0.64	77.86	0.51
189,868	72,636	12.8	22	10,588	4,633	0.0083	173	0.745	20	0.769	88	0.676	0.732	0.860	613.96	598.85	1.28	58.85	0.69
72,807	29,437	6.6	23	9,922	4,086	0.0058	172	0.781	24	0.806	66	0.696	0.742	0.890	307.95	462.41	0.71	85.30	0.50
81,903	33,985	15.5	25	10,139	4,460	0.0083	180	0.744	30	0.781	101	0.688	0.720	0.810	358.50	444.75	1.41	112.61	0.57
178,493	72,197	7.5	15	8,729	3,510	0.0396	232	0.753	31	0.775	95	0.699	0.743	0.760	496.96	746.63	0.86	14.46	0.57
144,946	57,063	16.0	36	9,727	3,537	0.0057	210	0.741	22	0.791	76	0.687	0.737	0.890	467.37	563.07	1.60	163.91	0.67
11,476	4,936	7.3	28	9,835	4,026	0.0062	175	0.795	20	0.844	132	0.690	0.801	0.490	201.28	149.52	0.55	95.24	0.27
11,476	4,936	7.3	28	9,835	4,026	0.0128	175	0.741	20	0.761	132	0.694	0.719	0.450	143.92	235.26	0.49	58.21	0.21
86,957	36,307	6.4	28	9,244	3,924	0.0067	231	0.724	28	0.746	50	0.656	0.715	0.880	410.83	494.88	0.57	34.27	0.46
199,821	75,420	9.4	40	10,958	4,927	0.0083	206	0.724	26	0.742	121	0.652	0.728	0.800	769.72	548.86	0.91	39.04	0.72
199,821	75,420	9.4	40	10,958	4,927	0.0500	206	0.724	26	0.742	121	0.690	0.728	0.650	542.78	659.28	1.08	6.56	0.61
154,934	66,681	14.4	38	10,491	4,065	0.0049	239	0.764	30	0.813	86	0.708	0.745	0.890	471.01	574.43	1.44	121.56	0.64
26,065	10,634	8.8	18	11,242	4,702	0.0085	234	0.745	34	0.772	68	0.673	0.731	0.740	284.77	295.04	0.55	40.74	0.38
124,702	45,628	6.6	30	11,026	4,066	0.0684	189	0.736	15	0.751	129	0.662	0.710	0.420	487.51	481.19	0.79	7.08	0.53
108,160	43,969	7.4	28	9,940	4,538	0.0054	236	0.756	12	0.777	58	0.683	0.763	0.920	370.48	451.91	0.98	120.59	0.73
175,326	70,282	7.3	40	11,316	5,510	0.0281	207	0.768	19	0.800	65	0.698	0.750	0.820	610.51	608.92	0.71	7.02	0.57
194,065	83,698	8.4	40	11,394	5,284	0.0914	233	0.763	27	0.800	109	0.672	0.752	0.470	719.95	486.74	0.90	3.37	0.67
119,126	42,630	6.3	32	11,185	4,477	0.0125	221	0.745	33	0.768	102	0.687	0.727	0.900	506.70	570.94	0.60	86.64	0.53
53,846	20,676	10.5	27	9,129	3,257	0.0053	237	0.795	29	0.811	114	0.717	0.752	0.800	302.35	375.08	1.01	168.19	0.51
22,982	8,433	12.8	19	9,424	4,210	0.0699	238	0.743	22	0.775	77	0.657	0.735	0.420	234.47	251.93	0.99	22.75	0.39
42,396	17,571	14.3	20	10,892	4,898	0.0089	175	0.726	30	0.732	103	0.658	0.714	0.720	311.95	320.78	1.09	88.57	0.50
32,311	12,034	9.2	21	11,156	4,463	0.0158	174	0.770	23	0.820	65	0.690	0.760	0.680	318.71	303.49	0.64	24.79	0.43
21,127	8,011	10.5	16	10,722	5,311	0.0062	224	0.801	18	0.832	114	0.730	0.770	0.680	230.70	281.27	0.69	83.36	0.36
21,127	8,011	10.5	16	10,722	5,311	0.0500	224	0.751	18	0.774	114	0.684	0.743	0.320	218.83	261.42	0.79	20.82	0.31
39,143	14,201	7.8	22	11,359	4,779	0.0091	233	0.777	12	0.798	133	0.697	0.755	0.610	327.13	298.45	0.69	41.88	0.44
61,471	23,387	11.5	25	9,383	3,225	0.0081	227	0.742	10	0.792	84	0.678	0.735	0.810	356.43	431.90	0.61	61.62	0.46
61,471	23,387	11.5	25	9,383	3,225	0.0800	227	0.742	10	0.767	84	0.691	0.735	0.530	309.29	531.41	0.69	9.71	0.42
68,061	27,092	14.3	26	11,743	5,465	0.0044	179	0.733	31	0.754	56	0.660	0.717	0.910	385.66	426.16	1.10	110.93	0.54
68,061	27,092	14.3	26	11,743	5,465	0.0076	179	0.733	31	0.754	96	0.678	0.717	0.800	348.77	405.35	1.27	80.63	0.57

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
15,398	5,782	7.2	16	11,166	5,482	0.0365	236	0.737	31	0.752	118	0.686	0.727	0.650	208.08	265.99	0.44	56.95	0.28
141,458	59,168	6.1	30	11,620	4,090	0.0071	188	0.786	15	0.818	65	0.718	0.780	0.880	575.64	534.39	0.58	20.56	0.57
12,409	4,583	7.2	15	11,013	4,893	0.0089	192	0.776	13	0.791	93	0.700	0.730	0.600	179.21	223.12	0.41	39.84	0.26
78,756	33,740	9.5	27	11,039	5,338	0.0024	234	0.802	29	0.858	124	0.725	0.787	0.830	453.46	381.85	0.81	102.39	0.58
37,936	16,192	13.7	20	10,198	4,837	0.0463	189	0.751	31	0.774	102	0.680	0.738	0.770	292.64	315.12	1.00	123.93	0.48
54,743	23,822	9.0	22	10,392	3,841	0.0147	186	0.791	20	0.845	71	0.730	0.789	0.800	359.37	407.23	0.64	22.72	0.41
87,330	32,523	14.6	30	10,611	4,774	0.0325	205	0.740	23	0.761	84	0.668	0.722	0.710	404.20	427.03	1.41	28.26	0.62
78,331	32,794	6.5	28	9,246	3,891	0.0032	188	0.769	29	0.820	73	0.706	0.764	0.910	329.20	377.56	0.67	151.58	0.55
21,702	9,321	13.2	16	8,852	3,299	0.0184	204	0.787	30	0.779	60	0.694	0.739	0.650	245.78	242.49	0.85	56.73	0.40
38,905	13,772	8.0	22	10,922	4,620	0.0076	213	0.736	30	0.754	122	0.666	0.717	0.700	314.94	327.81	0.67	64.47	0.44
121,606	47,939	15.0	32	10,469	4,707	0.0826	180	0.737	21	0.754	113	0.661	0.757	0.450	497.04	412.69	1.64	13.45	0.65
121,606	47,939	15.0	32	10,469	4,707	0.0826	180	0.737	21	0.754	113	0.690	0.720	0.540	400.61	529.16	1.59	13.71	0.59
104,305	41,298	8.2	30	10,454	4,468	0.0135	230	0.786	13	0.854	123	0.733	0.774	0.730	446.46	520.65	0.75	25.28	0.50
43,325	15,718	10.7	20	9,140	3,789	0.0055	215	0.754	14	0.761	73	0.709	0.738	0.870	268.34	430.99	0.83	157.60	0.42
58,181	25,554	13.1	24	11,571	5,303	0.0681	203	0.809	28	0.838	58	0.727	0.787	0.590	351.74	349.75	1.12	7.50	0.50
98,179	42,854	9.0	30	10,141	3,994	0.0063	212	0.734	27	0.766	67	0.697	0.715	0.910	323.35	520.32	0.94	95.99	0.50
158,277	64,761	8.0	35	10,035	3,853	0.0081	196	0.753	34	0.785	83	0.701	0.735	0.870	499.44	616.22	0.83	47.16	0.55
89,123	32,471	14.3	29	10,914	4,370	0.0471	216	0.800	25	0.819	83	0.731	0.764	0.660	336.07	464.28	1.60	19.78	0.63
59,171	24,185	13.6	28	8,577	4,235	0.0051	192	0.721	27	0.746	112	0.689	0.771	0.880	265.30	433.78	1.29	312.77	0.56
19,182	8,279	9.8	15	9,187	3,607	0.0076	189	0.756	35	0.775	121	0.692	0.748	0.660	210.12	266.85	0.63	105.31	0.34
149,260	54,031	11.9	40	8,669	3,837	0.0025	204	0.783	27	0.804	94	0.720	0.770	0.930	478.76	596.50	1.26	322.73	0.66
126,972	44,894	16.0	35	11,575	5,188	0.0098	231	0.798	27	0.847	113	0.734	0.790	0.760	578.42	553.85	1.57	50.34	0.72
177,441	74,106	11.4	40	8,898	3,587	0.0181	208	0.764	13	0.782	71	0.708	0.739	0.870	455.97	631.31	1.31	44.69	0.64
109,515	45,574	6.4	28	9,862	3,963	0.0976	199	0.732	34	0.740	65	0.693	0.718	0.690	393.19	562.24	0.66	4.72	0.44
109,515	45,574	6.4	28	9,862	3,963	0.0032	199	0.732	34	0.740	105	0.680	0.718	0.880	442.22	536.33	0.61	105.60	0.50
171,735	65,155	8.7	40	8,784	3,482	0.0049	220	0.781	25	0.809	123	0.717	0.763	0.860	551.18	603.38	0.94	105.39	0.65
42,094	15,663	15.1	26	11,140	5,135	0.0056	197	0.750	33	0.773	130	0.688	0.724	0.720	295.90	340.13	1.21	143.22	0.49
168,027	64,774	7.0	23	10,482	4,136	0.0066	187	0.785	23	0.824	76	0.723	0.791	0.890	459.44	496.46	1.11	72.95	0.82
13,172	4,760	8.2	16	8,554	3,878	0.0900	211	0.771	14	0.799	56	0.680	0.741	0.420	181.16	222.33	0.59	14.21	0.28
182,182	75,172	13.6	40	8,724	4,259	0.0511	198	0.756	15	0.782	119	0.673	0.729	0.680	522.40	572.28	1.51	21.01	0.71

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
182,182	75,172	13.6	40	8,724	4,259	0.0098	198	0.756	15	0.782	119	0.700	0.729	0.860	462.43	619.58	1.58	104.49	0.69
136,821	55,713	9.7	36	9,696	4,335	0.0084	238	0.808	33	0.849	90	0.735	0.796	0.850	506.76	546.57	0.94	49.79	0.59
167,912	67,518	15.2	39	8,467	4,134	0.0131	185	0.766	25	0.803	125	0.707	0.750	0.820	463.46	593.09	1.69	92.22	0.70
23,600	9,821	11.0	16	9,251	3,155	0.0185	181	0.806	17	0.768	51	0.690	0.733	0.750	213.69	318.42	0.71	50.56	0.35
144,599	56,904	8.3	37	8,968	4,066	0.0037	201	0.799	25	0.852	80	0.740	0.774	0.930	435.69	614.59	0.92	144.29	0.58
155,578	65,481	16.0	40	11,563	5,410	0.0037	202	0.757	12	0.788	81	0.694	0.749	0.910	549.99	557.19	1.41	110.13	0.67
121,199	48,643	9.0	29	11,440	5,047	0.0043	193	0.773	20	0.801	56	0.709	0.799	0.940	384.97	610.34	0.93	99.72	0.57
197,651	85,781	8.0	40	9,060	4,428	0.0032	197	0.749	25	0.766	130	0.667	0.744	0.890	660.56	568.85	0.81	109.84	0.68
114,232	40,234	9.2	30	10,517	4,665	0.0910	183	0.783	27	0.824	92	0.716	0.800	0.490	472.13	434.75	1.10	7.91	0.60
148,091	64,048	7.6	38	11,310	5,057	0.0051	201	0.789	34	0.812	131	0.736	0.794	0.830	496.92	510.71	0.87	51.52	0.64
49,590	18,477	9.2	22	8,906	3,543	0.0062	187	0.759	13	0.802	89	0.711	0.775	0.820	292.46	385.40	0.85	141.84	0.50
57,098	21,623	10.1	20	11,085	4,318	0.0444	234	0.802	27	0.829	79	0.733	0.772	0.580	319.82	393.46	0.94	12.43	0.47
152,846	53,742	12.2	37	9,974	3,507	0.0145	202	0.749	25	0.786	102	0.712	0.736	0.800	444.70	632.04	1.36	55.32	0.61
91,784	39,534	7.2	30	11,287	4,381	0.0092	171	0.783	28	0.813	104	0.696	0.750	0.740	455.71	385.61	0.91	34.91	0.59
161,485	66,077	12.0	40	9,228	3,514	0.0047	193	0.774	33	0.800	65	0.680	0.775	0.910	503.10	474.96	1.52	185.50	0.84
149,671	65,267	10.3	39	10,229	4,051	0.0084	216	0.754	25	0.783	70	0.674	0.740	0.870	556.23	540.72	0.94	46.37	0.61
125,414	49,873	11.1	35	8,941	3,834	0.0073	186	0.724	33	0.771	95	0.670	0.747	0.860	463.38	486.93	1.20	114.72	0.68
21,588	8,502	12.1	15	11,154	5,299	0.0067	227	0.737	20	0.754	117	0.672	0.718	0.660	235.91	275.93	0.77	69.87	0.37
49,614	21,218	14.5	24	8,747	3,761	0.0106	209	0.754	26	0.795	58	0.720	0.743	0.870	249.31	422.41	1.20	92.45	0.46
113,083	47,454	12.0	32	9,859	4,230	0.0049	221	0.784	28	0.831	86	0.713	0.764	0.880	450.85	499.54	1.11	77.62	0.60
123,326	45,977	13.1	33	10,365	4,575	0.0034	199	0.799	11	0.832	76	0.723	0.729	0.900	531.91	528.72	1.13	93.01	0.63
51,518	20,935	8.2	21	10,838	5,307	0.0187	198	0.761	22	0.801	110	0.701	0.745	0.640	332.36	370.32	0.70	16.81	0.44
51,518	20,935	8.2	21	10,838	5,307	0.0074	198	0.761	22	0.801	110	0.680	0.745	0.730	387.89	324.40	0.68	34.33	0.50
45,586	18,476	13.4	23	9,384	3,948	0.0022	221	0.767	22	0.817	106	0.668	0.770	0.840	369.79	312.33	0.98	252.64	0.56
110,794	40,917	9.5	34	10,763	4,385	0.0047	219	0.802	23	0.847	107	0.730	0.795	0.820	527.10	493.66	0.84	47.26	0.59
119,631	48,027	6.1	35	8,767	4,221	0.0357	172	0.733	14	0.742	124	0.650	0.718	0.680	495.09	503.42	0.63	14.37	0.56
14,076	5,911	7.8	15	8,838	3,557	0.0052	179	0.750	24	0.800	76	0.695	0.719	0.770	166.30	258.87	0.51	121.50	0.28
103,480	44,815	8.8	34	8,473	3,713	0.0120	174	0.748	11	0.768	108	0.706	0.733	0.830	356.93	530.06	0.92	65.37	0.53
47,543	18,230	12.2	25	9,289	4,053	0.0045	209	0.758	15	0.793	118	0.686	0.751	0.800	309.24	361.74	1.02	210.72	0.52
138,288	60,813	10.2	27	10,557	4,091	0.0091	191	0.733	27	0.741	105	0.677	0.726	0.820	489.87	538.70	0.98	48.27	0.59

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
59,716	21,522	11.4	27	8,695	3,928	0.0105	190	0.805	30	0.844	116	0.717	0.778	0.740	345.40	377.07	1.08	95.40	0.56
180,044	75,239	9.6	40	10,492	4,555	0.0073	228	0.768	20	0.801	102	0.707	0.740	0.850	572.58	570.16	1.02	49.09	0.66
27,366	11,778	14.5	20	10,549	3,934	0.0066	177	0.765	22	0.770	72	0.688	0.725	0.800	207.98	298.66	1.10	152.30	0.44
80,595	28,896	6.4	33	9,289	3,680	0.0088	236	0.769	11	0.772	81	0.696	0.753	0.850	389.43	477.24	0.62	50.06	0.52
136,558	59,271	8.2	35	8,621	3,568	0.0057	223	0.755	12	0.782	95	0.681	0.723	0.890	441.89	543.27	0.90	105.54	0.59
42,743	15,914	7.6	22	8,800	4,005	0.0166	219	0.734	18	0.743	56	0.666	0.720	0.840	281.25	393.73	0.64	46.26	0.44
86,122	34,006	9.8	31	9,200	3,779	0.0281	221	0.746	33	0.772	70	0.670	0.733	0.770	405.78	461.66	0.90	24.79	0.53
88,324	35,336	13.9	32	10,756	4,580	0.0065	174	0.783	16	0.804	86	0.705	0.804	0.840	380.51	363.02	1.64	132.49	0.78
88,324	35,336	13.9	32	10,756	4,580	0.0025	174	0.783	16	0.804	86	0.735	0.804	0.920	328.89	517.80	1.33	277.18	0.59
114,673	43,526	8.8	32	10,877	4,836	0.0369	175	0.770	26	0.800	124	0.704	0.759	0.550	502.62	496.36	0.84	10.89	0.54
104,896	41,437	7.5	30	8,578	3,362	0.0056	210	0.805	28	0.848	72	0.728	0.797	0.890	466.59	514.99	0.68	76.86	0.53
141,028	61,396	13.6	36	9,593	4,005	0.0129	212	0.742	24	0.785	51	0.668	0.736	0.910	411.83	574.19	1.43	70.15	0.65
12,935	5,525	6.2	15	10,789	4,714	0.0051	184	0.769	22	0.809	99	0.681	0.746	0.600	238.73	156.47	0.43	59.66	0.30
32,439	12,036	14.5	19	10,813	4,659	0.0163	192	0.739	21	0.773	89	0.668	0.740	0.620	292.06	284.25	1.08	56.17	0.47
26,817	10,103	6.9	16	9,477	4,484	0.0071	219	0.777	30	0.808	120	0.698	0.746	0.690	259.15	290.96	0.53	75.54	0.37
31,172	11,044	13.1	18	9,348	3,581	0.0043	233	0.769	34	0.795	107	0.719	0.765	0.770	252.12	349.55	0.93	238.99	0.43
13,726	4,933	10.1	15	10,771	4,682	0.0854	218	0.731	23	0.759	104	0.669	0.718	0.120	165.24	226.30	0.77	17.90	0.20
195,617	71,194	12.0	40	10,185	4,051	0.0092	212	0.755	21	0.831	79	0.688	0.733	0.870	582.84	641.00	1.27	58.97	0.67
154,234	62,517	9.8	38	11,257	4,698	0.0046	202	0.728	35	0.764	125	0.710	0.734	0.840	529.73	538.60	1.04	84.20	0.64
76,691	29,149	14.9	28	9,206	3,870	0.0228	230	0.758	23	0.780	115	0.702	0.778	0.670	347.88	409.33	1.52	62.82	0.60
167,458	65,418	15.9	26	11,522	4,984	0.0103	238	0.802	34	0.840	76	0.735	0.804	0.840	553.26	522.60	1.69	47.17	0.76
151,750	63,161	9.1	38	9,563	4,300	0.0367	199	0.790	12	0.821	103	0.682	0.754	0.680	557.43	488.09	0.99	13.88	0.68
75,989	29,956	10.3	26	9,296	4,027	0.0162	202	0.795	15	0.786	133	0.668	0.724	0.670	375.43	411.64	1.00	53.60	0.55
44,681	16,366	8.8	25	8,762	3,932	0.0084	223	0.736	30	0.744	76	0.655	0.741	0.830	326.38	365.95	0.70	94.99	0.48
21,352	8,575	9.7	19	10,207	4,260	0.0023	179	0.755	22	0.775	91	0.684	0.738	0.810	242.25	279.80	0.60	185.64	0.37
71,046	25,689	8.8	25	11,138	4,069	0.0088	175	0.749	31	0.788	102	0.668	0.771	0.680	526.16	351.90	0.72	37.01	0.53
162,938	70,300	14.2	39	10,584	5,243	0.0674	188	0.772	34	0.798	122	0.709	0.758	0.560	508.94	545.52	1.45	10.18	0.64
97,667	39,900	11.1	31	9,531	4,062	0.0038	216	0.757	30	0.766	95	0.696	0.734	0.890	391.28	491.35	1.08	180.61	0.57
22,904	9,497	8.3	17	8,968	3,599	0.0028	197	0.753	25	0.783	105	0.670	0.744	0.790	258.24	270.71	0.54	180.67	0.39
141,878	51,762	9.4	38	8,780	3,798	0.0153	234	0.742	26	0.742	68	0.688	0.748	0.890	406.00	621.53	1.12	59.67	0.61

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
45,404	19,150	7.3	24	10,080	4,090	0.0476	224	0.798	25	0.819	98	0.723	0.763	0.490	287.78	344.00	0.67	11.90	0.39
45,404	19,150	7.3	24	10,080	4,090	0.0059	224	0.741	25	0.761	98	0.681	0.751	0.790	320.46	361.18	0.57	71.79	0.44
187,857	68,691	15.1	40	10,188	4,748	0.0863	217	0.752	31	0.775	131	0.696	0.764	0.530	481.87	534.31	2.13	16.32	0.79
58,493	21,005	9.0	25	8,417	3,526	0.0422	235	0.737	35	0.742	63	0.702	0.718	0.770	288.14	460.15	0.85	25.35	0.44
58,493	21,005	9.0	25	8,417	3,526	0.0098	235	0.737	35	0.742	63	0.671	0.718	0.870	324.51	443.39	0.79	96.11	0.47
152,619	61,151	12.5	39	9,632	3,713	0.0202	214	0.781	13	0.809	115	0.712	0.794	0.730	441.52	465.83	1.75	54.81	0.83
108,600	44,235	8.6	32	11,179	4,555	0.0078	178	0.765	24	0.770	70	0.693	0.731	0.890	354.08	524.12	1.00	59.02	0.60
75,628	29,170	11.2	28	10,397	4,254	0.0044	228	0.804	24	0.809	119	0.713	0.763	0.810	399.07	377.85	1.11	126.37	0.60
81,277	31,367	9.2	30	9,219	3,390	0.0102	205	0.740	14	0.766	129	0.698	0.779	0.770	323.21	474.39	1.00	93.02	0.52
195,439	78,878	7.9	40	10,231	4,398	0.0140	219	0.763	14	0.785	93	0.665	0.784	0.820	509.29	427.35	1.45	53.32	1.03
154,683	64,752	8.4	37	9,611	4,576	0.0096	182	0.781	11	0.797	54	0.711	0.793	0.930	408.81	651.80	0.96	59.63	0.60
167,242	65,536	6.4	38	10,946	4,045	0.0364	226	0.758	20	0.758	90	0.670	0.718	0.750	437.25	582.47	0.91	12.46	0.69
36,086	14,126	13.7	21	10,906	4,600	0.0055	170	0.758	25	0.768	72	0.697	0.772	0.810	313.18	331.97	0.89	100.25	0.45
172,797	73,620	7.2	40	10,077	3,796	0.0404	198	0.743	11	0.775	112	0.668	0.748	0.620	581.80	520.37	0.82	10.47	0.64
27,147	10,077	9.9	19	9,266	3,290	0.0166	213	0.753	30	0.769	109	0.653	0.740	0.520	273.13	255.15	0.79	61.94	0.42
27,147	10,077	9.9	19	9,266	3,290	0.0040	213	0.753	30	0.769	109	0.680	0.765	0.750	272.39	293.30	0.69	187.51	0.42
93,268	35,348	12.5	30	9,718	4,102	0.0085	209	0.739	19	0.747	126	0.676	0.722	0.780	397.77	468.74	1.22	103.82	0.60
60,775	24,338	15.8	28	10,128	4,856	0.0132	204	0.764	34	0.781	68	0.670	0.737	0.820	351.84	360.25	1.38	73.84	0.60
180,481	74,375	12.2	40	11,871	4,486	0.0119	207	0.755	25	0.785	78	0.671	0.746	0.790	728.03	544.89	1.03	20.45	0.67
144,812	56,971	7.3	37	15,148	5,822	0.0080	219	0.740	24	0.777	131	0.685	0.740	0.660	744.06	488.36	0.61	9.19	0.57
131,515	51,207	8.6	36	13,380	5,136	0.0055	194	0.788	31	0.828	52	0.726	0.803	0.900	360.25	381.48	1.69	61.99	1.03
146,625	64,507	14.5	38	12,333	5,134	0.0196	174	0.743	14	0.763	56	0.693	0.731	0.850	489.53	560.76	1.33	19.08	0.58
132,076	52,350	11.7	36	12,982	4,854	0.0085	233	0.775	20	0.816	128	0.714	0.780	0.690	604.82	474.32	1.04	26.09	0.62
164,986	69,016	14.3	39	12,634	5,401	0.0271	196	0.738	22	0.800	78	0.667	0.736	0.720	687.06	520.12	1.18	10.02	0.65
20,323	8,547	9.4	16	14,704	5,699	0.0074	200	0.790	16	0.817	133	0.702	0.732	0.470	236.20	234.10	0.63	28.80	0.29
20,323	8,547	9.4	16	14,704	5,699	0.0111	200	0.790	16	0.817	133	0.695	0.766	0.300	274.14	159.80	0.80	23.60	0.27
97,606	41,526	7.9	30	14,692	6,228	0.0088	200	0.809	15	0.842	75	0.736	0.781	0.830	373.27	468.66	0.86	16.14	0.60
72,073	29,033	8.8	25	12,538	4,554	0.0059	220	0.764	12	0.786	113	0.687	0.735	0.710	463.75	359.13	0.76	38.16	0.52
72,073	29,033	8.8	25	12,538	4,554	0.0115	220	0.754	12	0.786	113	0.699	0.741	0.640	420.74	417.92	0.72	21.91	0.47
46,649	19,737	11.1	17	13,987	5,978	0.0080	177	0.784	11	0.817	77	0.715	0.769	0.720	370.29	325.16	0.79	21.90	0.48

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
46,649	19,737	11.1	17	13,987	5,978	0.0250	177	0.756	11	0.817	77	0.715	0.742	0.630	302.95	374.13	0.84	10.08	0.40
106,076	43,585	14.6	31	13,320	5,786	0.0228	217	0.734	26	0.747	103	0.665	0.722	0.610	522.16	412.70	1.29	16.17	0.63
106,076	43,585	14.6	31	13,320	5,786	0.0090	217	0.734	26	0.747	103	0.686	0.722	0.780	457.63	491.12	1.24	37.04	0.57
95,578	40,890	6.8	33	14,079	5,612	0.0069	196	0.738	29	0.791	119	0.706	0.786	0.680	601.32	361.99	0.60	13.25	0.49
186,182	78,456	6.3	40	11,937	4,608	0.0466	214	0.801	28	0.850	99	0.740	0.810	0.580	588.53	513.48	0.80	4.71	0.63
10,653	4,151	10.5	15	11,810	5,235	0.0094	198	0.746	19	0.751	59	0.694	0.719	0.730	167.59	234.84	0.57	46.59	0.27
106,219	37,811	15.3	31	12,003	5,358	0.0143	195	0.760	17	0.767	111	0.680	0.747	0.680	528.30	412.31	1.46	35.98	0.70
38,040	16,634	7.9	26	14,022	5,307	0.0072	210	0.733	14	0.746	109	0.677	0.726	0.630	335.93	297.25	0.58	23.96	0.41
119,040	43,592	6.0	30	14,891	6,245	0.0227	181	0.760	22	0.786	115	0.714	0.746	0.580	564.81	541.98	0.53	3.76	0.47
175,946	71,448	8.8	39	12,675	4,530	0.0336	211	0.756	19	0.792	112	0.710	0.748	0.610	572.40	622.95	0.86	5.67	0.54
56,847	22,587	7.5	25	12,059	5,433	0.0063	211	0.722	16	0.760	117	0.675	0.720	0.760	364.32	386.49	0.63	42.55	0.47
124,196	48,385	15.9	34	13,654	6,319	0.0563	171	0.745	18	0.782	98	0.691	0.756	0.500	538.96	442.90	1.52	7.32	0.61
167,725	64,002	8.2	38	14,249	6,985	0.0095	190	0.721	13	0.781	96	0.705	0.756	0.820	610.12	587.10	0.81	11.13	0.61
41,749	15,111	12.5	23	14,149	5,330	0.0091	178	0.783	33	0.796	100	0.701	0.768	0.550	420.87	254.17	0.97	26.08	0.49
66,477	23,764	10.6	25	14,511	6,980	0.0078	236	0.752	25	0.781	96	0.713	0.744	0.770	384.79	457.52	0.83	24.17	0.47
169,012	72,006	8.9	37	13,871	5,820	0.0251	204	0.766	15	0.793	67	0.684	0.765	0.680	865.06	513.50	0.65	3.49	0.60
18,186	6,695	7.9	16	12,891	4,925	0.0035	187	0.772	26	0.789	127	0.697	0.723	0.660	228.24	257.13	0.52	76.98	0.30
179,209	77,654	8.3	40	14,367	6,217	0.0031	222	0.781	25	0.801	123	0.722	0.757	0.840	640.59	547.67	0.61	27.68	0.61
150,935	56,036	9.4	37	12,961	5,444	0.0864	171	0.759	29	0.766	53	0.674	0.774	0.720	471.08	572.66	1.10	4.30	0.66
95,010	38,180	13.5	31	14,062	5,245	0.0926	186	0.746	32	0.767	64	0.687	0.718	0.860	340.28	520.12	1.34	40.62	0.57
33,687	12,508	8.8	21	12,515	5,431	0.0069	210	0.780	22	0.833	68	0.726	0.790	0.770	322.77	352.09	0.55	31.43	0.39
117,187	49,854	9.8	32	14,134	5,371	0.0078	173	0.737	11	0.764	122	0.697	0.759	0.780	344.32	371.16	1.70	60.18	0.89
85,255	34,793	9.6	29	14,964	7,234	0.0089	236	0.805	22	0.805	125	0.720	0.785	0.590	614.14	277.98	0.93	13.02	0.57
154,171	57,263	6.7	35	12,341	5,288	0.0022	195	0.764	32	0.783	85	0.694	0.751	0.900	696.20	574.45	0.57	52.16	0.58
124,121	45,837	6.1	32	13,785	5,699	0.0063	239	0.743	12	0.770	108	0.702	0.730	0.810	521.23	590.93	0.55	17.51	0.48
127,385	53,649	12.2	32	13,234	5,814	0.0201	239	0.751	32	0.777	112	0.680	0.781	0.580	620.76	358.07	1.28	15.37	0.67
75,363	27,957	15.1	27	12,474	6,135	0.0079	203	0.744	18	0.769	67	0.668	0.728	0.850	433.75	409.38	1.22	54.66	0.61
146,797	58,465	13.7	27	14,363	5,764	0.0075	216	0.786	34	0.812	128	0.727	0.764	0.710	580.08	494.97	1.30	25.00	0.65
75,058	32,801	8.5	25	13,621	5,219	0.0051	229	0.754	14	0.785	129	0.692	0.735	0.710	443.44	371.26	0.73	33.04	0.51
97,455	42,038	6.5	30	13,340	5,208	0.0891	195	0.792	21	0.836	98	0.736	0.810	0.740	424.03	358.77	0.83	20.77	0.66

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
83,829	32,387	6.5	28	13,397	5,494	0.0075	177	0.729	12	0.766	112	0.662	0.723	0.710	555.02	392.71	0.54	17.67	0.53
54,021	20,718	14.6	20	11,806	5,134	0.0094	239	0.763	22	0.790	79	0.672	0.736	0.750	384.95	327.55	1.18	55.43	0.57
20,795	7,376	9.9	25	12,080	4,406	0.0071	174	0.740	23	0.754	133	0.673	0.716	0.540	233.62	271.27	0.69	70.50	0.34
15,346	6,685	14.7	15	14,898	6,519	0.0051	202	0.755	26	0.794	78	0.713	0.769	0.660	241.02	231.35	0.68	36.92	0.32
68,906	28,872	9.6	26	13,736	5,581	0.0076	184	0.752	30	0.780	131	0.683	0.726	0.640	438.63	335.02	0.85	27.77	0.51
135,000	49,343	8.1	35	14,090	5,893	0.0400	173	0.790	12	0.846	134	0.720	0.792	0.360	640.69	374.10	0.98	4.61	0.58
123,715	44,384	7.1	34	13,061	5,829	0.0169	205	0.728	16	0.746	87	0.672	0.718	0.760	579.59	549.99	0.61	9.32	0.52
154,008	56,066	10.3	37	14,714	5,798	0.0039	235	0.758	11	0.788	60	0.701	0.735	0.930	423.66	680.01	1.14	53.23	0.61
24,224	8,668	14.0	17	14,140	6,261	0.0045	175	0.753	31	0.789	120	0.666	0.727	0.620	319.40	203.38	1.01	78.25	0.43
124,891	53,166	15.0	36	14,724	5,837	0.0089	223	0.750	27	0.774	72	0.688	0.761	0.820	464.25	388.70	1.80	37.62	0.84
96,011	36,044	13.3	32	13,715	5,625	0.0122	217	0.744	29	0.799	118	0.683	0.737	0.640	522.85	416.88	1.14	23.36	0.60
67,907	24,349	14.1	29	14,211	5,341	0.0037	180	0.745	31	0.746	56	0.690	0.728	0.910	323.49	487.29	1.19	93.46	0.56
16,862	6,769	10.2	15	12,168	5,152	0.0075	226	0.746	18	0.780	97	0.658	0.718	0.580	250.57	207.55	0.63	50.33	0.36
51,761	19,026	10.3	25	14,085	6,828	0.0087	214	0.745	26	0.769	131	0.683	0.750	0.620	430.36	303.21	0.84	25.42	0.51
28,685	11,764	13.6	17	12,474	5,583	0.0089	216	0.768	17	0.808	55	0.688	0.773	0.740	336.72	283.33	0.74	34.36	0.43
188,966	81,357	11.6	40	13,195	6,174	0.0138	177	0.761	20	0.802	119	0.698	0.757	0.750	674.88	557.28	1.06	17.02	0.65
105,531	39,776	14.9	35	11,867	5,593	0.0645	188	0.758	16	0.791	115	0.686	0.730	0.810	455.57	436.34	1.50	82.60	0.68
105,104	41,496	9.9	34	13,801	6,120	0.0212	199	0.780	13	0.824	63	0.721	0.794	0.800	350.81	348.32	1.69	21.53	0.93
143,279	51,081	8.5	36	14,664	5,952	0.0345	238	0.750	23	0.767	99	0.678	0.747	0.510	741.39	439.50	0.81	3.89	0.63
169,371	64,343	14.0	38	12,761	5,365	0.0068	203	0.753	21	0.784	95	0.691	0.722	0.870	441.51	610.18	1.69	73.71	0.74
69,882	28,178	12.5	28	11,995	5,420	0.0060	184	0.760	12	0.794	129	0.696	0.803	0.730	413.23	314.56	1.27	80.07	0.62
172,000	66,797	9.4	39	12,809	5,825	0.0156	199	0.760	12	0.760	100	0.670	0.746	0.700	811.60	473.81	0.85	10.27	0.71
172,000	66,797	9.4	39	12,809	5,825	0.0088	199	0.760	12	0.760	100	0.690	0.750	0.800	663.11	586.23	0.85	19.26	0.62
12,370	5,330	7.0	15	13,423	6,340	0.0097	181	0.777	34	0.811	127	0.695	0.739	0.420	202.66	176.31	0.46	23.68	0.23
91,850	39,961	15.5	31	12,310	6,195	0.0354	199	0.747	35	0.770	120	0.677	0.724	0.560	440.99	396.74	1.39	25.23	0.58
144,609	61,386	6.2	35	13,739	5,172	0.0064	203	0.760	22	0.794	92	0.691	0.789	0.830	430.92	310.28	1.38	51.84	1.08
62,540	27,514	11.8	27	14,193	6,305	0.0197	221	0.779	22	0.847	67	0.736	0.777	0.720	378.80	420.09	0.82	8.77	0.43
23,593	9,198	8.6	18	13,172	6,509	0.0084	219	0.751	29	0.791	50	0.701	0.769	0.810	255.22	271.05	0.59	31.32	0.40
69,810	29,877	11.6	26	12,105	5,798	0.0048	196	0.789	26	0.789	84	0.687	0.754	0.830	424.38	330.18	1.06	67.70	0.62
133,252	53,330	9.8	34	13,369	5,822	0.0075	210	0.784	30	0.858	126	0.730	0.809	0.720	541.71	410.19	1.16	30.65	0.70

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
46,776	19,366	6.7	22	14,980	6,672	0.0158	227	0.745	17	0.765	65	0.664	0.742	0.620	497.82	273.84	0.47	5.35	0.50
101,827	43,356	9.7	32	12,576	5,418	0.0491	198	0.770	24	0.809	80	0.711	0.760	0.600	480.87	479.12	0.81	4.96	0.49
95,046	40,672	7.4	30	14,126	5,969	0.0091	196	0.749	13	0.785	122	0.708	0.806	0.770	358.92	469.37	0.82	21.90	0.47
95,046	40,672	7.4	30	14,126	5,969	0.0061	196	0.749	13	0.785	122	0.690	0.755	0.710	575.80	388.75	0.62	16.85	0.52
54,681	22,762	12.2	24	12,602	4,330	0.0102	207	0.748	15	0.776	134	0.687	0.719	0.600	334.95	343.65	1.07	46.72	0.48
85,354	35,687	6.1	28	12,429	5,501	0.0014	180	0.774	29	0.777	79	0.684	0.795	0.890	589.69	315.61	0.57	90.70	0.61
73,956	29,267	7.1	28	11,948	5,294	0.0079	215	0.754	28	0.783	93	0.700	0.723	0.840	319.51	460.71	0.75	41.30	0.49
73,956	29,267	7.1	28	11,948	5,294	0.0040	215	0.754	28	0.775	93	0.680	0.731	0.850	438.06	411.04	0.61	50.75	0.52
38,773	15,177	10.0	23	13,860	6,507	0.0084	177	0.722	21	0.803	99	0.686	0.740	0.660	362.36	281.25	0.74	24.77	0.46
152,774	54,855	8.1	36	14,791	5,855	0.0144	216	0.767	30	0.792	85	0.700	0.764	0.680	826.76	530.45	0.61	4.96	0.58
173,174	71,709	12.4	38	13,825	6,585	0.0276	186	0.809	25	0.856	106	0.736	0.810	0.730	851.19	485.15	0.96	13.43	0.67
140,710	56,606	16.0	36	12,398	5,776	0.0203	176	0.781	27	0.817	110	0.717	0.779	0.660	580.02	491.12	1.42	18.27	0.66
88,998	33,678	11.9	29	12,035	5,086	0.0732	233	0.761	21	0.800	116	0.670	0.763	0.300	479.52	304.92	1.43	9.44	0.56
88,998	33,678	11.9	29	12,035	5,086	0.0067	233	0.756	21	0.795	116	0.692	0.763	0.750	480.75	408.83	1.06	54.41	0.60
82,572	35,138	14.3	29	14,062	6,755	0.0099	188	0.744	21	0.760	130	0.677	0.772	0.620	523.57	293.81	1.33	28.49	0.60
137,626	48,971	7.8	27	12,883	5,313	0.0022	183	0.756	33	0.793	122	0.688	0.798	0.870	514.22	329.92	1.39	168.76	0.96
92,356	35,507	11.3	31	12,958	5,983	0.0168	188	0.782	33	0.821	98	0.696	0.756	0.640	535.21	373.77	1.02	14.20	0.61
56,696	20,847	8.7	26	12,264	5,260	0.0043	194	0.753	22	0.775	68	0.688	0.742	0.860	399.92	428.78	0.61	50.45	0.46
114,109	45,626	11.8	33	13,002	4,582	0.0094	214	0.795	22	0.836	52	0.730	0.775	0.890	359.55	572.95	1.25	37.55	0.62
95,604	38,227	7.5	30	13,380	5,722	0.0081	230	0.733	10	0.762	118	0.668	0.733	0.710	547.87	398.85	0.68	20.29	0.61
95,604	38,227	7.5	30	13,380	5,722	0.0150	230	0.733	10	0.733	118	0.684	0.722	0.680	454.86	482.36	0.67	12.50	0.50
11,812	4,831	9.1	15	12,653	5,719	0.0056	213	0.762	10	0.806	72	0.700	0.784	0.660	229.00	196.60	0.46	39.72	0.29
28,280	10,746	14.6	20	14,088	5,452	0.0251	213	0.774	27	0.797	130	0.712	0.756	0.280	253.44	260.99	1.16	21.27	0.35
40,027	16,514	8.4	20	13,944	6,953	0.0081	228	0.755	28	0.786	82	0.675	0.734	0.710	396.11	279.13	0.59	18.02	0.46
38,341	16,266	7.4	18	14,562	6,232	0.0072	187	0.787	28	0.819	110	0.688	0.733	0.660	326.77	251.08	0.67	26.33	0.44
38,341	16,266	7.4	18	14,562	6,232	0.0100	187	0.747	28	0.773	110	0.670	0.728	0.510	414.33	223.41	0.59	13.94	0.40
81,918	34,518	9.9	28	13,712	6,436	0.0237	222	0.755	26	0.779	94	0.685	0.774	0.570	543.35	336.20	0.84	8.32	0.53
77,125	31,620	13.4	28	13,578	5,080	0.0089	213	0.747	31	0.755	98	0.693	0.715	0.780	327.43	430.68	1.34	48.75	0.56
39,245	17,077	12.4	20	14,297	5,176	0.0286	190	0.760	22	0.781	88	0.678	0.737	0.360	373.81	247.52	0.92	10.49	0.43
130,155	55,871	14.4	35	12,015	6,075	0.0333	228	0.731	10	0.776	51	0.683	0.723	0.850	459.85	558.79	1.28	13.01	0.57

