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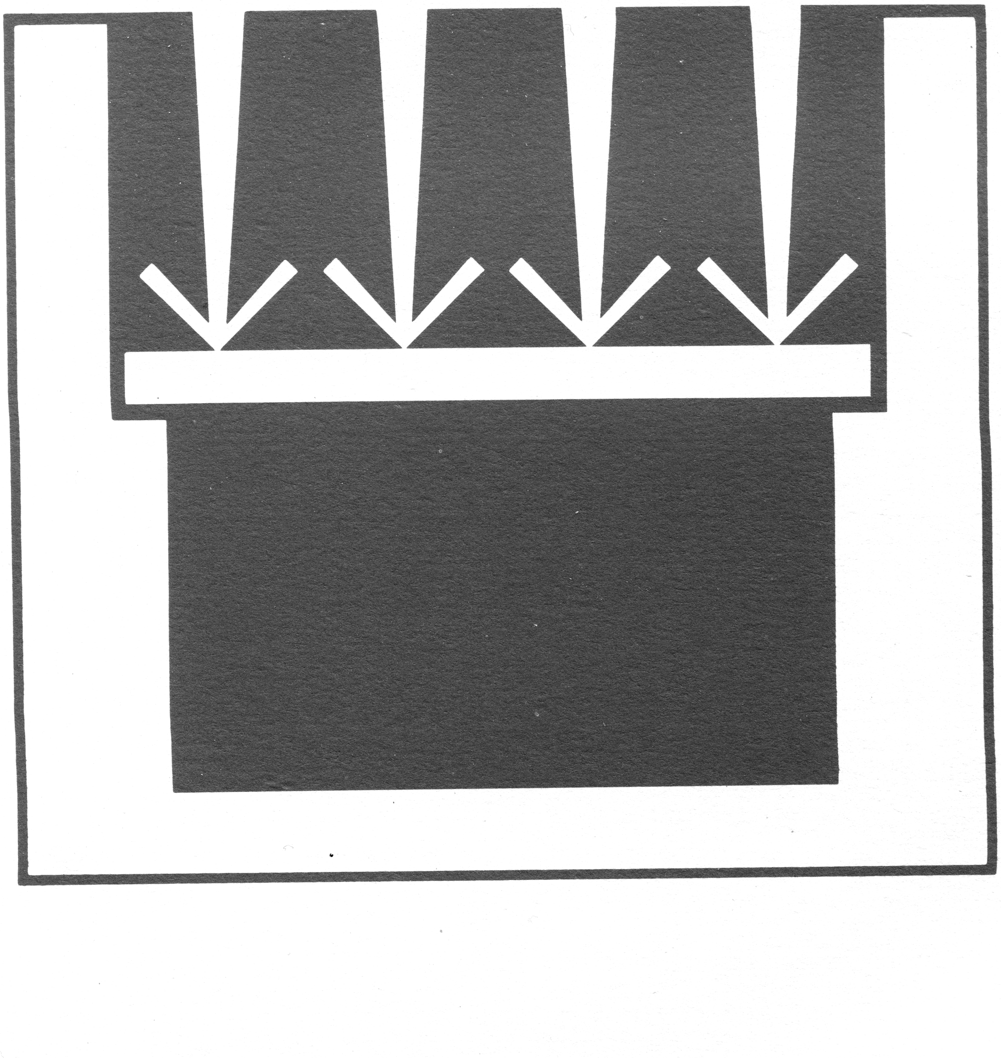

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DESIGN OF FLEXURAL MEMBERS

by

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ABSTRACT

Wood has adequate strength both in bending and shear. These, however, are accompanied by relatively low elastic properties, so that the design of timber beams is frequently controlled by deflection limitation. In some other cases, either bending or shear might be the critical action. Unfortunately, it cannot be predicted which one of these will govern the design, and therefore, a threefold procedure has to be carried out. In order to simplify and speed the procedure, design tables and charts were constructed based on the conventional design formulae.

LIST OF SYMBOLS

a	constant defined in Table B
b	width of the cross section
d	depth of the cross section
E	Young's modulus parallel to grain
I	moment of inertia
M	bending moment
L	effective span
Q	statical moment
P	concentrated force
S	sectional modulus
V	vertical shear force
W	general symbol for load
w	distributed load

Additional symbols are defined in the text, as and when they occur.

INTRODUCTION

The design of flexural members is based on either of two stipulations: either the actual stress components must be less than the permissible stresses, or the stress components obtained by some factorized load should not exceed the ultimate stresses. As long as the stress components are linear functions of the load, the two methods, in principle, provide the same result. Since the modulus of elasticity for wood is comparatively low, the above design procedures do not always satisfy the functional requirements demanded by the structural member. At times the deflection of the beam exceeds a practical limit (360th, 240th, 180th of the span), beyond which the deformed shape jeopardizes the function of the adjoining structural components, e.g., a door or window cannot open or close or the interior of the ceiling may crack, etc. These undesirable consequences could occur even if the beam were structurally safe. In order to conquer these inconveniences, the design must include additionally a deformational analysis. Unfortunately, of the three critical actions, shear, bending, and deformation, thus far we have not been able to precisely predict the critical one. Therefore, each of them must be examined separately. This threefold process is, however, fairly time consuming and, except for the solution of one specific problem, it does not provide a definite prediction for the critical effect in other cases. A satisfactory solution to this problem will simplify the design to a great extent, will provide more versatile results, and may elucidate the entire behavior and resistance of flexural members in the light of simultaneous shear, bending and deflection.

BACKGROUND TO COMPUTATIONS

In most applications, flexural wood members are designed as simply supported beams. The following formulae apply to rectangular cross-sections.

for *shear:*
$$f_s' = \frac{VQ}{Ib} < f_s \tag{1.a}$$

for *bending*
$$f_b' = \frac{M}{S} < f_b \tag{1.b}$$

for *deflection:*
$$\Delta' = A\frac{WL^\alpha}{IE} < \frac{L}{\beta} \tag{1.c}$$

where A is a constant, its value being determined by the type of loading; W denotes the loading which could be concentrated or distributed. The exponent α depends also on the loading system and it will be equal to 3 for concentrated and to 4 for distributed loads. The symbol β denotes the deflection limitation, which may be 360, 240, 180; and f'_s, f'_b, Δ' are the maximum effective shear stress, bending stress, and vertical displacement respectively, and f_s and f_b denote the allowable shear and bending stress.

Formulae 1.a, 1.b, and 1.c can now be rewritten into functional forms so that the ratios W/b play the role of independent variables while the ratios d/L are the functions. Thus we have:

for *shear:*
$$\frac{d}{L} = C_1 \frac{W}{b} \qquad (2.a)$$

for *bending:*
$$\frac{d}{L} = C_2 \sqrt{\frac{W}{b}} \qquad (2.b)$$

for *deflection:*
$$\frac{d}{L} = C_3 \sqrt[3]{\frac{W}{b}} \qquad (2.c)$$

The material properties f_s, f_b and E are incorporated in the constants C_1, C_2 and C_3 respectively. The representation of functions 2.a, 2.b, and 2.c in an orthogonal system of coordinates leads to a set of curves which consists of a straight line for shear, a second order parabola for bending, and a third order parabola for deflection as shown in Fig. 1. Thus the functional relationships represented by the set of curves prescribe three ratios of d/L for each value of the W/b ratio. On closer inspection one can identify three distinctly different divisions in the family of curves represented in Fig. 1.

