Informative Advertising and Strategic Entry Deterrence: A Cournot Model※

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1. Introduction

Can established firms strategically advertise to impede entry? This question has been long analyzed since Bain [1956] argued that product differentiation via advertising serves as an entry barrier. Recent developments in game theory have enabled economists to rigorously investigate this question1). Schmalensee [1983] was the first to examine whether an incumbent can prevent entry by engaging in informative advertising. Advertisements by a firm inform consumers of the existence of its brand and how to quote the price. Consumers can know the existence of brands only through their exposure to advertisements. The incumbent advertises before the entrant. Having observed the incumbent's advertising level, the entrant sets its advertising level. Finally, the firms simultaneously announce a quantity conditional on the firms' advertising levels. Under this setting, he claimed that the quantity-setting subgame always has pure-strategy Cournot-Nash equilibria. Then, he showed that the incumbent can optimally advertise less than if the threat of entry is absent to deter new entry.

This article reexamines Schmalensee's [1983] strategic entry-deterrence game. I first show that his characterization of the post-advertising Cournot equilibria is incorrect. Unlike his claim, the post-advertising Cournot-Nash equilibria can be in mixed strategies when the number of consumers knowing only one brand is relatively greater than that of consumers knowing only the other brand2). Then, based on the correct characterization of the post-advertising Cournot-Nash equilibria, I reconsider if the incumbent can deter entry via advertising in his game. At the sequential-move advertising stage, the entrant's profit initially declines with the incumbent's advertising investment. After attaining a minimum, it rises with the incumbent's advertising investment, but never exceeds the incumbent's

※ This article is a revision of part of Essay 1 of my Ph. D. dissertation submitted to Purdue University in December, 1998. I wish to thank my supervisors Dan Kovenock and William Novhsek for encouragement and suggestion. My dissertation research was made possible through a grant from Purdue Research Foundation.

1) See Fudenberg and Tirole [1984], Baldani and Masson [1984], and Bonnano [1986].

2) This finding interestingly contrasts the Nash equilibria in the price-setting duopolies under similar demand conditions. Narasimhan [1988], Deneckere, Kovenock and Lee [1992], Ireland [1993], and Golding and Slutsky [undated] show that the Bertrand-Nash equilibria are always in mixed strategies.

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monopoly profit. Thus, under certain conditions of the levels of sunk, fixed advertising cost, the incumbent can find an advertising level less than its monopoly advertising level (1) to make the entrant's overall profit zero (if entry occurs), and (2) to earn more than when entry is accommodated. That is, the incumbent can deter entry by advertising less than if the threat of entry is absent. This is what he showed on the basis of his incorrect characterization of the post-advertising equilibrium. Hence, this paper demonstrates that even if Schmalensee's [1983] analysis of the post-advertising quantity competition is incorrect, his finding regarding strategic entry deterrence via advertising by the incumbent is qualitatively correct.

The organization of this paper is as follows. Section 2 describes the structure of the model considered. Section 3 characterizes the Nash equilibrium of the post-advertising game. Section 4 examines the incumbent's strategic advertising to influence the entrant's entry decision. In Section 5, I conclude this article.

2. The Model

The basic structure of the model follows Schmalensee [1983] except that the variable cost function of advertising is quadratic. Formally, I examine the following three-stage game: at stage 1, an incumbent decides whether to enter a market, and if it does, it engages in advertising with sunk fixed and variable costs; at stage 2, an entrant chooses whether to enter the market, and if it does, it advertises with sunk fixed and variable costs; and finally at stage 3, the firms compete in prices (in Section 4) or quantities (in Section 5). The extensive form of this game is shown in Figure 1. My analysis focuses on subgame perfect equilibria.

Following the Schmalensee's notation, I assume that firm X is the incumbent and Y is the potential entrant. They produce a homogeneous product if they produce anything. The marginal cost of production, is a positive constant and less than 1 for both firms. There is a continuum of potential consumers. The total mass of these consumers is normalized to unity. Their common utility function is defined by

$$U(q_i) = q_0 + q_i - q_i^2/2 \quad \text{for } i = X,Y \tag{1}$$

where q_0 is the numéraire, q_i is the quantity demanded for brand i and p_i is brand i's market price. The consumers are price takers and maximize the utility not only subject to a budget constraint, $q_0 + p_i q_i \leq m$ where m is a sufficiently large income in terms of the numéraire but also subject to their exposure to advertisements by the firms.

In order to make any sales, firms X and Y must invest a fixed amount, f, to design and prepare to print leaflets. This cost is forever sunk. Firms, then, print leaflets and send them at random to consumers. Before receiving the leaflets, consumers are assumed not to know the existence of either of brands. Receiving one or more leaflets from a firm, a consumer knows the existence and the attribute of its brand and the telephone number and the location of the firm, namely, how to quote its price.

Advertising here creates a segmentation of the consumers. Suppose firm X has informed $0 \leq x \leq 1$ (fraction of) consumers of the existence of brand X, and firm Y has informed $0 \leq y \leq 1$ consumers of the existence of brand Y. Then, $x(1-y)$ consumers know the existence of only brand X; $y(1-x)$ consumers know the existence of only brand Y; xy consumers know the existence of brands X and Y; and $(1-x)(1-y)$ consumers know neither of the brands. The perfectly informed consumers can compare the prices set by both the firms. They each purchase $q_X = 1 - p_X, p_X \in [0, 1]$, units of brand X and nothing from firm Y if brand X's price is less than brand Y's; otherwise buy $q_Y = 1 - p_Y, p_Y \in [0, 1]$, units of brand Y and nothing from firm X. On the other hand, the asymmetrically informed consumers are insensitive to the price differences because they know the existence of only one of the brands. Thus, those who know only brand X (Y) have a demand function, $q_X = 1 - p_X, p_X \in [0, 1]$ $(q_Y = 1 - p_Y, p_Y \in [0, 1])$, which is obtained from the utility maximization subject to the budget constraint. They are virtually monopolized by either of the firms.

It would be worth mentioning that the properties of purchasing behavior of the perfectly and imperfectly informed consumers are, respectively, parallel to those of switchers and brand-loyal consumers. Narasimhan [1988], and Deneckere, Kovenock and Lee [1992] among others have analyzed the situation in which the population sizes of these consumers are exogenous. Therefore, this model is considered as a generalization of these models by endogenizing the population sizes of brand-loyal consumers and switchers.

As to variable cost of advertising, following Tirole [1988], I will employ a quadratic function,

$$e(z) = (\alpha/2)z^2, \alpha > 0, \tag{2}$$

where z = x or y and is a positive parameter. As I will show later, the model prediction does not depend on the specific form of the cost function of advertising as long as the marginal cost of advertising increases at a relatively lower rate. Thus, I will confine my analysis to the case where variable cost of advertising follows this quadratic function.

3. Post-advertising Cournot Equilibrium

In this section, I re-characterize the Cournot-Nash equilibria in the post-advertising game to disprove Schmalensee's [1983] claim that all the Cournot-Nash equilibria are in pure strategies. I first identify the profit function to derive the quantity best-response correspondences, given that the levels of advertising are fixed. Then, with those best-response correspondences, I fully characterize the Cournot-Nash equilibria.

Suppose that the firms are about to decide a level of output after they have already advertised. Those firms face the following market conditions (Schmalensee [1983] pp. 640-641, especially footnote 8): (a) there is a separate Cournot market for each brand; (b) each firm indirectly controls the market price by changing its production level; and (c) the perfectly informed consumers compare the market prices. Under this situation, firm X's profit function must be

$$\pi_X(q_X, q_Y \mid x, y) = \begin{cases} \left[1 - c - \dfrac{q_X}{x}\right] q_X & \text{if } 0 < 1 - \dfrac{q_X}{x} \leq 1 - \dfrac{q_Y}{y(1-x)} \\ \left[1 - c - \dfrac{q_X + q_Y}{x + y - xy}\right] q_X & \text{if } \begin{cases} 1 - \frac{q_X}{x} \geq 1 - \frac{q_Y}{y(1-x)} > 0 \\ 1 - \frac{q_Y}{y} \geq 1 - \frac{q_X}{x(1-y)} > 0 \end{cases} \\ \left[1 - c - \dfrac{q_X}{x(1-y)}\right] q_X & \text{if } 0 < 1 - \dfrac{q_Y}{y} \leq 1 - \dfrac{q_X}{x(1-y)} \end{cases} \qquad (3)$$

The meaning of the top and bottom sub-profit functions would be straightforward. Firm X earns a profit through the top sub-profit function when it produces a quantity large enough to undercut firm Y, and sells to all the perfectly informed consumers as well as firm X's captive consumers. The bottom one arises in the opposite situation where firm X meets the demand of only its captive consumers by limiting its output level so that brand X's market price is higher than brand Y's.

The middle sub-profit function arises when the following two conditions hold at the same time: (a) if all the perfectly informed consumers buy from firm X, brand X's price is larger than brand Y's, and (b) if all the perfectly informed consumers buy from firm Y, brand Y's price is larger than brand X's. In such a situation, the perfectly informed consumers must play an arbitrage role to equalize the market prices. Hence, $1-q_X/[x(1-y)+kxy] = 1-q_Y/[y(1-x)+(1-k)xy]$, or $k = [q_X/(q_X + q_Y)]/x - (1-y)[q_Y/(q_X + q_Y)]/y$ is an endogenous variable. Substituting k into $\{1 - c - q_X/[x(1-y) + kxy]\}q_X$ yields the middle sub-profit function above. In this situation, the firms determine an output level as if all the products were sold in a single Cournot market.

Now I will derive the quantity best-response correspondences. Without loss of generality, I focus on the derivation of firm X's quantity best-response correspondence. Under the parameter restriction of $x \geq (\sqrt{y} - y)/(2 + \sqrt{y} - y)$,[3] firm X's profit function is given in Figure 2a. When firm Y's quantity is less than $q_Y' := y(1-x)(1-c)/2$, firm X's typical profit function is ABDEFG. The q_Y' is determined by the condition that the

3) The meaning of this inequality restriction is explained later.

quantity level at which firm X's profit function changes from the middle to the top in (3) re aches the maximizer of the top sub-profit function in (3). At points B and D, firm X's profit function changes from the bottom to the middle, and from the middle to the top in (3), resp ectively. The best-response is $q_X^{**} := x(1-c)/2$, which is the maximizer of firm X's mo nopoly profit when firm X sells to all the consumers who know brand X. When $q_Y' \leq q_Y \leq q_Y'' := (x+y-xy)(1-x)y(1-c)/(2x+y-xy)$, firm X's typical profit functi on becomes ACEFG. The q_Y'' is determined by the condition that the quantity level at whic h firm X's profit function changes from the middle to the top in (3) coincides with the maxi mizer of the middle sub-profit function. The best response is $q_X^{****} := q_Y x/[y(1-x)]$. Notice that then it is indifferent to firm X whether firm X is a monopolist selli ng to all the consumers who know its brand, or firm X competes with firm Y as if both pro ducts were traded in a single Cournot market. When $q_Y'' \leq q_Y \leq q_Y''' := [x+y-xy-\sqrt{x+y-xy}\sqrt{x(1-y)}](1-c)$, firm X's typical profit function is AHFG. The q_Y''' is firm Y's quantity level where the maximum of the middle sub-profit function is equivalent to that of the bottom sub-profit function in (3). The best-response is $q_X^{***} = [(1-c)(x+y-xy)-q_Y]/2$. This is exactly the conventional Cournot best response. When $q_Y''' \leq q_Y$ firm X's profit function is AHIJG. The best response is $q_X^{*} = x(1-y)(1-c)/2$. Summarizing the findings above, I have firm X's quantity best-response correspondence:

$$q_X^*(q_Y \mid x,y) = \begin{cases} x(1-c)/2 & \text{if } 0 \leq q_Y \leq q_Y' \\ xq_Y/[y(1-x)] & \text{if } q_Y' \leq q_Y \leq q_Y'' \\ [(1-c)(x+y-xy)-q_Y]/2 & \text{if } q_Y'' \leq q_Y \leq q_Y''' \\ x(1-y)(1-c)/2 & \text{if } q_Y''' \leq q_{Y_Y} \end{cases} \tag{4}$$

Firm Y's quantity best-response correspondence can be obtained from the above with arguments transposed.

In the above, with the inequality condition, I considered the situation where firm X's best-response can be q_X^{***}. But, if the opposite inequality condition, $x \leq (\sqrt{y}-y)/(2+\sqrt{y}-y)$, holds, firm X's best response changes from q_X^{****} to q_X^{*}. The situation is shown in Figure 2b. Firm X chooses the same best responses as in the above until firm Y's quantity reaches $\hat{q}_Y := x(1-y)(1+\sqrt{x})(1-c)/2$. This critical value is determined by the condition that firm X's profit when firm X's produces is equal to the maximum of the bottom sub-profit function in (3). When firm Y's quantity is beyond $\hat{q}_Y$ firm X's optimal quantity is q_X^{*}. Thus, if $x \leq (\sqrt{y}-y)/(2+\sqrt{y}-y)$ holds, firm X's quantity best-response correspondence is

$$q_X^*(q_Y \mid x, y) = \begin{cases} x(1-c)/2 & \text{if } 0 \le q_Y \le q_Y' \\ xq_Y/[y(1-x)] & \text{if } q_Y' \le q_Y \le \hat{q}_Y. \\ x(1-y)(1-c)/2 & \text{if } \hat{q}_Y \le q_Y \end{cases} \tag{5}$$

Firm Y also has the equivalent best-response correspondence under the corresponding inequality condition.

