Population Growth Models of Forest Trees for Conservation Management: Case of Teak (Tectona Grandis) Forest in Begal, East Java, Indonesia

Novriana Sumarti*, Mharta A. Wardana, Nuning Nuraini

Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Bandung, Indonesia

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ABSTRACT

Based on 2010 FAO report, teak forest and plantation in Indonesia covers 1,269 million hectares or 7 per mill of total area of Indonesia. It can be found dominantly in Central and East Java. PT Perhutani, Indonesia has responsible for management of the government owned forests in the islands of Java and Madura. Based on 2007 data, the teak wood production is 517,627 m^3 and the highest percentage, which is 37% of total production, is coming from East Java. In this paper, we develop growth population models using Leslie Matrix and Markov Chain in order to predict the future condition based on the current condition. The models are implemented into data from Teak Forest in Begal, East Java, that covers 2,052.8 hectares and consists of 114 sites. The result from the first model using Leslie Matrix shows that it needs 16 years from year 2011 that the sustainable condition of the forest can be achieved. The result from the implementation of the second model using Markov Chain into the existing data shows that the condition of the teak forest can be classified as quite critical because the good condition part based on its density of the early age group (0 - 4 years) has potential to become the worst condition before its harvest time.

1. Introduction

Teak tree (Tectona grandis) is a tropical hardwood that becomes the most popular choice for furniture, exterior construction, carving, boat building, and others, due to its durability and water resistance. Native habitants of this tree are in south and Southeast Asia, mainly India, Sri Lanka, Indonesia, Malaysia, Thailand, Myanmar, and Bangladesh (US Agriculture Dept 2007). It has been studied another good benefit on having teak forest in the area by through its dendroclimatic potential, which is the prediction of past climates by observing the tree-ring-width index chronologies of teak wood (Ram 2008) and (D'Arrigo 2011).

Researchers had been studying to find the suitable growth model of teak tree. Author in (Nunifu 1997) presented a preliminary study at assessing the growth and yield potential and developing provisional yield models for the management of teak in Northern Ghana. The diameter distribution, modelled by the three-parameter Weibull function, showed positive skewness indicating there are more trees in smaller diameter classes. Initial planting spacing of 2 by 2 m could be reduced to accommodate initial mortality and to achieve optimum stocking levels in order to improve form and timber quality. This assessment resulted the potential for growing teak to acceptable timber size on good sites.

In (Tewari 2014), the state-space approach is used for modelling rates of change of three state-variables: dominant height, number of trees per hectare and stand basal area. The model includes a stand volume equation as an out-put function to estimate this variable at any point in time. The model is implemented into data from 22 teak sample plots at Karnataka State of Peninsular India.

Authors in (Shamaki 2016) discussed a model of height-diameter relationship on Teak tree and implemented into data from Nimbia Forest Reserve, Nigeria. The fitting methods being used are Chapman-Richards and Weibull. Chapman-Richards function produced the best goodness-of-fit statistics for Teak height-diameter modelling.

In this paper we consider a population growth model using Leslie Matrix, which is the matrix population models that useful in predicting population growth

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* Corresponding Author
    E-mail Address: novriana@math.itb.ac.id

(Leslie 1948) and (Caswell 2001). This method had been used in many applications including in (Crouse 1987) and (Smith 1994). Authors in (Crouse 1987) proposed a modification of the original Leslie Matrix by adding the element of probability of individuals in a group of various ages will survive staying in the same group. The method is implemented into data of loggerhead turtle population in southeastern United States. This modified matrix is also used to develop the population growth model of teak trees population in this paper.

In (Smith 1994), the method is implemented into the fecundity (fertility) and mortality data of wild rabbit Oryctolagus cuniculus populations in England. Its population growth rate was shown to be slightly greater than unity in an average year. It means the populations is increasing, so there is a need to implement two kind of control policies; removing more adults than juveniles and vice versa, which are proposed to be implemented respectively at particular and different times.

Another population growth model considered in this paper is using Markov Chain, which is the matrix containing probabilities of transition between two consecutive events. In 1907, A. A. Markov began the study the type of chance where the outcome of a given experiment can affect the outcome of the next experiment, which is later called a Markov chain (Kemenny 1976) and (Sheldon 2010).