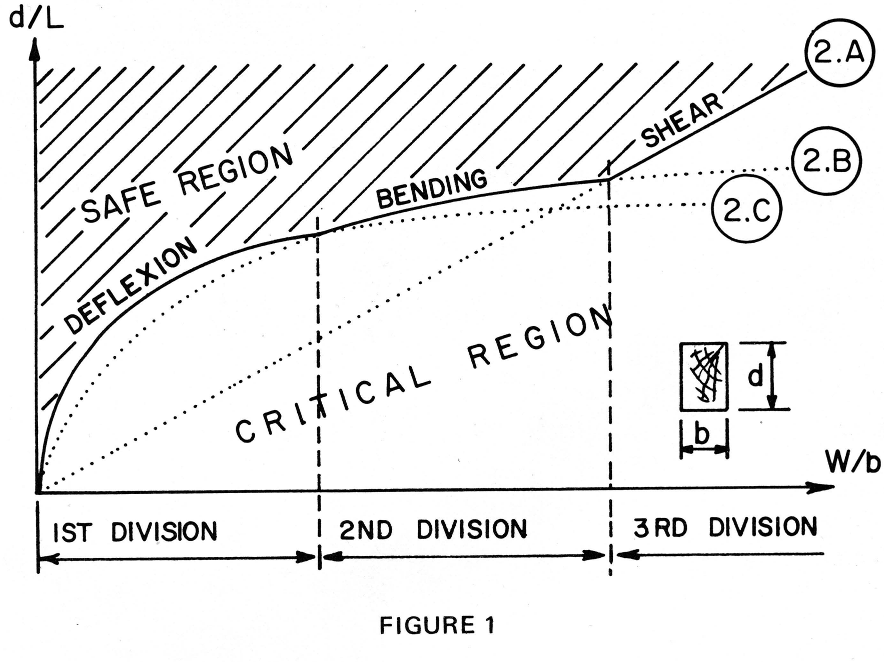


FIGURE 1

In the first division the set is enveloped by the deflexion line; in the second one by the bending line and in the third division by the shear line. Recognizing that by decreasing the ratio d/L the stiffness of the beam also will decrease, it follows that the points located above the enveloping curves represent a region of overdesign or safe region, while those below represent a critical region. It is evident now that the results of safe and most rational design are represented by those points which coincide with the enveloping curve. Thus the design will be controlled by deflection, bending, or shear if the ratio W/b falls into the first, second, or third division respectively. Since the load W and the span L are generally prefixed quantities for the structural design, an arbitrarily chosen width of b of the cross-section will automatically determine whether design is controlled by shear, bending or deflection. Balanced design with respect to either deflection and bending or bending and shear is obtained at the intersection of the respective portions of the enveloping curves.

The envelope line in Fig. 1 can now be used as a tool to determine the suitable depth d for any practically chosen width b of a cross-section to be designed. The method required, however, the numerical values of the constants C_1, C_2 and C_3 for various types of loads.

DETERMINATION OF THE CONSTANTS C_1, C_2 AND C_3

Three types of distributed and three types of concentrated loads will be examined. The allowable deflection will be considered as the 360th of the span. If the larger deflections are permitted, such as 240th or 180th of the span, then the constant C_3 obtained from the L/360 limitation must be multiplied by modifying factors derived later.

Distributed Loads

Consider first a uniformly distributed load w lbs. per foot run (Fig. 2). For maximum vertical shear, bending moment and deflection we have:

$$V = \frac{wL}{2}$$

$$M = \frac{wL^2}{8}$$

$$\Delta = \frac{5}{384}\,\frac{wL^4}{EI}$$

Substituting these into eqs. 1.a, 1.b, and 1.c we obtain for:

a. *shear:*

$$f_s' = \frac{3wL}{4bd} < f_s$$

from which

$$\frac{d}{L} = \frac{0.75}{f_s}\,\frac{w}{b} = C_1\,\frac{w}{b} \qquad (3.a)$$

where

$$C_1 = \frac{0.75}{f_s}$$

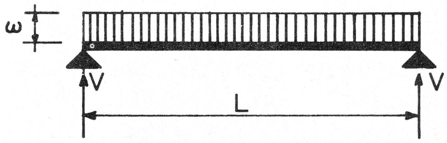


FIGURE 2

b. *bending*:

$$f_b' = 12\,\frac{wL^2/8}{bd^2/6} = \frac{9wL^2}{bd^2} < f_b$$

from which

$$\frac{d}{L} = \sqrt{\frac{9}{f_b}}\,\sqrt{\frac{w}{b}} = C_2\sqrt{\frac{w}{b}} \qquad (3.b)$$

where

$$C_2 = \sqrt{\frac{9}{f_b}}$$

c. *deflection:*

$$\Delta = \frac{5}{384}\,\frac{144wL^4}{bd^3E/12} < \frac{L}{360}$$

from which

$$\frac{d^3}{L^3} = 12^3\,\frac{1800}{384E}\,\frac{w}{b}$$

finally

$$\frac{d}{L} = 12\sqrt[3]{\frac{4.6875}{E}\,\frac{w}{b}} = C_3\sqrt[3]{\frac{w}{b}} \qquad (3.c)$$

where

$$C_3 = 12\sqrt[3]{\frac{4.6875}{E}}$$

Using similar procedures, C_1, C_2 and C_3 constants can be derived for symmetric and non-symmetric triangular loads. These are included in Table B.

Concentrated Loads

In order to express the ratio d/L as various functions of quantities f_s, f_b, E, P and b, it is necessary to introduce a fictitious, uniformly distributed load w_o so that:

$$w_o = \frac{\Sigma P}{L}$$

In addition, factors A_1, A_2 and A_3 have to be defined in such a way that:

$$V = A_1 V_o = A_1 \frac{w_o L}{2}$$

$$M = A_2 M_o = A_2 \frac{w_o L}{8}$$

$$\Delta = A_3 \Delta_o = A_3 \frac{5}{384} \frac{w_o L^4}{EI}$$

where V, M and Δ denote the maximum shear force, bending moment and deflection due to the corresponding type of point loads, while V_o, M_o and Δ_o are the same actions obtained when the beam is subjected to the fictitious, unformly distributed load w_o. Thus all the actions caused by the real concentrated forces are the same as those caused by a factorized fictitious, uniformly distributed load. The factors A_1, A_2 and A_3 should be determined now for various types of point loads. In order to find these, the ratios of shear forces, bending moments and deflections obtained from the real load to those of the fictitious load are determined. The results of these comparative calculations for three types of concentrated loads are presented in Table A. Consider first a single point load at mid-span of the beam (Fig. 3). The fictitious load and the maximum actions are:

$$w_o = \frac{P}{L}$$

$$V = \frac{w_o L}{2}$$

$$M = \frac{w_o L}{4}$$

$$\Delta = 1.6 \frac{5}{384} \frac{w_o L^4}{EI} = \frac{w_o L^4}{48EI}$$

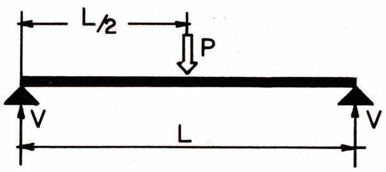


FIGURE 3

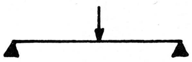
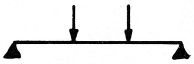
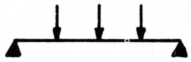
Point Loads	Factors		
	for Shear A_1	for Bending A_2	for Deflection A_3
	1.00	2.00	1.6000
	1.00	4 / 3	1.3626
	1.00	4 / 3	1.2666