These quantity best-response correspondences essentially imply that when firm j's output level is relatively low, firm i is better off by undercutting through greater production, and selling to all the consumers who know brand i. On the other hand, if firm j's output level is relatively great, firm i benefits from restricting its quantity to a lower level and selling to the consumers who know only brand i. Each firm never produces less than when it sells to the consumers who know only its product at the monopoly price. The presence of the second type of best response from the top in (4) and (5) suggests that quantities can be locally strategic complements unlike in the conventional Cournot model under the assumption that consumers know the existence of all homogeneous products.

With the quantity best-response correspondences, I can derive the Cournot-Nash equilibria of this game. Since the combination of x and y determines the quantity best-response correspondences, the type of Cournot-Nash equilibrium is totally identified by the location of (x, y) in the advertising strategy space. Figure 3a shows the map of the regions of (x, y) space in each of which the same (or similar) type of Cournot-Nash equilibrium emerges. Let me first present the equilibrium of each case[4)].

Proposition 1: (Post-advertising Cournot-Nash Equilibrium).

The post-advertising simultaneous-move quantity-setting game has the following Cournot-Nash equilibrium.

1. When (x, y) lies in Region A (of Figure 3a), the game has a pure-strategy Cournot-Nash equilibrium where the two firms produce an identical quantity (See Figure 4a). Firm i's profit is

$$\Pi_i^* = (x + y - xy)(1-c)^2/9, i = X, Y. \tag{6}$$

2. When (x, y) lies in Region B, the game has a mixed-strategy Cournot-Nash equilibrium where each firm randomizes two quantities (See Figures 4b and 4c). Firms X and Y earn

4) I will later discuss the Cournot-Nash equilibria when x and y are in Region F.

$$\Pi_X^* = x(1-y)(1-c)^2/4, \tag{7}$$

$$\Pi_Y^* = \frac{(1-c)^2 y(2x+3y-2xy)}{16\left(\left\{\frac{3\cdot 2^{2n-1}}{4^{n+1}-1}\right\}x(1-y)+y\right)}, \tag{8}$$

respectively, where n = 0, 1, 2, 3, ... and so on, and the "0" corresponds to the case of $q_Y^*(q_X^*(y(1-c)/2)) \geq q_Y'''$ the "1" corresponds to the case of $q_Y^*(q_X^*(q_Y^*(q_X^*(y(1-c)/2)))) \geq q_Y''' \geq q_Y^*(q_X^*(y(1-c)/2))$, and so on. Symmetric profits can be obtained when x and y are in Region E.

3. When (x, y) lies in Region C, the game has a mixed-strategy Cournot-Nash equilibrium where both firms randomize two quantities (See Figures 4d and 4e). Firms X and Y earn

$$\Pi_X^* = x(1-y)(1-c)^2/4, \Pi_Y^* = y\left\{1-\left(\frac{y}{2x(1-y)}\right)^2\right\}(1-c)^2/4, \tag{9}$$

respectively. Symmetric profits can be obtained when x and y are in Region D.

The proof of the equilibrium in each situation is its explicit derivation. Nevertheless, I will provide only a sketch of the detailed process of the derivation of each equilibrium since the derivation is not difficult but rather messy, and the procedure is in principle the same among all the cases. The method used to find the equilibrium is as follows. First, one identifies the firms' pure strategies that survive iterated deletion of strictly dominated strategies. Let firm i's set of the survived strategies be S_X^∞,i = X, Y. Then, one needs to check if every possible combination of the quantities in S_X^∞ and S_Y^∞ can constitute an mixed-strategy subgame perfect Nash equilibrium. One can prove that the mixed strategies under one combination constitute a subgame perfect Nash equilibrium by showing that the following two conditions are satisfied. (a) All the mixed-strategy equilibrium probabilities assigned to the quantities in the combination are non-negative. (b) Firm i has no incentive to produce different outputs in from the ones in the combination, given that firm j's (randomized) strategy of the quantities of S_j^∞ in the combination is fixed (i, j = X, Y; i ≠ j). For the sake of explanation, I often refer to the symbols in Figure 4a of the quantity best-response correspondences.

First of all, I will prove the equilibrium when x and y lie in Region A of Figure 3a. A typical pair of the quantity best-response correspondences is shown in Figure 4a. The values of x and y of Region A are restricted by the condition that the two quantity best-response correspondences have a unique intersection at $q_i^C = (x + y - xy)(1 - c)/3, i = X, Y$. That is, $q_i'' \leq q_i^C, \bar{q}_i^*(q_j''') \leq q_i^C, q_i^C \leq \bar{q}_i^*(q_j'')$, and $q_i^C \leq q_i'''$, or $5x/(5x + 4) \leq y \leq 4x/(5 - 5x)$. In this case, each S_i^∞ is a singleton, or contains only $(x + y - xy)(1 - c)/3$. Thus, when x and y are in Region A, the post-advertising game has the "conventional" pure-strategy Cournot equilibrium as if all the products were sold in a single market. The profits can be obtained by following the routine calculation.

Let me next prove the equilibria when x and y are in Region B (and Region E). The condition $q_X^*(q_Y''') \geq q_X^C$ or $5x/(5x + 4) > y$ implies that in Figure 4a, the line segment IJ is below the line segment CD. The condition $y \leq 2x^2/(1 - 2x)(1 - x)$ implies that in Figure 4a, $q_X^*(y(1 - c)/2)$ is the traditional Cournot type best response, or the line AB meets the line segment IJ. The condition of $x(\sqrt{17} - 1 - 4x)/[2\{2 - x(1 + 2x)\}] \leq y$ means that $q_Y^*(q_X^*(y(1 - c)/2))$ is the traditional Cournot type best response. The condition of $y \leq 2x(2x - 1 + 2\sqrt{4x^2 - 6x + 2})/(12x^2 - 20x + 7)$ ensures that $q_X^*(y(1 - c)/2)$ is smaller then q_X'''.

Given these conditions, suppose additionally $q_Y^*(q_X^*(y(1-c)/2)) \geq q_Y'''$ (or $y \leq 2x(14x - 1 - 4\sqrt{11})/(28x^2 - 4x - 25)$) holds (n = 0 in Proposition 1). A typical pair of the quantity best-response correspondences is shown in Figure 4b. Apparently, the game with those parameter values has no pure-strategy Cournot-Nash equilibrium. By deleting strictly dominated strategies sequentially, one can find that $S_X^\infty = \{x(1 - y)(1 - c)/2, q_X^*(y(1 - c)/2) = (2x + y - 2xy)(1 - c)/4\}$, and $S_Y^\infty = \{y(1 - c)/2, q_Y^*(q_X^*(y(1 - c)/2)) = (2x + 3y - 2xy)(1 - c)/8\}$. An important observation here is that in the reduced game, since firm Y always produces more than $y(1 - c)/2$, firm X can always earn $x(1 - y)(1 - c)^2/4$ by setting $x(1 - y)(1 - c)/2$. Because of this, firm X's expected profit in the mixed-strategy equilibrium must be $x(1 - y)(1 - c)^2/4$. Firm Y's expected profit can be obtained by explicit calculation: $(1 - c)^2 y(2x + 3y - 2xy)/8(x + 2y - xy)$.

Suppose now that the condition $q_Y^*(q_X^*(q_Y^*(q_X^*(y(1-c)/2)))) \geq q_Y''' \geq q_Y^*(q_X^*(y(1-c)/2))$ comes to hold (n = 1 in Proposition 1). This condition implies that the point of I is much closer to the line segment CD than in Figure 4b. A typical pair of the quantity best-response correspondences is given in Figure 4c. As shown in the figure, the number of the elements of S_i^∞ is now three: $S_X^\infty = \{x(1 - y)(1 - c)/2, q_X^*(q_Y^*(q_X^*(y(1 - c)/2))), q_X^*(y(1 - c)/2)\}$, and $S_Y^\infty = \{y(1 - c)/2, q_Y^*(q_X^*(y(1 - c)/2)), q_Y^*(q_X^*(q_Y^*(q_X^*(y(1 - c)/2))))\}$. A useful observation is that, as seen in the above, firm X can always earn $x(1 - y)(1 - c)^2/4$ by setting $x(1 - y)(1 - c)/2$ since firm Y is always expected to produce more than $y(1 - c)/2$. Thus, one can see that there is no mixed-strategy equilibrium where the firms use all three quantities survived iterated strict dominance. This is because there is no combination of the probabilities assigned to firm Y's three quantities so that firm X earns the same profit by producing the second quantity as by producing the third quantity. With careful examination of the non-negativity condition of the probabilities assigned to the strategies and the Nash condition of the strategies, one can show that the carrier of firm X's mixed-strategy equilibrium probability function is composed of $x(1-y)(1-c)/2$, and $q_X^*(q_Y^*(q_X^*(y(1-c)/2)))$; and that of firm Y's contains $y(1-c)/2$ and $q_Y^*(q_X^*(y(1-c)/2)))$. In the equilibrium, firm X's expected profit is $x(1-y)(1-c)^2/4$, firm Y's expected profit is $(1-c)^2 y(2x+3y-2xy)/16\{(2/5)x(1-y)+y\}$.

When the point of I becomes again much closer to the line segment CD, $q_Y^*(q_X^*(q_Y^*(q_X^*(q_Y^*(q_X^*(y(1-c)/2)))))) \geq q_Y''' \geq q_Y^*(q_X^*(q_Y^*(q_X^*(q_Y^*(y(1-c)/2))))$ may hold (n = 2 in Proposition 1). Under this condition, S_i^∞, i = X, Y, now contains four elements. But, by using a similar logic to the above, one can show that in the mixed-strategy equilibrium, firm X uses two quantities, $x(1-y)(1-c)/2$, and $q_X^*(q_Y^*(q_X^*(q_Y^*(q_X^*(y(1-c)/2))$, firm Y uses two quantities, $y(1-c)/2$ and $q_Y^*(q_X^*(y(1-c)/2)))$. Firm X's equilibrium profit is, as above, $x(1-y)(1-c)^2/4$, but firm Y's equilibrium expected profit becomes $(1-c)^2 y(2x+3y-2xy)/16\{(32/85)x(1-y)+y\}$.

Observing the changes in the firms' carriers of the mixed-strategy equilibrium probability functions from one case to another in the above, one can see that (a) firm X always chooses two quantities from S_X^∞ so that the average of the two quantities is smaller than that of any other two quantities in S_X^∞ and that (b) firm Y chooses the same two quantities. This finding makes it easy to identify the carriers of the firms' mixed-strategy equilibrium probability functions and to calculate firm Y's profit for n = 3, 4, ..., and so on. The general formula of firm Y's expected profit in Proposition 1 can be obtained from those profits of firm Y's by mathematical induction.

Now I turn to the derivation of the equilibrium when x and y are in Region C (also Region D). As in Region B, the parameter values are restricted by the condition $q_X^*(q_Y''')$ $\geq q_X^C$ or $5x/(5x+4) > y$ (the line segment IJ is below the line segment CD in Figure 4a); the condition $y \leq 2x^2/(1-2x)(1-x)$ (the line AB meets the line segment IJ in Figure 4a). But, the condition of $y \leq x(\sqrt{17}-4x-1)/[2\{2-x(2x+1)\}]$ means that firm Y's best-response to $q_X^*(y(1-c)/2)$ is on the line segment BC in Figure 4a. A typical pair of the quantity best-response correspondences of such a case is given in Figures 4d and 4e when both firms' quantity best-response correspondences are the type of (4), and firm X's is the type of (4) but firm Y's is the type of (5), respectively. Given these conditions, one can show that $S_X^\infty = \{x(1-y)(1-c)/2, q_X^*(y(1-c)/2) = (2x+y-2xy)(1-c)/4\}$, and $S_Y^\infty = \{y(1-c)/2, q_Y^*(q_X^*(y(1-c)/2)) = y(2x+y-2xy)(1-c)/[4x(1-y)]\}$. This case is equivalent in spirit to the situation when n = 0 in Region B. Likewise, one can obtain the expected profits shown in Proposition 1. This completes the proof of

Proposition 1.

Region F can be partitioned into small regions (R1, R2, ..., R12, and the blanked region around the origin) in which S_i^{∞} has different elements. The partition is shown in Figure 5. Table 1 has the list of the critical mathematical conditions of R1, R2, ..., R10 to determine the levels and the number of the elements of S_i^{∞}. A pair of quantity best-response correspondences for each of R1, R2, ..., R10 is given in Figures 6a-j. Each of Figures 6a-j indicates the quantities that survive iterated strict dominance by black dots.

