Horizontal Stratification of the Soil in Multi-Layer Using Non-Linear Optimization

Estratificação Horizontal do Solo em Várias Camadas Usando Otimização Não-Linear

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Resumo: O propósito principal deste trabalho é apresentar uma metodologia que utiliza um processo de otimização não-linear na obtenção dos parâmetros da estratificação horizontal do solo em várias camadas. O método utiliza uma curva de resistividade experimental proveniente de leituras feitas no solo. Esta curva experimental é comparada com uma outra curva de resistividade teórica produzida pelo processo de otimização. A curva teórica é embasada no algoritmo de Sunde que é exatamente o processo inverso da estratificação horizontal do solo em várias camadas. De posse das duas curvas, pode-se mensurar o erro produzido na estratificação horizontal do solo. Os parâmetros a serem estimados são os números de camadas N, a resistividade ρ_i e a espessura h_i de cada camada.

Palavras-chave: Resistividade Elétrica do Solo; Estratificação do Solo; Otimização Não-Linear.

Abstract: The main purpose of this paper is to present a methodology to obtain the stratification of the soil into horizontal multi layers from measuring results, obtained from the Wenner's method. A mathematical modeling is presented combining the horizontal stratification of the soil method in several layers, with the non-linear optimization of Quasi-Newton method. The process initiates, calculating the resistivity of the first layer in a non-traditional way, and then calculating the resistivity and the thickness of the following layers, based on the Sunde's algorithm, which is exactly the inverse process of soil stratification into multi layers. The parameters to be optimized in this methodology are: the resistivity ρ_i, the thickness h_i and the number of layers N. The results achieved point out advantages in comparison to other known methods.

Key words: Electrical Resistivity of Soil; Stratification of Soil; Non-Linear Optimization.

1 Introduction

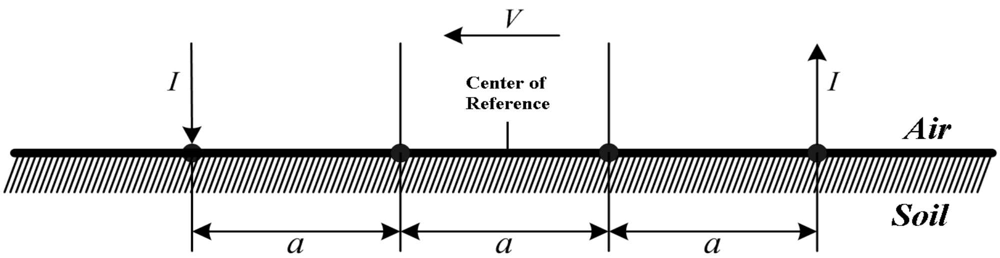
In an electric grounding project, it is essential to know the soil behavior, regarding to its electrical conduction properties. In metals, this property is represented by the value of its electric resistivity, which varies according to the metal temperature, impurities etc. However, variations on the electrical resistivity of electric conductors are relatively small, maintaining a great homogeneity along its dimensions. In case of the soil, the electrical resistivity variations are not only extremely large, as well as it has a heterogeneous behavior along its dimensions [1].

This characteristic represents a major obstacle for designers of electric grounding. Due to this difficulty, a model that represents the soil by its electricity conduction behavior is developed. The most common model is the one which represents the soil as horizontal layers, where each one of them has the same electrical resistivity throughout its points. Since an electric grounding extension does not overtake more than a hundreds of meters, this model can be applied with a good accuracy in the major part of the electric grounding designs, justifying its wide use.

With a soil stratification model adopted, it is only necessary to define the area to be stratified i.e. to a certain area addressed to build an electric grounding, it is necessary to know, (i) with how many layers its soil can be represented, and (ii) the values of thickness and resistivity of each soil layers.

Frank A. Wenner [2], carried out a measuring method, which the results contains the answers of those prior questions. Although this measuring method has some fluctuations, one of them will fit best in the electric grounding project [3]. The method configuration is shown in fig. 1, where four align and equally spaced electrodes are inserted into the soil.

Figure 1. Wenner's Method



Source: The authors

When a current I is forced into the external electrodes, the voltage V can be measured through the internal electrodes [4]. From [5, 6], the resistivity of a homogeneous soil is given by (1).

$$\rho_a(a) = \frac{4\pi aR}{1 + \frac{2a}{\sqrt{a^2 + (2P)^2}} - \frac{2a}{\sqrt{(2a)^2 + (2P)^2}}} \qquad (1)$$

Where a is the spacing between electrodes, considering the Wenner´s method, P is the depth in which the electrodes are immersed into the soil and R is the measured soil resistance. If the soil is homogeneous, when the distance a varies, the relation $R = \frac{V}{I}$ becomes inversely proportional, related to a, so ρ remains a same value, which characterizes a homogeneous soil with resistivity ρ.