TABLE A

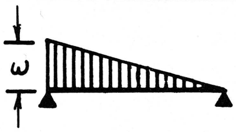
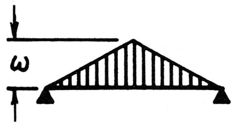
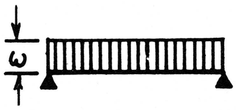
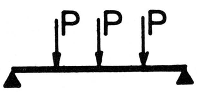
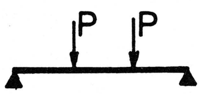
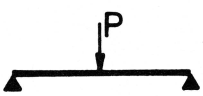
Loading	Constants			Reference Numbers
	for Shear C_1	for Bending C_2	for Deflection C_3	
	$\frac{0.500}{f_s}$	$\sqrt{\frac{4.608}{f_b}}$	$12\sqrt[3]{\frac{2.2680}{E}}$	1
	$\frac{0.375}{f_s}$	$\sqrt{\frac{6.000}{f_b}}$	$12\sqrt[3]{\frac{3.0000}{E}}$	2
	$\frac{0.750}{f_s}$	$\sqrt{\frac{9.000}{f_b}}$	$12\sqrt[3]{\frac{4.6875}{E}}$	3
	$\frac{0.750}{f_s}$	$\sqrt{\frac{12.000}{f_b}}$	$12\sqrt[3]{\frac{5.9372}{E}}$	4
	$\frac{0.750}{f_s}$	$\sqrt{\frac{12.000}{f_b}}$	$12\sqrt[3]{\frac{6.3871}{E}}$	5
	$\frac{0.750}{f_s}$	$\sqrt{\frac{18.000}{f_b}}$	$12\sqrt[3]{\frac{7.5000}{E}}$	6

TABLE B

Substituting these into eqs. 1.a, 1.b, and 1.c the following functions are obtained for:

a. *shear*:

$$f_s = \frac{3 w_o L}{4bd} < f_s$$

from which

$$\frac{d}{L} = \frac{0.75}{f_s} \frac{w_o}{b} = C_1 \frac{w_o}{b} \qquad (4.a)$$

where

$$C_1 = \frac{0.75}{f_s}$$

b. *bending:*

$$f_b' = \frac{12 w_o L^2 / 4}{bd^2/6} = \frac{18 w_o L^2}{bd^2} < f_b$$

from which

$$\frac{d}{L} = \sqrt{\frac{18}{f_b} \frac{w_o}{b}} = C_2 \sqrt{\frac{w_o}{b}} \qquad (4.b)$$

where

$$C_2 = \sqrt{\frac{18}{f_b}}$$

c. *deflection:*

$$\Delta = \frac{144 w_o L^4}{48bd^3/E12} = \frac{12^3 w_o L^4}{48bd^3 E} < \frac{L}{360}$$

from which

$$\frac{d^3}{L^3} = 12^3 \frac{7.5}{E} \frac{w_o}{b}$$

finally

$$\frac{d}{L} = 12 \sqrt[3]{\frac{7.5}{E} \frac{w_o}{b}} = C_3 \sqrt[3]{\frac{w}{b}} \qquad (4.c)$$

where

$$C_3 = 12 \sqrt[3]{\frac{7.5}{E}}$$

Using a similar procedure, C_1, C_2 and C_3 constants can be derived for two and three point loads acting at the third and at the quarter points of the beam. These also are included in Table B.

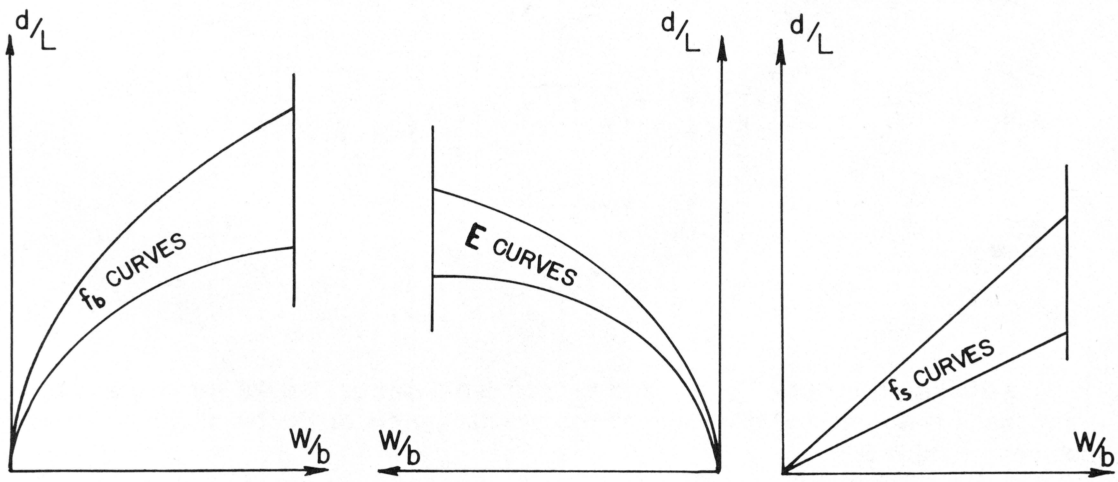
Having determined the constants C_1, C_2 and C_3, the numerical values of functions 2.a, 2.b, and 2.c can now be computed. These constants, however, include some material characteristics, such as allowable unit stress for shear and bending, and the

Young's modulus parallel to grain. These properties have different values for each species and grades of structural wood. The various design specification for stress-grade lumber indicates at least 15 values for shear, 33 values for bending and 23 values for Young's modulus. Considering all these, 71 different values of properties and the like number of curves would be necessary to cover the design completely for one loading case. Thus the six loading cases discussed require at least 426 curves. Ten ordinates for one curve provide, in general, sufficient accuracy in the graphical representation of such relations; consequently, the entire work requires the computation of 426 constants and the minimum of 4260 ordinates. This work was carried out on the IBM 360 digital computer of West Virginia University.

THE SUGGESTED METHOD OF DESIGN

The computer program provides 15 straight lines for shear, 33 second order parabola for bending and 23 third order parabola for deflection, altogether 71 curves for each of the six loading cases. The task before us now is to combine these curves into various related groups or, in other words, to form sets of curves similar to that of Fig. 1. According to the above data, the necessary number of sets to cover one loading case completely is at minimum 210. Thus, the representation of the six types of loads requires 1260 independent graphs. Such an interpretation of the results is evidently not suitable, and therefore the idea of Fig. 1 must be disregarded and another method must be developed.

Let us consider, therefore, three separate orthogonal systems of coordinates so that the ratios w/b or w_o/b are measured along the horizontal axes and the corresponding d/L ratios occupy the positive vertical axes. We restrict ourselves to one particular loading case and plot all the curves for bending in the first quadrant of the first system, all the curves for deflection in the second quadrant of the second system, and all the straight lines for shear in the first quadrant of the third system. Then we have the arrangement shown in Fig. 4.