By carefully examining the conditions (2) of all the cases in Table 1, one would notice the following invariant rule on the number of the elements of S_i^{∞}:

1. Under the condition that $q_i'''(\text{or } \hat{q}_i) < q_i^*(\ldots q_k^*(q_h^*(k(1-c)/2))\ldots)$ and $q_j'''(\text{or } \hat{q}_j) < q_j^*(\ldots q_h^*(q_k^*(h(1-c)/2))\ldots)$ $(i, j, k, h = X, Y \text{ or } x, y; i \neq j; k \neq h)$ where each initial value is alternately nested n times by the quantity best-response correspondences:
   (1) If i = k and j = h, the n must be a positive even number, and the firms have n + 1 strategies in S_i^{∞} (e.g., R5, R6, (R6') and R7; n = 2);
   (2) If i = h and j = k, the n must be a positive odd number, and the firms have 2n − 1 strategies in S_i^{∞} (e.g., R1, R2, (R2'), and R3; n = 1: R9, n = 3).
2. Under the condition that $q_i'''(\text{or } \hat{q}_i) < q_i^*(\ldots q_k^*(q_h^*(k(1-c)/2))\ldots)$ but $q_j'''(\text{or } \hat{q}_j) > q_j^*(\ldots q_h^*(q_k^*(h(1-c)/2))\ldots)$ $(i, j, k, h = X, Y \text{ or } x, y; i \neq j; k \neq h)$ where each initial value is alternately nested n times by the two quantity best-response correspondences:
   (1) If i = k and j = h, the n must be a positive even number, and the firms have (n/2) + 1 strategies in S_i^{∞} (e.g., R4 (R4') and R8 (R8'); n = 2);
   (2) If i = h and j = k, n, n ≥ 3, must be a positive odd number, and the firms have [(n + 1)/2] + 1 strategies in S_i^{∞} (e.g., R10 (R10'); n = 3).

By using these formulas (and the corresponding condition (1)), one can partition the unshaded region around the origin in Figure 10. As one can expect from Figure 5 and this rule, the partition of the region becomes finer as (x, y) approaches to the origin.

Figures 7a and 7b show that different mixed-strategy equilibrium can arise for the same S_i^{∞} for i = X, Y in R1, R2, ..., R8 of Region F[5]. The division of each R1, R2, ..., R8 is determined by the non-negativity conditions of the probabilities assigned to the quantities chosen in the equilibria and the optimal condition of the equilibrium strategies. I indicate which elements of S_i^{∞} constitute the carriers of the mixed-strategy equilibrium probability functions below Figures 7a and 7b by using the numbers assigned to the quantities of S_i^{∞} graphically shown in Figures 6a-h. Note that as in the case of Region B, not all the survived strategies are always used in the mixed-strategy equilibrium. With those quantities, one can easily obtain the mixed-strategy probability functions and the expected profits. By assuming that the marginal cost of advertising does not increase at a

5) Clearly, this is true of the other parts of Region F than R1, R2, ..., R8.

very fast rate, my analysis of the advertising stage in the next subsection focuses on the situation where x and y are out of Region F. Thus, I do not go too far into the characterization of each mixed-strategy advertising subgame perfect Nash equilibrium when x and y are in Region F.

Finally, I would like to conclude this subsection by discussing Schmalensee's [1983] characterization of the Cournot-Nash equilibria of the post-advertising game. In his characterization of the Cournot-Nash equilibria, he presumed that the presence of the perfectly informed consumers always leads to the equalization of the two Cournot market prices. Based on this, he claimed that there are two kinds of Cournot-Nash equilibria of the post-advertising subgame: (1) when $y/(1+y) \leq x \leq y/(1-y)$ or "at least half the informed buyers must know of X and at least half must know of Y" (Schmalensee [1983] p. 641); both firms sell to the perfectly informed consumers in addition to their own captive consumers by producing a quantity, $(x+y-xy)(1-c)/3$ as if both products were sold in a single Cournot market; and (2) when $y/(1-y) < x \leq 1 (0 \leq x \leq y/(1-y))$, firm X (Y) sells to the consumers knowing only brand X (Y), firm Y (X) sells to all the consumers who know brand Y (X), and because of the presence of the perfectly informed consumers, the products are sold at the same price: $p_X = 1-q_X/[x(1-y)] = p_Y = 1-q_Y/y$ $(p_X = 1-q_X/x = p_Y = 1-q_Y/[y(1-x)])$.

Apparently, my characterization has disproved his claim. In particular, the examination in this section has shown that his presumption about the role of the perfectly informed consumers on the market prices is incorrect. That is, the arbitrage role played by the perfectly informed buyers is not sufficient for the market price equalization in equilibrium. In any mixed-strategy equilibrium, the market price of brand X can be different from that of brand Y with a positive probability. The market prices of the two brands are always equal only when the equilibrium is in pure strategies.

4. Advertising Equilibrium and Entry Decisions

In this section, I examine the possibility of strategic entry deterrence via strategic precommitment to advertising investment by the incumbent. My analysis of the advertising stage is confined to the situation in which firms optimally choose relatively larger levels of advertising under the assumption that the marginal cost of advertising increases steadily (the exact condition follows later). In other words, I exclude the possibility that the firms compete at the quantity-setting stage given that x and y are in Region F of Figure 3a. In this sense, I must admit the conclusion of my analysis is not completely general. The firms' profit functions at the advertising stage have many discontinuities in Region F since small changes in advertising levels abruptly replace the elements of S_i^{∞}, and the mixed-strategy subgame perfect Nash equilibria (See Figures 7a and 7b). This makes the exact examination of strategic entry deterrence intolerably complicated when x and y are in

Region F.

I start the analysis with the derivation of the advertising best-response correspondences. For the sake of the convenience of explanation, without loss of generality, I will focus on the derivation of firm X's advertising best-response correspondence given that firm Y's advertising level is fixed. Using the results of Proposition 1, I can obtain firm X's (re-scaled) profit function at the advertising stage (Here, I have already ruled out the possibility that x and y are in Region F):

$$V_X(x,y) = \begin{cases} x(1-y) - \dfrac{A}{2}x^2 & \text{if } (x,y) \in \text{Regions B \& C} \\ \dfrac{4(x+y-xy)}{9} - \dfrac{A}{2}x^2 & \text{if } (x,y) \in \text{Regions A} \\ r(x,y,n) - \dfrac{A}{2}x^2 & \text{if } (x,y) \in \text{Regions E} \\ x\left\{1 - \left[\dfrac{x}{2y(1-x)}\right]^2\right\} - \dfrac{A}{2}x^2 & \text{if } (x,y) \in \text{Regions D} \end{cases} \tag{10}$$

where

$$r(n,x,y) = \frac{x(2y+3x-2xy)}{4\left(\left\{\frac{3\cdot 2^{2n-1}}{4^{n+1}-1}\right\}y(1-x)+x\right)}; \tag{11}$$

n is defined in the same way as in Proposition 1. As to handling the profit of (11), when n is larger than or equal to 4, I will assume that firm X's profit is $\lim_{n\to\infty} r(n,x,y)$. This simplification can be justified because, when n is beyond 4, firm Y's profit is very close to its limit, and the set of x and y with $n \geq 4$ is very thin (See Figure 3b). In particular, observe that $(3 \cdot 2^{2\cdot 4-3})/(4^{4+1}-1) = 128/341 = 0.375367$, and $\lim_{n\to\infty}[(3 \cdot 2^{2n-1})/(4^{n+1} - 1)] = 3/8 = 0.375$. Thus, this approximation should not change the qualitative prediction of this model—which will be explicitly shown later. To avoid the firms choosing x and y in Region F, I assume that $1 < A < 1.57618$. I have verified that when A is smaller than this upper bound, firm X never chooses x so that x and y are in Region F [6].

6) The upper boundary of A is determined by the condition that when y = 0.265278, firm X's best-response changes from (1 – y)/A to 5y/(5y +4), the boundary between Region A and Region E.
The exact process to ensure that given y, firm X never chooses x so that x and y are in Region F is as follows. First I assumed that when x and y are in Region F, firm X earns the monopoly profit by selling to all the consumers who know brand X. Notice that firm X never earns more than this monopoly profit in any mixed-strategy equilibrium arising in Region F, and the monopoly profit is fairly low when x is low. Then, I made Mathematica programs to generate the figures of firm X's profit functions for 0 ≤ y ≤ 0.1459, 0.1459 ≤ y ≤ 0.1798, 0.1798 ≤ y ≤ 0.1910, 0.1910 ≤ y ≤ 0.2, 0.2 ≤ y ≤ 0.2490, 0.2490 ≤ y ≤ 0.2616, 0.2616 ≤ y ≤ 0.2644, 0.2644 ≤ y ≤ 0.2652, and 0.265224 ≤ y ≤ 0.265278. The intervals are determined by considering the discontinuous change in firm X's profit function. Then, I ensured that (, y) is out of Region F under the restriction of the value of A.

Figure 8a and 8b have firm X's advertising profit functions with A being 12/11 when x is 0.3, 0.8, respectively. Figure 8a shows that when y is relatively low, firm X optimally chooses a relatively large x to earn the (re-scaled) advertising revenue $x(1-y) : x^* = (1-y)/A$. In the case of Figure 8b, firm X's best response is a boundary value between Region E and Region A, 5y/(5y + 4) (See Figure 8c)[7]. These two figures indicate a certain critical value of y, $\bar{y}$ at which firm X changes the type of its best-response advertising levels. The critical value can be identified by the condition

$$\left\{\frac{x(2y+3x-2xy)}{4\{(3/8)y(1-x)+x\}} - \frac{A}{2}x^2\right\}_{x=\frac{5y}{5y+4}} = \frac{(1-y)^2}{2A}. \tag{12}$$

When A = 12/11, $\bar{y}$= 0.322381.

From what I have shown, it is easy to derive the advertising best-response correspondences. A pair of the advertising best-response correspondences when A = 12/11 is shown in Figure 9. The shape of the advertising best-response correspondence can be shown to be similar for 1 < A < 1.57618. An immediate observation is that the simultaneous-move advertising subgame has no advertising subgame perfect equilibrium in pure strategies, but in mixed strategies[8]. The shape of the advertising best-response correspondences suggests that the firms' advertising levels are neither globally strategic complements nor strategic substitutes.

With the advertising best-response correspondences, I can now examine the possibility of strategic entry deterrence. Figure 10a has firm X's and Y's profit functions, $V_X(x, y^*(x))$, and the monopoly profit, $V_X^m(x) = x - Ax^2/2$ with $A = 12/11$. As seen from the figure, when F is larger than $V_X^m(x^m)$, even the incumbent does not enter the market since no firm earns more than the monopoly profit. When $V_Y(x^m, y^*(x^m)) \leq F \leq V_X^m(x^m)$, entry is blockaded since when firm X simply advertises as a monopolist, firm Y cannot earn an overall positive profit by entry. When $V_Y(\bar{x}, y^*(\bar{x})) \leq F \leq V_Y(x^m, y^*(x^m))$, firm X can find x^b to make firm Y's overall profit zero where $\bar{x} \leq x^b \leq x^m$ satisfies $V_Y(x^b, y^*(x^b)) = F$, and $V_X^m(x^b) \geq V_X(x^s, y^*(x^s))$ and $V_X^m(x^b) > F$. Thus, firm Y stays out of the market; entry is effectively impeded. This shows that the optimal entry deterrence of this game actually involves underinvestment in advertising by the incumbent, as Schmalensee [1983] originally claimed on the base of the incorrect characterization of

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7) Significantly, this result shows that the above approximation of firm Y's equilibrium profits does not change the overall implication of this model.

8) The same observation has been made by Schmalensee [1983]. In fact, he also derived similar advertising best-response correspondences based on his characterization of the post-advertising Cournot equilibria (Schmalensee 1983, p. 645).

the post-advertising Cournot equilibria. When $0 \leq F \leq V_Y(\bar{x}, y^*(\bar{x}))$, firm X has no strategies to make firm Y's overall profit non-negative, and sets x^S in anticipation of firm Y's entry. Thus, firm Y can profitably enter the market; entry is easy (See Figure 10b). It can be shown that qualitatively similar outcome above can hold for all $1 < A < 1.57618$; in particular, strategic entry deterrence is possible, and always involves strategic precommitment to underinvestment in advertising by the incumbent.

As a summary of what I have shown in this subsection, I establish the following proposition.

Proposition 2: (Subgame Perfect Entry Equilibrium).

In the game where, at stage 1, a firm (incumbent) decides whether to enter a market and a level of informative advertising; at stage 2, a firm (entrant) decides whether to enter the market and a level of informative advertising; and at stage 3, the two firms compete in quantities; the incumbent may strategically deter entry through underinvestment in advertising. Nevertheless, when the sunk fixed cost of advertising is relatively larger, entry may be blockaded. Besides, if the sunk fixed cost of advertising is relatively smaller, the incumbent may profitably enter the market.

The robustness of this proposition should be discussed before I end this section. First of all, I must refer to what would happen if the marginal advertising cost is relatively high, namely, A = 2, 3, 4, and so on. If A takes such a value, firm Y's advertising best-response correspondence may have a discontinuity that $(x, y^*(x))$ jumps from Region D into Region F. But, it would be true that $V_Y(x^m, y^*(x^m))$ is smaller than $V_X^m(x^m)$ and $V_Y(x, y^m(x))$ increases with x in the neighborhood of x^m. This indicates that even if A is large and the firms select x and y in Region F, strategic entry deterrence via advertising would involve underinvestment in advertising by the incumbent. Second, the choice of the quadratic advertising cost function is not critical to the result. For example, as Schmalensee [1983] originally assumed, even if the function is logarithmic, the qualitatively same equilibrium result of strategic entry deterrence can arise since the advertising best-response correspondence resembles the one I have derived above.