However, in a heterogeneous soil, when a varies, the relation $R = \frac{V}{I}$ can also varies, but not in the same way as in the prior case. Therefore, the resistivity ρ has different values for each distance a. This variable resistivity is going to be called apparent resistivity of the soil ρ_a, which is a function of a, i.e., $\rho_a(a)$. In fig. 1, varying the distance a, with a fixed reference point, it is possible to rise the experimental curve $\rho_a(a)$.This curve stores information about the number of layers with their respective electrical resistivity and thickness. The main purpose of this paper is to show an efficient and accurate alternative way to stratify the soil, i.e., determinate the number of layers and its respective values of electrical resistivity and thickness, using the Wenner's experimental curve, $\rho_a(a)$. The methodology employed applies a non-linear optimization process based on the mathematical modeling deduced from the Sunde's algorithm [7].

As it is presented throughout the paper, advantages are found in using this method of soil stratification rather than other methods already known.

2 Soil Stratification

The vast majority of electric grounding projects are a combination of vertical and horizontal ground conductors. Knowing the soil's layers, with their respective resistivities and thicknesses, the depth that the vertical conductor should achieve can be defined, assuring the best use of the soil.

If there is a thick first layer with low resistivity in comparison to the others, it is probably that the grounding should be built superficial, being more indicated the use of horizontal conductors [8]. If the opposite occurs, vertical ground conductors should be employed, leaving the horizontal conductors only with the role of electric linking. Whether are known the soil structure values, it is possible to model the grounding geometry, i.e., to work with vertical and horizontal conductors in a most efficient way, so much regard with the grounding resistance, as well as in the potentials, mainly the superficial ones.

When interpreted along with the values of soil's layers, those superficial potentials can define certain projects criteria. A common case is the vertical conductors. If it belongs to thick and low resistive layer, the current that flows from it tends to spread itself throughout the first layer, producing surface potentials until great distances [9, 10]. These phenomena smoothes the potentials curves, transforming the vertical conductor application less dangerous when related to step voltages[11].

However, if this same superficial layer has high resistivity values, the current will flow deeper, producing superficial potentials that are reduced in small distances [12], causing a abruptly decreasing in the potentials curves [13]. In this case, the vertical conductor employ, can become more dangerous regarding to step voltages.

Another uncountable number of analyses can be done relating the grounding conductors with the soil's structure in horizontal layers [14]. These analyses define extremely useful orientations criteria to the development of a good electric grounding project.

3 Mathematical Modeling

The soil stratification proposed on this paper has on its basis, the idea of comparison between the experimental Wenner's curve $\rho_{aE}(a)$ and the theoretical

Wenner's curve $\rho_{aT}(a)$. This last one, created from a known soil structure, i.e., number of layers N, thickness h_i and resistivity ρ_i, of a generic layer i. Therefore, the basic idea is: when one theoretical curve equal to the experimental curve is obtained, considering a maximum admissible error, the values of the soil structure that generates this theoretical curve correspond to the desired soil stratification. From [7], (2) can be written.

$$\rho_a(a) = 2\rho_1 a \int_0^\infty N_n(m)\left[J_0(ma) - J_0(2ma)\right] \quad (2)$$

The first layer resistivity is ρ_1, m is the auxiliary integration variable, J_0 the Bessel function of type one and order zero, and N_n is the Sunde's characteristic function. If the soil structure is known, it is possible to obtain its characteristic function by Sunde's algorithm [7], i.e., N, ρ_i and h_i. In Sunde's algorithm, equality (3) can be adopted.

$$E = e^{-2mh_1} \quad (3)$$

In (3) it is presented a variable change from m to E, which enables to express the characteristic function N_n in E, and, in this particular case, a polynomial expression described in (4) was chosen.

$$N_n = A_0 + A_1 E + A_2 E^2 + \ldots \quad (4)$$

It is possible to verify by Sunde's algorithm that for $E = 0$ it is obtained $N_n = 1$ and, therefore $A_0 = 1$. So (4) becomes (5).

$$N_n = 1 + A_1 E + A_2 E^2 + \ldots \quad (5)$$

Assuming, at first, a certain number of terms j to (5), we can choose j values for E, remembering that $0 < E \leq 1$. For each value E, it is possible to obtain the corresponding value N_n, using Sunde's algorithm. In this way, equation (5) gives a system of j equations and j unknown variables, from which the values for A_k with $k = 1,2,\ldots, j$, can be achieved.

