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# Scale and Unit Specification Influences in Harvest Scheduling with Maximum Area Restrictions

Alan T. Murray and Andrés Weintraub

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**ABSTRACT.** This article examines alternative approaches for representing a forest region to be scheduled for harvesting, where the primary concerns are maximizing return and imposing a maximum contiguous area of disturbance restriction. One approach assumes that any two adjacent management units exceed a regulated maximum area of disturbance. An alternative approach recognizes that management units may be substantially smaller than the maximum area restriction, so simultaneously disturbing two neighboring units does not necessarily represent a maximum area violation. The distinguishing feature of these two approaches is the way in which a forest is spatially represented. A single time period, 351 management unit harvest scheduling problem is utilized to investigate whether analysis results are subject to manipulation when forest representation, and associated modeling, is interpreted in different ways. Empirical results highlight significant economic and spatial variation in harvest schedules when maximum area restrictions are imposed using alternative approaches. *For. Sci.* 48(4):779–789.

**Key Words:** Harvest scheduling, adjacency restrictions, spatial analysis, modifiable areal unit problem.

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**H**ARVEST SCHEDULING continues to be an important component in the overall forest management process. It is at this level of analysis that public and private concerns typically collide. On one hand, the public supports sustaining our natural resources. That is, forests and other natural resources are the sources of inputs necessary in our everyday lives (e.g., timber, oil, coal, etc.), but they must be accessible for multiple uses, and they need to be in a healthy state. On the other hand, private companies are driven toward maximizing economic productivity when using natural resources. This is often viewed as being counter to either preservation or sustainability of natural resources. However, the continued viability of our natural resources is in the best interests of both private and public constituents. Harvest scheduling models have come to be an important part of forest planning, because they may be used to balance productivity and preservation considerations.

Harvest scheduling involves operational decisions associated with where forestry activities will occur and the extent of their impact (Thompson et al. 1973, Kirby et al. 1986, Lockwood and Moore 1993, Snyder and ReVelle 1997). While the driving factor in commercial harvest scheduling is maximizing the return in treating a region, careful environmental management of such activity is of major importance. Specific constraining conditions have become the defining characteristic of harvest scheduling. Some of these conditions involve spatial limitations on harvest activity and are usually referred to as maximum area restrictions. As such, harvest activity in a contiguous area is restricted from exceeding a specified bound. Limiting spatial disturbance has become standard practice in the management of public and private forest lands (Jones et al. 1991, Barrett et al. 1998, American Forest and Paper Association 2000). Further, spatially constrained harvest scheduling models are regularly

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used to construct logging and management plans at local and regional scales. This article is interested in the ways in which maximum area restrictions may be imposed in harvest scheduling modeling and the compatibility of alternative approaches for modeling this planning problem.

There are two basic modeling approaches for imposing maximum area restrictions in harvest scheduling (Murray 1999, Barrett and Gilless 2000). The classic approach is to assume that any two adjacent management units exceed a regulated maximum area of disturbance (Thompson et al. 1973). As an example, the Sustainable Forestry Initiative stipulates that average clearcuts should not exceed 120 ac, or 48.56 ha (American Forest and Paper Association 2000, Boston and Bettinger 2001). If our management units are greater than 25 and less than 48.56 ha in size, then it would not be possible to simultaneously harvest two neighboring units without violating the 48.56 ha maximum. Murray (1999) terms the imposition of this condition for harvest scheduling as the unit restriction model (URM). The key to the URM being applicable is that management units are defined appropriately (e.g., units are 25–48 ha in size if the maximum area restriction is 48.56 ha). This enables adjacency constraints to be structured and imposed in either an exact (Thompson et al. 1973, Kirby et al. 1986, Murray and Church 1996, Snyder and ReVelle 1996, 1997) or heuristic solution approach (O'Hara et al. 1989, Daust and Nelson 1993, Murray and Church 1995, Hoganson and Borges 1998). The second approach categorized in Murray (1999) is where management units are substantially smaller than the maximum area restriction, so that simultaneously disturbing two neighboring units does not necessarily represent a spatial violation (Hokans 1983). Relating this to our previous example, let the management units now range between 10–25 ha in size. In this case, there could potentially be up to four neighboring units simultaneously treated. Murray (1999) terms this approach to harvest scheduling as the area restriction model (ARM). Structuring and imposing area restrictions of this sort has been accomplished using heuristic (Hokans 1983, Lockwood and Moore 1993, Barrett et al. 1998, Barrett and Gilless 2000, Clark et al. 2000, Richards and Gunn 2000, Boston and Bettinger 2001) and exact solution approaches (Barrett and Gilless 2000, McDill and Braze 2000).

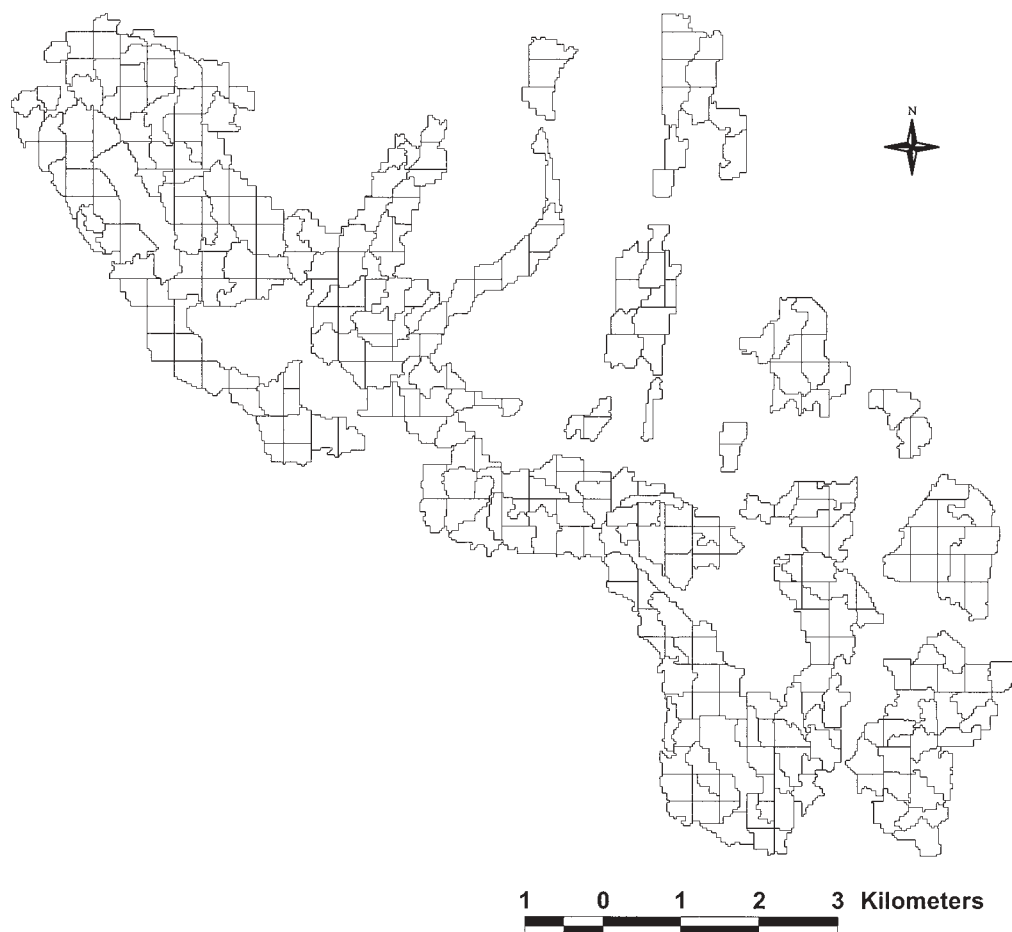
The existence of alternative approaches for imposing spatial restrictions in harvest scheduling raises numerous practical and theoretical issues. The purpose of this article is to examine these issues. Important considerations are spatial scale and unit specification implicitly distinguishing the URM and ARM. Another issue is the mathematical complexities related to solving either the URM or ARM. The URM has seen the most widespread use and application over the past 30 yr, but with greater access to more spatially detailed information this has necessitated a change. Here we wish to compare and contrast the URM and the ARM using a simplified forest application. This will enable us to assess the URM and ARM in relation to the maximum area restriction. The next section provides the background context of this research in forest modeling and discusses variation in spatial