5. Conclusions

Schmalensee [1983] examined whether an incumbent can prevent a new firm's entry by engaging in informative advertising when the strategic variables at the post-advertising stage are quantities. Assuming that the two brands are sold at the same price because of the presence of perfectly informed consumers, he claimed that the post-advertising Cournot game always has pure-strategy Nash equilibria. Then, based on his claim, he showed that optimal entry deterrence always involves strategic precommitment to underinvestment in informative advertising by the incumbent.

This article reexamined his strategic entry-deterrence model via advertising. I have first shown that when the number of consumers knowing of only one brand is relatively greater than that of consumers knowing of only the other brand as a result of the advertising competition, mixed-strategy Cournot-Nash equilibria can arise. This disproves his characterization of the post-advertising Cournot-Nash equilibria, and invalidates his conclusion that optimal entry deterrence may involve advertising less than when entry threat is absent. Yet, I have shown that his conclusion remains qualitatively true even if the firms use the correct Cournot-Nash equilibrium strategies at the post-advertising stage.

Received: June 22, 1999

Figures

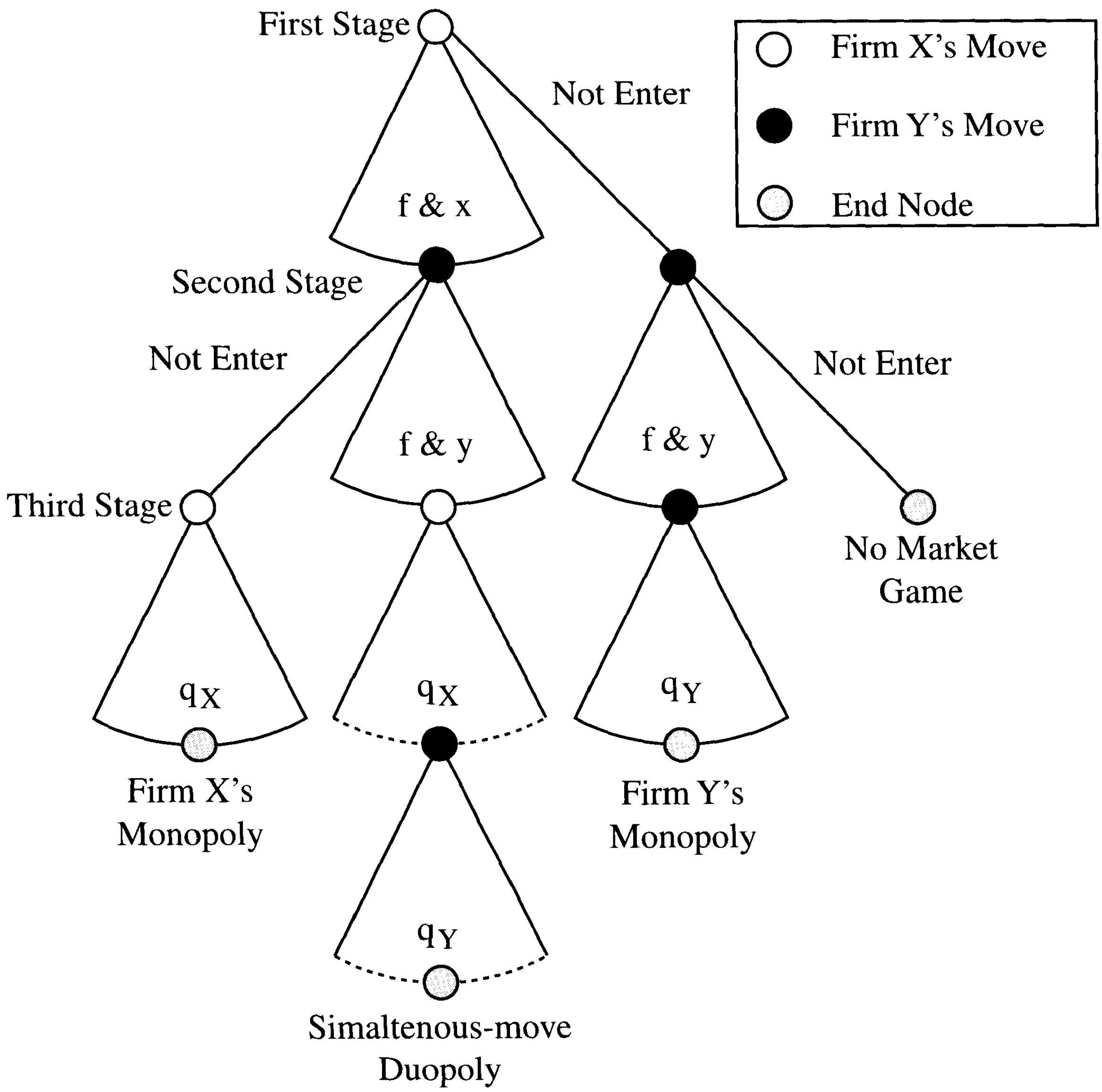


Figure 1. The Extensive Form of the Three-stage Game

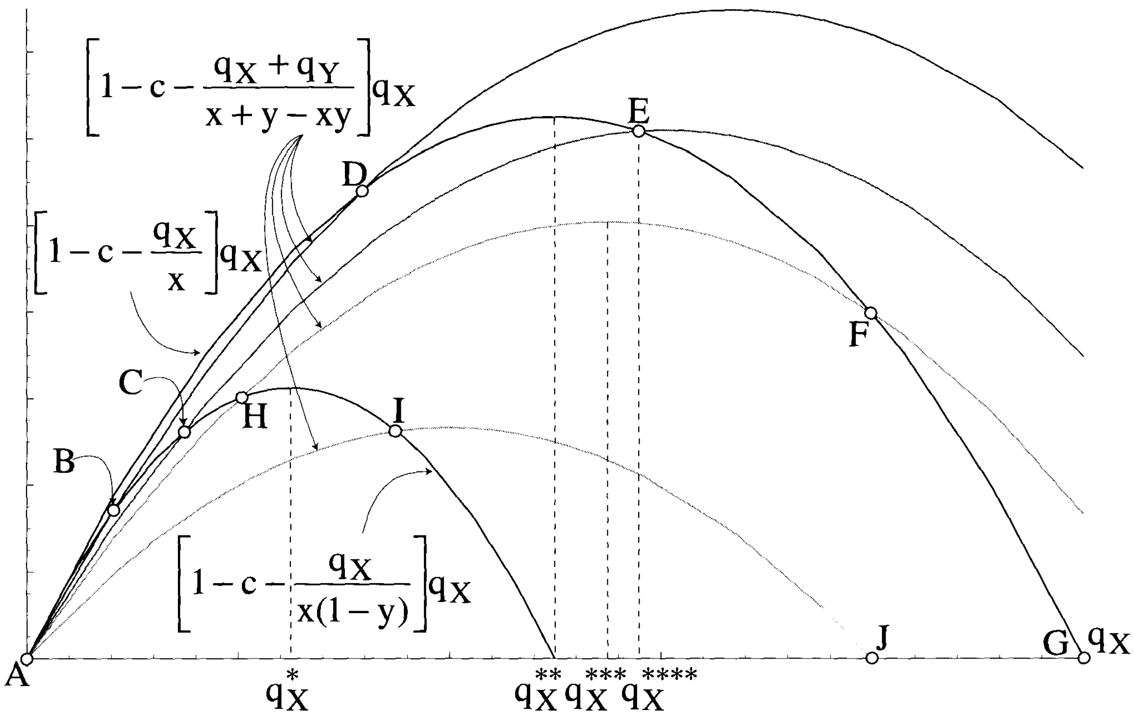


Figure 2a. Firm X's Profit Function of Quantities: $x \geq (\sqrt{y}-y)/(2+\sqrt{y}-y)$

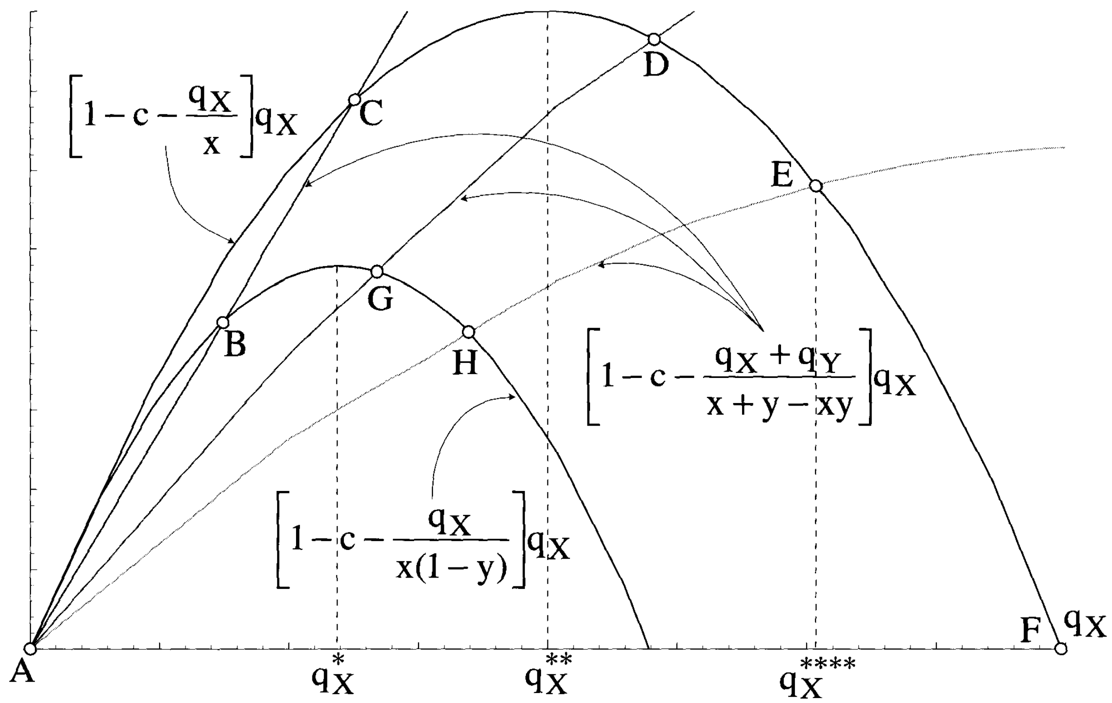


Figure 2b. Firm X's Profit Function of Quantities: $x \leq (\sqrt{y}-y)/(2+\sqrt{y}-y)$