2. Materials and Methods

2.1. Teak Forest

Based on the Constitution of Republic of Indonesia Number 41 Year 1999 Regarding Forestry, forest areas are designated and or specified by the government to be maintained as permanent forests. Efforts in maintaining sustainable forest areas can be interpreted as to maintain the existence of permanent forest as the original state. In this state, the forest plants generally are dominated by old trees, so their strong roots and wide leaves' canopy can maximize the function of forests as one of the factors supporting the cycle of hydrology. From a long time ago, the good condition of hydrology cycle reflected on a stable climate for years up to about the end of 19th century. Nowadays, people's activities on forest tree looting or illegal logging had given bad impact not also on the climate change but also on an economic loss. In Indonesia, this loss suffered by PT Perhutani, the company owned by the government that has duty and authority to organize the planning and management of the exploitation and protection at Indonesian forests. Table 1 shows the economic loss due to a quite massive illegal logging at PT Perhutani's teak forest from 1999 to 2001. Many of the stolen trees were in old ages so the lack of these old trees in the
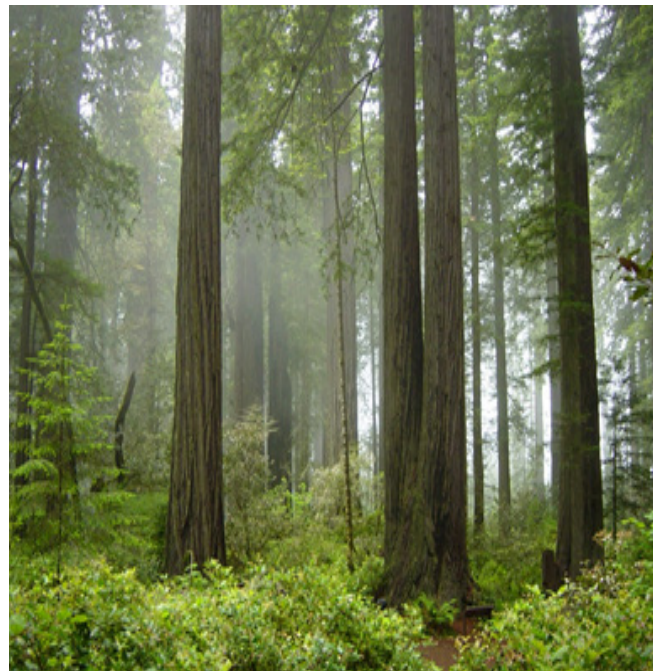

Figure 1. A picture sample of a teak forest (Redwood National Park wiki)

Table 1. Estimated economic loss

Year	Number of stolen trees	Economic loss (rupiah)
1999	3,179,973	55,851,584,000
2000	2,574,948	569,757,232,000
2001	2,675,161	613,924,367,000

forest could cause serious problems in maintaining its conversation.

Teak trees have age based stages and will be harvested after 60 years. The age classification of teak tree consist of classes Nursling/Young Plants (0-4), I (5-9), II (10 –19), III (20–29), IV (30–39), V (40–49), VI (50–59). In the period after class VI before harvesting, the tree is smoothed to turn off its cambium so the wood has high quality hardness. A part of forest area classified into the Nursling class can be interpreted as a void area where a severe looting activity had been occurred and induced deforestation in that area. This condition is undesirable and should be prevented by PT Perhutani. It is assumed that this area is allowed to exist only about one hectare of the total forest areas.

In order to improve wood quality and growth of individual high-quality trees, the management should do thinning and pruning not only in early stages but also often during later development of the plantation. PT Perhutani has used the Standard Density per hectare for Thinning (obtained from Buku Prosedur Kerja) that is shown in Table 2.