representation. This is followed by the structuring of the two modeling approaches. The study design using a forest in northern California is then detailed. Application results are presented that highlight differences between the URM and ARM approaches. The article ends with a discussion and conclusions.

## Background

Harvest scheduling has long been recognized as an important component of the forest planning process. Broadly defined interest in harvest scheduling includes work on hierarchical or multiscale modeling attempting to integrate strategic and operational (harvest scheduling) level plans (Hof and Baltic 1991, Kent et al. 1991, Nelson et al. 1991, Weintraub and Cholak 1991, Jamnick and Walters 1993, Church et al. 2000). This work has focused on reconciling plans across forests and regions in order to ensure that established targets and commitments are feasible on the ground. For example, a strategic plan may identify total land dedicated to different uses and specific quantities of different timber types to be harvested. At this level of planning, however, there is typically no attention paid to the exact geographic location where these activities will occur (Kent et al. 1991, Jamnick and Walters 1993). Alternatively, the operational level of planning focuses on where and when treatment, harvesting, preservation, etc., are to be prescribed or scheduled. The analysis framework responsible for linking these different levels is known as a planning hierarchy, where different decisions and issues are addressed at different levels, both spatial and organizational, of forest management processes (Hof 1993, Church et al. 2000). As such, hierarchical forest planning deals with varying spatial scales of analysis (bio-regions, forests, watersheds, stands, etc.). The intent of this area of research is to link decisions being made at the different levels of planning.

Another way in which scale of analysis may vary is through land aggregation or classification (Jamnick et al. 1990, Chong and Beck 1991, Daust and Nelson 1993, Murray 1999). Consider the 351 units shown in Figure 1. Assume for the moment that inventory information, vegetation structure, timber valuation, etc., is known for each of these units and maintained in a digital format. If we further stipulate that no finer level of spatial information exists for this region, this means that this information represents the most spatially detailed information that is available for analysis, provided that all necessary attributes are included. Given this delineation of spatial units, any aggregation of these units that produces less spatial units (<351) represents a change in scale. So, we are still looking at the same region, but have changed the number of units representing the region through the combination of one or more units, presumably with their neighboring units. This is the scale of analysis issue of most interest in this paper. There has in fact been forest research focused on the impacts and influences of changing regional scale, particularly with respect to harvest scheduling. Jamnick et al. (1990) found significant economic impacts when scale was altered by land unit aggregation in harvest scheduling analysis using linear programming. However, spatial restric-



**Figure 1. Northern California forest.**

tions were not evaluated. Related to this, Daust and Nelson (1993) examined the impacts of adjacency restrictions on harvested timber volume for different scales of analysis associated with unit aggregation.

There are currently at least two general approaches for determining area restriction-based harvest schedules using mathematical modeling. The fundamental difference distinguishing these two approaches is unit aggregation oriented scale of analysis. The URM has been the more commonly utilized approach in harvest scheduling, since its basic introduction by Thompson et al. (1973), and continues to be the focus of research aimed at improving solution capabilities (Weintraub et al. 2000). Alternatively, the ARM, first discussed in Hokans (1983), has been less relied on for harvest scheduling because of its inherent computational complexity (Murray 1999). Compared to the URM, there has been little research and development thus far on effective techniques for solving the ARM. Given these two approaches for imposing area restrictions in harvest scheduling, a number of important questions may be asked. Are there differences between the URM and ARM? What is the nature of these differences, if they exist? Both models have the same basic goals, but the distinguishing characteristic is the issue of scale and spatial unit representation. No previous forest research has attempted to explore this harvest scheduling issue, yet answers to the above questions are essential for understanding the appropriateness of models imposing maximum area restrictions.

Further highlighting the importance of this proposed comparison is geographic work on scale and unit specification. Aggregation based scale of analysis is a component of the modifiable areal unit problem (MAUP), a discussion of which may be found in Openshaw and Taylor (1981). In geographical modeling and analysis, one is typically faced with choosing an appropriate spatial unit of representation, given a defined geographic study region. This is a scale of analysis issue dependent on whether the planning/management focus requires fine or coarse spatial resolution. For example, suppose that we are examining a national forest and have detailed information for the ranger districts within this national forest. Is this suitable for our analysis objectives and intent or do we need information at a sub-ranger district level? We are still examining the national forest, but our spatial units used to report forest details may vary depending upon our needs. This example is representative of aggregation based variation in scale, because the sub-ranger district units (i.e., compartments) are combined to summarize the ranger district to which they belong. A related issue is unit specification. Given a particular scale of analysis, it is necessary to decide how to delineate (or partition) the region being considered. Are there naturally defined spatial units associated with a particular problem of interest? If not, then there is flexibility in the choice of how to structure spatial units. Forest stands are typically defined based upon vegetation, timber characteristics, existing roads and streams, topogra-

phy, and ownership (Barrett and Gilless 2000). Such units may be too small for analysis and modeling purposes. It then becomes necessary to aggregate (or block) them in order to create larger spatial entities. While there may be some limits on how units may be combined due to timber characteristics, topography, etc., it is likely that there would be many different ways aggregation could be accomplished. As an example of unit specification variation, suppose that we need to represent the region shown in Figure 1 using only 100 blocks, rather than the 351 units currently depicted. If we further require the reduced number of reported areas to be spatially contiguous combinations of our original 351 units, then we can consider numerous realizations of the 100 blocks that could be produced, as detailed in Barrett (1997). The MAUP reflects the ability to spatially define a given region, potentially, an infinite number of ways, through varying scale or block delineation.