It is possible by an iterative process to obtain the best values of j, minimizing numerical errors generated by the polynomial decomposition. If the values of A_k are known, the equation (5) express N_n as function of E. Considering this data, it is possible to solve the integral in (2) analytically. Solving the integral in (2) and using some algebraic manipulation (6) is obtained.

$$r_a(a) = r_1.\left(1+2\sum_{k=1}^{j} A_k.S_k(a)\right) \tag{6}$$

Where:

$$S_k(a) = S_1 + S_2 \tag{7}$$

In which:

$$S_1 = \frac{1}{\sqrt{4.b^2.k^2+1}} \tag{8}$$

$$S_2 = 2.\frac{1}{\sqrt{b^2.k^2+1}} \tag{9}$$

$$\beta = \frac{h_1}{a} \tag{10}$$

To obtain the theoretical curve $\rho_{aT}(a)$ equation (6) is used. To initiate the optimization process, saving processing time, it is interesting to obtain the initial values of the process coherent with the experimental Wenner's curve $\rho_{aE}(a)$. This became possible determining at first, the electric resistivity of the first layer. For this, the function $\rho_a(a)$ of (6) may be written assuming pairs of values of a, i.e., a_1 and a_2.

$$\rho_a(a_1) = \rho_1.\left(1+2\sum_{k=1}^{j} A_k.S_k(a_1)\right) \tag{11}$$

$$\rho_a(a_2) = \rho_1.\left(1 + 2\sum_{k=1}^{j} A_k.S_k(a_2)\right) \tag{12}$$

Dividing and subtracting (11) and (12), expressions (13) and (14), are respectively obtained.

$$r\rho_a(a_{1,2}) = \frac{\rho_a(a_1)}{\rho_a(a_2)} = \frac{1 + 2\sum_{k=1}^{j} A_k.S_k(a_1)}{1 + 2\sum_{k=1}^{j} A_k.S_k(a_2)} \tag{13}$$

$$d\rho_a(a_{1,2}) = \rho_a(a_1) - \rho_a(a_2) = 2.\sum_{k=1}^{j} \rho_1.A_k\left[S_k(a_1) - S_k(a_2)\right] \tag{14}$$

From (13) and (14), equations (15) and (16) can be obtained, respectively as:

$$\sum_{k=1}^{j} A_k.M_k(a_{1,2}) = r\rho_a(a_{1,2}) - 1 \tag{15}$$

$$\sum_{k=1}^{j} B_k.P_k(a_{1,2}) = \frac{d\rho_a(a_{1,2})}{2} \tag{16}$$

Where:

$$M_k(a_{1,2}) = S_k(a_1) - r\rho_a(a_{1,2}).S_k(a_2) \tag{17}$$

$$B_k = \rho_1.A_k \tag{18}$$

$$P_k(a_{1,2}) = S_k(a_1) - S_k(a_2) \tag{19}$$

Thus, ρ_1 can be obtained from (15) and (16). This process is explained in the next section.

4 Methodology

When a_1 and a_2 are both modified by the algorithm iteration, expressions (15) and (16) represents two system of equations, where the unknown variables are A_k, B_k and h_1.

Firstly, the goal is to get only the first layer resistivity ρ_1. This layer can be imagined as composed by two virtual horizontal layers, both with resistivity ρ_1. Considering that in the real world, the thickness of the first layer is usually superior to 10 cm, one can adopt the thickness of the first virtual layer as 10 cm. Therefore, for purposes of ρ_1 calculation, h_1 can be assumed to 10 cm, and thus, from (15) and (16), ρ_1 is obtained.

Knowing the ρ_1 value, the characteristic function N_n can be obtained by Sunde's algorithm. So, the A_k coefficients also can be obtained by using the polynomial (5). From expression (6) the theoretical Wenner's curve $\rho_{aT}(a)$ can be draw. In order to generate this curve it is necessary to adopt values of N, ρ_i and h_i, remembering that ρ_1 was definitely obtained. The comparison between the experimental Wenner's curve and the theoretical Wenner's curve shows if the assumed values of N, ρ_i and h_i represent a good stratification.

At this point, a non-linear optimization method is employed, so that by an iterative process, the set of values N, ρ_i and h_i are being modified, until a theoretical curve as close as possible of the experimental curve is obtained.

The usual employed methodologies estimate the number of layer N, regarding the number of inflections p presented at the experimental curve of apparent resistivity of the soil i.e. $N = p+1$. At the proposed methodology, the number N of layers is a parameter to be optimized.

The optimization method adopted is called "Quasi-Newton" [15], already known as the Secants Method its use is justified once it brings the simplicity of the Gradient method as much as the quickness of the the Newton's method [16]. Rather than calculating the inverse of Hessian matrix, as it is done in the Newton's method, this inverse process is approached by finite iterations using the first order differentiations, as it is done in the gradient method [15].

Assuming that ρ_{aEx} is the experimental resistivity curve and ρ_{aTh} the theoretical resistivity curve, the evaluation function $f(x)$ which is a metric is defined by:

$$f(x) = \frac{\rho_{aEx} - \rho_{aTh}}{\rho_{aEx}} \tag{20}$$

The Quasi-Newton algorithm updates the matrix with the parameters N, ρ_i and h_i to be optimized, minimizing the deviation value $f(x)$ found. The N value, is a integer number, distinct from the other values of ρ_i and h_i, due to this, there is a different treatment for the N value optimization.