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
140,412	49,844	8.2	36	13,509	6,844	0.0046	208	0.723	22	0.743	107	0.663	0.715	0.850	668.64	549.07	0.69	29.58	0.60
58,027	24,199	6.5	26	12,734	4,909	0.0510	203	0.775	11	0.786	114	0.689	0.745	0.330	383.79	327.74	0.62	5.60	0.42
178,715	74,774	14.8	40	13,789	6,047	0.0168	232	0.768	30	0.782	95	0.701	0.770	0.700	713.89	501.54	1.31	12.22	0.69
186,378	74,945	11.5	40	14,386	6,020	0.0058	202	0.760	18	0.760	83	0.697	0.752	0.810	770.47	559.87	0.95	18.84	0.66
32,895	12,547	6.4	27	12,353	5,093	0.0087	203	0.758	27	0.776	73	0.708	0.765	0.720	318.00	348.99	0.41	20.81	0.36
187,279	69,991	12.7	40	14,841	7,441	0.0091	237	0.808	11	0.838	77	0.740	0.812	0.820	637.85	469.54	1.56	19.63	0.87
70,177	25,647	8.8	29	14,821	6,919	0.0049	212	0.806	25	0.841	76	0.733	0.804	0.790	552.13	382.38	0.61	16.25	0.55
40,790	14,778	7.0	23	12,034	5,274	0.0079	212	0.775	19	0.800	86	0.707	0.758	0.750	244.17	361.68	0.51	27.15	0.42
191,023	75,871	9.9	40	12,938	5,757	0.0234	217	0.785	28	0.818	70	0.706	0.768	0.760	751.06	586.42	0.85	6.12	0.62
40,946	16,973	6.7	19	13,124	5,466	0.0107	198	0.735	33	0.775	107	0.680	0.741	0.600	369.90	311.88	0.49	16.48	0.43
165,566	57,994	9.3	39	13,633	5,208	0.0057	232	0.762	12	0.779	120	0.676	0.740	0.750	772.75	435.14	0.98	38.36	0.74
47,590	20,854	9.1	24	13,158	4,851	0.0044	182	0.753	32	0.790	86	0.688	0.747	0.770	391.05	338.59	0.61	40.15	0.47
119,983	40,722	14.8	32	14,348	6,601	0.0021	230	0.768	20	0.798	105	0.699	0.771	0.850	680.05	426.03	1.23	82.27	0.71
119,983	40,722	14.8	32	14,348	6,601	0.0099	230	0.768	20	0.798	105	0.712	0.771	0.720	606.15	480.18	1.22	20.61	0.63
144,904	54,421	10.2	36	14,770	5,523	0.0408	187	0.724	32	0.775	61	0.692	0.727	0.650	546.12	601.29	0.92	4.07	0.55
55,065	22,758	10.6	26	13,735	6,035	0.0129	227	0.756	34	0.788	73	0.688	0.781	0.710	383.72	277.06	1.03	23.73	0.60
55,065	22,758	10.6	26	13,735	6,035	0.0080	227	0.756	34	0.788	73	0.670	0.741	0.740	468.36	310.48	0.76	22.20	0.53
13,844	5,347	13.6	23	12,400	5,462	0.0046	220	0.750	30	0.750	52	0.668	0.750	0.780	249.38	219.99	0.64	75.41	0.37
144,657	52,414	14.8	37	12,084	5,112	0.0059	216	0.746	15	0.781	132	0.692	0.732	0.800	523.62	554.49	1.43	76.36	0.67
136,804	56,240	10.1	26	12,156	5,598	0.0372	183	0.768	26	0.789	124	0.672	0.732	0.530	603.52	405.79	1.09	9.14	0.65
198,572	81,462	16.0	40	15,079	6,797	0.0066	173	0.771	31	0.801	97	0.705	0.785	0.780	735.07	404.59	1.89	31.13	0.88
74,485	30,322	13.4	30	14,313	7,102	0.0089	205	0.798	33	0.847	134	0.732	0.809	0.670	406.54	325.56	1.38	31.65	0.69
101,421	37,762	11.0	30	12,035	5,018	0.0026	175	0.766	34	0.777	65	0.717	0.755	0.920	450.84	558.83	0.89	112.58	0.51
17,663	6,888	8.1	15	13,882	6,265	0.0079	207	0.768	28	0.787	95	0.689	0.768	0.560	256.58	184.25	0.61	28.84	0.39
46,436	17,064	7.6	20	12,393	5,277	0.0066	178	0.741	16	0.772	126	0.672	0.775	0.630	426.56	268.72	0.67	39.55	0.44
63,004	25,384	10.9	26	13,873	5,036	0.0173	214	0.797	27	0.844	103	0.739	0.805	0.500	441.26	346.49	0.85	11.27	0.48
191,880	67,358	9.2	40	14,637	6,270	0.0075	208	0.759	20	0.782	92	0.677	0.736	0.790	849.59	516.97	0.86	12.64	0.70
98,219	39,436	11.6	32	13,653	6,100	0.0232	204	0.773	18	0.799	52	0.688	0.772	0.740	621.41	432.03	0.81	6.66	0.57
24,951	8,837	12.7	17	13,471	6,127	0.0082	219	0.741	34	0.780	94	0.678	0.726	0.630	291.44	270.13	0.81	41.44	0.39
34,720	13,518	7.1	23	13,132	5,382	0.0259	187	0.762	18	0.796	132	0.705	0.758	0.350	294.42	299.84	0.59	11.21	0.35

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
83,189	31,047	8.1	29	14,640	5,982	0.0074	181	0.801	22	0.830	109	0.728	0.764	0.730	421.75	408.89	0.83	19.30	0.57
183,380	79,841	11.5	40	13,200	5,763	0.0512	186	0.756	11	0.789	111	0.721	0.766	0.640	399.00	481.46	1.98	11.71	0.88
168,000	62,160	7.5	40	13,200	5,763	0.0100	186	0.756	11	0.766	111	0.707	0.746	0.780	620.36	615.26	0.72	14.11	0.57
77,968	34,255	6.3	27	13,439	6,309	0.0472	214	0.791	21	0.820	89	0.714	0.773	0.470	497.15	361.44	0.54	2.77	0.47
98,955	35,313	11.0	29	12,253	5,715	0.0821	205	0.781	25	0.812	82	0.721	0.761	0.550	434.65	478.87	1.08	5.28	0.54
89,309	34,200	14.3	28	13,966	6,894	0.0664	219	0.749	14	0.755	87	0.711	0.741	0.570	392.15	487.17	1.26	5.49	0.50
40,500	16,363	12.6	21	11,898	5,223	0.0548	197	0.746	17	0.784	79	0.679	0.729	0.480	302.50	320.82	0.98	11.50	0.43
119,239	52,425	16.0	33	14,467	6,292	0.0056	218	0.733	23	0.756	75	0.674	0.727	0.840	570.30	492.73	1.14	33.25	0.59
90,308	39,520	8.7	17	12,793	5,657	0.0275	177	0.785	19	0.812	86	0.692	0.789	0.550	607.86	322.66	0.75	6.62	0.59
157,550	56,310	10.8	35	14,078	6,559	0.0968	229	0.768	29	0.789	84	0.687	0.738	0.580	485.56	559.79	1.30	3.60	0.73
116,384	45,603	8.0	37	12,442	5,258	0.0159	177	0.803	31	0.829	95	0.720	0.810	0.650	666.96	393.32	0.74	10.15	0.59
72,664	28,201	6.3	26	15,098	7,363	0.0506	207	0.755	28	0.771	62	0.683	0.757	0.540	567.07	368.82	0.47	1.62	0.51
135,642	49,121	8.6	36	12,062	4,820	0.0364	231	0.746	25	0.777	112	0.689	0.739	0.590	546.06	548.90	0.83	8.12	0.55
76,422	27,738	14.0	28	13,758	5,922	0.0365	206	0.738	19	0.785	69	0.671	0.733	0.620	463.91	400.08	1.13	9.48	0.55
96,239	35,443	7.6	22	13,908	6,278	0.0893	193	0.727	12	0.744	71	0.686	0.722	0.570	464.20	525.71	0.65	2.10	0.43
131,473	55,585	12.5	36	14,904	6,509	0.0669	189	0.731	29	0.768	119	0.655	0.744	0.260	667.26	290.42	1.54	4.70	0.59
118,514	47,027	9.0	36	13,350	6,504	0.0269	175	0.779	28	0.822	78	0.699	0.803	0.680	523.61	326.24	1.25	11.53	0.80
51,562	20,580	6.9	24	12,235	5,981	0.0165	206	0.728	24	0.828	60	0.652	0.717	0.820	393.62	382.10	0.49	11.72	0.46
64,918	28,243	13.9	30	14,995	6,055	0.0504	216	0.739	33	0.772	94	0.660	0.743	0.310	473.01	289.03	1.13	5.53	0.51
58,824	20,776	12.2	24	12,005	5,626	0.0302	180	0.770	22	0.770	59	0.695	0.745	0.730	365.01	395.99	1.01	15.14	0.52
145,432	62,032	9.6	38	15,044	6,451	0.0565	197	0.739	10	0.787	87	0.690	0.730	0.570	587.40	556.15	0.81	2.28	0.51
198,939	82,945	14.2	40	12,570	5,842	0.0104	184	0.751	12	0.782	61	0.699	0.743	0.870	590.22	613.01	1.40	30.68	0.67
71,474	28,102	11.9	29	11,866	5,142	0.0675	194	0.774	32	0.818	103	0.729	0.788	0.430	365.54	412.07	1.09	8.35	0.46
157,301	62,969	9.7	37	14,020	6,586	0.0085	208	0.784	14	0.826	91	0.716	0.802	0.810	497.40	364.60	1.66	25.39	0.98
41,604	17,439	8.7	19	13,034	5,589	0.0353	171	0.762	28	0.779	110	0.685	0.739	0.330	350.90	278.60	0.71	9.55	0.39
54,644	21,867	11.3	25	12,169	5,640	0.0066	205	0.752	12	0.772	110	0.664	0.713	0.750	367.20	321.28	1.00	61.86	0.53
117,630	44,171	8.8	30	13,654	5,910	0.0088	223	0.748	22	0.781	89	0.682	0.755	0.741	666.70	449.20	0.72	15.76	0.58
101,769	40,673	12.4	30	12,282	5,508	0.0159	189	0.735	27	0.776	115	0.674	0.739	0.702	529.70	454.90	1.02	21.44	0.57
87,629	31,388	12.8	28	12,388	5,289	0.0011	216	0.762	13	0.786	121	0.704	0.734	0.905	373.60	500.50	1.20	39.90	0.59
197,756	81,813	9.9	40	14,828	6,188	0.0043	205	0.798	13	0.835	74	0.734	0.810	0.900	452.17	373.04	2.23	76.67	1.27

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
107,238	42,922	8.5	21	5,068	2,069	0.0109	204	0.735	10	0.804	100	0.654	0.738	0.780	387.10	531.87	0.90	150.37	0.55
107,238	42,922	8.5	21	5,068	2,069	0.0500	204	0.750	10	0.844	100	0.690	0.738	0.610	345.26	548.80	0.97	43.33	0.53
178,443	74,888	6.6	30	5,640	1,146	0.0078	212	0.764	20	0.786	85	0.650	0.720	0.920	495.35	605.55	0.79	106.92	0.64
103,751	45,037	13.8	29	7,390	2,046	0.0411	234	0.751	32	0.777	51	0.710	0.730	0.790	380.22	482.08	1.39	30.44	0.61
98,898	42,715	6.0	18	6,953	2,553	0.0071	170	0.787	30	0.842	105	0.684	0.764	0.770	476.34	468.28	0.53	55.61	0.51
177,570	70,775	12.1	30	6,859	2,547	0.0012	179	0.756	16	0.794	130	0.662	0.780	0.920	452.90	464.69	1.94	141.42	0.78
27,805	10,824	11.1	26	5,138	2,102	0.0111	201	0.789	12	0.810	119	0.696	0.752	0.710	200.97	316.82	0.95	257.58	0.42
123,789	43,874	16.0	29	8,373	3,622	0.0268	219	0.741	33	0.750	83	0.706	0.730	0.790	373.16	579.16	1.78	50.63	0.64
123,789	43,874	16.0	29	8,373	3,622	0.0023	219	0.790	33	0.833	83	0.710	0.760	0.920	417.34	545.33	1.69	240.03	0.70
34,022	14,266	6.3	22	6,709	2,647	0.0365	173	0.749	20	0.836	64	0.671	0.787	0.680	250.09	311.40	0.58	26.86	0.41
34,022	14,266	6.3	22	6,709	2,647	0.0365	173	0.749	20	0.808	64	0.683	0.732	0.720	229.05	344.98	0.57	25.87	0.37
73,142	29,174	7.4	20	6,270	2,687	0.0241	195	0.759	11	0.802	115	0.695	0.758	0.680	331.79	471.47	0.73	45.18	0.46
73,142	29,174	7.4	20	6,270	2,687	0.0241	195	0.762	11	0.782	115	0.660	0.732	0.680	348.98	435.64	0.75	47.32	0.51
31,723	13,395	7.3	17	5,544	2,019	0.0454	192	0.772	24	0.844	112	0.674	0.738	0.480	218.56	322.37	0.64	41.42	0.35
135,988	50,101	7.2	29	6,321	2,173	0.0069	195	0.749	28	0.799	92	0.670	0.745	0.870	466.64	581.09	0.82	125.97	0.59
135,988	50,101	7.2	29	6,321	2,173	0.0069	195	0.767	28	0.819	92	0.704	0.734	0.880	413.81	607.47	0.89	133.96	0.57
164,619	59,908	9.0	30	7,795	3,621	0.0340	220	0.761	25	0.796	69	0.704	0.746	0.820	481.01	651.64	1.01	18.98	0.58
34,401	13,459	15.7	17	7,126	3,420	0.0010	197	0.767	24	0.776	126	0.658	0.752	0.890	256.85	338.63	1.13	140.43	0.49
28,261	10,653	6.6	18	6,888	2,724	0.0354	216	0.776	27	0.793	134	0.699	0.781	0.410	222.50	321.56	0.57	32.75	0.32
185,510	75,695	13.1	30	7,487	1,978	0.0059	227	0.748	26	0.754	85	0.707	0.730	0.900	397.24	731.14	1.54	198.11	0.73
120,923	45,143	7.8	28	7,040	2,385	0.0140	214	0.770	12	0.834	126	0.709	0.790	0.740	384.06	501.60	1.08	73.55	0.66
22,042	8,171	15.2	15	7,311	2,288	0.0072	177	0.740	20	0.764	52	0.703	0.739	0.840	191.59	321.69	1.02	225.31	0.39
65,387	27,331	11.5	20	8,324	3,814	0.0105	229	0.736	18	0.757	114	0.650	0.776	0.710	363.17	334.87	1.15	90.72	0.60
175,697	69,827	7.1	27	5,123	2,080	0.0057	180	0.760	22	0.780	60	0.671	0.740	0.890	489.18	646.39	0.83	184.31	0.61
24,376	10,491	8.4	20	7,295	2,927	0.0022	228	0.757	11	0.836	116	0.687	0.736	0.820	216.91	285.24	0.63	342.59	0.38
77,837	31,127	7.6	22	5,937	1,270	0.0100	213	0.732	12	0.821	80	0.668	0.771	0.870	288.05	453.30	0.94	169.27	0.59
98,653	42,370	13.1	26	6,459	1,484	0.0018	194	0.770	28	0.764	96	0.675	0.722	0.930	328.75	476.07	1.48	269.96	0.65
179,024	63,574	15.9	40	6,369	2,598	0.0066	184	0.779	19	0.817	103	0.701	0.767	0.900	471.61	610.67	1.93	279.08	0.79
140,287	57,933	10.6	28	6,027	2,501	0.0647	227	0.726	32	0.741	95	0.684	0.719	0.760	378.25	606.05	1.24	26.82	0.59
140,287	57,933	10.6	28	6,027	2,501	0.0065	227	0.800	32	0.840	95	0.720	0.820	0.900	373.09	555.49	1.37	249.80	0.69

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
14,030	6,004	7.9	28	6,644	2,094	0.0232	198	0.766	21	0.779	103	0.672	0.751	0.510	159.68	219.97	0.62	62.98	0.29
199,989	87,746	10.1	35	5,991	2,392	0.0085	184	0.775	32	0.792	105	0.700	0.780	0.900	437.76	642.79	1.32	160.30	0.71
68,869	29,072	8.8	21	6,563	2,947	0.0389	174	0.764	34	0.772	103	0.704	0.727	0.610	286.84	451.50	0.91	15.54	0.46
29,985	11,078	11.5	19	5,705	2,374	0.0049	189	0.756	29	0.780	72	0.665	0.767	0.890	244.12	333.68	0.86	395.75	0.45
87,206	34,011	14.6	25	7,443	3,191	0.0437	179	0.753	23	0.833	128	0.690	0.775	0.460	372.54	410.79	1.55	20.92	0.62
153,909	58,282	15.2	40	8,205	2,559	0.0084	210	0.753	18	0.802	101	0.666	0.760	0.840	457.76	510.94	1.89	149.60	0.82
80,226	28,440	8.2	25	5,568	2,500	0.0020	174	0.763	27	0.838	98	0.720	0.789	0.940	298.84	497.77	0.97	267.26	0.63
74,578	27,827	10.6	27	7,204	3,087	0.0891	209	0.751	32	0.769	122	0.676	0.757	0.500	326.82	428.41	1.16	17.91	0.53
74,522	30,104	14.1	30	7,886	3,935	0.0053	216	0.743	29	0.754	77	0.690	0.745	0.910	327.31	467.28	1.26	202.60	0.56
166,000	71,348	10.4	35	6,319	2,679	0.0123	170	0.785	34	0.820	85	0.708	0.749	0.890	434.64	596.37	1.24	90.37	0.65
77,599	33,481	9.1	24	6,985	2,750	0.0062	237	0.737	16	0.751	87	0.670	0.748	0.880	322.41	460.58	0.90	162.14	0.54
182,194	68,142	9.1	35	5,519	2,836	0.0045	226	0.743	17	0.766	82	0.675	0.718	0.950	460.08	675.69	1.12	305.42	0.64
153,438	63,730	7.6	35	7,738	3,663	0.0257	237	0.781	27	0.830	70	0.706	0.788	0.850	417.79	509.10	1.10	29.19	0.75
61,724	23,475	15.6	19	5,866	2,255	0.0295	213	0.747	28	0.792	57	0.657	0.744	0.810	309.06	415.81	1.41	91.02	0.56
167,771	69,779	7.5	36	7,695	3,613	0.0800	207	0.782	25	0.820	113	0.698	0.772	0.610	537.13	585.02	0.81	6.41	0.58
29,324	10,279	7.6	18	7,273	2,663	0.0067	214	0.756	34	0.795	101	0.696	0.749	0.420	223.08	330.67	0.67	19.12	0.34
43,280	16,305	6.7	27	5,175	1,971	0.0079	237	0.748	28	0.802	82	0.678	0.732	0.820	242.40	387.01	0.68	210.52	0.44
57,150	21,812	6.8	28	5,693	2,813	0.0342	212	0.747	34	0.814	76	0.683	0.727	0.910	272.20	422.22	0.74	41.57	0.51
46,020	16,425	6.3	19	6,872	1,985	0.0033	180	0.747	31	0.784	71	0.674	0.738	0.940	297.21	404.10	0.56	216.12	0.46
54,853	21,224	7.1	27	6,672	2,820	0.0035	187	0.739	34	0.769	105	0.668	0.736	0.940	281.93	438.08	0.67	250.21	0.48
58,855	21,050	6.7	18	5,726	2,000	0.0084	222	0.751	34	0.768	63	0.671	0.735	0.930	302.76	443.56	0.67	143.45	0.50
64,287	27,282	7.4	21	7,799	3,194	0.0027	212	0.747	10	0.803	99	0.688	0.734	0.870	306.99	426.46	0.71	230.88	0.50
57,791	24,137	6.4	24	7,716	3,000	0.0251	216	0.733	31	0.763	79	0.667	0.745	0.820	319.82	401.06	0.60	22.38	0.48
131,540	57,607	10.1	23	9,919	4,566	0.0099	213	0.734	26	0.788	130	0.687	0.725	0.900	447.92	562.16	0.97	51.62	0.57
141,795	50,306	7.5	33	11,587	5,719	0.0043	190	0.778	21	0.823	131	0.718	0.767	0.910	609.47	579.83	0.67	39.61	0.59
184,445	73,527	10.7	40	10,952	5,163	0.0396	196	0.774	28	0.797	51	0.706	0.740	0.870	440.98	637.59	1.37	13.67	0.65
121,311	48,249	12.2	30	9,857	4,552	0.0082	202	0.783	30	0.833	130	0.725	0.784	0.770	481.60	519.74	1.13	70.76	0.59
99,348	38,652	7.2	30	10,291	4,682	0.0275	191	0.782	23	0.818	76	0.718	0.748	0.800	352.22	516.31	0.83	18.12	0.54
172,898	74,331	12.8	22	10,642	4,480	0.0085	186	0.782	12	0.815	82	0.707	0.759	0.860	560.90	559.88	1.26	48.92	0.67
90,772	36,800	12.2	30	8,438	4,018	0.0073	203	0.746	17	0.765	94	0.679	0.733	0.870	381.94	474.36	1.15	147.30	0.58

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
195,831	73,060	6.3	19	9,994	4,830	0.0038	212	0.781	23	0.846	93	0.724	0.795	0.920	469.13	515.75	1.14	146.23	0.90
197,729	80,180	12.7	40	9,575	4,296	0.0062	230	0.759	29	0.788	100	0.702	0.742	0.880	569.24	627.00	1.32	99.11	0.68
124,702	45,628	10.1	30	11,026	4,066	0.0096	189	0.768	15	0.791	89	0.695	0.740	0.800	516.96	515.58	0.98	38.64	0.60
41,446	18,027	13.0	20	11,356	4,605	0.0155	215	0.768	23	0.791	107	0.691	0.754	0.550	315.61	294.33	0.96	35.59	0.45
78,756	33,740	9.5	27	11,039	5,338	0.0010	234	0.741	29	0.762	124	0.683	0.729	0.750	391.10	523.95	0.58	42.29	0.52
98,179	42,854	7.7	30	10,141	3,994	0.0554	212	0.734	27	0.766	67	0.675	0.735	0.710	367.62	427.53	0.93	12.20	0.61
19,159	7,171	7.7	15	11,779	5,558	0.0538	196	0.762	12	0.799	133	0.719	0.755	0.190	182.44	282.73	0.61	13.01	0.21
164,007	72,005	14.9	39	9,112	4,030	0.0314	186	0.761	22	0.789	106	0.672	0.733	0.700	556.38	530.76	1.42	25.53	0.67
90,027	36,172	11.7	30	9,772	4,467	0.0031	200	0.724	31	0.735	81	0.668	0.728	0.920	399.68	504.31	1.00	213.23	0.54
90,027	36,172	11.7	30	9,772	4,467	0.1000	200	0.724	31	0.735	81	0.682	0.728	0.630	350.60	494.10	1.16	10.21	0.51
117,249	43,798	6.8	34	8,584	3,637	0.0022	213	0.730	22	0.745	120	0.695	0.727	0.930	399.52	597.35	0.74	268.19	0.52
129,951	48,674	7.2	35	9,539	3,981	0.0043	174	0.764	16	0.785	56	0.684	0.743	0.930	510.99	560.23	0.71	84.64	0.56
80,075	29,321	7.8	27	11,715	4,429	0.0017	214	0.779	27	0.852	56	0.672	0.757	0.930	398.40	469.21	0.73	182.05	0.59
174,549	74,764	6.5	40	9,112	2,858	0.0929	229	0.784	33	0.822	55	0.715	0.763	0.800	460.30	665.15	0.75	4.78	0.55
182,575	79,810	15.8	40	11,499	5,424	0.0093	215	0.739	21	0.750	128	0.685	0.739	0.800	533.72	547.63	1.67	44.88	0.73
114,673	43,526	8.8	32	10,877	4,836	0.0040	175	0.770	26	0.800	124	0.680	0.759	0.800	621.04	418.53	0.81	67.90	0.65
76,691	29,149	6.2	28	9,206	3,870	0.0028	230	0.769	23	0.780	115	0.672	0.741	0.860	426.01	403.79	0.61	144.55	0.57
103,715	36,373	7.3	20	10,147	3,458	0.0075	229	0.780	19	0.858	81	0.695	0.777	0.810	573.13	493.57	0.60	36.05	0.55
21,352	8,575	9.7	19	10,207	4,260	0.0665	179	0.726	22	0.740	91	0.673	0.722	0.350	200.18	272.38	0.75	16.99	0.29
16,230	6,230	7.2	16	9,286	4,339	0.0075	174	0.791	27	0.839	131	0.726	0.750	0.620	186.98	268.76	0.49	85.74	0.28
167,242	65,536	6.4	38	10,946	4,045	0.0029	226	0.748	20	0.777	90	0.663	0.745	0.890	657.22	523.32	0.67	75.72	0.70
60,775	24,338	15.8	28	10,128	4,856	0.0088	204	0.790	34	0.841	68	0.728	0.810	0.840	347.92	378.09	1.33	95.76	0.57
121,578	51,226	13.8	34	12,006	5,988	0.0063	230	0.761	27	0.806	68	0.711	0.760	0.890	499.03	548.43	1.10	50.24	0.55
144,812	56,971	7.3	37	15,148	5,822	0.0654	219	0.759	24	0.777	131	0.685	0.725	0.310	579.29	406.14	0.94	2.80	0.52
87,209	32,744	16.0	30	12,676	5,495	0.0042	229	0.750	25	0.765	78	0.678	0.738	0.860	459.15	437.23	1.31	97.12	0.64
77,117	28,579	7.7	26	14,340	6,560	0.0062	215	0.791	18	0.826	84	0.727	0.778	0.780	495.62	418.42	0.61	15.90	0.52
63,845	24,662	8.3	24	12,421	5,319	0.0075	188	0.741	23	0.762	130	0.703	0.733	0.720	358.38	443.90	0.69	37.20	0.43
92,403	36,355	9.6	31	14,244	6,971	0.0106	189	0.767	32	0.786	134	0.729	0.757	0.700	439.84	496.57	0.81	15.11	0.48
67,870	27,288	10.8	26	11,924	5,046	0.0130	213	0.756	20	0.771	131	0.680	0.780	0.560	467.30	302.44	1.01	30.46	0.53
17,917	7,144	12.6	16	14,591	5,612	0.0045	197	0.758	13	0.770	122	0.701	0.732	0.590	218.77	250.94	0.75	57.95	0.32

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
198,609	70,236	8.3	40	14,206	5,218	0.0281	173	0.790	17	0.832	93	0.724	0.791	0.570	880.35	563.46	0.73	3.25	0.61
99,885	38,093	8.4	32	14,938	6,521	0.0173	182	0.767	35	0.795	117	0.687	0.751	0.540	622.92	372.91	0.75	6.66	0.55
188,074	81,062	10.5	23	15,170	6,375	0.0108	206	0.739	29	0.778	78	0.705	0.749	0.850	446.14	482.94	1.69	25.87	0.89
188,074	81,062	10.5	23	15,170	6,375	0.0108	206	0.739	29	0.778	78	0.670	0.730	0.770	864.14	556.66	0.76	6.85	0.60
105,104	41,496	9.9	34	13,801	6,120	0.0212	199	0.780	13	0.824	63	0.711	0.774	0.740	597.00	563.42	0.72	5.76	0.52
140,522	60,992	8.2	34	13,864	6,625	0.0059	227	0.733	27	0.757	73	0.670	0.718	0.870	617.28	535.88	0.67	18.48	0.55
14,895	6,502	10.9	16	13,109	6,232	0.0046	194	0.745	10	0.780	56	0.676	0.742	0.790	241.97	329.25	0.52	48.54	0.35
101,381	42,331	12.5	30	12,317	4,706	0.0069	238	0.770	18	0.811	70	0.709	0.765	0.840	496.35	492.55	0.95	37.72	0.54
37,526	13,886	8.5	28	14,131	6,254	0.0369	176	0.784	16	0.811	97	0.708	0.763	0.350	356.17	269.51	0.70	6.15	0.39
77,539	29,568	15.1	27	13,493	6,095	0.0184	232	0.792	16	0.795	97	0.712	0.767	0.720	473.89	355.01	1.30	35.30	0.62
16,463	6,241	7.1	17	13,804	6,221	0.0149	220	0.754	34	0.788	76	0.684	0.725	0.620	191.97	250.07	0.50	16.66	0.33
26,459	9,866	9.4	19	12,471	4,772	0.0039	216	0.746	23	0.772	117	0.655	0.718	0.670	315.66	235.42	0.68	82.13	0.42
164,467	70,312	7.9	38	13,423	5,370	0.0078	187	0.750	24	0.785	82	0.687	0.719	0.880	430.75	599.02	0.99	29.32	0.62
162,036	67,463	11.3	28	11,915	4,743	0.0159	222	0.746	32	0.782	100	0.680	0.726	0.750	574.96	554.58	1.07	19.27	0.63
33,060	12,905	8.0	18	13,145	7,203	0.0059	177	0.762	34	0.766	130	0.687	0.754	0.600	365.16	240.42	0.62	35.31	0.40
144,168	59,473	12.8	36	14,106	5,706	0.0147	217	0.757	33	0.782	92	0.686	0.768	0.660	714.25	424.42	1.12	12.16	0.66
117,352	47,589	8.0	32	13,436	6,096	0.0947	196	0.757	21	0.757	57	0.684	0.729	0.710	388.10	534.55	0.92	3.03	0.58
160,104	66,044	13.0	34	12,279	4,780	0.0083	238	0.736	31	0.755	105	0.668	0.719	0.780	614.55	536.80	1.16	35.80	0.65
84,687	30,246	6.1	20	12,188	5,933	0.0769	188	0.807	24	0.842	83	0.709	0.763	0.860	355.21	460.17	0.72	38.30	0.59
111,874	44,767	7.6	33	11,999	5,076	0.0143	180	0.753	15	0.763	87	0.661	0.770	0.710	618.55	354.07	0.80	16.47	0.67
17,186	6,148	9.9	15	12,882	6,553	0.0026	187	0.746	28	0.847	111	0.665	0.720	0.750	266.13	216.67	0.60	108.94	0.38
95,395	40,767	7.4	34	12,283	6,128	0.0082	179	0.730	10	0.766	132	0.695	0.727	0.800	386.99	516.02	0.70	29.91	0.47
183,786	71,066	12.8	40	11,980	4,343	0.0092	233	0.737	27	0.769	110	0.690	0.725	0.790	610.51	615.24	1.13	34.50	0.64
83,740	32,774	10.5	28	12,832	5,043	0.0075	213	0.800	11	0.835	99	0.736	0.806	0.722	497.10	411.50	0.85	26.76	0.55
187,802	73,241	7.9	40	14,368	6,209	0.0051	220	0.744	10	0.786	109	0.683	0.739	0.803	826.00	575.60	0.65	15.04	0.62
143,130	59,149	7.0	30	7,190	3,008	0.0026	223	0.756	18	0.762	57	0.688	0.737	0.960	449.48	605.50	0.76	192.71	0.55
190,339	69,496	10.5	40	7,715	3,286	0.0173	208	0.742	18	0.757	95	0.655	0.729	0.840	544.49	636.84	1.19	42.09	0.69
90,516	36,317	6.4	29	6,263	3,234	0.0466	225	0.800	27	0.837	102	0.720	0.783	0.730	361.16	494.77	0.69	17.84	0.50
115,178	50,068	10.7	17	6,455	2,434	0.0485	220	0.775	21	0.806	126	0.705	0.736	0.650	345.30	517.67	1.26	32.69	0.58
23,486	8,936	6.9	17	8,153	4,056	0.0392	204	0.747	33	0.763	108	0.700	0.755	0.510	207.13	306.03	0.54	19.95	0.31

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
72,178	31,305	8.5	26	8,385	4,000	0.0076	197	0.781	25	0.817	70	0.717	0.753	0.900	290.79	462.69	0.88	83.47	0.51
28,271	10,074	11.9	18	8,379	4,043	0.0196	200	0.738	11	0.760	55	0.678	0.742	0.800	235.33	345.12	0.85	51.25	0.42
145,050	53,324	6.8	35	7,998	3,420	0.0381	171	0.766	15	0.801	114	0.689	0.772	0.680	482.69	528.88	0.86	15.49	0.64
121,654	49,707	14.5	18	6,051	2,967	0.0385	178	0.778	28	0.813	61	0.721	0.763	0.850	368.94	547.70	1.58	50.91	0.63
193,889	68,500	6.4	40	6,712	2,660	0.0122	184	0.743	23	0.773	124	0.706	0.735	0.860	448.44	713.16	0.89	64.65	0.64
151,222	56,039	11.1	35	7,077	2,243	0.0096	237	0.765	29	0.786	91	0.699	0.764	0.890	417.14	596.33	1.37	117.28	0.70
101,406	40,871	15.3	15	7,580	2,095	0.0162	216	0.770	11	0.818	75	0.687	0.743	0.840	372.97	486.59	1.55	77.59	0.65
80,812	33,819	15.6	28	6,376	3,000	0.0314	227	0.742	28	0.758	115	0.707	0.760	0.690	286.35	467.30	1.62	50.81	0.60
158,087	58,326	9.1	27	5,998	2,344	0.0073	202	0.724	16	0.845	89	0.692	0.757	0.890	406.56	575.87	1.28	222.66	0.74
95,629	33,626	8.8	30	6,354	2,185	0.0135	206	0.761	28	0.785	107	0.684	0.756	0.800	385.65	508.90	0.94	88.60	0.57
74,410	28,380	10.3	27	6,926	2,797	0.0185	185	0.769	30	0.800	60	0.716	0.755	0.910	319.29	477.69	1.02	120.59	0.52
87,192	33,397	6.6	26	7,045	2,548	0.0463	230	0.780	34	0.825	98	0.696	0.751	0.650	366.92	478.97	0.72	16.15	0.50
53,715	19,751	7.8	15	7,473	3,562	0.0417	219	0.730	25	0.777	84	0.676	0.733	0.600	294.36	418.01	0.74	24.61	0.45
46,781	17,624	7.5	27	7,065	2,528	0.0313	238	0.739	13	0.800	79	0.671	0.738	0.620	268.06	391.29	0.72	36.33	0.43
63,974	25,440	7.0	25	6,546	2,523	0.0082	218	0.737	17	0.773	64	0.670	0.755	0.940	269.04	456.78	0.77	147.43	0.55
60,476	21,267	6.4	15	5,894	1,886	0.0302	223	0.757	32	0.787	96	0.670	0.743	0.770	326.93	433.68	0.63	38.16	0.50
52,204	19,080	7.7	25	6,095	2,807	0.0463	184	0.736	15	0.786	90	0.680	0.739	0.810	230.67	422.39	0.74	30.12	0.47
134,577	49,431	11.5	34	11,244	5,331	0.0181	173	0.755	18	0.778	128	0.684	0.736	0.680	554.10	506.55	1.11	23.84	0.64
85,455	30,269	9.2	28	10,422	4,626	0.0099	220	0.775	20	0.800	116	0.710	0.788	0.730	451.31	434.39	0.86	45.67	0.56
41,190	14,417	13.8	23	9,610	3,943	0.0103	219	0.741	12	0.773	79	0.689	0.748	0.790	272.87	375.54	1.12	116.12	0.49
120,297	52,477	8.5	33	10,886	4,741	0.0031	232	0.749	19	0.781	94	0.671	0.760	0.880	510.31	420.34	0.91	115.05	0.69
22,188	9,662	7.6	16	8,645	3,553	0.0072	232	0.738	20	0.750	58	0.671	0.720	0.830	221.19	307.68	0.43	80.42	0.33
74,119	30,297	12.7	28	11,409	5,000	0.0066	233	0.770	16	0.798	107	0.695	0.744	0.780	393.80	384.97	1.15	72.44	0.58
179,486	64,235	15.5	40	9,204	3,827	0.0393	187	0.761	20	0.784	132	0.663	0.727	0.620	557.78	530.68	1.83	32.15	0.80
161,567	59,776	10.1	39	10,908	4,737	0.0048	189	0.793	35	0.828	99	0.718	0.767	0.870	590.96	561.26	1.01	62.63	0.65
94,245	39,055	7.5	31	11,655	3,922	0.0100	178	0.756	27	0.763	52	0.702	0.766	0.890	349.74	556.24	0.73	32.35	0.49
51,250	17,978	11.0	24	8,469	4,045	0.0167	192	0.721	30	0.725	61	0.660	0.718	0.840	328.67	422.57	0.85	65.15	0.45
49,614	21,218	14.5	24	8,747	3,761	0.0500	209	0.769	26	0.821	138	0.715	0.766	0.440	253.83	366.05	1.36	39.45	0.46
65,926	24,260	14.3	25	10,119	3,288	0.0156	184	0.732	21	0.755	111	0.694	0.782	0.710	289.52	431.48	1.47	89.96	0.55
13,172	4,760	8.2	16	8,554	3,878	0.0093	211	0.771	14	0.799	56	0.703	0.741	0.790	181.06	265.52	0.49	82.84	0.29

