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At $T=0$ the full electron propagator is known to have an infrared anomalous dimension and thus a branch point at $P^2=m^2$ rather than a pole. An explicit calculation shows that if $0<eT\ll m$ the retarded self-energy is analytic in the vicinity of $P^2\approx m^2$, which includes the thermal mass shell. The low-temperature propagator has a simple pole. Only when $T=0$ is there a branch point at the mass shell.
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I. INTRODUCTION

In high temperature gauge theories the fermion propagator is quite different than at zero temperature [1] and calculations require the Braaten-Pisarski resummation of hard thermal loops [2–4]. One of the important quantities to be calculated is the imaginary part of the fermion self-energy, or damping rate, which has been computed in various circumstances. In QCD the massless gluons are so changed by thermal effects that a resummed gluon propagator is always necessary to compute thermal damping rates. Whether the quark masses are small or large compared to gT determines if a resummed quark propagator is required. The quark damping rate has been computed in both cases [2,5] and the potential infrared divergences are controlled by incorporating a magnetic screening mass. The absence of a magnetic screening mass in QED makes the electromagnetic damping rates more problematic [6]. It appears that at high temperature ($eT\gg m$) the electron propagator at large time does not decay exponentially [7]. The same behavior is observed in scalar QED [8].

In QED at low temperature ($eT\ll m$) neither the electron nor photon propagator require Braaten-Pisarski resummation. One would expect very little qualitative difference between low temperature and zero temperature. However, explicit calculation will show that there is an important difference: at $T=0$ the electron propagator has a branch cut at the mass shell but for $T>0$ the propagator has a simple pole.

A. Zero temperature

The scattering formalism of zero-temperature quantum field theory relies upon the assumption that asymptotically separated particles do not influence each other. Consequently propagators are supposed to have simple poles at the physical mass of the particle. However, this argument fails for charged particles because of the long range of the Coulomb force. Two charged particles that are arbitrarily far apart do not travel in straight lines. Instead their asymptotic trajectories are bent and the curvature grows logarithmically with their separation. Thus charged particles that are infinitely separated cannot be treated as free particles. The consequences of this for QED were first investigated by Abrikosov, Chung, and Kibble [9–11]. They showed that the electron propagator actually has a branch point at $P^2=m^2$ rather than a pole. The propagator has the behavior

$$S'(P)\to \frac{Z_2}{m^{2\gamma}}\frac{\not{P}+m}{(P^2-m^2)^{1-\gamma}} \tag{1}$$

in the vicinity $P^2\approx m^2$. The value of γ depends upon the choice of gauge. If the free photon propagator is

$$D^{\mu\nu}(K)=-\frac{g^{\mu\nu}}{K^2}+(1-\xi)\frac{K^\mu K^\nu}{(K^2)^2},$$

then

$$\gamma=(\xi-3)\alpha/2\pi. \tag{2}$$

Only when $\xi=3$ (Yennie gauge) does the electron propagator have a simple pole. The exponent γ is the infrared anomalous dimension of the electron propagator. A recent calculation [12] of γ for the propagator and other multi-fermion Green functions shows that Eq. (1) is valid provided the anomalous dimension in the range $-1<\gamma<1/2$, which corresponds to an enormous ξ range: $-857<\xi<433$. Although γ is an infrared anomalous dimension and arises from the infrared behavior of the gauge boson propagator, γ itself is not infrared divergent.

B. Nonzero temperature

The question that will be investigated here is the effect of massless photons on the near mass-shell behavior of the finite-temperature electron propagator. At finite temperature the location of the singularity in the propagator is temperature-dependent. It is convenient to deal with the retarded propagator. The inverse of the full retarded thermal propagator is

$$S_R'^{-1}(P)=\not{P}-m-\Sigma_R, \tag{3}$$

where Σ_R is defined to contain the $T=0$ mass counterterm δm. In the rest frame of the plasma, rotational invariance requires that Σ_R be a linear combination of the matrices $1,\gamma_0,\vec{\gamma}\cdot\vec{p}$, and $\gamma_0\vec{\gamma}\cdot\vec{p}$. It is then straightforward to compute the inverse of Eq. (3). The result may be expressed compactly by defining

$$\tilde{\Sigma}_R = \Sigma_R - \frac{1}{2}\mathrm{Tr}[\Sigma_R].$$

Then $\mathrm{Tr}[\tilde{\Sigma}_R] = -\mathrm{Tr}[\Sigma_R]$. The inversion of Eq. (3) gives for the full propagator

$$S'_R(P) = \frac{\not{P} + m + \tilde{\Sigma}_R}{P^2 - m^2 - \Pi_R(P)}, \tag{4}$$

where the scalar self-energy in the denominator is

$$\Pi_R(P) = \frac{1}{2}\mathrm{Tr}[(\not{P}+m)\Sigma_R] - \frac{1}{4}\mathrm{Tr}[\Sigma_R \tilde{\Sigma}_R]. \tag{5}$$

