

An Equilibrium Pricing for OTC Derivatives with Collateralization. Application of Economic Premium Principle

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Abstract

In this article, we propose an equilibrium pricing rule for the contingent claims by applying the economic premium principle initiated by Bühlmann (1980). The derivative markets in our model are over-the-counter (OTC) markets and have counterparty risks. We reconstruct the economic premium principle to explicitly handle the concrete form of the payoff function and the claim volume, and then we provide the equilibrium pricing rule for the OTC derivatives with the counterparty risks and the collateral agreements. We also demonstrate whether our pricing approach is consistent with an another equilibrium pricing rule in the point of the sensitivity of derivative prices.

JEL Classification: G10, G12, G13

Keywords: OTC derivative markets, counterparty risk, collateral, economic premium principle

1. Introduction

In this article, we consider the OTC derivative pricing model with a collateral agreement. We especially provide a pricing rule by applying the economic premium principle. The economic premium principle was initiated by Bühlmann (1980) and some researchers have discussed this principle recently (Iwaki et al. 2001, Iwaki 2002, Karatzas and Shreve 1998 and Kijima et al. 2010). The concept of this pricing method is to determine the pricing kernel or state price density through the market equilibriums.

After the financial crisis in 2008, the counterparty risk has been in focus for many practitioners and researchers (Acharya and Bisin 2011, Duffie and Zhu 2011, Fujii and Takahashi 2013, Gregory 2010 and Takino 2013a). The collateralization is one of the methods used to reduce such a risk as used in the money market. Recently, G20 in 2013 decided to make collateralization obligatory in the OTC swap market. The derivative pricing models with the collateralization have been studied

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by some researchers. Johannes and Sundaresan (2007) examined the effects of the collateralization on the swap rate. They argued and showed that the collateralization increases the swap rate through the collateral cost. Fujii and Takahashi (2013) consider more practical model including the asymmetric and imperfect collateral agreement, and they showed that the derivative price is affected by the adjustment of the collateral cost arising from the imperfect collateral agreement. These pricing approaches are provided under the risk-neutral pricing method.

An equilibrium pricing approach has been recently used to price the derivatives with collateral. Takino (2015b) derived the equilibrium pricing rule from the demand and supply function for the derivative and examined the effects of the collateralization on the derivative transactions. He showed that the effect of the collateralization on the option price and the swap rate are monotone and collateral amount increases the option price. He also argued that the impact on the swap rate is not significant rather than the option case because the counterparty risk bilaterally arises in the swap contract. The equilibrium rule in Takino (2015b) is determined by maximizing the investor's expected utility for her/his wealth, and then the collateralization affects the price through the demand and supply function influenced by the collateralization if the wealth is reflecting the collateral amount.

In contrast, there is the equilibrium pricing method without the demand and supply function in explicit. This approach was the so-called economic premium principle proposed by Bühlmann (1980), where the pricing kernel or state price density is determined from the market equilibrium. The method given by Bühlmann was extended to a multiperiod model by Iwaki et al. (2001) and the pricing approach, where the pricing kernel is derived from the utility maximization for the consumption (Iwaki 2002, Karatzas and Shreve 1998). Kijima et al. (2010) further applied this approach to evaluate the emission credit in the point of general equilibrium. The economic premium principle provides the linear pricing method like an arbitrage pricing theory. In this work, we consider the utility maximization problem for the wealth and the problem that explicitly treats the volume of the claims. We then provide the equilibrium pricing rule by determining the pricing kernel under the market equilibrium. This formation enables us to consider various derivative payoff formations, then we can construct the pricing rule that takes into account the counterparty risks and the collateralizations. Of course, our formula is able to accommodate the incomplete market models. In this study, we assume that the collateral amount is not accounted into the participant's wealth to utilize the economic premium principle and to eliminate possibility of default for the delivery of the collateral. The agent can receive the collateral if the counterparty defaults. These settings enable us to identify the pricing kernel. Our pricing approach is the same as those provided in previous researches (Fujii and Takahashi 2013, Johannes and Sundaresan 2007) except that the change of measure (state price density) is given by the equilibrium criterion. Thus, our

study gives an another equilibrium pricing rule different from Takino (2015b), and verifies how the effects of the collateralization on the derivative price depend on the pricing rule included in the setting of the collateralization.