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Final Proppant Concentration	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
26,443	10,717	7.1	18	8,619	3,514	0.0057	203	0.737	26	0.764	111	0.709	0.795	0.740	247.10	313.09	0.51	120.27	0.37
38,905	13,772	8.0	22	10,922	4,620	0.0022	213	0.756	30	0.780	122	0.693	0.748	0.820	323.04	358.58	0.60	164.49	0.44
108,731	43,810	16.0	29	10,665	4,610	0.0096	198	0.772	13	0.807	89	0.700	0.768	0.790	491.33	466.79	1.36	62.22	0.64
13,484	5,576	8.2	16	11,652	4,566	0.0014	209	0.752	29	0.794	105	0.683	0.732	0.780	223.27	215.94	0.46	189.27	0.32
188,001	69,195	6.4	40	9,448	4,284	0.0055	189	0.795	28	0.828	51	0.730	0.762	0.930	448.89	696.80	0.87	82.48	0.63
66,648	27,583	7.7	26	11,407	5,218	0.0033	224	0.769	25	0.794	91	0.696	0.739	0.880	318.81	433.18	0.74	101.44	0.52
72,811	26,280	12.7	25	11,666	5,503	0.0200	200	0.741	29	0.750	128	0.661	0.717	0.590	430.10	364.13	1.18	30.71	0.57
63,904	23,454	9.0	33	13,581	4,800	0.0092	214	0.744	27	0.784	65	0.691	0.750	0.780	441.30	434.80	0.63	14.54	0.45
131,847	54,885	15.2	33	12,633	5,383	0.0134	218	0.759	31	0.786	121	0.677	0.742	0.636	617.00	406.10	1.41	27.09	0.70
163,288	62,450	6.9	36	12,389	6,155	0.0279	180	0.764	17	0.764	105	0.716	0.762	0.712	521.80	604.20	0.77	7.90	0.59
125,329	45,593	14.3	33	14,288	5,619	0.0077	172	0.728	24	0.749	90	0.680	0.726	0.786	562.10	516.90	1.19	30.57	0.60
72,764	28,687	8.8	27	13,231	5,878	0.0245	236	0.759	14	0.798	51	0.730	0.767	0.814	296.40	363.10	1.21	19.52	0.72
85,069	34,904	10.9	26	13,737	6,087	0.0075	238	0.746	25	0.800	65	0.670	0.726	0.830	465.00	415.30	0.91	20.27	0.65
42,478	15,271	8.5	24	12,243	4,611	0.0173	187	0.741	10	0.767	113	0.687	0.732	0.562	312.90	354.20	0.70	22.71	0.41
60,306	26,086	14.5	29	14,526	5,297	0.0096	219	0.759	23	0.794	86	0.671	0.733	0.653	455.80	284.80	1.16	24.66	0.57
92,967	36,445	7.0	26	12,578	5,876	0.0085	233	0.735	30	0.761	124	0.674	0.710	0.740	467.00	447.20	0.66	24.19	0.51
97,897	42,612	12.0	31	14,708	5,366	0.0122	233	0.758	30	0.785	89	0.692	0.749	0.660	551.40	400.90	0.94	12.18	0.58
59,244	25,005	15.0	25	12,385	5,664	0.0098	183	0.808	14	0.845	73	0.734	0.792	0.770	376.99	354.24	1.16	37.77	0.55
16,862	6,769	10.2	15	12,168	5,152	0.0075	226	0.746	18	0.780	97	0.685	0.728	0.610	222.18	249.57	0.59	49.79	0.32
47,262	17,139	11.5	21	14,195	6,662	0.0195	191	0.764	25	0.785	66	0.691	0.741	0.710	325.15	357.92	0.94	14.21	0.49
158,009	58,547	6.1	36	13,559	5,917	0.0437	210	0.747	14	0.762	108	0.702	0.741	0.590	597.05	612.62	0.59	3.06	0.51
58,789	22,477	12.8	28	12,071	5,567	0.0094	175	0.761	12	0.789	77	0.692	0.756	0.790	399.85	390.38	0.94	38.47	0.52
132,726	53,099	10.4	35	13,772	5,370	0.0231	191	0.742	15	0.783	111	0.693	0.757	0.610	559.54	533.81	0.90	8.71	0.54
72,636	28,066	9.8	28	13,146	6,259	0.0034	236	0.793	12	0.827	64	0.725	0.802	0.880	453.95	417.29	0.76	52.39	0.56
78,995	30,523	6.4	23	15,116	6,122	0.0289	221	0.759	34	0.775	131	0.687	0.743	0.330	533.14	302.30	0.68	4.72	0.44
22,895	9,260	7.7	17	12,821	4,469	0.0058	199	0.781	21	0.811	121	0.711	0.757	0.560	259.65	251.29	0.54	40.51	0.36
67,775	26,579	12.1	27	14,417	5,777	0.0376	229	0.742	26	0.766	67	0.701	0.734	0.650	366.20	436.69	0.98	6.84	0.45
111,207	44,736	13.4	31	15,034	6,549	0.0198	230	0.768	12	0.812	78	0.710	0.779	0.710	446.59	382.34	1.61	14.99	0.77
125,772	55,148	9.6	35	12,195	5,620	0.0179	220	0.806	27	0.843	85	0.724	0.791	0.720	592.28	465.92	0.81	9.96	0.60
77,992	28,393	8.3	25	8,393	4,189	0.0082	192	0.727	24	0.845	125	0.698	0.781	0.770	437.00	401.91	0.80	59.02	0.59

Table A-1: Data Generated for Ramp Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
88,692	34,851	3,365	2.0	9.2	1.8	27	8,064	3,386	0.0077	219	0.770	16	0.791	129	0.673	0.745	0.79	395.54	454.47	1.24	177.47	0.64
191,270	76,844	7,152	1.0	7.0	1.5	40	6,420	2,666	0.0087	217	0.782	13	0.823	98	0.668	0.753	0.89	510.46	628.12	1.08	155.59	0.74
167,331	69,140	6,137	1.5	7.5	1.5	36	6,149	2,736	0.0448	224	0.738	25	0.771	105	0.65	0.721	0.92	430.25	600.22	1.24	413.36	0.73
101,823	42,426	3,712	2.0	9.6	1.9	30	6,846	3,004	0.0160	223	0.753	10	0.785	71	0.677	0.744	0.88	321.18	514.49	1.55	175.57	0.68
48,253	18,700	1,847	2.0	9.6	1.9	22	6,314	2,815	0.0602	170	0.760	16	0.799	91	0.681	0.740	0.64	255.92	391.63	1.27	62.60	0.52
27,236	11,440	987	1.5	7.5	1.5	17	6,661	2,403	0.0197	215	0.810	28	0.856	74	0.727	0.796	0.70	219.95	307.97	0.78	60.77	0.39
16,528	7,069	591	1.5	6.3	1.2	15	7,376	3,534	0.0072	235	0.766	12	0.774	50	0.67	0.732	0.86	192.80	261.17	0.53	100.78	0.33
27,760	10,520	1,077	1.5	9.5	2.0	20	5,422	2,100	0.0588	187	0.735	25	0.774	54	0.662	0.753	0.73	202.41	316.52	1.12	59.59	0.44
173,821	72,413	6,338	2.0	8.4	1.6	38	5,407	2,460	0.0202	201	0.788	31	0.820	134	0.691	0.777	0.81	422.28	591.88	1.55	107.65	0.76
104,788	43,737	3,816	2.0	9.6	1.9	32	8,004	4,036	0.0015	220	0.790	12	0.845	104	0.663	0.765	0.93	474.68	456.65	1.18	424.82	0.71
180,025	75,718	6,519	2.0	8.8	1.7	39	7,175	2,998	0.0713	223	0.745	23	0.808	73	0.663	0.723	0.78	454.46	627.18	1.46	17.10	0.69
59,386	25,650	2,108	1.0	9.8	2.2	25	8,168	3,832	0.0136	233	0.753	34	0.758	79	0.676	0.741	0.82	309.23	410.67	1.09	69.21	0.53
94,060	41,354	3,294	2.0	8.0	1.5	29	8,276	4,095	0.0428	217	0.803	24	0.852	123	0.698	0.799	0.57	423.87	404.53	1.12	17.89	0.61
108,578	46,831	3,859	1.5	7.5	1.5	24	7,672	3,797	0.0082	235	0.761	31	0.796	126	0.689	0.734	0.84	382.60	549.40	0.99	87.68	0.56
122,925	44,117	4,925	1.5	6.7	1.3	23	7,000	2,942	0.0442	234	0.750	30	0.782	71	0.673	0.743	0.83	362.34	534.51	1.23	40.39	0.71
139,728	53,214	5,407	1.5	8.7	1.8	34	6,663	3,171	0.0075	226	0.768	14	0.768	87	0.651	0.764	0.90	432.84	569.74	1.34	187.83	0.74
175,550	62,789	7,048	1.0	9.4	2.1	39	5,214	2,031	0.0877	230	0.768	34	0.814	66	0.697	0.753	0.83	376.89	660.49	1.81	34.27	0.79
90,824	38,781	3,253	1.0	7.0	1.5	30	7,310	3,340	0.0098	170	0.792	31	0.830	66	0.7	0.762	0.89	376.75	498.04	0.84	63.36	0.54
165,516	71,887	5,852	1.0	6.6	1.4	35	8,226	4,013	0.0221	239	0.763	35	0.793	88	0.68	0.728	0.84	457.29	649.41	0.90	22.45	0.60
154,202	65,967	5,515	1.0	6.6	1.4	36	7,605	3,608	0.0746	172	0.802	34	0.841	126	0.676	0.797	0.81	620.73	485.89	0.84	52.56	0.71
40,846	15,071	1,611	1.0	9.8	2.2	23	7,413	3,109	0.0213	215	0.765	28	0.813	51	0.656	0.738	0.82	290.88	362.92	0.66	36.20	0.45
43,430	17,781	1,603	1.5	7.1	1.4	20	7,409	3,498	0.0073	197	0.772	24	0.805	100	0.683	0.757	0.81	292.73	361.37	0.77	108.92	0.47
92,675	33,081	3,725	2.0	8.8	1.7	27	6,885	2,776	0.0576	184	0.760	35	0.807	71	0.667	0.766	0.73	350.41	431.68	1.56	36.45	0.72
94,157	33,027	3,821	2.0	10.8	2.2	30	7,714	3,022	0.0049	210	0.749	11	0.796	134	0.659	0.728	0.84	384.65	482.47	1.57	278.28	0.70
72,096	29,227	2,679	1.0	9.8	2.2	28	5,995	3,002	0.0015	236	0.768	33	0.768	112	0.694	0.750	0.94	301.18	470.75	1.25	266.97	0.58
72,096	29,227	2,679	1.0	9.8	2.2	28	5,995	3,002	0.0100	236	0.768	33	0.768	112	0.66	0.750	0.85	320.04	431.56	1.29	208.09	0.62
175,294	74,707	6,287	2.0	10.0	2.0	39	7,392	3,264	0.0062	205	0.775	11	0.776	127	0.686	0.778	0.87	448.87	598.01	1.67	184.08	0.77
72,688	29,095	2,725	2.0	6.8	1.2	26	6,433	3,058	0.0176	217	0.800	15	0.836	58	0.716	0.790	0.89	286.51	427.15	1.14	87.01	0.63
64,194	23,156	2,565	1.5	9.5	2.0	26	7,319	3,523	0.0072	174	0.754	18	0.770	77	0.668	0.721	0.89	318.03	443.69	1.20	177.33	0.57
86,684	30,752	3,496	1.5	6.7	1.3	28	8,313	4,038	0.0285	183	0.793	18	0.840	83	0.724	0.796	0.77	340.78	439.93	1.13	31.46	0.65

Table A - 2: Data Generated for Six Stage Treatment

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
127,892	50,786	4,819	2.0	6.0	1.0	34	6,270	3,018	0.0076	182	0.746	29	0.789	127	0.675	0.726	0.88	400.84	591.91	0.93	140.96	0.59
25,295	10,756	909	1.0	8.2	1.8	17	6,617	3,114	0.0055	228	0.760	27	0.760	51	0.669	0.763	0.90	209.17	305.31	0.73	223.70	0.40
51,639	18,462	2,074	2.0	8.4	1.6	24	6,393	2,977	0.0360	192	0.783	33	0.814	72	0.708	0.753	0.77	270.62	414.81	1.13	51.60	0.51
57,418	22,110	2,207	1.5	6.7	1.3	15	6,835	3,296	0.0091	229	0.741	18	0.795	88	0.663	0.724	0.86	303.81	432.14	0.81	117.81	0.50
192,068	67,787	7,768	2.0	8.4	1.6	40	7,410	3,512	0.0044	239	0.800	25	0.836	113	0.699	0.810	0.90	463.58	497.86	2.06	326.27	1.07
113,767	46,837	4,183	1.0	9.0	2.0	30	6,242	3,065	0.0421	216	0.757	26	0.796	55	0.658	0.786	0.87	373.39	558.88	1.23	38.21	0.62
167,327	61,179	6,634	1.5	9.5	2.0	37	8,265	3,887	0.0135	221	0.767	29	0.789	115	0.683	0.746	0.94	491.81	619.30	1.60	669.39	0.81
16,628	7,118	594	1.5	6.7	1.3	16	6,830	2,854	0.0075	230	0.736	29	0.770	59	0.668	0.718	0.84	182.98	274.71	0.57	136.62	0.31
186,129	68,403	7,358	2.0	7.6	1.4	38	7,928	3,874	0.0326	182	0.765	25	0.787	103	0.664	0.747	0.76	541.62	572.33	1.33	25.01	0.77
196,711	77,037	7,480	1.0	8.2	1.8	40	6,686	2,907	0.0094	204	0.750	11	0.806	116	0.68	0.732	0.88	459.02	642.34	1.42	140.29	0.77
46,697	19,200	1,719	2.0	8.0	1.5	21	5,217	2,791	0.0047	208	0.751	17	0.809	121	0.664	0.729	0.89	254.14	394.28	1.01	495.04	0.51
108,509	42,380	4,133	1.5	7.5	1.5	32	7,876	3,745	0.0707	236	0.772	19	0.817	60	0.694	0.753	0.79	373.31	528.02	1.12	12.71	0.61
147,603	54,413	5,824	2.0	8.8	1.7	36	5,931	3,009	0.0439	211	0.800	12	0.839	75	0.709	0.771	0.84	393.44	587.39	1.61	47.69	0.75
65,424	26,285	2,446	1.5	8.7	1.8	24	6,012	3,038	0.0168	191	0.820	16	0.859	100	0.729	0.822	0.79	311.68	431.90	1.11	97.50	0.56
78,561	30,989	2,973	1.5	7.5	1.5	24	5,930	1,983	0.0075	190	0.743	16	0.780	117	0.663	0.730	0.83	334.55	486.65	0.98	210.89	0.54
39,809	15,277	1,533	2.0	6.4	1.1	19	7,285	2,948	0.0109	214	0.780	17	0.819	108	0.662	0.760	0.71	297.54	317.69	0.79	87.83	0.50
54,769	23,039	1,983	1.5	8.3	1.7	26	7,716	2,027	0.0063	236	0.752	20	0.752	122	0.67	0.727	0.81	285.13	394.06	1.03	166.04	0.53
95,473	38,871	3,538	2.0	7.2	1.3	31	7,066	1,751	0.0229	214	0.766	19	0.793	78	0.684	0.737	0.82	360.49	517.69	1.01	42.36	0.56
95,473	38,871	3,538	1.5	7.5	1.5	31	7,066	1,751	0.0060	214	0.766	19	0.793	78	0.658	0.737	0.89	392.61	494.29	0.97	147.97	0.61
71,673	28,925	2,672	1.5	7.5	1.5	28	6,620	3,741	0.0529	206	0.794	12	0.838	96	0.729	0.775	0.74	300.61	473.01	1.00	23.52	0.51
120,936	44,271	4,792	1.5	9.1	1.9	34	5,436	2,760	0.0014	225	0.800	25	0.830	59	0.724	0.803	0.97	366.31	572.53	1.45	287.00	0.70
110,535	45,964	4,036	2.0	6.8	1.2	30	6,318	3,236	0.0077	178	0.800	29	0.800	116	0.711	0.780	0.87	384.21	537.65	1.00	134.14	0.61
75,819	28,867	2,934	1.0	6.2	1.3	26	7,474	2,959	0.0343	191	0.769	30	0.800	96	0.669	0.733	0.68	372.48	447.72	0.76	20.64	0.51
68,086	27,685	2,525	1.5	8.3	1.7	24	5,232	2,301	0.0081	233	0.782	14	0.827	51	0.661	0.774	0.93	283.35	413.69	1.26	328.64	0.66
68,086	27,685	2,525	1.5	8.3	1.7	24	5,232	2,301	0.0110	233	0.750	14	0.800	51	0.661	0.731	0.92	285.27	458.33	1.13	215.48	0.57
182,226	72,229	6,875	1.5	8.3	1.7	37	7,251	2,993	0.0082	220	0.768	33	0.789	65	0.692	0.742	0.93	437.93	635.63	1.45	124.91	0.73
70,960	25,832	2,821	2.0	8.8	1.7	30	5,116	2,554	0.0114	199	0.800	26	0.800	111	0.655	0.772	0.84	333.38	428.27	1.26	222.06	0.65
32,203	12,292	1,244	2.0	6.8	1.2	19	5,424	2,106	0.0044	177	0.774	11	0.813	69	0.708	0.754	0.90	223.03	371.05	0.77	400.52	0.41
75,563	29,370	2,887	2.0	8.0	1.5	26	6,837	2,871	0.0032	205	0.814	28	0.855	93	0.711	0.809	0.89	350.47	403.90	1.19	362.63	0.66
124,341	43,555	5,049	1.0	8.2	1.8	30	8,271	4,047	0.0117	212	0.740	23	0.777	116	0.679	0.733	0.83	406.04	561.65	1.24	78.22	0.66

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
133,685	48,297	5,337	2.0	9.6	1.9	34	7,329	3,497	0.0077	185	0.765	20	0.804	103	0.666	0.751	0.87	438.74	522.28	1.60	169.34	0.78
188,759	70,603	7,385	1.0	7.0	1.5	39	7,260	3,450	0.0078	208	0.780	30	0.794	63	0.691	0.758	0.93	471.27	639.95	1.18	101.86	0.75
128,414	50,907	4,844	1.5	7.1	1.4	31	6,908	3,396	0.0904	198	0.755	30	0.778	93	0.666	0.736	0.68	435.80	559.33	1.01	11.49	0.59
132,994	57,705	4,706	2.0	10.8	2.2	32	6,570	2,521	0.0549	240	0.786	29	0.827	116	0.681	0.751	0.64	399.90	503.54	1.79	38.08	0.72
30,147	12,067	1,130	1.5	7.5	1.5	18	7,186	3,517	0.0024	204	0.734	17	0.734	64	0.65	0.734	0.93	237.38	352.58	0.72	439.22	0.44
43,431	19,060	1,523	1.5	8.7	1.8	25	7,993	3,677	0.0448	236	0.759	19	0.799	121	0.676	0.738	0.54	250.88	342.41	1.08	29.68	0.46
167,067	69,721	6,084	1.0	9.0	2.0	37	7,401	3,103	0.0194	186	0.786	11	0.831	88	0.655	0.767	0.80	554.70	537.14	1.25	40.30	0.76
146,938	57,198	5,609	1.0	9.0	2.0	33	5,299	2,517	0.0080	170	0.743	16	0.782	98	0.652	0.723	0.91	401.75	587.15	1.45	282.61	0.72
102,382	38,468	3,995	2.0	6.0	1.0	27	7,110	2,917	0.0346	223	0.755	32	0.800	99	0.689	0.762	0.67	352.02	484.22	1.07	22.14	0.61
109,815	47,791	3,876	1.0	8.6	1.9	30	7,727	3,041	0.0031	232	0.754	27	0.769	123	0.671	0.737	0.88	400.83	526.27	1.08	241.98	0.62
11,435	4,795	415	1.0	5.8	1.2	15	7,652	2,632	0.0049	207	0.733	18	0.762	113	0.651	0.714	0.67	155.30	207.51	0.46	153.42	0.27
60,058	22,146	2,370	1.5	6.7	1.3	27	6,972	2,893	0.0058	236	0.792	17	0.792	63	0.691	0.768	0.90	317.30	437.69	0.83	152.50	0.54
121,250	48,410	4,553	1.0	8.6	1.9	33	7,605	3,541	0.0182	193	0.753	18	0.778	78	0.675	0.737	0.86	389.58	533.31	1.28	56.07	0.67
87,407	33,169	3,390	2.0	9.2	1.8	28	6,924	2,712	0.0244	238	0.761	21	0.786	73	0.65	0.743	0.80	369.17	459.30	1.32	64.87	0.66
198,340	74,682	7,729	2.0	10.0	2.0	40	5,371	2,418	0.0061	177	0.744	32	0.768	105	0.654	0.728	0.93	422.51	670.37	1.94	475.58	0.83
91,474	37,990	3,343	1.5	7.5	1.5	29	7,685	2,929	0.0017	229	0.812	24	0.850	74	0.719	0.830	0.94	354.37	401.72	1.26	545.36	0.76
91,474	37,990	3,343	1.5	7.5	1.5	29	7,685	2,929	0.0017	229	0.770	24	0.770	74	0.673	0.760	0.94	372.24	498.09	0.97	428.30	0.60
171,302	70,674	6,289	1.5	6.7	1.3	40	5,393	2,588	0.0212	197	0.753	15	0.797	121	0.69	0.736	0.85	383.46	631.82	1.26	88.39	0.70
185,771	69,343	7,277	1.0	6.6	1.4	40	6,221	2,257	0.0015	189	0.745	25	0.850	121	0.675	0.770	0.93	571.75	637.49	0.91	468.21	0.75
172,291	64,630	6,729	2.0	7.2	1.3	40	8,336	3,700	0.0490	198	0.772	28	0.798	68	0.67	0.771	0.91	493.36	482.82	1.50	170.74	0.91
12,951	5,150	488	1.0	6.2	1.3	15	5,485	2,331	0.0088	235	0.776	19	0.793	58	0.682	0.746	0.82	157.69	237.49	0.50	173.90	0.30
38,916	16,388	1,408	1.0	7.4	1.6	20	7,236	3,000	0.0206	221	0.790	20	0.827	135	0.694	0.741	0.60	244.62	345.63	0.84	55.71	0.43
124,069	50,252	4,614	2.0	7.6	1.4	20	7,653	3,211	0.0067	208	0.757	20	0.783	72	0.675	0.752	0.91	832.63	489.83	1.38	161.95	0.77
75,636	30,988	2,790	2.0	8.0	1.5	27	5,654	2,439	0.0159	204	0.800	13	0.816	115	0.71	0.784	0.79	313.62	464.00	1.12	117.63	0.56
72,183	29,467	2,670	1.5	7.1	1.4	26	7,425	2,664	0.0052	235	0.762	29	0.781	96	0.65	0.742	0.84	390.71	416.40	0.79	127.81	0.57
78,096	27,672	3,152	1.0	7.4	1.6	28	6,526	3,066	0.0914	222	0.743	28	0.771	74	0.66	0.725	0.72	340.53	487.29	0.97	16.40	0.53
116,223	44,374	4,491	1.5	9.5	2.0	32	7,874	3,645	0.0059	180	0.762	14	0.783	91	0.659	0.762	0.88	421.28	444.14	1.59	197.57	0.81
63,572	23,744	2,489	2.0	8.4	1.6	24	5,055	2,507	0.0065	221	0.777	14	0.804	87	0.674	0.757	0.90	302.78	446.72	1.12	364.33	0.57
22,408	7,862	909	2.0	6.4	1.1	17	6,124	2,993	0.0210	228	0.747	25	0.782	115	0.671	0.740	0.65	199.08	308.26	0.68	85.97	0.36
172,553	67,458	6,568	2.0	10.4	2.1	38	6,342	2,846	0.0128	195	0.769	19	0.811	91	0.689	0.744	0.89	413.62	617.31	1.90	162.21	0.79

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
156,511	63,727	5,799	1.0	7.4	1.6	35	7,974	4,003	0.0130	188	0.764	12	0.784	68	0.677	0.756	0.90	437.74	517.89	1.30	64.05	0.79
57,828	24,294	2,096	1.5	6.7	1.3	24	5,025	2,031	0.0297	187	0.751	18	0.785	122	0.677	0.760	0.70	270.00	432.06	0.87	72.00	0.47
104,355	44,047	3,769	2.0	8.8	1.7	30	6,739	2,964	0.0061	187	0.761	28	0.769	104	0.653	0.756	0.87	388.36	479.77	1.29	226.57	0.69
16,438	6,875	598	1.5	9.9	2.1	16	8,212	3,369	0.0333	234	0.750	33	0.755	81	0.68	0.739	0.52	171.58	240.46	0.96	46.40	0.34
61,164	22,966	2,387	1.5	6.3	1.2	30	7,621	3,811	0.0219	205	0.774	33	0.811	98	0.681	0.760	0.84	359.49	419.38	0.72	75.48	0.51
29,610	11,197	1,151	1.0	7.8	1.7	19	5,786	2,486	0.0083	186	0.751	17	0.784	132	0.676	0.719	0.76	205.36	341.28	0.88	268.86	0.42
153,157	60,133	5,814	1.5	7.5	1.5	34	7,706	3,252	0.0456	199	0.732	35	0.775	118	0.667	0.758	0.69	401.84	511.60	1.51	29.40	0.78
141,748	54,604	5,447	2.0	10.0	2.0	34	6,927	2,938	0.0205	237	0.780	22	0.813	98	0.705	0.748	0.83	391.20	598.64	1.66	74.20	0.70
141,748	54,604	5,447	2.0	10.0	2.0	34	6,927	2,938	0.0089	237	0.750	22	0.813	98	0.67	0.748	0.88	386.41	533.87	1.88	221.67	0.82
193,646	75,250	7,400	1.5	9.1	1.9	40	8,146	4,085	0.0638	230	0.810	27	0.848	89	0.705	0.789	0.81	549.47	604.94	1.42	30.06	0.77
49,113	18,130	1,936	1.5	5.9	1.1	23	5,822	2,382	0.0088	196	0.760	11	0.760	90	0.661	0.755	0.84	281.13	399.51	1.10	252.47	0.54
107,314	42,306	4,063	1.5	7.1	1.4	32	5,610	2,094	0.0685	184	0.744	15	0.807	133	0.678	0.755	0.62	342.96	535.76	1.12	31.88	0.56
90,013	38,842	3,198	1.5	7.9	1.6	18	6,364	3,120	0.0113	205	0.727	13	0.751	70	0.65	0.715	0.97	317.47	501.44	1.13	142.68	0.59
16,987	6,292	668	2.0	8.0	1.5	17	7,095	2,653	0.0038	216	0.749	32	0.758	72	0.674	0.747	0.84	201.45	275.94	0.70	308.93	0.36
30,029	11,541	1,155	1.5	8.3	1.7	18	6,721	3,218	0.0079	220	0.762	13	0.755	129	0.726	0.762	0.76	224.79	333.12	0.90	210.69	0.44
108,394	46,597	3,862	1.0	8.2	1.8	21	7,004	3,054	0.0052	183	0.794	24	0.838	105	0.726	0.801	0.89	339.96	469.10	1.35	228.95	0.71
108,394	46,597	3,862	1.0	8.2	1.8	21	7,004	3,054	0.0700	183	0.794	24	0.838	105	0.726	0.801	0.87	382.01	446.93	1.26	202.28	0.72
80,953	30,503	3,153	1.5	6.3	1.2	26	7,441	2,820	0.0404	176	0.766	25	0.816	111	0.661	0.745	0.59	404.91	439.69	0.81	19.48	0.53
73,745	26,926	2,926	1.0	7.8	1.7	25	7,222	3,332	0.0094	225	0.763	27	0.796	125	0.69	0.746	0.80	338.78	475.94	0.97	103.30	0.54
44,310	17,914	1,650	2.0	8.0	1.5	21	8,009	4,045	0.0226	231	0.776	31	0.770	70	0.675	0.755	0.77	288.62	353.30	0.94	40.56	0.51
194,045	75,332	7,420	2.0	8.8	1.7	20	7,434	3,678	0.0100	227	0.786	28	0.793	85	0.676	0.756	0.90	452.50	563.00	1.85	131.44	0.91
123,330	46,910	4,776	2.0	7.6	1.4	28	5,730	2,203	0.0070	189	0.788	30	0.854	132	0.702	0.790	0.84	388.24	538.82	1.33	265.88	0.68
136,317	56,542	4,986	1.5	7.5	1.5	30	8,272	3,687	0.0184	187	0.781	11	0.782	120	0.667	0.750	0.71	538.05	495.89	1.00	27.27	0.67
38,769	16,387	1,399	1.5	8.7	1.8	19	6,263	2,886	0.0050	204	0.784	23	0.848	52	0.686	0.739	0.93	223.47	373.44	1.03	361.05	0.46
91,395	39,453	3,246	1.5	9.1	1.9	27	5,786	2,059	0.0067	208	0.767	15	0.789	94	0.685	0.757	0.88	321.96	502.15	1.28	297.09	0.62
112,541	45,228	4,207	1.0	9.8	2.2	29	6,348	2,739	0.0379	179	0.757	30	0.787	89	0.675	0.747	0.77	372.39	523.09	1.43	50.40	0.65
172,148	61,618	6,908	1.0	7.8	1.7	36	8,047	3,448	0.0112	207	0.774	26	0.801	131	0.67	0.766	0.78	577.43	559.86	1.14	56.83	0.75
119,806	47,120	4,543	1.5	8.3	1.7	30	5,330	2,636	0.0098	234	0.736	27	0.770	77	0.65	0.716	0.92	357.95	553.64	1.34	237.34	0.68
168,472	70,442	6,127	2.0	6.4	1.1	38	8,373	4,261	0.0603	239	0.783	25	0.825	64	0.705	0.771	0.82	420.00	553.14	1.32	13.17	0.76
136,695	51,597	5,319	1.5	9.1	1.9	32	7,666	3,188	0.0381	223	0.764	28	0.805	73	0.658	0.734	0.78	493.86	577.30	1.00	17.90	0.62

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
83,269	33,529	3,109	1.5	7.5	1.5	28	7,717	3,370	0.0085	199	0.750	19	0.796	124	0.685	0.729	0.71	346.52	488.14	0.98	95.85	0.54
83,269	33,529	3,109	1.5	7.5	1.5	28	7,717	3,370	0.0200	199	0.750	19	0.796	124	0.665	0.729	0.79	374.86	457.00	0.97	77.49	0.57
183,199	78,135	6,567	2.0	9.2	1.8	40	6,024	2,933	0.0221	218	0.776	32	0.809	132	0.686	0.756	0.81	464.26	620.39	1.51	74.86	0.73
113,140	49,061	4,005	1.0	7.0	1.5	32	7,056	2,941	0.0797	187	0.745	25	0.820	78	0.657	0.733	0.70	413.19	525.73	0.89	11.10	0.55
56,758	24,942	1,988	2.0	9.6	1.9	18	6,601	2,918	0.0655	178	0.810	24	0.833	77	0.697	0.791	0.61	307.41	376.09	1.18	25.41	0.52
119,927	44,611	4,707	1.0	7.4	1.6	29	7,190	3,117	0.0474	211	0.750	14	0.783	103	0.674	0.729	0.70	411.53	576.27	1.01	20.56	0.57
40,213	14,121	1,631	1.5	9.5	2.0	24	6,366	3,174	0.0043	222	0.752	28	0.796	57	0.664	0.751	0.93	271.94	377.39	1.05	427.90	0.51
22,122	8,546	849	1.0	7.8	1.7	17	8,008	3,348	0.0634	188	0.797	18	0.852	79	0.71	0.770	0.43	207.09	274.27	0.80	19.40	0.34
13,316	4,793	533	1.0	5.8	1.2	15	6,664	2,509	0.0034	197	0.758	29	0.806	96	0.674	0.739	0.78	175.30	234.59	0.46	283.51	0.30
195,925	83,815	7,007	2.0	9.6	1.9	40	7,812	2,866	0.0304	170	0.770	31	0.772	79	0.685	0.766	0.81	461.88	632.39	1.65	34.98	0.74
195,925	83,815	7,007	2.0	9.6	1.9	40	7,812	2,866	0.0090	170	0.770	31	0.772	79	0.66	0.750	0.88	528.60	592.83	1.54	94.22	0.79
51,267	20,969	1,894	1.0	9.0	2.0	22	8,130	3,586	0.0081	185	0.769	26	0.791	110	0.672	0.736	0.78	307.38	360.49	1.04	113.29	0.54
115,970	46,968	4,313	2.0	9.6	1.9	31	8,345	3,694	0.0085	220	0.815	27	0.855	82	0.718	0.802	0.85	423.34	483.69	1.44	97.26	0.72
25,602	9,867	983	1.5	6.3	1.2	17	6,485	2,725	0.0077	228	0.781	20	0.827	123	0.702	0.769	0.72	214.07	313.38	0.67	160.54	0.38
125,788	53,138	4,541	1.5	7.1	1.4	32	8,032	4,000	0.0674	177	0.758	32	0.780	62	0.672	0.746	0.85	408.99	493.60	1.14	28.46	0.68
167,240	63,085	6,510	1.0	7.4	1.6	37	7,992	3,826	0.0091	193	0.808	13	0.851	90	0.705	0.816	0.87	470.92	464.01	1.51	98.86	0.94
165,864	65,147	6,295	2.0	10.0	2.0	36	6,238	2,678	0.0924	203	0.757	27	0.757	97	0.681	0.733	0.70	426.12	616.80	1.71	23.81	0.71
23,392	8,354	940	1.0	8.6	1.9	18	7,439	3,209	0.0858	180	0.780	22	0.829	93	0.701	0.775	0.78	229.05	298.26	0.79	181.74	0.41
56,043	23,564	2,030	2.0	6.4	1.1	23	6,027	2,390	0.0141	227	0.752	24	0.758	86	0.671	0.751	0.82	280.71	421.86	0.83	104.43	0.50
193,972	75,701	7,392	2.0	10.0	2.0	36	7,747	3,384	0.0010	223	0.769	30	0.780	115	0.666	0.737	0.84	546.36	633.61	1.52	87.16	0.77
65,720	24,467	2,578	2.0	10.0	2.0	28	7,247	3,247	0.0924	222	0.752	25	0.831	68	0.652	0.755	0.88	334.35	399.66	1.37	172.05	0.64
151,075	66,255	5,301	1.5	8.3	1.7	35	6,878	2,124	0.0923	203	0.800	26	0.800	96	0.728	0.800	0.63	434.00	548.57	1.30	11.58	0.66
80,822	33,791	2,939	1.5	8.7	1.8	28	6,584	3,085	0.0072	210	0.781	33	0.822	95	0.708	0.774	0.88	330.06	280.09	1.13	177.56	0.57
76,245	30,990	2,828	2.0	9.2	1.8	26	5,931	2,488	0.0403	202	0.816	23	0.857	116	0.715	0.808	0.83	335.30	449.95	1.24	242.67	0.60
156,790	58,927	6,116	1.5	7.5	1.5	36	5,969	2,619	0.0404	203	0.775	19	0.797	66	0.68	0.767	0.85	407.95	630.52	1.27	41.54	0.68
128,118	54,073	4,628	2.0	8.0	1.5	32	5,819	2,207	0.0084	203	0.753	14	0.773	131	0.659	0.730	0.84	397.59	561.63	1.21	193.72	0.64
173,776	74,112	6,229	2.0	9.2	1.8	37	5,277	2,053	0.0258	206	0.740	14	0.816	66	0.681	0.766	0.89	384.72	626.63	1.71	101.34	0.74
170,423	65,942	6,530	1.5	8.7	1.8	38	5,952	3,040	0.0462	239	0.800	33	0.858	109	0.716	0.806	0.90	432.00	579.46	1.59	280.77	0.82
40,758	14,899	1,616	1.0	6.2	1.3	22	6,312	3,040	0.0095	188	0.723	22	0.742	134	0.65	0.714	0.79	256.34	389.98	0.70	145.18	0.44
73,420	31,104	2,645	2.0	8.8	1.7	30	5,045	2,445	0.0192	194	0.763	19	0.799	77	0.685	0.745	0.91	288.43	469.30	1.24	277.74	0.58

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
73,420	31,104	2,645	2.0	8.8	1.7	30	5,045	2,445	0.0400	194	0.763	19	0.799	77	0.65	0.745	0.82	307.10	446.08	1.23	65.76	0.59
99,827	43,104	3,545	2.0	8.4	1.6	29	8,016	4,061	0.0034	224	0.748	12	0.749	100	0.666	0.732	0.92	371.16	492.04	1.18	246.72	0.65
24,563	9,068	968	1.0	6.6	1.4	18	5,838	2,804	0.0109	205	0.778	33	0.805	121	0.664	0.751	0.72	219.33	387.03	0.69	152.90	0.40
29,011	10,831	1,136	2.0	6.4	1.1	17	7,463	3,211	0.0436	208	0.750	15	0.781	106	0.651	0.736	0.48	243.38	304.04	0.70	27.44	0.38
45,485	18,843	1,665	1.5	6.7	1.3	22	8,168	3,370	0.0062	234	0.804	16	0.850	75	0.686	0.809	0.82	330.99	311.47	0.77	96.46	0.55
118,445	50,784	4,229	1.5	9.5	2.0	31	6,147	2,134	0.0775	237	0.765	30	0.794	74	0.672	0.744	0.73	373.14	539.54	1.39	24.17	0.62
83,545	31,755	3,237	1.5	9.5	2.0	27	5,398	2,439	0.0383	208	0.753	19	0.781	115	0.651	0.725	0.73	323.49	475.64	1.39	77.31	0.62
19,623	8,309	707	1.0	6.6	1.4	16	6,334	2,654	0.0915	179	0.795	28	0.832	103	0.674	0.774	0.27	175.87	241.62	0.67	24.32	0.28
54,204	21,647	2,035	1.5	7.5	1.5	26	8,132	3,385	0.0075	234	0.774	22	0.808	128	0.705	0.786	0.77	306.46	397.35	0.89	102.08	0.49
17,043	7,043	625	1.0	7.0	1.5	15	6,874	2,809	0.0062	220	0.749	13	0.770	111	0.677	0.733	0.73	166.04	250.38	0.64	205.28	0.33
29,981	11,408	1,161	2.0	10.0	2.0	19	5,419	2,757	0.0758	182	0.746	11	0.788	105	0.671	0.727	0.58	189.92	332.43	1.31	61.92	0.45
28,479	11,364	1,070	2.0	7.2	1.3	17	7,626	3,730	0.0099	222	0.793	11	0.828	98	0.707	0.760	0.75	230.79	309.33	0.79	95.09	0.42
26,935	10,354	1,036	1.5	7.5	1.5	18	7,916	2,210	0.0276	173	0.764	31	0.765	80	0.657	0.731	0.62	241.67	282.74	0.80	38.33	0.42
12,118	4,815	456	1.0	9.0	2.0	15	6,541	2,605	0.0049	187	0.725	21	0.736	84	0.657	0.717	0.80	148.18	223.92	0.75	392.73	0.32
40,314	15,344	1,561	1.5	7.5	1.5	30	6,412	2,965	0.0137	186	0.756	30	0.807	85	0.674	0.752	0.80	273.09	373.46	0.82	102.21	0.46
67,309	23,720	2,724	1.5	8.3	1.7	25	8,021	3,318	0.0442	218	0.809	30	0.817	62	0.688	0.810	0.70	354.87	393.41	1.14	23.64	0.63
74,247	27,644	2,913	2.0	6.8	1.2	27	5,971	2,514	0.0197	176	0.752	13	0.780	133	0.676	0.756	0.74	323.46	476.70	0.96	84.89	0.53
77,596	29,415	3,011	1.0	8.2	1.8	28	6,561	2,489	0.0087	179	0.763	12	0.794	60	0.665	0.724	0.91	331.56	489.42	1.04	153.64	0.56
74,783	31,752	2,689	1.5	8.7	1.8	26	7,939	3,368	0.0901	230	0.789	29	0.820	88	0.677	0.740	0.53	361.95	396.10	1.14	12.06	0.56
74,783	31,752	2,689	1.5	8.7	1.8	26	7,939	3,368	0.0100	230	0.770	29	0.800	88	0.7	0.760	0.83	332.82	461.08	1.07	86.67	0.55
37,409	14,262	1,447	2.0	10.0	2.0	20	6,580	3,103	0.0081	223	0.742	25	0.754	71	0.664	0.720	0.87	241.84	364.50	1.17	262.48	0.50
182,362	74,303	6,754	2.0	8.8	1.7	38	6,110	2,801	0.0076	178	0.753	22	0.782	86	0.673	0.754	0.92	414.71	660.74	1.57	241.17	0.71
12,037	4,549	468	1.0	5.8	1.2	15	6,004	2,351	0.0074	224	0.778	35	0.815	50	0.699	0.744	0.84	154.83	233.64	0.46	156.52	0.28
176,221	65,616	6,913	2.0	6.0	1.0	38	6,181	2,819	0.0269	174	0.769	12	0.788	132	0.65	0.735	0.77	510.13	617.37	1.01	39.30	0.68
103,796	40,077	3,982	1.0	8.2	1.8	30	5,976	2,589	0.0080	203	0.784	31	0.785	99	0.683	0.770	0.87	382.27	526.50	1.11	183.34	0.64
130,770	50,270	5,031	2.0	7.6	1.4	33	6,698	3,006	0.0283	173	0.761	12	0.805	73	0.69	0.740	0.85	370.49	555.72	1.37	53.04	0.69
92,719	40,469	3,266	1.0	8.6	1.9	30	5,343	2,446	0.0050	222	0.799	16	0.829	120	0.693	0.800	0.89	340.31	472.48	1.19	376.67	0.65
54,421	19,628	2,175	1.0	6.6	1.4	22	6,739	2,925	0.0089	170	0.748	34	0.788	109	0.654	0.720	0.80	323.63	417.65	0.74	114.95	0.49
33,924	13,547	1,274	1.0	5.4	1.1	17	7,482	3,284	0.0367	174	0.750	23	0.774	97	0.657	0.738	0.56	276.80	324.87	0.46	16.71	0.38
143,695	55,173	5,533	2.0	6.8	1.2	35	7,794	1,854	0.0099	176	0.792	19	0.824	64	0.671	0.768	0.90	508.65	510.95	1.08	62.62	0.72