Let $D(P) = P^2 - m^2 - \Pi_R(P)$ be the denominator of Eq. (4). The location at which $1/D$ is singular (either a pole or a branch point) gives a complicated, temperature-dependent relation between p_0 and p of the form

$$P^2 = m^2 + a(P). \tag{6}$$

The denominator of the propagator has the structure

$$D(P) = [P^2 - m^2 - a(P)]^{1-c(P)}[1 + b(P)], \tag{7}$$

with a, b, c generally complex. The question of whether the propagator has a branch point at the thermal dispersion relation (6) is therefore a question of whether the function $c(P)$ vanishes when Eq. (6) is satisfied.

Order α Approximation for $eT \ll m$: In a perturbative expansion the functions a, b, c are each of order α or smaller. If $eT \ll m$ then $a(P) \ll m^2$. To first order in α the denominator, is

$$D(P) \approx P^2 - m^2 - a(P) + b(P)(P^2 - m^2) - c(P)(P^2 - m^2)\ln(P^2 - m^2). \tag{8}$$

Although the thermal mass-shell condition is given by Eq. (6), the possibility of a branch point at the thermal mass shell is reduced to finding whether there is a term of the form $c(P)(P^2 - m^2)\ln(P^2 - m^2)$. To first order in α this only requires computing

$$\Pi_R(P) = \frac{1}{2}\mathrm{Tr}[(\not{P}+m)\Sigma_R] + \mathcal{O}(\alpha^2). \tag{9}$$

Note that the logarithmic term in Eq. (8) is quite small at the mass-shell (6), $\mathcal{O}(\alpha^2 \ln(\alpha))$ and was not examined in previous calculations [13].

Section II gives the explicit result for the one-loop electron denominator both in Feynman gauge and in general covariant gauges. The detailed calculations are contained in the appendixes.

II. ONE-LOOP SELF-ENERGY

To compute the one-loop self-energy Σ_R it will be useful to use free propagators that are themselves retarded or advanced. The free retarded propagator for the electron is

$$S_R(P) = \frac{\not{P} + m}{P^2 - m^2 + i\eta p_0},$$

and for the photon in a general covariant gauge is

$$D_R^{\mu\nu}(K) = \left[-g^{\mu\nu} + (1-\xi)\frac{K^\mu K^\nu}{2k}\frac{\partial}{\partial k}\right]\frac{1}{K^2 + i\eta k_0}. \tag{10}$$

Since $eT \ll m$ neither propagator requires resummation. The advanced propagators are obtained by reversing the sign of the infinitesimal imaginary part in the denominators. Dimensional regularization will be used to control the zero-temperature ultraviolet divergences. The one-loop self-energy has the structure

$$\Sigma_R(P) = \Sigma_R^e(P) + \Sigma_R^\gamma(P) + \delta m, \tag{11}$$

with δm the $T=0$ mass counterterm. The first contribution has the internal electron on shell:

$$\Sigma_R^e(P) = \frac{ie^2\mu^\epsilon}{2}\int \frac{d^{4-\epsilon}K}{(2\pi)^4}\tanh((p_0 - k_0)/2T) \times D_R^{\mu\nu}(K)\gamma_\mu[S_R(P-K) - S_A(P-K)]\gamma_\nu. \tag{12}$$

At $T=0$ this contribution does not contain a term of the form $(P^2 - m^2)\ln(P^2 - m^2)$. Although the magnitude of the temperature-dependent part is exponentially suppressed by $\exp(-m/T)$, that would not rule out a branch cut with a small coefficient. Appendix A proves that there is no such term.

The important contribution is the second term in Eq. (11). It has the internal photon on shell:

$$\Sigma_R^\gamma(P) = \frac{ie^2\mu^\epsilon}{2}\int \frac{d^{4-\epsilon}K}{(2\pi)^4}\coth(k_0/2T)\gamma_\mu S_R(P-K)\gamma_\nu \times [D_R^{\mu\nu}(K) - D_A^{\mu\nu}(K)].$$

Inserting the photon propagator (10) gives

$$\Sigma_R^\gamma(P) = \frac{e^2\mu^\epsilon}{2}\int \frac{d^{4-\epsilon}K}{(2\pi)^3}\coth(|k_0|/2T)\gamma_\mu S_R(P-K)\gamma_\nu \times \left[-g^{\mu\nu} + \frac{(1-\xi)K^\mu K^\nu}{2k}\frac{\partial}{\partial k}\right]\delta(K^2). \tag{13}$$

The contribution of this term to the denominator of the electron propagator will be labeled

$$\Pi^\gamma(P) = \frac{1}{2}\mathrm{Tr}[(\not{P}+m)\Sigma_R^\gamma(P)]. \tag{14}$$

Most of the paper is devoted to this computation.

