WestVirginiaUniversity
THE RESEARCH REPOSITORY @ WVU

Faculty Scholarship

2014

Gravitational Waves And Stalled Satellites From Massive Galaxy Mergers At Z $\leqslant$ 1

Sean T. McWilliams

Jeremiah P. Ostriker

Frans Pretorius

Follow this and additional works at: https://researchrepository.wvu.edu/faculty_publications

GRAVITATIONAL WAVES AND STALLED SATELLITES FROM MASSIVE GALAXY MERGERS AT $z \leqslant 1$

SEAN T. MCWILLIAMS[1,2], JEREMIAH P. OSTRIKER[3], AND FRANS PRETORIUS[1]
[1] Department of Physics, Princeton University, Princeton, NJ 08544, USA; Sean.McWilliams@mail.wvu.edu
[2] Department of Physics and Astronomy, West Virginia University, Morgantown, WV 26505, USA
[3] Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA

ABSTRACT

We present a model for merger-driven evolution of the mass function for massive galaxies and their central supermassive black holes at late times. We discuss the current observational evidence in favor of merger-driven massive galaxy evolution during this epoch, and demonstrate that the observed evolution of the mass function can be reproduced by evolving an initial mass function under the assumption of negligible star formation. We calculate the stochastic gravitational wave signal from the resulting black hole binary mergers in the low redshift universe ($z \leqslant 1$) implied by this model, and find that this population has a signal-to-noise ratio $2\times$ to $5\times$ larger than previous estimates for pulsar timing arrays, with a $\{2\sigma, 3\sigma\}$ lower limit within this model of $h_c(f = 1\,\mathrm{yr}^{-1}) = \{1.1 \times 10^{-15}, 6.8 \times 10^{-16}\}$. The strength of this signal is sufficient to make it detectable with high probability under conservative assumptions within the next several years. A principle reason that this result is larger than previous estimates is our use of a recent recalibration of the black hole–stellar mass correlation for the brightest cluster galaxies, which increases our estimate by a factor of $\sim$2 relative to past results. For cases where a galaxy merger fails to lead to a black hole merger, we estimate the probability for a given number of satellite black holes to remain within a massive host galaxy, and interpret the result in light of ULX observations. We find that in rare cases, wandering supermassive black holes may be bright enough to appear as ULXs.

Key words: black hole physics – galaxies: evolution – galaxies: kinematics and dynamics – galaxies: luminosity function, mass function – gravitational waves – pulsars: general

Online-only material: color figures

1. INTRODUCTION

The growth of supermassive black holes (SMBHs) at the centers of massive galaxies is thought to have occurred primarily at large redshifts, with the cessation of quasar activity at $z \leqslant 2$ signaling the end of SMBH growth as well (see Natarajan 2004 for a review). More specifically, SMBHs are thought to grow primarily through gas accretion at higher redshifts; gas-rich galaxy mergers would trigger the accretion episodes through which SMBHs would grow, with the actual merging of multiple SMBHs being a subdominant mode of SMBH growth (Volonteri & Natarajan 2009; Treister et al. 2010). At lower redshifts, gas depletion would result in a greater relative importance of merging black holes to SMBH mass growth. This epoch is associated with lower rates of growth in SMBH mass density overall. However, mergers can redistribute the mass density at late times ($z \leqslant 1$), leading to significant growth of the most massive black holes, even in the absence of observable amounts of gas accretion. The increased importance of mergers at late times, if true, will have two inescapable consequences: the stochastic gravitational-wave background from merging SMBHs will be larger than otherwise expected, and any black holes that fail to merge will spiral through their host galaxy indefinitely, accreting gas and stars and emitting X-rays as they go.

Recent evidence indicates that, although these mostly "red and dead" massive galaxies undergo little star formation at late times, they continue to merge unabated, so that mergers dominate the evolution of the black hole mass function for large masses at $z \leqslant 1$. Although some papers have indicated that massive black holes grow by as little as $\sim$1/3 of their

original mass since $z = 1$ (e.g., Brown et al. 2007; Yoo et al. 2007; de Ravel et al. 2009), more recent results of both observational campaigns (Zhang et al. 2012; Laporte et al. 2013) and numerical simulations (Oser et al. 2012) strongly suggest that massive galaxies have grown still more rapidly, and may have as much as doubled their mass since $z = 1$. We will adopt the assumption of mass doubling since $z = 1$, with the obvious caveat that a smaller amount of growth for massive ellipticals will imply a proportionally smaller number of merger events. However, since the gravitational wave signals from these events will add incoherently, we expect the full range of predicted merger rates to cause no more than a $\sim\sqrt{1/3}$ difference in the predicted signal. Both observations and simulations further indicate that the growth of massive galaxies since $z = 1$ occurred primarily through mergers with $M_\bullet^s/M_\bullet^h \geqslant 1/10$, where the black hole of mass $M_\bullet^s$ from the satellite galaxy merges with the host black hole of mass $M_\bullet^h$ through dynamical friction, stellar hardening (or some alternative process), then gravitational wave emission, and the gas and stars from the satellite are deposited in the outer reaches of the host galaxy.

The anomalously large optical luminosity observed in the brightest cluster galaxies (BCGs) has long suggested that mergers play a critical role in their evolution, providing another indication of the importance of mergers for massive galaxies. The Bautz–Morgan classification (Bautz & Morgan 1970) of galaxy clusters depends primarily on the magnitude difference, Δm_{12}, between the first and second brightest galaxies in a cluster; the large size of this difference in some clusters had been a noted anomaly (see Abell 1965 and references therein). Tremaine & Richstone (1977) invented a statistical test that implicated a physical process, rather than a statistical variation,

as the cause of the large value of Δm_{12}. Ostriker & Hausman (1977) and Hausman & Ostriker (1978) argued that a plausible origin was "galactic cannibalism" driven by the process of dynamical friction, which would cause a massive satellite galaxy to spiral into the center of a cluster and merge with the BCG residing there. Many subsequent studies have affirmed the likelihood of this scenario.

In this work, we will discuss the observable consequences of the fact that these merging massive galaxies will generally contain SMBHs. The satellite black holes will be detectable as accreting X-ray sources as they orbit within the cannibal BCG, and then as gravitational wave sources if they reach the more massive black hole at the center of the BCG. We will demonstrate that the observed evolution of the galaxy and black hole mass functions can be reproduced by a calculation that assumes only mergers drive the evolution. By comparing the calculation with the observed evolution, we can find the total number of mergers necessary to reproduce the observations. With this number in hand, we perform a Monte Carlo simulation of the population of merging black holes, from which we can directly extract the total expected gravitational wave signal. Our results, which are the first to be derived primarily from observational data,[4] indicate that the signal within the frequency band of pulsar timing arrays (PTAs) is substantially larger than previously estimated, and is in fact at the threshold of detectability for the current generation of PTAs. Furthermore, the signal extends to higher frequencies than previous estimates, potentially reaching the lowest sensitive frequencies for space-based gravitational wave observatories.

It is worth noting what ingredients of our approach are essential in producing a stronger gravitational-wave signal than was previously predicted. The details of our method are far less important than three ingredients which are based on both accumulated and recent observational studies: the mass density in stars in massive galaxies (Kormendy & Ho 2013), the recent calibrations of the black hole–stellar mass correlation (McConnell & Ma 2013),[5] and the evidence from both observations (Bell et al. 2006; Robaina et al. 2010; van Dokkum et al. 2010; Trujillo et al. 2011) and simulations (Naab et al. 2009; Oser et al. 2012; Kulier et al. 2013) for a significant late mass growth for massive ellipticals via merging. It is the calibration of our computation with these observational results that is primarily responsible for the higher estimated values of late gravitational radiation emission and the resultant prediction for pulsar timing strain measurements.

In addition to the galaxy mergers that yield gravitational wave signals, many galaxy mergers will result in what we will call a "stalled" satellite, where the smaller black hole cannot make its way to the host core in less than a Hubble time. We calculate this population as well, and compare their quantity and expected characteristics to those observed in ultraluminous X-ray sources (ULXs).

In Section 2, we will describe our model for the black hole mass function, first presenting the explicit redshift-dependent expressions based primarily on observational data in Section 2.1, then discussing our construction of a set of evolution equations for propagating an initial mass function to later redshifts without explicitly enforcing any redshift dependence in Section 2.2. We calculate the dynamics of the black holes after galaxy merger to determine whether the black holes will merge in Section 3. We then apply our model to first calculate the stochastic gravitational-wave signal from merging SMBH binaries in Section 4, elaborating on the results presented in McWilliams et al. (2012). We then use our model to predict the number of stalled satellite black holes from failed mergers and their potential appearance as ULXs in Section 5. We summarize and conclude in Section 6.

2. THE MASS FUNCTION

2.1. The Schechter Parameters

Any attempt to calculate the stochastic background of gravitational waves from SMBHs requires a model for the mass and redshift distribution of galaxy mergers. In order to anchor our model in reality, we wish to calibrate it using current observations of the mass and redshift distribution of galaxies, typically referred to as the galaxy stellar mass function. The information contained in observations is condensed by fitting the data to established models of the mass function, which are generally empirical. The stellar bulge and black hole mass functions are frequently parameterized using a Schechter function of the form (Schechter 1976),

$$\phi(M)\,dM = \varphi\,M^\alpha\,\exp(-M)\,dM, \tag{1}$$

where $\phi\,dM \equiv (\partial n/\partial M)\,dM$ is the comoving number density of either black holes or bulges with masses between M and $M+dM$, with $M \equiv M_\bullet/\mathcal{M}_\bullet$ for black holes or $M_\star/\mathcal{M}_\star$ for bulges. $\mathcal{M}$, φ, and α are the three Schechter parameters, and the subscripts "$\bullet$" and "$\star$" refer to black holes and stellar bulges, respectively. However, Equation (1) fails to accurately reproduce the observed mass function when BCGs are included. In order to represent both BCGs and less massive galaxies (subscripted "low"), we combine Equation (1) with a Gaussian component representing BCGs (Lin et al. 2010),

$$\phi(M)\,dM \equiv (\phi_{\rm low} + \phi_{\rm BCG})\,dM = \varphi\,M^\alpha\,\exp(-M)\,dM$$
$$+ \tilde{\varphi}\exp\left[-\frac{1}{2}\left(\frac{2.5\log M}{\sigma_M}\right)^2 - 1\right]\,dM, \tag{2}$$

where $\tilde{\varphi}$ normalizes and σ_M determines the e-folding of the high-mass BCG-populated portion of the mass function. To avoid ambiguity, we will use $\phi_{\rm tot} \equiv \phi$ to represent the full mass function in any expressions that include both the full function and the individual components.