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
190,722	75,303	7,214	2.0	10.0	2.0	40	7,904	3,130	0.0172	221	0.819	31	0.849	77	0.652	0.805	0.84	499.38	549.19	1.87	62.05	0.89
102,916	37,532	4,087	1.5	7.5	1.5	30	6,382	2,678	0.0074	208	0.767	16	0.808	83	0.686	0.774	0.90	341.35	532.83	1.20	217.63	0.64
102,916	37,532	4,087	1.5	7.5	1.5	30	6,382	2,678	0.0074	208	0.767	16	0.808	83	0.658	0.750	0.89	391.92	495.43	1.13	189.30	0.67
60,037	25,382	2,166	1.0	8.2	1.8	23	6,742	2,947	0.0087	231	0.770	27	0.771	126	0.668	0.755	0.79	306.90	391.64	1.01	148.81	0.56
17,300	6,490	676	1.0	5.8	1.2	15	6,943	2,500	0.0123	171	0.750	25	0.764	91	0.69	0.723	0.67	175.74	252.31	0.54	91.76	0.32
94,185	36,819	3,585	1.5	9.5	2.0	30	6,047	2,523	0.0048	195	0.770	23	0.799	116	0.682	0.767	0.88	346.59	489.83	1.40	419.02	0.67
184,760	72,680	7,005	1.5	8.3	1.7	21	7,451	3,157	0.0054	187	0.738	18	0.761	106	0.65	0.716	0.90	481.12	610.75	1.39	181.20	0.77
80,585	31,636	3,059	2.0	8.4	1.6	28	7,108	2,793	0.0645	215	0.765	32	0.782	108	0.671	0.746	0.58	342.85	441.89	1.23	24.15	0.57
19,227	7,247	749	2.0	6.8	1.2	17	6,802	2,496	0.0926	193	0.755	31	0.755	85	0.683	0.722	0.40	167.75	285.93	0.80	24.26	0.30
158,466	61,272	6,075	1.5	6.3	1.2	36	6,822	2,621	0.0186	183	0.763	34	0.784	111	0.667	0.740	0.79	489.12	599.36	0.95	44.02	0.65
175,812	76,924	6,181	1.5	7.9	1.6	39	5,899	2,268	0.0091	224	0.784	26	0.817	61	0.675	0.758	0.93	422.37	582.24	1.41	174.33	0.76
49,123	21,449	1,730	1.0	6.6	1.4	23	6,620	3,011	0.0517	175	0.759	15	0.791	106	0.675	0.764	0.60	279.11	383.63	0.74	23.15	0.43
125,620	46,814	4,925	2.0	7.6	1.4	35	7,408	3,585	0.0546	228	0.784	20	0.798	63	0.665	0.760	0.80	419.46	500.30	1.31	20.24	0.74
103,795	45,303	3,656	1.5	9.1	1.9	31	7,482	3,826	0.0133	227	0.772	23	0.799	115	0.676	0.756	0.81	400.58	493.74	1.17	66.24	0.62
175,513	61,468	7,128	1.0	7.8	1.7	40	7,392	2,985	0.0558	203	0.811	32	0.851	115	0.714	0.793	0.64	513.16	600.34	1.07	13.52	0.65
30,719	13,344	1,086	1.0	7.8	1.7	24	6,246	2,416	0.0706	211	0.767	20	0.797	113	0.685	0.730	0.75	207.38	318.09	0.88	161.94	0.42
65,662	28,008	2,353	2.0	9.2	1.8	28	5,505	2,513	0.0082	188	0.797	22	0.822	84	0.723	0.788	0.89	279.10	447.32	1.25	271.64	0.57
65,662	28,008	2,353	2.0	9.2	1.8	28	5,505	2,513	0.0082	188	0.797	22	0.822	84	0.69	0.788	0.88	304.79	422.80	1.21	256.28	0.59
140,431	61,570	4,929	1.5	5.9	1.1	35	5,842	2,813	0.0715	193	0.779	31	0.779	127	0.682	0.779	0.66	406.83	559.12	0.94	18.10	0.61
195,170	74,115	7,566	1.0	6.6	1.4	29	6,855	2,941	0.0481	193	0.758	32	0.791	89	0.671	0.747	0.86	468.39	601.68	1.23	59.86	0.79
61,918	22,340	2,474	1.5	8.3	1.7	27	7,555	2,998	0.0750	211	0.764	12	0.781	51	0.656	0.727	0.91	326.66	448.35	0.98	125.72	0.55
104,873	44,650	3,764	2.0	7.6	1.4	30	5,537	2,209	0.0423	207	0.817	30	0.817	135	0.684	0.800	0.62	390.89	469.38	1.15	42.93	0.62
98,393	34,991	3,963	1.0	6.2	1.3	29	8,744	4,155	0.0066	201	0.752	18	0.780	56	0.683	0.758	0.93	363.25	600.10	0.79	176.43	0.50
17,692	7,655	627	1.5	9.9	2.1	15	9,481	3,739	0.0095	202	0.790	15	0.852	58	0.73	0.782	0.75	196.85	265.41	0.75	86.75	0.34
76,983	32,181	2,800	2.0	10.0	2.0	26	11,754	5,377	0.0081	205	0.755	14	0.804	99	0.676	0.723	0.78	396.46	425.44	1.17	56.21	0.58
106,730	39,066	4,229	1.0	7.0	1.5	30	9,789	3,776	0.0315	216	0.799	13	0.799	51	0.695	0.776	0.80	453.45	491.91	0.92	14.32	0.64
174,450	68,218	6,640	2.0	8.4	1.6	39	9,818	3,832	0.0097	224	0.739	26	0.786	134	0.669	0.718	0.79	535.73	647.56	1.17	64.99	0.66
115,180	46,996	4,262	1.5	7.5	1.5	32	10,743	4,674	0.0248	224	0.740	30	0.740	98	0.654	0.729	0.70	487.21	470.12	0.99	20.88	0.64
40,712	15,377	1,583	2.0	10.0	2.0	22	9,501	3,532	0.0099	174	0.741	31	0.779	107	0.678	0.746	0.70	287.81	351.72	1.11	120.75	0.49
79,981	33,003	2,936	1.0	7.0	1.5	28	11,274	4,961	0.0487	226	0.748	18	0.799	56	0.688	0.740	0.75	352.92	456.68	0.88	9.78	0.53

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
169,657	69,956	6,231	1.5	9.5	2.0	38	10,776	4,427	0.0088	193	0.761	32	0.800	84	0.703	0.755	0.86	471.56	551.20	1.61	77.41	0.76
169,657	69,956	6,231	1.5	9.5	2.0	38	10,776	4,427	0.0150	193	0.761	32	0.786	84	0.68	0.750	0.80	564.15	533.71	1.39	36.59	0.74
155,303	63,736	5,723	1.0	7.0	1.5	35	10,892	3,859	0.0698	202	0.810	27	0.833	75	0.723	0.799	0.57	579.04	525.53	0.91	4.76	0.62
107,413	37,779	4,352	1.0	6.6	1.4	29	11,788	5,208	0.0174	195	0.800	28	0.839	97	0.739	0.785	0.70	514.06	553.67	0.70	12.51	0.51
190,522	67,270	7,703	1.5	7.5	1.5	40	8,847	4,372	0.0046	230	0.763	34	0.793	85	0.67	0.735	0.92	564.56	632.38	1.16	147.57	0.73
51,669	22,597	1,817	1.0	6.2	1.3	26	10,019	4,941	0.0064	224	0.770	28	0.813	124	0.678	0.772	0.74	386.34	307.66	0.66	61.47	0.52
125,339	51,044	4,643	1.5	8.7	1.8	30	10,997	4,842	0.0757	221	0.761	34	0.806	126	0.672	0.785	0.33	543.63	158.15	1.57	11.19	0.63
136,108	51,350	5,297	2.0	10.0	2.0	34	10,806	5,350	0.0085	212	0.784	24	0.784	128	0.71	0.751	0.81	453.06	579.51	1.43	70.30	0.68
127,366	45,048	5,145	1.0	9.0	2.0	32	9,012	3,862	0.0155	197	0.800	28	0.851	80	0.734	0.800	0.83	403.09	503.06	1.55	75.41	0.76
101,532	40,464	3,817	2.0	10.8	2.2	29	10,784	4,332	0.0230	206	0.817	10	0.844	78	0.721	0.800	0.69	483.57	457.86	1.32	23.78	0.66
14,980	5,432	597	1.5	7.5	1.5	15	9,591	4,430	0.0098	175	0.762	24	0.806	67	0.665	0.739	0.69	242.94	221.59	0.60	67.73	0.36
12,718	4,621	506	1.5	7.1	1.4	16	9,723	3,755	0.0266	174	0.808	25	0.831	78	0.708	0.803	0.36	213.55	187.86	0.65	32.81	0.34
25,765	10,219	972	1.0	6.2	1.3	20	8,591	4,363	0.0078	197	0.752	28	0.781	57	0.686	0.725	0.87	218.02	324.50	0.53	92.87	0.35
25,765	10,219	972	1.0	6.2	1.3	20	8,591	4,363	0.0078	197	0.752	28	0.770	57	0.667	0.746	0.86	259.14	307.86	0.53	88.96	0.41
67,586	24,022	2,723	1.0	8.2	1.8	25	9,831	4,888	0.0113	232	0.805	18	0.853	79	0.737	0.794	0.81	360.56	451.87	0.93	54.52	0.54
98,358	37,011	3,834	1.0	7.0	1.5	28	9,851	3,936	0.0207	188	0.738	31	0.761	105	0.656	0.718	0.69	475.44	491.76	0.80	25.35	0.56
62,729	25,364	2,335	2.0	10.8	2.2	26	10,087	4,556	0.0479	219	0.760	21	0.796	106	0.699	0.748	0.56	304.68	404.42	1.45	27.96	0.54
12,992	5,007	499	1.0	7.4	1.6	16	9,127	4,492	0.0049	191	0.810	19	0.810	51	0.714	0.780	0.84	188.21	218.95	0.54	131.15	0.34
76,874	28,395	3,030	1.5	7.9	1.6	24	10,777	4,423	0.0098	192	0.758	25	0.775	132	0.663	0.731	0.66	465.96	350.15	1.04	57.37	0.60
123,582	48,891	4,668	1.5	9.5	2.0	33	10,609	4,845	0.0927	211	0.800	30	0.851	86	0.713	0.786	0.51	527.42	492.73	1.19	6.27	0.61
181,340	71,809	6,846	2.0	10.0	2.0	40	10,292	4,738	0.0293	180	0.730	24	0.751	113	0.663	0.717	0.73	505.65	593.64	1.63	31.05	0.74
181,624	65,899	7,233	1.5	8.3	1.7	38	11,728	3,881	0.0053	201	0.796	31	0.841	105	0.727	0.786	0.82	606.38	588.88	1.18	58.76	0.71
74,044	29,683	2,773	1.5	7.9	1.6	28	11,079	4,805	0.0062	237	0.762	34	0.752	103	0.68	0.750	0.79	433.05	403.49	0.89	63.61	0.58
69,767	27,882	2,618	1.5	8.3	1.7	26	8,434	3,508	0.0056	195	0.760	18	0.786	93	0.69	0.738	0.86	335.39	467.31	0.98	175.36	0.53
151,126	59,569	5,722	1.0	8.2	1.8	35	11,681	3,967	0.0092	228	0.737	27	0.772	76	0.675	0.741	0.85	458.13	476.44	1.55	66.90	0.83
40,321	15,786	1,533	1.0	6.6	1.4	21	9,361	3,733	0.0169	209	0.738	20	0.757	100	0.655	0.711	0.71	277.20	344.91	0.73	46.84	0.46
67,758	27,749	2,501	2.0	8.4	1.6	26	10,269	4,599	0.0316	201	0.796	11	0.835	108	0.729	0.773	0.86	350.98	455.72	0.95	162.64	0.52
67,758	27,749	2,501	2.0	8.4	1.6	26	10,269	4,599	0.0100	201	0.796	11	0.835	108	0.7	0.773	0.72	402.11	372.42	1.02	56.25	0.59
197,190	72,144	7,815	1.5	7.5	1.5	18	8,508	3,858	0.0210	198	0.759	29	0.786	91	0.66	0.741	0.81	573.73	588.09	1.24	43.44	0.78
102,109	37,453	4,041	1.5	8.3	1.7	23	10,605	3,598	0.0122	222	0.788	12	0.825	63	0.688	0.762	0.81	487.26	484.74	1.00	37.78	0.63

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
44,601	17,227	1,711	1.0	7.0	1.5	20	8,840	3,544	0.0065	237	0.763	19	0.792	73	0.676	0.742	0.83	314.60	384.57	0.69	102.81	0.47
161,192	70,842	5,647	1.0	6.2	1.3	36	8,988	3,639	0.0151	236	0.751	31	0.789	82	0.677	0.768	0.85	408.48	442.43	1.35	66.10	0.91
69,820	28,822	2,562	1.0	7.4	1.6	24	9,349	4,456	0.0661	202	0.748	29	0.781	94	0.666	0.752	0.53	399.52	403.15	0.81	11.04	0.49
100,159	39,284	3,805	1.5	6.7	1.3	30	9,844	3,882	0.0059	205	0.805	32	0.857	69	0.722	0.808	0.87	422.62	419.90	1.04	97.73	0.69
120,479	49,182	4,456	2.0	10.8	2.2	30	11,378	4,652	0.0126	206	0.763	31	0.797	76	0.687	0.738	0.82	428.76	565.40	1.40	46.30	0.65
157,973	63,982	5,874	1.0	6.2	1.3	36	10,459	4,506	0.0084	184	0.735	17	0.775	94	0.66	0.731	0.84	538.51	514.71	0.92	49.92	0.70
74,648	28,930	2,857	1.5	8.3	1.7	28	11,759	4,850	0.0327	198	0.765	15	0.790	124	0.695	0.753	0.49	394.09	398.03	1.06	16.29	0.53
101,071	37,146	3,995	2.0	10.0	2.0	30	9,313	3,774	0.0088	179	0.742	31	0.768	78	0.677	0.724	0.86	388.43	519.13	1.41	127.11	0.64
169,536	60,380	6,822	2.0	10.0	2.0	35	10,146	5,029	0.0284	205	0.810	34	0.834	117	0.73	0.817	0.67	517.45	476.68	1.97	34.90	0.88
105,355	40,101	4,078	1.0	6.2	1.3	28	9,771	4,384	0.0228	200	0.757	32	0.795	123	0.694	0.750	0.64	471.15	545.27	0.69	19.19	0.49
60,301	26,056	2,140	1.0	6.2	1.3	24	11,783	4,707	0.0094	214	0.765	26	0.788	112	0.677	0.763	0.59	469.53	271.75	0.72	26.37	0.54
145,654	59,585	5,379	2.0	10.0	2.0	24	11,398	4,801	0.0506	192	0.739	22	0.785	57	0.68	0.721	0.80	389.50	623.89	1.57	15.22	0.67
95,451	34,820	3,789	1.5	7.5	1.5	29	8,782	4,600	0.0072	235	0.786	22	0.816	88	0.704	0.766	0.87	406.36	503.12	0.99	104.87	0.61
163,563	60,340	6,451	2.0	8.8	1.7	38	10,913	4,033	0.0064	237	0.735	19	0.750	122	0.666	0.717	0.81	512.67	595.03	1.34	95.41	0.72
127,207	49,982	4,827	1.5	9.1	1.9	31	11,780	5,308	0.0065	240	0.754	28	0.772	111	0.66	0.747	0.78	549.32	448.44	1.24	61.23	0.73
190,845	69,126	7,607	2.0	10.4	2.1	40	8,479	4,001	0.0172	193	0.799	25	0.837	82	0.725	0.767	0.87	458.28	687.09	1.78	73.59	0.77
190,845	69,126	7,607	2.0	10.4	2.1	40	8,479	4,001	0.0072	193	0.799	25	0.837	82	0.7	0.800	0.90	496.97	494.82	2.28	222.82	1.04
101,068	44,020	3,566	1.0	7.0	1.5	30	10,159	4,325	0.0015	180	0.763	23	0.796	114	0.668	0.788	0.88	509.85	332.91	1.01	300.69	0.72
82,026	30,730	3,206	1.0	8.2	1.8	27	10,301	4,059	0.0215	193	0.739	24	0.777	106	0.67	0.724	0.64	414.64	478.57	0.90	28.69	0.51
97,300	40,876	3,527	2.0	10.4	2.1	28	10,435	4,364	0.0097	205	0.759	27	0.797	133	0.673	0.738	0.68	458.49	445.04	1.28	70.39	0.63
17,700	6,915	674	1.0	6.6	1.4	15	9,324	3,745	0.0035	192	0.762	12	0.792	83	0.678	0.737	0.76	220.87	241.12	0.51	166.68	0.34
115,235	42,884	4,522	1.5	7.5	1.5	32	8,408	3,639	0.0188	233	0.753	30	0.807	103	0.686	0.762	0.77	382.82	478.12	1.32	76.44	0.71
104,252	43,375	3,805	2.0	10.0	2.0	30	9,215	3,755	0.0058	217	0.800	24	0.800	130	0.699	0.780	0.77	464.02	463.38	1.26	129.72	0.67
49,057	19,858	1,825	2.0	10.0	2.0	20	9,878	3,790	0.0085	228	0.749	11	0.772	67	0.683	0.731	0.82	301.19	409.91	1.06	105.15	0.49
67,246	28,116	2,446	1.5	9.1	1.9	29	9,277	4,274	0.0770	229	0.766	29	0.819	114	0.687	0.766	0.46	317.39	400.00	1.23	17.50	0.49
148,238	56,237	5,750	1.0	9.4	2.1	25	8,940	3,722	0.0433	233	0.768	35	0.833	99	0.717	0.779	0.65	496.29	596.68	1.23	19.70	0.61
97,423	41,777	3,478	1.5	8.7	1.8	26	11,151	3,661	0.0136	177	0.749	19	0.781	113	0.704	0.736	0.68	388.45	547.98	1.00	35.20	0.49
127,934	51,769	4,760	2.0	10.0	2.0	32	10,859	5,411	0.0083	196	0.770	22	0.791	112	0.691	0.769	0.79	504.56	455.01	1.47	73.83	0.74
40,881	15,968	1,557	1.5	7.5	1.5	21	11,511	4,898	0.0061	184	0.780	35	0.814	113	0.696	0.785	0.62	392.42	257.54	0.81	56.75	0.48
40,881	15,968	1,557	1.5	7.5	1.5	21	11,511	4,898	0.0061	184	0.780	35	0.814	113	0.73	0.785	0.68	320.67	375.65	0.68	54.39	0.40

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
82,151	32,381	3,111	1.0	7.0	1.5	30	10,597	4,276	0.0098	200	0.782	16	0.808	68	0.729	0.767	0.84	347.49	489.37	0.88	47.65	0.53
62,501	26,776	2,233	2.0	10.4	2.1	23	10,389	3,451	0.0156	197	0.756	22	0.778	98	0.664	0.735	0.61	374.89	353.21	1.24	55.53	0.56
130,004	56,403	4,600	1.5	9.5	2.0	34	8,916	3,245	0.0042	218	0.747	19	0.759	91	0.658	0.713	0.89	430.98	564.74	1.25	207.25	0.66
69,172	25,215	2,747	1.5	7.5	1.5	26	11,230	4,383	0.0055	210	0.810	17	0.847	60	0.728	0.820	0.86	340.25	332.10	1.29	113.74	0.76
142,866	57,845	5,314	1.0	6.2	1.3	36	9,421	4,061	0.0078	199	0.784	34	0.812	52	0.701	0.803	0.92	432.87	652.48	0.82	64.40	0.54
166,886	62,388	6,531	1.0	7.0	1.5	38	10,008	4,718	0.0061	206	0.772	35	0.803	113	0.71	0.755	0.86	517.86	638.32	0.96	72.39	0.63
187,736	72,941	7,175	1.5	9.5	2.0	40	8,763	4,095	0.0049	214	0.763	30	0.763	118	0.672	0.756	0.88	528.68	614.54	1.46	191.92	0.78
103,262	39,362	3,994	1.5	8.7	1.8	32	9,011	3,657	0.0037	194	0.772	12	0.784	117	0.687	0.770	0.85	412.54	449.25	1.32	265.91	0.72
89,625	38,813	3,176	1.5	6.3	1.2	26	10,075	4,767	0.0109	210	0.758	13	0.794	78	0.681	0.743	0.81	420.89	469.20	0.74	37.29	0.54
64,566	26,708	2,366	1.0	6.6	1.4	24	10,138	4,385	0.0319	221	0.753	29	0.794	63	0.668	0.719	0.75	333.06	450.83	0.72	16.41	0.46
173,277	61,468	6,988	2.0	10.8	2.2	37	8,946	3,791	0.0064	219	0.775	32	0.798	101	0.686	0.780	0.87	483.16	521.82	2.12	232.47	0.95
113,777	41,437	4,521	2.0	10.0	2.0	27	11,382	5,130	0.0235	217	0.744	27	0.785	63	0.667	0.753	0.78	428.45	399.12	1.88	41.62	0.85
162,048	57,940	6,507	1.0	5.8	1.2	38	10,921	3,804	0.0188	235	0.759	18	0.796	94	0.697	0.741	0.83	536.58	638.06	0.77	35.65	0.60
99,232	39,588	3,728	1.5	7.5	1.5	17	11,132	5,234	0.0068	220	0.811	11	0.849	108	0.698	0.816	0.74	617.97	299.51	1.07	46.40	0.72
147,231	58,795	5,527	1.5	6.3	1.2	25	8,458	4,042	0.0023	190	0.753	18	0.789	101	0.659	0.742	0.92	508.20	515.99	0.96	309.18	0.72
174,671	68,343	6,646	2.0	10.8	2.2	40	10,334	4,618	0.0068	209	0.759	27	0.782	97	0.663	0.722	0.87	532.58	621.15	1.54	102.03	0.77
92,169	32,514	3,728	1.5	7.5	1.5	28	11,440	4,308	0.0524	212	0.764	19	0.781	106	0.679	0.746	0.44	469.41	398.28	1.06	10.78	0.57
186,947	71,222	7,233	1.5	6.7	1.3	39	9,861	4,183	0.0033	211	0.766	10	0.789	84	0.683	0.760	0.92	532.17	526.36	1.25	185.90	0.86
186,947	71,222	7,233	1.5	6.7	1.3	39	9,861	4,183	0.0100	211	0.766	10	0.789	84	0.66	0.760	0.84	625.07	502.20	1.12	51.55	0.84
49,932	20,317	1,851	2.0	10.0	2.0	23	10,228	4,639	0.0074	204	0.775	29	0.800	76	0.672	0.735	0.81	335.99	330.14	1.18	107.81	0.57
178,971	73,458	6,595	1.0	7.4	1.6	38	10,478	4,239	0.0506	210	0.797	21	0.798	107	0.666	0.778	0.47	802.59	364.62	1.14	8.81	0.75
59,437	25,376	2,129	2.0	10.4	2.1	23	9,785	3,410	0.0372	206	0.747	18	0.774	110	0.679	0.762	0.50	316.56	366.90	1.35	37.34	0.52
32,289	13,545	1,171	2.0	6.0	1.0	19	10,340	3,558	0.0076	203	0.769	24	0.796	90	0.662	0.741	0.67	308.86	250.36	1.08	110.64	0.51
189,393	71,301	7,381	2.0	9.6	1.9	40	8,887	3,869	0.0807	204	0.766	30	0.810	79	0.677	0.764	0.70	492.31	522.37	1.97	18.76	0.89
32,901	12,710	1,262	1.5	9.5	2.0	18	11,729	5,639	0.0165	193	0.767	19	0.830	85	0.707	0.801	0.59	307.52	265.92	1.01	34.40	0.48
98,928	38,581	3,772	1.5	7.5	1.5	27	9,692	3,967	0.0136	179	0.731	33	0.736	59	0.659	0.729	0.86	393.52	549.84	0.93	51.09	0.54
133,229	47,216	5,376	2.0	8.0	1.5	34	10,019	5,006	0.0064	237	0.768	15	0.814	125	0.708	0.755	0.85	438.88	571.02	1.26	112.81	0.67
157,918	61,484	6,027	1.0	7.0	1.5	32	11,344	4,732	0.0539	237	0.830	31	0.836	119	0.73	0.820	0.37	678.73	370.78	1.16	6.79	0.65
18,161	7,851	644	1.0	5.8	1.2	29	8,696	3,495	0.0028	225	0.764	15	0.817	132	0.688	0.731	0.75	194.01	254.65	0.47	230.39	0.32
29,118	12,673	1,028	1.5	7.5	1.5	19	9,653	4,463	0.0078	208	0.744	13	0.748	79	0.665	0.727	0.79	257.40	319.46	0.66	83.17	0.41

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
35,574	12,931	1,415	1.5	8.3	1.7	19	10,236	4,999	0.0056	225	0.781	35	0.813	122	0.712	0.770	0.73	297.28	332.63	0.84	113.35	0.45
63,518	27,689	2,239	1.0	9.0	2.0	25	9,784	4,688	0.0047	173	0.810	24	0.838	127	0.737	0.825	0.78	352.82	368.74	1.05	137.42	0.57
118,230	49,996	4,265	1.5	7.5	1.5	29	9,217	3,627	0.0317	224	0.743	22	0.782	111	0.676	0.723	0.67	429.02	554.60	0.96	22.97	0.54
41,979	17,553	1,527	2.0	6.0	1.0	20	9,714	4,025	0.0056	220	0.794	32	0.827	110	0.662	0.785	0.67	425.86	200.70	0.82	88.52	0.52
170,978	74,466	6,032	1.0	6.2	1.3	36	9,657	4,745	0.0078	182	0.743	29	0.776	102	0.673	0.744	0.87	495.87	509.01	1.03	70.66	0.75
149,265	53,385	5,993	1.0	6.6	1.4	34	11,086	4,775	0.0955	216	0.755	22	0.785	89	0.683	0.746	0.54	561.81	560.58	0.87	4.33	0.58
112,220	44,101	4,257	1.5	8.3	1.7	30	11,796	5,038	0.0068	174	0.800	14	0.800	110	0.71	0.764	0.77	534.61	403.85	1.15	45.11	0.66
99,277	38,469	3,800	2.0	8.0	1.5	28	11,326	4,059	0.0092	232	0.820	22	0.851	52	0.727	0.820	0.85	430.00	397.41	1.30	52.86	0.76
135,818	53,772	5,128	1.0	7.4	1.6	32	11,646	5,487	0.0223	233	0.765	22	0.814	58	0.68	0.751	0.81	535.78	487.63	1.00	14.95	0.68
134,321	55,470	4,928	2.0	10.0	2.0	25	10,487	4,327	0.0026	208	0.730	32	0.760	132	0.66	0.719	0.86	471.73	543.00	1.37	269.69	0.69
68,704	29,932	2,423	2.0	8.0	1.5	26	9,340	4,031	0.0152	225	0.791	11	0.826	98	0.716	0.787	0.73	339.17	409.81	1.02	52.26	0.55
187,205	73,227	7,124	2.0	10.0	2.0	38	9,797	4,341	0.0021	232	0.754	14	0.802	87	0.665	0.735	0.93	541.82	593.42	1.58	369.80	0.81
194,481	76,127	7,397	1.5	7.5	1.5	40	10,089	4,236	0.0031	217	0.783	25	0.819	122	0.706	0.781	0.88	568.66	542.45	1.28	169.32	0.81
36,855	15,740	1,320	1.5	9.5	2.0	19	11,202	4,181	0.0048	202	0.761	21	0.801	92	0.675	0.726	0.76	295.93	299.70	0.98	117.67	0.48
124,663	49,764	4,681	1.5	8.7	1.8	30	11,508	4,620	0.0283	192	0.793	32	0.819	80	0.72	0.800	0.68	473.98	416.50	1.44	19.26	0.74
56,700	20,438	2,266	1.5	7.5	1.5	25	11,591	4,470	0.0052	233	0.810	31	0.831	132	0.729	0.822	0.67	424.25	308.57	0.92	67.39	0.55
53,849	21,654	2,012	2.0	8.0	1.5	23	9,922	4,454	0.0194	193	0.771	26	0.823	101	0.679	0.771	0.61	381.89	329.86	0.94	36.42	0.53
56,547	22,151	2,150	2.0	7.2	1.3	24	11,225	3,997	0.0228	196	0.757	23	0.757	122	0.66	0.723	0.78	405.09	292.60	0.96	157.95	0.57
137,659	57,701	4,997	2.0	10.0	2.0	35	11,149	4,399	0.0074	237	0.782	15	0.829	75	0.696	0.762	0.86	475.10	533.67	1.40	70.25	0.72
41,130	14,578	1,660	1.5	7.5	1.5	21	9,851	4,511	0.0272	239	0.745	32	0.756	82	0.655	0.723	0.64	327.11	328.02	0.83	29.43	0.48
155,273	65,754	5,595	2.0	10.0	2.0	37	8,857	4,329	0.0020	178	0.785	18	0.797	63	0.699	0.770	0.96	457.05	666.16	1.31	401.12	0.65
144,521	53,404	5,695	2.0	10.0	2.0	33	8,734	4,832	0.0860	237	0.766	21	0.798	112	0.67	0.805	0.62	427.64	472.63	2.01	23.43	0.83
39,765	15,562	1,513	2.0	8.8	1.7	20	10,636	4,443	0.0281	237	0.786	24	0.815	84	0.708	0.747	0.87	274.36	388.11	0.90	208.03	0.47
60,643	21,623	2,439	1.0	9.0	2.0	24	11,116	4,630	0.0190	229	0.789	16	0.828	73	0.684	0.774	0.67	437.13	345.97	0.98	26.15	0.59
182,800	80,319	6,405	1.0	6.2	1.3	40	9,365	4,323	0.0051	209	0.801	19	0.830	56	0.703	0.787	0.93	552.08	545.69	0.92	79.20	0.71
73,652	26,278	2,961	1.0	7.0	1.5	27	11,434	4,592	0.0346	221	0.735	28	0.764	74	0.666	0.737	0.67	373.10	398.76	0.95	16.66	0.58
70,437	26,705	2,733	1.5	7.5	1.5	26	9,034	4,348	0.0099	223	0.795	30	0.841	104	0.729	0.800	0.78	354.53	427.31	0.96	79.43	0.55
70,437	26,705	2,733	1.5	7.5	1.5	26	9,034	4,348	0.0190	223	0.795	30	0.841	104	0.69	0.800	0.66	433.56	365.17	0.92	37.28	0.59
69,649	27,369	2,642	2.0	10.0	2.0	23	11,014	5,249	0.0077	192	0.751	32	0.806	101	0.68	0.751	0.40	390.91	350.90	1.36	12.54	0.57
129,726	49,249	5,030	2.0	10.0	2.0	33	9,048	3,774	0.0239	195	0.820	28	0.831	106	0.713	0.811	0.70	434.73	434.76	1.90	60.53	0.84

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
49,689	21,686	1,750	2.0	10.8	2.2	20	10,409	4,660	0.0066	235	0.748	31	0.759	86	0.664	0.749	0.78	357.11	327.16	1.14	113.71	0.55
69,596	24,400	2,825	2.0	8.8	1.7	25	11,194	3,849	0.0893	174	0.782	27	0.831	97	0.686	0.776	0.27	381.58	283.79	1.65	13.43	0.55
82,829	36,360	2,904	2.0	9.6	1.9	29	10,388	3,827	0.0044	197	0.767	15	0.786	123	0.689	0.731	0.82	364.35	456.37	1.20	161.29	0.59
79,449	27,904	3,222	1.5	7.5	1.5	26	9,629	3,915	0.0084	178	0.800	25	0.838	107	0.702	0.810	0.72	482.42	373.71	0.95	69.02	0.63
66,136	27,938	2,387	2.0	10.4	2.1	24	11,633	4,438	0.0034	235	0.830	31	0.830	112	0.73	0.830	0.73	477.45	359.05	1.37	117.62	0.65
66,136	27,938	2,387	2.0	10.4	2.1	24	11,633	4,438	0.0200	235	0.790	31	0.820	112	0.73	0.780	0.57	350.35	400.73	1.25	29.98	0.52
161,342	60,513	6,302	2.0	9.2	1.8	35	10,626	5,134	0.0042	171	0.777	22	0.793	60	0.675	0.758	0.92	609.92	520.33	1.31	114.51	0.76
45,959	19,629	1,646	1.0	6.6	1.4	22	10,251	4,691	0.0180	191	0.736	31	0.784	120	0.664	0.719	0.60	320.48	350.67	0.67	29.21	0.43
96,124	33,947	3,886	1.5	9.5	2.0	27	9,036	4,171	0.0408	176	0.800	17	0.851	126	0.664	0.719	0.56	407.70	478.20	1.32	28.80	0.59
183,701	78,149	6,597	2.0	10.0	2.0	39	10,191	4,421	0.0069	213	0.774	20	0.825	133	0.69	0.766	0.80	579.86	556.95	1.45	86.61	0.77
18,913	7,021	743	1.5	5.9	1.1	24	8,690	3,405	0.0054	212	0.751	17	0.819	131	0.67	0.754	0.65	231.53	256.10	0.55	152.66	0.35
122,633	44,973	4,854	1.5	6.3	1.2	30	11,659	4,859	0.0136	220	0.768	29	0.801	105	0.693	0.808	0.69	610.87	513.73	0.70	15.64	0.57
101,716	36,185	4,096	2.0	8.0	1.5	30	9,720	4,216	0.0119	180	0.810	32	0.850	118	0.729	0.791	0.72	470.27	496.00	1.03	48.77	0.60
29,840	10,792	1,191	1.0	7.0	1.5	16	10,282	3,743	0.0072	237	0.770	18	0.782	61	0.679	0.756	0.78	309.97	299.82	0.62	65.76	0.44
29,840	10,792	1,191	1.0	7.0	1.5	16	10,282	3,743	0.0072	237	0.747	18	0.782	61	0.705	0.756	0.83	200.95	290.78	0.88	136.59	0.49
168,510	66,691	6,364	1.5	7.5	1.5	37	9,019	4,018	0.0395	228	0.759	22	0.786	78	0.681	0.723	0.81	457.60	683.64	1.09	18.19	0.63
124,834	50,796	4,627	1.5	9.5	2.0	33	8,606	4,354	0.0108	213	0.721	25	0.762	71	0.668	0.718	0.90	392.59	521.84	1.49	131.82	0.71
115,603	42,864	4,546	1.0	7.0	1.5	29	9,897	4,138	0.0380	235	0.759	27	0.782	65	0.656	0.729	0.68	551.36	434.30	0.91	8.57	0.64
113,915	48,319	4,100	1.5	7.5	1.5	31	11,779	5,741	0.0083	204	0.784	18	0.818	96	0.704	0.789	0.78	520.38	406.83	1.15	41.02	0.70
157,572	57,689	6,243	2.0	10.4	2.1	36	10,669	4,958	0.0079	209	0.750	17	0.765	116	0.679	0.740	0.82	503.19	549.20	1.67	101.41	0.78
186,839	78,230	6,788	1.5	7.5	1.5	19	10,709	4,578	0.0039	192	0.768	15	0.780	55	0.66	0.754	0.88	768.79	507.36	0.93	66.01	0.75
149,847	60,898	5,559	2.0	10.4	2.1	34	11,238	4,241	0.0426	239	0.810	12	0.840	80	0.724	0.800	0.65	473.78	497.45	1.74	16.19	0.80
190,559	83,105	6,716	1.0	6.2	1.3	40	8,913	3,646	0.0032	216	0.725	31	0.756	132	0.654	0.717	0.89	526.39	596.78	0.93	176.79	0.69
156,628	64,221	5,775	1.0	6.6	1.4	35	8,939	3,714	0.0175	227	0.746	17	0.781	65	0.669	0.720	0.88	439.68	640.59	0.94	37.16	0.63
192,522	73,413	7,444	1.0	9.0	2.0	40	10,714	3,597	0.0279	199	0.779	13	0.781	105	0.684	0.792	0.63	624.58	505.87	1.44	21.80	0.81
86,275	34,354	3,245	1.0	6.2	1.3	28	10,557	4,165	0.0079	193	0.820	24	0.820	66	0.73	0.830	0.84	408.03	471.31	0.73	42.11	0.55
152,237	61,486	5,672	2.0	9.6	1.9	36	10,489	4,467	0.0174	193	0.801	21	0.841	120	0.695	0.810	0.65	661.10	408.14	1.44	36.76	0.79
174,788	73,927	6,304	1.0	7.0	1.5	36	10,211	4,042	0.0077	189	0.761	16	0.791	97	0.694	0.751	0.84	525.74	568.40	1.02	57.80	0.68
137,153	48,526	5,539	1.5	7.5	1.5	33	11,216	4,075	0.0720	206	0.744	34	0.775	112	0.681	0.747	0.45	515.16	501.98	1.14	8.31	0.58
44,209	18,765	1,590	1.0	7.0	1.5	19	9,486	4,573	0.0081	222	0.784	25	0.784	55	0.674	0.762	0.84	338.59	327.10	0.69	63.54	0.51