Table 2. Standard density for teak tree thinning management

| Age	Density	Age	Density	Age	Density	Age	Density	Age	Density
0 – 4	2045	16	667	28	387	40	269	52	201
5	1769	17	634	29	373	41	263	53	197
6	1617	18	600	30	359	42	256	54	194
7	1466	19	567	31	348	43	251	55	190
8	1314	20	524	32	338	44	245	56	187
9	1163	21	513	33	328	45	240	57	180
10	1012	22	492	34	319	46	234	58	176
11	949	23	471	35	308	47	230	59	173
12	887	24	450	36	300	48	225		
13	825	25	430	37	292	49	221		
14	763	26	416	38	284	50	208		
15	701	27	401	39	277	51	204		
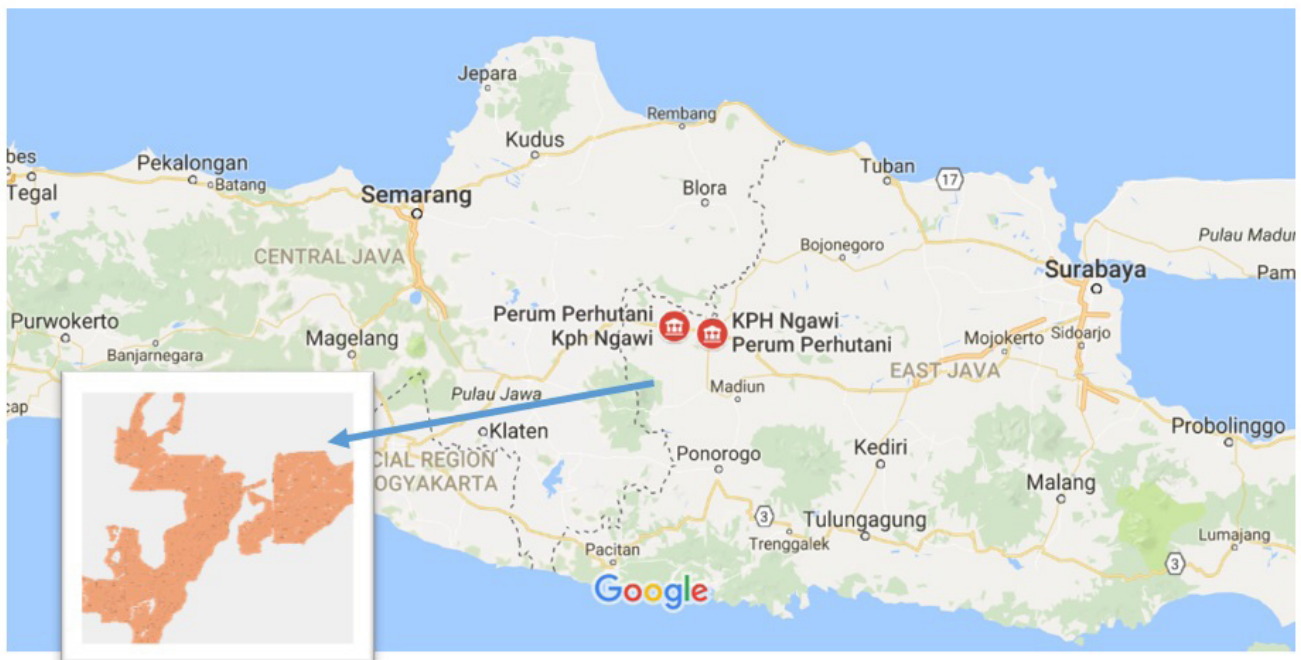
Figure 2. Map of KPH Ngawi PT Perhutani (Google Maps)

2.2. Model of Growth Population

2.2.1. Leslie Matrix

The growth model of teak trees population in a forest area will be derived based on modified Leslie Matrix that had been implemented into the population model of Loggerhead Turtle (Crouse 1987). The Leslie Model is powerful tool to determine the growth of a population as well as the age distribution within a population over time. The Leslie matrix population model is a discrete (or finite, i.e time goes in steps as opposed to continuously) and Age-Structured Population (construction of the model consider only age).

Let m be the number of groups containing particular age stages in a population, d_i be the group duration that is the age difference between the oldest and the youngest individuals in group i, $n(t)$ be a population vector containing $n_1(t)$, $n_2(t)$,..., $n_m(t)$ representing the numbers of individuals in each group at time t, and $n(t+1)$ be a population vector in the next year.

$$n(t+1)=A\,n(t) \qquad (1)$$

Where A is the modified Leslie matrix, based on (Crouse et al. 1987).

$$A = \begin{pmatrix} P_1 & F_1 & F_2 & \cdots & F_{m-1} & F_m \\ G_1 & P_2 & 0 & \cdots & 0 & 0 \\ 0 & G_2 & \ddots & & \vdots & \vdots \\ \vdots & & \ddots & \ddots & & \\ 0 & & & \ddots & P_{m-1} & 0 \\ 0 & 0 & \cdots & & G_{m-1} & P_m \end{pmatrix} \qquad (2)$$

Note that *A* is not tridiagonal matrix. The possible nonzero entries are only in the first row, the diagonal, and one place below diagonal. Fertility F_i is the number of offspring of a group i to $i + 1$ in a unit of time *t* that will survive to the next age stage group at time *t* + 1. Survival G_i is the probability that individuals in group i at time t will survive to group $i + 1$ at time *t* + 1. Survival P_i is the probability that individuals in group i at time *t* will survive getting older but they are still in group *i* at time *t* + 1. For example, if *m* = 3, equation (1) will be in the form below.