The experimental and theoretical curves were obtained from the interpolation of the soil apparent resistivity values. This interpolation is made with groups of few points, obtained through a lower degree polinomial, imposing conditions for the approach function to be continuous and having continuous derivatives until a certain order [17]. This makes the curve to not have peaks and neither abruptly changes of curvature in its knots, softening the curve.

5 Results

This section presents the results from six different soil stratifications process. Among them, two has resulted in a two layers stratification, other one has resulted in a three layers, and the rest of them resulted in a four layers stratification. Small deviation values are obtained between the experimental and theoretical curves of apparent resistivity of the soil. The Wenner's curves, which are illustrated from 2 to 7 were obtained through an interpolation of the experimental curves points. The resistivity experimental curve, proceeds from field measuring, and the theoretical curve, from the Sunde's algorithm.

In the Wenner's method application, several measures are done in the field at different directions and, in general, some are discarded after analysis. As the measure harvested in the field reflects the soil real configuration and, what we aim here, is to demonstrate a new methodology to stratify the soil, in this work, the measures were harvested just in one direction, for the Wenner's method application.

A larger amount of measured points, i.e. for a larger amount of a values utilized to produce the experimental curve of apparent resistivity of the soil, heads to lowest deviations between the experimental and theoretical curves of apparent resistivity of the soil.

For the process of Soil Stratification 1, the data of the experimental and theoretical curves of apparent resistivity of the soil, are given at tab. 1, with the Wenner's curve represented in fig. 2 and the results of the stratification process (thickness and resistivity of each layer), given at tab. 2. The same way is used for the others stratifications processes, being tab. 3 and tab. 4 with the fig. 3, related to the Soil Stratification 2, the tab. 5 and tab. 6 with the fig. 4 to Soil Stratification 3, tab. 7 and tab. 8 with the fig. 5 to Soil Stratification 4.

For stratifications numbers 5 and 6, two distinct soil stratification methods were used. One of them is the proposed method, and the other is known as the complex image method [18, 19].

At tab. 9 are listed the theoretical and experimental values of apparent resistivity of the soil from the the stratification process number 5, using the proposed method. tab. 10 contains the theoretical and experimental values of apparent resistivity of the soil from the process of stratification number 5, using the method of the complex image. fig. 6 presents the curves interpolated with the points listed at tab. 9 and tab. 10, the results of thickness and resistivity of each layer, found by the proposed method and the method of the complex image, are presented in tab. 11.

The same sequence described above were made for the soil stratification process number 6. The theoretical and experimental values of soil resistivity found by the proposed method are listed at tab. 12, and the values found with the complex image method are listed at tab. 13, the interpolated curves are plotted on fig. 7, and the results of this stratification, found with the proposed method and the complex image method, are presented at tab. 14.

5.1 Soil Stratification 1

Table 1. Stratification 1 - Experimental and Theoretical Curves

a (m)	**Experimental Curve** (Ωm)	**Theoretical Curve** (Ωm)	**Deviation** (%)
1.0	641.83	653.43	1.81
2.0	996.62	1014.29	1.77
4.0	1437.62	1456.71	1.33
8.0	1887.08	1850.27	−1.95
16.0	2091.32	2098.57	0.35

Figure 2: Curve of Apparent Resistivity of the Soil from Stratification 1

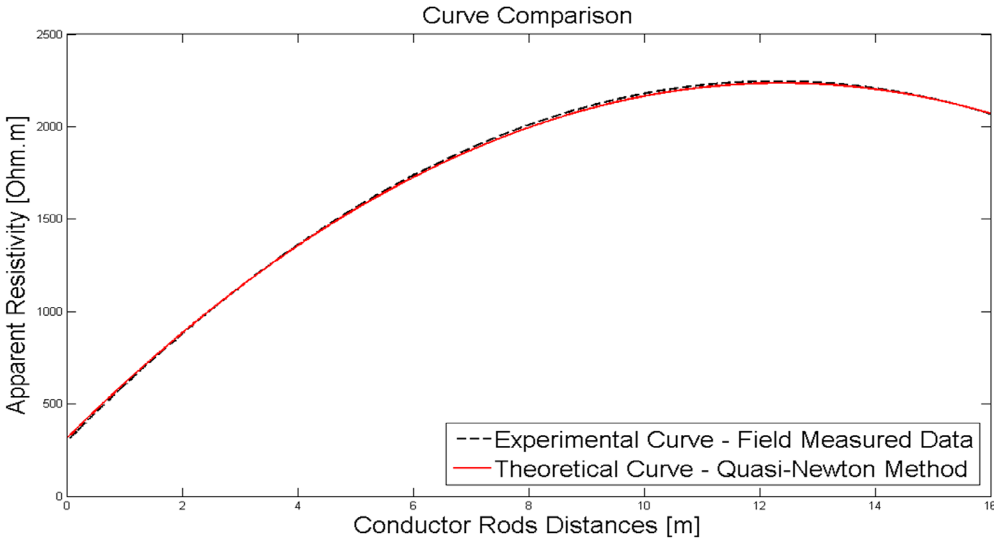


Table 2. Stratification 1 - Results

Layer Number	**Layer Thickness** (m)	**Layer Resistivity** (Ωm)
1st	0.72	408.99
2nd	*inf*	2257.95

Through the tab. 1, one can notice that the major deviation between the theoretical and the experimental curve is -1.95% which corresponds to point 8 *meters*.