Of most significance for harvest scheduling is that there are recognized analytic consequences of the MAUP (Openshaw and Taylor 1981, Fotheringham and Wong 1991). In particular, Openshaw and Taylor (1981) note that common statistical measures and spatial analytic models have been shown to vary significantly merely by altering geographic representation. What this means is that one may find a correlation between two forest attributes at the ranger district level, as an example, but when examining these attributes at the sub-ranger district level, the correlation is not found. The statistical test is the same and the variables have not changed. The only change is in spatial representation. This is, of course, problematic, because the validity of the analysis becomes questionable and subject to manipulation. Evidence of scale and/or unit specification effects in harvest scheduling is reported in Jamnick et al. (1990) and Daust and Nelson (1993), where economic and harvest flow rates, respectively, were found to change when scale varied. Such findings are of general concern given the relative ease with which a forest analyst can alter spatial representation using commercial geographical information system (GIS) software. If data have been aggregated, we cannot easily establish confidence in reported findings unless sensitivity analysis has been carried out. Further, given the existence of alternative approaches for structuring and solving a harvest scheduling problem with area restrictions (URM vs. ARM), potential MAUP effects may not be readily evident.

Tobler (1989) discusses potential MAUP effects in spatial analysis and gives further motivation for examining ARM and URM differences. Suggested in Tobler (1989) is that if a particular method of analysis is subject to manipulation based on a change of geographical scale or unit specification, then the method is inappropriate. Tobler (1989) issued a call for frame independent spatial analysis approaches. Such approaches would give consistent results irrespective of the scale or unit definition relied on. This would effectively eliminate MAUP issues. While the focus of discussion in Tobler (1989) was directed at statistical analysis, there are implications for harvest scheduling models. We have at least two approaches for addressing harvest planning subject to maximum area restrictions. Does either of these models

better reflect the notion of frame independence? If the answer is yes, then one approach would likely be less susceptible to MAUP effects. It is recognized in geographical research that one must explicitly test for MAUP effects in order to understand the behavior of a model (or models) with respect to changes in scale or unit definition. If sensitivities are found, then two choices exist for an analyst: (1) use the most disaggregate information available, which is the typical response to MAUP effects; or (2) look for a more appropriate spatial model as suggested by Tobler (1989), if one exists. If sensitivity is *not* found, then the approach is frame independent as called for by Tobler (1989).

It is clear that MAUP effects and frame independence are important to assess in spatial analysis, including harvest scheduling. Based on previous forest research, harvest scheduling results are known to be sensitive to variation in scale (Daust and Nelson 1993). However, this is only one component of the MAUP. The other component is unit specification, where a region may be delineated in different ways. This has not been explored in harvest scheduling subject to maximum area restrictions. With respect to frame independence, there have been few attempts to date to compare alternative approaches for imposing maximum area restrictions in harvest scheduling.

## Structuring Maximum Contiguous Area Restrictions

The major difference between the URM and ARM is how maximum area restrictions are structured. In order to fully appreciate this distinction, reviewing the mathematical specification of both models is helpful. Without loss of generality, temporal considerations will be excluded from the present discussion. If  $x_i$  is our harvesting decision variable for spatial unit  $i$ , then  $x_i = 1$  represents a decision to harvest unit  $i$  and  $x_i = 0$  indicates a decision to leave unit  $i$  untreated. We can also define  $\alpha_i$  as the economic benefit of harvesting unit  $i$ . Note also that we could readily view  $\alpha_i$  as a measure of environmental benefit. With this notation, the typical objective of a harvest scheduling model is:

$$\text{Maximize } Z = \sum_i \alpha_i x_i \quad (1)$$

Given this objective along with the binary decision variables ( $x_i = \{0, 1\}$ ), the major constraining condition of interest is the maximum disturbed area limitation. If we assume that a URM representation is appropriate, then a possible constraint would be of the following form (see Murray and Church 1996):

$$x_i + x_j \leq 1 \quad \forall i, j \in N_i \quad (2)$$

where  $N_i$  = set of units adjacent to unit  $i$ .

The constraint structured in (2) will prohibit simultaneous treatment in two adjacent units. Of course the assumption in the URM is that the combined area of units  $i$  and  $j$  exceeds the maximum allowable contiguous area of disturbance.

As mentioned previously, an alternative interpretation of this constraining condition in harvest scheduling is



where the spatial units are considerably smaller than the maximum area restriction. Because spatial units are smaller, two or more neighboring units may not violate the maximum allowable contiguous area of disturbance (Hokans 1983). In this case, the ARM would be required for harvest scheduling. A linear constraint for imposing maximum area restrictions in the ARM was suggested in Barrett and Gilles (2000), using subgraph adjacency, and McDill and Braze (2000), based on enumerating area violations. This constraint is structured as follows:

$$\sum_{j \in C_k} x_j \leq |C_k| - 1 \quad \forall k \quad (3)$$

where  $C_k$  = set of units representing a maximum contiguous area violation.

So, the ARM is comprised of objective (1) subject to constraint (3) and integer restrictions on the decision variables. The use of this constraint form in the ARM requires one to devise an approach for identifying all potential violations of the maximum contiguous area restriction. Effectively it necessitates an enumeration scheme, but the spatial nature of the restriction is such that total enumeration is avoided. There may, however, be circumstances where the computational complexity is too great, and identifying the necessary constraints would not be feasible. Nevertheless, constraint (3) provides an approach for mathematically structuring the ARM.

It is important to note at this point that extensions to the URM and ARM are possible. Most harvest scheduling applications do in fact build on these basic models. Potential extensions include temporal considerations (multiple time periods), upper/lower total harvest output in each time period, bounds on economic return, road building and maintenance associated with accessing harvest units, green-up conditions, etc.

