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IN MEMORY OF ARKADY VIKTOROVICH KRYAZHIMSKIY (1949–2014)

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Abstract: The article is devoted to the description of Academician Arkady Kryazhimskiy's life path. The facts of the scientific biography of Acad. Kryazhimskiy are presented with the emphasis on his outstanding contribution into the theory of dynamic inversion, the theory of differential games, and control theory. His personal talents in different spheres are also marked out.

Key words: Arkady Viktorovich Kryazhimskiy.

Arkady Viktorovich Kryazhimskiy was born on January 2, 1949, in Qingdao, China. In 1971 he graduated from the Department of Mathematics and Mechanics of Gor'kii Ural State University in Sverdlovsk (now, Ekaterinburg) and entered a postgraduate program under the supervision of Yurii Sergeevich Osipov. In 1971 Osipov completed the development of the foundations of positional game theory for control systems with delayed argument and suggested Kryazhimskiy to study an pursuit–evasion differential game for a target set given in an infinite-dimensional phase space of a delay system. That was the time when convex analysis in Hilbert spaces, a division of functional analysis, was actively developed. Kryazhimskiy used technique from this research area to design solution methods for the described problem. He carried out a comprehensive study of an pursuit–evasion game with a functional target. Based on the results of these studies, Kryazhimskiy defended his candidate's dissertation "Some Game Problems of Pursuit–Evasion" in 1974.

In July 1972, the Laboratory (later, Department) of Differential Equations, headed by Osipov, was created at the Institute of Mathematics and Mechanics of the Ural Scientific Center of the Russian Academy of Sciences (in Sverdlovsk). Kryazhimskiy worked at this department from its creation till the beginning of the 1990s. In the 1970s, after defending his candidate's dissertation, he abandoned his work on differential games for delayed systems and turned to studying differential games for "ordinary" systems with incomplete information as well as infinite-dimensional control systems. Numerous workshops were held on these topics at the Laboratory of Differential Equations. One of difficult important problems was to extend the basic principles of the theory of positional differential games to "ordinary" systems whose right-hand sides did not satisfy the

Lipschitz condition in the phase variable. Working on this problem, Kryazhimskiy designed a "universal" implementation of the extremal shift principle, which was independent of the specifics of a control system's phase space. The implementation was based on an interesting idea: if a control system is looked upon as a "control–trajectory" transformation, then the extremal shift rule can be specified in the space of "inputs," i.e., controls, rather than in the space of "outputs," i.e., trajectories. This idea showed a way that later led Kryazhimskiy to the following solution of a differential game for non-Lipschitz "ordinary" systems: passage to the infinite-dimensional functional control space, search for an adequate criterion for the deviation between the "true" and "target" controls, and implementation of extremal shift in terms of this criterion. These results served as a base for Kryazhimskiy's doctoral dissertation "Differential Games for Non-Lipschitz Systems" (1981).

By the beginning of the 1980s, the development of fundamental issues of the theory of positional differential games related to finding general solvability criteria of game problems and describing the general structure of their solutions has been mostly completed. Scientists at the Institute of Mathematics and Mechanics who worked in the area of differential games had to decide on the directions on further studies. One of the possible directions was the development of the theory of positional differential games "into the depth" by designing new solution methods for differential game problems; another, the search for new problems and the development of new theoretical approaches. Kryazhimskiy and his colleagues from the department chose the new research area. They intended to search for topical problems at the interface of subject areas. At that time, along with the studies on control theory and differential games, other research directions were developed successfully at the institute; one of them was concerned with the theory of ill-posed problems. Despite the remoteness of this theory from differential games, specialists within game theory were familiar with the idea of regularization, which played an important role in the theory of ill-posed problems. In particular, the widely known method of positional control with a guide proposed by Nikolai Nikolaevich Krasovskii in the first half of the 1970s was based on the effect of regularization, i.e., elimination of instability in the presence of small information noise. Kryazhimskiy and Osipov set a goal to find an application direction for methods of the theory of differential games in the area of ill-posed problems. Finding a specific direction of applications was an extremely difficult task, since it required problems of a principally new class. In the theory of ill-posed problems, the so-called inverse problems of control systems are closest to objects studied in control theory. A typical inverse problem consists in finding a control implementing a specified trajectory of a system or a given signal from a trajectory. Similar problems in the presence of trajectory perturbations are close to the process of observing a real trajectory of a system generated by an unknown control, which in this case loses the traditional meaning of "control," i.e., rational influence aimed at the optimization of motion, since the control is replaced by an unobservable and uncontrolled "input" fed to the system from the environment. According to the ideology of the theory of ill-posed problems, the unobservable input is to be recovered, and the recovery error must be arbitrarily small for sufficiently small observation error. Since a direct observation of perturbing inputs, as a rule, was not possible, a new problem of dynamic inversion arose, which consisted in the real-time recovery of current values of unobservable inputs from an available signal about the trajectory. Later, the problem of dynamic inversion became an "inversion block" in the general "inversion–control" scheme, in which, in the process of operation of a control system exposed to the action of unobservable inputs, current values of inputs are recovered approximately in real time from current, generally speaking, inaccurate observations of states of the system (the "inversion block"); these values, together with the results of direct observations of the system's states are fed to an automatic regulator, which produces current values of the control parameter (the "control block").