$$\begin{pmatrix} n_1(t+1) \\ n_2(t+1) \\ n_3(t+1) \end{pmatrix} = \begin{pmatrix} P_1 & F_1 & F_2 \\ G_1 & P_2 & 0 \\ 0 & G_2 & P_3 \end{pmatrix} \begin{pmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \end{pmatrix} = \begin{pmatrix} P_1 n_1(t) + F_1 n_2(t) + F_2 n_3(t) \\ G_1 n_1(t) + P_2 n_2(t) \\ G_2 n_2(t) + P_3 n_3(t) \end{pmatrix}$$

For $n_1 (t + 1)$, the individuals in age group 1 at time $(t + 1)$ consist of the individuals at time *t* that survive getting older but still in the age group 1, and individuals at time *t* that is born from the fertile individuals in age groups 2 and 3. For $n_2 (t + 1)$, the individual in age group 2 at time $(t + 1)$ consist of individuals at time *t* that survive getting older from the age group 1 and 2. For $n_3 (t + 1)$, the individual in age group 3 at time $(t + 1)$ consist of individuals at time *t* that survive getting older from the age group 2 and 3.

In a group, there are individuals who have been in this group for $1, 2, \ldots, d_i$ years. Denote p_i the probability of an individual in group *i* surviving to one year ahead. We assume that all individuals within a group are subject to the same survival probability and group duration. The probability of an individual surviving for d_i years is $p_i^{d_i}$, and assume that the survival probability of the youngest individual is 1. Assume that the population is stationary and the age distribution within groups is stable. In the time interval from *t* to *t* + 1, the oldest individuals will transfer, if survive, to the next group, and the younger individuals will remain in the group. The proportion of all remaining individuals at *t* + 1 and at time *t* is following.

$$P_i = \frac{p_i + p_i^2 + \cdots + p_i^{d_i - 1}}{1 + p_i + p_i^2 + \cdots + p_i^{d_i - 1}} = \frac{1 - p_i^{d_i - 1}}{1 - p_i^{d_i}} p_i \qquad (3)$$

Where each series in numerator and denominator is a geometric series with ratio factor $p_i < 1$, so its partial summation is $S_n = \frac{1 - p_i^{n+1}}{1 - p_i}$. The proportion of the oldest individuals survive and the remaining individuals is following.

$$G_i = \frac{p_i^{d_i}}{1 + p_i + p_i^2 + \cdots + p_i^{d_i - 1}} = \frac{p_i^{d_i}(1 - p_i)}{1 - p_i^{d_i}} \qquad (4)$$

From equation (1), the number of population at a long time *k* will depend on the number of initial population or $n(k) = A^k n(0)$. An analysis of matrix A is done by calculating its eigenvalues λ_i, $i = 1, 2, \ldots, m$ by solving.

$$|A - \lambda I| = 0$$

Where *I* is the identity matrix. Consider $\tilde{\lambda} = \max \lambda_i$. If $\tilde{\lambda} = 1$ or A^k tends to be an identity matrix, so the whole population is stationary, which means the number of population is constant. If $\tilde{\lambda} > 1$ or A^k tends to be infinity matrix, so the population will blow up. The harvesting effort could be done in order to keep the population stable. If $\tilde{\lambda} < 1$ or A^k tends to be a zero matrix, so the population will be diminishing. The annual rate of increase of the population is given by the logarithm of the maximum eigenvalue,

$$r = \ln \tilde{\lambda} \qquad (5)$$

2.2.2. Markov Chain Model

The second model derived in this paper needs Markov Chain model to estimate the long-run behavior. Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables. This sequence has Markov Chain property if

$$P(X_{n+1} = j | X_n = i, X_{m_1} = k_1, \ldots, X_{m_s} = k_s), \qquad m_i < n, i = 1, 2, \ldots, s$$

is a probability determined only by *n, i,* and *j,* which is the transition probability from *i* to *j* at time *n.* It means that the occurrence of next event at *n* + 1 is dependent only on the current event at *n,* but not on events occurred before *n.* If this probability is also independent of any current time *n,* then the Markov Chain is homogeneous and the transition probability is denoted $p_{ij} = P(X_{n+1} = j | X_n = i)$. It is obvious that for $\sum_{k=1}^{j} p_{ik} = 1$ for all *i.*

Let $X_1, X_2, X_3, \ldots$ be a homogeneous Markov chain, then the s-step transition probabilities are defined for any k as following.

$$p_{ij}^s = P(X_{k+s} = j | X_k = i)$$

We define p_{ij}^s or simplified by p^s be N × N matrix that is called s-step transition matrix. Here *N* is the number of events or states. p^s is s-th power of the transition probability matrix. The Chapman – Kolmogorov theorem will be used in determining many steps ahead.