An appealing feature of the URM is its straightforward problem formulation and structure. As a result, a range of proven techniques exist for solving this model using exact and heuristic approaches. In terms of exact approaches, the clique constraints structured in (2), and higher ordered cliques, have been utilized to successfully solve medium-to-large problem instances on a personal computer (Murray and Church 1996, Snyder and ReVelle 1996). The ability to solve the URM by exact methods has arguably furthered heuristic solution development for the URM as well. In essence, heuristics have been developed for the URM which have been demonstrated to perform exceptionally well (Murray and Church 1995). Unlike the URM, solution technique development has only recently focused on the ARM. Heuristics have been the major approach thus far for solving the ARM (Hokans 1983, Lockwood and Moore 1993, Barrett et al. 1998, Barrett and Gilles 2000, Clark et al. 2000, Richards and Gunn 2000). Barrett and Gilles (2000) and McDill and Braze (2000) proposed an exact approach for solving the ARM. The effectiveness of this formulation for general application remains to be proven, however.

The existence of exact techniques for solving both the URM and ARM optimally make it possible to carry out an empirical assessment of these alternative approaches. This will allow MAUP effects to be examined and frame independence evaluated. In addition, it will be possible to investigate relative computational considerations in the application and analysis of the URM and ARM for harvest scheduling.

## Comparison

In order to highlight the practical issues and operational differences between the URM and ARM approaches, the forest shown in Figure 1 will be utilized for application. There are 351 management units in this forest, averaging 10 ha in size. Each unit may be harvested; the maximum allowed disturbance for a contiguous area is 48.56 ha. We have elected to isolate the two fundamental characteristics of this particular harvest scheduling problem: economic return and maximum contiguous area limitation. Only a single time period with no additional constraints will be considered in this assessment. The most important aspects of this comparison are objective function performance, spatial distribution of harvests, and computational effort.

The analysis was carried out on a Pentium III/600 personal computer. ArcView version 3.2 was utilized to manage and manipulate the spatial representation of this forest. Using ArcView, an Avenue script generates the associated URM optimization problem file. The problem is then solved externally to ArcView using CPLEX version 6.53. A result file is exported from CPLEX and read into ArcView for subsequent display and analysis. For the ARM, the addition of a Fortran program, coded by the first author, was needed to generate the required area restrictions specified in constraint (3).

The ability to make a comparative assessment of the URM and ARM is needed if we are to examine MAUP effects, frame independence, and computational considerations. An obvious basis for comparison is to examine objective function performance between the URM and ARM. After all, both models have exactly the same objective function, but differ only with respect to the underlying spatial representation. As a start we first explore the application of the URM. One approach is to apply the URM to this region using the disaggregated spatial units shown in Figure 1. Of course the average unit size is only 10 ha and the maximum area restriction is 48.56 ha, so clearly the URM results in this case are overly restricted. In fact, this gives a lower bound on the return possible in harvesting this region. Another approach using the URM would be to create larger planning units (or blocks), so that any two neighboring blocks exceed the 48.56 ha maximum. Doing this provides the context in which MAUP effects associated with scale and unit specification may be evaluated.

Spatial unit aggregation (or blocking) approaches have been and continue to be of interest in spatial analysis (Goodchild 1979, Jamnick et al. 1990, Fotheringham and Wong 1991, Murray and Gottsegen 1997). Recent research in forestry may be found in Barrett (1997). We have chosen two spatial aggregation approaches to include in this research. The first is referred to as the Thiessen approach (equivalent

to the S-mosaic in Goodchild 1979 and the Voronoi tessellation method in Barrett 1997). This method combines spatial units by first selecting at random a specified number of “seed” units. The number of seeds selected is generally a function of desired block size. All spatial units are then combined with their closest seed unit to form a block. This must be followed by an *a posteriori* analysis to ensure that generated blocks do not violate maximum area restrictions. The randomization component in selecting seed units means that if a different set of seed units is utilized, an entirely different set of aggregated units (or blocks) will result. Thus, the Thiessen aggregation approach enables us to assess unit specification effects associated with the MAUP, because numerous different blockings (aggregations) may be created for an individual forest region.

The second aggregation approach utilized is referred to as Block Building, and is discussed in Goodchild (1979) and Murray and Gottsegen (1997). The process involves the following steps:

1. Consider all units unblocked
2. Randomly select a unit not yet merged in a block
3. Aggregate unmerged neighbors (randomly) to form a block until the size limit is reached
4. If all units are not a member of a block (or merged), return to step (2)

The Block Building aggregation scheme represents a diffusion process where an initial unit is selected and then it is merged with neighbors, neighbors of neighbors, etc., until the maximum area of disturbance is reached. The randomization element, as with the Thiessen approach, means that an entirely different aggregation of units (or blocks) will be produced if the process is run multiple times with different seed units, so numerous different blockings (aggregations) of the forest may be created by this process. Differing from the Thiessen approach, however, the use of the Block Building approach enables us to assess both scale and unit specification issues associated with the MAUP in this analysis. The reason for this is that a variable number of blocks is likely to be created using this approach, whereas the Thiessen approach creates a specified number of blocks. This feature of varying numbers of created blocks is similar to the strategies proposed in Borges and Hoganson (1999).

In this research, 100 aggregation instances were generated using each approach. The 100 Thiessen aggregation instances were produced using a random seeding of 80 units. This results in the creation of 80 blocks in the aggregation process for each new forest representation. Of course there will be 100 different representations of our forest region using the Thiessen approach. Average block size for the Thiessen instances is 44.5 ha. For the URM applied to these forest representations, the implication is that two neighboring blocks violate the 48.56 ha maximum. For the Block Building approach, 100 aggregation instances of the forest region were also created. The number of resulting blocks using the Block Building approach ranged between 113–127 for the 100 aggregation instances. Average block size for the

Block Building instances ranged between 28–31.6 ha, so that two neighboring blocks would likely violate the 48.56 ha maximum. Collectively there are 201 data realizations for the forest region—the original 351 unit instance, the 100 aggregation instances produced using the Thiessen approach, and the 100 aggregation instances generated using the Block Building approach.

## Application Results

The URM was initially applied to the original 351 unit problem instance shown in Figure 1. The resulting objective function value was 5854.25, which required 0.26 seconds to solve using CPLEX (591 iterations and zero branches). This is the lower bound on the best possible harvest scheduling objective function value. The spatial configuration of this solution is displayed in Figure 2. The relatively small disturbed areas (10 ha on average) are distributed throughout the forest. If one is interested solely in limiting the spatial extent of harvest activity, this is clearly an effective approach. However, economic production considerations suggest that harvesting such small areas would be prohibitive with respect to fixed costs in this case.