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
198,452	79,847	7,413	2.0	10.0	2.0	37	8,898	3,835	0.0589	223	0.768	34	0.800	121	0.668	0.770	0.67	580.54	514.87	1.77	39.30	0.86
160,919	58,514	6,400	1.5	7.1	1.4	32	11,613	4,595	0.0088	231	0.777	23	0.821	100	0.709	0.780	0.78	548.41	494.75	1.20	43.48	0.76
84,838	35,099	3,109	1.5	7.5	1.5	26	9,132	4,220	0.0067	194	0.756	22	0.800	129	0.68	0.746	0.79	413.16	471.62	0.86	91.69	0.55
84,838	35,099	3,109	1.5	7.5	1.5	26	9,132	4,220	0.0110	194	0.756	22	0.790	129	0.65	0.736	0.72	442.14	379.71	0.99	64.06	0.62
112,191	42,213	4,374	1.0	6.2	1.3	31	9,185	4,720	0.0662	183	0.800	25	0.828	96	0.714	0.787	0.60	509.08	511.27	0.73	7.28	0.55
64,418	22,685	2,608	2.0	10.0	2.0	24	11,549	4,626	0.0113	234	0.762	26	0.794	112	0.709	0.761	0.69	357.32	429.03	1.20	54.83	0.53
107,540	42,430	4,069	1.5	7.9	1.6	30	9,494	4,727	0.0313	226	0.782	32	0.841	130	0.694	0.753	0.62	452.75	469.91	1.07	23.48	0.61
22,483	9,871	788	1.5	7.5	1.5	16	10,891	4,715	0.0051	190	0.755	23	0.810	102	0.664	0.726	0.68	266.97	310.70	0.73	96.11	0.40
188,946	78,296	6,916	1.5	9.5	2.0	40	9,837	3,640	0.0373	239	0.764	25	0.799	65	0.673	0.737	0.81	453.29	652.76	1.55	25.09	0.77
130,494	55,742	4,672	1.0	5.8	1.2	30	9,956	3,777	0.0072	220	0.801	14	0.822	76	0.7	0.786	0.83	579.69	496.94	0.66	35.55	0.62
41,499	14,916	1,661	2.0	9.2	1.8	24	9,395	4,615	0.0568	214	0.749	12	0.778	97	0.686	0.735	0.56	263.83	355.33	1.18	26.94	0.48
41,499	14,916	1,661	2.0	9.2	1.8	24	9,395	4,615	0.0100	214	0.749	12	0.778	97	0.66	0.735	0.76	312.67	320.93	1.10	109.03	0.55
159,096	56,017	6,442	1.5	7.5	1.5	32	9,867	4,682	0.0251	183	0.747	11	0.785	100	0.661	0.766	0.74	590.50	597.32	0.98	20.24	0.65
63,543	25,763	2,361	2.0	9.2	1.8	24	9,103	3,664	0.0122	230	0.747	27	0.783	67	0.679	0.744	0.83	308.13	399.71	1.27	103.23	0.60
65,468	24,381	2,568	1.5	8.7	1.8	27	8,587	4,001	0.0959	197	0.776	26	0.776	92	0.677	0.770	0.51	348.69	382.90	1.18	15.58	0.57
32,873	13,169	1,232	1.5	8.7	1.8	18	9,638	4,081	0.0023	224	0.757	31	0.797	131	0.683	0.735	0.80	274.69	321.76	0.85	334.22	0.45
162,815	64,132	6,168	1.5	7.5	1.5	34	8,857	3,577	0.0088	226	0.728	20	0.747	81	0.65	0.710	0.88	473.52	584.82	1.19	103.71	0.71
113,047	41,883	4,448	1.5	8.3	1.7	28	11,634	3,949	0.0099	238	0.810	18	0.846	83	0.735	0.800	0.76	478.30	480.68	1.12	37.81	0.66
177,334	66,432	6,931	1.5	7.5	1.5	38	10,359	4,589	0.0149	239	0.786	18	0.835	119	0.702	0.798	0.74	554.94	463.69	1.44	43.42	0.86
61,381	26,314	2,192	2.0	10.8	2.2	18	9,555	4,240	0.0056	194	0.781	11	0.752	119	0.671	0.727	0.80	324.68	393.86	1.31	192.41	0.58
147,680	61,448	5,389	1.5	7.5	1.5	38	8,829	3,642	0.0260	200	0.744	21	0.766	75	0.665	0.722	0.82	448.71	573.92	1.12	32.44	0.65
151,581	64,434	5,447	1.0	6.2	1.3	35	9,784	4,677	0.0477	197	0.774	16	0.774	126	0.686	0.786	0.80	585.07	523.67	0.78	42.05	0.65
44,463	16,158	1,769	2.0	10.8	2.2	24	8,737	3,870	0.0108	218	0.804	35	0.809	51	0.721	0.782	0.86	289.46	378.58	1.23	117.87	0.53
13,471	5,630	490	1.0	5.4	1.1	15	10,446	3,465	0.0056	236	0.765	16	0.814	104	0.676	0.765	0.54	230.47	166.42	0.48	75.15	0.31
116,966	49,734	4,202	1.5	9.1	1.9	29	10,768	4,759	0.0256	191	0.758	15	0.793	84	0.671	0.755	0.69	502.64	441.81	1.20	22.39	0.67
42,603	16,148	1,653	1.5	7.5	1.5	21	11,265	5,224	0.0023	171	0.795	27	0.845	52	0.725	0.820	0.92	266.32	258.85	1.27	319.64	0.72
52,419	18,705	2,107	1.0	5.4	1.1	23	9,058	4,228	0.0141	221	0.755	24	0.817	133	0.691	0.732	0.68	319.44	421.62	0.60	43.76	0.43
52,419	18,705	2,107	1.0	5.4	1.1	23	9,058	4,228	0.0060	221	0.755	24	0.790	133	0.661	0.732	0.76	369.97	362.99	0.60	86.88	0.51
48,800	19,737	1,816	2.0	8.0	1.5	23	11,259	4,308	0.0249	216	0.763	11	0.800	56	0.667	0.726	0.75	296.98	359.99	0.99	25.27	0.53
185,989	77,110	6,805	1.0	7.0	1.5	39	8,598	3,940	0.0079	216	0.779	24	0.804	72	0.662	0.747	0.90	619.58	592.42	0.90	62.04	0.69

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
94,834	39,373	3,466	1.5	7.9	1.6	32	10,172	3,778	0.0382	237	0.779	27	0.810	78	0.711	0.761	0.69	396.11	506.46	0.97	15.23	0.53
90,144	36,736	3,338	1.5	7.1	1.4	24	9,968	4,561	0.0061	220	0.762	20	0.799	105	0.681	0.770	0.82	418.21	397.35	1.02	101.69	0.66
137,196	59,023	4,886	1.0	8.2	1.8	31	9,803	3,983	0.0168	208	0.773	29	0.821	84	0.716	0.752	0.80	435.68	577.61	1.09	36.03	0.58
112,916	41,396	4,470	1.5	6.7	1.3	30	10,782	4,710	0.0082	238	0.731	22	0.757	67	0.657	0.715	0.88	451.66	513.51	0.93	59.60	0.63
181,047	75,961	6,568	2.0	9.6	1.9	40	10,964	5,079	0.0084	232	0.737	32	0.850	127	0.685	0.819	0.70	859.41	323.38	1.62	46.90	0.88
68,007	24,738	2,704	1.0	7.0	1.5	27	8,973	4,265	0.0669	206	0.830	30	0.839	133	0.708	0.830	0.34	397.22	308.00	1.07	16.35	0.53
167,922	69,171	6,172	2.0	10.0	2.0	34	9,842	3,941	0.0014	240	0.746	14	0.788	131	0.664	0.750	0.90	532.58	490.00	1.68	604.87	0.85
67,429	27,276	2,510	2.0	9.6	1.9	28	10,517	4,310	0.0168	213	0.759	21	0.795	73	0.677	0.765	0.76	337.86	347.91	1.46	62.19	0.69
70,962	29,166	2,612	1.0	7.0	1.5	26	10,128	4,067	0.0061	175	0.751	12	0.792	83	0.709	0.740	0.83	399.97	433.66	0.73	68.00	0.52
143,297	63,012	5,018	2.0	9.6	1.9	35	9,211	4,250	0.0056	221	0.807	11	0.833	117	0.72	0.797	0.84	465.47	508.28	1.46	143.33	0.75
170,274	62,778	6,719	2.0	10.0	2.0	40	10,832	4,763	0.0595	234	0.749	20	0.792	119	0.665	0.724	0.57	559.13	560.15	1.53	12.23	0.74
30,456	11,423	1,190	2.0	10.4	2.1	18	9,891	4,128	0.0194	204	0.740	15	0.758	111	0.655	0.748	0.55	261.33	284.14	1.18	75.79	0.47
96,458	39,567	3,556	1.0	7.0	1.5	30	10,198	4,425	0.0391	229	0.761	20	0.788	80	0.698	0.750	0.71	381.78	497.05	0.91	15.17	0.55
128,654	48,579	5,005	1.5	6.3	1.2	30	13,141	6,059	0.0642	215	0.788	12	0.799	116	0.688	0.763	0.34	622.61	326.17	1.13	3.48	0.60
141,855	60,338	5,095	1.5	9.1	1.9	35	14,043	6,145	0.0071	196	0.750	13	0.821	124	0.674	0.730	0.72	660.07	438.63	1.12	17.87	0.69
72,754	27,943	2,801	1.5	6.7	1.3	28	15,188	6,617	0.0164	191	0.810	28	0.858	122	0.724	0.805	0.39	611.12	232.82	0.94	5.49	0.51
72,754	27,943	2,801	1.5	6.7	1.3	28	15,188	6,617	0.0164	191	0.810	28	0.858	122	0.735	0.805	0.41	583.69	269.63	0.85	5.04	0.53
18,752	6,918	740	1.5	7.9	1.6	17	13,037	5,708	0.0678	226	0.760	18	0.785	54	0.67	0.727	0.75	276.78	216.21	0.69	30.65	0.39
121,447	47,620	4,614	1.5	7.5	1.5	32	11,923	5,929	0.0091	226	0.805	14	0.842	89	0.7	0.783	0.78	633.27	386.08	1.01	19.46	0.72
84,208	33,863	3,147	1.5	6.7	1.3	27	14,628	7,162	0.0096	204	0.724	19	0.769	120	0.663	0.718	0.66	561.11	331.09	0.81	11.12	0.56
63,997	25,511	2,405	1.0	8.2	1.8	26	12,041	5,261	0.0077	205	0.755	23	0.800	89	0.675	0.726	0.78	403.84	319.79	1.04	38.00	0.59
133,526	53,220	5,019	1.0	7.0	1.5	34	13,570	6,619	0.0985	175	0.766	29	0.766	95	0.665	0.724	0.43	622.07	365.40	1.06	1.98	0.61
22,145	8,824	833	1.5	8.7	1.8	17	12,524	5,882	0.0041	200	0.744	25	0.768	131	0.656	0.714	0.67	267.93	203.27	0.93	85.52	0.42
41,886	18,373	1,470	1.0	8.2	1.8	20	12,838	6,274	0.0146	228	0.755	18	0.767	62	0.676	0.750	0.73	351.79	307.35	0.76	13.84	0.48
107,394	38,682	4,294	2.0	8.0	1.5	30	14,970	6,767	0.0066	226	0.782	23	0.816	73	0.69	0.748	0.83	556.92	423.79	1.05	15.38	0.64
90,211	38,770	3,215	1.0	5.8	1.2	29	13,873	6,086	0.0485	186	0.765	21	0.801	115	0.7	0.748	0.39	489.74	395.05	0.67	2.64	0.51
131,337	52,810	4,908	2.0	6.0	1.0	31	14,907	7,425	0.0086	239	0.778	29	0.778	91	0.702	0.764	0.74	725.29	398.42	0.77	6.83	0.64
70,972	28,733	2,640	2.0	9.6	1.9	26	12,009	5,581	0.0061	177	0.759	11	0.790	97	0.675	0.753	0.77	459.57	371.54	1.07	46.01	0.61
60,099	22,521	2,349	2.0	8.8	1.7	24	15,026	6,959	0.0046	197	0.747	28	0.781	94	0.688	0.722	0.80	404.81	339.29	1.07	31.51	0.54
191,058	80,385	6,917	1.5	7.5	1.5	19	13,635	6,630	0.0144	211	0.765	24	0.785	73	0.677	0.740	0.81	739.16	549.25	0.91	6.32	0.68

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
136,466	49,386	5,443	2.0	9.6	1.9	32	12,875	6,228	0.0286	225	0.778	21	0.794	82	0.696	0.779	0.68	552.94	416.97	1.62	11.12	0.81
55,024	22,940	2,005	1.0	7.8	1.7	24	15,000	6,501	0.0080	186	0.767	14	0.796	121	0.7	0.739	0.61	392.21	309.11	0.87	14.89	0.49
113,497	47,620	4,117	2.0	9.2	1.8	29	14,181	6,977	0.0098	224	0.820	27	0.840	53	0.733	0.821	0.82	585.21	390.48	1.18	12.36	0.72
143,666	60,164	5,219	2.0	10.4	2.1	35	14,834	6,631	0.0098	193	0.776	18	0.775	104	0.674	0.754	0.65	815.35	299.06	1.57	26.09	0.78
177,515	69,848	6,729	2.0	8.0	1.5	38	12,086	5,931	0.0144	171	0.751	25	0.772	57	0.672	0.734	0.87	638.70	564.92	1.09	27.55	0.68
184,572	80,419	6,510	1.0	6.6	1.4	40	11,956	5,598	0.0195	227	0.774	20	0.799	116	0.663	0.788	0.60	925.56	308.62	1.05	15.44	0.78
112,669	48,223	4,028	2.0	6.4	1.1	30	13,178	5,939	0.0521	189	0.755	28	0.778	122	0.686	0.745	0.40	537.64	404.59	0.89	6.81	0.54
108,037	43,704	4,021	2.0	6.4	1.1	29	13,265	6,614	0.0063	197	0.788	13	0.828	79	0.714	0.764	0.84	552.95	470.75	0.75	28.15	0.56
15,316	5,989	583	1.0	5.8	1.2	15	13,521	6,164	0.0019	174	0.769	18	0.816	121	0.666	0.726	0.70	272.93	191.28	0.54	139.46	0.34
35,290	13,252	1,377	2.0	8.4	1.6	20	12,998	5,985	0.0088	192	0.769	30	0.816	117	0.669	0.740	0.57	373.12	198.30	1.13	64.70	0.50
23,678	9,740	871	2.0	7.2	1.3	17	12,363	6,164	0.0051	221	0.767	18	0.791	127	0.68	0.745	0.64	284.76	198.94	0.83	101.19	0.40
199,962	71,246	8,045	1.5	7.5	1.5	40	14,489	6,887	0.0083	213	0.757	35	0.784	112	0.672	0.733	0.74	922.74	451.05	1.03	18.93	0.75
155,859	54,907	6,309	2.0	8.8	1.7	38	13,521	6,294	0.0816	193	0.760	17	0.760	133	0.677	0.757	0.32	611.66	298.40	2.19	10.45	0.74
177,555	76,901	6,291	2.0	9.2	1.8	37	13,771	5,991	0.0261	204	0.752	23	0.797	105	0.687	0.735	0.63	652.71	594.92	1.07	10.20	0.62
23,051	8,843	888	1.5	7.9	1.6	16	15,055	7,015	0.0364	236	0.793	21	0.846	52	0.72	0.824	0.40	343.74	227.02	0.60	5.45	0.35
39,798	14,322	1,592	1.5	7.1	1.4	21	13,348	6,058	0.0094	216	0.737	13	0.772	54	0.668	0.740	0.81	353.44	358.08	0.63	31.21	0.44
160,639	62,733	6,119	1.0	9.0	2.0	36	12,091	5,220	0.0095	224	0.751	24	0.802	88	0.678	0.727	0.83	573.42	642.66	1.01	41.77	0.62
46,619	19,662	1,685	1.5	9.5	2.0	22	13,736	6,660	0.0962	171	0.760	12	0.784	127	0.683	0.724	0.57	334.66	285.37	1.16	53.99	0.49
30,505	13,412	1,068	1.5	7.5	1.5	22	12,289	5,641	0.0175	232	0.771	22	0.815	132	0.709	0.752	0.47	255.02	294.28	0.75	32.21	0.36
66,638	24,106	2,658	1.5	8.7	1.8	26	14,648	6,337	0.0072	226	0.770	34	0.792	69	0.678	0.767	0.72	620.66	288.70	0.89	27.30	0.60
66,638	24,106	2,658	1.5	8.7	1.8	26	14,648	6,337	0.0072	226	0.760	34	0.780	69	0.694	0.750	0.79	445.42	401.33	0.89	34.86	0.54
113,183	43,732	4,341	1.5	9.9	2.1	28	12,011	5,044	0.0254	187	0.736	35	0.761	118	0.673	0.717	0.60	479.38	530.46	1.17	27.30	0.58
166,794	60,902	6,618	1.0	7.0	1.5	31	14,228	6,242	0.0431	203	0.760	26	0.782	62	0.66	0.735	0.64	884.41	476.61	0.75	3.39	0.70
26,289	9,742	1,034	2.0	6.8	1.2	18	13,580	6,079	0.0085	237	0.775	15	0.792	99	0.715	0.746	0.64	263.30	283.72	0.69	41.04	0.37
187,858	78,668	6,824	1.5	8.3	1.7	40	12,073	5,911	0.0079	198	0.755	29	0.787	113	0.67	0.770	0.79	707.30	408.10	1.39	58.59	0.86
157,950	64,934	5,813	2.0	10.0	2.0	38	13,753	6,139	0.0184	177	0.760	22	0.760	80	0.69	0.740	0.76	552.40	569.02	1.31	19.17	0.69
77,929	29,772	3,010	1.0	7.8	1.7	28	14,874	7,272	0.0174	214	0.757	27	0.788	71	0.686	0.739	0.71	526.31	349.58	0.86	11.24	0.56
40,985	14,665	1,645	1.0	7.4	1.6	20	13,893	6,177	0.0095	234	0.754	16	0.772	55	0.69	0.751	0.79	302.33	306.55	0.80	40.33	0.51
44,906	18,939	1,623	2.0	10.0	2.0	22	12,673	5,446	0.0678	239	0.768	31	0.768	84	0.67	0.745	0.32	360.18	253.91	1.26	13.72	0.48
144,556	59,965	5,287	2.0	8.0	1.5	37	13,634	6,524	0.0096	228	0.809	26	0.849	76	0.728	0.784	0.83	583.60	554.33	0.95	19.93	0.63

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
132,820	47,349	5,342	2.0	8.4	1.6	32	12,122	5,135	0.0490	176	0.739	23	0.744	102	0.659	0.716	0.54	589.79	496.36	1.11	12.29	0.64
70,182	25,479	2,794	1.0	7.8	1.7	25	14,403	6,822	0.0166	215	0.739	19	0.798	113	0.682	0.767	0.50	576.35	274.65	0.94	16.59	0.57
26,733	10,917	988	1.5	7.5	1.5	15	13,185	5,749	0.0073	202	0.761	25	0.798	52	0.7	0.741	0.82	261.77	342.94	0.58	41.39	0.36
122,435	46,409	4,752	1.5	8.7	1.8	34	12,252	5,631	0.0374	176	0.788	20	0.830	110	0.713	0.764	0.68	541.46	515.36	1.04	22.46	0.62
150,870	59,217	5,728	1.0	8.2	1.8	33	14,806	6,518	0.0100	191	0.766	22	0.800	129	0.69	0.766	0.59	842.74	341.49	1.10	18.30	0.68
22,626	9,177	841	1.0	7.0	1.5	16	14,793	6,900	0.0066	199	0.724	33	0.746	90	0.655	0.710	0.62	298.43	185.56	0.60	35.60	0.37
60,820	25,859	2,185	1.0	7.8	1.7	24	12,679	6,050	0.0994	227	0.820	30	0.850	94	0.732	0.813	0.22	405.62	267.15	1.08	5.80	0.39
90,239	33,590	3,541	1.5	7.5	1.5	28	12,496	5,613	0.0887	173	0.770	20	0.770	108	0.67	0.792	0.74	443.81	437.84	0.97	50.44	0.61
91,913	35,364	3,534	1.0	7.4	1.6	30	14,996	7,319	0.0635	177	0.749	25	0.755	90	0.671	0.751	0.45	550.58	407.64	0.79	3.49	0.52
55,843	22,222	2,101	2.0	6.8	1.2	24	13,045	5,720	0.0212	198	0.798	33	0.837	122	0.719	0.793	0.41	444.94	284.82	0.84	16.95	0.48
146,251	54,035	5,763	1.0	6.2	1.3	34	13,109	6,408	0.0084	203	0.750	14	0.780	85	0.714	0.743	0.85	501.81	549.90	0.90	35.33	0.66
174,584	75,367	6,201	2.0	6.8	1.2	38	14,373	7,144	0.0050	192	0.769	20	0.796	74	0.674	0.737	0.90	603.53	513.38	1.01	34.55	0.74
29,174	11,584	1,099	2.0	6.8	1.2	18	14,198	6,865	0.0053	236	0.732	12	0.785	128	0.665	0.712	0.64	310.32	236.56	0.74	58.66	0.41
30,589	11,287	1,206	1.0	7.0	1.5	21	14,514	6,384	0.0076	224	0.800	23	0.820	98	0.728	0.779	0.59	319.02	221.66	0.64	26.63	0.43
190,821	68,147	7,667	1.0	7.4	1.6	23	12,201	5,484	0.0065	222	0.777	20	0.805	56	0.692	0.750	0.92	587.09	720.28	0.92	48.17	0.64
61,807	21,811	2,500	1.0	6.2	1.3	23	11,847	4,978	0.0605	171	0.798	11	0.811	71	0.695	0.765	0.80	486.65	342.86	0.65	47.55	0.54
10,544	4,524	376	2.0	9.2	1.8	15	12,035	5,266	0.0046	187	0.751	32	0.773	90	0.672	0.743	0.66	193.69	177.11	0.74	151.30	0.33
96,107	35,425	3,793	2.0	6.8	1.2	30	11,944	5,763	0.0542	209	0.735	30	0.746	89	0.655	0.714	0.60	501.35	474.71	0.81	9.23	0.53
51,652	18,241	2,088	2.0	9.2	1.8	27	13,290	6,510	0.0766	205	0.786	13	0.810	58	0.686	0.765	0.74	423.73	309.96	1.04	24.67	0.59
86,527	33,029	3,344	2.0	10.0	2.0	27	14,933	6,648	0.0319	196	0.782	29	0.828	129	0.71	0.800	0.31	511.98	253.24	1.81	14.11	0.59
181,941	71,919	6,876	1.5	7.5	1.5	28	13,913	6,201	0.0110	170	0.787	29	0.813	122	0.684	0.786	0.86	975.07	323.17	1.16	138.64	0.81
181,941	71,919	6,876	1.5	7.5	1.5	28	13,913	6,201	0.0500	170	0.787	29	0.813	122	0.72	0.786	0.46	688.97	536.37	0.99	4.80	0.62
55,767	21,134	2,165	2.0	6.0	1.0	25	12,468	5,327	0.0034	221	0.772	23	0.800	123	0.706	0.750	0.78	379.25	366.65	0.71	91.88	0.50
162,809	64,016	6,175	1.5	8.7	1.8	36	14,017	6,251	0.0082	211	0.731	19	0.757	113	0.651	0.715	0.73	771.83	450.06	1.08	27.85	0.72
183,464	69,200	7,141	1.5	7.5	1.5	40	14,977	7,125	0.0093	221	0.762	16	0.796	94	0.691	0.749	0.78	772.87	546.45	0.90	13.98	0.66
87,086	30,515	3,536	2.0	8.4	1.6	27	14,834	7,144	0.0114	176	0.786	29	0.811	101	0.712	0.770	0.64	592.99	351.95	1.02	18.54	0.61
150,266	59,538	5,671	1.5	9.9	2.1	38	14,489	6,942	0.0330	179	0.741	13	0.755	78	0.683	0.732	0.71	576.25	530.77	1.27	9.76	0.66
61,746	22,893	2,428	1.5	8.3	1.7	25	13,195	6,438	0.0093	238	0.788	20	0.800	80	0.706	0.744	0.79	380.90	430.64	0.86	36.67	0.50
147,246	62,951	5,268	2.0	6.4	1.1	34	12,411	5,994	0.0112	209	0.754	30	0.775	99	0.682	0.755	0.79	586.37	457.85	0.95	35.40	0.69
169,729	72,310	6,089	1.5	7.5	1.5	38	13,275	6,592	0.0251	193	0.728	15	0.741	89	0.667	0.714	0.74	708.17	607.84	1.09	12.57	0.69

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
137,308	48,675	5,540	1.5	7.5	1.5	35	13,765	6,568	0.0076	221	0.747	12	0.788	116	0.676	0.750	0.75	649.30	429.70	1.05	36.51	0.72
68,446	26,951	2,593	1.5	9.5	2.0	26	12,300	5,873	0.0243	184	0.770	12	0.770	63	0.696	0.756	0.75	359.21	441.11	1.08	25.50	0.57
27,338	11,395	996	1.5	5.9	1.1	18	14,740	6,871	0.0075	204	0.764	14	0.783	106	0.674	0.738	0.59	307.62	224.49	0.60	26.20	0.37
74,720	29,191	2,846	2.0	7.2	1.3	27	12,957	6,277	0.0220	191	0.779	25	0.850	110	0.697	0.765	0.53	523.54	330.15	0.88	14.94	0.55
14,636	6,367	517	2.0	10.0	2.0	17	13,787	5,969	0.0111	193	0.747	23	0.774	114	0.688	0.729	0.44	193.21	209.08	0.88	54.98	0.30
180,408	67,604	7,050	1.5	7.5	1.5	40	14,364	6,613	0.0089	222	0.811	23	0.844	51	0.733	0.820	0.89	502.81	407.23	1.83	45.36	1.10
124,281	44,082	5,012	1.0	5.8	1.2	33	13,527	5,915	0.0072	194	0.743	23	0.755	101	0.689	0.728	0.78	629.83	518.00	0.62	22.62	0.57
96,919	34,225	3,918	1.0	8.6	1.9	30	14,997	7,185	0.0335	238	0.819	15	0.858	119	0.732	0.806	0.35	604.28	280.96	1.34	8.21	0.58
39,101	13,914	1,574	1.0	6.6	1.4	21	12,114	5,851	0.0091	227	0.772	15	0.807	56	0.683	0.750	0.81	359.77	319.65	0.63	39.65	0.47
32,800	12,292	1,282	1.5	7.5	1.5	19	12,702	5,620	0.0391	234	0.740	19	0.773	94	0.659	0.724	0.37	327.88	265.78	0.78	16.44	0.44
64,640	25,154	2,468	2.0	8.0	1.5	28	12,367	5,962	0.0722	187	0.749	25	0.791	84	0.681	0.756	0.79	433.10	367.12	0.91	60.42	0.58
34,184	14,960	1,201	1.5	5.9	1.1	20	13,147	6,520	0.0090	209	0.731	17	0.732	69	0.685	0.717	0.80	280.51	356.72	0.52	28.56	0.36
13,384	5,658	483	1.0	5.8	1.2	15	14,126	6,827	0.0730	235	0.735	33	0.751	110	0.657	0.713	0.51	227.43	138.43	0.53	38.40	0.28
95,300	39,509	3,487	2.0	8.8	1.7	29	14,507	7,230	0.0083	232	0.820	26	0.820	134	0.735	0.820	0.59	642.94	257.83	1.33	27.26	0.63
95,300	39,509	3,487	2.0	8.8	1.7	29	14,507	7,230	0.0083	232	0.780	26	0.780	134	0.73	0.780	0.70	429.69	448.79	1.14	34.39	0.59
65,751	26,099	2,478	1.5	7.5	1.5	25	14,647	6,537	0.0272	170	0.750	27	0.819	88	0.684	0.768	0.47	528.38	323.56	0.77	8.62	0.49
65,751	26,099	2,478	1.5	7.5	1.5	25	14,647	6,537	0.0272	170	0.750	27	0.778	88	0.699	0.731	0.59	400.97	435.29	0.75	8.98	0.44
125,871	53,910	4,498	2.0	6.4	1.1	34	14,165	6,495	0.0162	225	0.744	23	0.786	89	0.687	0.729	0.74	563.43	546.16	0.70	10.31	0.52
99,223	43,403	3,489	2.0	8.8	1.7	31	14,826	7,285	0.0063	195	0.776	18	0.811	89	0.72	0.770	0.78	548.36	429.71	0.93	27.09	0.59
29,848	11,482	1,148	2.0	7.6	1.4	19	14,468	7,109	0.0074	179	0.770	12	0.800	79	0.696	0.769	0.67	367.72	230.77	0.74	32.20	0.45
29,851	10,687	1,198	1.0	7.8	1.7	18	13,763	6,120	0.0185	178	0.740	18	0.766	134	0.671	0.721	0.39	294.24	264.02	0.80	28.57	0.37
192,767	72,861	7,494	2.0	10.8	2.2	38	12,934	5,590	0.0119	200	0.757	10	0.772	55	0.655	0.733	0.88	599.85	658.53	1.44	38.66	0.75
167,438	66,247	6,324	2.0	10.0	2.0	36	13,039	6,251	0.0030	230	0.768	22	0.798	127	0.671	0.751	0.81	804.11	361.20	1.55	113.17	0.84
76,019	29,705	2,895	2.0	9.2	1.8	25	12,828	6,276	0.0096	233	0.752	32	0.771	56	0.688	0.729	0.88	343.21	516.32	1.08	55.82	0.55
104,014	45,159	3,678	1.0	9.0	2.0	32	13,199	6,043	0.0354	202	0.750	22	0.784	105	0.698	0.717	0.67	363.29	554.88	1.11	13.66	0.51
45,048	19,270	1,611	1.0	9.0	2.0	23	13,389	6,487	0.0082	196	0.758	25	0.775	123	0.71	0.734	0.66	329.12	310.64	0.95	49.97	0.48
181,866	77,049	6,551	1.0	6.2	1.3	39	14,939	6,819	0.0077	239	0.777	15	0.828	96	0.697	0.778	0.73	968.85	440.04	0.66	9.80	0.65
61,114	23,758	2,335	2.0	9.6	1.9	28	14,187	6,261	0.0090	188	0.742	22	0.783	75	0.68	0.744	0.77	389.96	360.29	1.13	47.20	0.58
143,658	52,017	5,728	2.0	10.0	2.0	35	13,617	5,973	0.0086	178	0.740	13	0.799	69	0.698	0.731	0.87	445.72	573.56	1.59	65.19	0.70
175,473	67,087	6,774	1.5	5.9	1.1	39	14,897	6,802	0.0122	235	0.769	23	0.822	113	0.715	0.768	0.71	729.75	624.51	0.64	8.10	0.56

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
160,390	66,278	5,882	1.0	8.6	1.9	35	14,258	6,791	0.0477	225	0.820	18	0.851	95	0.725	0.821	0.39	858.05	319.87	1.25	4.69	0.67
97,150	38,084	3,692	1.5	7.5	1.5	30	13,998	6,371	0.0586	191	0.800	10	0.840	96	0.729	0.815	0.39	522.55	342.81	1.10	5.82	0.57
187,284	75,234	7,003	1.5	6.3	1.2	40	14,081	6,802	0.0071	212	0.732	16	0.773	64	0.687	0.730	0.91	578.55	696.23	0.79	23.84	0.58
29,364	10,733	1,164	1.0	5.8	1.2	18	12,674	5,764	0.0157	231	0.739	34	0.759	100	0.678	0.725	0.55	283.79	258.30	0.56	29.09	0.39
111,345	43,375	4,248	2.0	8.0	1.5	31	13,190	5,965	0.0425	173	0.765	16	0.765	80	0.689	0.751	0.60	566.96	475.67	0.92	7.31	0.58
18,301	7,120	699	2.0	8.0	1.5	16	14,681	6,774	0.0075	226	0.777	22	0.799	73	0.683	0.763	0.56	337.90	148.49	0.78	33.85	0.39
190,839	83,231	6,726	1.5	7.5	1.5	28	14,219	6,505	0.0052	181	0.757	24	0.795	106	0.692	0.746	0.81	771.69	563.11	0.82	25.62	0.64
54,663	22,357	2,019	2.0	7.6	1.4	27	12,121	5,946	0.0242	220	0.798	29	0.825	97	0.716	0.785	0.57	393.99	325.21	0.88	19.06	0.52
106,727	39,900	4,177	1.5	9.1	1.9	30	14,611	6,560	0.0370	185	0.730	35	0.770	109	0.652	0.718	0.42	645.54	341.13	1.20	9.89	0.60
53,538	19,025	2,157	2.0	8.4	1.6	25	13,349	5,920	0.0305	170	0.743	14	0.779	74	0.686	0.737	0.64	345.37	381.30	1.00	17.91	0.50
75,160	27,545	2,976	1.5	7.1	1.4	28	14,923	6,627	0.0100	182	0.729	33	0.766	115	0.679	0.719	0.67	480.90	451.86	0.69	19.19	0.48
132,604	55,798	4,800	1.5	9.5	2.0	34	14,245	6,796	0.0096	175	0.820	24	0.845	62	0.728	0.817	0.79	753.77	376.82	1.11	16.53	0.69
47,365	17,462	1,869	2.0	8.8	1.7	24	13,464	6,657	0.0071	237	0.765	21	0.792	83	0.685	0.764	0.74	425.08	296.92	0.93	49.81	0.55
44,286	16,572	1,732	1.0	6.6	1.4	24	12,674	5,792	0.0095	213	0.780	23	0.799	66	0.71	0.768	0.78	357.48	360.59	0.72	34.69	0.47
85,625	31,309	3,395	1.5	7.5	1.5	24	11,850	4,216	0.0510	196	0.734	23	0.772	97	0.676	0.717	0.77	407.97	467.60	0.97	56.10	0.55
119,340	42,076	4,829	2.0	10.0	2.0	31	14,454	7,083	0.0076	239	0.757	29	0.788	100	0.689	0.744	0.76	617.59	497.00	1.11	33.39	0.65
80,784	31,961	3,051	2.0	10.8	2.2	25	12,735	5,432	0.0421	186	0.801	16	0.835	82	0.74	0.780	0.68	386.76	465.97	1.36	15.12	0.55
80,784	31,961	3,051	2.0	10.8	2.2	25	12,735	5,432	0.0090	186	0.801	16	0.835	82	0.731	0.790	0.76	417.67	436.10	1.35	58.53	0.61
86,893	32,446	3,403	2.0	9.6	1.9	27	13,296	5,488	0.0055	196	0.741	34	0.765	103	0.696	0.745	0.80	385.89	427.37	1.40	110.86	0.64
33,342	12,414	1,308	1.5	7.5	1.5	18	15,004	5,639	0.0063	192	0.772	16	0.811	53	0.722	0.765	0.80	290.44	338.17	0.67	32.96	0.40
176,337	67,816	6,783	1.0	7.0	1.5	40	11,903	5,619	0.0063	234	0.752	20	0.791	56	0.69	0.736	0.88	562.45	636.30	0.92	57.62	0.64
176,337	67,816	6,783	1.0	7.0	1.5	40	11,903	5,619	0.0288	234	0.741	20	0.770	56	0.67	0.736	0.76	569.63	543.91	1.06	17.25	0.73
168,726	61,220	6,719	1.5	7.5	1.5	33	14,207	6,526	0.0066	181	0.780	32	0.780	94	0.678	0.750	0.79	818.76	427.15	1.02	24.28	0.74
161,772	61,957	6,238	1.0	9.0	2.0	38	14,831	6,869	0.0955	233	0.744	30	0.783	78	0.681	0.725	0.58	589.30	641.39	1.00	2.20	0.58
52,385	20,310	2,005	1.0	7.0	1.5	25	14,874	6,992	0.0109	216	0.776	25	0.817	59	0.678	0.772	0.78	353.47	353.34	0.67	17.33	0.47
164,702	72,459	5,765	2.0	9.2	1.8	38	13,056	6,518	0.0084	201	0.737	21	0.761	76	0.677	0.784	0.90	467.44	670.42	1.21	47.93	0.63
131,217	57,151	4,629	1.0	9.0	2.0	33	13,284	5,390	0.0066	182	0.782	21	0.822	67	0.725	0.799	0.88	367.66	349.32	2.20	123.17	1.07
131,217	57,151	4,629	1.0	9.0	2.0	33	13,284	5,390	0.0066	182	0.782	21	0.822	67	0.707	0.762	0.85	558.59	544.06	0.93	33.49	0.59
23,450	8,965	905	1.5	7.5	1.5	17	14,679	6,951	0.0074	207	0.748	27	0.770	112	0.684	0.728	0.58	284.43	221.54	0.71	38.35	0.39
188,248	68,518	7,483	1.0	8.6	1.9	40	12,064	5,267	0.0493	232	0.766	27	0.786	115	0.684	0.735	0.57	657.36	584.07	1.14	10.01	0.70
55,664	20,813	2,178	1.0	7.8	1.7	24	13,207	5,456	0.0053	207	0.772	14	0.818	83	0.71	0.785	0.80	328.37	322.86	1.07	94.40	0.62

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
167,104	65,680	6,339	1.0	7.0	1.5	39	14,918	6,071	0.0184	194	0.759	17	0.790	117	0.681	0.733	0.59	745.04	433.25	0.94	9.24	0.68
191,724	75,167	7,285	2.0	10.0	2.0	40	14,566	5,768	0.0659	184	0.800	23	0.800	60	0.724	0.786	0.63	700.50	566.60	1.30	3.66	0.71
130,360	52,062	4,894	2.0	10.0	2.0	34	13,422	5,599	0.0084	240	0.766	19	0.800	85	0.697	0.764	0.80	517.68	463.45	1.44	53.09	0.74
14,349	5,685	542	1.0	7.0	1.5	15	13,082	6,523	0.0033	219	0.744	27	0.774	58	0.674	0.722	0.83	216.80	200.67	0.49	87.54	0.33
153,311	58,585	5,920	1.0	7.4	1.6	35	14,082	5,602	0.0184	180	0.738	29	0.779	55	0.703	0.749	0.86	388.26	448.75	1.71	33.18	0.94
197,243	78,623	7,414	2.0	8.8	1.7	40	12,090	6,010	0.0216	236	0.782	31	0.814	121	0.718	0.765	0.71	565.16	644.68	1.29	22.23	0.70
197,243	78,623	7,414	2.0	8.8	1.7	40	12,090	6,010	0.0216	236	0.782	31	0.814	121	0.695	0.756	0.67	717.20	464.08	1.41	20.77	0.79
52,242	18,503	2,109	2.0	7.6	1.4	24	13,502	5,278	0.0125	202	0.761	27	0.767	125	0.684	0.743	0.52	411.60	291.27	0.96	31.58	0.50
183,060	71,211	6,991	1.5	7.5	1.5	40	13,285	6,360	0.0076	199	0.777	18	0.822	134	0.709	0.750	0.77	666.84	565.86	0.99	28.58	0.67
33,746	12,296	1,341	1.0	6.2	1.3	21	13,151	6,321	0.0594	178	0.733	21	0.758	93	0.675	0.719	0.60	310.74	320.29	0.58	17.89	0.39
98,534	35,290	3,953	2.0	9.2	1.8	21	12,996	5,808	0.0096	193	0.780	16	0.783	101	0.676	0.746	0.66	621.70	365.62	1.15	41.02	0.65
98,534	35,290	3,953	2.0	9.2	1.8	21	12,996	5,808	0.0055	193	0.780	16	0.783	101	0.69	0.746	0.76	574.89	468.38	0.97	56.22	0.59
169,164	65,866	6,456	1.5	7.9	1.6	39	14,272	6,387	0.0088	194	0.820	17	0.820	116	0.74	0.810	0.68	834.13	385.94	1.12	17.28	0.73
132,057	55,112	4,809	1.5	7.9	1.6	34	12,193	6,004	0.0262	214	0.767	14	0.805	120	0.688	0.745	0.62	570.56	439.82	1.07	16.27	0.65
65,704	26,020	2,480	2.0	10.0	2.0	27	14,937	7,424	0.0783	210	0.820	11	0.840	124	0.737	0.820	0.18	381.30	215.51	2.13	9.12	0.48
65,704	26,020	2,480	2.0	10.0	2.0	27	14,937	7,424	0.0078	210	0.766	11	0.796	124	0.689	0.742	0.66	459.92	289.52	1.31	42.51	0.60
146,538	59,124	5,463	2.0	8.0	1.5	35	13,695	6,160	0.0098	187	0.753	34	0.757	127	0.677	0.757	0.79	406.88	550.47	1.30	52.42	0.65
146,538	59,124	5,463	2.0	8.0	1.5	35	13,695	6,160	0.0098	187	0.753	34	0.757	127	0.677	0.727	0.75	513.52	513.82	1.10	36.20	0.67
153,299	54,829	6,154	1.0	9.0	2.0	37	15,073	6,059	0.0152	225	0.806	15	0.806	67	0.725	0.780	0.87	630.89	512.62	1.16	30.63	0.71
67,844	24,569	2,705	1.0	7.0	1.5	29	13,304	5,975	0.0080	217	0.739	26	0.739	58	0.664	0.725	0.91	380.98	393.29	0.76	110.90	0.57
163,775	63,923	6,241	1.5	9.5	2.0	37	12,375	5,337	0.0052	218	0.791	11	0.829	133	0.716	0.753	0.81	553.28	556.43	1.34	76.95	0.71
116,640	41,188	4,716	2.0	10.0	2.0	30	11,834	5,864	0.0052	227	0.739	28	0.765	118	0.672	0.727	0.83	501.72	562.22	1.19	117.73	0.64
116,640	41,188	4,716	1.5	7.5	1.5	30	11,834	5,864	0.0642	227	0.761	28	0.794	118	0.697	0.759	0.48	511.77	479.70	1.03	9.53	0.56
188,943	72,870	7,255	2.0	10.0	2.0	40	12,637	5,378	0.0094	198	0.762	25	0.803	132	0.678	0.739	0.73	681.19	553.34	1.37	43.32	0.75
51,251	18,058	2,075	1.0	8.2	1.8	27	12,058	5,011	0.0094	224	0.742	16	0.773	110	0.677	0.755	0.70	340.78	339.52	1.00	78.59	0.56
18,197	7,519	667	1.5	7.5	1.5	16	14,324	6,696	0.0065	222	0.750	33	0.779	121	0.686	0.729	0.55	241.02	192.31	0.73	50.74	0.33
119,860	48,833	4,439	2.0	9.6	1.9	31	14,710	6,929	0.0104	198	0.765	33	0.796	98	0.688	0.793	0.67	656.33	292.58	1.58	31.42	0.78
89,181	38,304	3,180	1.0	7.0	1.5	27	12,573	5,733	0.0089	204	0.738	25	0.767	75	0.668	0.715	0.84	441.90	466.56	0.74	34.24	0.53
29,031	10,637	1,150	2.0	8.0	1.5	17	14,417	7,051	0.0062	230	0.751	25	0.788	83	0.69	0.743	0.71	338.54	285.94	0.62	36.76	0.41
29,031	10,637	1,150	2.0	8.0	1.5	17	14,417	7,051	0.0090	230	0.732	25	0.732	83	0.671	0.719	0.69	310.94	286.46	0.67	32.10	0.41
91,574	34,017	3,597	1.5	9.5	2.0	30	12,206	5,490	0.0092	216	0.782	31	0.813	82	0.733	0.774	0.83	378.52	492.22	1.28	67.99	0.60