For this study, we set $\tilde{\varphi} = \varphi$ and $\sigma_M \equiv 0.58$ at all redshifts. The choices of $\tilde{\varphi}$ and σ_M are motivated by a desire to include at least one galaxy as massive as M87 in our sample; when we use a value for σ_M taken from Lin et al. (2010), we find that setting $\tilde{\varphi} \sim \varphi$ results in $\mathcal{O}(1)$ instance of an M87-mass galaxy. However, we note that the uncertainties in this choice are poorly understood, as it governs a regime with a relatively low number density of galaxies, and Lin et al. (2010) do not provide error bars. We have endeavored to be conservative, as including more massive galaxies will increase the strength of the overall gravitational-wave signal, but this

---

[4] After completion of this manuscript, we became aware of an independent calculation of the gravitational-wave signal from PTAs (Sesana 2013), which combines the observed galaxy mass function and pair fraction to predict the gravitational wave signal. This new estimate is moderately larger than previous estimates.

[5] Sesana (2013) also employs the results of McConnell & Ma (2013). However, he additionally includes several other correlations, all of which predict systematically much lower black hole masses, so that the lower limit is not affected by the inclusion of McConnell & Ma (2013).

must nonetheless be considered a central uncertainty in our overall analysis.

Since observational data are generally available for stellar bulges rather than for massive black holes directly, we relate the Schechter parameters for the central black holes to the parameters for their concomitant stellar bulges by assuming a redshift-independent $M_\bullet$–$M_\star$ relation given by $\mathcal{M} \equiv \mathcal{M}_\bullet = 2.9 \times 10^{-3}\mathcal{M}_\star$ (McConnell & Ma 2013), $\alpha \equiv \alpha_\bullet = \alpha_\star$, and $\varphi \equiv \varphi_\bullet = \varphi_\star$, which are the only choices consistent with negligible star formation and negligible accretion of gas onto the central black holes. We emphasize that recent observational results indicate that simple scaling relationships like $M_\bullet$–$M_\star$ may systematically underestimate the mass of black holes in BCGs by a factor as large as 10 (Hlavacek-Larrondo et al. 2012), so we would be equally justified in enhancing the high-mass portion of the distribution in a similar fashion by including a mass dependence in the proportionality constant of the $M_\bullet$–$M_\star$ relation. There are a number of other corollaries for merger-driven evolution: the black hole mass–stellar-bulge mass ($M_\bullet$–$M_\star$) relation and the black hole and stellar-bulge densities, $\rho_\bullet$ and $\rho_\star$, are all redshift independent; $\mathcal{M}(z=0) \geqslant \mathcal{M}(z>0)$; and $\alpha(z=0) \geqslant \alpha(z>0)$.

We note that the observational data on the redshift dependence of the Schechter parameters have substantial variance, which provides the motivation for developing theoretically motivated dependencies to inform choices for our model. For instance, although we assume a constant $M_\bullet$–$M_\star$ relationship, the literature varies wildly, using $M_\bullet \approx 10^{-3}M_\star(1+z)^a$ with a as large as 2, although larger values are typically drawn from fits to data that span to higher redshifts. For $\mathcal{M}$, we use

$$\mathcal{M} \equiv \frac{1.2 \times 10^8}{1+z}\, M_\odot. \tag{3}$$

Though consistent with the observational data indicating mass-doubling since $z = 1$ (Robaina et al. 2010; van Dokkum et al. 2010), this choice of redshift dependence also makes sense in light of our assumption of merger-driven evolution, since massive galaxies with $M_\star \sim \mathcal{O}(\mathcal{M}_\star)$ should become more massive at lower redshifts as they consume less massive satellites.

We choose a constant value for φ,

$$\varphi \equiv 3 \times 10^{-3}\,\mathrm{Mpc}^{-3}, \tag{4}$$

the magnitude of which is consistent with the literature (Cole et al. 2001) at low redshifts when the final *Wilkinson Microwave Anisotropy Probe* (*WMAP*) cosmological parameters are used (Komatsu et al. 2011), and is also consistent with merger-driven evolution, except in the unlikely scenario that most mergers occur between nearly equal mass binaries. We note that a constant φ is slightly inconsistent with observational data in the literature, which typically measures φ to be a modestly decreasing function of redshift. We suspect sampling effects may account for this to some degree, with galaxies evolving from blue to red, and effectively "appearing" in a survey at lower redshifts, thereby increasing the apparent number density of massive red galaxies. In contrast, our approach conserves, by construction, the mass density of old stars. Nonetheless, this uncertainty again has a negligible effect on the total gravitational-wave signal; the observational uncertainties in the overall magnitude of φ play a much greater role, and are

therefore included in our estimated uncertainty for the amplitude of the gravitational-wave signal.

To find α, we first integrate the mass function for total comoving mass density,

$$\int_0^\infty M n_\bullet\, dM = \mathcal{M}\varphi\,\Gamma[\alpha+2] = \langle\rho_\bullet\rangle = \text{const.} \tag{5}$$

In choosing a mean mass density $\langle\rho_\bullet\rangle$ for black holes during this epoch, we adopt a conservative estimate (Yu & Tremaine 2002), but correct it to account for the more recent $M_\bullet$–$M_\star$ relationship for BCGs found by McConnell & Ma (2013), to arrive at a density of $6 \times 10^5\, M_\odot\,\mathrm{Mpc}^{-3}$. Due to the observed constancy of the $M_\bullet$–$M_\star$ relationship for BCGs, and the small fractional change in density from accretion for massive black holes (Yu & Tremaine 2002), we treat $\langle\rho_\bullet\rangle$ as constant. This choice is consistent with observational results (Hopkins et al. 2007; Zhang et al. 2012; Sesana 2013) out to $z = 1$.

Finally, we solve for the redshift dependence of α that satisfies mass conservation. We expand the resulting α to $\mathcal{O}(1/(1+z))$, and find that

$$\alpha \approx -2.0 + \frac{0.52}{1+z}, \tag{6}$$

is accurate to within a percent for $0 \leqslant z \leqslant 1$. We note that the redshift dependence of α agrees reasonably well with the literature (see, e.g., Kajisawa et al. 2009); however, the overall magnitude of α is larger than most estimates in the literature. This may be a result of a sensitivity-induced bias toward observing more massive galaxies. Regardless, we must emphasize that our choice of a steeper α has little effect on our calculation of the total gravitational wave emission, since massive galaxies will still gain the same amount of mass via mergers in our simulations, although it will effect the statistics for the expected mass ratio of mergers.

The principle theoretical reason for choosing a constant φ is that mergers involving massive galaxies are unlikely to be equal mass mergers, given the rarity of black holes with $M_\bullet \sim \mathcal{M}_\bullet$. To intuit why this implies a constant φ, and what behaviors we might expect for $\mathcal{M}$ and α, we can use a very simple model of galaxy evolution solely for the purpose of gaining intuition about the evolution of the Schechter parameters. We note that van Dokkum et al. (2010) present essentially the same arguments for why mergers should not change φ unless galaxies with mass $\mathcal{M}$ only merge with each other. Instead of using linear-in-mass bins for the mass function, we can imagine that mass doubling occurs only through equal mass mergers, whereas mass doubling of high-mass bins occurs through the depletion of the lowest-mass bins in the minor-merger scenario. For the major-merger scenario, after doubling, every log-mass bin will simply shift toward larger mass by one log-mass unit and will halve their number density, doubling $\mathcal{M}$, halving φ, and leaving α constant. However, for the minor-merger scenario, the number of very massive galaxies remains fixed while they accrete smaller satellites, so that after mass doubling, massive number-density bins will simply shift by one log-mass unit toward higher mass at fixed number density, whereas low-mass bins will shift toward higher mass with their number density being depleted by mergers with more massive galaxies, and replenished by mergers between lower mass members. If we require that the distribution maintains a single power-law mass distribution for less massive galaxies, then in the minor-merger scenario the low-mass bins must have their number density depleted relative to the high-mass bins, in which case the slope of the mass function at low

masses would evolve toward shallower, less negative α values as we evolve toward lower redshift.