The URM was next applied to each of the 100 Block Building aggregation instances. These individual problems required less than 0.2 seconds to solve using CPLEX. The objective value performance of the URM using the 100 Block Building instances ranged from a low of 6218.00 to a high of 7231.86, with an average value of 6700.99. The best URM solution found for the 100 Block Building instances, with an objective of 7231.86, is displayed in Figure 3. In contrast to Figures 1 and 2, the spatial units (blocks) in Figure 3 are substantially larger. The result of these larger blocks is that harvest activity is more concentrated in local areas. Further, undisturbed areas are also more concentrated in Figure 3.

The URM was then applied to the 100 Thiessen aggregation instances. These individual problems also required less than 0.2 seconds to solve using CPLEX. The objective value performance of the URM for the 100 Thiessen instances ranged from a low of 6004.19 to a high of 8139.57, with an average value of 6935.88. The best URM solution found for the 100 Thiessen instances, with an objective of 8139.57, is displayed in Figure 4. As in Figure 3, the blocks in Figure 4 are considerably larger than the units shown in Figure 2. The result is again a concentration of harvest activity as well as a concentration of nonactivity. In comparing scheduled harvests in Figures 3 and 4 to that shown in Figure 2, it is fairly easy to see why objective value performance has increased in the latter two cases. Much more activity appears to be taking place in Figures 3 and 4 across the forest, which means that the economic return is likely to be higher (and it is in both cases). In fact, the lowest values found for the Block Building and Thiessen approaches, 6218.00 and 6004.19 respectively, are both higher than the objective value of 5854.25 associated with applying the URM to the original 351 units.

The best solution found for the 201 applications of the URM had an objective function value of 8139.57 (see

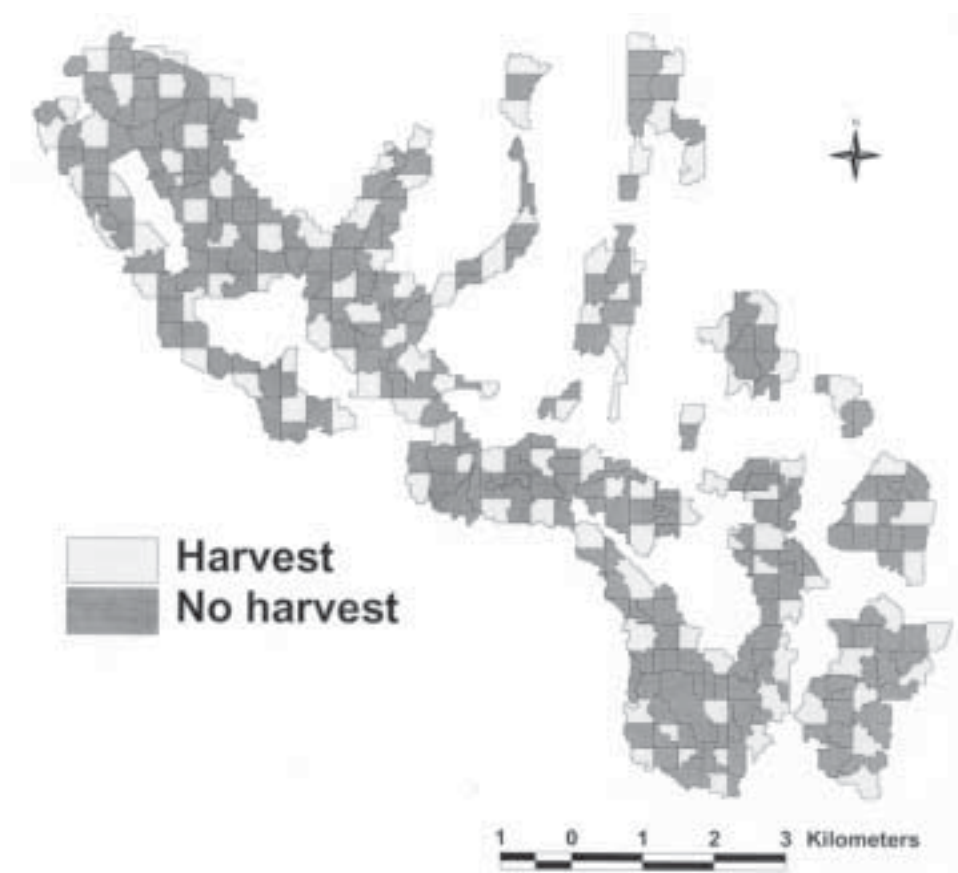


Figure 2. Solution for URM applied to original 351 units.