At this point, we have an interest in whether our equilibrium pricing rule is consistent with another equilibrium methods. Are the characteristics of the pricing rule given by former studies maintained in our approach? So, we examine the sensitivity analysis for both pricing approaches i.e., our formula and the pricing rule provided by Takino (2015b). As a result, we show that the effects of collateralization on the option price and the swap rate are almost the same as those demonstrated by Takino (2015b).

The remainder of the article is organized as follows: In the next section, we set the financial market model with the collateral agreement. In Section 3, we derive the equilibrium prices for derivatives after determining the pricing kernel. In Section 4, we examine the sensitivity analysis of the derivative prices with respect to the collateral amount. Section 5 summarizes this work.

2 Model and Collateralization

2.1 Financial Markets with Counterparty Risk

There are J market participants in our financial market, and we denote the set of market participants by \mathcal{J} , i.e., $\mathcal{J} = \{1, 2, \dots, J\}$. They, respectively, invest their money in the portfolio consisted of the risk free asset and the risky business and also trade the derivatives. The motivation to enter the derivative contract is to hedge or eliminate the business risk as considered in Kijima et al. (2010), for instance. We denote by S^j the risky business value at time t ($0 \leq t \leq T$) invested from the participant $j \in \mathcal{J}$ where T denotes the maturity date of the derivatives introduced in the following. We assume that the risky business is traded in the large market and the agents can invest their money in the risky business at the unit price S^j at time t . Note that, because we examine the partial equilibrium for the derivative contracts in this study, we suppose that the participant j can trade S^j only for convenience. In order to extend the general equilibrium, the assumption is eased such that some or all market participants are able to invest their money into other businesses. The values of the risky businesses are correlated with a common asset price which is assumed to be *nontraded* in the market and the price process is denoted $\{Y_t\}_{0 \leq t \leq T}$. This assumption is one of the incomplete market models and the asset Y corresponds to the price indices of the stock markets, the weather or energy indices (e.g., Bessembinder and Lemmon (2002), Cao and Wei (2004), Kijima et al., (2010), Lee and Oren (2009), Yamada (2007)) and so on. The market participants are also supposed to trade the European-type derivatives written on Y and its payoff function at maturity T is defined as follows:

$$H(T) := H(T, Y_T).$$

We consider the call option and the swap contract and assume that those contracts are entered at time 0. We suppose that they behave as the *price taker* in the financial markets that included the derivative market like risky business markets. The remaining money is deposited into the bank account with the interest rate r (i.e., risk-free asset). The value of the bank account at time t is $B_t = e^{rt}$ with $B_0 = 1$.

The counterparty risk in the derivative contract is the possibility that the participants fail to provide full payout of $H(T)$. We assume that the default event and payment depend on the counterparty's business value at maturity as modeled by Henderson and Liang (2014). They modeled the counterparty risk with a so-called constructed form model as examined by Merton (1974). That is, the default event of agent j by 1_{D_j} is represented by

$$1_{D_j} = 1_{S_T^j < L}$$

for a certain level L . The payment received by agent $j \in \mathcal{J}$ is represented by $\eta_i(S_T^i)H(T)$ ($i \neq j$) when the counterparty $i \in \mathcal{J}$ defaults, where $\eta_i(\cdot)$ is the recovery function for the participant i 's default. At this point, for the option contract, we suppose that the buyer of the option does not fail to pay the option fee when the contract is entered. Furthermore, there are possibilities for both counterparties to fail to pay for the swap contract. We express the long holder and the short holder of the derivatives by $j = l$ and $j = s$, respectively.