A. Self-energy in Feynman gauge

In the Feynman gauge, $\xi = 1$, the photon propagator is simplest. The trace in Eq. (14) yields

$$\Pi^{\gamma}(P)=c_0+\frac{e^2T^2}{6}+f(P), \tag{15}$$

where c_0 is a temperature-independent, divergent constant that is canceled by the mass counterterm in Eq. (11). Thus the mass shell condition (6) is essentially $P^2\approx m^2+e^2T^2/6$, which is well known [13].

The P dependence is contained in the function

$$f(P)=\frac{A\mu^{\epsilon}}{2\pi^2}\int d^{4-\epsilon}K\,\frac{\delta(K^2)\coth(|k_0|/2T)}{(P_c-K)^2-m^2}, \tag{16}$$

$$A=\alpha(P^2-3m^2), \tag{17}$$

where $P_c=(p_0+i\eta,\vec{p})$ because of the retarded prescription. The imaginary part of the denominator is $2\eta(p_0-k_0)$ and will generally change sign as k_0 is integrated. Appendix B shows that the denominator does not change sign if p_0 is positive time-like:

$$p_0>p=|\vec{p}|. \tag{18}$$

Then P in Eq. (16) can be taken real and m replaced by $m_c^2=m^2-i\eta$. The integration is performed in Appendix B with the result

$$f(P)=\frac{A}{2\pi}\left(\frac{P^2-m^2}{P^2}\right)\left[-\frac{1}{\epsilon}+\ln\left(\frac{2\pi T}{\mu}\right)\right]$$
$$-i\frac{AT}{p}\left[\frac{1}{2}\ln\left(\frac{p_0+p}{p_0-p}\right)+\ln\left(\frac{\Gamma(Z_+)}{\Gamma(Z_-)}\right)\right], \tag{19}$$

$$Z_{\pm}\equiv 1+i\frac{m_c^2-P^2}{4\pi T(p^0\pm p)}. \tag{20}$$

The ultraviolet divergent term, $1/\epsilon$, is absorbed into the wave-function renormalization factor. Various properties are discussed below.

Analyticity at $P^2\approx m^2$: The most important result is that there is no term of the form $(P^2-m^2)\ln(P^2-m^2)$. In the vicinity of $P^2\approx m^2$ the variables $Z_{\pm}$ are close to 1. Therefore, $\ln\Gamma(Z_{\pm})$ is analytic near the mass shell. This means that when $T\neq 0$ the electron propagator has a simple pole at $P^2\approx m^2+e^2T^2/6$ and not a branch cut.

Zero-Temperature Limit: It is rather surprising that Eq. (19) does have a logarithmic branch point precisely at $T=0$. This comes about because as $T\to 0$, the arguments $Z_{\pm}\to\infty$ in Eq. (20). Using the Stirling approximation $\ln\Gamma(Z)\to Z\ln(Z)-Z$ gives the zero-temperature limit

$$\lim_{T\to 0}\frac{-iT}{p}\ln\left(\frac{\Gamma(Z_+)}{\Gamma(Z_-)}\right)=\frac{P^2-m^2}{2\pi P^2}\left[\ln\left(\frac{im_c^2-iP^2}{4\pi T\sqrt{P^2}}\right)\right.$$
$$\left.-1+\frac{p_0}{2p}\ln\left(\frac{p_0+p}{p_0-p}\right)\right]. \tag{21}$$

Therefore, the zero-temperature limit of Eq. (19) is

$$f(P)|_{T=0}=\frac{A(P^2-m^2)}{2\pi P^2}\left[-\frac{1}{\epsilon}+\ln\left(\frac{im_c^2-iP^2}{2\mu\sqrt{P^2}}\right)\right.$$
$$\left.-1+\frac{p_0}{2p}\ln\left(\frac{p_0+p}{p_0-p}\right)\right]. \tag{22}$$

This does contain the term $(P^2-m^2)\ln(P^2-m^2)$. The logarithmic contribution to the electron denominator function, $D(P)=P^2-m^2-f(P)$, is

$$D(P)\approx P^2-m^2+\frac{\alpha}{\pi}(P^2-m^2)\ln(P^2-m^2). \tag{23}$$

This agrees with the standard result in Eq. (2).

Analyticity for $\mathrm{Im}\,p_0>0$: A further check of the result is the requirement that the retarded self-energy be analytic in the upper half of the complex p_0 plane. The only singularities in Eq. (19) that occur at complex p_0 come from poles in $\Gamma(Z_{\pm})$. These occur at $Z_{\pm}=1-n$, for n a positive integer and require that p_0 satisfy $p_0^2=E^2-i4\pi nT(p_0\pm p)$. The complex roots of this equation can be written $p_0=p_{0r}+ip_{0i}$ and satisfy

$$p_{0r}^2=E^2+4\pi nTp_{0i}+p_{0i}^2,$$

$$p_{0i}=2\pi nT\left(-1\pm\frac{p}{p_{0r}}\right).$$

If there were a root with $p_{0i}>0$, then the first equation implies that $|p_{0r}|>E$ so that $p/p_{0r}<1$. But then the second equation implies that $p_{0i}<0$ contrary to the hypothesis. Hence there are no branch cuts for $\mathrm{Im}\,p_0>0$.