5.2 Soil Stratification 2

Table 3. Stratification 2 - Experimental and Theoretical Curves

a (m)	Experimental Curve (Ωm)	Theoretical Curve (Ωm)	Deviation ($\%$)
1.0	3582.92	3663.81	2.26
2.0	3354.57	3258.08	−2.88
4.0	1872.05	1927.73	2.97
8.0	2104.58	2106.74	0.10
16.0	2322.47	2210.28	−4.83

Figure 3. Curve of Apparent Resistivity of the Soil from Stratification 2

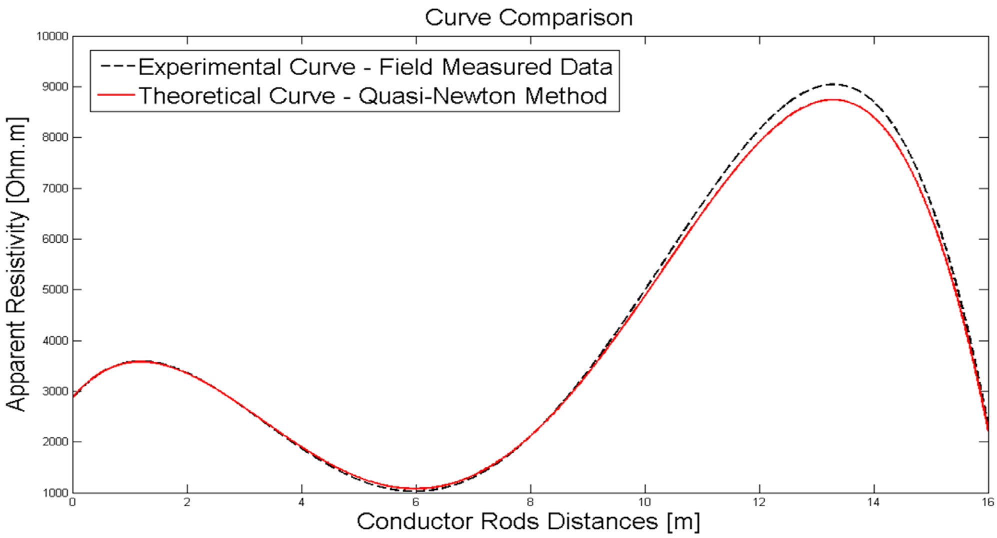


Table 4. Stratification 2 - Results

Layer Number	Layer Thickness (m)	Layer Resistivity (Ωm)
1st	1.31	4120.35
2nd	2.21	1089.34
3rd	2.75	4220.83
4th	*inf*	2007.31

Through tab. 3, one can notice that the major deviation between the theoretical and experimental curve is -4.83% which corresponds to point 16 *meters*.

5.3 Soil Stratification 3

Table 5. Stratification 3 - Experimental and Theoretical Curves

a (m)	Experimental Curve (Ωm)	Theoretical Curve (Ωm)	Deviation (%)
1.0	16841.29	16120.21	−4.28
2.0	20715.20	19966.26	−3.61
4.0	15381.67	15422.58	0.27
8.0	9483.99	8999.18	−5.11
16.0	24970.15	23716.15	−5.02