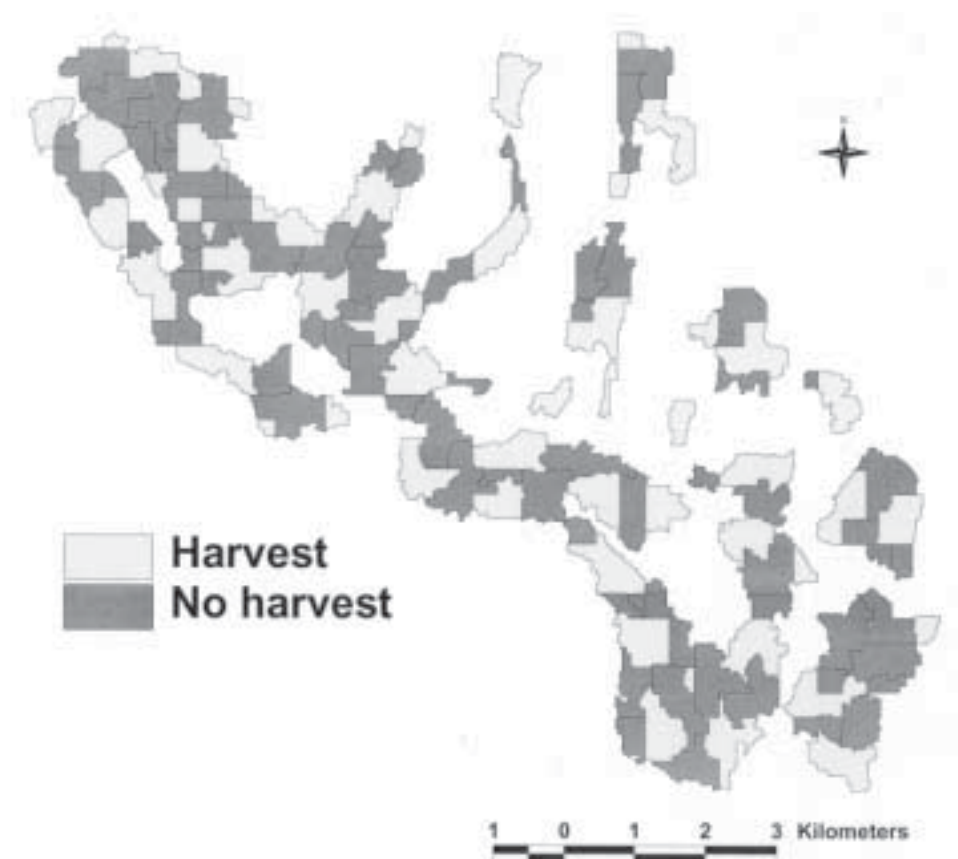
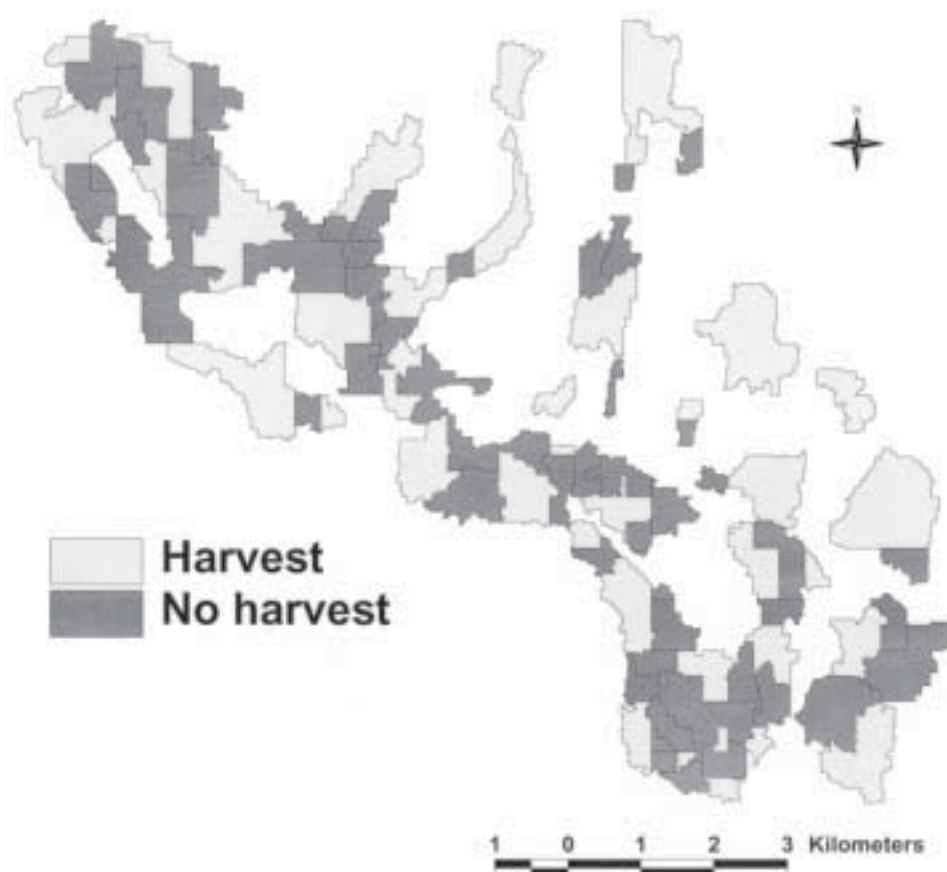


Figure 3. URM solution using Block Building approach to aggregate spatial units.





**Figure 4.** URM solution using Thiessen approach to aggregate spatial units.

Figure 4). This provides a basis for comparison to the ARM. Given that the original 351 units of the forest shown in Figure 1 average approximately 10 ha in size and that a maximum area of contiguous disturbance is limited to 48.56 ha, we need to structure and solve the ARM. Using the Fortran program described previously, 64,034 constraints of the form specified in (3) were found to be necessary for this single time period problem. It required 35.76 hr of processing time to identify these constraints. [The Fortran program enumerated all unique contiguous spatial violations. This involves numerous levels of subloops and verification of uniqueness when violations are found. It is conceivable that other programming languages, such as C++, might be considerably more efficient in carrying out this enumeration process.] CPLEX was used to solve this ARM, but a considerable optimality gap existed after 19.43 hr (equating to 959,793 iterations and 17,208 branches). The best found feasible solution had an objective function value of 9482.41 and is shown in Figure 5. The optimality gap at termination was 15.22%, so this solution may be characterized as being within 15.22% of the optimal solution. An interesting comparison may be made between the results shown in Figure 5 to those displayed in Figures 2–4. The spatial extent of harvest activity in Figure 5 is more sizeable than that found in Figure 2, but this is expected. Interestingly, the concentrated patterns shown in Figures 3 and 4 are not unlike the spatial patterns found in Figure 5.

## Discussion

One striking finding is that none of the 201 different spatial representations of this forest region solved as a URM application could be interpreted as being a competitive ARM approximation. The best ARM solution found was 16.50% higher than the best URM solution. In terms of economic return, this is a significant difference. Figures 2–5 illustrate substantial differences in the spatial extent of harvest activities as well. Based on these results, one cannot expect to easily or readily solve a derived URM (by blocking or otherwise) that is equivalent to an associated ARM. The differences between the two approaches would likely be less when constraining the models further through the addition of time considerations, volume flows, road building/maintenance, etc.

A major motivation for this research was to assess MAUP effects in harvest scheduling subject to maximum contiguous area restrictions. Previous research has shown that URM results are influenced by changes in geographic representation (Daust and Nelson 1993) and different blocking strategies (Borges and Hoganson 1999). The analysis using the Block Building aggregation instances of the forest region supports, to some degree, sensitivity of the URM to altered scale, because the number of blocks generated varies between 113 and 127 and the associated objectives range between 6218.00 and 7231.86. The more significant contribution, however, is the finding that unit