2.2 Collateral Agreement

To hedge the loss due to the counterparty risk, the agent who has a positive exposure could receive the cash collateral from the counterparty with a negative exposure. We assume that the positive or negative exposure is determined at the marked-to-market (MtM) date, and the MtM is priced through the pricing rule, which is independent of the agent's risk preference. We denote the value of the MtM at time t by V_t . If the MtM value of the derivative contract held by one market participant is positive, she/he could receive the collateral with the counterparty's default. We also introduce the coverage ratio ϕ (≥ 0), and then the collateral amount is calculated by

$$C(\phi) = \phi V_t \tag{2.1}$$

where t means the MtM date. We finally suppose that the cash collateral is deposited into the account aside from the wealth accounts of the participants. This assumption implies that the collateralization does not affect the agent's wealth.

2.2.1 Option Payoff with Collateral Agreement

The buyer (or long holder) of the option always has positive exposure. So she/he is entitled to receive the collateral at maturity when the seller of the option defaults. We set the MtM date by $t = 0$,

which is the contract date of the option. The collateral amount is then

$$C(\phi) = \phi V_0.$$

Under these conditions, we provide the payoff $g_{opt}(T)$ of the option subject to the collateral agreement at maturity. We formulate the value of $g(T)$ from the point of the long holder, that is,

$$g_{opt}(T) = H(T)(1 - 1_{D_s}) + (\eta_s(S_T^s)H(T) + C(\phi))1_{D_s} \quad (2.2)$$

where 1_{D_s} denotes the default indicator function for the option seller. The first term is the payoff of the option without defaults. The second term corresponds to the default payment of the option. If the participants default, the long holders have the default payments of $\eta_s(S_T^s)H(T)$ and additionally obtain the collateral amounts. For the short holder, the formula is given by adding minus sign to g_{opt} .

2.2.2 Swap Payoff with Collateral Agreement

The counterparty risk arises from both sides in the swap contract unlike the option contract. The standard swap valuation determines the swap rate such that the present value of the contract equals to zero. This implies that the exposures of the derivative contract for both counterparties are vanished. As introduced in Johannes and Sundareshan (2003), we consider the two-period model.

We suppose that the MtM is done once for $(0, T)$ and the date of MtM is denoted by $t \in (0, T)$. Then, the collateral amount at the MtM date t is given by

$$C(\phi) = \phi V_t.$$

The payoff $g_{swp}(T)$ of this swap contract to the long holder is represented by

$$g_{swp}(T) = Y_T(1 - 1_{D_s}) + (\eta_s(S_T^s)Y_T + C(\phi))1_{D_s} - K(1 - 1_{D_l}) - (\eta_l(S_T^l)K + C(\phi))1_{D_l}. \quad (2.3)$$

We rewrite (2.3) as

$$g_{swp}(T) = g_Y(T) - K g_\perp(T)$$

where

$$g_Y(T) = Y_T\{1 - (1 - \eta(S_T^s))1_{D_s}\} + C(\phi)(1_{D_s} - 1_{D_l}), \quad (2.4)$$

$$g_\perp(T) = 1 - (1 - \eta_l(S_T^l))1_{D_l}.$$

The long holder (short holder) receives Y_T (K) if the seller does not default at maturity and obtain $\eta_s(S_T^s)Y_T + C(\phi)$ ($\eta_l(S_T^l)K + C(\phi)$) if the seller defaults. The payoff function for the short holder is given by adding minus sign to g_{swp} .

2.3 Participant's Total Wealth

We derive the equilibrium price by solving the utility maximization problem for the terminal wealth which is constructed with the portfolio and the derivative positions. To this end, we set the wealth equation for the market participant.