Figure 4. Curve of Apparent Resistivity of the Soil from Stratification 3

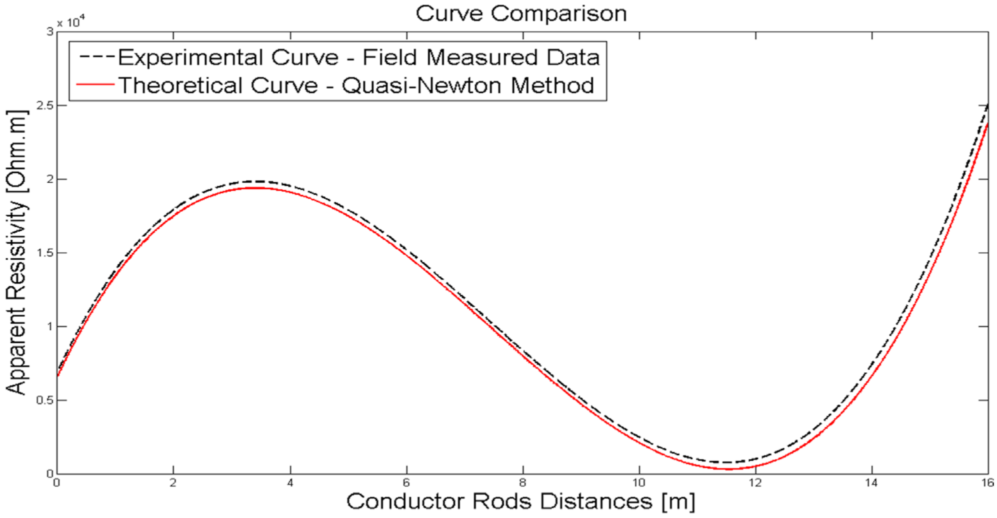


Table 6. Stratification 3 - Results

Layer Number	Layer Thickness (m)	Layer Resistivity (Ωm)
1st	1.06	14315.09
2nd	0.78	21814.81
3rd	*inf*	13535.38

Through tab. 5, one can notice that the major deviation between the theoretical and experimental curve is -5.11% which corresponds to point 8 *meters*.

5.4 Soil Stratification 4

Table 7. Stratification 4 - Experimental and Theoretical Curves

a (m)	Experimental Curve (Ωm)	Theoretical Curve (Ωm)	Deviation (%)
1.0	9560.68	9281.51	−2.92
2.0	11135.90	11105.60	−0.26
4.0	7328.86	7073.66	3.48
8.0	11591.73	11589.17	−0.02
16.0	11990.35	11994.37	0.03

Figure 5. Curve of Apparent Resistivity of the Soil from Stratification 4

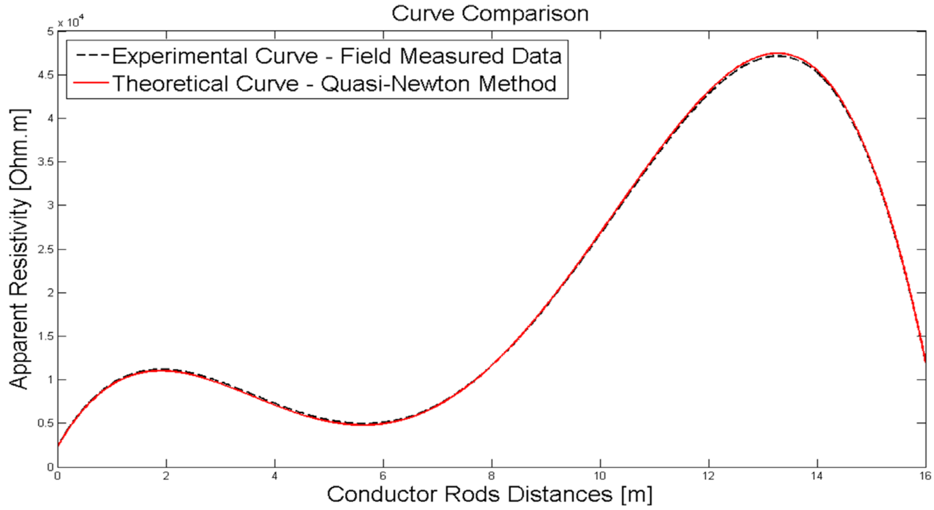


Table 8. Stratification 4 - Results

Layer Number	Layer Thickness (m)	Layer Resistivity (Ωm)
1st	0.59	8126.57
2nd	3.30	9174.68
3rd	4.58	17388.55
4th	*inf*	10257.32

Through tab. 7, one can notice that the major deviation between the theoretical and experimental curve is 3.48% which corresponds to point 4 *meters*.

5.5 Soil Stratification 5

Table 9. Stratification 5 - Experimental and Theoretical Curves (Quasi-Newton Method)

a (m)	Experimental Curve (Ωm)	Theoretical Curve Quasi-Newton Method (Ωm)	Deviation (%)
1.0	1338.76	1324.15	−1.09
2.0	1360.86	1438.28	5.69
4.0	1472.59	1459.73	−0.87
8.0	1300.99	1301.90	0.07
16.0	1078.87	1075.90	−0.28

Table 10. Stratification 5 - Experimental and Theoretical Curves (Complex image Method)

a (m)	Experimental Curve (Ωm)	Theoretical Curve Complex Image Method (Ωm)	Deviation (%)
1.0	1338.76	1373.34	2.58
2.0	1360.86	1379.78	1.39
4.0	1472.59	1389.64	−5.63
8.0	1300.99	1302.39	0.11
16.0	1078.87	1073.92	−0.46

The values of each layer resistivities ρ_1, ρ_2, ρ_3 and ρ_4, and the respectives thickness values h_1, h_2, h_3 and h_4 obtained from each stratification method, are compared at tab. 11.