Imaginary Part: At $P^2=m^2$ the self-energy (19) is pure imaginary:

$$f(P)|_{P^2=m^2}=i\frac{\alpha m^2T}{p}\ln\left(\frac{E+p}{E-p}\right). \tag{24}$$

This is an artifact of not having an infrared regularization as shown by Rebhan in a different context [5]. Even the sign of Eq. (24) is opposite what it should be for a retared self-energy. To check that it is an infrared effect, one can return to Eq. (16) and compute the imaginary part directly:

$$\mathrm{Im}\,f(P)=\frac{-A}{2\pi}\int\frac{d^3k}{2k}\coth\left(\frac{k}{2T}\right)\epsilon(p_0-k_0)$$
$$\times\delta(P^2-m^2-2P\cdot K)|_{k_0=\pm k}.$$

At $P^2=m^2$ the δ function becomes

$$\delta(2P\cdot K)|_{k_0=\pm k}=\frac{\delta(k)}{2(E\mp\vec{p}\cdot\hat{k})}.$$

Even though the support is at $k=0$ the integral does not vanish because because of the Bose-Einstein enhancement of $k=0$:

$$\int_0^\infty kdk \coth\left(\frac{k}{2T}\right)\delta(k)=T.$$

The remaining angular integration is

$$\mathrm{Im}\, f(P)=\frac{\alpha m^2 TE}{2\pi}\int d\Omega \frac{1}{E^2-(\vec{p}\cdot\hat{k})^2},$$

and reproduces Eq. (24). The entire effect comes from the point $k=0$. If the infrared behavior is regulated there will be no imaginary part at $P^2=m^2$.

B. Self-energy in general covariant gauge

In a general covariant gauge with $\xi\neq 1$, the second term in Eq. (13) must be computed. A prime will be used to denote this contribution:

$$\Sigma_R'(P)=(1-\xi)\frac{e^2\mu^\epsilon}{2}\int \frac{d^{4-\epsilon}K}{(2\pi)^3}\frac{\partial\delta(K^2)}{\partial k}\coth\left(\frac{|k_0|}{2T}\right) \times\frac{K^\mu K^\nu}{2k}\gamma_\mu S_R(P-K)\gamma_\nu. \tag{25}$$

The trace necessary for the electron denominator is

$$f'(P)=\frac{1}{2}\mathrm{Tr}[(\not{P}+m)\Sigma_R'(P)], \tag{26}$$

which yields

$$f'(P)=\frac{B\mu^\epsilon}{4\pi^2}\int \frac{d^{4-\epsilon}K}{k}\frac{\partial\delta(K^2)}{\partial k}\coth\left(\frac{|k_0|}{2T}\right)\frac{P\cdot K-K^2}{(P_c-K)^2-m^2}, \tag{27}$$

$$B=\alpha(1-\xi)(P^2-m^2). \tag{28}$$

In Appendix C this is computed for $p_0>p$ with the result

$$\begin{aligned} f'(P)=\frac{B}{2\pi}\bigg[&\frac{1}{\epsilon}-1+\ln\left(\frac{\mu}{2\pi T}\right)+\frac{i2\pi p_0 T}{P^2-m_c^2} \\ &+\frac{i\pi T}{2p}\ln\left(\frac{p_0+p}{p_0-p}\right)+\frac{i\pi T}{p}\ln\left(\frac{\Gamma(Z_+)}{\Gamma(Z_-)}\right) \\ &-\frac{1}{2}\left(1+\frac{P^2-m^2}{2p(p_0+p)}\right)\psi(Z_+) \\ &-\frac{1}{2}\left(1-\frac{P^2-m^2}{2p(p_0-p)}\right)\psi(Z_-)\bigg]. \end{aligned} \tag{29}$$

Analyticity at $P^2\approx m^2$: As was the case in Feynman gauge, there is no explicit $\ln(im^2-iP^2)$. Since $\Gamma(Z_\pm)$ and $\psi(Z_\pm)$ are analytic near $Z_\pm\approx 1$, the entire function $f'(P)$ is analytic near the mass shell. There is no branch point.