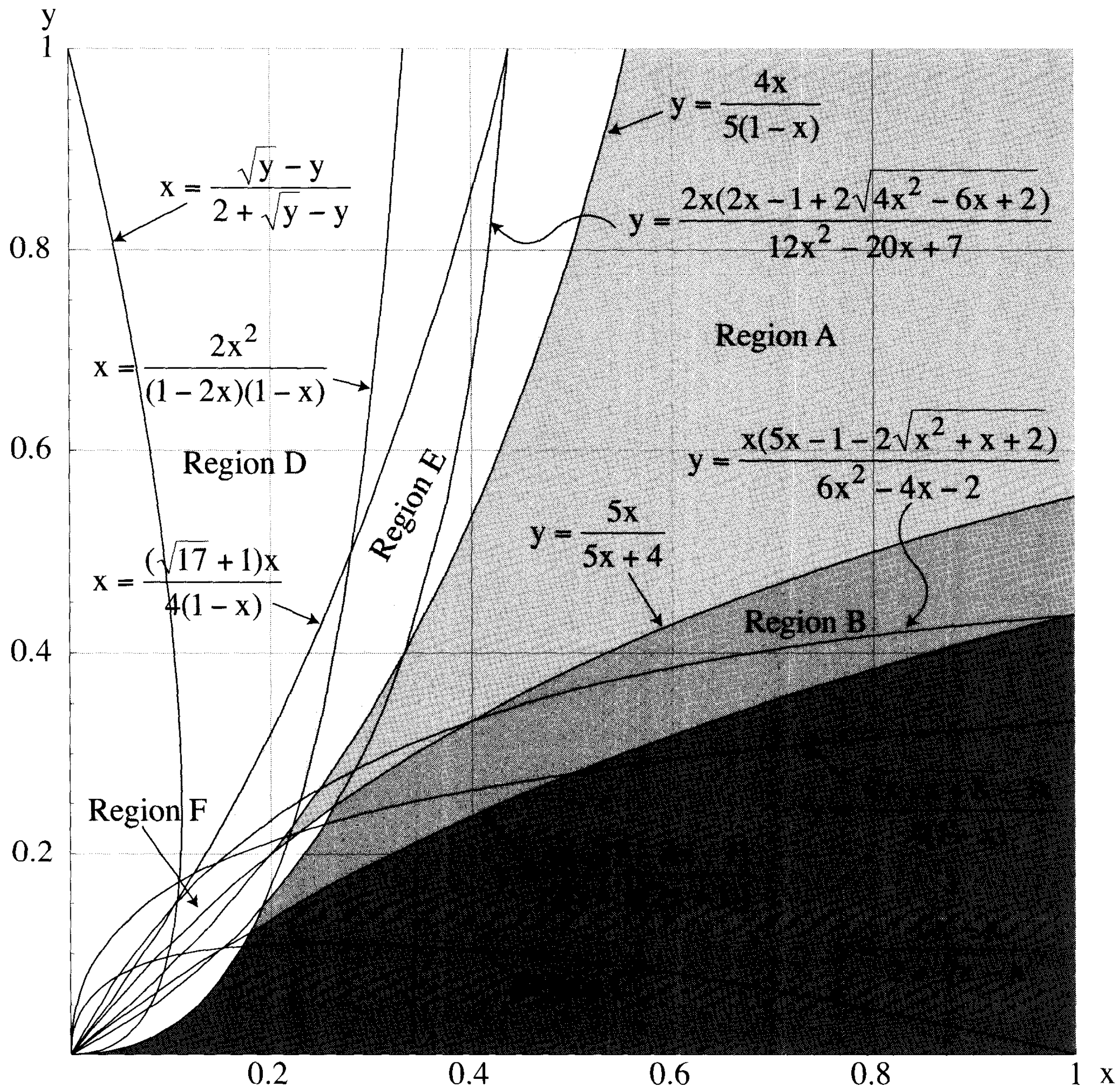


Figure 3a. The Cournot-Nash Equilibrium of the Post-advertising Game

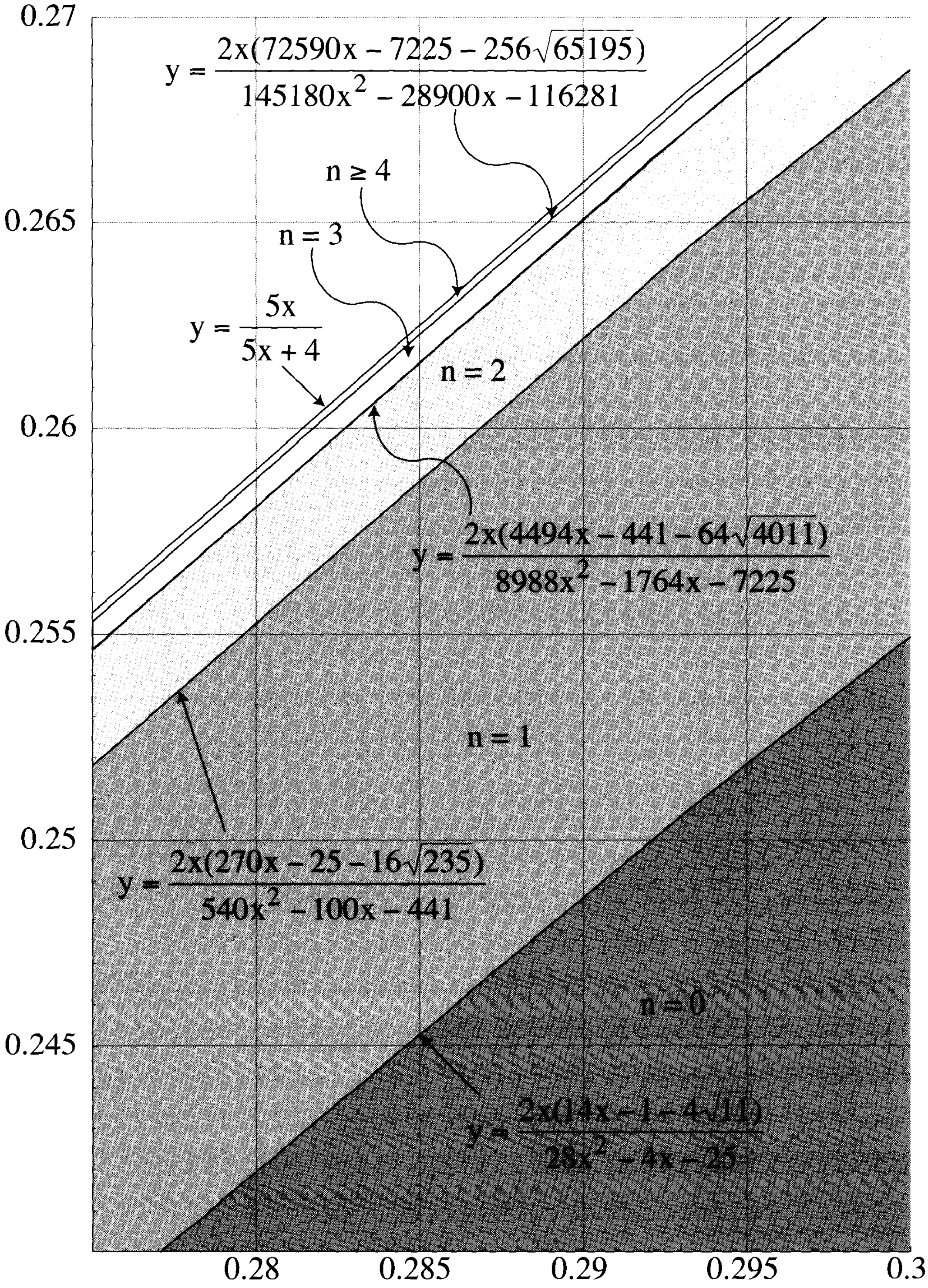


Figure 3b. The Cournot-Nash Equilibria of the Post-advertising Game: Region B

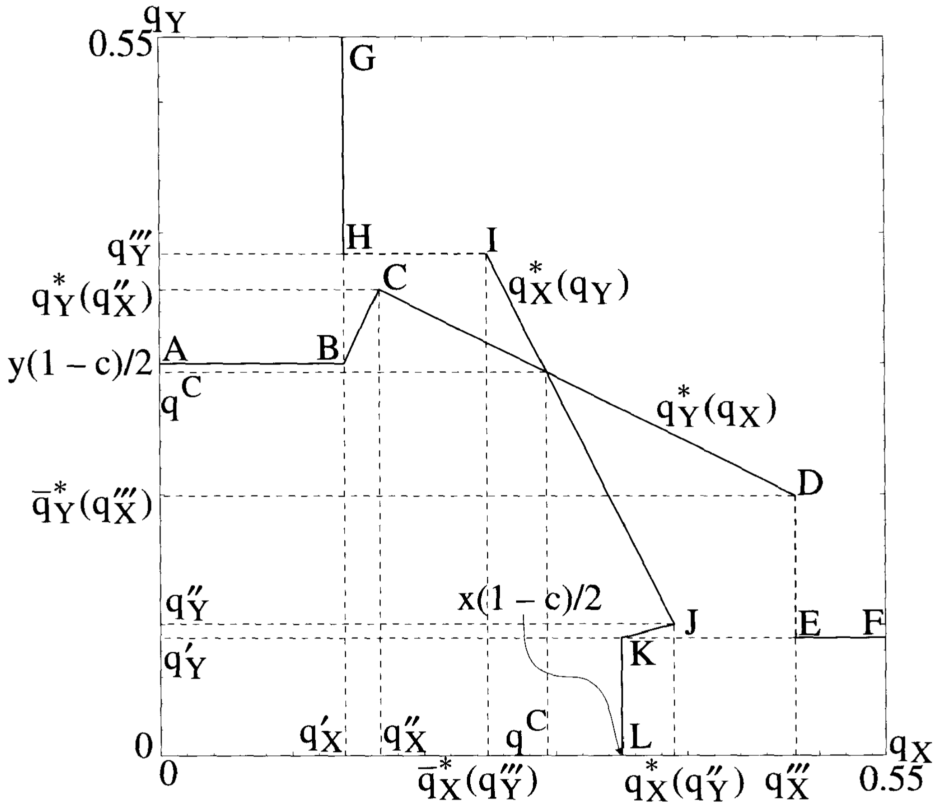


Figure 4a. Quantity Best-response Correspondences: Region A (x = 0.7 and y = 0.6)

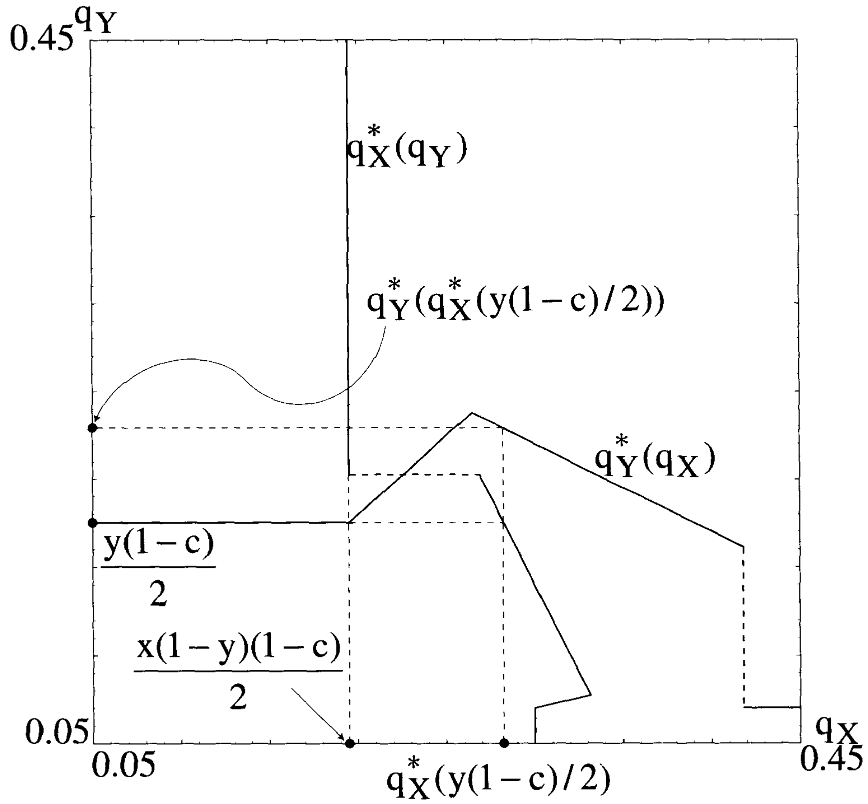


Figure 4b. Quantity Best-response Correspondences: Region B (x = 0.6 and y = 0.35)

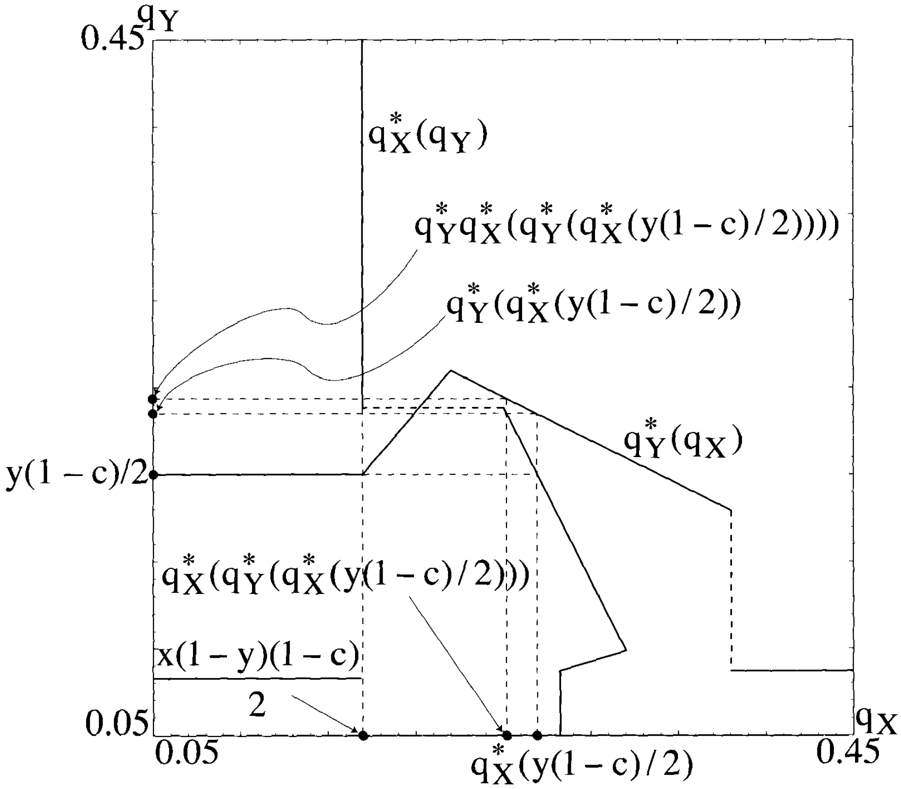


Figure 4c. Quantity Best-response Correspondences: Region B (x = 0.565 and y = 0.4)

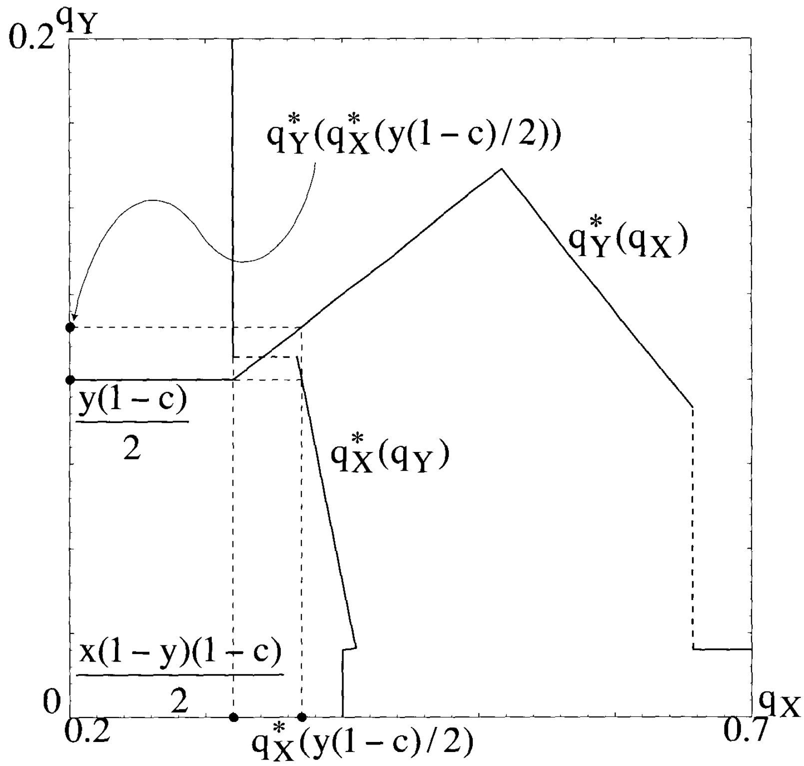


Figure 4d. Quantity Best-response Correspondences: Region C (x = 0.8 and y = 0.2)