2.2. Evolution of the Mass Function

In this section, we investigate how closely the observed evolution with redshift of the mass function can be reproduced with a simple model that assumes merger-driven evolution over the redshift interval $z \leqslant 1$. The evolution of the mass function generally results from some combination of mergers, star formation, and mass loss (e.g., through winds):

$$\frac{\partial}{\partial z} (\phi \, dM) \, dz = \left[\frac{\partial}{\partial z} (\phi \, dM) \, dz \right]_m - \frac{\partial}{\partial M} \left(\phi \left\langle \frac{\partial M}{\partial z} \right\rangle \right) dM \, dz,$$
(7)

where $\langle \partial M / \partial z \rangle \approx \partial \mathcal{M} / \partial z$ is the mean rate of change of mass at fixed number densities. To find the contribution from mergers, $[\partial / \partial z (\phi \, dM) dz]_m$, we need to know the number of mergers between a black hole with $\mathcal{M}$-normalized mass between M' and $M' + dM'$ and another with normalized mass between M'' and $M'' + dM''$ at a redshift between z and $z + dz$. This requires some care, since BCGs will not merge with each other, so they must be handled separately. Since we are assuming merger-dominated evolution, the mass dependence of the merger probability distribution will be proportional to the number densities of the two merging black holes. The redshift dependence $P(z)$ can be chosen to maximize the agreement between our model and the observed evolution of the mass function. We therefore express the number density of mergers between black holes of $\mathcal{M}$-normalized mass $M' \equiv M'_\bullet / \mathcal{M}_r$ and $M'' \equiv M''_\bullet / \mathcal{M}_r$ at redshift z (where $\mathcal{M}_r \equiv (1+z)\mathcal{M}$ is the redshifted characteristic black hole mass) as

$$\frac{\partial^3 \phi_{\{low, BCG\}}}{\partial M' \partial M'' \partial z} dM' dM'' dz$$
$$= P(z) dz \, \phi_{\{tot, BCG\}}(M') \, dM' \, \phi_{\{tot, low\}}(M'') \, dM'', \quad (8)$$

where in the notation $\phi_{\{a, b\}}$, the first arguments all form an equation for ϕ_a, and the second arguments form an equation for ϕ_b. In addition, $P(z)$ can be separated into a purely cosmological component and a comoving merger probability component as $P(z) \equiv (dp/dV_c)((dV_c/dz)/n(z))$, (dp/dV_c) is the merger probability in a fixed comoving volume, $n(z)$ is the total number density at a given redshift, and dV_c/dz accounts for cosmological expansion and is given by

$$\frac{dV_c}{dz} = \frac{4 \pi c D_L^2}{H_o (1+z)^2 \sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}}.$$
(9)

We assume a flat cosmology with *WMAP* seven-year values for the cosmological parameters (Komatsu et al. 2011); therefore, the luminosity distance is given by

$$D_L(z) = \frac{c(1+z)}{H_o} \int_0^z \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}}$$
(10)

(see Hogg 1999 and references therein). We can then find the number of black holes that leave (the sink term "si") and enter (the source term "so") a mass bin at $M_\bullet$ during the interval dz,
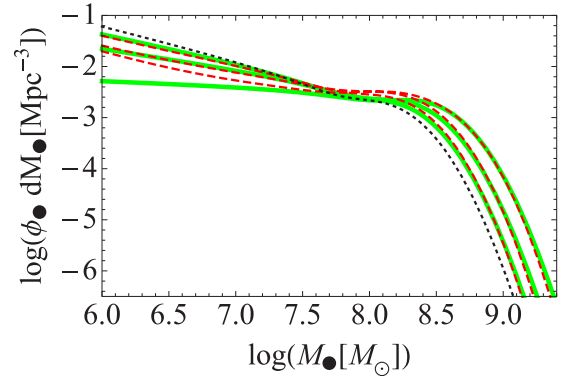

Figure 1. Evolution of the black hole mass function (equivalently, the stellar mass function using $M_\star = 347 \, M_\bullet$) from $z = 1$ to $z = 0$. The dotted (black) curve shows the initial $z = 1$ mass function, and the solid (green) and dashed (red) curves show the analytic (described in Section 2.1) and evolved (described in Section 2.2) mass functions, respectively, at $z = \{2/3, 1/3, 0\}$. We note that the curves cross at $\log(M_\bullet) \approx 7.9$, so that galaxies above this mass are experiencing a net increase in number density through merging, while galaxies below this mass experience a net depletion. Our model predicts an excess in number density due to mergers below $\log(M_\bullet) \approx 7$ at late times, but this mass range does not affect the gravitational-wave signal.

(A color version of this figure is available in the online journal.)

$$\left[\frac{\partial}{\partial z} \phi_{\{low, BCG\}}(\hat{M}) \right]_{si} = P(z) \int_{10^6 M_\odot / \mathcal{M}_r}^{10^{11} M_\odot / \mathcal{M}_r} \phi_{\{tot, BCG\}}$$
$$\times (\hat{M}) \phi_{\{tot, low\}}(M') \, dM'$$

$$\left[\frac{\partial}{\partial z} \phi_{\{low, BCG\}}(\hat{M}) \right]_{so} = P(z) \int_{10^6 M_\odot / \mathcal{M}_r}^{(\hat{M} - 10^6 M_\odot)/\mathcal{M}_r} \int_{10^6 M_\odot / \mathcal{M}_r}^{(\hat{M} - 10^6 M_\odot)/\mathcal{M}_r}$$
$$\times \delta(\hat{M} - M' - M'') \phi_{\{low, BCG\}}$$
$$\times (M') \, dM' \phi_{\{low, low\}}(M'') \, dM''$$
$$= P(z) \int_{10^6 M_\odot / \mathcal{M}_r}^{(\hat{M} - 10^6 M_\odot)/\mathcal{M}_r} \phi_{\{low, BCG\}}$$
$$\times (M') \phi_{\{low, low\}}(\hat{M} - M') \, dM'.$$
(11)

We will consider the mass range $10^6 \, M_\odot \leqslant M_\bullet \leqslant 10^{11} \, M_\odot$ throughout this work, although the precise choices do not significantly affect any of the results that we present. All that remains is to determine whether dp/dV_c can be well-approximated (and replaced in our evolution equation) by a constant value. We find that fixing $dp/dV_c = 260 \, \text{Mpc}^{-3}$ in our evolution equation reproduces the redshift-dependence of the mass function using our redshift-dependent Schechter parameters remarkably well (see Figure 1). In Figure 1, we see that systems with $\log(M_\bullet) \gtrsim 7.9$ experience a net increase in number density due to mergers. As expected, our model predicts an excess in the number density of systems with $\log(M_\bullet) \lesssim 7$ at $z \leqslant 1/3$, because it predicts too many mergers between two low mass systems. However, as we will show, these low mass mergers have a completely negligible contribution to the gravitational-wave signal.

It is noteworthy that, despite the good agreement between our model based on galaxy evolution through mergers and our adopted Schechter-based mass function, the evolved solutions can only approximate Equation (2). The analytic form of the source term involves generalized hypergeometric functions

among others, so that the evolved mass function does not simplify to the functional form of Equation (2). The shape of the transition from less massive black holes to the BCG component as well as the exponential decay for very large masses in the evolved model depend sensitively on the location of the break mass $\mathcal{M}$, which sets both the "knee" location and the e-folding, so a sufficiently fine redshift sampling (or continuous integration, as we have done) is necessary to accurately approximate the target mass function at each redshift.

3. SATELLITE BLACK HOLE DYNAMICS WITHIN THE HOST GALAXY

Having established a method for calculating the distribution of galaxy mergers, we must now determine whether those mergers lead to the merger of the concomitant SMBHs and thereby contribute to the gravitational wave background. In the case of major mergers, the stellar bulges and their central black holes will usually merge in less than a Hubble time. We loosely define major mergers as having mass ratios $q \equiv M^s/M^h \gtrsim 1/4$, where M^h is the larger "host" mass, and M_s is the "satellite" mass (either black hole or bulge). For minor mergers, the satellite black hole will likely be stripped of its stellar bulge, which is deposited outside the host bulge and explains the exceedingly large increase in the apparent size of galaxies since $z = 1$ (van Dokkum et al. 2010) in this model. However, the satellite black hole must be able to migrate to the central core of the host under dynamical friction by the present day to contribute to the gravitational wave signal. To determine whether this occurs, we use the well-known expression for the dynamical friction timescale (Chandrasekhar 1943; Tremaine et al. 1975) in a convenient form (Equation (8.12) in Binney & Tremaine 1987),

$$
t_{DF} = \frac{19\,\text{Gyr}}{\ln(1 + M_\star^h/M_\bullet^s)} \left(\frac{R_e}{5\,\text{kpc}}\right)^2 \frac{\sigma(R_e)}{200\,\text{km s}^{-1}} \frac{10^8 M_\odot}{M_\bullet^s}
$$
$$
\approx \frac{4.5\,\text{Gyr}}{q(6.9 - \ln q)} \left(\frac{M_\bullet^h}{10^8 M_\odot}\right)^{2/3} (1+z)^{-3/2}, \quad (12)
$$

where R_e is the half-light radius of the host galaxy and σ is the local velocity dispersion, for which we use

$$
R_e = 2.5\,\text{kpc} \left(\frac{M_\bullet}{10^8 M_\odot}\right)^{0.73} (1+z)^{-1.44} \text{ and}
$$
$$
\sigma(R_e) = 190\,\text{km s}^{-1} \left(\frac{M_\bullet}{10^8 M_\odot}\right)^{0.2} (1+z)^{0.44}, \quad (13)
$$

where the mass-dependence comes from fits to Sloan Digital Sky Survey (SDSS) data (Nipoti et al. 2009), and the redshift dependence comes from fits to simulation results which were shown to be consistent with various surveys in Oser et al. (2010) within the redshift range we consider.

We note that Equation (12) differs in multiple ways from what is often used in merger-tree models (see Equation (9) in Volonteri et al. 2003). Specifically, it is often assumed that t_{DF} depends only on the mass ratio of the merging pair, i.e., dynamical friction does not depend on the mass scale. This would be a very surprising result for multiple reasons. The appropriate t_{DF} for merger-trees should incorporate both the merging dark matter halos and, subsequently, the stellar bulges. The mass of the host and the inclusion or exclusion of its stellar bulge will affect its equilibrium density distribution, and will therefore affect the tidal stripping of the satellite. Even if the

inclusion of the bulge does not affect t_{DF} significantly as argued in Boylan-Kolchin et al. (2008), the radius of injection for the satellite and the orbital kinematics of the satellite should depend directly on the host mass. Since we need an expression for t_{DF} that applies for $t_{DF} \approx t_H$ in order to determine whether the host and satellite will merge, the scenario of a "naked" satellite black hole moving through the host bulge by dynamical fraction is precisely the case of interest. Therefore, in Equation (12), the relevant mass ratio for our purpose is $M_\star^h/M_\bullet^s$, rather than $M_\star^h/M_\star^s$ as is often used in dark matter merger trees, such as the model of Volonteri et al. (2003) which has been employed frequently for gravitational wave calculations. We emphasize, however, that unlike the other effects we will detail, this difference in our method compared to others in the literature will serve to decrease our overall merger rate relative to past estimates.