$$p^{n+m} = p^n p^m, \qquad n, m \geq 0$$

For long run behavior, the Markov Chain is needed to be in a canonical form. Consider an arbitrary absorbing Markov chain that containing absorbing states, where its probability to be in the same state is 1, and the transition probability to or from other state is zero. Renumber the states so that the absorbing states come first. If there are r absorbing states and t transient or unabsorbing states, the transition matrix will have the following canonical form.

$$\tilde{P} = \begin{pmatrix} I & | & 0 \\ -- & | & -- \\ R & | & Q \end{pmatrix} \qquad (6)$$

Where I is an $r \times r$ identity matrix of absorbing states, 0 is an $r \times t$ zero matrix, R is a nonzero $t \times r$ transition matrix from transient states to absorbing states, and Q is an $t \times t$ transition matrix from and to transient states. The first r states are absorbing and the last t states are transient. For 2-steps and 3-steps transition matrices, we obtain respectively.

$$\tilde{P}^{(2)} = \begin{pmatrix} I & 0 \\ R+QR & Q^2 \end{pmatrix}, \qquad \tilde{P}^{(3)} = \begin{pmatrix} I & 0 \\ R+QR+Q^3R & Q^3 \end{pmatrix}$$

Eventually, the n-steps transition matrix is following.

$$\tilde{P}^{(n)} = \begin{pmatrix} I & 0 \\ R+QR+Q^3R+...+Q^{n-1}R & Q^n \end{pmatrix} \qquad (7)$$

For $n \to \infty$, it yields $Q^n \to 0$ since the transient probability $p<1$ so $p^n \to 0$. On the other part, if $n \to \infty$, $(1 + Q + Q^3 + Q^4 +...) R = NR$, where $N = (1 - Q)^{-1}$ that is called the fundamental matrix. The entry n_{ij} of matrix N is the expected number of times the chain is in state j, given that it starts in state i. We can have simulation on this entry in order to predict the number of periods for which a desired state of the event is kept in the same state before it transform to the undesired absorbing state.

3. Results

Total numbers of trees in years 2011–2015 from teak forest managed by PT Perhutani at Begal, East Java, Indonesia are shown in Table 3. There are 148

sites being observed with various areas from 0.3–84.5 hectares.

3.1. Leslie Matrix for the Teak Tree Forest

Based on the age classification of teak tree described in Section 2, we have time intervals $d_0 = d_1 = 5$, $d_i = 10$, $i = 2, 3, ..., 6$. The annual survival probability is defined by following formula.

$$p_i = \frac{1}{d_i}\sum_{k=1}^{d_i-1} \frac{n_i(k+1)}{n_i(k)}, \quad i = 0,1,...,6 \qquad (8)$$

In this research, fertility elements are not considered, $F_i = 0$, because the plantation of the new trees is deliberately done by the worker of PT Perhutani. It is assumed that the decreasing of number of trees is only due to normal thinning. The values of p_i in (8), P_i in (3), and G_i in (4) are shown in Table 4. Note that they are unitless values of probabilities.

Its Leslie matrix (2) is following.

$$A = \begin{pmatrix} 0.7396 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2334 & 0.7526 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1408 & 0.8649 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0661 & 0.8802 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0789 & 0.8861 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0842 & 0.8895 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0881 & 0.8887 \end{pmatrix}$$

The maximum eigenvalue of this matrix is $\tilde{\lambda}=0.8895 < 1$ and the annual rate of increase of the population r in (5) is ln 0.8895 = -0.1171, so the teak trees population for all age groups is diminishing. See Figure (2 - 3). It is normal because the plantation of new tree is controlled by the Perhutani staffs. Trees with an appropriate age will be harvested also by them. Using this matrix, the number of teak trees for years in the future can be approximated. We calculate $n_i(t + 1)$, $i = 0, 1, 2, 3, 4, 5, 6$ in (2) iteratively for $t = 0, 1, 2,$

Table 3. Numbers of teak trees at begal

Age group	2011	2012	2013	2014	2015
Nursling	762923	449093	228613	134598	85998
I	507611	761691	873816	663968	631445
II	88716	150332	211462	265625	343980
III	43870	31067	31067	24920	24920
IV	25078	37881	36586	42112	42112
V	19521	19521	16272	12833	12833
VI	9457	9457	14001	17314	17314

Table 4. Probability values for all age groups

	0	1	2	3	4	5	6
p_i	0.973007	0.893426	0.930967	0.959082	0.97069	0.977674	0.9759
P_i	0.739647	0.752593	0.864896	0.880181	0.886092	0.889538	0.888671
G_i	0.23336	0.140833	0.066071	0.078901	0.084598	0.088136	0.08723

3. We use year 2011 data as the initial value $n_i(0)$ and the calculated values $n_i(t+1)$ are validated using the exact data $\tilde{n}_i(t+1)$ from 2012 - 2015 in Table 3 by calculating relative errors between the approximated and exact data $\tilde{n}_i(t)$, $i = 0, 1, 2, 3, 4, 5, 6$ using the formula below.

$$relErr_i(t+1) = \left| \frac{\tilde{n}_i(t+1) - n_i(t+1)}{\tilde{n}_i(t+1)} \right|$$

The results show that the range of their relative errors is 0.0018-2.5898, which are quite good considering that it is rather uneasy task to collect the data from the resource.