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Stage 1 Proppant Concentration	Stage 5 Proppant Concentration	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Closure Stress Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Dimensionless Conductivity Ratio	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output	output
91,574	34,017	3,597	1.5	9.5	2.0	30	12,206	5,490	0.0070	216	0.790	31	0.830	82	0.733	0.800	0.83	400.37	398.46	1.49	98.51	0.73
91,574	34,017	3,597	1.5	9.5	2.0	30	12,206	5,490	0.0070	216	0.790	31	0.830	82	0.7	0.780	0.79	512.23	416.59	1.11	65.07	0.67
112,561	47,054	4,094	1.5	7.5	1.5	30	12,431	4,853	0.0066	196	0.758	27	0.781	119	0.684	0.729	0.76	494.41	475.61	1.15	67.41	0.63
108,469	39,590	4,305	1.5	9.5	2.0	31	13,156	5,529	0.0318	174	0.792	16	0.825	55	0.726	0.769	0.79	400.61	584.61	1.22	14.72	0.60
82,028	34,929	2,944	1.0	7.0	1.5	29	15,159	7,360	0.0597	202	0.741	27	0.799	73	0.66	0.732	0.45	610.99	318.69	0.68	2.88	0.53
82,028	34,929	2,944	1.5	7.5	1.5	29	15,159	7,360	0.0100	202	0.741	27	0.776	73	0.681	0.727	0.79	506.81	441.07	0.57	11.46	0.48
92,673	40,241	3,277	2.0	10.0	2.0	28	14,177	7,066	0.0109	237	0.764	20	0.784	135	0.687	0.755	0.62	558.87	303.63	1.37	34.58	0.64
92,673	40,241	3,277	2.0	8.0	1.5	28	14,177	7,066	0.0050	237	0.764	20	0.784	135	0.702	0.742	0.76	468.34	405.33	1.00	48.71	0.58
105,837	44,424	3,838	1.0	10.8	2.2	30	14,224	6,990	0.0082	177	0.736	11	0.741	77	0.669	0.720	0.83	490.16	486.51	1.26	4.90	0.63
71,082	31,006	2,505	1.0	7.0	1.5	26	13,639	6,405	0.0710	211	0.767	34	0.810	131	0.692	0.760	0.20	424.21	263.13	1.08	7.23	0.40
71,082	31,006	2,505	1.0	7.0	1.5	26	13,639	6,405	0.0071	211	0.767	34	0.810	131	0.716	0.745	0.71	388.98	425.81	0.73	32.78	0.46
173,074	64,946	6,758	2.0	10.4	2.1	30	14,353	5,715	0.0032	213	0.758	30	0.777	80	0.679	0.760	0.86	726.58	449.18	1.52	86.85	0.84
144,963	54,414	5,659	1.0	7.8	1.7	35	12,956	5,064	0.0034	224	0.800	35	0.825	71	0.725	0.780	0.88	640.78	572.84	0.80	52.23	0.60
16,226	5,972	641	1.0	7.0	1.5	15	14,638	5,622	0.0086	172	0.745	32	0.765	68	0.681	0.758	0.49	303.01	154.40	0.54	27.18	0.34
161,441	60,613	6,302	2.0	10.8	2.3	37	13,480	5,054	0.0088	236	0.757	13	0.759	79	0.683	0.744	0.82	561.39	536.36	1.67	57.20	0.81
14,329	5,956	523	1.5	7.5	1.5	16	12,758	5,465	0.0041	186	0.766	31	0.797	84	0.706	0.743	0.71	208.61	203.60	0.64	98.71	0.34
119,014	49,771	4,328	2.0	12.0	2.5	30	13,715	5,318	0.0147	179	0.791	23	0.797	108	0.717	0.764	0.64	522.59	470.19	1.47	27.40	0.67
125,610	52,655	4,560	2.0	10.0	2.0	32	14,204	6,575	0.0053	201	0.755	25	0.789	120	0.68	0.721	0.79	540.90	433.28	1.38	59.34	0.71
74,077	28,646	2,839	1.5	7.5	1.5	26	13,138	5,510	0.0092	194	0.732	13	0.763	91	0.667	0.716	0.75	448.92	435.82	0.78	34.35	0.52
39,936	16,115	1,489	1.0	7.0	1.5	24	12,661	5,592	0.0057	179	0.759	33	0.759	88	0.663	0.713	0.77	341.33	263.73	0.79	70.21	0.48
81,866	34,383	2,968	2.0	10.0	2.0	28	12,844	4,683	0.0113	180	0.796	31	0.810	88	0.686	0.769	0.67	557.23	308.84	1.23	35.48	0.66
81,866	34,383	2,968	2.0	10.0	2.0	28	12,844	4,683	0.0080	180	0.796	31	0.810	88	0.706	0.769	0.75	490.84	365.28	1.18	54.41	0.66
163,612	68,097	5,970	2.0	10.8	2.2	37	13,963	5,337	0.0037	216	0.760	11	0.794	124	0.668	0.744	0.76	786.70	346.83	1.66	85.01	0.83
53,531	19,551	2,124	1.5	9.5	2.0	25	13,010	5,574	0.0099	232	0.750	12	0.787	71	0.67	0.731	0.76	386.48	359.39	1.01	50.06	0.55
24,016	10,322	856	1.5	7.5	1.5	19	13,264	5,029	0.0051	171	0.740	24	0.760	66	0.669	0.714	0.80	266.48	250.78	0.68	66.01	0.40
48,466	18,684	1,861	1.0	7.4	1.6	23	12,054	5,936	0.0018	227	0.804	11	0.842	132	0.726	0.784	0.82	378.17	305.89	0.82	193.62	0.52
65,049	25,865	2,449	1.5	7.5	1.5	24	14,965	7,131	0.0086	184	0.746	15	0.779	71	0.672	0.749	0.75	498.66	324.27	0.79	23.02	0.57
90,933	37,865	3,317	1.5	7.5	1.5	27	13,927	5,726	0.0691	177	0.761	21	0.783	74	0.678	0.755	0.73	643.74	316.86	0.86	21.60	0.60
126,660	54,398	4,516	1.0	7.0	1.5	35	15,013	5,615	0.0085	184	0.730	11	0.754	69	0.666	0.721	0.80	608.16	493.30	0.72	16.65	0.58
79,103	30,023	3,068	1.5	7.1	1.4	27	13,804	6,050	0.0075	228	0.754	24	0.788	72	0.679	0.751	0.79	491.50	386.43	0.82	33.99	0.59
107,734	41,750	4,124	1.5	9.1	1.9	20	12,124	5,962	0.0180	229	0.801	12	0.850	124	0.719	0.796	0.61	539.80	398.87	1.22	27.66	0.66

Table A-2: Data Generated for Six Stage Treatment, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
96,569	38,287	2,534	1.3	30	5,572	2,482	0.0119	182	0.750	13	0.801	117	0.680	0.764	0.88	311.40	474.82	1.52	0.73
37,356	11,554	1,122	1.0	15	13,321	6,435	0.0205	178	0.758	36	0.762	132	0.679	0.766	0.53	362.95	227.19	0.86	0.51
37,356	11,554	1,122	1.0	1	13,321	6,435	0.0205	178	0.778	36	0.832	132	0.720	0.796	0.59	283.27	349.79	0.72	0.44
75,897	29,751	2,006	1.0	28	6,603	2,772	0.0157	225	0.739	25	0.750	88	0.701	0.729	0.88	285.49	538.68	1.01	0.52
98,274	32,194	2,873	1.0	32	11,747	5,515	0.0050	176	0.768	17	0.782	120	0.712	0.730	0.88	380.81	502.88	1.01	0.62
158,845	49,756	4,743	1.3	38	15,021	6,611	0.0214	193	0.727	13	0.734	104	0.685	0.724	0.76	486.02	595.64	1.61	0.75
31,685	9,884	948	1.1	19	5,645	2,556	0.0028	224	0.718	42	0.742	65	0.696	0.715	0.96	202.30	378.90	1.05	0.54
34,322	12,333	956	1.2	22	15,053	6,985	0.0106	232	0.776	31	0.833	98	0.682	0.742	0.71	354.86	177.67	1.14	0.60
126,759	49,130	3,375	1.5	27	11,558	4,949	0.0276	209	0.730	29	0.777	124	0.686	0.723	0.68	440.35	578.00	1.37	0.60
66,538	24,128	1,844	1.5	30	14,789	6,665	0.0144	231	0.758	38	0.758	58	0.710	0.742	0.86	311.55	447.21	1.18	0.56
158,609	52,390	4,618	1.5	40	12,912	5,631	0.0230	223	0.789	16	0.868	78	0.719	0.816	0.83	380.32	343.51	1.80	1.54
158,609	52,390	4,618	1.5	40	12,912	5,631	0.0230	223	0.770	16	0.868	78	0.710	0.760	0.81	517.13	587.90	1.63	0.75
117,823	43,153	3,247	1.0	35	5,477	2,378	0.0724	182	0.747	56	0.793	93	0.684	0.728	0.88	349.08	560.76	1.09	0.66
140,826	46,874	4,085	1.3	40	7,880	3,552	0.0091	224	0.763	27	0.763	93	0.700	0.736	0.92	380.19	614.27	1.59	0.75
102,227	42,658	2,590	1.2	33	11,797	5,793	0.0048	240	0.721	11	0.768	61	0.661	0.717	0.94	383.73	469.88	1.19	0.67
126,596	48,209	3,408	1.1	36	5,141	1,746	0.0076	196	0.769	34	0.792	79	0.709	0.745	0.94	332.09	575.64	1.48	0.71
89,155	30,896	2,533	1.0	33	6,451	2,239	0.0452	191	0.705	45	0.733	87	0.643	0.716	0.81	305.73	477.80	1.42	0.70
31,969	12,709	837	1.0	25	12,053	6,142	0.0029	207	0.747	18	0.769	129	0.705	0.729	0.87	221.00	323.18	0.72	0.44
31,969	12,709	837	1.0	25	12,053	6,142	0.0029	207	0.747	18	0.769	129	0.725	0.739	0.88	204.26	358.37	0.71	0.41
90,178	27,234	2,737	1.0	30	14,463	6,977	0.0114	192	0.788	54	0.843	99	0.730	0.763	0.82	402.34	518.04	1.08	0.65
39,970	16,781	1,008	1.5	17	8,394	3,771	0.0148	220	0.741	19	0.759	70	0.680	0.720	0.85	234.77	357.72	1.28	0.55
151,081	54,449	4,201	1.0	38	10,277	4,392	0.0093	200	0.796	28	0.852	81	0.724	0.769	0.90	465.52	629.15	0.94	0.66
151,081	54,449	4,201	1.0	38	10,277	4,392	0.0093	200	0.796	28	0.796	81	0.700	0.769	0.87	579.19	532.32	0.89	0.71
107,264	39,514	2,946	1.0	29	5,094	1,652	0.0031	217	0.750	24	0.776	63	0.680	0.743	0.96	316.06	549.26	1.29	0.68
84,334	30,612	2,336	1.0	28	11,138	4,655	0.0065	177	0.728	30	0.739	98	0.682	0.720	0.87	375.98	484.25	0.84	0.57
25,960	8,952	739	1.1	15	7,054	2,592	0.0647	233	0.729	28	0.760	80	0.675	0.730	0.66	200.21	312.29	0.87	0.49
25,960	8,952	739	1.1	15	7,054	2,592	0.0647	233	0.729	28	0.750	80	0.690	0.730	0.68	183.35	325.57	0.92	0.48
109,625	40,000	3,027	1.3	34	8,500	3,661	0.0404	188	0.733	35	0.773	132	0.685	0.734	0.67	247.72	515.21	1.66	0.70
143,102	44,697	4,278	1.5	40	12,252	5,757	0.0068	175	0.788	13	0.825	54	0.711	0.778	0.92	499.57	494.89	1.85	0.87
83,966	28,172	2,426	1.1	27	7,299	2,533	0.0047	177	0.718	18	0.714	88	0.660	0.705	0.90	337.58	527.88	1.01	0.57

Table A - 3: Data Generated for Eight Stage Treatment Type I

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
20,073	7,777	535	1.0	18	5,950	2,498	0.0277	235	0.727	18	0.781	99	0.683	0.725	0.64	163.65	284.63	0.71	0.33
111,577	33,743	3,384	1.5	35	9,205	4,170	0.0065	179	0.754	18	0.827	65	0.703	0.746	0.92	360.53	520.94	1.83	0.77
51,721	20,141	1,373	1.0	26	8,096	3,248	0.0231	230	0.724	20	0.778	73	0.655	0.718	0.78	300.09	419.16	0.73	0.46
112,175	48,219	2,781	1.5	27	8,208	3,814	0.0255	187	0.748	32	0.808	74	0.690	0.740	0.82	384.14	539.91	1.36	0.59
60,558	24,269	1,578	1.0	25	10,789	5,075	0.0729	195	0.764	22	0.813	79	0.709	0.750	0.57	338.79	439.52	0.71	0.41
54,223	17,794	1,584	1.2	21	6,753	2,847	0.0309	203	0.781	35	0.786	98	0.725	0.766	0.70	276.23	445.24	1.04	0.48
40,716	13,864	1,167	1.1	17	9,268	3,697	0.0579	231	0.720	17	0.779	65	0.663	0.717	0.61	281.18	386.85	0.79	0.41
155,035	65,837	3,878	1.2	40	7,076	2,585	0.0045	171	0.761	28	0.816	109	0.719	0.769	0.91	346.17	530.37	1.71	0.86
139,116	56,853	3,577	1.4	36	9,518	3,982	0.0178	204	0.723	36	0.757	73	0.670	0.714	0.86	388.51	551.14	1.58	0.71
56,578	23,839	1,423	1.1	28	13,445	6,142	0.0161	171	0.716	22	0.769	134	0.652	0.709	0.61	409.31	289.15	0.89	0.51
156,768	49,007	4,685	1.5	40	10,615	2,485	0.0414	207	0.729	14	0.782	93	0.675	0.726	0.69	429.86	543.42	0.98	0.54
99,524	41,564	2,520	1.3	36	10,458	4,702	0.0065	194	0.729	28	0.810	78	0.666	0.730	0.88	407.96	473.20	1.14	0.61
138,147	47,893	3,924	1.2	39	13,268	6,374	0.0073	172	0.765	34	0.822	133	0.719	0.746	0.79	520.23	635.81	0.96	0.57
56,473	21,659	1,514	1.0	24	8,159	3,844	0.0972	204	0.734	39	0.733	114	0.678	0.721	0.50	278.94	428.12	0.86	0.42
56,473	21,659	1,514	1.2	21	13,232	6,000	0.0089	226	0.704	23	0.761	107	0.649	0.704	0.70	408.27	390.49	0.77	0.48
47,806	15,488	1,405	1.0	19	14,687	7,317	0.0333	195	0.741	20	0.822	72	0.696	0.744	0.58	356.18	364.90	0.67	0.42
90,198	38,777	2,236	1.3	29	9,836	4,870	0.0027	175	0.726	10	0.726	90	0.643	0.722	0.91	416.36	465.56	1.01	0.62
63,571	18,444	1,962	1.2	26	13,890	6,774	0.0055	210	0.707	38	0.751	83	0.658	0.704	0.89	408.75	441.35	0.88	0.53
63,571	18,444	1,962	1.0	28	5,396	1,833	0.0757	238	0.788	33	0.838	82	0.694	0.746	0.68	295.56	471.33	0.95	0.51
127,387	51,962	3,279	1.0	27	7,542	2,786	0.0332	216	0.739	26	0.806	65	0.700	0.738	0.85	342.81	547.15	1.18	0.68
91,245	37,946	2,317	1.0	30	10,055	4,387	0.0297	176	0.733	19	0.788	91	0.683	0.821	0.73	377.28	526.29	0.79	0.49
32,010	13,751	794	1.1	17	10,889	4,813	0.0105	184	0.712	16	0.756	102	0.676	0.710	0.71	242.83	372.25	0.65	0.34
75,747	23,778	2,260	1.0	25	5,118	2,070	0.0678	185	0.753	26	0.772	124	0.687	0.732	0.62	306.55	506.71	0.98	0.51
75,747	23,778	2,260	1.1	31	12,048	4,505	0.0085	224	0.708	27	0.781	134	0.656	0.707	0.70	423.16	474.43	0.80	0.51
101,470	30,675	3,078	1.3	32	15,127	7,192	0.0123	237	0.749	39	0.806	98	0.710	0.744	0.74	468.71	559.93	1.03	0.54
133,070	52,874	3,487	1.2	37	6,357	2,447	0.0085	187	0.745	32	0.745	106	0.696	0.745	0.88	365.48	600.33	1.29	0.66
159,630	52,005	4,679	1.1	40	7,847	3,911	0.0050	199	0.715	22	0.753	101	0.651	0.712	0.93	405.42	567.72	1.51	0.85
140,614	48,526	4,004	1.0	37	9,297	4,375	0.0352	232	0.730	28	0.789	91	0.672	0.722	0.78	439.26	585.03	1.05	0.63
83,548	30,075	2,325	1.1	32	6,901	3,182	0.0071	211	0.731	37	0.775	135	0.680	0.718	0.87	343.51	544.18	0.92	0.51
80,951	34,392	2,024	1.0	26	10,881	3,441	0.0174	189	0.743	20	0.842	128	0.711	0.749	0.64	320.95	485.30	0.88	0.47

Table A-3: Data Generated for Eight Stage Treatment Type I, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
82,354	31,867	2,195	1.2	28	8,726	4,309	0.0568	180	0.741	26	0.741	125	0.653	0.733	0.56	392.97	436.26	1.04	0.56
89,805	29,072	2,641	1.3	35	10,231	4,439	0.0538	224	0.744	27	0.816	118	0.689	0.734	0.56	380.22	500.64	1.22	0.55
81,323	28,375	2,302	1.1	30	11,989	5,441	0.0283	171	0.733	19	0.731	55	0.663	0.729	0.80	381.38	491.50	0.91	0.56
114,520	42,181	3,145	1.0	22	5,957	2,216	0.0046	198	0.714	19	0.795	100	0.647	0.715	0.91	358.16	533.95	1.11	0.66
102,640	39,499	2,745	1.0	29	10,895	5,186	0.0957	199	0.756	40	0.842	105	0.671	0.729	0.49	471.74	514.48	0.76	0.53
81,199	30,767	2,193	1.0	26	12,319	5,895	0.0101	185	0.739	16	0.820	111	0.673	0.730	0.73	485.63	446.16	0.68	0.52
36,077	12,254	1,036	1.0	20	7,415	3,993	0.0697	214	0.748	38	0.859	123	0.702	0.750	0.47	247.47	363.00	0.78	0.39
107,728	37,945	3,034	1.0	28	11,842	5,275	0.0262	185	0.750	29	0.785	73	0.710	0.740	0.77	402.06	549.61	0.93	0.54
107,728	37,945	3,034	1.3	33	13,173	5,275	0.0118	222	0.713	31	0.769	129	0.655	0.714	0.67	532.86	493.35	1.01	0.60
38,430	15,952	977	1.0	27	9,348	4,194	0.0041	190	0.751	14	0.818	74	0.725	0.770	0.88	210.09	322.18	0.90	0.54
129,894	41,746	3,833	1.4	37	14,051	6,933	0.0284	224	0.727	36	0.780	115	0.683	0.729	0.66	487.05	531.40	1.40	0.65
64,773	22,781	1,826	1.1	25	10,555	4,885	0.0075	214	0.723	46	0.748	60	0.676	0.715	0.90	295.60	436.79	1.05	0.58
42,511	16,679	1,123	1.0	28	15,014	7,024	0.0052	237	0.760	38	0.760	71	0.715	0.750	0.82	306.37	371.79	0.61	0.42
39,375	11,446	1,214	1.3	19	8,463	4,245	0.0311	215	0.727	30	0.772	100	0.690	0.724	0.59	252.93	397.66	0.75	0.38
147,989	56,644	3,971	1.2	40	10,108	4,388	0.0089	192	0.766	36	0.843	85	0.719	0.760	0.87	472.93	595.57	1.14	0.62
130,306	42,326	3,825	1.3	38	10,539	4,376	0.0014	179	0.730	20	0.791	66	0.703	0.741	0.98	323.55	492.46	2.10	1.03
130,306	42,326	3,825	1.0	38	10,539	4,376	0.0014	179	0.735	20	0.790	66	0.680	0.731	0.97	432.67	509.69	1.17	0.75
82,716	35,349	2,059	1.1	26	13,743	6,503	0.0087	189	0.752	29	0.807	93	0.705	0.751	0.79	423.51	466.02	0.77	0.50
118,245	37,385	3,516	1.1	35	6,753	2,814	0.0198	225	0.738	20	0.815	65	0.642	0.708	0.89	369.43	517.94	1.37	0.75
143,466	59,884	3,634	1.3	40	12,532	5,453	0.0643	213	0.765	14	0.849	130	0.700	0.755	0.44	540.54	515.23	1.14	0.58
69,029	21,969	2,046	1.0	25	13,454	6,394	0.0088	204	0.729	38	0.741	104	0.674	0.733	0.74	425.10	389.67	0.83	0.56
76,174	26,516	2,159	1.2	30	14,381	6,719	0.0824	233	0.739	15	0.807	72	0.688	0.730	0.56	399.32	463.36	0.94	0.50
85,528	25,285	2,619	1.0	32	7,149	3,422	0.0284	205	0.765	33	0.765	80	0.710	0.776	0.58	306.08	514.60	1.34	0.57
88,268	26,818	2,672	1.2	27	7,137	2,911	0.0074	173	0.720	37	0.800	109	0.660	0.712	0.85	379.80	541.87	1.05	0.56
117,128	34,790	3,580	1.3	33	9,307	3,891	0.0139	192	0.721	33	0.788	109	0.660	0.718	0.78	462.60	558.55	1.18	0.63
151,417	57,233	4,095	1.2	40	10,001	4,409	0.0069	226	0.744	14	0.744	80	0.677	0.825	0.90	448.55	653.35	1.13	0.65
151,417	57,233	4,095	1.0	40	12,380	5,597	0.0976	240	0.718	15	0.789	79	0.650	0.717	0.62	518.00	510.36	1.04	0.67
59,955	18,204	1,815	1.0	25	9,683	4,232	0.0352	229	0.767	10	0.857	81	0.730	0.769	0.71	286.12	442.14	0.97	0.52
81,401	27,889	2,327	1.0	26	5,770	2,102	0.0107	174	0.728	19	0.753	51	0.676	0.721	0.92	305.66	549.21	0.93	0.53
49,438	18,568	1,342	1.1	22	13,557	6,743	0.0093	222	0.752	36	0.828	114	0.717	0.764	0.66	379.36	422.04	0.62	0.40

Table A-3: Data Generated for Eight Stage Treatment Type I, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
39,706	16,591	1,005	1.0	20	8,798	3,411	0.0916	208	0.734	20	0.772	80	0.692	0.721	0.51	236.59	380.51	0.75	0.35
151,572	48,891	4,464	1.1	40	8,173	3,204	0.0282	190	0.724	40	0.740	68	0.655	0.731	0.87	402.70	710.83	1.16	0.63
126,338	51,255	3,264	1.0	36	8,555	4,126	0.0190	180	0.722	31	0.752	74	0.686	0.726	0.89	351.80	651.28	0.96	0.57
85,390	35,345	2,176	1.0	27	6,427	2,742	0.0483	182	0.736	15	0.761	105	0.694	0.741	0.71	299.84	528.11	0.93	0.50
144,061	51,028	4,045	1.1	15	10,169	4,857	0.0055	215	0.741	20	0.770	76	0.692	0.733	0.92	422.42	536.22	1.32	0.77
104,749	42,977	2,686	1.1	37	12,216	5,770	0.0161	221	0.721	38	0.811	104	0.668	0.721	0.75	472.62	546.80	0.84	0.54
142,948	51,849	3,961	1.0	23	14,710	5,799	0.0677	218	0.754	22	0.834	69	0.701	0.755	0.72	428.08	487.32	1.28	0.75
142,948	51,849	3,961	1.0	35	14,710	5,799	0.0067	218	0.724	22	0.800	69	0.650	0.728	0.88	532.54	287.89	1.04	0.75
143,371	49,805	4,068	1.1	38	9,359	3,696	0.0207	188	0.758	27	0.758	90	0.674	0.740	0.78	484.02	585.41	1.07	0.68
63,181	24,765	1,670	1.3	23	14,370	6,313	0.0176	227	0.732	13	0.778	107	0.658	0.720	0.53	490.78	300.04	0.92	0.52
47,978	17,910	1,307	1.0	20	12,705	6,009	0.0367	171	0.765	16	0.848	79	0.730	0.787	0.61	255.20	304.75	1.05	0.54
102,656	43,619	2,567	1.5	30	7,657	3,668	0.0013	197	0.733	11	0.763	93	0.683	0.729	0.96	318.72	522.94	1.72	0.69
107,318	34,040	3,186	1.5	28	7,733	3,808	0.0200	240	0.767	33	0.797	127	0.685	0.798	0.77	371.19	448.02	1.94	0.85
72,402	29,151	1,880	1.5	26	5,244	1,743	0.0019	180	0.734	30	0.734	117	0.675	0.716	0.87	270.71	465.46	1.51	0.64
148,715	47,951	4,381	1.0	40	9,990	4,086	0.0028	211	0.793	18	0.825	102	0.690	0.779	0.93	623.98	500.52	0.91	0.77
148,715	47,951	4,381	1.0	40	9,990	4,086	0.0028	211	0.755	18	0.755	102	0.650	0.720	0.94	575.57	498.28	0.99	0.79
150,770	53,323	4,237	1.5	40	12,960	5,159	0.0332	201	0.749	13	0.763	105	0.682	0.736	0.70	528.99	542.16	1.52	0.76
89,086	29,864	2,575	1.5	25	13,782	6,620	0.0093	201	0.717	30	0.800	90	0.698	0.734	0.95	378.70	470.36	1.42	0.69
89,086	29,864	2,575	1.5	25	13,782	6,620	0.0020	201	0.740	30	0.740	90	0.670	0.724	0.91	504.64	454.11	1.10	0.64
145,089	48,858	4,184	1.2	36	5,927	2,066	0.0016	183	0.770	36	0.781	120	0.714	0.764	0.95	336.05	559.29	1.35	0.71
145,089	48,858	4,184	1.2	36	5,927	2,066	0.0016	183	0.741	36	0.781	120	0.714	0.730	0.96	314.69	586.42	1.38	0.72
125,109	42,960	3,572	1.5	27	7,405	3,644	0.0052	220	0.759	12	0.750	133	0.681	0.727	0.91	375.52	565.17	1.69	0.78
143,896	51,315	4,025	1.0	40	8,236	3,597	0.0104	189	0.787	33	0.794	114	0.712	0.783	0.87	404.93	560.68	1.46	0.78
159,816	51,809	4,696	1.0	40	11,259	4,239	0.0065	234	0.753	43	0.801	127	0.681	0.737	0.83	605.64	582.17	0.94	0.70
59,815	21,493	1,666	1.2	22	8,465	3,296	0.0122	202	0.748	30	0.796	104	0.661	0.730	0.90	339.28	352.30	1.25	0.67
42,963	13,226	1,293	1.3	27	12,001	5,508	0.0054	227	0.734	19	0.736	74	0.651	0.711	0.88	296.44	348.40	1.16	0.60
25,349	8,418	736	1.0	19	5,065	1,556	0.0063	238	0.727	36	0.741	128	0.670	0.711	0.91	179.08	307.50	0.92	0.50
134,445	53,376	3,525	1.0	35	8,112	3,803	0.0307	200	0.712	19	0.736	60	0.644	0.700	0.90	375.61	520.55	1.14	0.74
47,689	15,023	1,420	1.0	22	12,612	5,321	0.0077	179	0.745	21	0.805	86	0.713	0.740	0.84	286.35	411.58	0.75	0.46
47,689	15,023	1,420	1.0	22	12,612	5,321	0.0077	179	0.745	21	0.805	86	0.660	0.740	0.79	422.45	306.38	0.74	0.59

Table A-3: Data Generated for Eight Stage Treatment Type I, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
91,374	26,888	2,804	1.2	26	11,188	4,996	0.0720	172	0.762	40	0.784	129	0.707	0.758	0.51	372.84	423.39	1.41	0.67
91,374	26,888	2,804	1.2	26	11,188	4,996	0.0072	172	0.762	40	0.784	129	0.650	0.758	0.74	625.40	304.76	1.17	0.76
91,230	38,545	2,291	1.2	29	9,860	4,383	0.0103	174	0.741	24	0.743	72	0.656	0.723	0.88	399.92	450.26	1.12	0.64
44,175	14,896	1,273	1.0	21	5,705	2,005	0.0115	222	0.763	38	0.808	122	0.688	0.732	0.83	230.43	378.70	1.90	0.58
120,129	34,841	3,708	1.0	29	5,108	1,840	0.0081	179	0.715	37	0.774	129	0.650	0.710	0.89	375.17	588.02	1.14	0.67
33,043	10,294	989	1.0	19	8,763	4,073	0.0210	215	0.747	25	0.758	55	0.690	0.735	0.84	218.79	344.49	0.81	0.49
61,050	22,396	1,681	1.0	24	13,483	6,649	0.0080	194	0.763	35	0.793	125	0.687	0.746	0.77	416.55	318.94	0.83	0.61
158,130	47,266	4,820	1.0	40	14,715	6,566	0.0182	231	0.711	36	0.750	108	0.664	0.705	0.79	519.89	618.95	1.01	0.69
158,130	47,266	4,820	1.0	40	14,715	6,566	0.0070	231	0.728	36	0.750	108	0.650	0.710	0.81	774.88	456.12	0.92	0.77
142,353	47,857	4,109	1.0	38	10,399	5,171	0.0177	193	0.770	24	0.799	54	0.710	0.761	0.93	402.00	633.15	1.33	0.67
99,299	40,857	2,541	1.1	30	5,269	2,500	0.0103	206	0.722	10	0.769	76	0.651	0.710	0.94	309.93	618.77	1.17	0.64
99,299	40,857	2,541	1.1	34	11,597	5,225	0.0103	231	0.767	19	0.726	92	0.695	0.748	0.85	430.02	471.80	0.90	0.62
35,653	12,483	1,007	1.0	21	9,152	3,776	0.0129	208	0.754	19	0.771	108	0.683	0.731	0.80	240.58	332.11	0.88	0.55
44,699	16,204	1,239	1.0	16	13,646	6,615	0.0186	201	0.754	12	0.790	92	0.681	0.745	0.70	372.18	304.36	0.73	0.54
85,348	36,644	2,118	1.0	29	11,270	4,773	0.0099	239	0.768	17	0.800	72	0.700	0.745	0.87	372.42	429.99	0.87	0.61
97,639	28,783	2,994	1.0	18	12,690	5,664	0.0227	224	0.756	15	0.752	64	0.700	0.735	0.84	391.63	549.28	0.94	0.60
82,735	27,150	2,417	1.5	25	10,385	4,812	0.0097	173	0.740	31	0.765	102	0.688	0.736	0.84	338.76	490.53	1.49	0.66
62,508	25,107	1,626	1.2	25	14,726	7,158	0.0112	227	0.738	25	0.783	88	0.658	0.720	0.79	464.29	377.99	1.38	0.76
62,508	25,107	1,626	1.0	27	6,902	2,965	0.0010	229	0.706	22	0.746	70	0.643	0.700	0.97	272.92	440.51	0.91	0.57
71,773	30,485	1,795	1.0	22	6,191	2,259	0.0011	201	0.787	37	0.815	65	0.703	0.770	0.97	311.86	451.07	0.86	0.55
95,991	40,166	2,427	1.1	28	10,595	5,154	0.0137	171	0.719	30	0.734	109	0.652	0.707	0.82	415.39	472.16	0.89	0.60
37,153	11,417	1,119	1.0	19	12,473	5,368	0.0069	210	0.759	11	0.769	102	0.674	0.718	0.81	283.06	345.67	0.77	0.51
42,002	15,364	1,158	1.1	21	6,857	3,132	0.0095	224	0.715	25	0.736	114	0.662	0.709	0.87	229.29	382.14	0.91	0.55
54,393	23,198	1,356	1.4	25	11,442	5,293	0.0088	170	0.720	24	0.753	65	0.670	0.711	0.90	297.48	406.98	0.88	0.50
83,797	34,897	2,126	1.0	25	13,022	5,704	0.0077	215	0.799	32	0.869	62	0.712	0.820	0.78	544.78	306.07	0.95	0.68
127,080	42,404	3,682	1.0	36	14,082	6,579	0.0248	194	0.741	12	0.818	134	0.662	0.741	0.84	552.54	356.81	1.25	0.90
89,254	36,928	2,275	1.0	27	10,879	4,635	0.0092	235	0.739	19	0.781	113	0.652	0.733	0.79	460.43	392.55	0.85	0.65
89,254	36,928	2,275	1.5	23	12,745	4,525	0.0147	221	0.777	12	0.794	120	0.691	0.754	0.67	454.09	362.22	1.40	0.72
47,391	18,008	1,278	1.2	20	8,809	3,493	0.0202	204	0.757	41	0.756	60	0.675	0.754	0.84	270.84	376.34	0.98	0.57
96,354	28,937	2,931	1.3	26	6,741	2,928	0.0146	218	0.778	28	0.806	91	0.708	0.750	0.86	358.96	516.12	1.46	0.70

Table A-3: Data Generated for Eight Stage Treatment Type I, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
62,569	18,651	1,909	1.0	25	10,078	4,975	0.0122	228	0.789	37	0.819	126	0.646	0.755	0.72	480.64	262.83	1.00	0.70
99,552	41,595	2,520	1.2	28	14,967	6,779	0.0510	221	0.776	27	0.776	126	0.705	0.750	0.45	486.35	344.35	1.17	0.62
62,569	18,651	1,909	1.2	22	13,215	5,702	0.0069	197	0.747	24	0.795	82	0.674	0.736	0.81	457.57	391.54	0.86	0.58
98,812	35,841	2,738	1.5	30	13,586	6,101	0.0088	233	0.748	13	0.768	73	0.666	0.722	0.86	476.10	461.61	1.25	0.70
85,941	36,577	2,146	1.2	20	9,716	3,896	0.0206	205	0.794	38	0.841	96	0.720	0.784	0.83	387.93	456.63	0.91	0.58
157,458	52,417	4,567	1.0	40	10,546	4,715	0.0365	188	0.755	16	0.773	67	0.670	0.732	0.78	569.78	569.45	0.95	0.71
89,844	30,775	2,568	1.1	24	8,082	3,972	0.0054	218	0.781	27	0.799	65	0.686	0.757	0.92	400.40	470.99	1.10	0.64
98,592	36,237	2,711	1.4	24	7,479	2,861	0.0905	234	0.746	36	0.765	91	0.671	0.730	0.86	369.14	508.64	1.44	0.66
80,827	31,731	2,135	1.0	26	13,755	6,103	0.0047	227	0.754	26	0.769	114	0.659	0.728	0.79	555.37	286.58	0.91	0.66
41,149	13,016	1,223	1.0	18	12,441	5,818	0.0043	189	0.782	22	0.778	51	0.658	0.769	0.89	421.81	283.27	0.69	0.57
122,102	38,404	3,639	1.4	38	9,247	3,942	0.0449	221	0.776	16	0.813	95	0.727	0.782	0.77	344.32	545.10	1.82	0.78
122,102	38,404	3,639	1.4	38	9,247	3,942	0.0044	221	0.770	16	0.810	95	0.680	0.750	0.93	456.11	524.92	1.43	0.75
120,919	49,626	3,100	1.0	33	10,940	5,401	0.0074	203	0.719	22	0.761	65	0.659	0.700	0.93	404.76	571.26	0.90	0.59
146,848	58,004	3,863	1.4	19	11,936	5,015	0.0054	177	0.795	10	0.834	109	0.696	0.769	0.84	604.67	454.51	1.47	0.83
69,759	26,936	1,862	1.2	23	6,236	2,219	0.0028	186	0.724	15	0.760	93	0.650	0.710	0.93	302.77	450.91	1.07	0.59
69,759	26,936	1,862	1.0	26	14,109	6,610	0.0090	213	0.730	23	0.766	111	0.669	0.709	0.78	402.41	385.91	0.77	0.56
90,378	27,901	2,716	1.5	29	15,021	6,930	0.0101	209	0.778	23	0.793	89	0.708	0.778	0.77	508.09	385.60	1.40	0.75
90,378	27,901	2,716	1.0	28	10,233	4,936	0.0254	239	0.798	46	0.867	70	0.733	0.800	0.81	398.55	458.00	1.09	0.60
83,310	26,695	2,462	1.0	30	14,595	6,433	0.0052	177	0.735	27	0.784	103	0.675	0.718	0.84	507.25	440.74	0.74	0.59
133,416	44,950	3,846	1.8	39	10,946	5,118	0.0243	179	0.766	36	0.782	134	0.670	0.741	0.69	579.59	452.72	1.47	0.81
156,706	57,898	4,296	1.0	40	14,934	6,370	0.0752	173	0.733	35	0.737	126	0.669	0.735	0.45	583.06	378.57	1.93	0.81
34,200	12,099	961	1.0	24	6,832	2,754	0.0147	209	0.739	14	0.747	119	0.683	0.714	0.82	202.53	345.72	0.92	0.52
116,444	37,792	3,420	1.2	35	5,863	2,430	0.0155	222	0.756	38	0.753	57	0.663	0.731	0.93	360.39	561.71	1.32	0.72
158,063	58,809	4,315	1.5	40	5,634	2,791	0.0543	216	0.764	37	0.804	71	0.706	0.759	0.89	346.96	617.37	1.50	0.80
158,063	58,809	4,315	1.5	40	5,634	2,791	0.0050	216	0.720	37	0.720	71	0.640	0.705	0.97	376.40	618.14	1.39	0.75
151,254	64,968	3,752	1.0	21	9,821	3,525	0.0211	193	0.730	21	0.772	90	0.663	0.716	0.82	459.61	571.54	0.96	0.65
110,501	43,768	2,901	1.0	36	13,988	6,129	0.0023	202	0.760	25	0.787	122	0.715	0.750	0.86	488.09	550.23	0.73	0.54
110,501	43,768	2,901	1.1	31	9,941	4,862	0.0669	179	0.728	40	0.804	80	0.667	0.719	0.68	456.92	552.65	0.85	0.50
86,011	34,879	2,223	1.0	26	9,770	4,522	0.0547	209	0.758	21	0.758	100	0.690	0.747	0.59	392.82	485.20	0.79	0.49
63,916	24,180	1,728	1.1	30	5,588	2,093	0.0118	190	0.719	38	0.799	129	0.661	0.710	0.80	284.72	470.45	0.96	0.50