33 CURVES FOR BENDING — 23 CURVES FOR DEFLEXION — 15 CURVES FOR SHEAR

FIGURE 4

To design a flexural member for a given load, span, and a specified grade of lumber, one should select a suitable value of b, find the ratio w/b or w_o/b on the horizontal axis of each of the three coordinate systems, and read the d/L ratios corresponding to the appropriate shear (f_s), bending (f_b), and deflection (E) curves. The greatest of these ratios provides the critical action that will govern the design. From this ratio the required depth d of the beam can be calculated; i.e.: d (in) = ratio x L (ft).

The three families of curves in Fig. 4 can be condensed to a complex nomogram or can be converted to a tabulated form, so that one loading case will be represented entirely by one nomogram or by one table. Each of the nomograms and each of the tables is provided with a reference number (from 1 to 6) which corresponds to one particular type of load. The reference number and the related loads also are shown in Table B.

MODIFYING FACTORS

For a long time it has been recognized that the bending strength of a wood member is significantly influenced by various factors, among which the duration of the load and the depth of the beam are the most important. Therefore, the various codes and design specifications for stress-grade lumber and laminated beams introduce modifying factors in order to adjust the allowable stresses in the beam when the duration of the load is different from the normal (10 years) or when the depth of the beam is greater than 12 inches. The presented design process which is based on the suggested nomograms or tables, however, results directly in the effective depth of the beam, and therefore the modifying factors given by the codes should be converted to provide modifications applied directly to the depth of the beam.

Modification Due to Duration of the Load

Since tests have shown that wood has the property of carrying substantially greater loads for shorter periods than for long duration, the allowable working stresses specified for normal duration should be multiplied by a factor k_{12}, when the duration is different from the normal. The value of k_{12} is specified by the code for various durations.

For normal duration, when k_{12} = 1, equation 2.b stands as:

$$\frac{d}{L} = C_2\sqrt{\frac{w}{b}} \quad \text{and} \quad C_2 = \sqrt{\frac{a}{f_b}}$$

In the last equation a is a constant and depends on the type of load as shown in Table B. If the duration of the load is different from the normal, then instead of C'_2 one may write C'_2 and consequently:

$$C'_2 = \sqrt{\frac{a}{k_{12} f_b}} = \sqrt{\frac{1}{k_{12}}}\sqrt{\frac{a}{f_b}} = \frac{1}{\sqrt{k_{12}}} C_2$$

Then obviously the required $\frac{d}{L}$ ration will be:

$$\frac{d}{L} = C'_2\sqrt{\frac{w}{b}} = \sqrt{\frac{1}{k_{12}}}\, C_2\sqrt{\frac{w}{b}} \qquad (5)$$

It follows now that if the duration is different from the normal, the result obtained from the nomograms or tables should be multiplied by $1/\sqrt{k_{12}}$. The values of k_{12} and $1/\sqrt{k_{12}}$ for the usual durations are given in Table C.

Duration	Application	k_{12}	$1/\sqrt{k_{12}}$
Permanent (50 years)	Self Weight	0.90	1.0540
Normal (10 years)	Life load	1.00	1.0000
Two week	Snow load	1.15	0.9325
7 day	Long term wind	1.25	0.8944
1 day	Medium term wind	1.33	0.8671
1 hour	Short term wind	1.50	0.8165
1 second	Impact	2.00	0.7011

TABLE C

Modification Due to Increasing Depth

The allowable stresses for bending and for sawed lumber include an adjustment for depths up to 12 in. As the sizes of the glued-laminated beams continued to increase to depths exceeding almost 80 in., it becomes particularly important to define precisely the size-strength phenomenon. A recent study on the subject based on a statistical-strength theory by B. Bohannan suggests to multiply the allowable stress by a depth factor k (equation 6) if the depth of the laminated beam exceeds 12 in.

$$k = \sqrt[9]{\frac{12}{d}} \qquad (6)$$

Unfortunately this formula is true only for a span:depth ratio equal to 21 and for a loading case which consists of two equal concentrated loads applied on the third points of the beam. Consequently, the formula (equation 7) suggested by British Standard Code of Practice CP112 for deep beams, is much more suitable for considering the depth effect. According to which:

$$k_{16} = 0.81\,\frac{d^2 + 143}{d^2 + 83} \qquad (7)$$

If d is less than 12 in., then $K_{16} = 1$ and equation 2.6 stands as:

$$\frac{d}{L} = C_2\sqrt{\frac{w}{b}} \quad \text{and} \quad C_2 = \sqrt{\frac{a}{f_b}}$$

where a is a factor which depends on the type of the load as shown in Table B. If d is greater than 12 in., then an adjustment on the allowable stress should be carried out. Then one may write C'_2 instead of C_2, and obviously:

$$C'_2 = \sqrt{\frac{a}{k_{16} f_b}} = \sqrt{\frac{1}{k_{16}}}\sqrt{\frac{a}{f_b}} = \sqrt{\frac{1}{k_{16}}}\,C_2$$

and the required depth:span ratio will be:

$$\frac{d}{L} = \frac{1}{\sqrt{k_{16}}} C_2 \sqrt{\frac{w}{b}} \qquad (8)$$

It follows now that if d is greater than 12 in., then the depth obtained from the nomograms or from the tables should be multiplied by $\sqrt{1/k_{16}}$. The values of k_{16} and $\sqrt{1/k_{16}}$ for realistic depths are given in Table D.

Depth	k_{16}	$1/\sqrt{k_{16}}$
12.0	1.0020	0.9995
12.5	0.9924	1.0038
13.0	0.9833	1.0084
13.5	0.9748	1.0128
14.0	0.9669	1.0170
14.5	0.9594	1.0210
15.0	0.9523	1.0247
15.5	0.9457	1.0283
16.0	0.9395	1.0317
16.5	0.9337	1.0349
17.0	0.9282	1.0380
17.5	0.9230	1.0409
18.0	0.9181	1.0436
18.5	0.9135	1.0462
19.0	0.9092	1.0487
19.5	0.9051	1.0511
20.0	0.9013	1.0533

TABLE D

Modification Due to Deflection Limitation

During the derivation of the equations of the present design nomograms and tables, a deflection limitation of $\Delta = L/360$ was assumed. Sometimes, however, the design may satisfy weaker limitations, i.e. L/240 or even L/180. If this is the case, then the depth obtained from the limitation L/360 must be adjusted to correspond to these weaker limitations. The adjustment should be carried out by multiplying the depth obtained from the L/360 limitation by a factor. The values of this factor will be determined next for L/240 and L/180 deflection limitations.

If the deflection limitation is L/360, then equation 2.3 stands as

$$\frac{d}{L} = C_3 \sqrt[3]{\frac{w}{b}} \quad \text{and} \quad C_3 = 12 \sqrt[3]{\frac{a}{E}}$$

In the second equation a is a constant that depends on the type of loading as shown in Table B. But a also includes the deflection limitation L/360 so that

$$a = 360\ a_1$$

and so

$$C_3 = 12\sqrt[3]{\frac{360\ a_1}{E}}$$

If the deflection limitation is considered as L/240 or L/180, then one may write C'_3 and C''_3, respectively, instead of C_3, and therefore:

$$C'_3 = 12\sqrt[3]{\frac{360\ a_1}{E} \cdot \frac{240}{360}} = 12\sqrt[3]{\frac{2}{3} \cdot \frac{a}{E}} = 0.8735\ C_3$$

$$C''_3 = 12\sqrt[3]{\frac{360\ a_1}{E} \cdot \frac{180}{360}} = 12\sqrt[3]{\frac{1}{2} \cdot \frac{a}{E}} = 0.7937\ C_3$$

and the required depth:span ratio for:

L/240 limitation is $$\frac{d}{L} = 0.8735\ C_3\sqrt[3]{\frac{w}{b}} \qquad (9)$$

L/180 limitation is $$\frac{d}{L} = 0.7937\ C_3\sqrt[3]{\frac{w}{b}} \qquad (10)$$

Since the product $C_3\sqrt[3]{\frac{w}{b}}$ is obtained from the nomograms or tables for a limitation of L/360, the effective depth of the beam for the other two limitations can be obtained directly from equations 9 and 10.