Zero-temperature limit: To evaluate $f'(P)$ at zero temperature requires using Eq. (21) and the asymptotic behavior $\psi(Z_\pm)\to\ln(Z_\pm)$ in order to obtain

$$f'(P)|_{T=0}=\frac{B}{2\pi}\left[\frac{1}{\epsilon}-\frac{P^2+m^2}{2P^2}+\ln\left(\frac{2\mu\sqrt{P^2}}{im^2-iP^2}\right)\right]. \tag{30}$$

This does contain the term $(P^2-m^2)\ln(P^2-m^2)$ with coefficient

$$-\frac{\alpha}{2\pi}(1-\xi)(P^2-m^2)\ln(P^2-m^2).$$

Subtracting this from the Feynman-gauge contribution (23) gives

$$D(P)\approx P^2-m^2+\frac{\alpha}{2\pi}(3-\xi)(P^2-m^2)\ln(P^2-m^2). \tag{31}$$

This agrees with the general result (2).

Analyticity for $\mathrm{Im}\, p_0>0$: Since the only singularities in $\Gamma(Z_\pm)$ or $\psi(Z_\pm)$ are when $Z_\pm$ is zero or a negative integer, the same analysis as before shows that Eq. (29) is analytic in the upper-half of the complex p_0 plane.

Imaginary Part: The factor B vanishes at $P^2=m^2$. However, from the first line of Eq. (29) it appears that $f'(P)\to i\alpha(1-\xi)ET$ as $P^2\to m^2$. As before this term would not survive if the infrared behavior had been regulated [5].

III. COMMENTS

The branch point in the $T=0$ electron propagator is a major complication. It indicates the impossibility of an electron being isolated. It will always have a cloud of photons and will, therefore, not be an eigenstate of the mass operator [14]. To treat charged particles properly it is necessary to employ a Hilbert space containing an infinite number of coherent photons [10,11,15]. The Lehmann-Symanzik-Zimmermann (LSZ) asymptotic conditions and reduction formulas are modified [16]. It is possible to avoid having a branch point by using a more complicated, gauge-invariant field operator for the electron [17].

It is remarkable that for $0<eT\ll m$ the electron self-energy does not contain a term $(P^2-m^2)\ln(P^2-m^2)$. The reason for this difference is that the coordinate-space photon propagator $D^{\mu\nu}(x)$ at large time-like separations falls exponentially, $\exp(-2\pi T(t-r))$, whereas at zero-temperature it falls like a power, $1/(t^2-r^2)$. This approach will be presented in a subsequent publication.

The electron propagator thus has a simple pole at the thermal mass shell $P^2\approx m^2+e^2T^2/6$. This result does not automatically carry over to QCD at low temperature. No matter how large the quark masses are that break chiral symmetry, the gluon propagator requires Braaten-Pisarski resummation.

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APPENDIX A: ANALYSIS OF Σ_R^e

One can eliminate the self-energy contribution in Eq. (12) as a possibility for producing a branch cut at $P^2=m^2$ rather easily:

$$\Sigma_R^e(P)=\frac{e^2\mu^\epsilon}{2}\int\frac{d^{4-\epsilon}K}{(2\pi)^3}\delta[(P-K)^2-m^2]\times\tanh\left(\frac{|p_0-k_0|}{2T}\right)D_R^{\mu\nu}(K)\gamma_\mu(\not{P}-\not{K}+m)\gamma_\nu .$$

The δ function constraint sets $k_0=p_0\pm\Omega$ where $\Omega=[m^2+(\vec{p}-\vec{k})^2]^{1/2}$ so that

$$\Sigma_R^e(P)=\frac{e^2\mu^\epsilon}{16\pi^3}\int\frac{d^{3-\epsilon}k}{2\Omega}\tanh\left(\frac{\Omega}{2T}\right)\times D_R^{\mu\nu}(K)\gamma_\mu(\not{P}-\not{K}+m)\gamma_\nu|_{k_0=p_0\pm\Omega} .$$

If this were to contain a term $(P^2-m^2)\ln(P^2-m^2)$ then the derivative with respect to p_0 would be logarithmically divergent at $p_0=E$. The case $k_0=p_0+\Omega$ does not have this behavior because as $p_0\to E$ the denominator of the photon propagator is infrared safe. That leaves the case $k_0=p_0-\Omega$. The largest contribution to the derivative of the self-energy comes from differentiating $1/K^2$:

$$\left.\frac{\partial\Sigma_R^e(P)}{\partial p_0}\right|_{p_0=E}\sim\int\frac{d^{3-\epsilon}k}{2\Omega}\frac{E-\Omega}{[(E-\Omega)^2-k^2]^2}.$$

The important region is k small, in which case $\Omega\approx E-\vec{v}\cdot\vec{k}$, where $\vec{v}=\vec{p}/E$ is the electron velocity. This gives

$$\int\frac{d^{3-\epsilon}k}{2E}\frac{\vec{v}\cdot\vec{k}+\mathcal{O}(k^2)}{[(\vec{v}\cdot\vec{k})^2-k^2]^2}.$$

By power counting this integration could give a logarithmic divergence. However, the numerator $\vec{v}\cdot\vec{k}$ is odd in $\vec{k}$ and thus the angular integral gives zero. The neglected terms are all $d^3k k^2/k^4$ and are finite. Thus Σ_R^e cannot contain a term $(P^2-m^2)\ln(P^2-m^2)$.