Finally, in the scenario of a stripped satellite, the satellite black hole is effectively deposited at R_e within the host galaxy, and spirals toward the center through dynamical friction. R_e is far smaller than the virial radius, which is the fiducial distance scale used in dark-matter merger-tree simulations (Volonteri et al. 2003), where the merger of two halos is considered, and the bulges are ignored when calculating t_{DF}.

At each redshift, we use Equation (12) to determine which black holes will merge. In order to calculate the total gravitational wave signal from the merging population using the Monte Carlo method, we need to sample from the probability distribution $d^4p/dM_1dM_2dz\,df$. Equation (8) provides most of what we need, since $d^3p/dM_1dM_2dz = (1/n(z))(d^3n/dM_1dM_2dz)$. The probability of finding a binary in a given frequency interval df is found by observing that the quadrupole frequency rate $\dot{f} \propto f^{11/3}$, so that $dp/df \propto f^{-11/3}$, with the normalization being determined by the range of possible frequencies.

It is known that dynamical friction becomes ineffective once the binary reaches a separation of $\mathcal{O}(1\,\text{pc})$, so we must assume a mechanism for solving the "last parsec problem" (Merritt & Milosavljević 2005) in order to determine the minimum frequency where gravitational radiation will drive the evolution of the binary. Following Quinlan (1996), we assume the binary hardens through the repeated scattering of stars in the core of the host, until gravitational radiation becomes the dominant process at

$$
f_{\min} = 2.7 \times 10^{-9}\,\text{Hz} \left(\frac{M_\bullet^h M_\bullet^s}{(10^8 M_\odot)^2}\right)^{-0.3} \left(\frac{M_\bullet^h + M_\bullet^s}{2 \times 10^8 M_\odot}\right)^{0.2}.
$$
$$
(14)
$$

The maximum frequency is set to the innermost stable circular orbit (ISCO) for a test particle orbiting a Schwarzschild black hole with the same total mass,

$$
f_{\max} = f_{ISCO} = 1.1 \times 10^{-4}\,\text{Hz} \left(\frac{M_\bullet^h + M_\bullet^s}{2 \times 10^8 M_\odot}\right)^{-1}, \quad (15)
$$

though we emphasize that the results are insensitive to the precise choice of $f_{\max}$, since we perform a random Monte Carlo draw from an $f^{-11/3}$ distribution, so only $f_{\min}$ plays a critical role. Using a Monte Carlo sample to set the starting frequency, we invert

$$
f(t) = 1.4 \times 10^{-6}\,\text{Hz} \left(\frac{(10^8 M_\odot)^2}{M_\bullet^h M_\bullet^s} \frac{10\,\text{yr}}{t_m - t}\right)^{3/8}
$$
$$
\times \left(\frac{M_\bullet^h + M_\bullet^s}{2 \times 10^8 M_\odot}\right)^{1/8}, \quad (16)
$$

to find the corresponding time before merger, then reapply Equation (16) to find the frequency after a $T = 5$ yr observation, where t_m approximates the time of the final merger. We note that all of the frequency expressions given above are appropriate for the rest frame of the source, f_r, and must be redshifted to give the measured gravitational wave frequency, f_m, using $f_m = f_r/(1 + z)$. With the range of permitted frequencies now determined by the dynamics of each binary, we can generate gravitational wave signals from each, and combine them to form our estimate of the stochastic gravitational wave background.

4. GRAVITATIONAL WAVES FROM SMBH MERGERS SINCE $z = 1$

In order to calculate the total gravitational wave signal from all SMBHBs in the PTA band, we must combine our knowledge of the merger probability distribution d^3p/dM_1dM_2dz and the total number of mergers N with the signal strength of each source, given in units of dimensionless strain h. If the resulting span of frequencies $\Delta f \equiv f(t) - f(t - T) < T^{-1}$ for a given source, then we assign a gravitational wave strain amplitude appropriate for a monochromatic source, which is given within the quadrupole approximation by Equation (58) of Thorne (1987),

$$
\begin{aligned}
h = 3.2 \times 10^{-17} & \left(\frac{M_\bullet^h M_\bullet^s}{(10^8\, M_\odot)^2} \right) \\
& \times \left(\frac{2 \times 10^8\, M_\odot}{M_\bullet^h + M_\bullet^s} \right)^{1/3} \left(\frac{1\, \mathrm{Gpc}}{D_L} \right) \left(\frac{f}{10^{-7}\, \mathrm{Hz}} \right)^{2/3},
\end{aligned} \quad (17)
$$

where this amplitude is already averaged over source polarization. Sources with $\Delta f > T^{-1}$ are potentially resolvable, and are not part of the stochastic background, since they introduce a correlation in the signal between adjacent frequency bins. In addition, sufficiently nearby sources may be sufficiently bright to be resolved even if they are monochromatic, but these sources would also need to be removed before the data could be considered stochastic. Therefore, we exclude both of these types of sources from the current study. We note that these resolvable sources, although much rarer than sources contributing to the stochastic signal, will nonetheless dominate the total energy density in gravitational waves, and are the only means by which we might gain knowledge about individual binary systems. Resolvable sources are therefore of great interest as well; however, since they require a much larger and more computationally expensive Monte Carlo sampling due to their rarity, we leave them for future work.

Finally, we calculate the full stochastic gravitational wave signal by summing the power of each source, thereby approximating

$$
\begin{aligned}
h_c^2(f) = \int_0^1 dz \int_{10^6\, M_\odot}^{10^{11}\, M_\odot} dM_2 \\
\times \int_{10^6\, M_\odot}^{M_2} dM_1 N h^2 \frac{d^4 p}{dM_1 dM_2 dz\, df},
\end{aligned} \quad (18)
$$

where

$$
N \equiv \int_0^1 dz \int_{10^6\, M_\odot}^{10^{11}\, M_\odot} dM \left[\frac{\partial}{\partial z} \phi \right]_{\mathrm{so}} \quad (19)
$$

is the total number of mergers during the full observation, and the frequency minimum and resolution are set assuming

a 5 yr observation. Since we have shown that our evolution duplicates the observed mass function over the mass-range of interest, we draw merging pairs from the probability distribution formed using the analytical fit to the observed mass function at each redshift, rather than from the numerical mass function generated through our evolution. Due to the extremely large number, $\mathcal{O}(10^{11})$, of mergers required to duplicate the observed evolution of the mass function, and the fact that low mass black holes cannot be excluded a priori since they dominate the overall number density, we have performed a Monte Carlo for $N_{\mathrm{MC}} = 5 \times 10^6$ binaries, and scaled the result appropriately. To scale, we first fit the Monte Carlo result to the form

$$
h_{\mathrm{fit}} \equiv h_o \exp\left[\left(\frac{f}{f_{\mathrm{LF}}} \right)^\beta \right] \left(\frac{f}{f_o} \right)^{-2/3} \left(1 + \frac{f}{f_o} \right)^\gamma, \quad (20)
$$

where $h_o, f_{\mathrm{LF}}, f_o, \beta,$ and γ are tunable parameters ("LF" refers to low frequency). The $f^{-2/3}$ term is the usual spectrum for stochastic binaries (Phinney 2001; Jaffe & Backer 2003). The final term in Equation (20) is the same one used in Sesana et al. (2008) and accounts for the discrete nature of the sources and its effect at high frequencies where the number of sources diminishes. The exponential term is introduced here for the first time, and is primarily motivated by the suppression of high mass sources the mass dependence of f_{min} in Equation (14), which can lead to biasing against massive merging systems, so that the mechanism for solving the last parsec problem can influence this region of the spectrum. We will use the parameterization of Equation (20) to first fit, then scale the results from our Monte Carlo simulation.

After calculating the best-fit parameters from Equation (20), we multiply the resulting f_o by $(N/N_{\mathrm{MC}})^{3/11}$ (since $N \propto dp/df \propto f^{-11/3}$ as previously mentioned), and we multiply the overall amplitude of h_{fit} by $\sqrt{N/N_{\mathrm{MC}}}$ (equivalently, we multiply h_o by $(N/N_{\mathrm{MC}})^{7/22}$, since $h_{\mathrm{fit}} \propto h_o f_o^{2/3}$ for $f \ll f_o$). We find the lowest frequency bin at which the Monte Carlo simulation only predicts a single source, we again multiply by $(N/N_{\mathrm{MC}})^{3/11}$, and we terminate the scaled fit at that frequency, since the signal is clearly no longer stochastic. The parameters $f_{\mathrm{LF}}, \beta,$ and γ remain fixed in our rescaling, since there is no apparent physical grounds for these parameters to depend on N, and empirically we find that fixing these values from one Monte Carlo provides a good fit for other simulations with different values of N.

Figure 2 demonstrates the effectiveness of the overall scaling procedure by comparing two different Monte Carlo simulations. One simulation has $N_{\mathrm{MC}} = 5 \times 10^6$, and the other has $N_{\mathrm{MC}} = 10^5$. We show the raw strain data from both simulations, along with the best fits of each simulation using Equation (20). We then scale the simulation with $N_{\mathrm{MC}} = 10^5$ according to the scaling rules we have described, in order to compare with the actual fit of the $N_{\mathrm{MC}} = 5 \times 10^6$ simulation. Despite the scatter in the data, particularly at large frequencies, the scaled fit agrees extremely well with the larger Monte Carlo fit, indicating that the scaling of the mean gravitational-wave strain is very robust.