NUMERICAL EXAMPLES

In order to show the versatility of the nomograms or tables, the following numerical examples are presented.

Example 1

Design the cross-section of a simply supported beam, if the following information is available: f_b = 2100 psi., f_s = 140 psi., E = 1,700,000 psi., L = 17.5 ft., w = 80 lb/ft. uniformly distributed load.

Select, for example, b = 1.5 in.;

$$\text{then } \frac{w}{b} = \frac{80}{1.5} = 53.25$$

From nomogram 3 or Table 3 one obtains the following ratios:

For *shear:* 0.285

For *bending:* 0.477

For *deflection:* 0.635

The critical effect is the deflection, and the critical ratio is 0.635. Then the required depth:

$$d = 0.635 \times 17.5 = 11.10 \text{ in., say } 11.25 \text{ in.}$$

Example 2

A glue-laminated beam of 20 ft. span is loaded by a uniformly distributed load w = 450 lb./ft. Design the beam if the thickness and width of one layer are 0.5 in. and 5 in. respectively and the available material has the following properties: f_b = 1300 psi., f_s = 95 psi., E = 1,700,000. Then $\frac{w}{b} = \frac{450}{5} = 90$. From nomogram 3 or Table 3 we obtain the following ratios:

For *shear:* 0.707

For *bending:* 0.789

For *deflection:* 0.754

The critical effect is the bending and therefore:

$$d = 0.789 \times 20 = 15.78 \text{ in.}$$

Since d is greater than 12 in., modification of the depth is necessary. From Table D we can interpolate the corresponding factor, 1.0334. Then the modified depth becomes:

$$d = 1.0334 \times 15.78 = 16.3070 \text{ say } 16.50$$

and the number of layers: $\frac{16.5}{0.5} = 33$

Example 3

A simply supported beam, having a span of 25 ft., is loaded by two equal, concentrated loads (P = 375 lb.) acting at the third points of the beam. Design the beam to carry this load safely if the self weight of the beam is neglected and the available lumber has the following properties: f_b = 1900 psi., f_s = 125 psi., E = 1,520,000 psi.

The fictitious load is $\frac{2 \times 375}{25}$ = 30 lb./ft. Assuming 1.5 in. for the width of the beam, we have: $\frac{w_o}{b} = \frac{30}{1.5} = 20$. Then from nomogram 5 or Table 5 one obtains the following ratios:

For *shear:* 0.120

For *bending:* 0.355

For *bending:* 0.526

The critical effect is the deflection, and the required depth will be:

$$d = 0.526 \times 25 = 13.15 \text{ in. say } 13.5 \text{ in.}$$

Example 4

A simply supported beam is loaded by three equal, concentrated loads (P = 400 lb.) acting at the quarter points of the beam. The span of the beam is equal to 20 ft and the cross-section is given by b = 1.5 in. and d = 9.25 in. Find the effective stresses and determine whether the cross-section is adequate for deflection when E = 1,700,000 psi. If not, redesign the cross-section and state the effective stresses. Neglect the self weight of the beam.

The fictitious load is $\frac{3 \times 400}{20} = 60$ lb./ft.

$$\text{Then} \quad \frac{w_o}{b} = \frac{60}{1.5} = 40$$

$$\text{And} \quad \frac{d}{L} = \frac{9.25}{20} = 0.462$$

By using nomogram 4 or Table 4, the above ratios determine one f_b and one f_s curve which will provide the effective stresses in the beam and these are: f_b = 2250 psi. and f_s is less than 70 psi.

Checking the deflection, one finds that the required Young's modulus is much greater than 1,910,000 psi. which is the highest available figure according to the codes. Consequently, the beam is not adequate for deflection. Considering the given Young's modulus (E = 1,700,000 psi.), the necessary ratio for deflection becomes 0.623 and the required depth:

$$d = 0.623 \times 20 = 12.46 \text{ in., say } 13.25 \text{ in.}$$

Then the effective stresses obtained by the corresponding f_b and f_s curves are: f_b = 1240 psi. and f_s = less than 70 psi.

Example 5

A simply supported beam is loaded by a non-symmetric triangular load having a maximum intensity of 200 lb./ft. The cross-section of the beam is given as b = 2.5 in. and d = 9.25 in. Find the maximum span for which the beam can be applied safely if f_b = 1900 psi., f_s = 140 psi., and E = 1,700,000 psi.

The load-width ratio is $\frac{w}{b} = \frac{220}{2.5} = 88$

Using Nomogram 1 or Table 1, one can find the following ratios:

For *shear:* 0.314
For *bending:* 0.461
For *deflection:* 0.587

The critical effect is the deflection, and therefore the maximum safe span:

$$L = \frac{9.25}{0.587} = 15.75 \text{ ft.}$$

Example 6

A beam, simply supported over a span of 25 ft., is loaded by a uniformly distributed load of 60 lb./ft. and by a concentrated load of 400 lb. applied at the middle of the span. Design the cross-section of the beam if the allowable unit stresses for shear and for bending are 125 psi., and 2100 psi. respectively, and the Young's modulus is 1,500,000 psi.

The design is carried out in two steps. First, consider the concentrated load, for which the fictitious load is:

$$w_o = \frac{400}{25} = 16 \text{ lb./ft.}$$

Assume now 1.75 in. for the initial width of the beam. Then:

$$\frac{w_o}{b} = \frac{16}{1.5} = 9.15$$

and using Nomogram 6 or Table 6 we obtain the following ratios:

For *shear:*	0.055
For *bending:*	0.259
For *deflection:*	0.435

Then obviously: d = 0.435 x 25 - 10.86 in., say 11.50 in.