APPENDIX B: CALCULATION OF Σ_R^γ IN FEYNMAN GAUGE

The integral displayed in Eq. (16) for the Feynman-gauge self-energy is performed explicitly in this Appendix. The answer for the zero-temperature contribution is displayed in Eq. (B6); for the thermal contribution, in Eq. (B8). The sum of the two gives the result quoted in Eq. (19).

To analyze Eq. (16) the integration over k_0 and over angles can be performed with the result

$$f(P)=\frac{A\mu^\epsilon}{4\pi p}\int_0^\infty\frac{dk}{k^\epsilon}\ln\left[\frac{(k+r)(k-s)}{(k-r)(k+s)}\right]\coth\left(\frac{k}{2T}\right),$$

$$r=\frac{P_c^2-m^2}{2(p_c^0+p)},\quad s=\frac{P_c^2-m^2}{2(p_c^0-p)},\tag{B1}$$

where $p_c^0=p^0+i\eta$. The imaginary parts of r and s are always positive for any real values of p_0 and p. For p_0 positive and time-like, i.e., $p_0>p$, the denominators of Eq. (B1) are positive so that r and s can be replaced by

$$r'=\frac{P^2-m^2+i\eta}{2(p_0+p)},\quad s'=\frac{P^2-m^2+i\eta}{2(p_0-p)}.$$

This is the same as using a Feynman prescription, $m^2-i\eta$, in the original denominator of Eq. (16). Thus,

$$f(P)=\frac{A\mu^\epsilon}{2\pi^2}\int d^{4-\epsilon}K\frac{\delta(K^2)\coth(|k_0|/2T)}{P^2-m^2-2P\cdot K+i\eta},\tag{B2}$$

which will be easier to compute. Because the imaginary part of the denominator no longer changes sign one can use the parametric representation

$$\frac{1}{X+i\eta}=-i\int_0^\infty ds e^{i(X+i\eta)s},\tag{B3}$$

and interchange the order of integrations to get

$$f(P)=-i\int_0^\infty ds e^{i(P^2-m^2+i\eta)s}J(s),$$

$$J(s)\equiv\frac{A\mu^\epsilon}{2\pi^2}\int d^{4-\epsilon}K\delta(K^2)\coth(|k^0|/2T)e^{-i2P\cdot Ks}.$$

The k^0 integration can be performed using the Dirac delta function and the angular integrals are elementary:

$$J(s)=\frac{A\mu^\epsilon}{2\pi ps}\int_0^\infty dk k^{-\epsilon}\coth(k/2T)\times\{\sin[2s(p^0+p)k]-\sin[2s(p^0-p)k]\}.\tag{B4}$$

At zero temperature there are ultraviolet divergences (regularized by $k^{-\epsilon}$); at finite temperature there are not. The relation $\coth(k/2T)=1+2n(k/T)$, where

$$n(x)=\frac{1}{\exp(x)-1},\tag{B5}$$

allows the temperature-independent part of the integration to be isolated and leads to

$$J(s)=J_0(s)+J_T(s).$$

Zero-Temperature: At $T=0$ the momentum integration in Eq. (B4) gives

$$J_0(s)=Ns^{\epsilon-2},$$

$$N\equiv\frac{A(2\mu)^\epsilon}{4\pi p}\Gamma(1-\epsilon)\cos\left(\frac{\epsilon\pi}{2}\right)[(p^0+p)^{\epsilon-1}-(p^0-p)^{\epsilon-1}].$$

The zero-temperature contribution to $f(P)$ is

$$f_0(P) = -iN\int_0^\infty ds e^{i(P^2-m^2+i\eta)s} s^{\epsilon-2}$$
$$= -iN\Gamma(\epsilon-1)(im^2-iP^2)^{1-\epsilon}.$$

In the limit $\epsilon\to 0$ this has the expected $1/\epsilon$ ultraviolet divergence plus finite terms:

$$f_0(P) = \frac{A(m^2-P^2)}{2\pi P^2}\left[\frac{1}{\epsilon}+1-\frac{p^0}{2p}\ln\left(\frac{p^0+p}{p^0-p}\right) + \ln\left(\frac{2\mu\sqrt{P^2}}{im^2-iP^2}\right)\right]. \tag{B6}$$

There is no branch point at $p_0=p$ because of a cancellation between the two logarithms. It does have a logarithmic branch cut $P^2=m^2$ as expected.