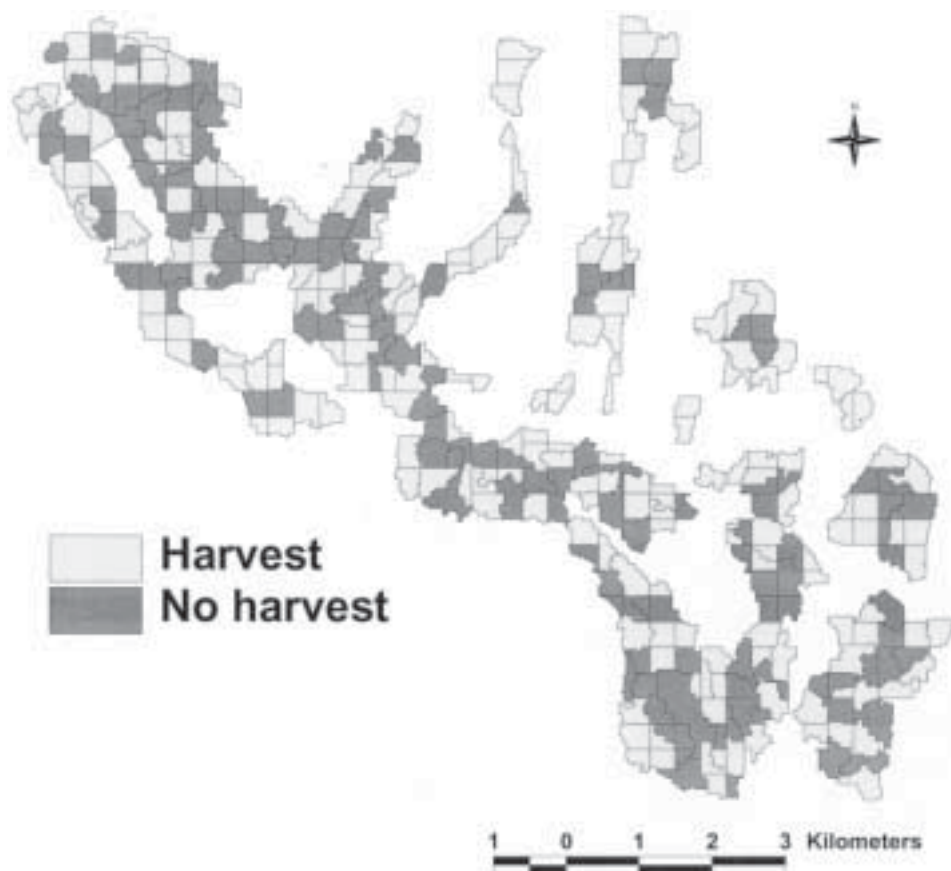


Figure 5. Best ARM solution found.

specification is very influential in the application of the URM. Using the Thiessen aggregation approach, 100 aggregation instances of the forest region were generated. Each instance contained 80 blocks by aggregating (or combining) the original 351 units in a spatially contiguous manner. If the URM were not sensitive to unit specification effects (change in blocking), we would expect to get the same objective value for each problem instance. However, we found a range of objective function values, from 6004.19 to 8139.57 with an average (6935.88) much lower than the maximum. This is at best 14.16% less than the best ARM solution found, but could end up being more than 36%, depending on which aggregation instance was utilized in the analysis using the URM. The URM results using the Block Building aggregation instances support this finding as well. Clearly, the URM is highly influenced by scale and unit specification (blocking) variation, both characteristics of the MAUP. In contrast to the URM, the ARM does not need *a priori* spatial unit aggregation (blocking) in order for it to be applied in the context of the maximum area restriction. Thus, it is technically free from MAUP effects because it can rely on the most disaggregate spatial data available. The URM typically cannot, and must be subject to blocking (or aggregation). Nevertheless, if the ARM is applied to aggregate data, then it will be sensitive to MAUP effects. The basis for this conclusion is given in Murray (1999), who showed that the URM is mathematically a special case of the ARM.

Another issue of importance in this work was the assessment of frame independence. As noted previously, Tobler (1989) called for spatial analysis approaches that were frame independent. The ARM would appear to be somewhat scale invariant, at least in comparison to the URM. The reason is that we clearly find MAUP effects in the application of the URM, whereas the ARM may be applied to the most disaggregate data available. Tobler's point is particularly relevant in the evaluation and comparison of these two approaches for harvest scheduling subject to maximum area restrictions. Further, this highlights that one should always be looking for alternative representations of spatial models and problems. Doing so could help to ensure better management efficiency as well as long term viability in natural resource use. The empirical analysis supports this in that a timber company could reduce harvest volume output targets, thereby decreasing the total regional impact of harvest activities, using the ARM and still be able to achieve as good or better returns than any of the URM plans.

The solvability of the ARM is, however, an issue, even for the relatively small 351 unit forest utilized in this research. Recall that 64,034 constraints were found to be necessary and required over 35 hr to identify. Further, only a feasible solution could be identified for the ARM after nearly 20 hr of processing using CPLEX. The resulting optimality gap was 15.22%. The structure of constraint (3) is not tight or integer-friendly in the sense that constraint (2) has been shown to be (Murray and Church 1996), which explains why we were not

successful in solving the ARM exactly. It would be unrealistic to expect to solve more difficult problems (either larger in terms of the number of spatial units, multiple time periods, volume bounds or roading considerations) when this simplified problem instance cannot be optimally solved. More work is clearly needed on exact approaches for the ARM in order to address larger problems with multiple time periods and other constraining conditions. Clearly, heuristic solution approaches are essential at this point in time. While there has been some progress made in the development of heuristics for the ARM (Murray and Snyder 2000), there is no work demonstrating the superiority of any one approach.

A final comment is that the ARM is an important problem and represents a frame independent approach. However, the spatial implications/impacts appear to be much more pronounced (see Figure 5) when compared with the URM schedules shown in Figures 3 and 4. Given this, questioning whether this is a desired intention seems reasonable. In fact, the URM may be more appealing in this regard, but it is highly susceptible to MAUP effects.

## Conclusions

Limiting spatial disturbance is a significant consideration in natural resource management. In this article, we have evaluated two representation and modeling possibilities for generating harvest schedules in forest management. Both approaches attempt to maximize associated return while restricting spatial activity. The unit restriction model (URM) assumes that two neighboring spatial units violate established maximum contiguous area limitations. The area restriction model (ARM), on the other hand, imposes limits only with respect to total contiguous area, rather than make an assumption regarding spatial unit size. The issue of scale and unit definition, the modifiable areal unit problem (MAUP), was shown to be fundamental in distinguishing these two approaches. The empirical assessment provided comparative results (based on solving 202 different optimization problems) associated with objective function performance, spatial impact, and computational effort for the two approaches. The URM was found to be influenced by MAUP effects. While the URM may be more easily solved, the identified harvest schedules do not favorably compare with the best found ARM results as measured by objective function performance. In addition, the ARM has the added benefit of being frame independent. Nevertheless, the appropriateness of the URM and/or the ARM would depend on the application intent and underlying management goals. This research further highlights differences in the two approaches, both conceptually and computationally. Further, this work also demonstrates that exact techniques for the ARM are currently limited.

The general implication of these findings is that there is a definite need for continued research on the ARM. Not only is this justified by the empirical results, but this makes sense given trends in spatial information sciences. The information age has witnessed the availability of plentiful digital information on where people live, their shopping habits, and our natural resources. Detailed, reliable spatial information will

be the standard in the near future, reflecting the significant investments in creating forest inventories and further integration of remote sensing technologies. Modeling approaches capable of application to spatially explicit, fine resolution information is needed now, and more so in the future.