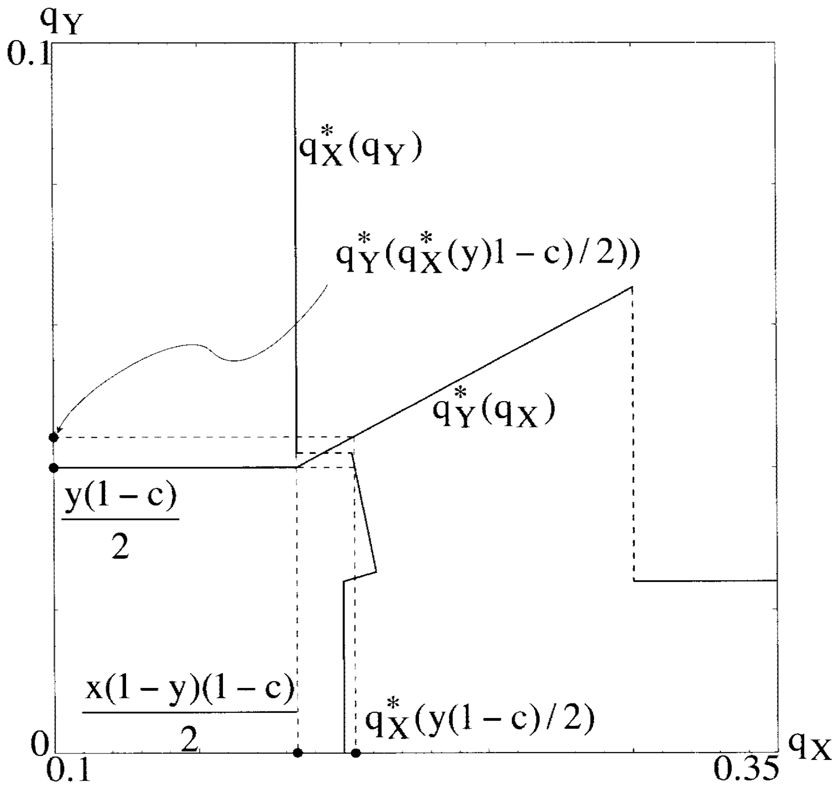


Figure 4e. Quantity Best-response Correspondences: Region C (x = 0.4 and y = 0.08)

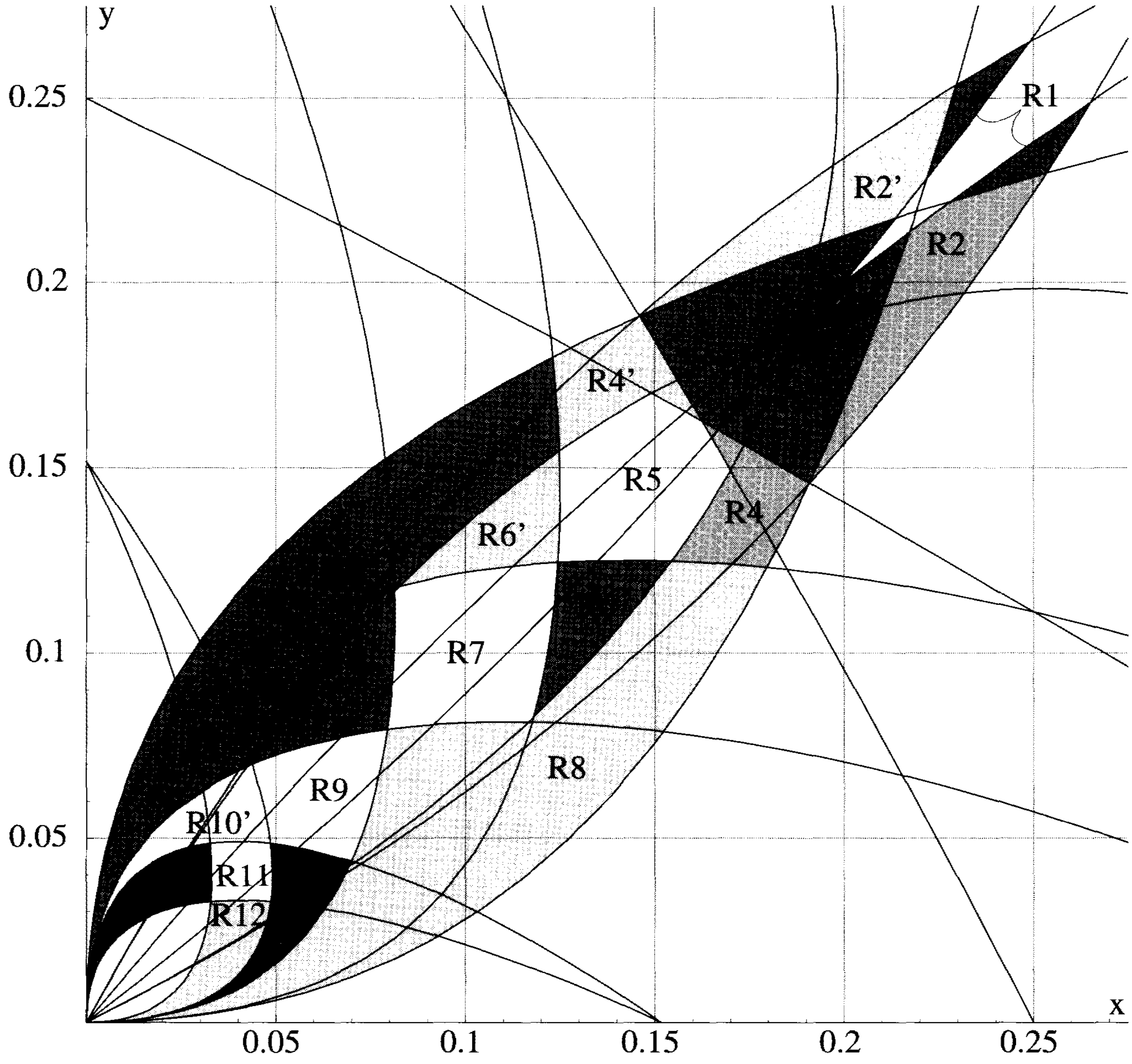


Figure 5.The Regions in Which Firms Have Different Pure Strategies that Survive Iterated Strict Dominance: Region F

Table 1. The Regions in Which the Firms Have Different Pure Strategies that Survive Iterat ed Strict Dominance: the Critical Mathematical Conditions.

When (x, y) is in	The Mathematical Conditions are Satisfied:
R1	(1) $x(1-c)/2 \leq q''_X$ and $y(1-c)/2 \leq q''_Y$;
	(2) $q'''_X < q^*_X(y(1-c)/2)$ and $q'''_Y < q^*_Y(x(1-c)/2)$
R2†	(1) $x(1-c)/2 \geq q''_X$ and $y(1-c)/2 \leq q''_Y$;
	(2) $q'''_X < q^*_X(y(1-c)/2)$ and $q'''_Y < q^*_Y(x(1-c)/2)$.
R3	(1) $x(1-c)/2 \leq q''_X$ and $y(1-c)/2 \leq q''_Y$;
	(2) $q'''_X < q^*_X(y(1-c)/2)$ and $q'''_Y < q^*_Y(x(1-c)/2)$.
R4†	(1) $x(1-c)/2 \leq q''_X$ and $y(1-c)/2 \leq q''_Y$;
	(2) $q'''_X < q^*_X[q^*_X(y(1-c)/2)]$ and $q^*_X[q^*_Y(x(1-c)/2)] \leq q'''_X$
R5	(1) $x(1-c)/2 < q''_X$ and $y(1-c)/2 < q''_Y$;
	(2) $q'''_X < q^*_Y[q^*_X(y(1-c)/2)]$ and $q'''_Y < q^*_X[q^*_Y(x(1-c)/2)]$
R6†	(1) $x(1-c)/2 < q''_X$ and $y(1-c)/2 < q''_Y$;
	(2) $q'''_X < q^*_Y[q^*_X(y(1-c)/2)]$ and $q'''_Y < q^*_X[q^*_Y(x(1-c)/2)]$.
R7	(1) $x(1-c)/2 < q''_X(\text{or } \hat{q}_X)$ and $y(1-c)/2 < q''_Y(\text{or } \hat{q}_Y)$;
	(2) $q'''_X < q^*_X[q^*_X(y(1-c)/2)]$ and $q'''_X(\text{or } \hat{q}_X) < q^*_X[q^*_Y(x(1-c)/2)]$.
R8†	(1) $x(1-c)/2 \leq q''_X(\text{or } \hat{q}_X)$ and $y(1-c)/2 \leq q''_Y$
	(2) $q'''_X \leq q^*_Y[q^*_X(y(1-c)/2)]$ and $q^*_X[q^*_Y(x(1-c)/2)] \leq q'''_X(\text{or } \hat{q}_X)$
R9	(1) $x(1-c)/2 < \hat{q}_X$ and $y(1-c)/2 < \hat{q}_Y$;
	(2) $\hat{q}_X < q^*_X(q^*_Y[q^*_X(y(1-c)/2)])$ and $\hat{q}_Y < q^*_Y(q^*_X[q^*_Y(x(1-c)/2)])$.
R10†	(1) $x(1-c)/2 \leq q''_X$ and $y(1-c)/2 \leq q''_Y$
	(2) $\hat{q}_Y \leq q^*_X(q^*_Y[q^*_X(y(1-c)/2)])$ and $q^*_Y(q^*_X[q^*_Y(x(1-c)/2)]) \leq \hat{q}_X$

Note: † Symmetric conditions must hold for R2', R4', R6', R8', and R10', respectively.

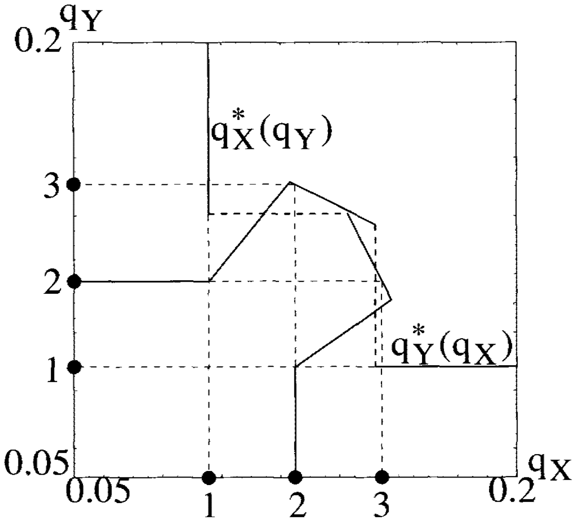


Figure 6a. R1: (x, y) = (0.25, 0.235)

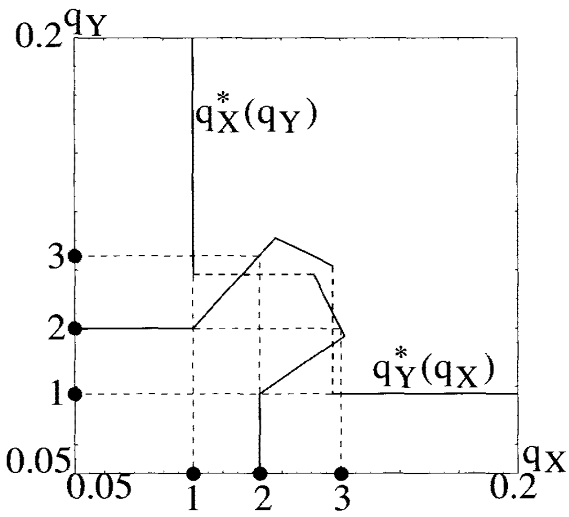


Figure 6b. R2: (x, y) = (0.225, 0.2)

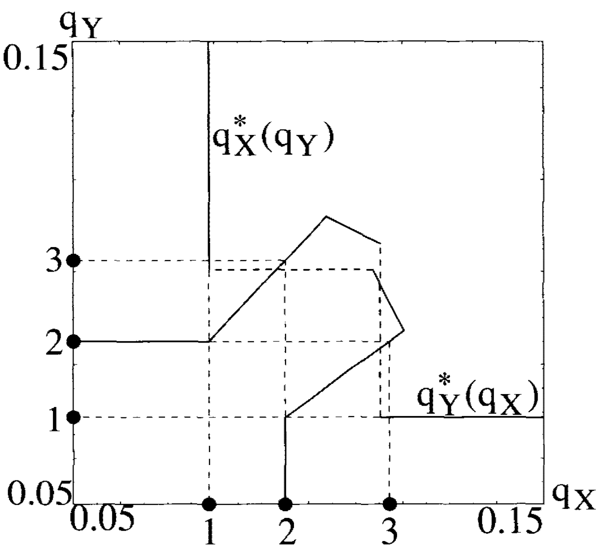


Figure 6c. R3: (x, y) = (0.19, 0.17)