Thermal Contribution: The remainder of Eq. (B4) is temperature-dependent:

$$J_T(s) = \frac{A\mu^\epsilon}{\pi p s}\int_0^\infty dk k^{-\epsilon} \times \frac{\sin[2s(p^0+p)k]-\sin[2s(p^0-p)k]}{\exp(k/T)-1}.$$

This can be performed using the useful integral [18]

$$\int_0^\infty dx x^{\nu-1}\frac{\exp(ax)}{\exp(bx)-1} = \frac{\Gamma(\nu)}{b^\nu}\zeta\left[\nu,1-\frac{a}{b}\right], \tag{B7}$$

which is valid for positive real b and $\mathrm{Re}\, a<b$. The result is

$$J_T(s) = \frac{AT}{ps}\left[\frac{-1}{4\pi T(p^0+p)s} + n[4\pi T(p^0+p)s] + \frac{1}{4\pi T(p^0-p)s} - n[4\pi T(p^0-p)s]\right].$$

The final integration over s requires

$$f_T(P) = -i\int_0^\infty ds e^{i(P^2-m^2+i\eta)s} J_T(s).$$

Although various pieces of $J_T(s)$ behave like s^{-2} for small s, the complete function is completely finite at $s=0$. To integrate over s it is convenient to regulate the small s behavior of the individual terms by multiplying the integrand by s^ν with $\nu>1$. The terms $s^{\nu-2}$ integrate to Γ functions. The exponential parts can be integrated by using Eq. (B7) again. The full integration has no singularity at $\nu=1$ or at $\nu=0$. After setting $\nu\to 0$ the result is

$$f_T(P) = \frac{A(m^2-P^2)}{2\pi P^2}\left[-1+\frac{p_0}{2p}\ln\left(\frac{p^0+p}{p^0-p}\right) + \ln\left(\frac{im^2-iP^2}{4\pi T\sqrt{P^2}}\right)\right] - \frac{iAT}{p}\left[\frac{1}{2}\ln\left(\frac{p^0+p}{p^0-p}\right) + \ln\left(\frac{\Gamma(Z_+)}{\Gamma(Z_-)}\right)\right], \tag{B8}$$

where the arguments of the Γ function are

$$Z_\pm \equiv 1+i\frac{m^2-P^2}{4\pi T(p^0\pm p)}. \tag{B9}$$

Although it is not apparent, $f_T(P)$ does vanishes $T=0$. The most important feature is the $(m^2-P^2)\ln(im^2-iP^2)$ term that exactly cancels the zero-temperature contribution (B6). The sum of Eqs. (B6) and (B8) is given in Eq. (19).

APPENDIX C: CALCULATION OF $\Sigma_R'(P)$ IN COVARIANT GAUGE

This appendix computes the self-energy integral (27), which is present in covariant gauges in which $\xi\neq 1$. The $T=0$ result is displayed in Eq. (C3) and the temperature-dependent part in Eq. (C4).

The analysis begins with the observation that the denominators of Eq. (27) have singularities in k at the locations (B1). Therefore, for $p^0>p$ the infinitesimal positive imaginary part can be omitted from p_c^0 and replaced by a negative imaginary part on the mass: $m^2\to m_c^2=m^2-i\eta$ as was done in Eq. (B2). After an integration by parts, Eq. (27) can be written

$$f'(P) = \frac{B\mu^\epsilon}{4\pi^2}\int\frac{d^{4-\epsilon}K}{k^2}\delta(K^2)\coth(|k_0|/2T) \times\left[1-\epsilon+k\frac{\partial}{\partial k}\right]\frac{-P\cdot K+K^2}{(P-K)^2-m_c^2}. \tag{C1}$$

It is convenient to put $\sigma=1-\epsilon$. Computing the derivatives gives

$$f'(P) = \frac{B\mu^\epsilon}{4\pi^2}\int\frac{d^{4-\epsilon}K}{k^2}\delta(K^2)\coth(|k_0|/2T) \times\left[\frac{-\sigma P\cdot K+\vec{p}\cdot\vec{k}-2k^2}{(P-K)^2-m_c^2}+\frac{2P\cdot K(\vec{p}\cdot\vec{k}-k^2)}{[(P-K)^2-m_c^2]^2}\right].$$

The denominators can be exponentiated using Eq. (B3) so that

$$f'(P) = -i\int_0^\infty ds e^{i(P^2-m^2+i\eta)s} J'(s),$$

$$J'(s) \equiv \frac{B\mu^{\epsilon}}{4\pi^2}\int \frac{d^{4-\epsilon}K}{k^2}\,\delta(K^2)\coth(|k_0|/2T)e^{-i2P\cdot Ks}$$
$$\times[-\sigma P\cdot K+\vec{p}\cdot\vec{k}-2k^2-is2P\cdot K(\vec{p}\cdot\vec{k}-k^2)].$$