Agent $j \in \mathcal{J}$ has the initial wealth x_0^j and first allocates it to the risky business and the derivative

contract. The rest of the money is deposited in the bank account with a constant interest rate r . The money amount invested in the risky business by agent $j \in \mathcal{J}$ is denoted by $\pi_j := \pi_j(0)$. We assume that the agents do not change the position at all for $(0, T]$. The volume or the position of the claim which the participant $j \in \mathcal{J}$ is willing to trade, is denoted by $\delta_j k_j$ ($k_j \geq 0$). Where $\delta_j = 1$ corresponds the case of the participant $j = l$, and $\delta_j = -1$ relates the case of $j = s$. Recall, l and s mean the long holder and the short holder, respectively. The unit price of the claim g is given by the formula

$$E[\mathcal{E}(T)g(T)] \quad (2.5)$$

where $\mathcal{E}(T)$ is a pricing kernel or state price density at time T . We determine \mathcal{E} through the market equilibrium.

The money w_0^j deposited into the risk-free asset for the participant $j \in \mathcal{J}$ at time 0 is

$$w_0^j = x_0^j - \pi_j - \delta_j k_j E[\mathcal{E}(T)g(T)].$$

And the terminal wealth is given by

$$X_j(T) = w_0^j B_T + \frac{\pi_j}{S_0^j} S_T^j + \delta_j k_j g(T) = (x_0^j - \pi_j - \delta_j k_j E[\mathcal{E}(T)g(T)])B_T + \frac{\pi_j}{S_0^j} S_T^j + \delta_j k_j g(T) \quad (2.6)$$

for the claim g .

3 Equilibrium Price

In this section, we provide the pricing formula based on the economic premium principle (Bühlmann 1980). The pricing formula is given by (2.5), which is sufficient to determine the pricing kernel \mathcal{E} .

We suppose that the preference of the market participant $j \in \mathcal{J}$ is represented by the exponential utility function with the risk-averse coefficient γ_j , that is

$$U_j(x) = -\frac{1}{\gamma_j} e^{-\gamma_j x}.$$

We denote the inverse function of U_j by I_j , that is,

$$I_j(x) = (U_j')^{-1}(x).$$

Agent $j \in \mathcal{J}$ maximizes her/his expected utility from the terminal wealth with respect to the claim volume. The objective for the participant j is then given by

$$E[U_j(X_j(T))] \longrightarrow \text{maximize w.r.t. } k_j$$

where X_j is given in (2.6).

In order to derive the market equilibrium price, we need the clearing condition.

Definition 3.1. *The market equilibrium is represented by the following conditions:*

1. $\sum_{j=1}^J (x_0^j - \pi_j) = R_0$
2. $\sum_{j=1}^J \frac{\pi_j}{S_0^j} S_T^j = R_T$

$$3. \sum_{j=1}^J \delta_j k_j = 0 \quad (\text{market clearing condition of the derivatives})$$

where $k_j \geq 0$ for all j .

Under Definition 3.1, we provide the pricing kernel.

Theorem 3.1. *We suppose that our market satisfies the above assumptions and Definition 3.1. The terminal wealth of the participant j is given by (2.6). Under the equilibrium, the pricing kernel \mathcal{E} is then given by*

$$\mathcal{E}(T) = \frac{e^{-\gamma R_T}}{B_T E[e^{-\gamma R_T}]} \quad (3.1)$$

$$\text{where } \frac{1}{\gamma} = \sum_{j=1}^J \frac{1}{\gamma_j}.$$

Proof. The first-order condition of the utility maximization is

$$E[U'_j(X_j(T))g(T)] = E[U'_j(X_j(T))]E[\mathcal{E}(T)g(T)].$$

From this equation, we have

$$\mathcal{E}(T) = \frac{U'_j(X_j(T))}{E[U'_j(X_j(T))]} =: \frac{U'_j(X_j(T))}{M_j}$$

where M_j is a constant. Thus it holds

$$(x_0^j - \pi_j - \delta_j k E[\mathcal{E}(T)g(T)])B_T + \frac{\pi_j}{S_0^j} S_T^j + \delta_j k_j g(T) = I_j(M_j \mathcal{E}(T)) \quad (3.2)$$

for $j \in \mathcal{J}$.