Table A-3: Data Generated for Eight Stage Treatment Type I, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
54,965	21,990	1,434	1.2	22	6,878	2,962	0.0377	226	0.763	17	0.786	132	0.728	0.755	0.61	246.59	441.48	1.07	0.45
52,844	18,190	1,507	1.0	24	9,021	3,736	0.0092	210	0.751	16	0.751	72	0.701	0.751	0.85	278.82	464.48	0.78	0.46
93,749	33,085	2,638	1.1	29	10,984	4,104	0.0027	222	0.731	28	0.790	113	0.671	0.744	0.86	409.36	430.16	1.11	0.68
116,296	43,488	3,166	1.0	32	10,072	4,418	0.0087	184	0.720	22	0.780	103	0.652	0.710	0.90	349.83	543.12	1.46	0.70
116,296	43,488	3,166	1.3	29	5,272	2,418	0.0087	184	0.754	22	0.793	103	0.652	0.718	0.85	355.45	487.01	1.21	0.72
135,314	45,000	3,927	1.3	35	12,394	2,337	0.0236	209	0.725	31	0.770	86	0.678	0.736	0.75	457.36	488.66	1.54	0.77
59,262	19,551	1,727	1.0	23	11,616	5,047	0.0087	186	0.758	26	0.833	113	0.712	0.768	0.71	379.53	428.22	0.72	0.47
77,242	24,319	2,301	1.2	26	10,456	4,532	0.0938	184	0.791	28	0.842	67	0.736	0.805	0.60	368.39	484.95	1.04	0.49
77,242	24,319	2,301	1.2	26	10,456	4,532	0.0098	184	0.742	28	0.822	67	0.684	0.740	0.84	394.94	468.59	1.00	0.55
132,783	56,636	3,311	1.4	36	9,034	3,806	0.0068	222	0.726	13	0.761	111	0.659	0.726	0.86	396.67	497.24	1.58	0.76
132,783	56,636	3,311	1.0	30	11,203	5,057	0.0427	197	0.739	34	0.790	73	0.690	0.742	0.74	440.60	515.66	0.98	0.59
31,152	11,621	849	1.0	22	6,989	3,199	0.0132	170	0.756	35	0.835	77	0.724	0.754	0.81	218.83	368.86	0.71	0.37
138,491	56,705	3,556	1.5	36	11,915	5,708	0.0121	199	0.732	39	0.797	95	0.700	0.741	0.83	406.87	536.71	1.65	0.71
133,674	44,909	3,859	1.1	31	7,792	2,970	0.0934	191	0.738	40	0.774	118	0.703	0.734	0.58	387.31	622.91	1.19	0.55
133,674	44,909	3,859	1.1	31	7,792	2,970	0.0060	191	0.738	40	0.774	118	0.670	0.734	0.85	428.06	557.70	1.20	0.69
102,697	36,334	2,885	1.2	35	6,899	3,146	0.0329	197	0.735	36	0.760	62	0.685	0.721	0.87	327.38	573.86	1.24	0.60
99,620	40,480	2,571	1.5	30	9,328	3,954	0.0062	213	0.744	36	0.775	81	0.677	0.744	0.88	360.68	535.43	1.35	0.61
153,392	65,146	3,837	1.0	40	14,852	6,850	0.0031	213	0.707	18	0.735	84	0.663	0.730	0.91	477.38	479.25	1.13	0.76
153,392	65,146	3,837	1.0	35	14,852	6,850	0.0031	213	0.715	18	0.782	84	0.674	0.725	0.91	457.72	461.30	1.23	0.79
95,804	33,330	2,716	1.3	33	9,325	4,662	0.0057	188	0.732	38	0.767	130	0.694	0.738	0.86	333.61	494.23	1.44	0.68
159,801	59,885	4,344	1.4	40	11,026	5,239	0.0079	217	0.712	17	0.766	91	0.645	0.722	0.88	471.38	464.96	1.87	0.92
159,801	59,885	4,344	1.0	40	7,061	3,001	0.0984	201	0.727	38	0.792	98	0.674	0.715	0.78	424.59	651.34	1.06	0.61

Table A-3: Data Generated for Eight Stage Treatment Type I, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
63,679	19,251	1,932	1.0	26	5,716	1,901	0.0920	232	0.737	30	0.741	128	0.659	0.724	0.63	269.75	434.16	1.08	0.63
46,004	17,749	1,228	1.0	22	6,907	2,544	0.0086	192	0.773	16	0.803	84	0.695	0.740	0.88	241.12	364.76	0.97	0.57
74,291	28,315	1,999	1.0	25	9,341	4,359	0.0163	200	0.727	18	0.773	105	0.640	0.716	0.81	371.23	404.97	1.05	0.65
86,063	32,393	2,333	1.1	30	10,837	5,295	0.0215	174	0.774	35	0.828	105	0.713	0.757	0.77	395.30	476.09	0.81	0.57
93,207	29,605	2,765	1.3	35	8,042	3,983	0.0065	177	0.746	13	0.836	126	0.643	0.700	0.90	356.54	498.00	1.30	0.70
90,379	36,043	2,362	1.6	36	13,534	6,518	0.0189	190	0.712	17	0.741	110	0.650	0.706	0.77	422.11	447.15	1.36	0.67
95,902	40,866	2,393	1.0	38	6,300	2,200	0.0081	229	0.750	15	0.786	82	0.694	0.733	0.90	300.12	490.88	1.02	0.63
95,902	40,866	2,393	1.5	40	6,733	3,207	0.0086	189	0.768	28	0.800	135	0.694	0.733	0.89	296.42	472.78	1.62	0.72
16,783	5,892	474	1.0	18	8,969	3,882	0.0154	237	0.765	23	0.779	114	0.688	0.749	0.74	168.72	227.32	0.71	0.45
143,195	50,714	4,021	1.0	40	7,328	3,255	0.0135	235	0.741	28	0.740	125	0.668	0.713	0.87	404.33	602.89	1.06	0.70
138,075	55,006	3,612	1.5	38	10,967	4,672	0.0398	196	0.772	35	0.814	106	0.710	0.767	0.71	466.51	563.39	1.22	0.65
112,681	38,202	3,238	1.2	34	14,770	6,617	0.0073	211	0.766	32	0.807	58	0.693	0.740	0.91	450.46	586.68	0.96	0.58
64,651	26,983	1,638	1.0	26	8,862	4,222	0.0022	220	0.736	19	0.732	114	0.669	0.714	0.93	291.90	414.83	0.87	0.58
64,651	26,983	1,638	1.1	29	13,338	6,559	0.0126	182	0.740	17	0.740	100	0.660	0.721	0.79	395.46	333.09	0.95	0.63
159,298	62,937	4,190	1.0	40	12,715	5,816	0.0509	184	0.763	33	0.807	122	0.695	0.754	0.59	633.63	522.44	0.87	0.66
41,663	16,074	1,113	1.0	23	12,272	5,959	0.0118	176	0.721	30	0.783	108	0.641	0.704	0.77	345.55	288.72	0.72	0.55
159,298	62,937	4,190	1.0	40	9,540	4,459	0.0672	188	0.749	29	0.777	122	0.681	0.753	0.88	461.22	509.11	1.15	0.80
55,850	17,308	1,676	1.5	26	5,707	1,887	0.0124	206	0.740	38	0.768	107	0.646	0.733	0.85	271.87	392.26	1.52	0.70
78,597	26,964	2,245	1.1	25	11,453	5,033	0.0880	181	0.782	40	0.795	100	0.689	0.770	0.47	437.63	316.59	1.13	0.66
78,597	26,964	2,245	1.0	26	11,543	4,968	0.0291	221	0.748	36	0.746	79	0.666	0.745	0.75	417.89	427.44	0.78	0.64
89,330	27,864	2,672	1.0	28	5,424	1,885	0.0190	208	0.765	24	0.814	101	0.672	0.756	0.84	339.81	486.98	1.27	0.69
80,016	32,486	2,067	1.1	30	8,815	4,159	0.0095	229	0.766	11	0.765	101	0.687	0.761	0.87	340.49	437.40	1.00	0.64
40,603	14,235	1,146	1.0	20	10,807	5,017	0.0040	192	0.740	12	0.761	115	0.672	0.716	0.86	275.74	334.03	0.95	0.56
132,289	51,246	3,524	1.5	40	11,707	5,011	0.0115	177	0.764	17	0.799	94	0.704	0.767	0.85	434.47	461.12	1.57	0.82
122,749	37,186	3,720	1.0	34	12,278	5,331	0.0271	195	0.727	32	0.727	80	0.662	0.722	0.80	444.02	543.13	1.10	0.70
122,749	37,186	3,720	1.5	37	10,831	4,546	0.0122	175	0.760	21	0.807	56	0.760	0.804	0.96	397.65	445.16	2.18	0.98
41,891	13,280	1,244	1.1	20	7,562	3,028	0.0139	221	0.738	16	0.782	66	0.650	0.710	0.88	254.35	375.07	0.92	0.54
155,634	45,430	4,791	1.1	40	14,804	6,770	0.0086	196	0.762	29	0.830	69	0.680	0.767	0.85	740.11	456.50	1.00	0.78
138,732	43,383	4,146	1.4	37	14,083	6,389	0.0187	201	0.732	39	0.777	117	0.675	0.718	0.73	588.92	569.18	1.10	0.66
149,426	61,524	3,822	1.5	39	9,460	3,785	0.0075	211	0.781	23	0.833	55	0.704	0.780	0.93	425.42	496.47	1.62	0.83

Table A - 4: Data Generated for Eight Stage Treatment Type II

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
138,678	45,982	4,030	1.2	38	13,813	6,481	0.0561	235	0.782	17	0.788	126	0.705	0.775	0.51	603.97	376.33	1.37	0.77
138,678	45,982	4,030	1.0	38	13,813	6,481	0.0561	235	0.780	17	0.812	126	0.725	0.770	0.77	615.85	570.18	0.74	0.60
143,276	59,155	3,657	1.2	40	7,612	3,183	0.0168	226	0.747	27	0.760	76	0.655	0.734	0.87	423.11	554.25	1.20	0.69
124,417	50,200	3,227	1.1	38	11,961	5,261	0.0913	217	0.715	36	0.794	111	0.666	0.722	0.45	522.88	523.96	0.25	0.50
124,417	50,200	3,227	1.0	38	9,227	4,034	0.0337	222	0.755	38	0.760	69	0.679	0.745	0.81	405.11	583.47	0.98	0.58
54,454	20,158	1,491	1.0	25	10,267	4,870	0.0783	224	0.793	29	0.798	105	0.731	0.774	0.44	311.46	403.84	0.76	0.41
73,005	26,591	2,018	1.0	32	14,455	7,046	0.0115	201	0.799	40	0.862	83	0.727	0.789	0.70	546.61	415.01	0.70	0.53
73,005	26,591	2,018	1.5	29	11,124	4,642	0.0069	224	0.718	25	0.752	90	0.663	0.709	0.83	372.04	501.85	0.96	0.51
54,454	20,158	1,491	1.0	27	14,298	6,803	0.0082	203	0.757	15	0.817	118	0.695	0.739	0.67	407.05	344.67	0.66	0.48
147,206	48,441	4,294	1.0	40	9,472	4,378	0.0192	238	0.703	33	0.733	74	0.642	0.700	0.87	410.63	535.58	1.25	0.79
92,236	38,942	2,317	1.5	35	9,958	4,837	0.0386	198	0.742	39	0.775	120	0.688	0.731	0.65	337.97	485.20	0.93	0.47
92,236	38,942	2,317	1.0	27	14,995	7,049	0.0141	235	0.751	29	0.800	122	0.698	0.743	0.55	560.35	298.94	0.82	0.54
121,937	37,916	3,653	1.0	37	7,749	3,673	0.0099	231	0.726	21	0.754	74	0.670	0.724	0.92	354.57	570.72	1.30	0.72
115,429	44,021	3,105	1.0	34	6,988	2,876	0.0076	195	0.783	22	0.807	102	0.712	0.746	0.88	379.60	591.92	0.99	0.57
56,836	24,116	1,423	1.0	27	13,089	5,557	0.0026	192	0.711	27	0.754	92	0.642	0.706	0.84	446.75	367.02	0.68	0.50
37,592	13,923	1,029	1.0	20	14,280	6,606	0.0195	226	0.789	40	0.822	104	0.731	0.781	0.57	357.18	293.92	0.54	0.40
158,466	53,708	4,555	1.5	40	10,086	4,753	0.0042	172	0.748	28	0.778	129	0.691	0.723	0.88	487.39	658.03	1.19	0.67
158,466	53,708	4,555	1.0	35	10,086	4,753	0.0042	172	0.748	28	0.778	129	0.660	0.730	0.84	673.23	542.27	0.81	0.68
59,632	18,033	1,809	1.0	26	11,219	5,407	0.0056	230	0.763	36	0.806	84	0.681	0.743	0.83	433.59	406.77	0.66	0.53
79,217	30,064	2,137	1.1	30	11,623	5,810	0.0068	190	0.789	24	0.796	76	0.712	0.764	0.85	445.00	446.26	0.77	0.54
88,360	35,166	2,313	1.2	28	13,706	6,631	0.0097	235	0.761	35	0.781	111	0.690	0.735	0.72	471.06	404.43	0.90	0.58
81,818	29,503	2,275	1.3	25	11,299	4,905	0.0551	239	0.743	14	0.801	53	0.682	0.723	0.79	344.22	523.07	1.00	0.53
110,542	38,403	3,136	1.0	35	6,498	2,589	0.0159	173	0.726	26	0.774	125	0.651	0.708	0.79	391.91	570.15	1.11	0.60
128,712	49,712	3,435	1.5	35	7,768	3,663	0.0026	178	0.770	10	0.796	110	0.681	0.741	0.92	430.53	572.50	1.27	0.69
91,748	33,408	2,537	1.0	28	8,724	4,074	0.0085	217	0.731	35	0.771	67	0.653	0.712	0.90	379.30	541.50	0.79	0.55
91,748	33,408	2,537	1.2	27	9,358	4,330	0.0805	227	0.770	27	0.770	60	0.680	0.756	0.70	385.65	503.95	1.01	0.59
91,748	33,408	2,537	1.1	30	5,732	2,737	0.0073	215	0.774	28	0.818	117	0.709	0.743	0.88	372.57	599.30	0.99	0.57
106,618	34,870	3,119	1.3	35	10,995	4,947	0.0095	173	0.710	13	0.752	60	0.651	0.702	0.90	401.64	588.90	1.04	0.58
104,289	39,812	2,803	1.0	32	8,304	3,619	0.0235	193	0.787	39	0.785	69	0.704	0.765	0.83	377.20	546.75	1.07	0.60
141,482	46,557	4,127	1.1	38	13,274	6,369	0.0193	234	0.768	23	0.795	76	0.682	0.746	0.82	714.21	425.05	0.94	0.72

Table A-4: Data Generated for Eight Stage Treatment Type II, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
127,660	41,749	3,735	1.2	34	13,011	6,391	0.0273	214	0.713	30	0.756	103	0.651	0.702	0.69	608.30	564.67	0.81	0.57
70,750	22,410	2,102	1.0	22	9,273	4,310	0.0039	203	0.738	32	0.771	62	0.657	0.727	0.91	390.04	430.88	0.89	0.58
70,750	22,410	2,102	1.2	22	5,158	2,509	0.0066	219	0.746	32	0.742	59	0.663	0.720	0.94	306.76	557.90	0.91	0.50
137,020	47,245	3,903	1.0	25	10,735	4,945	0.0714	238	0.766	31	0.803	77	0.700	0.744	0.67	535.73	606.61	0.78	0.55
140,536	42,451	4,265	1.0	35	6,096	2,176	0.0024	235	0.721	13	0.788	108	0.670	0.701	0.94	366.81	608.13	1.23	0.74
140,536	42,451	4,265	1.5	40	14,005	6,596	0.0099	209	0.722	34	0.784	119	0.645	0.736	0.70	765.67	411.92	1.28	0.76
136,536	50,324	3,748	1.0	36	13,710	6,473	0.0072	172	0.741	35	0.794	130	0.684	0.732	0.84	582.24	566.45	0.81	0.57
153,097	62,888	3,922	1.5	19	6,552	3,227	0.0023	216	0.768	16	0.768	126	0.691	0.729	0.93	400.49	674.56	1.29	0.65
67,375	28,188	1,704	1.0	24	14,171	6,611	0.0079	236	0.745	16	0.778	84	0.650	0.722	0.71	570.48	267.73	0.72	0.55
67,375	28,188	1,704	1.0	24	14,171	6,611	0.0079	236	0.730	16	0.770	84	0.685	0.715	0.80	392.71	462.02	0.60	0.43
67,375	28,188	1,704	1.0	28	9,864	4,003	0.0173	228	0.760	26	0.816	72	0.695	0.753	0.79	339.15	430.14	0.82	0.51
67,375	28,188	1,704	1.0	22	11,088	4,591	0.0047	217	0.721	15	0.786	134	0.670	0.733	0.77	350.39	402.98	0.95	0.53
22,930	8,145	643	1.2	15	6,502	2,865	0.0494	200	0.705	25	0.768	53	0.665	0.709	0.74	178.86	332.28	0.73	0.35
127,168	39,055	3,831	1.0	36	14,234	6,743	0.0097	187	0.735	39	0.789	76	0.673	0.720	0.82	568.61	648.98	0.74	0.56
105,250	42,105	2,745	1.0	37	13,001	6,150	0.0053	194	0.730	16	0.782	80	0.666	0.718	0.87	516.62	506.02	0.67	0.55
120,705	37,616	3,613	1.3	40	7,167	3,208	0.0036	216	0.730	18	0.768	73	0.657	0.708	0.95	366.54	553.18	1.52	0.76
151,397	45,473	4,605	1.0	23	13,333	6,480	0.0910	233	0.773	15	0.811	63	0.707	0.761	0.68	539.22	555.16	0.99	0.67
72,128	27,230	1,952	1.0	25	5,699	2,554	0.0036	222	0.743	32	0.792	56	0.677	0.730	0.95	300.55	486.75	0.86	0.53
72,128	27,230	1,952	1.3	27	7,533	2,871	0.0634	194	0.715	17	0.796	84	0.652	0.704	0.66	323.30	483.80	0.98	0.48
72,128	27,230	1,952	1.0	24	9,016	3,836	0.0041	200	0.730	21	0.793	119	0.681	0.716	0.85	351.18	514.13	0.69	0.45
98,112	40,371	2,510	1.4	27	14,020	6,293	0.0084	213	0.723	27	0.762	98	0.651	0.715	0.72	622.92	324.59	1.04	0.63
35,402	14,439	911	1.3	21	15,141	6,905	0.0913	223	0.743	19	0.800	81	0.700	0.740	0.28	283.76	318.56	0.82	0.33
97,235	32,869	2,799	1.0	30	6,538	2,303	0.0069	212	0.705	14	0.784	128	0.672	0.708	0.86	332.94	550.14	1.00	0.59
51,315	20,035	1,360	1.2	20	10,149	4,465	0.0034	231	0.715	26	0.782	72	0.678	0.713	0.92	373.69	534.99	1.08	0.57
51,315	20,035	1,360	1.0	25	10,869	5,232	0.0431	186	0.785	29	0.825	61	0.726	0.800	0.70	281.24	323.40	0.96	0.58
139,592	46,170	4,062	1.2	34	10,651	5,313	0.0047	197	0.731	17	0.783	66	0.702	0.721	0.93	402.85	583.90	1.28	0.70
49,533	18,841	1,334	1.1	24	11,292	5,603	0.0574	219	0.710	22	0.788	129	0.672	0.704	0.78	315.88	455.21	0.64	0.39
49,533	18,841	1,334	1.0	22	5,520	1,776	0.0153	202	0.733	34	0.791	60	0.650	0.717	0.84	282.58	430.37	0.70	0.42
49,533	18,841	1,334	1.0	27	7,886	2,337	0.0073	237	0.764	17	0.802	76	0.715	0.746	0.85	260.27	437.73	0.75	0.43
25,949	7,835	788	1.0	22	10,413	4,640	0.0177	171	0.727	27	0.796	100	0.677	0.712	0.59	249.48	330.15	0.68	0.35

Table A-4: Data Generated for Eight Stage Treatment Type II, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
93,082	31,905	2,660	1.0	31	8,758	4,018	0.0054	224	0.758	57	0.819	105	0.707	0.788	0.86	402.16	580.01	0.82	0.50
97,549	39,482	2,525	1.1	35	10,608	5,252	0.0019	192	0.705	34	0.762	84	0.646	0.702	0.93	420.58	492.47	0.88	0.57
39,915	14,788	1,092	1.0	23	13,103	6,063	0.0086	173	0.758	27	0.835	99	0.707	0.747	0.66	349.27	377.88	0.53	0.38
121,612	45,805	3,296	1.0	35	14,651	6,627	0.0147	186	0.740	35	0.791	104	0.666	0.726	0.65	693.96	442.04	0.69	0.61
44,691	16,562	1,223	1.0	25	11,677	5,478	0.0569	236	0.710	27	0.769	80	0.660	0.702	0.53	318.50	380.37	0.72	0.39
44,691	16,562	1,223	1.0	22	8,210	6,266	0.0053	173	0.755	18	0.755	78	0.705	0.741	0.88	268.16	487.37	0.74	0.38
77,369	26,883	2,195	1.0	27	10,622	4,334	0.0050	174	0.726	23	0.788	98	0.658	0.718	0.82	426.34	455.68	0.89	0.55
43,515	15,612	1,213	1.0	17	7,302	3,080	0.0136	186	0.740	16	0.738	90	0.677	0.730	0.75	284.44	421.76	0.65	0.40
77,369	26,883	2,195	1.0	25	5,708	2,407	0.0199	201	0.779	38	0.845	106	0.710	0.770	0.76	339.87	510.03	0.90	0.50
42,098	12,493	1,287	1.5	20	7,389	3,165	0.0754	225	0.750	37	0.753	133	0.667	0.746	0.36	244.62	357.79	1.31	0.45
42,098	12,493	1,287	1.0	24	15,046	6,647	0.0750	206	0.760	24	0.800	87	0.684	0.724	0.69	380.39	337.30	0.58	0.43
39,794	15,310	1,065	1.0	25	9,408	4,272	0.0052	237	0.768	27	0.812	56	0.693	0.743	0.85	293.31	403.63	0.58	0.40
102,076	35,351	2,901	1.5	36	9,828	4,688	0.0138	232	0.728	14	0.803	94	0.671	0.714	0.81	390.30	546.76	1.25	0.61
123,820	52,370	3,107	1.0	39	7,338	3,283	0.0045	201	0.746	13	0.789	101	0.677	0.716	0.91	377.59	582.03	1.01	0.60
45,898	16,136	1,294	1.5	29	5,452	2,700	0.0043	228	0.767	18	0.767	98	0.709	0.776	0.90	244.21	420.78	1.12	0.50
68,692	29,038	1,724	1.1	25	12,656	6,164	0.0131	202	0.746	29	0.777	88	0.682	0.730	0.72	434.66	422.92	0.66	0.47
91,813	27,917	2,778	1.0	26	6,687	2,354	0.0297	228	0.757	13	0.678	96	0.743	0.773	0.71	386.44	534.08	0.96	0.55
68,692	29,038	1,724	1.0	21	10,770	5,477	0.0079	181	0.743	28	0.799	97	0.689	0.730	0.79	379.84	487.19	0.67	0.45
149,147	55,050	4,091	1.0	35	13,126	5,649	0.0304	203	0.755	11	0.817	91	0.695	0.746	0.66	586.54	564.92	0.79	0.58
114,965	42,156	3,166	1.1	30	7,269	3,554	0.0553	202	0.729	26	0.787	129	0.686	0.723	0.68	369.49	596.66	1.02	0.52
69,356	22,999	2,016	1.1	27	7,622	2,985	0.0178	174	0.734	36	0.795	125	0.700	0.725	0.73	305.45	509.15	0.92	0.47
49,091	18,817	1,316	1.2	22	9,835	4,386	0.0634	200	0.749	18	0.792	51	0.688	0.738	0.71	296.57	404.58	0.85	0.45
93,789	28,640	2,833	1.1	28	12,446	5,251	0.0322	191	0.744	23	0.744	118	0.672	0.746	0.50	525.90	413.41	0.94	0.57
69,356	22,999	2,016	1.0	24	10,951	4,770	0.0439	189	0.773	16	0.849	70	0.731	0.783	0.69	305.79	401.44	1.05	0.59
92,167	37,456	2,379	1.1	30	11,889	5,818	0.0616	186	0.732	15	0.782	127	0.689	0.720	0.52	383.77	525.36	0.83	0.44
155,619	61,119	4,109	1.0	40	12,858	5,946	0.0047	226	0.744	40	0.781	114	0.696	0.746	0.85	519.30	542.93	0.94	0.66
92,167	37,456	2,379	1.0	26	7,352	3,273	0.0179	236	0.738	31	0.755	65	0.700	0.740	0.88	314.93	589.51	0.82	0.51
152,139	49,896	4,445	1.0	38	6,497	2,727	0.0834	174	0.748	28	0.818	96	0.708	0.745	0.74	372.45	619.68	1.24	0.70
109,039	41,433	2,939	1.1	31	14,175	6,963	0.0261	238	0.777	25	0.857	71	0.722	0.777	0.72	483.22	483.88	0.89	0.56
109,039	41,433	2,939	1.0	35	8,963	4,134	0.0056	226	0.760	36	0.764	106	0.673	0.758	0.86	433.27	502.59	0.87	0.65

Table A-4: Data Generated for Eight Stage Treatment Type II, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
47,834	19,869	1,216	1.0	22	7,462	3,470	0.0182	202	0.757	37	0.783	59	0.698	0.748	0.84	273.81	412.45	0.69	0.44
150,612	58,126	4,021	1.0	36	12,819	5,643	0.0144	223	0.718	11	0.784	102	0.649	0.711	0.74	643.30	537.34	0.75	0.62
95,175	37,670	2,500	1.2	27	8,362	3,627	0.0076	189	0.755	11	0.755	94	0.677	0.742	0.85	383.42	522.73	0.96	0.58
158,413	48,534	4,777	1.1	36	13,538	6,481	0.0302	231	0.749	35	0.775	75	0.684	0.730	0.82	508.02	612.85	1.08	0.67
65,664	20,350	1,970	1.0	22	10,847	4,874	0.0123	204	0.726	25	0.796	128	0.677	0.719	0.68	393.53	481.21	0.67	0.44
136,859	49,791	3,786	1.0	33	12,609	6,291	0.0164	206	0.708	31	0.733	133	0.657	0.709	0.73	493.55	519.57	1.05	0.65
94,723	27,559	2,920	1.2	24	9,865	4,024	0.0446	180	0.739	36	0.763	114	0.686	0.736	0.56	416.77	524.30	1.03	0.52
38,145	14,872	1,012	1.5	20	6,766	3,085	0.0758	199	0.701	35	0.769	127	0.642	0.702	0.46	228.27	350.32	1.22	0.44
41,528	13,474	1,220	1.0	22	9,265	4,000	0.0064	206	0.799	14	0.855	66	0.736	0.782	0.88	296.95	412.38	0.64	0.43
103,936	39,027	2,822	1.1	34	5,717	2,296	0.0846	200	0.766	22	0.830	116	0.716	0.775	0.64	329.02	541.53	1.12	0.57
126,478	49,374	3,352	1.0	39	11,937	5,169	0.0114	211	0.745	40	0.812	78	0.684	0.746	0.83	502.59	515.85	0.79	0.58
120,086	37,040	3,611	1.2	35	9,734	3,875	0.0299	218	0.750	28	0.748	51	0.692	0.748	0.86	380.49	670.43	1.09	0.55
91,899	26,981	2,823	1.0	27	5,702	2,127	0.0194	199	0.718	38	0.756	99	0.647	0.708	0.75	395.79	535.74	0.86	0.55
42,476	14,372	1,222	1.2	23	9,453	4,169	0.0685	234	0.766	24	0.764	106	0.719	0.760	0.48	250.83	490.29	0.96	0.41
55,611	19,308	1,578	1.2	28	12,930	5,587	0.0157	240	0.740	23	0.783	71	0.671	0.723	0.72	404.24	410.16	0.70	0.47
55,611	19,308	1,578	1.5	26	9,692	4,543	0.0073	213	0.788	25	0.788	129	0.714	0.765	0.74	326.14	429.47	1.04	0.52
102,282	37,386	2,822	1.0	30	14,459	6,655	0.0255	193	0.715	37	0.761	124	0.662	0.705	0.52	557.43	495.74	0.81	0.51
65,586	25,214	1,755	1.0	28	10,596	5,427	0.0073	171	0.728	36	0.796	93	0.677	0.720	0.80	426.27	433.90	0.61	0.47
65,586	25,214	1,755	1.3	25	12,087	6,013	0.0056	210	0.733	25	0.779	112	0.680	0.727	0.78	391.95	453.55	0.78	0.49
83,256	29,570	2,334	1.1	29	8,053	3,205	0.0117	219	0.750	30	0.750	75	0.662	0.737	0.84	365.06	523.34	0.86	0.56
33,212	9,875	1,015	1.0	29	11,470	4,850	0.0044	230	0.716	16	0.767	127	0.660	0.710	0.74	291.98	367.49	0.67	0.41
149,723	57,602	4,005	1.0	40	5,420	1,909	0.0055	206	0.763	15	0.828	69	0.684	0.753	0.94	371.51	538.03	1.44	0.80
123,416	47,500	3,301	1.1	37	8,083	3,516	0.0187	184	0.787	34	0.789	109	0.700	0.768	0.83	434.29	577.43	0.93	0.63
69,828	21,423	2,105	1.5	26	7,717	3,748	0.0087	210	0.768	32	0.811	77	0.707	0.754	0.87	342.24	501.70	1.09	0.53
69,828	21,423	2,105	1.0	27	8,289	3,347	0.0026	223	0.742	33	0.784	85	0.673	0.730	0.90	373.44	487.93	0.74	0.52
81,374	34,176	2,052	1.4	32	8,737	4,187	0.0281	178	0.718	10	0.769	88	0.655	0.709	0.73	330.37	464.65	1.22	0.57
81,374	34,176	2,052	1.1	30	10,869	5,402	0.0010	201	0.758	16	0.809	119	0.679	0.744	0.90	477.35	417.33	0.72	0.56
145,639	43,204	4,454	1.0	40	7,899	3,392	0.0091	203	0.794	28	0.794	96	0.713	0.776	0.87	444.04	651.25	1.10	0.67
80,444	32,311	2,093	1.0	29	5,201	2,598	0.0047	170	0.741	31	0.744	87	0.654	0.726	0.93	329.28	510.75	0.80	0.53
80,444	32,311	2,093	1.0	24	10,546	4,018	0.0053	187	0.795	26	0.843	58	0.720	0.790	0.82	414.25	423.09	0.94	0.57

Table A-4: Data Generated for Eight Stage Treatment Type II, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
50,211	20,140	1,307	1.0	28	11,328	4,890	0.0080	194	0.752	30	0.756	105	0.675	0.738	0.71	373.64	325.62	0.68	0.48
52,826	18,210	1,505	1.0	23	8,358	3,190	0.0020	186	0.735	25	0.800	109	0.660	0.720	0.87	354.36	427.12	0.70	0.47
80,504	28,978	2,240	1.0	28	12,475	6,184	0.0072	172	0.755	20	0.811	123	0.681	0.733	0.72	504.13	349.88	1.00	0.59
58,543	19,779	1,685	1.0	27	5,209	1,829	0.0146	182	0.735	38	0.799	99	0.655	0.736	0.88	304.30	469.97	0.76	0.47
124,914	50,409	3,239	1.0	32	6,980	3,073	0.0071	175	0.747	17	0.742	52	0.675	0.721	0.94	377.40	616.82	1.00	0.57
58,543	19,779	1,685	1.0	29	9,756	3,390	0.0186	207	0.742	26	0.813	118	0.682	0.722	0.62	329.07	433.51	0.74	0.44
88,586	37,607	2,217	1.5	26	7,196	3,468	0.0014	238	0.777	22	0.843	91	0.697	0.768	0.95	357.75	484.58	1.14	0.59
88,586	37,607	2,217	1.0	30	12,667	5,359	0.0041	180	0.716	15	0.713	120	0.661	0.707	0.80	480.16	472.99	0.63	0.62
72,751	27,560	1,965	1.2	22	9,260	4,093	0.0052	176	0.748	23	0.804	75	0.683	0.731	0.87	378.14	490.47	0.80	0.49
72,751	27,560	1,965	1.1	29	11,103	4,492	0.0175	224	0.728	20	0.785	63	0.649	0.717	0.78	386.37	423.12	0.90	0.56
72,751	27,560	1,965	1.0	25	14,203	6,358	0.0022	205	0.766	13	0.766	121	0.699	0.742	0.84	483.23	348.12	0.74	0.54
42,518	15,003	1,196	1.0	22	12,634	6,279	0.0487	208	0.770	29	0.826	112	0.675	0.788	0.26	364.25	272.68	0.77	0.36
104,817	42,057	2,729	1.0	33	6,384	2,654	0.0091	196	0.750	23	0.803	78	0.660	0.742	0.88	383.62	485.49	1.05	0.63
40,965	16,469	1,065	1.5	25	9,938	4,469	0.0031	172	0.725	28	0.785	53	0.672	0.723	0.92	282.59	384.61	0.87	0.45
132,165	50,812	3,537	1.0	26	14,429	6,420	0.0223	177	0.764	18	0.822	72	0.684	0.751	0.75	749.51	386.49	0.87	0.66
86,244	32,615	2,332	1.5	20	12,143	5,988	0.0097	202	0.756	38	0.828	53	0.705	0.747	0.89	373.15	545.50	1.11	0.57
119,614	42,724	3,343	1.0	26	10,655	4,331	0.0857	181	0.767	10	0.794	100	0.655	0.749	0.66	423.15	498.00	1.22	0.64
59,099	25,195	1,474	1.0	19	12,995	6,136	0.0337	221	0.759	34	0.818	82	0.689	0.780	0.61	381.82	424.66	0.64	0.43
144,233	46,207	4,262	1.3	22	14,254	6,728	0.0618	201	0.771	12	0.745	54	0.736	0.735	0.65	480.94	669.01	0.85	0.54
102,037	43,794	2,532	1.2	36	14,780	6,395	0.0182	190	0.751	22	0.797	66	0.692	0.740	0.79	466.98	479.09	0.77	0.51
86,016	35,617	2,191	1.0	24	9,045	3,773	0.0057	214	0.733	24	0.783	82	0.647	0.720	0.85	421.44	454.28	0.90	0.57
46,520	15,424	1,352	1.0	25	14,705	6,927	0.0336	215	0.712	40	0.761	96	0.656	0.708	0.42	423.02	345.26	0.59	0.44
46,520	15,424	1,352	1.5	20	6,063	2,383	0.0050	187	0.755	27	0.828	81	0.660	0.750	0.86	303.46	394.23	1.09	0.52
156,719	48,984	4,684	1.0	40	6,603	2,498	0.0199	198	0.751	23	0.816	65	0.695	0.729	0.90	392.79	639.60	1.44	0.73
39,830	15,687	1,050	1.0	20	8,033	3,965	0.0093	228	0.777	30	0.777	104	0.699	0.769	0.75	284.47	360.17	0.73	0.43
140,572	41,351	4,314	1.0	38	8,750	3,922	0.0247	194	0.767	26	0.801	116	0.719	0.756	0.77	418.74	631.42	1.05	0.63
121,642	51,158	3,064	1.0	29	5,746	2,077	0.0403	227	0.739	21	0.762	67	0.670	0.714	0.83	360.36	584.09	0.93	0.56
80,665	24,969	2,422	1.0	24	9,015	4,203	0.0137	210	0.744	33	0.790	109	0.704	0.744	0.78	334.57	505.42	0.92	0.54
80,665	24,969	2,422	1.2	27	7,785	3,744	0.0402	216	0.729	18	0.726	74	0.689	0.729	0.81	312.14	563.49	1.06	0.53
154,167	55,952	4,270	1.5	37	8,755	3,484	0.0263	215	0.719	26	0.762	110	0.669	0.715	0.77	402.60	591.23	1.60	0.73

Table A-4: Data Generated for Eight Stage Treatment Type II, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
57,765	19,846	1,649	1.1	25	10,350	4,242	0.0189	209	0.731	27	0.788	97	0.673	0.724	0.69	359.15	438.85	0.74	0.45
127,363	50,235	3,353	1.0	33	9,014	4,274	0.0417	222	0.736	36	0.761	117	0.684	0.721	0.68	445.74	591.34	0.82	0.53
141,600	45,664	4,171	1.2	37	14,707	7,264	0.0073	237	0.725	11	0.726	102	0.656	0.725	0.80	644.85	434.45	1.15	0.77
35,078	12,763	970	1.0	18	6,261	2,950	0.0957	201	0.704	36	0.736	117	0.658	0.702	0.48	219.61	372.95	0.76	0.36
52,007	21,146	1,342	1.3	20	5,146	2,003	0.0327	232	0.723	15	0.795	89	0.666	0.716	0.75	244.48	421.25	1.09	0.48
73,325	21,591	2,249	1.0	28	9,720	4,493	0.0081	202	0.755	22	0.750	62	0.684	0.750	0.89	337.12	525.65	1.00	0.56
94,657	37,154	2,500	1.2	31	14,018	6,438	0.0223	240	0.713	20	0.751	116	0.646	0.710	0.57	555.40	381.84	0.91	0.58
70,573	23,229	2,058	1.0	25	11,967	5,514	0.0086	186	0.782	39	0.780	73	0.725	0.780	0.83	362.48	515.07	0.71	0.48
122,772	52,504	3,055	1.0	30	12,932	6,390	0.0435	226	0.774	38	0.850	133	0.727	0.770	0.46	528.66	526.45	0.74	0.47
80,154	26,011	2,354	1.1	30	11,943	5,647	0.0535	205	0.743	27	0.819	62	0.702	0.746	0.73	368.46	494.78	0.91	0.49
26,021	8,145	777	1.0	19	8,146	3,429	0.0289	191	0.758	30	0.784	113	0.685	0.739	0.52	231.85	320.49	0.67	0.36
113,346	36,042	3,361	1.1	30	7,916	3,439	0.0075	192	0.742	13	0.828	87	0.679	0.741	0.90	376.94	508.12	1.29	0.75
65,591	22,774	1,862	1.0	26	9,610	4,445	0.0322	176	0.730	34	0.780	83	0.685	0.717	0.73	335.51	487.03	0.81	0.45
157,130	55,704	4,410	1.0	40	10,423	2,631	0.0117	171	0.712	11	0.712	80	0.653	0.713	0.89	457.39	688.16	0.90	0.58
85,480	25,777	2,596	1.0	29	7,702	3,443	0.0272	230	0.733	20	0.803	131	0.685	0.728	0.56	342.82	528.55	0.92	0.49
79,688	31,811	2,082	1.0	26	10,858	5,086	0.0093	207	0.751	23	0.820	76	0.677	0.737	0.82	442.41	464.72	0.73	0.52
81,656	27,304	2,363	1.0	25	11,396	4,725	0.0540	237	0.720	32	0.766	89	0.672	0.724	0.61	367.47	454.01	0.91	0.53
129,718	40,822	3,865	1.1	34	7,900	3,423	0.0204	177	0.725	27	0.848	55	0.675	0.718	0.90	379.45	675.87	1.07	0.58
91,737	29,964	2,686	1.0	27	11,673	5,835	0.0093	191	0.762	30	0.781	80	0.687	0.755	0.81	492.04	440.40	0.80	0.61
63,266	21,085	1,834	1.0	28	12,826	5,556	0.0109	176	0.750	37	0.787	61	0.704	0.725	0.85	348.34	523.31	0.65	0.44