Integration over k_0 and the angles of $\vec{k}$ give

$$J'(s)=\frac{B\mu^{\epsilon}}{4\pi p}\int_0^{\infty}dk k^{-\epsilon}\coth(k/2T)\Bigg\{\Bigg[2ip(p_0+p)-\frac{1}{s}D_+-\frac{1}{s^2}\frac{i\epsilon D_-}{2k^2}\Bigg]\sin[2sk(p_0+p)]+\Bigg[2ip(p_0-p)+\frac{1}{s}D_+-\frac{1}{s^2}\frac{i\epsilon D_-}{2k^2}\Bigg]\sin[2sk(p_0-p)]\Bigg\}, \tag{C2}$$

where $D_{\pm}\equiv 1\pm s\partial/\partial s$. Note that the integral is convergent at both small and large k. The term linear in ϵ cannot be omitted as it will lead to a nonvanishing contribution after the s integration. As before use $\coth(k/2T)=1+2n(k/T)$ to obtain the separation

$$J'(s)=J_0'(s)+J_T'(s).$$

Zero-Temperature: The zero-temperature integration of Eq. (C2) contains powers of k times a sin function. The result is

$$J_0'(s)=\frac{B\mu^{\epsilon}}{4\pi}\Gamma(1-\epsilon)\cos\left(\frac{\epsilon\pi}{2}\right)\frac{[2(p_0+p)]^{\epsilon}}{p}\left[\frac{-\epsilon s^{\epsilon-2}}{2(p_0+p)}+i[p-\epsilon(p_0+p)]s^{\epsilon-1}\right]+(p\to -p).$$

Because only powers of s appear, the final integration over s

$$f_0'(P)=-i\int_0^{\infty}ds e^{i(P^2-m^2+i\eta)s}J_0'(s)$$

is straightforward. The result in the limit $\epsilon\to 0$ is

$$f_0'(P)=\frac{B}{2\pi}\left[\frac{1}{\epsilon}-\frac{P^2+m^2}{2P^2}+\ln\left(\frac{2\mu\sqrt{P^2}}{im^2-iP^2}\right)\right]. \tag{C3}$$

Thermal Contribution: Since there are no ultraviolet divergences in the thermal part of Eq. (C2) one may set $\epsilon=0$:

$$J_T'(s)=\frac{B}{2\pi p}\int_0^{\infty}dk n(k/T)\times\Bigg\{\Bigg[2ip(p_0+p)-\frac{1}{s}D_+\Bigg]\sin[2sk(p_0+p)]+\Bigg[2ip(p_0-p)+\frac{1}{s}D_+\Bigg]\sin[2sk(p_0-p)]\Bigg\}.$$

This can be performed using Eq. (B7) and gives

$$J_T'(s)=B\left(\frac{-i}{2\pi s}+ip_0T\right)+BT\left[i(p_0+p)-\frac{1}{2ps}\left(1+T\frac{\partial}{\partial T}\right)\right]n(4\pi T(p_0+p)s)+BT\left[i(p_0-p)+\frac{1}{2ps}\left(1+T\frac{\partial}{\partial T}\right)\right]\times n(4\pi T(p_0-p)s).$$

For later convenience the s derivatives have been converted to T derivatives using

$$s\frac{\partial}{\partial s}n(aTs)=T\frac{\partial}{\partial T}n(aTs).$$

The remaining integration is

$$J_T'(P)=-i\int_0^{\infty}ds e^{i(P^2-m^2+i\eta)s}J_T'(s).$$

It is easy to check that $J_T'(s)$ vanishes at $s=0$. However, various pieces behave as s^{-2} and s^{-1} at small s. It is therefore convenient to multiply $J_T(s)$ by a factor s^{ν} where initially $\nu>1$. Individual terms will have singularities at $\nu=1$ and at $\nu=0$. All these singularities cancel when the terms are combined, at which point one can put $\nu=0$. The integration is performed using Eq. (B7). The T derivatives of Euler gamma functions give ψ functions. The final result is

$$f_T'(P)=\frac{B}{2\pi}\Bigg[\ln\left(\frac{im^2-iP^2}{4\pi T\sqrt{P^2}}\right)-\frac{P^2-m^2}{2P^2}+\frac{i2\pi p_0T}{P^2-m_c^2}+\frac{i\pi T}{2p}\ln\left(\frac{p_0+p}{p_0-p}\right)+\frac{i\pi T}{p}\ln\left(\frac{\Gamma(Z_+)}{\Gamma(Z_-)}\right)-\frac{1}{2}\left(1+\frac{P^2-m^2}{2p(p_0+p)}\right)\psi(Z_+)-\frac{1}{2}\left(1-\frac{P^2-m^2}{2p(p_0-p)}\right)\psi(Z_-)\Bigg] \tag{C4}$$

with $Z_{\pm}$ as in Eq. (B9). The sum of Eqs. (C3) and (C4) is displayed in Eq. (29).

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