The objective of the reservation management is to make the number of trees of group Nursling (0 - 4) is decreasing to only 1% of the initial number (objective 1) or to only very small number of trees (objective 2), for example it remains only 100 trees. We assume that the planted trees has been grown normally and the forest is well protected. We calculate equation (2) iteratively for $t = 0, 1, 2, …, T^*$ such that for the first time $n_0(T^*) < 0.01\, n_0(0)$ in objective 1, or $n_0(T^*) < 100$ in objective 2. The calculation results of equation (2) from both objectives is shown in Figure 3.

The calculated time T^* for objective 1 is 16 years from 2011. For objective 2, the calculated time T^* is 30 years from 2011. At the end of periods, there will be a good composition of trees' ages, which represents the sustainable condition.

3.2. Markov Chain Model for Teak Forest

The first step in deriving model using Markov Chain is the determination of states. We know that the number of teak trees is decreasing due to normal thinning process or illegal logging activities. The last

cause that massively done by irresponsible people could give serious problem to the environment's sustainability. We define the Degree of Planting Density (DPD) that is the ratio between the number of trees in the observed area and the standard density for thinning in Table 2. The unit of density is per one hectare. The younger plant has shorter diameter than the older one so the density of younger plants larger than the density of older plants.

We define state 1 (Good Condition) with *DPD* > 0.6 where there are enough trees at the site. State 2 (Bad Density) is with $0.3 < DPD \le 0.6$ where there are not enough trees at the site. State 3 (Open Area) is $DPD \le 0.3$ where the site is almost a barren area with barely not more than 60 teak trees on a hectare area are planted.

Now we observe 104 sites of Nursling age group in year 2011 and they are mostly still in this group until 2015. We do not observe for other age groups because all their sites are only in state 1 and 2 so we cannot construct a Markov Chain matrix for these age groups. Except for age group IV (30 – 39), its Markov Chain matrix is an identity matrix that means they are all absorbing states, which is explained later.

The classification of states each year for Nursling age group is shown in Table 5. The number of sites in state 3 in year 2011 increase in year 2012, and then the number is decreasing for other years. The increase is suspiciously due to illegal logging activities at a particular site. So we concern whether this will bad impact in years ahead. The decrease is due to planting activities by PT Perhutani.

The transition probability matrices for (2011-2012), (2012–2013), (2013–2014), and (2014–2015) are shown as P_1, P_2, P_3, P_4 in the following matrices.
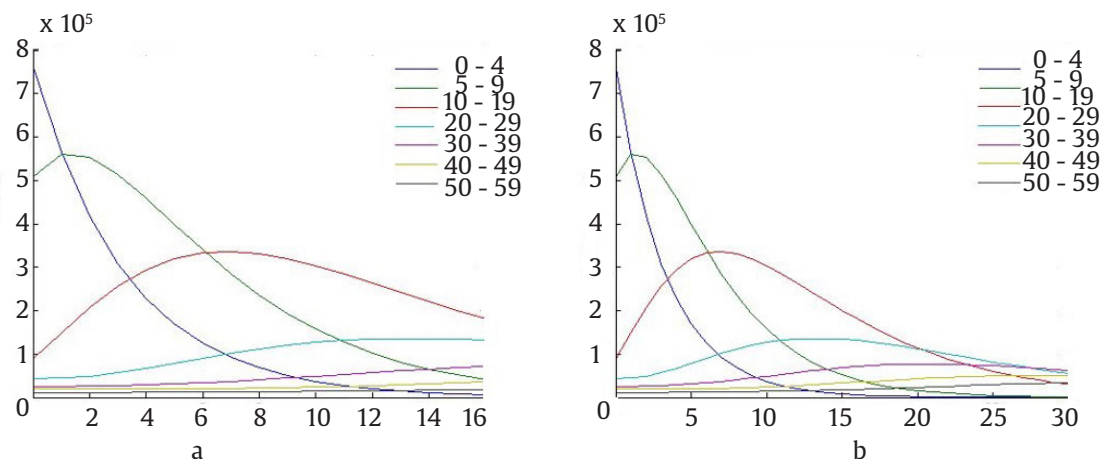