In Figure 3, a Monte Carlo with $N_{\mathrm{MC}} = 5 \times 10^6$ scaled to the actual number of expected binary sources ($N = 4.75 \times 10^{11}$ using Equation (19)) is shown, where the scaled-fit values (i.e., our estimate for h_c that we would expect to observe from massive merging binaries at $z \leqslant 1$) are $h_o = 8.4 \times 10^{-16}$, $f_o = 2.5 \times 10^{-6}$ Hz, $f_{\mathrm{LF}} = 6.3 \times 10^{-9}$ Hz, $\gamma = -1.3$, and $\beta = -4$, with the final frequency cutoff occurring at 6.2×10^{-6} Hz. We also estimated our uncertainty in h_c as
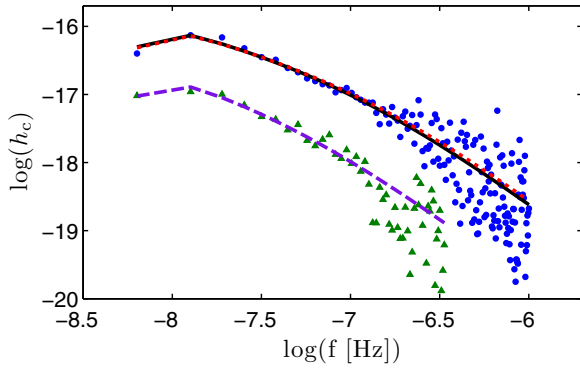
Figure 2. Gravitational wave strain demonstrating the effectiveness of scaling our Monte Carlo results to cases with a larger number of sources. The (green) triangular markers show the result of a Monte Carlo where $N_{MC} = 10^5$, and the (blue) circular markers show another Monte Carlo with $N_{MC} = 5 \times 10^6$. We fit the signal from the smaller sample to Equation (20), resulting in the (purple) dashed line, then scale the resulting amplitude, reference frequency, and final frequency to $N = 5 \times 10^6$ as discussed in the text. The result is the (red) dotted curve, which provides a very accurate representation of the Monte Carlo result from the larger sample. To demonstrate this, we also fit the signal from the larger sample using Equation (20), with the (black) solid line being the result, which agrees well with the scaled fit to the signal from the smaller Monte Carlo sample.

(A color version of this figure is available in the online journal.)

follows. We first assumed a $\pm 10\,\%$ standard deviation in $\log(\mathcal{M})$, φ, $\log(\rho_\bullet)$, and σ, consistent with experimental ranges (Drory & Alvarez 2008; Kajisawa et al. 2009; note that the value for N also changes as we vary any of the other parameters in our model). We then ran Monte Carlo simulations for all eight variations, and calculated best-fit parameters for h_c using Equation (20). However, since all analyses of PTA data to-

date have assumed a simplified form for the spectrum of the stochastic background, which we label h_s and which is given by (Jaffe & Backer 2003)

$$h_s \equiv h_o \left(\frac{f}{\mathrm{yr}^{-1}} \right)^{-2/3}, \tag{21}$$

we must convert our range of h_{fit} functions to a range of functions of the form given by Equation (21). We therefore approximate the expected and lower-limit strain spectra by requiring that $h_s \leqslant \langle h_{\mathrm{fit}} \rangle$ and $h_s \leqslant \min(h_{\mathrm{fit}})$, respectively, over the full band of the stochastic signals, using all 17 Monte Carlo realizations (one central parameter set, and 16 more from perturbing all four Schechter parameters up and down at both the 2σ and 3σ levels). We likewise require that $h_s \geqslant \max(h_{\mathrm{fit}})$ for the upper-limit strain spectrum. In this way, the uncertainty region shown in Figure 3 encompasses the full range of spectra resulting from the 2σ and 3σ confidence interval of best-fit Schechter functions to observational data. Because h_{fit} is the trend of the Monte Carlo results by construction, the fluctuations around the trend evident in Figure 2 are not included in the uncertainty interval estimate shown in Figure 3. Given the large number of total sources, and their concentration at $f \leqslant \mathrm{yr}^{-1}$, we do not expect the variance σ^2 of the spectra to significantly affect the constraint for the particular Monte Carlo realization produced by nature, since $\sigma^2(f) \approx h_c(f)/N(f)$.

We must emphasize that the procedure we have just described is only a rough estimate of how our model should translate to confidence intervals derived assuming a simple $f^{2/3}$ spectrum. First, the observational limits can only be used to imply detectability thresholds, as we have done, if the limits are derived from the same statistic (namely, the cross-correlation of signals from different pulsars) that must be used for detection.
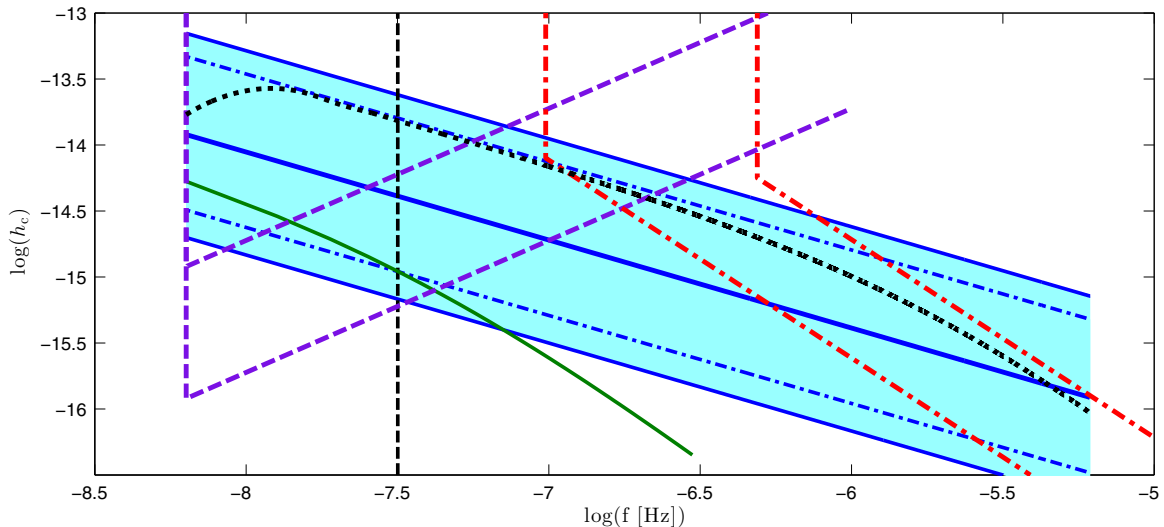

Figure 3. Gravitational wave strain and strain sensitivities for a 5 yr integration. The strain signals are orientation-averaged, and the sensitivities are sky-averaged. The dotted (black) line shows the scaled Monte Carlo results for the stochastic signal from merging massive black holes at $z \leqslant 1$ using our fiducial mass function parameters, and the (cyan) shaded region shows our estimate of the possible range of stochastic signals based on uncertainties in our model and assuming a conventional $f^{-2/3}$ spectrum, with the median signal shown as a thick solid (blue) line, and the optimistic and conservative $\{2\sigma, 3\sigma\}$ limits shown with thin {dash-dotted, solid} (blue) lines (see the text for details on our approximation of the uncertainty intervals). The thin solid (green) curved spectral line shows the signal predicted in Sesana et al. (2008), which is $\sim 4\times$ weaker than our median estimate at $f = 1\ \mathrm{yr}^{-1}$. The PTA sensitivities are scaled to match the 95% confidence limit of current PTAs and a future instrument like the SKA with $10\times$ better timing accuracy, where the current and future sensitivities are shown as dashed (purple) lines with lesser and greater sensitivity, respectively, and the typical PTA reference frequency of $f = 1\ \mathrm{yr}^{-1}$ is shown as a vertical thin dashed (black) line. We note that a detectable signal (at 95% confidence) will intersect the PTA limit at $f = 1\ \mathrm{yr}^{-1}$, indicating that much of the parameter space for the mass function has already been excluded in our model, and that a likely detection in the near future is an overall prediction of our model. Finally, the dash-dotted (red) lines show an optimistic estimate of LISA's and SGO's low frequency sensitivity (less and more sensitive, respectively), which indicates that SMBHB mergers at $z \leqslant 1$ may be a potential source of detectable stochastic gravitational waves below $f \sim 10^{-5}$ Hz for space-based gravitational wave detectors.

(A color version of this figure is available in the online journal.)

If, for instance, a tighter constraint is found through the use of autocorrelation of the signals from individual pulsars, we cannot assume that this constraint implies detectability of a signal at that level. This is not an issue for the constraints we quote, but may become an issue with future constraints. Second, it is important to bear in mind that any constraint assuming a simple $f^{2/3}$ spectrum will be dominated by the signal content at $f \approx T_{obs}^{-1}$, the inverse observation time. However, as is clear from the form of h_{fit} and from Figure 3, at least for some choices of Schechter parameters, the signal strength becomes diminished at the lowest frequencies, due to the efficiency of three-body interactions in bringing some binaries in our sample into the PTA band, particularly those with less massive satellites, so that $f_{min} \gtrsim T_{obs}^{-1}$ for those systems. The detectability of the signal will depend very sensitively on the behavior of the actual spectral shape of the signal at $f \sim T_{obs}^{-1}$ that is realized in nature. For cases where the signal does dip, the sensitivity will no longer accumulate as $T_{obs}^{13/6}$ (Jenet et al. 2005), but instead will accumulate like the SNR of the most sensitive frequency, which will be fixed and independent of T_{obs}.

