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Composite Input-Output Production Functions

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An algorithm to linear combination of subsector cost shares

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Abstract

Abstract. This document describes the algorithm used for creating an aggregated linear production function for an industry by weighting subsector production functions. The result can be used as a column in an interindustry ($I \times I$) coefficients table or in a standard Use table ($C \times I$) depending on the units (C or I) of the input data.

Introduction

Each power generating technology $k \in K$ has a corresponding production function. When the production function is assumed to be linear, each technology's production function corresponds to a set of cost shares whose sum over all inputs is 1. Define

$$A_j^k = \begin{bmatrix} a_{1j}^k \\ a_{2j}^k \\ \vdots \\ a_{n-1j}^k \\ a_{nj}^k \end{bmatrix} \tag{1}$$

be the cost shares for technology subsector k in power generation industry j. Then let Z_j^k be the contribution of subsector k to industry j output. Z

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can be expressed in dollar terms or in proportionate weights. The industry j composite cost shares can be computed as

$$A_j = \frac{\sum_{k=1}^{K} A_j^k Z_j^k}{\sum_{k=1}^{K} Z_j^k} \tag{2}$$

In matrix notation, A is a normalized cost or cost share matrix with N industries and K technologies, z is a K dimensional vector of the weights of the respective sectors in the composite sector, x is the sum of the weights, and i is an appropriately dimensioned summing vector. Then the compositing function is

$$\frac{1}{x_i} A \hat{z}_i \tag{3}$$

Supporting Algorithm(s)/Code

techagg.m

```
function [t] = techagg(A,z)
% PURPOSE: create an aggregated input-output column from subsectors,
% given subsector coefficient matrix and weights vector
% ---------------------------------------------------------
% USAGE: t = techagg(A,z)
%         where A is an nxk matrix of coefficient cost shares
%           and z is a k dimensional vector of weights, either shares or
%           levels
% INPUT:
% -> A is an nxk matrix of coefficient cost shares
% -> z is a k dimensional vector of weights, either shares or levels
% OUTPUT:
% -> t is an n dimensional vector of aggregate cost shares
% ---------------------------------------------------------
% REFERENCES: None
% ---------------------------------------------------------
% Written by: Randy Jackson, 08/07/2013
% Current e-mail: info@econalyze.com
wtsum=sum(sum(z)');
t=(A*diag(z)/wtsum)*ones(length(z),1);
% ---------------------------------------------------------
```