From Definition 3.1, under the market equilibrium, summing up (3.2) for all j leads

$$R_0 B_T + R_T = \sum_{j=1}^J I_j(M_j \mathcal{E}(T)). \quad (3.3)$$

For the exponential utility case defined above, the inverse function I_j is

$$I_j(x) = -\frac{1}{\gamma_j} \ln x.$$

(3.3) is then rewritten as

$$\frac{1}{\gamma} \ln \mathcal{E}(T) = \bar{M} - R_T \quad (3.4)$$

where $\frac{1}{\gamma} = \sum_{j=1}^J \frac{1}{\gamma_j}$ and \bar{M} are constants. So we have

$$\mathcal{E}(T) = e^{\gamma(\bar{M} - R_T)}. \quad (3.5)$$

Taking expectation both sides of (3.5) gives

$$E[\mathcal{E}(T)] = e^{\gamma \bar{M}} E[e^{-\gamma R_T}].$$

Since $E[\mathcal{E}(T)] = B_T^{-1}$, the constant $\frac{1}{\gamma} \ln \mathcal{E}(T) = \bar{M} - R_T$ is given by

$$\bar{M} = \frac{1}{\gamma} \ln \left(\frac{1}{B_T E[e^{-\gamma R_T}]} \right).$$

Substituting this into (3.5) completes proof.

Theorem 3.1 with (2.5) immediately leads the equilibrium option price and swap rate respectively.

Corollary 3.1. We suppose that there are one long holder and one short holder for the derivatives in our market, i.e., $\mathcal{J} = \{l, s\}$. The equilibrium option price $p(\phi)$ with the coverage ratio ϕ is represented by

$$p(\phi) = E[\mathcal{E}(T)H(T)\{1 - (1 - \eta_s(S_T^s))1_{D_s}\}] + \phi V_0 E[\mathcal{E}(T)1_{D_s}]. \quad (3.6)$$

The equilibrium swap rate $K(\phi)$ with the coverage ratio ϕ is given by

$$K(\phi) = \frac{E[\mathcal{E}(T)(Y_T\{1 - (1 - \eta(S_T^s))1_{D_s}\}] + \phi E[\mathcal{E}(T)V_t(1_{D_s} - 1_{D_l})]]}{E[\mathcal{E}(T)\{1 - (1 - \eta_l(S_T^l))1_{D_l}\}]} \quad (3.7)$$

4 Sensitivity Analysis

In this section, we examine the effects of the collateralization on the derivative prices through the sensitivity analysis.

4.1 Option Case

We first consider the effect of the counterparty risk on the option price without the collateral. Substituting $\phi = 0$ into (3.6) yields the equilibrium option price $p(0)$ without the collateral

$$p(0) = E[\mathcal{E}(T)H(T)\{1 - (1 - \eta_s(S_T^s))1_{D_s}\}].$$

The option price p_{woc} without the counterparty risk is given by

$$p_{woc} = E[\mathcal{E}(T)H(T)].$$

From the fact that

$$1 - (1 - \eta_s(S_T^s))1_{D_s} \leq 1,$$

it holds

$$p_{woc} \geq p(0).$$

Therefore, the counterparty risk decreases the option price according to the recovery rate and the degree of the default risk.

Next we show the impact of the collateralization on the option price. From (3.6), we have

$$\frac{\partial p(\phi)}{\partial \phi} = V_0 E[\mathcal{E}(T)1_{D_s}] > 0. \quad (4.1)$$

(4.1) means that the option price increases with the increase of the coverage ratio. Thus, the collateralization increases the option price. From the above arguments, the effects of the collateralization on the option price are summarized as follows.