In addition to our estimate for h_c, we show the results of Sesana et al. (2008), which agree with Sesana & Vecchio (2010). The large discrepancy between these results and our expected strain spectra (our mean estimate is $3.3\times$ larger at $f = 0.2 \, yr^{-1}$ and $5.1\times$ larger at yr^{-1}) requires an explanation, so we will address this disagreement before we discuss the remainder of Figure 3.[6] Sesana et al. (2008) construct $d^3n/dM_1 dM_2 dz$ using the methods described in Volonteri et al. (2003); specifically, the extended Press–Schechter formalism (Bond et al. 1991) is used to construct a dark-matter-halo merger tree, which is populated with black holes using the $M_\bullet - \sigma_\star$ or $M_\bullet - M_\star$ relationships, with the baryon dynamics treated as we described in our discussion of dynamical friction and the last parsec problem. Sesana et al. (2008) also employ other models than that in Volonteri et al. (2003), but these models differ in ways that are not relevant to our discussion (such as details in the way that feedback mechanisms are included), and the results from all of the models considered in Sesana et al. (2008) agree with each other far more than they agree with our results. Sesana & Vecchio (2010) use a different method altogether, yet arrive at a nearly identical result. They use $d^3n/dM_1 dM_2 dz$ by populating the catalog of merged galaxies constructed in Bertone et al. (2007) with black holes based again on the $M_\bullet - \sigma_\star$ or $M_\bullet - M_\star$ relationships. Bertone et al. (2007) used the Millennium simulation (Springel et al. 2005) to determine the merger rates of dark matter halos, and used semi-analytic techniques to approximate detailed baryonic physics such as star formation, accretion, stellar and supernova wind feedback, and the effects of metallicity. However, while Bertone et al. (2007) do not explain in detail their treatment of baryon interactions during galaxy mergers, they do state that the "outcome" of a galaxy merger depends only on the mass ratio in their model, and no mention is ever made of black holes stripped of their stellar bulge. Therefore, we conclude that the treatment of dynamical friction and the last parsec problem is qualitatively no different from that used in Volonteri et al. (2003), and therefore suffers from the same potential shortcomings for cases where the satellite black hole is stripped of its bulge. Since this is seen to occur in galaxy merger simulations of massive galaxies (Bellovary et al. 2010), we feel this is a likely source

of discrepancy between our result and past estimates. We also note that, even in the analysis of Sesana (2013), the same set of assumptions regarding tidal stripping and its impact on dynamical friction must be made in order to relate the galaxy merger rate implied by the galaxy mass function and pair fraction to the black hole merger rate (see, e.g., Jiang et al. 2013).

In addition to issues with the treatment of satellite dynamics within the host galaxy, two other factors that are shared by the treatments in Sesana et al. (2008) and Sesana & Vecchio (2010) contribute to their disagreement with our result; the first issue relates to the initial conditions of the dark matter simulations that are used, and the second issue arises from the nontrivial relationship between halo merger rates and galaxy merger rates. The Millennium simulation used by Sesana & Vecchio (2010) took the observed fluctuations in the power spectrum from the first year of *WMAP* data (Bennett et al. 2003) and used the data to determine the initial distribution of dark matter particles at $z = 127$. The model presented in Volonteri et al. (2003) and used in Sesana et al. (2008) predates *WMAP*, so that the abundance of X-ray emitting clusters in the local universe as measured in Eke et al. (1996) was used to normalize a theoretical fit to the cosmic microwave background power spectrum (Bardeen et al. 1986; Sugiyama 1995). In both cases, the amount of power contained in high-ℓ multipole moments/small angular scales was underestimated in comparison to results from the completed seven year *WMAP* survey (Komatsu et al. 2011). Since the black holes that we are interested in observing affect the power spectrum on small angular scales, this could have a significant impact on the total number of black holes, and therefore the number of mergers, that these models would predict.

Finally, we consider the relative merger rates of dark matter halos with and without cores of baryonic matter (i.e., the stellar component of the galaxy). From Equation (12), we see that the dynamical friction timescale $t_{df} \propto 1/M^s$. If a satellite halo is spiraling through a large host halo, as would be the case in a galaxy cluster, then the time it will take for the satellite to spiral to the center of the cluster scales inversely with M^s. The size and mass of the satellite halo will be limited by tidal stripping, which will be more severe in the absence of a stellar core. The remaining halo, though still larger than the stellar core, is far less dense, so the core may dominate the total satellite halo mass. Therefore, the inclusion of the stellar core could greatly decrease t_{df} and thereby increase the merger rate. By using the observed evolution of galaxies to calibrate $d^3n/dM_1 dM_2 dz$, our model automatically includes the correct baryonic physics, whereas including the correct physics semi-analytically in dark matter simulations is far from trivial. Therefore, the primary cause for the difference between our merger rate and the rate found in prior publications is that we estimate the merger rate using the observed density of massive galaxies, whereas most prior estimates have been based on dark-matter merger trees.

Returning to our description of Figure 3, we also show the rms strain sensitivity h_{rms}, averaged over sky location, for the current European/NANOGrav/Parkes PTA (using the European constraint and assuming 100 ns timing accuracy for all pulsars), and the Square Kilometer Array (SKA; 10 ns accuracy), assuming all arrays observe 20 pulsars for 5 yr (see Sesana et al. 2008 for details on calculating PTA sensitivities). Since a significant fraction of the overall range of our signal estimates is above the current PTA sensitivity at $f = yr^{-1}$, it is likely that a rigorous analysis of actual PTA data using a model that better represents the behavior of the Monte Carlo

---

[6] We note that the recent results in Sesana (2013) are somewhat less in conflict with our constraints. In this case, a lower implied merger rate may explain a large fraction of the overall difference.

results will already exclude large regions of the parameter space of Schechter parameters, assuming the overall validity of our merger-dominated assumption. Furthermore, our most conservative estimated level for the stochastic background would provide a detection with the SKA monitoring 20 pulsars for 5 yr at a far greater statistical significance than the expected strain level from Sesana et al. (2008) and Sesana & Vecchio (2010). If our model is valid, the SKA should not be necessary for a detection, as continued observation of the current set of pulsars with the current level of timing accuracy has a high likelihood of yielding a detection long before the SKA begins collecting data.

We also show the rms sensitivity for both the original Laser Interferometer Space Antenna (LISA) design, and a revised lower cost design (SGO Mid) currently under consideration by NASA, both calculated using Larson (2003) with the specifications for each design found in Stebbins (2011; a design with near-identical sensitivity called NGO is also being considered by the European Space Agency). We show sensitivity estimates calculated directly from Larson (2003), although we note that the sensitivity of these designs below $\sim 3 \times 10^{-5}$ Hz is highly uncertain, and the results from Larson (2003) are likely a best-case scenario. In addition to the stronger predicted signal within the PTA band, we predict the potential for significant signal within the LISA/SGO band as well, if the lowest frequency sensitivities from Larson (2003) are realized. This would provide a new class of signal for space-based gravitational wave observatories in the form of a stochastic ensemble of SMBH binaries, particularly for concepts with longer detector arms and lower frequency sensitive bandwidths (see, e.g., Folkner 2011), beyond the expected observation of individual sources sweeping through the band.

We note that the gravitational wave signal we have estimated does not truncate abruptly as might be assumed from Figure 3. Rather, the individual sources with $\Delta f > T^{-1}$ that we mentioned earlier will sweep through the band toward higher frequencies. They will no longer constitute a stochastic signal, but will have a significant impact on the event rate of resolvable sources. Given the small fraction of total mergers represented in our Monte Carlo, we do not adequately represent these more rare "chirping" signals. A much larger sample, or limiting the parameter range from which the sample is taken, would allow us to better represent the full merger population and predict the event rate for individually resolvable merger events, which we plan for future work. In addition to the expectation of an increased merger rate for SMBHs, the increased galaxy merger rate implies that a large number of satellite black holes fail to merge with their host, and may be observable as ULXs. This provides a second definitive prediction for our model, and will be explored in the next section.

In Figures 4 and 5, we show the relative contribution to the total gravitational-wave signal of different total masses and mass ratios, respectively. Since a histogram of strain would depend on the binning, we show a normalized version of the strain defined by

$$\hat{h}_c(x) = h_c(x) \Big/ \int_0^\infty h_c(x) \, dx, \qquad (22)$$

where x is any independent variable. The signal is dominated by the most massive binaries undergoing mergers, with masses above $10^8 \, M_\odot$ contributing over 99% of the signal. Satellite-to-host mass ratios of $\sim 1/2$ dominate the signal, with 99% of the signal coming from systems with mass ratios between $1/10$ and 1. We note that some simulations (e.g., Oser et al.
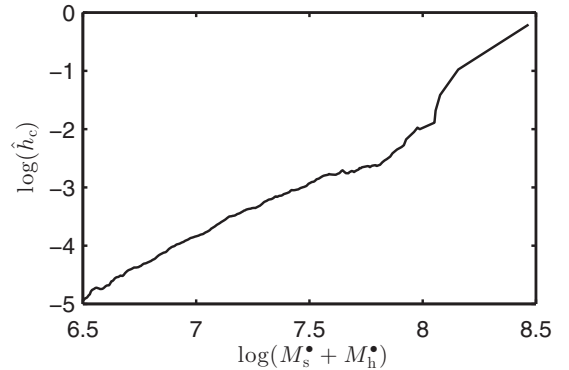

Figure 4. Relative contribution of the full range of combined host and satellite masses to the total gravitational-wave signal. Masses above $10^8 \, M_\odot$ contribute over 99% of the signal.
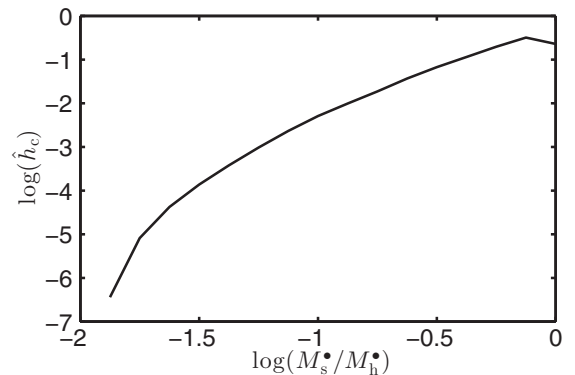

Figure 5. Relative contribution of the full range of satellite-to-host mass ratios to the total gravitational-wave signal. Mass ratios between $1/10$ and 1 contribute over 99% of the signal.