In the second step of the design we consider 11.50 in. for the depth; then the corresponding d/L ratio is 0.46. We find now a suitable width to carry the uniformly distributed load. Therefore, Nomogram 3 or Table 3 will be used which provides the following three values for the w/b ratios:

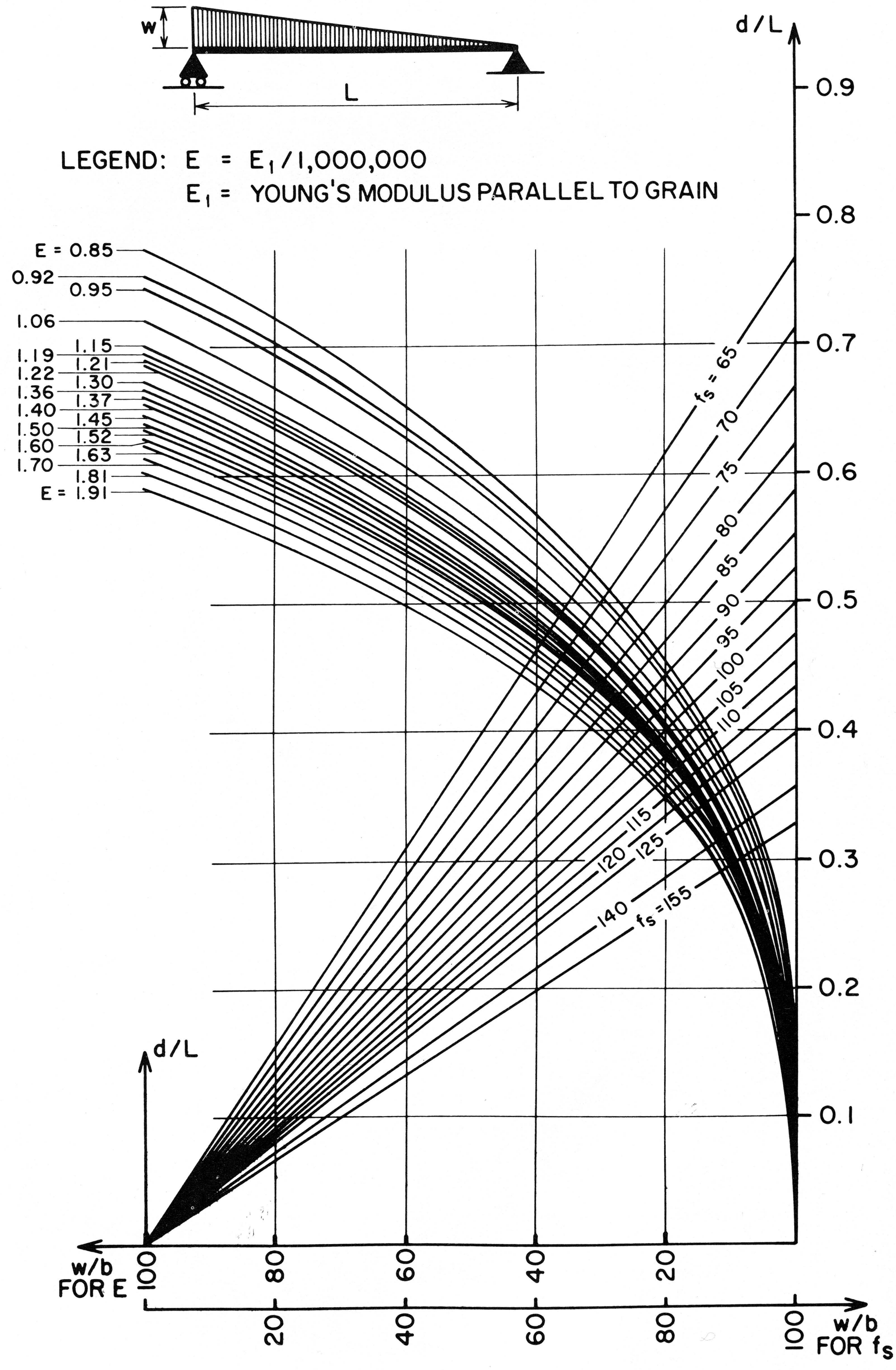
For *shear:*	76.66
For *bending:*	49.38
For *deflection:*	18.40

The critical effect is the deflection, consequently:

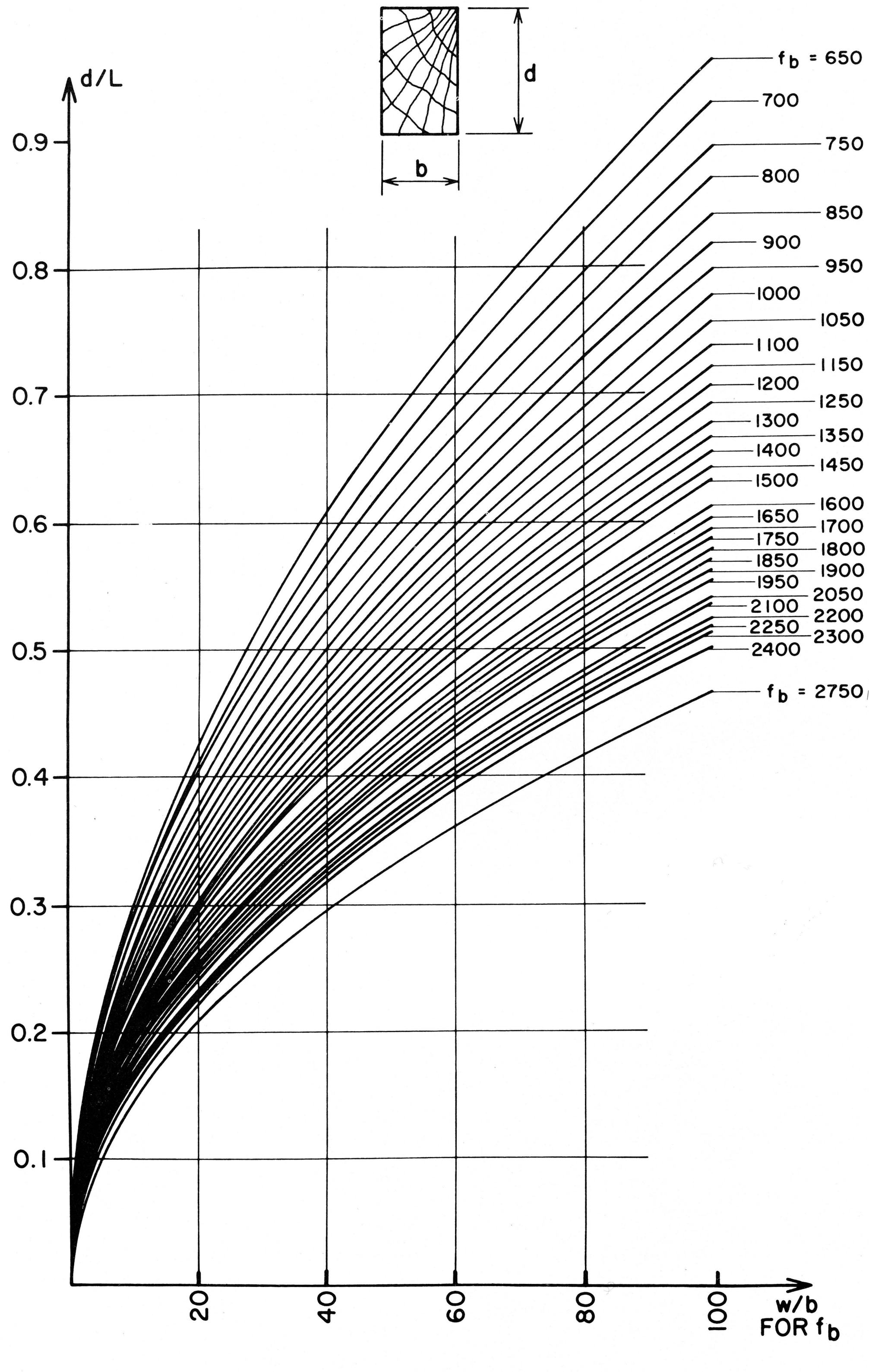
$$b = \frac{60}{18.40} = 3.36 \text{ in.}$$

Finally, the total width is equal to 1.75 + 3.36 = 5.01, say 5.50 in. and the depth is 11.5 in.