Proposition 4.1. *In the option contract, the collateralization affects the derivative price in the following ways:*

1. *The counterparty risk decreases the equilibrium option premium*
2. *The collateralization monotonically raises the option price*

4.2 Swap Case

From (3.7), we have

$$\frac{\partial K(\phi)}{\partial \phi} = \frac{E[\mathcal{E}(T)V_t(1_{D_s} - 1_{D_l})]}{E[\mathcal{E}(T)\{1 - (1 - \eta_t(S_T^l))1_{D_l}\}]} \quad (4.2)$$

The sign of (4.2) is dependent on the difference between $E[\mathcal{E}(T)V_t1_{D_s}]$ and $E[\mathcal{E}(T)V_t1_{D_l}]$ because the denominator of (4.2) is a positive. However, (4.2) does not depend on the coverage ratio. This implies that the effect of the collateralization on the swap rate is a monotone as demonstrated by Takino (2015b). We also have an interest in whether our pricing approach is consistent with another equilibrium pricing approaches. To check this, we consider the pricing formula of Takino (2015b) as an example and implement the price change. In the next section, we implement the signs for both formula under a certain model.

4.2.1 Numerical Result

We use the two-period multinomial tree model demonstrated by Takino (2015b), the model is based on Musiela and Zariphopoulou (2004).

Set two time periods $t = 0, \frac{1}{2}T, T$ and suppose that the economy varies four states in a time period. Time $t = \frac{1}{2}T$ is the MtM date. The finite probability space is defined by $(\Omega, \mathcal{F}, \mathbb{P})$ with $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and $\mathcal{F} = 2^\Omega$. The canonical probability measure of the state ω_i ($i = 1, 2, \dots, 4$) is assumed

$$P_i := \mathbb{P}(\omega_i).$$

The variations of prices S^j ($j = l, s$) and Y are modeled by a multinomial tree model. The variation of S^j ($j = l, s$) for a period is given by

$$\Delta S^l(\omega_i) = \frac{S_{t+\Delta t}^l}{S_t^l} = \begin{cases} u_l, & i = 1, 2, \\ d_l, & i = 3, 4, \end{cases} \quad \Delta S^s(\omega_i) = \frac{S_{t+\Delta t}^s}{S_t^s} = \begin{cases} u_s, & i = 1, 4, \\ d_s, & i = 2, 3, \end{cases}$$

where $\Delta t = \frac{1}{2}T$ and

$$u_j = e^{\sigma_j \sqrt{\Delta t}}, \quad d_j = e^{-\sigma_j \sqrt{\Delta t}}$$

for $j = l, s$. And, the variation of Y is given by

$$\Delta Y(\omega_i) = \frac{Y_{t+\Delta t}^l}{Y_t^l} = \begin{cases} u_Y, & i = 1, 3, \\ d_Y, & i = 2, 4 \end{cases}$$

where

$$u_Y = e^{\sigma_Y \sqrt{\Delta t}}, \quad d_Y = e^{-\sigma_Y \sqrt{\Delta t}}.$$

Next, we set the default events and the recovery rate. We use the constructed model to model the default event, that is, there exists threshold $L_j (> 0)$ ($j = l, s$), the agent fails to provide full payment of the claim if the terminal business value S_T^j is less than L_j . Then, the default indicator function is represented by

$$1_{D_j} = 1_{S_T^j < L_j}$$

for $j = l, s$. We also assume $S_0^j d_j^2 < L_j$ ($j = l, s$). The recovery rate is defined by

$$\eta(S_T^j) = \eta_j \frac{S_T^j}{L_j}$$

where $\eta_j \in [0, 1]$ is a constant for $j = l, s$.