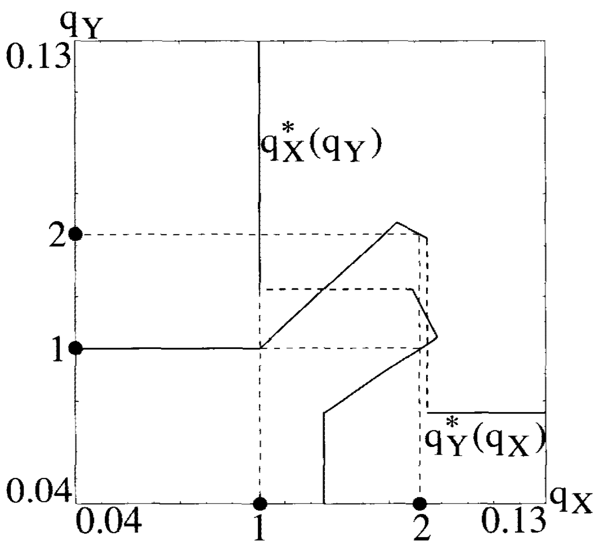


Figure 6d. R4: (x, y) = (0.175, 0.14)

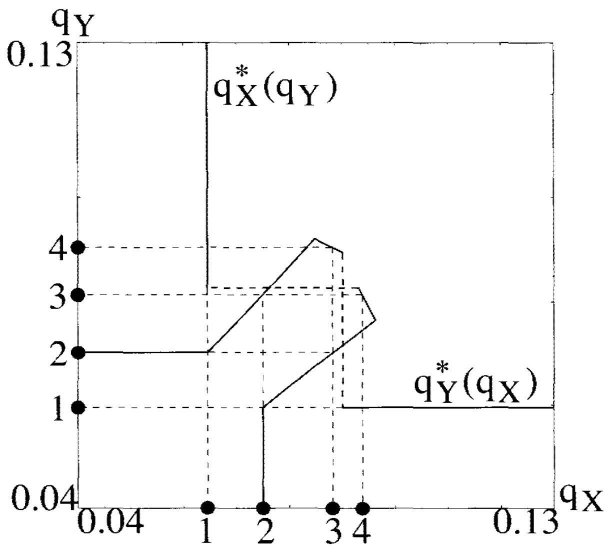


Figure 6e. R5: (x, y) = (0.15, 0.14)

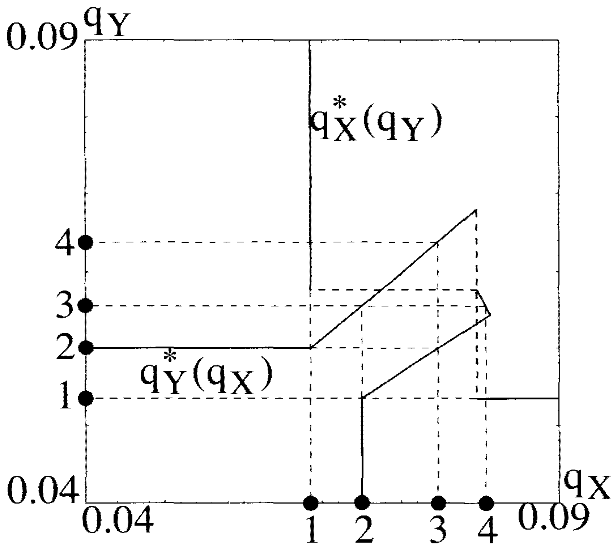


Figure 6f. R6: (x, y) = (0.13, 0.1)

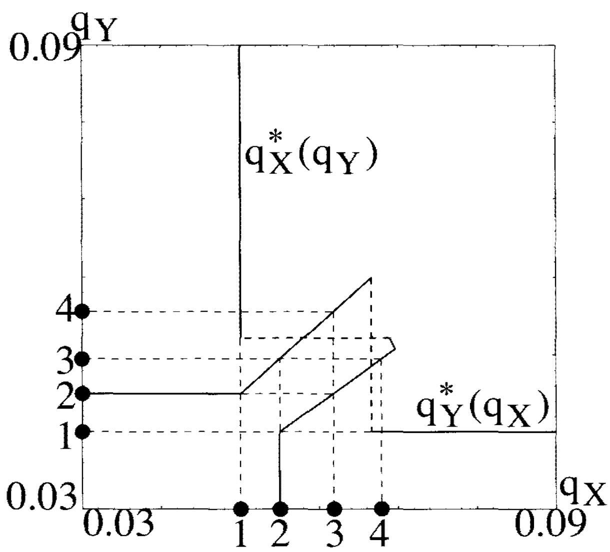


Figure 6g. R7: (x, y) = (0.11, 0.09)

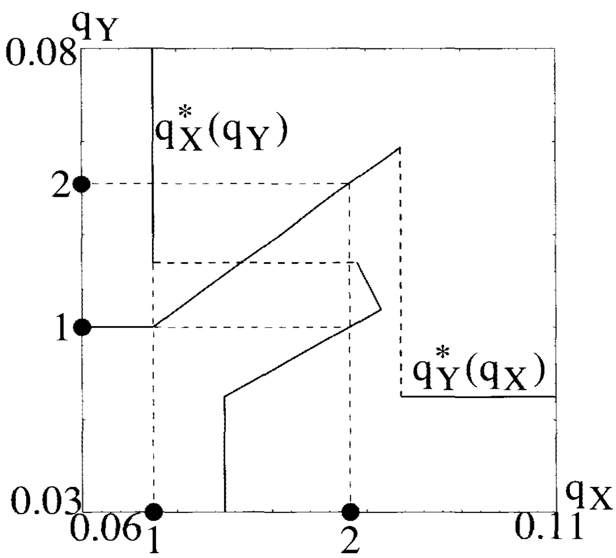
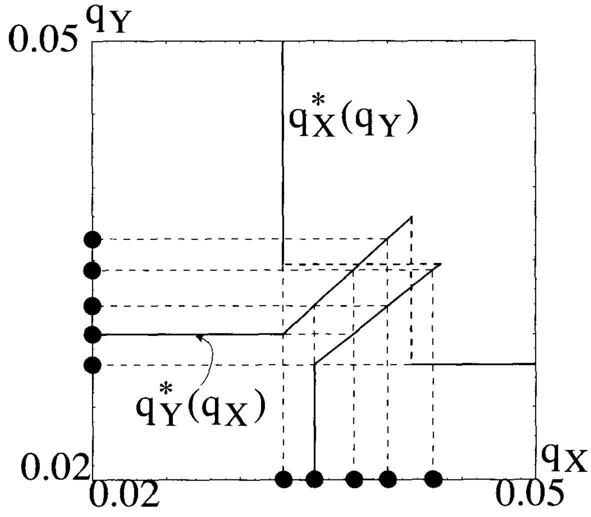


Figure 6h. R8: (x, y) = (0.15, 0.1)



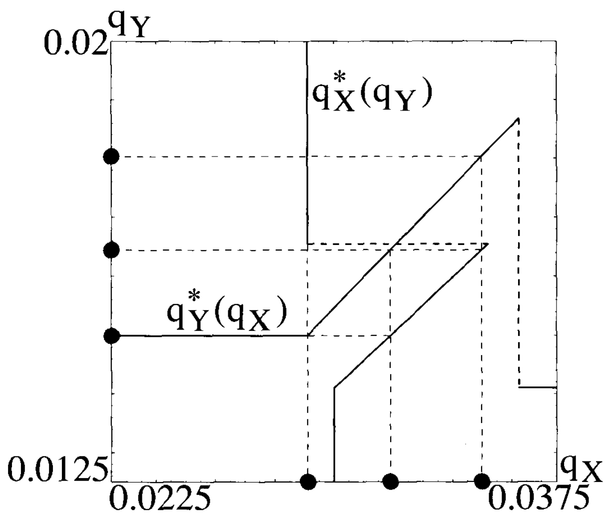
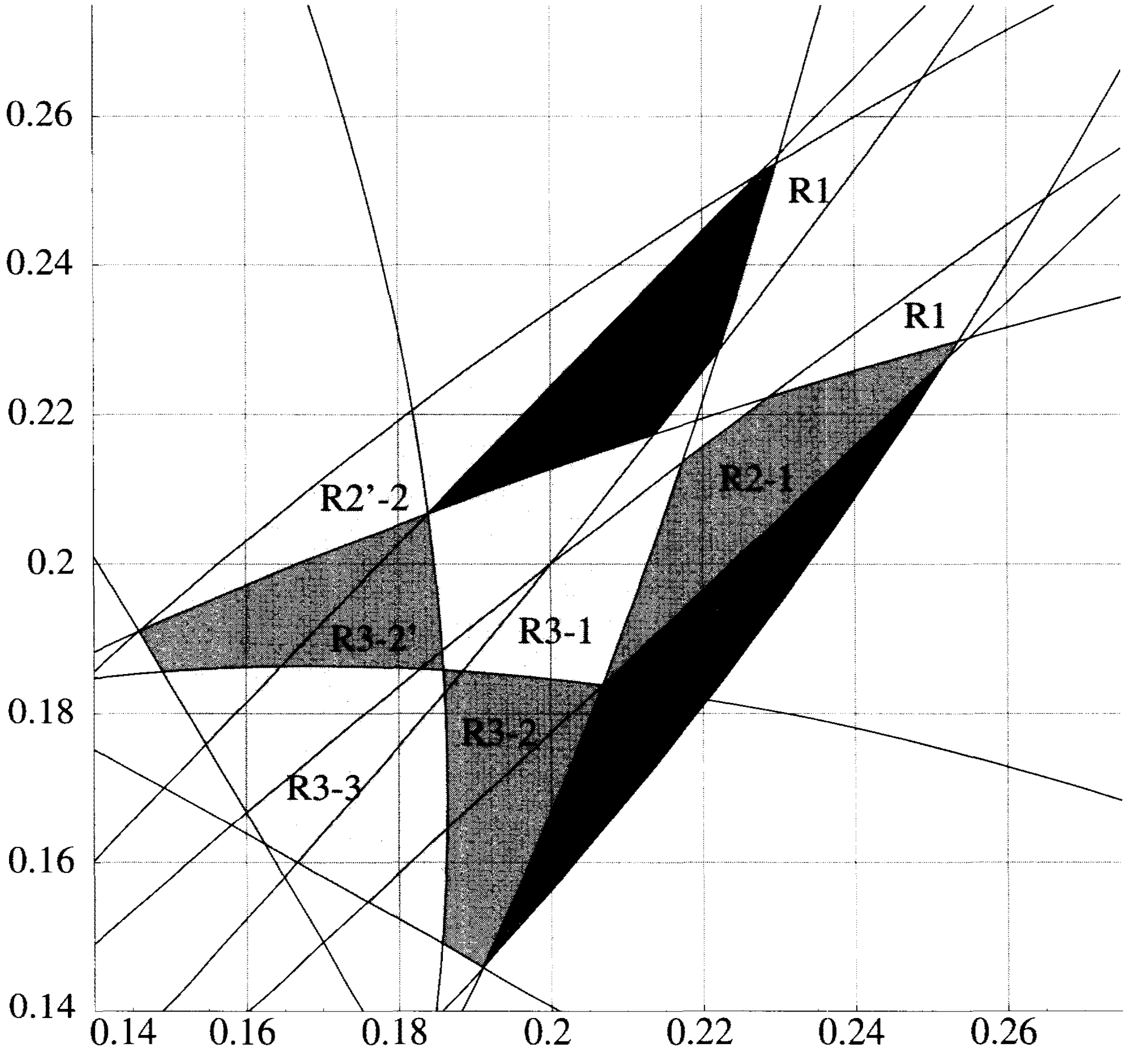


Figure 6i. R9: (x, y) = (0.07, 0.06)

Figure 6j. R10: (x, y) = (0.06, 0.03)