APPENDIX


w
L
d/L
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
LEGEND: E = E1/1,000,000
E1 = YOUNG'S MODULUS PARALLEL TO GRAIN
E = 0.85
0.92
0.95
1.06
1.15
1.19
1.21
1.22
1.30
1.36
1.37
1.40
1.45
1.50
1.52
1.60
1.63
1.70
1.81
E = 1.91
fs = 65
70
75
80
85
90
95
100
105
110
115
120
125
140
fs = 155
d/L
w/b
FOR E
100
80
60
40
20
20
40
60
80
100
w/b
FOR fs

NOMOGRAM I


d
b
d/L
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
20
40
60
80
100
w/b
FOR f_b
f_b = 650
700
750
800
850
900
950
1000
1050
1100
1150
1200
1250
1300
1350
1400
1450
1500
1600
1650
1700
1750
1800
1850
1900
1950
2050
2100
2200
2250
2300
2400
f_b = 2750

NOMOGRAM 1

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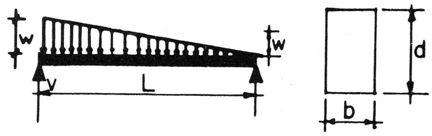


LEGEND: f_s = allowable unit stress for shear
f_b = allowable unit stress for bending
E = Young's modulus II to grain

SHEAR

f_s \ w/b	10	20	30	40	50	60	70	80	90	100
155	0·032	0·065	0·097	0·129	0·161	0·194	0·226	0·258	0·290	0·323
140	0·036	0·071	0·107	0·143	0·173	0·214	0·250	0·286	0·321	0·357
125	0·040	0·080	0·120	0·160	0·200	0·240	0·280	0·320	0·360	0·400
120	0·042	0·083	0·125	0·167	0·208	0·250	0·292	0·333	0·375	0·417
110	0·045	0·091	0·136	0·182	0·227	0·273	0·318	0·364	0·409	0·455
100	0·050	0·100	0·150	0·200	0·250	0·300	0·350	0·400	0·450	0·500
90	0·056	0·111	0·167	0·222	0·278	0·333	0·389	0·444	0·555	0·556
80	0·062	0·125	0·187	0·250	0·312	0·375	0·437	0·500	0·562	0·625
70	0·071	0·143	0·214	0·286	0·357	0·429	0·500	0·571	0·643	0·714