2010) predict a peak at somewhat smaller mass ratios. However, there is observational evidence (Kormendy & Bender 2013) that major mergers dominate massive galaxy evolution. Our result that major mergers dominate, but the contribution from minor mergers is definitely nonnegligible, should be considered to be consistent with both observations and simulations to a level well within the intrinsic uncertainties of each.

5. STALLED BLACK HOLE MERGERS AND ULXs

We have noted that, for very massive galaxies such as BCGs, all but the most comparable mass galaxy mergers stall prior to black hole merger. This has little effect on the total gravitational wave signal, since it is dominated by major and near-major mergers. However, the possibility of "stalled" mergers, where a smaller satellite black hole resides in the outer regions of a massive host galaxy, is an interesting candidate as a model for ULXs. Indeed, several authors have studied the dynamics of stalled mergers and their viability as ULX sources (Islam et al. 2003, 2004a, 2004b, 2004c; Volonteri & Perna 2005). Given the rarity of very massive galaxies, the probability of a merger with a comparable-mass galaxy is small. Therefore, very massive galaxies will often retain some of the black holes from merged satellite galaxies, since they will be unable to reach parsec-scale separations from the host black hole through dynamical friction in less than a Hubble time. For an extremely massive system such as M87 with $M_\bullet^h = 6.6 \times 10^9 \, M_\odot$, even very massive satellites with $M_\bullet^s \lesssim 5 \times 10^9 \, M_\odot$ would stall according to Equation (12), so essentially any merger would result in a stalled satellite. We
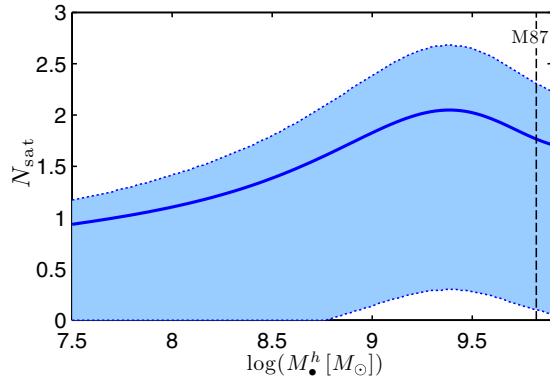
Figure 6. Expected number of satellite black holes with masses above $10^6 M_\odot$, $N_{\rm sat}$, for a given host black hole mass $M_\bullet^h$ is shown with a solid (blue) line. The central three quintiles of the probability distribution for $N_{\rm sat}$ are shaded and bound by dotted (blue) lines. The dashed (black) line shows the mass of M87 for reference.

(A color version of this figure is available in the online journal.)

can calculate the expected number of satellites for a given host mass, $\langle N_{\rm sat}(M_\bullet^h)\rangle \equiv N_{\rm sat,\,total}(M_\bullet^h)/N_{\rm host}(M_\bullet^h)$, where

$$N_{\rm sat,\,total}(M_\bullet^h) = \int_0^1 dz\, P(z) \int_{10^6 M_\odot}^{M_\bullet^h} \phi\,(M_\bullet^h)\,\phi_{\rm low}(M')\,\Theta$$
$$\times\, [t_{\rm H} - t_{\rm DF}(M_\bullet^h, M', z)]dM', \qquad (23)$$

$\Theta(t)$ is 1 for $t > 0$ and 0 otherwise, and

$$N_{\rm host}(M_\bullet^h) = \int_0^1 dz\, \frac{dV_c/dz}{n(z)}\,\phi(M_\bullet^h). \qquad (24)$$

Since $N_{\rm sat}$ will be Poisson-distributed, calculating $\langle N_{\rm sat}(M_\bullet^h)\rangle$ allows us to calculate the full distribution of stalled satellites for a given host. We show the result in Figure 6, including the expectation value and the middle three quintiles of the distribution.

It is interesting to note that the mean number of satellites for a massive host galaxy is ~1–2, consistent with the observed population of ULXs. That these satellites would be found in the outer regions of the host galaxy, and would likely be accompanied by enhanced star formation due to the interaction of the host and satellite bulges, is also consistent with the observed characteristics of ULXs. It has been noted that multiple ULXs are preferentially found in actively merging galaxies, so identified through morphology and kinematics (see e.g., Kaaret & Feng 2009 and references therein). Because ULXs exceed the Eddington luminosity, L_E, for a 20 $M_\odot$ black hole, which is the largest mass expected through typical stellar evolutionary channels (Fryer & Kalogera 2001), intermediate-mass black holes have been proposed as ULX central engines (Colbert & Ptak 2002).

In the noteworthy case of M82, which is likely to have undergone a recent merger, it has been suggested that the ULX M82 X-1 may be a low-luminosity active galactic nuclei powered by an intermediate-mass black hole (IMBH) from the core of a dwarf galaxy that M82 cannibalized (King 2004). Alternatively, it has been suggested that ULXs are simply low mass X-ray binaries that are either accreting beyond the Eddington limit, or whose emission is being beamed. However, SMBH satellites, spiraling toward the host core and accreting at the Bondi–Hoyle–Lyttleton rate, would naturally explain
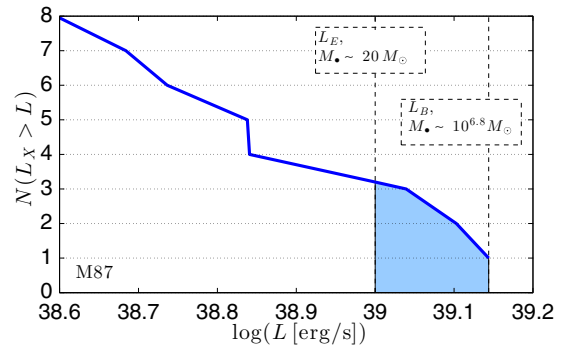

Figure 7. Luminosity distribution for M87 (Jordán et al. 2004) is shown with a solid (blue) line. The Eddington luminosity, L_E, of a 20 $M_\odot$ black hole, and the Bondi–Hoyle–Lyttleton luminosity, L_B, of a $10^{6.8} M_\odot$ black hole are given by the dashed (black) lines. The number of ultraluminous X-ray sources is consistent with the number of SMBH satellites expected in M87, as shown in Figure 6.

(A color version of this figure is available in the online journal.)

the luminosity of ULXs without resorting to super-Eddington accretion, or the as-yet-uncertain existence of IMBHs. This possibility has not been widely considered, given the common assumption that sufficiently massive satellites would sink to the host core in less than a Hubble time. However, as we have shown, this is far from the case in general, and especially so for BCGs. The luminosity of a satellite black hole is given by

$$L_B = \eta \dot{M}_B c^2 = 4\pi\eta\left(cGM_\bullet^s\right)^2 \rho_\infty v^{-3} = 10^{39}\,{\rm erg\ s}^{-1}\left(\frac{\eta}{0.01}\right)$$
$$\times \left(\frac{M_\bullet^s}{4\times 10^6 M_\odot}\right)^2 \left(\frac{n}{1\,{\rm cm}^{-3}}\right)\left(\frac{v}{190\,{\rm km\ s}^{-1}}\right)^{-3}, \quad (25)$$

where η is the efficiency of converting accreted gas into luminosity, $\dot{M}_B$ is the Bondi–Hoyle–Lyttleton accretion rate, ρ_∞ and n are the background gas and particle number density, respectively, and $v \equiv \sqrt{v_\bullet^2 + \sigma^2 + c_s^2}$ is the total relative velocity of the satellite and the background gas, with $v_\bullet$ the velocity of the satellite with respect to the mean gas flow, σ the local velocity dispersion, and c_s the local sound speed. We have normalized the mass so that L_B matches the bolometric Eddington luminosity of a 20 $M_\odot$ black hole.

In Figure 7, we show the tail of the luminosity function for M87, the most massive nearby example of a BCG. We follow the convention of assuming that an X-ray luminosity of 10^{39} erg s^{-1} for a 20 $M_\odot$ black hole implies a total bolometric luminosity beyond the Eddington limit. We note that, under this assumption, M87 contains ~2 ULXs, whose luminosity could be explained by the presence of an SMBH satellite, although their luminosities are less extreme than M82 X-1, and could also be explained by an unusually massive stellar-mass black hole or by marginally super-Eddington accretion. In the case of M82 X-1, the observed X-ray luminosity of ~10^{41} erg s^{-1} could be explained by the presence of a Sagittarius A*-sized satellite black hole spiraling through M82 under dynamical friction. Any ULX could also potentially be explained by the aforementioned models of a super-Eddington or beamed emission from a stellar-mass black hole, or an IMBH accreting near the Eddington limit. There is as-yet no unambiguous way to distinguish these scenarios from our supermassive satellite scenario; for example, since the emission mechanism is unclear, we cannot link variability to mass as conclusively as is done with active galactic nuclei. Furthermore, although quasi-periodic oscillations are observed in some ULXs, they are not a clear indicator of a binary

companion, and could arise generically in black hole accretion (see, e.g., Dolence et al. 2012). Our scenario has the advantage of corroborating observational evidence in the form of merging galaxies and enhanced star formation in galaxies hosting ULXs, as well as requiring only conventional astrophysics in the form of dynamical friction and Bondi–Hoyle–Lyttleton accretion to explain the observations. The abundance of ULXs may therefore be an additional indicator of the merger-driven nature of the evolution of the mass function for $z \leqslant 1$.