In the problem of dynamic inversion, an important requirement on the solution algorithm is its dynamic property, i.e., the real–time mode of operation. With reference to the theory of ill-posed problems, this requirement restricts the class of admissible regularizing algorithms, and the problem of dynamic inversion can be referred to as a dynamic regularization problem. Kryazhimskiy and Osipov proposed a new methodological approach to dynamic regularization, which became known as the principle of regularized extremal shift. It is based on the procedure of control with a guide from the theory of positional differential games and consists in the following. The process of dynamic recovery of an unobservable input is interpreted as the process of control of an auxiliary dynamic system (model). The model, which is often a copy of the original system, is essentially different from the latter in that it is controllable: the uncontrolled input is replaced by a control parameter. Current values of the model control are formed in real time by means of the feedback principle, as a reaction to the "real" (inaccurate) information on the current state of the original system and to the accurate information on the current state of the model. The feedback in the control loop of the model is chosen so that the implementation of the model control as a function of time track accurately enough the implementation of the input of the original system.

The described scheme was developed initially for "ordinary" finite-dimensional systems affine in the input variable. For such systems, the choice of a model feedback guaranteeing the proximity of trajectories of the model and of the system was not difficult: it was sufficient to use the standard rule of extremal shift of the model's current state toward the current signal about the system's state. The crucial step, which shaped the further development of the approach, was the understanding that an appropriate regularization of the extremal shift rule provides the required much stronger property—the proximity of the model control to the input of the system in the mean-square metric. The proposed regularization involves the combination of the basic criterion—the extremal shift—with an auxiliary criterion—the minimum criterion for the norm of the current value of the control parameter. The basic version of the method includes the auxiliary criterion by adding to the main, linear, shift criterion a quadratic smoothing function multiplied by a small regularization parameter. The regularized extremal shift, which consists in the minimization of the resulting criterion in the control variable, corresponds exactly to the application of Tikhonov's regularization method to the extremal shift method. Thus, at the interface of the theory of ill-posed problems and the theory of positional control, a new range of problems was found—dynamic regularization problems—and an approach to their solution was proposed—the method of regularized extremal shift.

The studies on the dynamic inversion of "ordinary" finite-dimensional systems carried out by Kryazhimskiy mainly in the 1980s were summarized in the famous monograph, which presents to the reader a deep theory covering a wide range of issues, from the formulation of dynamic inversion problems, investigation of their solvability, and comparison of the possibilities of dynamic and a posteriori methods to the construction of optimal algorithms and detailed implementation of the "inversion–control" scheme, which played an important motivating role at the initial stage of research. The theory is based on the methods of regularized extremal shift, which combine, as mentioned above, approaches from the theory of positional differential games and the theory of ill-posed problems. Certain divisions of the theory involve methods of the theory of differential equations, control theory (in particular, the techniques of generalized controls), estimation theory, functional analysis, convex analysis, and function approximation theory. Explicit descriptions of algorithms of inversion and inversion–control, which are ready for immediate application and are accompanied by accuracy estimates, are combined with the study of delicate theoretical issues, such as regularizability, order optimality, asymptotic optimality, etc.