Figure 6. Curve of Apparent Resistivity of the Soil from Stratification 5

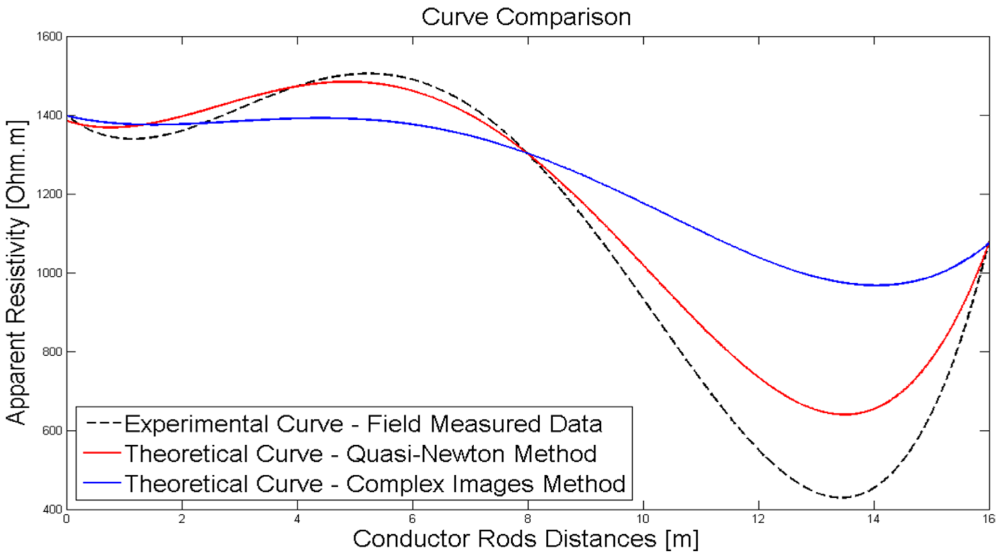


Table 11. Stratification 5 - Results

Layer	Quasi-Newton Method	Complex Image Method
ρ_1	1137.94	1371.99
ρ_2	1538.89	1866.52
ρ_3	336.23	137.99
ρ_4	9208.28	962.96
h_1	0.52	3.74
h_2	7.78	7.11
h_3	7.31	7.66
h_4	*inf*	*inf*

As can be observed at the tab. 9, tab. 10 and fig. 6, there is a great proximity between the results obtained by the stratifications made with the proposed method and the complex image method. However, smaller deviation between the curve of apparent resistivity of the soil obtained with the proposed method and the experimental one, were found.

5.6 Soil Stratification 6

Table 12. Stratification 6 - Experimental and Theoretical Curves (Quasi-Newton Method)

a (m)	**Experimental Curve** (Ωm)	**Theoretical Curve Quasi-Newton** Method (Ωm)	**Deviation** (%)
2.0	3389.00	3389.17	0.01
4.0	1900.00	1609.22	−15.30
8.0	585.00	585.05	0.01
16.0	568.00	574.55	1.15
32.0	823.00	771.82	−6.22

Table 13. Stratification 6 - Experimental and Theoretical Curves (Complex Image Method)

a (m)	**Experimental Curve** (Ωm)	**Theoretical Curve** Complex Image **Method** (Ωm)	**Deviation** (%)
2.0	3389.00	3226.46	−4.80
4.0	1900.00	2294.48	20.76
8.0	585.00	1105.38	88.95
16.0	568.00	690.07	21.49
32.0	823.00	640.72	−22.15

Figure 7. Curve of Apparent Resistivity of the Soil from Stratification 6

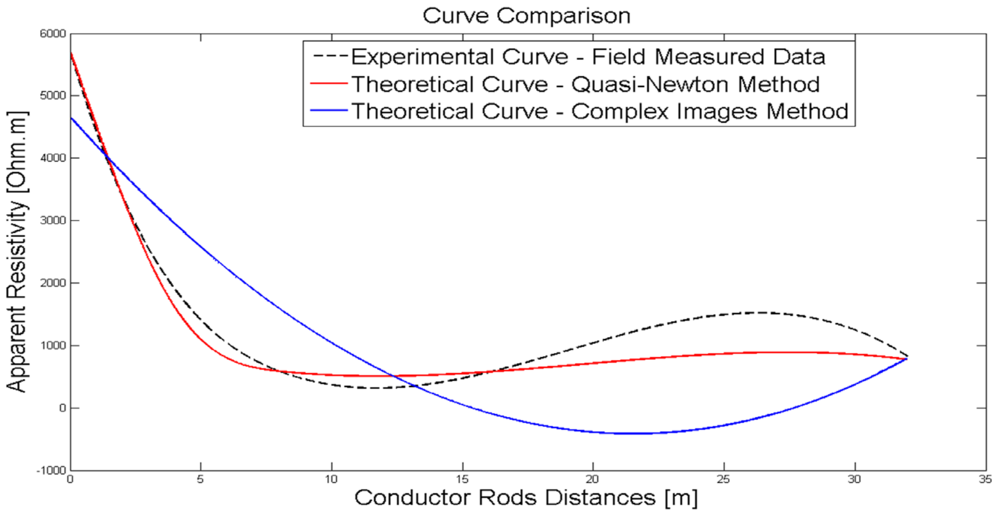


Table 14. Stratification 6 - Results

Layer	Quasi-Newton Method	Complex Image Method
ρ_1	99905.15	3550.00
ρ_2	570.77	630.00
h_1	0.36	3.10
h_2	*inf*	*inf*