Figure 3. (a) Year 2011 data for group 0 - 4 becomes < 1% of initial number; (b) Year 2011 data for group 0 - 4 becomes < 100 per hectare

$$P_1 = \begin{pmatrix} 0.9286 & 0.0714 & 0 \\ 0.0714 & 0.9048 & 0.0238 \\ 0 & 0 & 1 \end{pmatrix}, P_2 = \begin{pmatrix} 0.9818 & 0.0182 & 0 \\ 0.2143 & 0.7857 & 0 \\ 0 & 0.7143 & 0.2857 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0.9206 & 0.0635 & 0.0159 \\ 0.0769 & 0.8718 & 0.0513 \\ 0 & 0.5 & 0.5 \end{pmatrix}, P_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0.1538 & 0.8462 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The entries in the first row of the matrix P_k, k=1, 2, 3 represent the probabilities for the various kinds of states following state 1 (Good Condition). Similarly, the entries in the second and third rows represent the probabilities for the various kinds of states following state 2 (Bad Condition) and state 3 (Open Area), respectively.

In the third row of matrix P_1, there is probability 1 that means state 3 is absorbing state where the condition is always in this state forever. This case also occurs in the first and third rows of matrix P_4. The states other than the absorbing state are called transient states, and from any of these it is possible to reach the absorbing states 3 and then to be stuck in state 3 forever.

It also shown all transition matrices have dominant values on the diagonal of the matrices, where the sites are still grow naturally. There are exception on the third row of matrices P_2 and P_3, which means there might be plantation activities by by PT Perhutani at the sites with state 3 in those years, so their state changes to state 2.

In transition matrix P_3, there is a probability 0.0159 that the sites with state 1 are degraded to state 3, which means there might be illegal logging or looting activities at that sites.

In order to determine the probability that in 2011 state i occurs and it will be in state j two years later, the 2-steps matrix from 2011 to 2013 is obtained by the following formulas.

$$p_{ij}^{(2)} = P(X_3 = j|X_1 = i)$$

$$= P(X_3 = j|X_1 = i, X_2 = 1)P(X_2 = 1|X_1 = i)$$
$$+ P(X_3 = j|X_1 = i, X_2 = 2)P(X_2 = 2|X_1 = i)$$
$$+ P(X_3 = j|X_1 = i, X_2 = 3)P(X_2 = 3|X_1 = i)$$

For all i, j = 1, 2, 3, the resulted matrix $P^{(2)}$ can be obtained by doing the matrix multiplication P_1 and P_2.

Table 5. The number of sites for each state

Year	State 1	State 2	State 3
2011	56	42	6
2012	55	42	7
2013	63	39	2
2014	62	39	3

$$P^{(2)} = \begin{pmatrix} p_{11}^{(2)} & p_{12}^{(2)} & p_{13}^{(2)} \\ p_{21}^{(2)} & p_{22}^{(2)} & p_{23}^{(2)} \\ p_{31}^{(2)} & p_{32}^{(2)} & p_{33}^{(2)} \end{pmatrix} = P_1 P_2$$

For example, $p_{11}^{(2)}$ = 0.9286 × 0.9818 + 0.0714 × 0.2143 = 0.9270. The 3-steps matrix from 2011 to 2014 is obtained by similar procedures.

Using Chapman – Kolmogorov theorem, the transition probability matrices for more than one year can be approximated by using the multiplications of transition matrix 2011. For example,

$$p_{11}^{(2)} = p_{11}p_{11} + p_{12}p_{21} + p_{13}p_{31},$$
$$p_{12}^{(2)} = p_{11}p_{12} + p_{12}p_{22} + p_{13}p_{31},$$
$$p_{13}^{(2)} = p_{11}p_{13} + p_{12}p_{23} + p_{13}p_{31},$$

which are the elements on the first row of matrix P_1^2.

$$P_1^2 = P_1 P_1 = \begin{pmatrix} 0.8673 & 0.1310 & 0.0017 \\ 0.1310 & 0.8237 & 0.0454 \\ 0 & 0 & 1 \end{pmatrix}$$

Having validated this approximation, the differences or errors between elements of P_1^2 and $P_1 P_2$ are in range 0.0017 - 0.7143.

Now we do simulation on data so the condition of teak forest in Begal for long run can be estimated. There are 3 scenarios being exercised, the first one is based on the existing data, and the other two are simulations by doing a small change from the existing data deliberately on particular sites in order to obtain desired objectives in the long-run.