In the process of creating the theory of dynamic inversion, its authors developed a new approach to the investigation of some divisions of the theory of solution of operator equations, function approximation theory, etc. One of the strongest developments concerned the application of the dynamic inversion ideology to problems of the classical infinite-dimensional optimization. Studies in this direction began in the second half of the 1980s, when a new iterative algorithm for solving a linear–convex problem of optimal control under phase constraints was proposed. The algorithm was based on the principle of regularized extremal shift applied to an artificially designed dynamic model with discrete time. An original optimal control problem is interpreted as an optimization problem in the space of artificially created processes under constraints of the type of equality (provided by the equation of the system) and inclusion (provided by the original phase constraints and the original constraints on the control). A special Lyapunov functional related to the Tikhonov regularization method was introduced and stabilized by means of extremal shift; as a result, the model's states converge to the required solution. In further studies, similar algorithmic schemes, based on regularization ideas, were refined and extended to various classes of extremal problems. This series of papers was mainly concerned with the advance into the area of methods of global nonconvex optimization. One of the papers was devoted to studying a wide class of nonconvex optimization problems with constraints; for these problems, the regularized extremal shift principle produces a converging iterative solution algorithm. Problems of this class are characterized by a geometric condition on the separability of the graph of the "perturbed optimal value" function.

In the middle of the 1980s, Kryazhimskiy started investigations related to defense projects. Until the collapse of the Soviet Union in 1991, the researchers of the Department of Differential Equations of the Institute of Mathematics and Mechanics who worked in the sector headed by Kryazhimskiy took part in joint studies with their colleagues from NPO Energiya (Korolev, Moscow region) and NPO Avtomatika (Sverdlovsk). These studies were devoted to processes of interaction of dynamic systems under incomplete and varying information.

In the beginning of the 1990s, Kryazhimskiy moved to Laxenburg, Austria, where he started to work at the International Institute of Applied Systems Analysis. Until the end of 2012, he headed the Dynamic Systems Program (later integrated into the Advanced Systems Analysis Program). The systemic, comprehensive, approach to the solution of difficult interdisciplinary problems, which is a sort of trademark of the institute, was natural to Kryazhimskiy to the full extent. His role in the investigation of certain large-scale problems, such as various applied game problems, economic growth modeling, finding optimal ways of sustainable development on the global scale, modeling of innovation market dynamics, optimal gas transportation, etc., cannot be overestimated.

Since 1996, Kryazhimskiy had worked at the Steklov Institute of Mathematics of the RAS: first, as a leading researcher and, from 1997, as a chief researcher. Simultaneously, he had been a lecturer at the Department of Optimal Control of the Faculty of Computational Mathematics and Cybernetics at Moscow State University. His lectures were very popular with the students because of their rich content, informative value, and clarity of presentation. The lecturer's personal charm was of no less importance.

The wide scientific scope and industriousness at the highest intellectual level were Kryazhimskiy's important traits. He was successful at solving problems from the most diverse divisions of mathematics and borderline disciplines. His main motives in choosing new problems were the synthesis of disciplines and rich practical content.

A result of Kryazhimskiy's fruitful work and acknowledgement of his remarkable contribution to the development of Russian science was his election to the Academy of Sciences. He has been a corresponding member of the academy since May 1997 and a full member since May 2006.

A talented person is talented in everything. This popular saying is Kryazhimskiy's best characterization. He had been attracted to music and literature since his childhood, being a brilliant guitar player and an author of poetry and songs. Arkady Viktorovich was a responsive and warm-hearted person. He was open to people and did not dominate because of his position and well-deserved authority. His relatives and colleagues noticed his tact, enthusiasm, polymathy, and amazing willingness to both share his ideas and appreciate and discuss ideas of other people.

All who knew him were shocked to receive the news of Arkady Viktorovich's untimely death on 3rd of November 2014.

The International Conference in memory of Arkady Viktorovich Kryazhimskiy was organized by the Krasovskii Institute of Mathematics and Mechanics and the Ural Federal University. The conference named "System analysis: modeling and control" was held in Ekaterinburg in October 2016 to get together the colleagues of Kryazhimskiy from different countries for discussing actual scientific problems. The event was a success: more than fifty participants, more than thirty plenary reports. Several selected papers presented at the conference are published in this issue of the journal.

KRYAZHIMSKIY'S MAIN SCIENTIFIC PAPERS

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