The values of each layer resistivities ρ_1 and ρ_2, and the respectives thickness values h_1 and h_2 obtained from each stratification method, are compared at tab. 14.

As can be observed at the tab. 12, tab. 13 and fig. 7, there is a great proximity between the results obtained by the stratifications made with the proposed method and the complex image method. However, smaller deviations between the curve of apparent resistivity of the soil obtained experimentally and through the proposed method, were found, showing a improvement in the proposed stratification method, when compared with the complex image method, which is a very usual employed method, in soil stratifications processes.

In the soil stratification process number 6, the number of layers N was forced into $N = 2$. Analyzing the curve of experimental apparent resistivity of the soil, fig. 7, it can be notice that the number of curve inflections $p = 2$, using the described equation in section 4, $N = p + 1$, would be already enough to estimate a number of layers $N > 2$. At tab. 12 and tab. 13 the listed deviations presented to be larger than the acceptable in grounding projects. This horizontal soil stratification is a typical case of stratification, where the number of layers N is forced in a fixed number, in this case, $N = 2$. Whether this stratification was made considering a number of $N = 3$ layers, the deviations would have a considerably decrease, both to the two applied methods. This fact, evidences the necessity of a methodology that optimizes the number of layers N in the soil stratifications.

Nevertheless, even forcing the number of layers to $N = 2$, it is observed through tab. 12, tab. 13 and fig. 7 that the proposed method achieved lower deviations values than that obtained through the complex image method.

6 Conclusion

This paper has presented a methodology and a mathematical modeling for the soil stratification in multi layers, where the number of layers, the resistivity and the thickness of each layer are calculated and optimized from the experimental curve of apparent resistivity of the soil.

Several cases were processed. In all cases in which the stratification was made by the proposed methodology, the results have presented lower or equal deviations to those found in the pertinent literature. This is explained by the facts that differs this methodology, i.e.:

(i) The most common method employed to determinate the first layer resistivity is the extrapolation of Wenner's experimental curve, where $\rho_a(a)$ is extrapolated until the point where a is equal to zero. Knowing that a exploitation method is purely mathematical, when the curve $\rho_a(a)$ does not have a good behavior, something that frequently occurs, the obtained results can reach errors which overtakes 200%. A such order error is unacceptable for a ρ_1 value, invalidating the rest of the stratification. Some error at ρ_1 can be tolerable, but, depending on its order, it can propagate throughout the stratification, turning its accuracy not reliable. In the proposed methodology, ρ_1 is obtained from the derivations expressions of (6), representing the physical signification in attendance with Wenner's measuring method. This ensures that ρ_1 accuracy depends mostly on the adopted model to represent the soil.

(ii) Soil stratifications considering a fixed number of layers are common. Actually, soil stratification involves three unknown variables: number of layers, resistivity and thickness of each layer. Therefore, the number of layers is not a given date of the problem, but an unknown variable. By the proposed methodology, the set of unknown variables are: N, ρ_i and h_i.

Finally, the accuracy obtained at this work, opens up to new investigations possibilities. If the stratification accuracy is tied up to the project and the electric grounding construction, there is no sense in using errors lower than one unit percentage. An error around 10% is perfectly feasible, constructions imperfections of electric grounding justify this order of magnitude. Still, there is something more to be analyzed. In electrical engineering, the stratification of the soil for electric grounding construction project, adopts the soil as a model, considering its electrical conductibility in horizontally and uniformly layers. Although, some soils behave differently. This explains why sometimes the accumulated error of stratification is not due only to the numeric process of curve adjustment. One parcel of this error, sometimes the major one, can be caused by the fact that the soil does not have uniformity in its horizontal layers.

Restrictions on the proposed method can appear with the existence of a vertical or oblique rift in the layer, this is enough to interfere on the results of Wenner's method. In this situation, a very accurate stratification process is important. If it is assured that the error made at the curve adjustment is small, but a high error is still verified in the values obtained at the stratification, it is concluded that the soil model made by horizontal layers is not applicable to this type of soil. So, the great amount of small errors that appear in this paper, motivates further studies in order to obtain lowest errors on soil stratification, what justifies, for example, a more appropriated study on the optimization process.

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