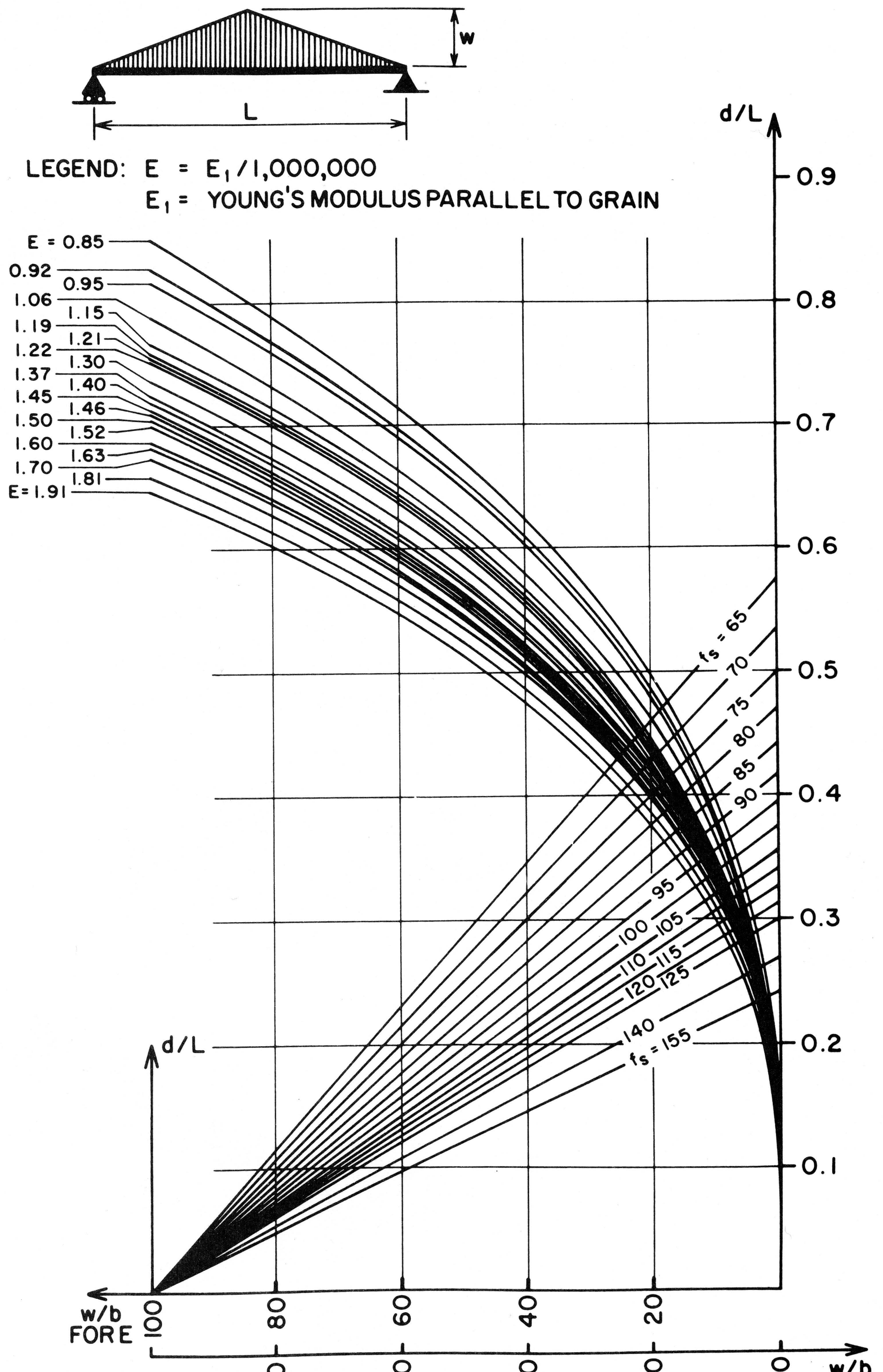
BENDING

f_b \ w/b	10	20	30	40	50	60	70	80	90	100
2750	0·129	0·183	0·224	0·259	0·289	0·317	0·342	0·366	0·388	0·409
2400	0·139	0·196	0·240	0·277	0·310	0·339	0·367	0·392	0·416	0·438
2300	0·142	0·200	0·245	0·283	0·317	0·347	0·374	0·400	0·425	0·448
2200	0·145	0·205	0·251	0·289	0·324	0·355	0·383	0·409	0·434	0·458
2100	0·148	0·209	0·257	0·296	0·331	0·363	0·392	0·419	0·444	0·468
2000	0·152	0·214	0·263	0·303	0·339	0·372	0·402	0·429	0·455	0·480
1900	0·156	0·220	0·270	0·311	0·348	0·381	0·412	0·440	0·467	0·492
1800	0·160	0·226	0·277	0·320	0·358	0·392	0·423	0·453	0·480	0·506
1700	0·165	0·233	0·285	0·329	0·368	0·403	0·436	0·466	0·494	0·521
1600	0·170	0·240	0·294	0·339	0·379	0·416	0·449	0·480	0·509	0·537
1500	0·175	0·248	0·304	0·351	0·392	0·429	0·464	0·496	0·526	0·554
1400	0·181	0·257	0·314	0·363	0·406	0·444	0·480	0·513	0·544	0·574
1300	0·188	0·266	0·326	0·377	0·421	0·461	0·498	0·533	0·565	0·595
1200	0·196	0·277	0·339	0·392	0·438	0·480	0·518	0·554	0·588	0·620
1100	0·205	0·289	0·355	0·409	0·458	0·501	0·542	0·579	0·614	0·647
1000	0·215	0·304	0·372	0·429	0·480	0·526	0·568	0·607	0·644	0·679
900	0·226	0·320	0·392	0·453	0·506	0·554	0·599	0·640	0·679	0·716
800	0·240	0·339	0·416	0·480	0·537	0·588	0·635	0·679	0·720	0·759
700	0·257	0·363	0·444	0·513	0·574	0·628	0·679	0·726	0·770	0·811

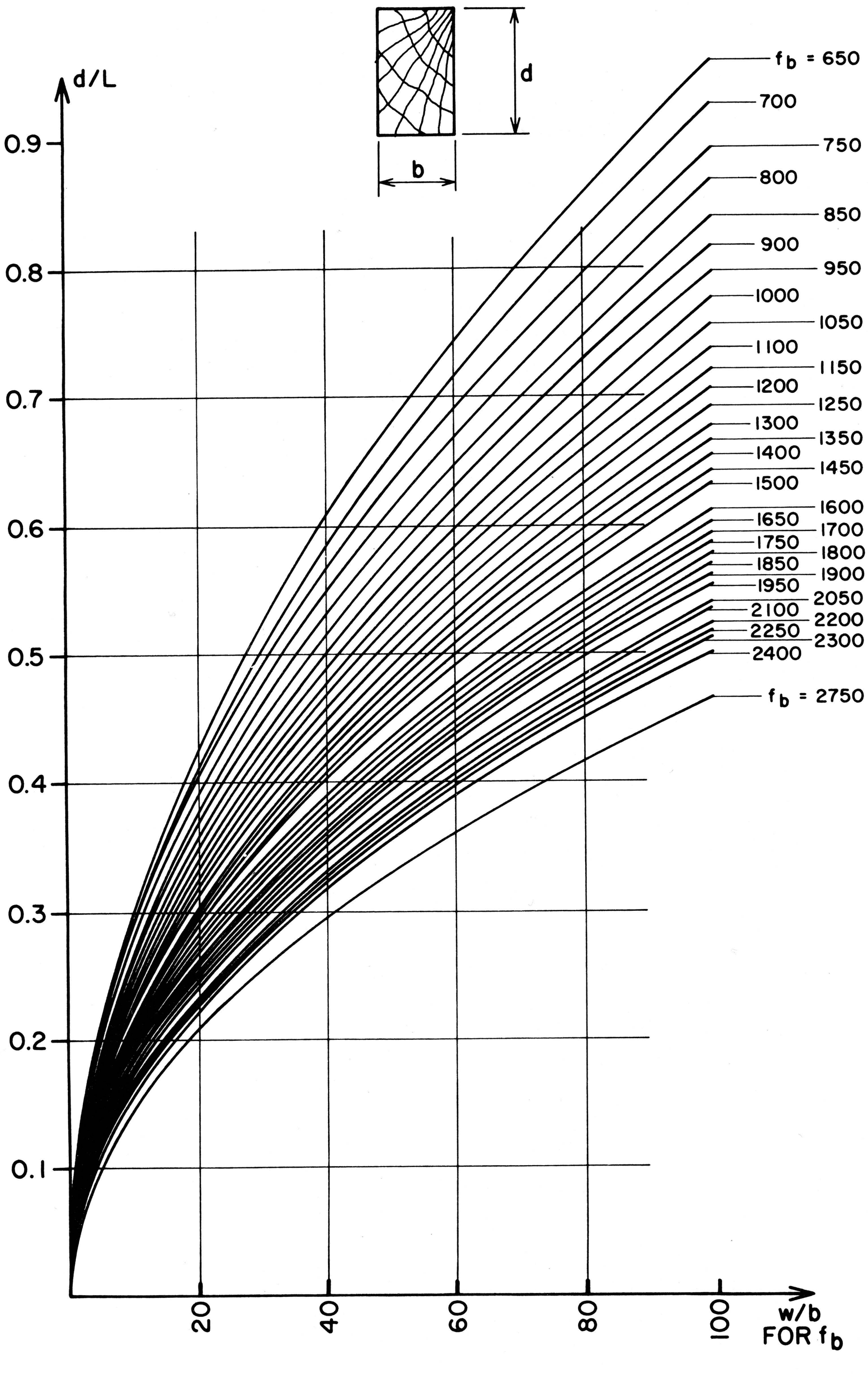
DEFLECTION

E \ w/b	10	20	30	40	50	60	70	80	90	100
1.91×10^6	0·274	0·345	0·395	0·435	0·468	0·497	0·524	0·548	0·569	0·590
1.81×10^6	0·279	0·351	0·402	0·442	0·477	0·506	0·533	0·557	0·580	0·600
1.70×10^6	0·285	0·359	0·410	0·452	0·487	0·517	0·544	0·569	0·592	0·613
1.60×10^6	0·290	0·366	0·419	0·461	0·497	0·528	0·528	0·581	0·604	0·626
1.50×10^6	0·297	0·374	0·428	0·471	0·507	0·539	0·568	0·593	0·617	0·639
1.40×10^6	0·304	0·383	0·438	0·482	0·519	0·552	0·581	0·607	0·632	0·654
1.30×10^6	0·311	0·392	0·449	0·494	0·532	0·566	0·595	0·622	0·647	0·671
1.20×10^6	0·320	0·403	0·461	0·507	0·546	0·580	0·611	0·639	0·665	0·689
1.10×10^6	0·330	0·416	0·474	0·523	0·564	0·598	0·630	0·660	0·686	0·717
0.95×10^6	0·346	0·435	0·498	0·548	0·591	0·628	0·661	0·691	0·719	0·744
0.85×10^6	0·359	0·452	0·517	0·569	0·613	0·652	0·686	0·717	0·746	0·773

TABLE 1


w
L
LEGEND: E = E1 / 1,000,000
E1 = YOUNG'S MODULUS PARALLEL TO GRAIN
d/L
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
E = 0.85
0.92
0.95
1.06
1.15
1.19
1.21
1.22
1.30
1.37
1.40
1.45
1.46
1.50
1.52
1.60
1.63
1.70
1.81
E = 1.91
fs = 65
70
75
80
85
90
95
100
105
110
115
120
125
140
fs = 155
w/b FOR E
100 80 60 40 20
w/b FOR fs
20 40 60 80 100

NOMOGRAM 2


d/L
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
d
b
f_b = 650
700
750
800
850
900
950
1000
1050
1100
1150
1200
1250
1300
1350
1400
1450
1500
1600
1650
1700
1750
1800
1850
1900
1950
2050
2100
2200
2250
2300
2400
f_b = 2750
20
40
60
80
100
w/b
FOR f_b

NOMOGRAM 2