Table A-4: Data Generated for Eight Stage Treatment Type II, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
150,920	58,932	3,999	1.2	26	11,296	5,131	0.0050	199	0.768	15	0.795	122	0.691	0.750	0.82	607.37	548.52	0.90	0.65
120,588	48,621	3,129	1.5	35	12,317	5,564	0.0300	199	0.780	30	0.842	129	0.730	0.770	0.58	467.25	552.50	1.14	0.54
166,561	57,026	4,762	1.0	40	8,775	3,945	0.0176	228	0.726	35	0.770	99	0.667	0.710	0.85	474.33	652.50	0.96	0.63
87,117	28,665	2,541	1.3	30	8,219	3,311	0.0085	209	0.733	36	0.733	89	0.670	0.722	0.87	343.01	537.45	1.13	0.59
173,529	56,395	5,093	1.5	39	10,588	3,578	0.0126	226	0.709	19	0.782	72	0.645	0.704	0.86	457.38	564.84	1.85	0.88
53,308	21,036	1,403	1.0	25	5,844	2,827	0.0172	227	0.724	39	0.761	55	0.650	0.716	0.91	274.29	424.56	0.75	0.48
149,441	45,261	4,530	1.5	40	7,390	3,559	0.0085	224	0.745	13	0.825	78	0.688	0.732	0.92	375.32	580.88	1.96	0.87
143,020	49,409	4,070	1.0	38	8,345	3,986	0.0017	231	0.728	38	0.795	78	0.665	0.722	0.96	444.29	581.15	0.99	0.66
115,293	35,510	3,469	1.0	33	5,457	2,171	0.0108	218	0.751	13	0.751	82	0.673	0.755	0.90	370.19	619.39	0.95	0.60
54,051	21,529	1,414	1.3	22	14,071	6,769	0.0043	185	0.745	27	0.809	71	0.646	0.742	0.84	583.49	331.47	0.59	0.51
158,002	48,685	4,753	1.5	39	6,273	2,446	0.0056	235	0.765	34	0.798	84	0.697	0.749	0.93	382.84	664.47	1.76	0.78
40,412	16,744	1,029	1.0	23	12,807	6,343	0.0063	192	0.723	24	0.783	63	0.679	0.723	0.86	299.51	375.18	0.57	0.40
117,574	44,512	3,177	1.4	29	7,855	3,873	0.0096	176	0.717	18	0.763	120	0.667	0.715	0.89	355.02	533.61	1.47	0.70
60,132	23,060	1,612	1.1	27	13,049	5,813	0.0169	236	0.773	36	0.816	86	0.720	0.776	0.68	368.58	393.97	0.76	0.49
39,445	14,333	1,092	1.0	20	6,208	2,125	0.0010	236	0.788	20	0.793	94	0.701	0.780	0.93	278.36	386.86	0.64	0.45
71,789	24,287	2,065	1.6	22	6,180	2,989	0.0485	177	0.743	11	0.803	69	0.666	0.756	0.79	279.86	425.32	1.74	0.72
154,047	49,734	4,535	1.3	17	9,749	4,856	0.0080	193	0.793	17	0.864	53	0.699	0.808	0.93	437.87	418.19	2.02	1.10
182,032	66,949	5,004	2.0	40	7,201	2,959	0.0163	220	0.745	36	0.780	103	0.672	0.734	0.82	430.53	609.43	2.39	0.87
77,004	25,387	2,244	1.2	30	5,224	2,360	0.0032	178	0.763	38	0.824	94	0.701	0.741	0.93	324.61	532.59	0.98	0.53
67,442	19,753	2,073	1.0	26	11,440	6,258	0.0060	180	0.744	26	0.805	73	0.687	0.722	0.90	392.21	532.28	0.62	0.46
77,959	28,224	2,162	1.5	28	8,692	2,730	0.0134	196	0.711	12	0.760	71	0.667	0.705	0.87	290.81	458.33	1.53	0.66
54,463	21,307	1,442	1.0	25	15,187	7,000	0.0056	197	0.750	21	0.803	56	0.679	0.723	0.89	301.92	413.81	0.72	0.50
60,139	24,443	1,552	1.1	26	7,297	3,112	0.0145	175	0.716	36	0.782	106	0.650	0.700	0.78	308.45	439.06	0.79	0.47
178,023	49,882	5,571	1.5	31	13,146	5,779	0.0084	222	0.746	38	0.807	95	0.688	0.737	0.83	607.52	621.65	1.39	0.73
39,518	15,000	1,066	1.0	20	5,587	1,919	0.0167	191	0.717	26	0.738	73	0.645	0.710	0.81	246.12	385.75	0.70	0.41
82,811	25,508	2,491	1.5	30	14,369	6,819	0.0146	239	0.750	27	0.787	69	0.690	0.735	0.78	440.10	477.59	1.11	0.58
160,396	59,530	4,385	1.4	40	10,185	5,035	0.0054	206	0.751	23	0.751	114	0.642	0.743	0.83	690.44	455.87	1.22	0.80
75,397	24,241	2,224	1.0	27	12,831	6,019	0.0163	207	0.781	11	0.781	59	0.721	0.786	0.82	358.00	551.99	0.70	0.47
72,240	27,732	1,935	1.0	26	11,246	5,385	0.0074	201	0.713	20	0.760	77	0.670	0.714	0.87	333.12	437.95	0.83	0.54
54,130	19,583	1,502	1.0	23	5,209	1,715	0.0027	221	0.731	12	0.779	117	0.683	0.730	0.90	263.10	462.29	0.77	0.46

Table A - 5: Data Generated for Eight Stage Treatment Type III

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
99,350	30,838	2,979	1.2	35	12,133	5,079	0.0072	234	0.730	33	0.777	86	0.644	0.726	0.81	546.77	427.37	0.96	0.68
103,791	41,044	2,728	1.0	31	14,140	6,475	0.0159	196	0.744	27	0.740	64	0.680	0.728	0.79	547.63	502.01	0.62	0.52
177,597	69,592	4,696	1.3	40	7,277	3,396	0.0089	199	0.768	22	0.837	82	0.695	0.770	0.91	407.71	534.55	1.75	0.91
186,757	56,300	5,672	1.1	40	13,737	6,037	0.0076	174	0.759	13	0.809	109	0.676	0.736	0.76	815.34	465.74	0.84	0.70
74,092	24,456	2,158	1.2	27	7,110	3,433	0.0096	172	0.779	35	0.830	52	0.712	0.770	0.92	326.23	471.23	1.06	0.58
130,445	37,493	4,041	1.4	34	12,983	5,847	0.0066	212	0.717	35	0.783	130	0.657	0.708	0.77	620.07	570.97	0.99	0.64
169,498	50,147	5,189	1.3	38	9,171	3,459	0.0032	219	0.751	23	0.837	70	0.717	0.750	0.94	393.25	604.62	1.78	0.89
172,163	66,852	4,579	1.0	37	11,624	5,017	0.0441	215	0.727	24	0.772	92	0.679	0.716	0.71	505.19	636.56	0.89	0.58
78,979	30,898	2,090	1.0	25	7,775	3,026	0.0129	215	0.755	39	0.797	123	0.671	0.734	0.73	388.26	465.50	0.73	0.52
142,588	50,393	4,008	1.1	36	8,784	3,505	0.0080	203	0.749	23	0.824	81	0.686	0.745	0.89	432.01	550.90	1.16	0.71
106,428	34,060	3,146	1.5	32	10,222	4,298	0.0101	202	0.717	20	0.806	127	0.644	0.758	0.75	491.57	512.33	1.18	0.64
174,501	64,493	4,783	1.6	40	9,916	4,629	0.0117	170	0.720	34	0.720	79	0.677	0.720	0.90	420.09	733.78	1.56	0.65
128,272	53,339	3,258	1.0	37	5,359	2,454	0.0705	185	0.713	37	0.760	58	0.667	0.710	0.87	320.97	600.48	1.06	0.64
73,443	21,102	2,276	1.1	30	5,943	2,405	0.0448	207	0.751	23	0.807	107	0.685	0.744	0.70	323.24	501.60	0.97	0.52
113,562	35,617	3,389	1.4	36	11,518	2,347	0.0034	179	0.731	12	0.774	113	0.672	0.724	0.86	464.11	501.02	1.28	0.71
184,928	56,620	5,579	1.6	37	14,329	6,660	0.0178	183	0.760	26	0.839	72	0.673	0.746	0.76	881.70	580.17	1.09	0.71
87,803	26,176	2,679	1.2	26	14,257	6,794	0.0014	193	0.750	30	0.748	62	0.680	0.745	0.93	502.58	495.11	0.81	0.59
47,449	17,512	1,302	1.0	24	6,757	2,831	0.0030	208	0.766	36	0.823	129	0.696	0.741	0.86	280.91	406.82	0.72	0.46
50,218	21,117	1,265	1.0	22	10,314	3,143	0.0097	171	0.711	17	0.780	83	0.645	0.701	0.76	315.00	383.72	0.65	0.46
139,169	45,204	4,085	1.5	31	7,766	3,799	0.0033	237	0.720	21	0.862	79	0.674	0.720	0.94	360.18	547.78	1.93	0.87
94,298	31,438	2,733	1.0	29	5,300	2,645	0.0470	220	0.767	38	0.769	62	0.651	0.759	0.85	355.83	533.33	0.90	0.59
128,876	52,076	3,339	1.4	34	13,201	6,039	0.0100	193	0.783	23	0.865	81	0.708	0.780	0.78	603.85	485.64	1.00	0.63
115,072	43,506	3,112	1.1	33	11,381	4,746	0.0040	202	0.749	15	0.804	87	0.695	0.753	0.89	409.06	458.60	1.14	0.72
88,942	37,294	2,246	1.5	26	7,484	3,287	0.0135	220	0.740	11	0.740	115	0.640	0.728	0.77	387.77	458.24	1.19	0.62
37,767	11,990	1,121	1.0	20	13,097	5,660	0.0070	172	0.736	31	0.781	79	0.675	0.728	0.73	392.99	355.50	0.50	0.40
40,772	12,593	1,225	1.0	21	10,534	6,085	0.0029	223	0.723	36	0.722	117	0.664	0.714	0.88	327.01	404.18	0.58	0.44
185,701	69,173	5,066	1.1	40	8,246	3,947	0.0028	193	0.714	33	0.801	118	0.689	0.738	0.93	452.29	615.21	1.26	0.75
185,900	75,494	4,800	1.5	40	13,218	6,022	0.0329	232	0.738	24	0.731	134	0.681	0.730	0.61	551.64	620.40	1.32	0.68
158,850	52,850	4,609	1.3	35	6,903	2,923	0.0062	181	0.734	15	0.764	108	0.698	0.733	0.91	390.21	725.13	1.32	0.67
184,893	73,374	4,849	1.1	40	6,407	2,786	0.0065	171	0.770	25	0.770	74	0.710	0.773	0.94	416.00	730.35	1.10	0.62

Table A-5: Data Generated for Eight Stage Treatment Type III, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
92,152	38,288	2,342	1.6	33	11,832	5,561	0.0046	190	0.775	14	0.777	80	0.720	0.765	0.88	372.60	541.02	1.16	0.56
83,557	32,857	2,204	1.2	30	8,906	3,463	0.0190	174	0.744	18	0.800	66	0.680	0.725	0.83	359.93	502.12	0.92	0.53
147,152	59,630	3,805	1.0	36	7,280	3,624	0.0013	220	0.728	25	0.741	108	0.670	0.711	0.96	396.19	643.09	0.94	0.61
126,829	49,021	3,383	1.0	33	12,907	5,663	0.0149	216	0.747	13	0.745	60	0.645	0.735	0.79	732.82	373.55	0.77	0.70
134,633	50,264	3,668	1.6	36	12,555	6,086	0.0100	201	0.785	22	0.845	125	0.725	0.770	0.75	533.15	589.45	1.17	0.63
79,118	28,969	2,180	1.0	35	7,996	3,284	0.0051	184	0.734	36	0.807	97	0.669	0.728	0.88	381.84	506.02	0.71	0.51
165,985	46,716	5,186	1.3	35	12,065	5,645	0.0207	201	0.780	24	0.851	51	0.679	0.757	0.86	625.02	641.49	1.05	0.70
97,140	40,613	2,458	1.1	28	6,014	2,914	0.0055	200	0.722	30	0.721	77	0.664	0.728	0.93	323.92	560.60	0.93	0.54
55,973	22,241	1,467	1.0	28	9,111	4,416	0.0076	186	0.734	31	0.790	55	0.681	0.718	0.91	294.50	478.26	0.65	0.44
146,112	55,154	3,955	1.5	35	10,171	5,048	0.0405	190	0.736	33	0.810	67	0.676	0.738	0.80	449.14	528.17	1.57	0.72
79,448	34,010	1,976	1.0	31	5,589	1,797	0.0095	236	0.753	24	0.805	58	0.695	0.743	0.92	283.01	481.42	0.91	0.55
87,290	32,207	2,395	1.3	29	7,133	3,382	0.0052	206	0.771	37	0.828	75	0.731	0.800	0.92	305.15	554.82	1.15	0.62
78,961	30,087	2,125	1.0	27	14,968	7,261	0.0022	187	0.741	18	0.846	80	0.687	0.729	0.90	456.94	542.60	0.54	0.46
36,207	13,801	974	1.0	22	11,704	5,684	0.0283	193	0.774	16	0.779	115	0.714	0.766	0.49	287.57	331.03	0.64	0.38
72,864	27,325	1,980	1.0	25	10,544	4,935	0.0075	190	0.717	18	0.747	135	0.651	0.711	0.76	427.86	442.24	0.65	0.50
189,449	66,239	5,357	1.3	40	7,743	3,731	0.0067	239	0.728	28	0.789	95	0.686	0.718	0.93	403.18	656.20	1.65	0.81
116,607	45,699	3,083	1.3	29	13,435	6,504	0.0165	207	0.717	14	0.781	63	0.662	0.710	0.83	482.17	520.33	1.00	0.58
103,278	35,418	2,950	1.6	33	8,022	3,142	0.0154	187	0.734	23	0.747	104	0.693	0.741	0.81	337.28	555.86	1.58	0.64
127,489	47,640	3,472	1.0	39	9,214	4,398	0.0080	172	0.789	28	0.847	90	0.730	0.809	0.88	378.57	443.07	1.30	0.84
49,125	17,897	1,358	1.0	39	15,163	6,938	0.0073	174	0.729	18	0.785	71	0.678	0.717	0.82	374.83	378.13	0.55	0.42
93,682	31,711	2,694	1.1	22	13,552	6,053	0.0045	231	0.797	18	0.800	69	0.729	0.806	0.85	448.18	547.65	0.76	0.52
61,888	20,416	1,803	1.0	25	11,077	4,566	0.0221	200	0.705	26	0.763	75	0.641	0.707	0.72	384.07	406.87	0.72	0.50
37,678	15,411	968	1.1	20	6,225	2,721	0.0200	176	0.750	17	0.826	104	0.715	0.749	0.73	217.72	383.68	0.80	0.39
97,258	29,179	2,960	1.2	33	7,115	3,541	0.0039	208	0.775	23	0.869	135	0.710	0.765	0.88	401.06	564.45	0.98	0.57
35,922	13,672	967	1.0	22	9,127	4,539	0.0905	216	0.754	18	0.750	110	0.704	0.743	0.43	231.62	361.76	0.72	0.34
52,240	21,634	1,331	1.2	24	8,100	3,723	0.0081	178	0.710	20	0.746	85	0.666	0.706	0.86	269.77	426.34	0.87	0.47
173,765	72,504	4,403	1.5	38	7,853	3,766	0.0038	233	0.758	26	0.791	73	0.690	0.728	0.95	394.33	663.11	1.58	0.72
40,354	16,364	1,043	1.2	20	12,893	6,420	0.0035	188	0.715	37	0.809	64	0.640	0.712	0.88	349.52	428.87	0.52	0.44
49,905	18,084	1,384	1.0	27	10,939	5,179	0.0208	189	0.794	0.744	0.822	135	0.697	0.742	0.59	318.51	417.05	0.65	0.40
99,867	37,752	2,701	1.3	30	8,014	3,410	0.0162	232	0.749	11	0.745	62	0.675	0.727	0.88	363.04	603.69	1.00	0.54

Table A-5: Data Generated for Eight Stage Treatment Type III, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
112,288	35,047	3,358	1.1	35	11,438	5,233	0.0307	207	0.799	25	0.800	68	0.735	0.780	0.92	442.60	576.16	0.91	0.61
162,365	46,949	5,018	1.1	39	8,138	3,391	0.0043	182	0.728	28	0.784	88	0.644	0.730	0.92	453.58	509.73	1.50	0.93
110,880	45,489	2,843	1.0	35	5,010	2,442	0.0057	205	0.711	35	0.752	88	0.647	0.705	0.94	331.19	567.30	0.95	0.60
102,088	40,643	2,672	1.2	26	8,883	3,880	0.0035	237	0.759	39	0.810	117	0.646	0.741	0.83	556.00	419.80	0.86	0.65
68,767	28,426	1,754	1.0	29	13,799	6,875	0.0072	181	0.732	33	0.784	68	0.688	0.727	0.86	397.09	469.35	0.59	0.44
184,863	65,931	5,171	1.5	40	10,652	5,069	0.0135	234	0.722	33	0.796	127	0.657	0.730	0.75	679.89	589.10	1.21	0.75
46,882	13,760	1,440	1.0	18	7,704	4,271	0.0177	209	0.733	38	0.721	73	0.653	0.714	0.79	298.85	447.41	0.67	0.44
153,417	60,104	4,057	1.0	39	11,141	5,441	0.0242	196	0.787	31	0.844	114	0.725	0.780	0.70	557.14	585.12	0.78	0.58
147,378	56,166	3,966	1.2	38	14,944	6,680	0.0050	199	0.754	35	0.838	134	0.687	0.745	0.73	779.14	348.86	1.10	0.73
53,013	21,102	1,388	1.0	23	12,331	5,619	0.0089	188	0.715	38	0.713	75	0.662	0.706	0.82	322.14	438.38	0.62	0.44
159,742	45,028	4,988	1.5	35	13,478	6,361	0.0338	188	0.730	34	0.831	111	0.681	0.719	0.68	498.94	649.07	1.45	0.69
93,700	37,102	2,461	1.2	34	9,308	4,374	0.0044	198	0.747	39	0.795	106	0.678	0.738	0.88	414.25	502.99	0.89	0.58
164,387	56,529	4,689	1.4	35	11,683	5,099	0.0083	198	0.764	31	0.847	118	0.699	0.756	0.77	635.34	609.48	1.06	0.66
41,053	13,410	1,202	1.1	20	9,365	4,619	0.0065	198	0.719	27	0.791	69	0.650	0.719	0.86	335.36	381.37	0.65	0.45
79,452	32,868	2,025	1.0	28	9,720	4,731	0.0088	209	0.790	16	0.853	115	0.731	0.809	0.79	341.17	395.69	0.94	0.61
170,133	67,606	4,458	1.1	28	5,050	2,322	0.0297	187	0.724	25	0.779	60	0.677	0.724	0.92	371.26	707.03	1.17	0.69
43,148	12,897	1,315	1.0	24	10,995	4,815	0.0085	200	0.729	39	0.769	90	0.664	0.717	0.76	361.54	386.88	0.59	0.44
56,838	16,294	1,763	1.1	29	14,076	6,401	0.0326	218	0.768	30	0.768	51	0.707	0.768	0.76	332.01	484.84	0.75	0.46
44,404	16,648	1,207	1.0	20	7,769	3,006	0.0026	225	0.742	31	0.808	52	0.698	0.736	0.94	283.17	440.11	0.61	0.39
176,885	52,064	5,427	1.4	32	13,200	6,411	0.0183	232	0.725	22	0.770	54	0.661	0.711	0.88	580.56	611.64	1.34	0.73
106,367	44,369	2,696	1.5	27	10,645	4,366	0.0167	216	0.733	11	0.730	128	0.691	0.722	0.71	364.08	573.91	1.21	0.54
138,805	39,985	4,297	1.1	35	14,511	6,271	0.0453	237	0.774	15	0.775	129	0.705	0.760	0.41	596.61	395.17	1.26	0.65
73,253	20,800	2,281	1.2	25	15,098	6,626	0.0174	218	0.726	12	0.788	123	0.681	0.711	0.57	440.65	488.79	0.80	0.47
57,389	18,595	1,687	1.2	23	11,556	6,833	0.0293	233	0.756	37	0.797	65	0.704	0.746	0.79	351.77	445.00	0.81	0.47
84,568	33,971	2,200	1.1	36	12,467	6,718	0.0058	209	0.714	24	0.787	111	0.665	0.711	0.86	420.23	491.67	0.74	0.53
148,744	57,500	3,967	1.0	40	15,013	6,649	0.0100	233	0.729	29	0.753	84	0.701	0.725	0.85	464.73	627.82	0.85	0.53
47,068	16,121	1,346	1.0	21	14,390	6,789	0.0181	173	0.728	28	0.764	55	0.682	0.728	0.77	308.16	343.92	0.73	0.48
86,037	28,306	2,510	1.2	31	9,057	4,337	0.0151	233	0.742	13	0.743	78	0.693	0.730	0.86	332.32	564.46	1.00	0.54
50,480	14,433	1,567	1.0	22	7,307	2,825	0.0058	205	0.787	13	0.787	97	0.711	0.789	0.83	307.88	438.76	0.73	0.50
66,187	26,775	1,714	1.2	29	14,205	6,654	0.0158	210	0.745	13	0.795	85	0.680	0.740	0.69	431.62	386.34	0.77	0.53

Table A-5: Data Generated for Eight Stage Treatment Type III, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
139,304	52,298	3,783	1.0	33	9,949	4,201	0.0181	183	0.738	18	0.738	129	0.651	0.721	0.70	589.35	520.98	0.77	0.63
70,317	21,555	2,120	1.0	25	14,518	6,850	0.0468	220	0.725	23	0.781	121	0.665	0.729	0.35	460.06	323.03	0.89	0.48
124,558	39,693	3,690	1.3	36	12,160	6,840	0.0830	199	0.750	13	0.750	88	0.706	0.747	0.65	390.99	613.33	1.25	0.58
48,264	14,064	1,487	1.0	26	10,545	5,098	0.0170	196	0.725	26	0.794	61	0.678	0.724	0.85	317.36	418.20	0.70	0.46
99,962	37,253	2,726	1.5	29	12,920	5,756	0.0165	204	0.770	11	0.770	68	0.692	0.763	0.72	462.08	491.96	1.13	0.64
53,957	22,096	1,385	1.0	23	7,685	3,491	0.0421	211	0.725	12	0.725	94	0.684	0.712	0.70	263.71	451.52	0.73	0.41
184,179	62,120	5,307	1.0	40	11,359	5,085	0.0251	189	0.772	26	0.776	128	0.677	0.774	0.58	839.38	372.56	1.07	0.76
167,944	58,464	4,760	1.2	38	14,099	6,073	0.0349	235	0.772	18	0.854	91	0.729	0.800	0.73	367.35	368.34	2.65	1.30
168,516	49,092	5,192	1.5	35	11,070	5,155	0.0099	209	0.767	11	0.812	99	0.709	0.785	0.85	417.41	436.88	2.68	1.23
79,145	27,069	2,264	1.0	30	5,329	1,984	0.0083	184	0.797	29	0.849	51	0.715	0.800	0.93	297.26	487.33	0.98	0.61
173,101	52,839	5,229	1.5	32	11,226	5,370	0.0099	240	0.729	31	0.744	74	0.683	0.720	0.90	439.68	684.30	1.64	0.74
134,063	53,813	3,489	1.4	38	8,282	3,154	0.0172	205	0.713	29	0.742	86	0.640	0.705	0.84	395.02	519.48	1.49	0.74
159,037	46,347	4,900	1.0	35	12,244	5,127	0.0065	193	0.766	29	0.814	106	0.711	0.754	0.82	593.44	626.86	0.83	0.61
48,512	20,114	1,235	1.1	22	14,378	6,330	0.0261	215	0.726	35	0.780	89	0.668	0.717	0.51	388.14	338.20	0.59	0.42
61,026	22,422	1,678	1.1	28	11,804	5,642	0.0037	232	0.740	13	0.791	73	0.693	0.729	0.90	319.57	450.61	0.80	0.50
83,119	26,925	2,443	1.0	31	12,403	5,469	0.0501	206	0.796	32	0.796	94	0.733	0.789	0.68	643.16	303.77	0.78	0.63
71,952	21,099	2,211	1.3	28	14,899	6,535	0.0311	198	0.746	32	0.809	107	0.668	0.741	0.39	552.31	280.58	1.16	0.57
178,436	59,039	5,191	1.2	40	10,342	4,739	0.0218	193	0.731	24	0.759	74	0.670	0.720	0.86	443.21	598.94	1.47	0.80
109,755	37,667	3,134	1.0	29	13,196	6,085	0.0053	184	0.710	37	0.710	122	0.653	0.709	0.80	516.20	478.43	0.79	0.62
185,786	60,952	5,428	1.0	40	10,607	4,959	0.0182	195	0.758	11	0.794	104	0.677	0.742	0.77	646.90	601.94	0.87	0.69
90,647	31,381	2,577	1.2	30	8,318	4,212	0.0046	186	0.779	30	0.847	78	0.731	0.775	0.91	357.06	504.21	1.08	0.58
153,845	50,282	4,503	1.5	32	7,567	6,439	0.0294	205	0.729	13	0.725	63	0.664	0.725	0.91	409.28	679.17	1.52	0.67
81,730	29,009	2,292	1.0	25	10,627	4,758	0.0049	219	0.753	27	0.835	122	0.695	0.766	0.79	431.56	437.45	0.76	0.55
156,355	61,286	4,133	1.4	37	6,096	2,570	0.0062	173	0.740	14	0.740	77	0.702	0.740	0.94	365.03	589.56	1.44	0.68
144,015	48,403	4,157	1.2	40	13,935	6,193	0.0149	225	0.738	20	0.788	91	0.671	0.719	0.75	653.60	571.25	0.84	0.61
55,251	22,745	1,413	1.0	21	5,694	2,804	0.0148	171	0.720	29	0.750	91	0.676	0.724	0.85	262.53	450.36	0.75	0.45
122,734	38,609	3,658	1.3	29	9,770	4,067	0.0075	228	0.784	17	0.800	60	0.726	0.772	0.90	395.07	275.72	1.38	0.72
75,944	25,721	2,184	1.2	24	10,884	5,127	0.0133	186	0.778	18	0.845	97	0.703	0.794	0.84	348.15	341.57	1.38	0.79
165,048	67,378	4,246	1.0	34	15,122	6,937	0.0063	171	0.746	14	0.820	130	0.700	0.734	0.76	672.72	607.34	0.65	0.55
78,923	27,258	2,246	1.5	27	9,653	4,782	0.0015	219	0.726	21	0.726	78	0.673	0.726	0.95	329.52	549.37	1.17	0.54

Table A-5: Data Generated for Eight Stage Treatment Type III, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
113,711	32,249	3,542	1.1	28	9,450	4,220	0.0386	204	0.727	28	0.785	116	0.677	0.718	0.65	455.17	611.67	0.90	0.52
53,318	16,034	1,621	1.0	24	8,105	4,304	0.0156	219	0.713	37	0.767	61	0.654	0.707	0.81	337.81	453.64	0.66	0.44
120,942	45,710	3,271	1.1	37	7,177	3,432	0.0068	193	0.797	40	0.847	96	0.729	0.780	0.90	399.50	577.91	0.98	0.60
109,061	41,378	2,943	1.0	33	14,918	6,643	0.0023	213	0.748	31	0.749	88	0.692	0.740	0.88	537.54	475.52	0.72	0.59
141,214	57,588	3,636	1.7	35	13,104	6,447	0.0559	220	0.720	24	0.800	128	0.670	0.714	0.89	511.59	588.00	1.29	0.64
66,811	19,553	2,055	1.2	30	14,535	6,445	0.0164	181	0.719	28	0.796	85	0.642	0.707	0.65	558.18	346.40	0.80	0.58
86,034	30,044	2,434	1.0	30	8,464	4,179	0.0066	170	0.739	37	0.752	96	0.680	0.733	0.88	350.89	523.88	0.83	0.55
56,779	17,507	1,708	1.0	24	10,850	4,799	0.0166	194	0.750	14	0.750	73	0.693	0.741	0.78	325.09	471.31	0.70	0.47
94,393	33,359	2,654	1.5	28	12,896	6,078	0.0078	236	0.742	20	0.748	114	0.695	0.733	0.79	386.70	498.82	1.29	0.62
67,248	26,422	1,775	1.0	24	11,843	5,170	0.0591	198	0.739	12	0.733	103	0.687	0.730	0.48	364.64	451.24	0.67	0.41
182,204	58,417	5,382	1.5	40	13,730	6,688	0.0184	209	0.719	21	0.777	105	0.673	0.717	0.77	537.83	587.78	1.60	0.77
54,164	16,304	1,646	1.1	26	12,781	5,608	0.0025	192	0.742	10	0.800	109	0.696	0.742	0.85	344.10	512.48	0.80	0.51
139,286	40,539	4,293	1.0	33	8,142	3,951	0.0265	188	0.714	18	0.767	126	0.662	0.710	0.77	418.32	598.82	1.07	0.66
182,393	65,912	5,064	1.4	28	15,000	7,446	0.0044	204	0.772	19	0.841	99	0.723	0.766	0.86	648.74	611.55	1.12	0.66
159,377	52,415	4,651	1.1	39	6,180	2,459	0.0065	217	0.774	19	0.856	109	0.714	0.768	0.90	391.93	609.10	1.35	0.77
94,263	33,578	2,638	1.3	32	13,299	5,912	0.0588	223	0.734	16	0.783	101	0.696	0.726	0.55	404.81	529.41	1.00	0.49
76,433	25,839	2,200	1.5	26	7,119	2,598	0.0286	222	0.775	26	0.847	129	0.709	0.759	0.64	319.90	470.21	1.38	0.58
78,638	26,805	2,254	1.1	27	11,989	5,918	0.0119	214	0.777	19	0.775	95	0.715	0.762	0.79	388.48	472.41	0.85	0.56
176,441	64,824	4,853	1.5	40	8,808	3,960	0.0705	177	0.795	30	0.859	79	0.732	0.784	0.73	457.12	606.26	1.65	0.73
123,915	49,381	3,241	1.0	30	13,118	6,262	0.0037	174	0.791	16	0.782	78	0.728	0.752	0.91	404.98	558.68	0.90	0.58
94,343	27,439	2,909	1.3	30	7,608	3,711	0.0351	172	0.741	31	0.805	94	0.702	0.740	0.78	343.43	545.78	1.26	0.60
115,589	38,512	3,351	1.5	31	12,349	5,815	0.0031	178	0.709	17	0.706	113	0.648	0.702	0.87	482.91	553.92	1.18	0.66
51,937	19,659	1,403	1.3	24	8,958	4,366	0.0092	227	0.761	22	0.761	128	0.712	0.758	0.76	283.80	467.67	0.86	0.43
108,183	38,021	3,051	1.2	34	7,141	2,686	0.0317	187	0.728	30	0.778	123	0.677	0.719	0.72	365.25	571.20	1.10	0.56
174,858	50,402	5,411	1.6	40	7,193	3,312	0.0055	182	0.789	27	0.788	130	0.727	0.764	0.89	431.87	696.55	1.80	0.78
37,794	14,034	1,033	1.0	19	11,458	4,772	0.0123	223	0.732	27	0.765	106	0.685	0.724	0.64	289.61	373.45	0.58	0.37
144,053	51,831	4,010	1.5	34	5,646	1,853	0.0548	239	0.714	31	0.765	132	0.660	0.713	0.68	376.99	614.18	1.63	0.66
141,803	54,610	3,791	1.1	37	13,314	6,621	0.0303	219	0.768	15	0.765	124	0.726	0.760	0.66	453.57	647.65	0.89	0.53
132,701	41,922	3,947	1.2	36	8,446	3,738	0.0147	182	0.732	36	0.806	105	0.693	0.731	0.83	404.16	608.38	1.21	0.62
87,583	28,694	2,560	1.0	25	5,086	2,051	0.0652	238	0.768	23	0.841	117	0.719	0.757	0.66	317.76	549.78	0.92	0.50

Table A-5: Data Generated for Eight Stage Treatment Type III, continued

Total Volume	Pad Volume	Fluid Increment	Proppant Increment	Injection Rate	Depth to Middle of Pay Zone	Net Pay Pressure	Permeability	Top Layer Thickness	Top Layer Stress Closure Gradient	Overburden Layer Thickness	Overburden Layer Closure Stress Gradient	Pay Zone Thickness	Pay Zone Closure Stress Gradient	Bottom Layer Closure Stress Gradient	Fracture Efficiency	Propped Fracture Length	Fracture Height	Proppant Concentration	Max Fracture Width
input	input	input	input	input	input	input	input	input	input	input	input	input	input	input	output	output	output	output	output
53,257	14,999	1,663	1.2	26	9,804	3,911	0.0441	219	0.730	22	0.800	88	0.678	0.724	0.62	316.64	434.65	0.91	0.46
80,243	26,863	2,321	1.1	28	9,021	4,477	0.0286	210	0.718	11	0.768	117	0.651	0.718	0.70	382.05	470.95	0.89	0.55
140,348	50,259	3,917	1.0	36	7,851	3,663	0.0076	225	0.724	28	0.789	70	0.671	0.720	0.93	388.83	561.84	1.12	0.70
184,176	62,261	5,301	1.6	39	14,730	7,269	0.0038	212	0.713	40	0.746	88	0.669	0.703	0.91	575.36	649.32	1.42	0.69
53,543	19,374	1,486	1.2	21	14,349	6,481	0.0061	216	0.723	19	0.784	89	0.677	0.715	0.77	391.71	439.48	0.65	0.44
55,673	23,041	1,419	1.0	25	9,363	4,369	0.0085	220	0.756	27	0.752	93	0.691	0.748	0.82	304.98	442.40	0.66	0.47
158,764	65,425	4,058	2.0	33	9,673	4,224	0.0123	171	0.788	11	0.844	65	0.730	0.774	0.88	446.68	581.53	0.98	0.64
43,563	15,073	1,239	1.1	18	5,256	1,744	0.0361	230	0.718	20	0.718	87	0.674	0.710	0.72	232.57	423.68	0.87	0.43
92,892	29,650	2,750	1.1	27	11,151	5,202	0.0255	220	0.726	40	0.784	111	0.693	0.753	0.70	332.79	420.72	1.35	0.71
138,697	52,563	3,745	1.5	33	8,421	3,748	0.0156	211	0.762	22	0.836	83	0.729	0.827	0.87	324.96	482.96	2.24	1.01
71,580	29,455	1,832	1.1	31	14,796	6,461	0.0254	178	0.701	21	0.743	108	0.657	0.705	0.58	404.03	414.64	0.68	0.46
131,268	50,470	3,513	1.4	36	10,174	4,621	0.0059	228	0.728	34	0.783	107	0.694	0.729	0.88	378.70	576.14	1.41	0.66
100,509	33,936	2,894	1.0	30	10,135	4,806	0.0326	203	0.779	23	0.869	92	0.683	0.718	0.76	362.61	533.21	0.94	0.55
47,747	18,199	1,285	1.2	25	9,481	4,710	0.0185	240	0.740	20	0.767	124	0.682	0.738	0.68	289.60	391.70	0.86	0.46
56,877	16,091	1,773	1.0	26	11,487	5,106	0.0071	198	0.713	35	0.756	61	0.678	0.710	0.88	314.32	447.94	0.79	0.49
115,159	34,332	3,514	1.2	24	13,800	5,999	0.0181	229	0.734	31	0.734	119	0.674	0.734	0.63	513.05	497.11	1.04	0.64
188,167	54,259	5,822	1.2	40	7,627	3,360	0.0120	229	0.709	16	0.708	99	0.652	0.709	0.89	448.02	773.76	1.26	0.66
75,043	28,827	2,009	1.1	29	7,996	3,929	0.0481	219	0.780	18	0.836	86	0.738	0.777	0.79	283.27	463.25	1.06	0.56
91,399	33,013	2,539	1.2	30	14,596	6,736	0.0300	216	0.736	18	0.760	125	0.663	0.726	0.44	577.55	337.01	0.98	0.56
77,793	27,146	2,202	1.2	27	10,522	4,492	0.0109	233	0.725	36	0.790	85	0.674	0.724	0.81	392.50	534.50	0.86	0.51
131,627	39,039	4,026	1.1	38	13,120	6,053	0.0889	182	0.730	29	0.729	84	0.672	0.734	0.62	449.79	608.92	1.01	0.59
88,605	34,296	2,361	1.1	25	8,174	3,812	0.0335	226	0.741	33	0.740	117	0.680	0.740	0.68	349.55	518.87	0.90	0.52

Table A-5: Data Generated for Eight Stage Treatment Type III, continued

CURRICULUM VITAE

Andrei Popa was born November 30, 1970, in Campina, Romania. He is the son of Sergiu Popa and Stela Popa.

He received his B.S. degree in Petroleum and Natural Gas Engineering from University of Ploiesti, Romania, in 1995. He achieved a Master of Science in Petroleum and Natural Gas Engineering in 1998 and his Doctor of Philosophy degree in Petroleum and Natural Gas Engineering in 2004, both from West Virginia University.

Andrei is a member of the Honor Society of Petroleum Engineers (Pi Epsilon Tau) and member of the Society of Petroleum Engineers (SPE). He has served as technical reviewer for different publications and currently serves as member of the Board of Directors for the San Joaquin Valley SPE Section. He has authored/coauthored more than a dozen technical papers on the subject of Artificial Intelligence applications in the energy industry for SPE, IEEE, Elsevier, and others.

Prior to coming to the United States, he was a Petroleum Engineer for PETROMAR, the offshore division of Romanian Oil Company, and then a Reservoir Engineer for the Institute of Research and Technology in Campina, Romania. During this time, he worked on various projects involving reservoir simulation, forecasting, and economic evaluation for several oil and gas fields in Romania.

While working towards his PhD, Andrei was also employed as a Research Associate at West Virginia University. His projects included pioneer research in the application of Artificial Intelligence to the oil and natural gas industry.

Currently he is a Petroleum Engineer with ChevronTexaco where he is responsible for the production and operation of over 1,500 wells in the Kern River field in Kern County, California. He is also involved in steamflood and workover optimization and continues to find new and innovative ways to apply his expertise in Artificial Intelligence for increased oil production opportunities.

He is married to Carrie Goddard Popa, and they currently reside in Bakersfield, California.

He plans to continue his research and application of Artificial Intelligence techniques to topics related to Petroleum and Natural Gas Engineering.