6. CONCLUSIONS

We have demonstrated that the mass function for massive galaxies since $z = 1$ is consistent with merger-driven evolution. Massive galaxies, though rarer than their smaller counterparts, are critical when considering overall luminosity, star formation rates, gravitational wave emission, etc. since the majority of total black hole mass is contained in individual black holes with masses above $\sim 5 \times 10^8 \, M_\odot$ at $z = 0$ (according to the mass function used in this work). Furthermore, despite the domination of the mass function by the number of smaller galaxies, our results indicate that the strain-weighted mean mass ratio between a satellite galaxy and its host is $\sim 1/2$. The novel expectation of at least one, and potentially several mergers with a comparable-mass companion for these massive galaxies (Trujillo et al. 2011; Hopkins et al. 2010) has dramatic implications for gravitational wave observations. We have shown that if this new paradigm of merger-driven evolution holds true, then the gravitational wave signal from the stochastic population of merging SMBH binaries is much stronger than previously expected, exceeding previous signal-to-noise estimates by a factor of ~ 2–5, depending on the choice of parameters in the initial mass function.[7] Indeed, our mean estimate of $h_o \geqslant 4.1 \times 10^{-15}$ is near the 95% confidence limit of the European PTA (6×10^{-15}) and the NANOGrav PTA (7×10^{-15}) and is larger than the most recent limit from the Parkes PTA (2.4×10^{-15}; Shannon et al. 2013), suggesting that PTAs may detect gravitational waves in the very near future, with the caveat that a dip in the signal at the lowest observable frequencies, as evidenced in many of our Monte Carlo realizations, would limit the sensitivity of PTAs, and also limit the rate at which the PTA constraint would improve if analyzed optimally.

In addition to an expected increase in black hole mergers, our assumption of merger-driven galaxy evolution also implies an increase in failed black hole mergers. For sufficiently massive host galaxies and/or sufficiently disparate mass ratios between the host and satellite, dynamical friction will fail to deliver the satellite to the core of the host. These stalled satellites would be observed as luminous X-ray sources. We find that, for the brightest cluster galaxy component of the mass function, 1–2 of these stalled satellites would be expected, with a large majority of hosts having 0–3 satellites. We have shown that this result is consistent with existing observations of ultraluminous X-ray sources, such as those found in M87 and, notably, M82 X-1, although the nature of these sources is not yet known conclusively and will require further study. Nonetheless, a validation of our model through the observation of gravitational waves with pulsar timing arrays in the immediate future would

also strongly suggest that at least some of these merging satellites would stall, and would therefore be observable as ULXs.

REFERENCES

Abell, G. O. 1965, ARA&A, 3, 1
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Bautz, L. P., & Morgan, W. W. 1970, ApJL, 162, L149
Bell, E. F., Phleps, S., Somerville, R. S., et al. 2006, ApJ, 652, 270
Bellovary, J. M., Governato, F., Quinn, T. R., et al. 2010, ApJL, 721, L148
Bennett, C. L., Halpern, M., Hinshaw, G., et al. 2003, ApJS, 148, 1
Bertone, S., De Lucia, G., & Thomas, P. A. 2007, MNRAS, 379, 1143
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press),
Bond, J. R., Cole, S., Efstathiou, G., & Kaiser, N. 1991, ApJ, 379, 440
Boylan-Kolchin, M., Ma, C.-P., & Quataert, E. 2008, MNRAS, 383, 93
Brown, M. J. I., Dey, A., Jannuzi, B. T., et al. 2007, ApJ, 654, 858
Chandrasekhar, S. 1943, ApJ, 97, 255
Colbert, E. J. M., & Ptak, A. 2002, ApJS, 143, 25
Cole, S., Norberg, P., Baugh, C. M., et al. 2001, MNRAS, 326, 255
de Ravel, L., Le Fèvre, O., Tresse, L., et al. 2009, A&A, 498, 379
Dolence, J. C., Gammie, C. F., Shiokawa, H., & Noble, S. C. 2012, ApJL, 746, L10
Drory, N., & Alvarez, M. 2008, ApJ, 680, 41
Eke, V. R., Cole, S., & Frenk, C. S. 1996, MNRAS, 282, 263
Folkner, W. M. 2011, A Non-drag-free Gravitational Wave Mission Architecture, http://pcos.gsfc.nasa.gov/studies/rfi/GWRFI-0003-Folkner.pdf
Fryer, C. L., & Kalogera, V. 2001, ApJ, 554, 548
Hausman, M. A., & Ostriker, J. P. 1978, ApJ, 224, 320
Hlavacek-Larrondo, J., Fabian, A. C., Edge, A. C., & Hogan, M. T. 2012, MNRAS, 424, 224
Hogg, D. W. 1999, arXiv:astro-ph/9905116v4
Hopkins, P. F., Croton, D., Bundy, K., et al. 2010, ApJ, 724, 915
Hopkins, P. F., Richards, G. T., & Hernquist, L. 2007, ApJ, 654, 731
Islam, R. R., Taylor, J. E., & Silk, J. 2003, MNRAS, 340, 647
Islam, R. R., Taylor, J. E., & Silk, J. 2004a, MNRAS, 354, 427
Islam, R. R., Taylor, J. E., & Silk, J. 2004b, MNRAS, 354, 443
Islam, R. R., Taylor, J. E., & Silk, J. 2004c, MNRAS, 354, 629
Jaffe, A. H., & Backer, D. C. 2003, ApJ, 583, 616
Jenet, F. A., Hobbs, G. B., Lee, K. J., & Manchester, R. N. 2005, ApJL, 625, L123
Jiang, C., Jing, Y., & Han, J. 2013, arXiv:1307.3322
Jordán, A., Côté, P., Ferrarese, L., et al. 2004, ApJ, 613, 279
Kaaret, P., & Feng, H. 2009, ApJ, 702, 1679
Kajisawa, M., Ichikawa, T., Tanaka, I., et al. 2009, ApJ, 702, 1393
King, A. R. 2004, MNRAS, 347, L18
Komatsu, E., Smith, K. M., Dunkley, J., et al. 2011, ApJS, 192, 18
Kormendy, J., & Bender, R. 2013, ApJL, 769, L5
Kormendy, J., & Ho, L. C. 2013, ARA&A, 51, 511
Kulier, A., Ostriker, J. P., Natarajan, P., Lackner, C. N., & Cen, R. 2013, arXiv:1307.3684
Laporte, C. F. P., White, S. D. M., Naab, T., & Gao, L. 2013, MNRAS, 435, 901
Larson, S. L. 2003, Sensitivity Curves for Spaceborne Gravitational Wave Observatories, http://www.srl.caltech.edu/~shane/sensitivity
Lin, Y.-T., Ostriker, J. P., & Miller, C. J. 2010, ApJ, 715, 1486
McConnell, N. J., & Ma, C.-P. 2013, ApJ, 764, 184
McWilliams, S. T., Ostriker, J. P., & Pretorius, F. 2012, arXiv:1211.4590
Merritt, D., & Milosavljević, M. 2005, LRR, 8, 8
Naab, T., Johansson, P. H., & Ostriker, J. P. 2009, ApJL, 699, L178
Natarajan, P. 2004, in Supermassive Black Holes in the Distant Universe, ed. A. J. Barger (Astrophysics and Space Science Library, Vol. 308; Dordrecht: Kluwer), 127
Nipoti, C., Treu, T., Auger, M. W., & Bolton, A. S. 2009, ApJL, 706, L86
Oser, L., Naab, T., Ostriker, J. P., & Johansson, P. H. 2012, ApJ, 744, 63
Oser, L., Ostriker, J. P., Naab, T., Johansson, P. H., & Burkert, A. 2010, ApJ, 725, 2312
Ostriker, J. P., & Hausman, M. A. 1977, ApJL, 217, L125
Phinney, E. S. 2001, arXiv:astro-ph/0108028
Quinlan, G. D. 1996, NewA, 1, 35

---

[7] We note that the most recent estimates from Sesana (2013) using observational data and from Kulier et al. (2013) using simulations are all consistent at the 2σ level with our estimate. Given the likelihood that systematic errors will be comparable in magnitude to the statistical errors for all of these approaches, we believe this level of statistical agreement indicates that the various approaches are all mutually consistent.

Robaina, A. R., Bell, E. F., van der Wel, A., et al. 2010, ApJ, 719, 844
Schechter, P. 1976, ApJ, 203, 297
Sesana, A. 2013, MNRAS, 433, L1
Sesana, A., & Vecchio, A. 2010, CQGra, 27, 084016
Sesana, A., Vecchio, A., & Colacino, C. N. 2008, MNRAS, 390, 192
Shannon, R. M., Ravi, V., Coles, W. A., et al. 2013, Sci, 342, 334
Springel, V., White, S. D. M., Jenkins, A., et al. 2005, Natur, 435, 629
Stebbins, R. T. 2011, A LISA-Like Concept for the Space-based Gravitational-wave Observatory (SGO), http://pcos.gsfc.nasa.gov/studies/rfi/GWRFI-0014-Stebbins.pdf
Sugiyama, N. 1995, ApJS, 100, 281
Thorne, K. S. 1987, in 300 Years of Gravitation, ed. S. Hawking & W. Israel (Cambridge: Cambridge Univ. Press), 392

Treister, E., Natarajan, P., Sanders, D. B., et al. 2010, Sci, 328, 600
Tremaine, S. D., Ostriker, J. P., & Spitzer, L., Jr. 1975, ApJ, 196, 407
Tremaine, S. D., & Richstone, D. O. 1977, ApJ, 212, 311
Trujillo, I., Ferreras, I., & de La Rosa, I. G. 2011, MNRAS, 415, 3903
van Dokkum, P. G., Whitaker, K. E., Brammer, G., et al. 2010, ApJ, 709, 1018
Volonteri, M., Haardt, F., & Madau, P. 2003, ApJ, 582, 559
Volonteri, M., & Natarajan, P. 2009, MNRAS, 400, 1911
Volonteri, M., & Perna, R. 2005, MNRAS, 358, 913
Yoo, J., Miralda-Escudé, J., Weinberg, D. H., Zheng, Z., & Morgan, C. W. 2007, ApJ, 667, 813
Yu, Q., & Tremaine, S. 2002, MNRAS, 335, 965
Zhang, X., Lu, Y., & Yu, Q. 2012, ApJ, 761, 5