## Literature Cited

- AMERICAN FOREST AND PAPER ASSOCIATION. 2000. Sustainable forestry initiative standard. <http://www.afandpa.org/>.
- BARRETT, T.M. 1997. Voronoi tessellation methods to delineate harvest units for spatial forest planning. *Can. J. For. Res.* 27:903–910.
- BARRETT, T.M., AND J.K. GILLESS. 2000. Even-aged restrictions with sub-graph adjacency. *Ann. Op. Res.* 95:159–175.
- BARRETT, T.M., J.K. GILLESS, AND L.S. DAVIS. 1998. Economic and fragmentation effects of clearcut restrictions. *For. Sci.* 44:569–577.
- BORGES, J.G., AND H.M. HOGANSON. 1999. Assessing the impact of management unit design and adjacency constraints on forestwide spatial conditions and timber revenues. *Can. J. For. Res.* 29:1764–1774.
- BOSTON, K., AND P. BETTINGER. 2001. The economic impact of green-up constraints in the southeastern United States. *For. Ecol. Manage.* 145:191–202.
- CHONG, S.-K., AND J.A. BECK. 1991. The effect of land classification and stratification on derivation of timber supply and allowable cut in harvest scheduling. *Can. J. For. Res.* 21:1334–1342.
- CHURCH, R.L., A.T. MURRAY, M.A. FIGUEROA, AND K.H. BARBER. 2000. Support system development for forest ecosystem management. *Eur. J. Op. Res.* 121:247–258.
- CLARK, M.M., R.D. MELLER, AND T.P. McDONALD. 2000. A three-stage heuristic for harvest scheduling with access road network development. *For. Sci.* 46:204–218.
- DAUST, D.K., AND J.D. NELSON. 1993. Spatial reduction factors for strata-based harvest schedules. *For. Sci.* 39:152–165.
- FOTHERINGHAM, A.S., AND D.W.S. WONG. 1991. The modifiable areal unit problem in multivariate analysis. *Environ. Plan. A* 23:1025–1044.
- GOODCHILD, M.F. 1979. The aggregation problem in location-allocation. *Geograph. Anal.* 11:240–255.
- HOF, J. 1993. Coactive forest management. Academic Press, San Diego. 189 p.
- HOF, J., AND T. BALTIC. 1991. A multilevel analysis of production capabilities of the national forest system. *Op. Res.* 39:543–552.
- HOGANSON, H.M., AND J.G. BORGES. 1998. Using dynamic programming and overlapping subproblems to address adjacency in large harvest scheduling problems. *For. Sci.* 44:526–538.
- HOKANS, R.H. 1983. Evaluating spatial feasibility of harvest schedules with simulated stand-selection decisions. *J. For.* 81:601–603, 613.
- JAMNICK, M.S., L.S. DAVIS, AND J.K. GILLESS. 1990. Influence of land classification systems on timber harvest scheduling models. *Can. J. For. Res.* 20:172–178.
- JAMNICK, M.S., AND K.R. WALTERS. 1993. Spatial and temporal allocation of stratum-based harvest schedules. *Can. J. For. Res.* 23:402–413.
- JONES, J.G., B. MENEGHIN, AND M. KIRBY. 1991. Formulating adjacency constraints in linear optimization models for scheduling projects in tactical planning. *For. Sci.* 37:1283–1297.
- KENT, B., B.B. BARE, R. FIELD, AND G. BRADLEY. 1991. Natural resource land management planning using large-scale linear programs: The USDA Forest Service experience with FORPLAN. *Op. Res.* 39:13–27.
- KIRBY, M.W., W.A. HAGER, AND P. WONG. 1986. Simultaneous planning of wildland management and transportation alternatives. *TIMS Studies Man. Sci.* 21:371–387.
- LOCKWOOD, C., AND T. MOORE. 1993. Harvest scheduling with spatial constraints: A simulated annealing approach. *Can. J. For. Res.* 23:468–478.

- McDILL, M., AND J. BRAZE. 2000. Harvest scheduling with area-based adjacency constraints. Penn State Sch. of For. Resour. Work. Pap.
- MURRAY, A.T. 1999. Spatial restrictions in harvest scheduling. *For. Sci.* 45:45–52.
- MURRAY, A.T., AND R.L. CHURCH. 1995. Heuristic solution approaches to operational forest planning problems. *OR Spektrum* 17:193–203.
- MURRAY, A.T., AND R.L. CHURCH. 1996. Analyzing cliques for imposing adjacency restrictions in forest models. *For. Sci.* 42:166–175.
- MURRAY, A.T., AND J.M. GOTTSEGEN. 1997. The influence of data aggregation on the stability of p-median location model solutions. *Geograph. Anal.* 29:200–213.
- MURRAY, A.T., AND S. SNYDER. 2000. Spatial modeling in forest management and natural resource planning. *For. Sci.* 46:153–156.
- NELSON, J., J.D. BRODIE, AND J. SESSIONS. 1991. Integrating short-term, area-based logging plans with long-term harvest schedules. *For. Sci.* 37:101–122.
- OPENSHAW, S., AND P.J. TAYLOR. 1981. The modifiable areal unit problem. P. 60–69 in *Quantitative geography: A British view*, Wrigley, N., and R. Bennett (eds.). Routledge and Kegan Paul, London.
- O'HARA, A., B.H. FAALAND, AND B.B. BARE. 1989. Spatially constrained timber harvest scheduling. *Can. J. For. Res.* 19:715–724.
- RICHARDS, E.W., AND E.A. GUNN. 2000. A model and tabu search method to optimize stand harvest and road construction schedules. *For. Sci.* 46:188–203.
- SNYDER, S., AND C.S. REVELLE. 1996. Temporal and spatial harvesting of irregular systems of parcels. *Can. J. For. Res.* 26:1079–1088.
- SNYDER, S., AND C.S. REVELLE. 1997. Dynamic selection of harvests with adjacency restrictions: the share model. *For. Sci.* 43:213–222.
- THOMPSON, E.F., B.G. HALTERMAN, T.S. LYON, AND R.L. MILLER. 1973. Integrating timber and wildlife management planning. *For. Chron.* 47:247–250.
- TOBLER, W.R. 1989. Frame independent spatial analysis. P. 115–122 in *The Accuracy of Spatial Databases*, Goodchild, M., and S. Gopal (eds.). Taylor and Francis, New York.
- WEINTRAUB, A., AND A. CHOLAKY. 1991. A hierarchical approach to forest planning. *For. Sci.* 37:439–460.
- WEINTRAUB, A., R.L. CHURCH, A.T. MURRAY, AND M. GUIGNARD. 2000. Forest management models and combinatorial algorithms: analysis of state of the art. *Annals of Operations Research* 96:271–285.