3.2.1. First Scenario

Certain changes of states obtained from data in years 2011 – 2012 are shown in Table 6.

Matrix $\tilde{P}$, which is the canonical form of matrix P_1, and its fundamental matrix are following.

$$\tilde{P} = \begin{pmatrix} p_{33} & p_{31} & p_{32} \\ p_{13} & p_{11} & p_{12} \\ p_{23} & p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.9286 & 0.0714 \\ 0.0238 & 0.0714 & 0.9048 \end{pmatrix}, N = \begin{pmatrix} 56 & 42 \\ 42 & 42 \end{pmatrix}$$

Table 6. States changes of original data

State Change	Site names	Number of sites
1 → 2	10D, 18G, 33B, 34A	4
2 → 1	26C, 27F, 32A	3
2 → 3	9F	1

From the fundamental matrix, n_{11} = 56 is the number of years needed that state 1 being kept in the same state before being absorbed. Note that the harvesting time for teak tree is 60 years, so there is a possibility on which the transition from state 1 to state 3 will occur before the harvest time. That will be a big loss to PT Perhutani, so we can claim that the current forest condition is critical.

3.2.2. Second Scenario

If site 10B, which is in state 2 for both years, is to be maintained by full monitoring program in order to be upgraded to state 1 in year 2012. The transition probability matrix is then recalculated.

Its canonical matrix $\tilde{P}$ and fundamental matrix N are following.

From the fundamental matrix, n_{11} = 70 > 60 is the time needed that state 1 being in the same state before being absorbed. It is enough time to harvest the wood before it will transformed to be open areas with small number of trees.

3.2.3. Third Scenario

If site 10D, which is in state 1 in 2011, is to be maintained by full monitoring in order to avoid illegal logging or looting so it keeps to be in state 1. The transition probability matrix is recalculated.

Its canonical matrix $\tilde{P}$ and fundamental matrix N are following.

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.9464 & 0.0536 \\ 0.0238 & 0.0714 & 0.9028 \end{pmatrix}, N = \begin{pmatrix} 74.6269 & 40.0168 \\ 55.9701 & 40.0168 \end{pmatrix}$$

From the fundamental matrix, n_{11} = 74.6269 > 60 is the time needed that state 1 being in the same state before being absorbed. It is enough time to harvest the wood before it will transformed to be open areas with small number of trees.

Another attempt to upgrade site 9F in order to be kept in state 2, not degrading as in the data. However, the matrix Q will be singular so its matrix fundamental cannot be obtained.

4. Discussion

We compare these factors below for both developed models.
1. Usefulness
   Leslie Matrix model could predict the period of time needed from year 2011 for the forest to

Table 7. States changes with modified site 10B

State change	Sites	Number of sites
1 – 2	10D, 18G, 33B, 34A	4
2 – 1	26C, 27F, 32A, 10B	4
2 – 3	9F	1

Table 8. States changes with modified site 10D

State change	Sites	Number of sites
1 – 2	18G, 33B, 34A	3
2 – 1	26C, 27F, 32A	3
2 – 3	9F	1

achieve sustainable condition that there will be a good composition of the trees' ages. Markov Chain model could propose improvement on managing the sites so the teak trees can be lasting until the harvest time.
2. Input of the model
   Leslie Matrix model needs classification of age groups of teak trees and the numbers of trees for each group. Markov Chain model needs determination of DPD (Degree of Planting Density) per hectare of areas.
3. Advantages
   Leslie Matrix model can be implemented into data of any forest because it is constructed from the Table of Standard Density for Thinning made from good practices on managing the teak tree forest. In Markov Chain, the transition probability matrix can be constructed easily based on the desired change of DPD (Degree of Planting Density) on a certain age group.
4. Disadvantages
   Leslie Matrix model has assumption that there is no extreme illegal logging and looting activities along the period of time. In constructing Markov Chain matrix, all determined states and transition between states should occur in the observed age group. If not, the matrix will be not existing or singular.

To conclude, both models have advantages in predicting the condition of the teak tree forest in the future. The results show a quite critical condition in current state. The first model using Leslie Matrix shows that the current condition is not guaranteed to sustain because it needs 16 years from year 2011 that the sustainable condition of the forest can be achieved. The second model using Markov Chain shows that the good condition part based on its density of the early age group (0 - 4 years) has potential to become the worst condition before its harvest time. Some effort are needed to be done in order to repair the current

condition. However, models developed in this paper still have not very small errors in the validation process because the recorded data is not enough and updated. If the data is available in enough number, we could choose the appropriate initial values that represent the real condition of the teak forest.

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