The MtM for the swap contract is priced by the arbitrage pricing method. The risk-neutral martingale measure \mathbb{Q} is obtained by ad-hoc way as follows: By solving

$$\begin{cases} \sum_{i=1}^4 Q_i Y_{\Delta t}(\omega_i) = B_{\Delta t} Y_0 \\ \sum_{i=1}^4 Q_i = 1 \end{cases}$$

where $Q_i := \mathbb{Q}(\omega_i)$ ($i = 1, 2, \dots, 4$), we obtain the marginal probability

$$Q := Q_1 + Q_3 = \frac{B_{\Delta t} - d_Y}{u_Y - d_Y}.$$

We use the parameters used in Takino (2015b), and these are as follows: $P_2 = 0.15$, $P_3 = 0.05$, $S_0^l = 100.0$, $\sigma_l = 0.1$, $\gamma_l = 0.0002$, $S_0^s = 100.0$, $\sigma_s = 0.4$, $\gamma_s = 0.0001$, $\pi_l = -154048.32$, $\pi_s = 26590.17$, $Y_0 = 100.0$, $\sigma_Y = 0.2$, $\eta_l = 0.5$, $L_l = 90.0$, $\eta_s = 0.5$, $L_s = 90.0$, $r = r_c = 0.05$ and $T = 1.0$. We implement for various P_4 such that

$$P_1 = 1 - (P_2 + P_3 + P_4). \quad (4.3)$$

Under this parameter set, we implement the sensitivity to the swap rate of the collateral amount i.e., $\partial K(\phi)/\partial \phi$ for both pricing formulae. We calculate $\partial K(\phi)/\partial \phi$ for the pricing rule of (2015b) and $E[\mathcal{E}(T)V_t(1_{D_s} - 1_{D_l})]$ for our work. Note that, it is sufficient to check $E[e^{-\gamma R_T} V_t(1_{D_s} - 1_{D_l})]$ only since $E[e^{-\gamma R_T}]$ is a constant. The result is presented in Table 1. The column of Takino (2015b) expresses the value of $\partial K^*/\partial \phi$ for the formula given in Takino (2015b). The column of EPP presents the value of $E[e^{-\gamma R_T} V_t(1_{D_s} - 1_{D_l})]$. From the table, we observe that signs of both models almost are equal. Therefore, there exists the case that the effects of collateralization on the swap rate are the same as the another equilibrium pricing approach.

P_4	Takino (2015b)	EPP
0.00	0.6324	0.0000
0.05	0.2459	0.0001
0.10	-0.0182	-0.0002
0.15	-0.2456	-0.0097
0.20	-0.4524	-0.0524
0.25	-0.6429	-0.1686
0.30	-0.8179	-0.4222
0.35	-0.9770	-0.9205
0.40	-1.1190	-1.8605
0.45	-1.2423	-3.6364
0.50	-1.3451	-7.1177
0.55	-1.4252	-14.4509
0.60	-1.4801	-31.7709
0.65	-1.5071	-80.4210
0.70	-1.5024	-258.5313
0.75	-1.4605	-1243.5795
0.80	-1.3715	-9541.5459

Table 1: Sensitivity of swap rate for change in coverage ratio ϕ . The column of Takino (2015b) expresses the value of $\partial K^*/\partial\phi$ for the formula given in Takino (2015b). The column of EPP presents the value of $E[e^{-\gamma R_T} V_t(1_{D_s} - 1_{D_l})]$.

5 Summary

In this article, we have constructed an equilibrium pricing method for the pricing of the OTC derivatives with the collateralizations, and then have investigated the effects of collateralization on the derivative prices through examining the sensitivity analyses of derivative prices to the collateral amount. We especially examined whether the sensitivity results for our pricing rule have the same as those provided by the previous research. While the equilibrium pricing rule based on the demand and supply function has been used by some researchers, our study constructed an equilibrium pricing criterion based on the economic premium principle and provided the pricing rule with the pricing kernel. For the option contract case, we showed that the collateral amount monotonically increases the option price as shown in the previous research. For the swap contract case, our numerical results demonstrated that the sensitivities of the swap rate to the collateral amount under various parameters almost equal to those for previous approach. Therefore, our research showed that we are able to use different equilibrium pricing approaches to investigate the effect of collateralization on the derivative prices.

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