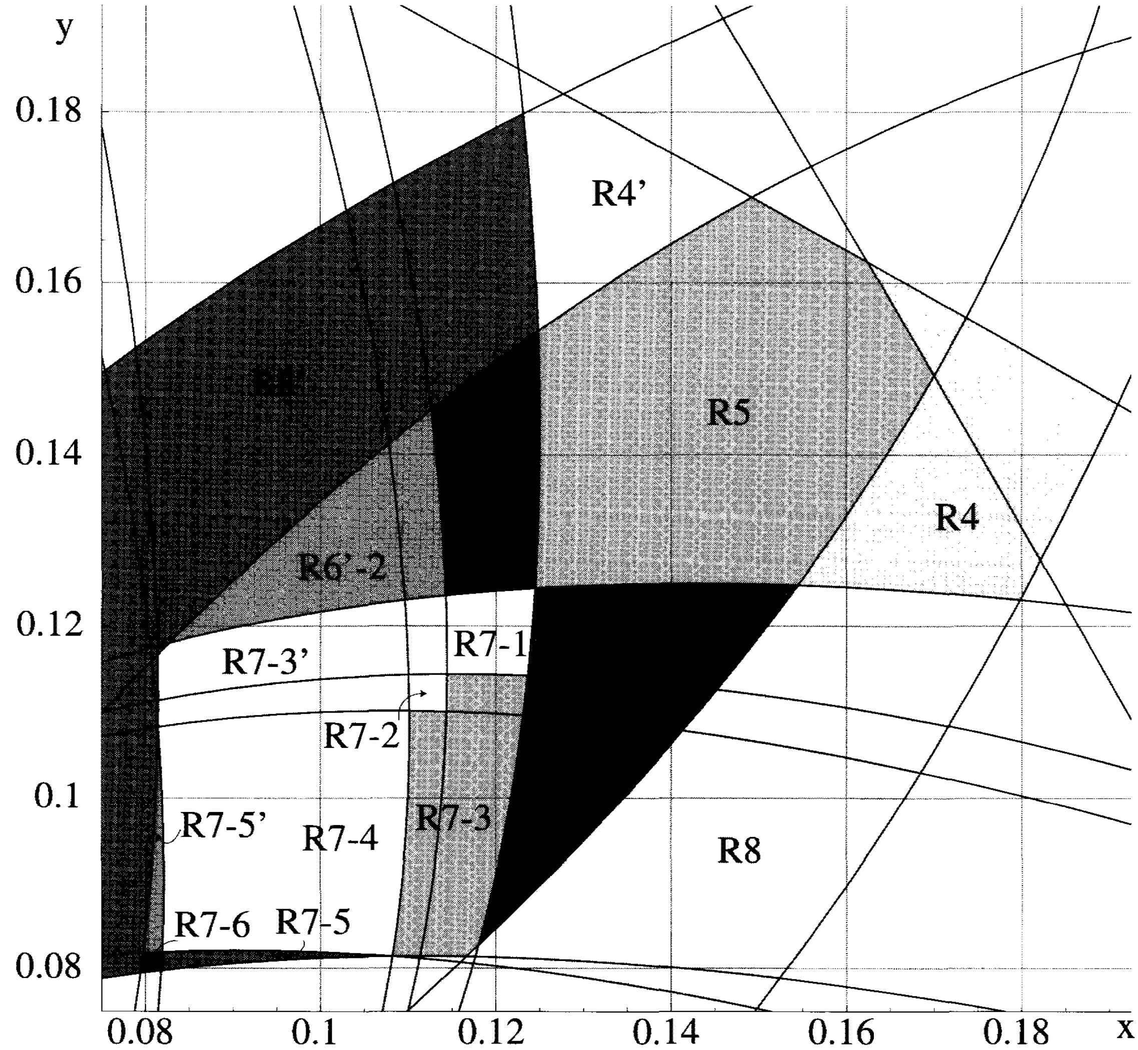
Figure 6. Quantity Best-response Correspondences: Region F



When (x, y) is in	The Carrier of the Mixed-strategy Equilibrium: {(Firm X's Carrier), (Firm Y's Carrier)}
R1, R2-1, (R2'-1), R3-1	{(1, 2, 3), (1, 2, 3)}
R2-2, R3-2, R3-3	{(1, 3), (2, 3)}
R3-2', (R2'-2), R3-3	{(2, 3), (1, 3)}
R3-3	{(1, 3), (1, 3)}

Note: When x and y are in R3-3, the post-advertising Cournot game has three mixed-strategy equilibria.

Figure 7a. The Cournot-Nash Equilibria of the Post-advertising Game: Region F



When (x, y) is in	The Carrier of the Mixed-strategy Equilibrium: {(Firm X's Carrier), (Firm Y's Carrier)}
R4, (R4'), R8, (R8'), R7-6	{(1, 2), (1, 2)}
R5, R6-1, (R6'-1), R7-1	{(1, 3, 4), (1, 3, 4)}
R6-2, R7-2, R7-3,	{(1, 3, 4), (2, 3, 4)}
R6'-2, R7-2, R7-3'	{(2, 3, 4), (1, 3, 4)}
R7-4	{(2, 3, 4), (2, 3, 4)}
R7-5, R7-6	{(2, 3, 4), (1, 2, 4)}
R7-5', R7-6	{(1, 2, 4), (2, 3, 4)}
R7-2	{(1, 2, 3, 4), (1, 2, 3, 4)}

Note: When x and y are in R7-2, or R7-6, the post-advertising Cournot game has three mixed-strategy equilibria.

Figure 7b. The Cournot-Nash Equilibria of the Post-advertising Game: Region F

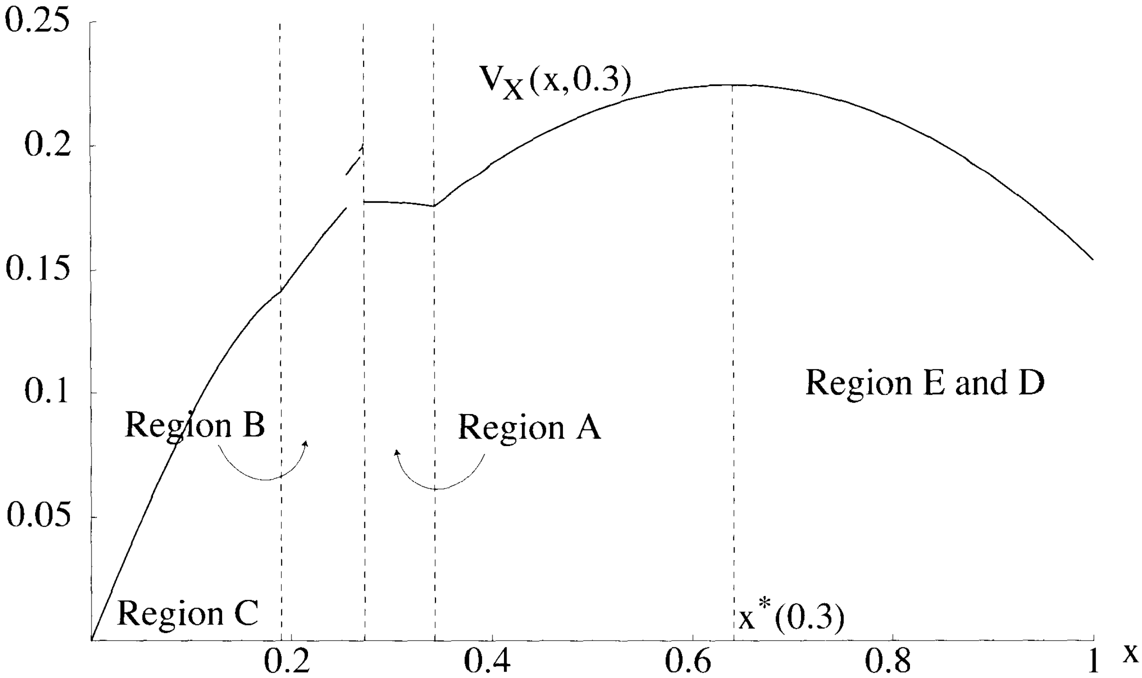


Figure 8a. Firm X's Advertising Profit Function of x given y: A =12/11 and y = 0.3

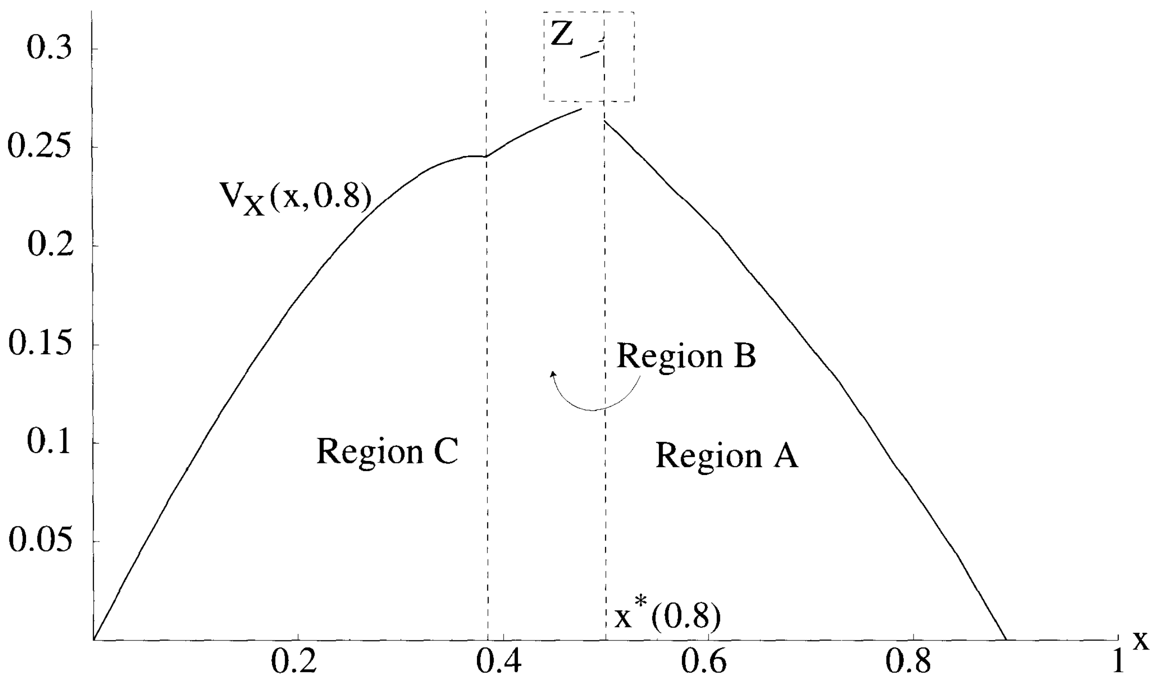


Figure 8b. Firm X's Advertising Profit Function of x given y: A =12/11 and y = 0.8

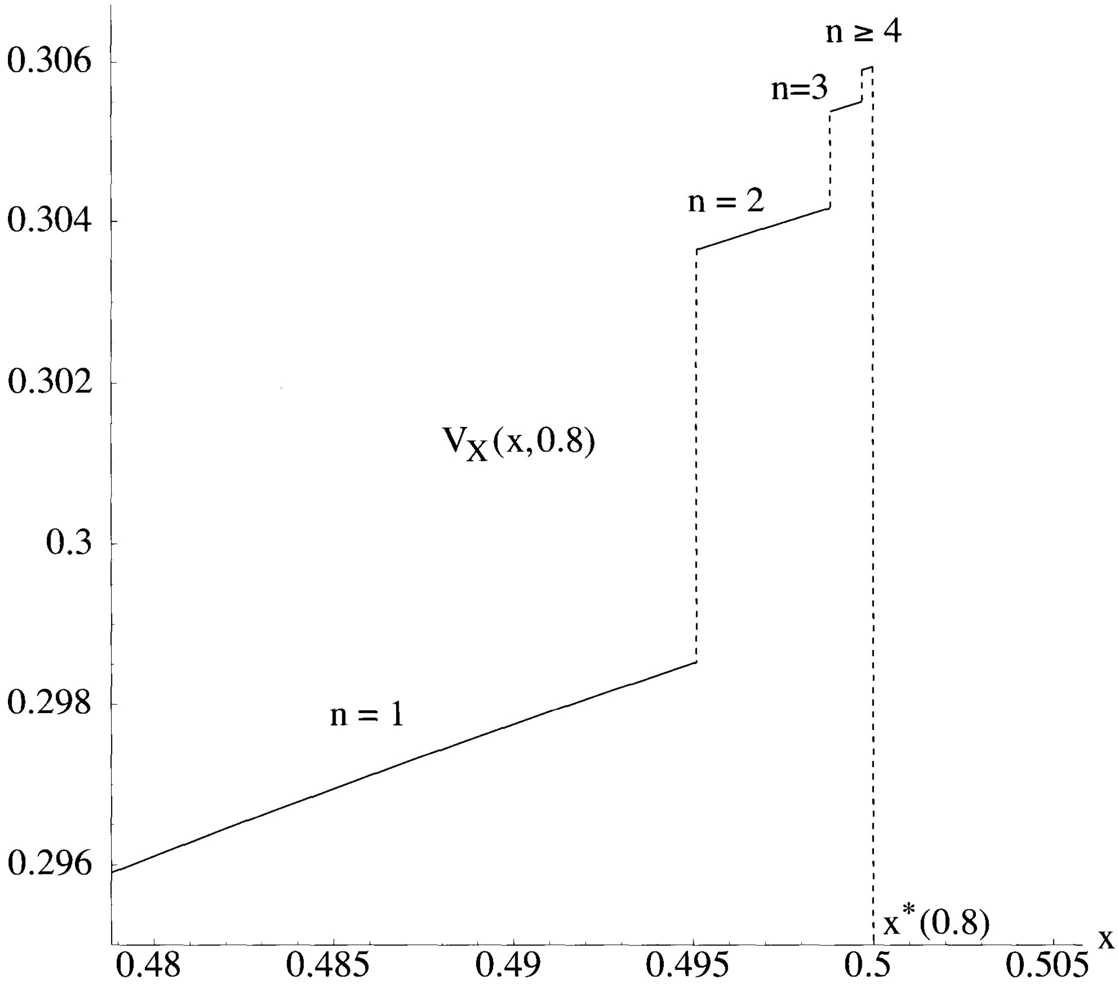


Figure 8c. Magnified part Z of Figure 8b

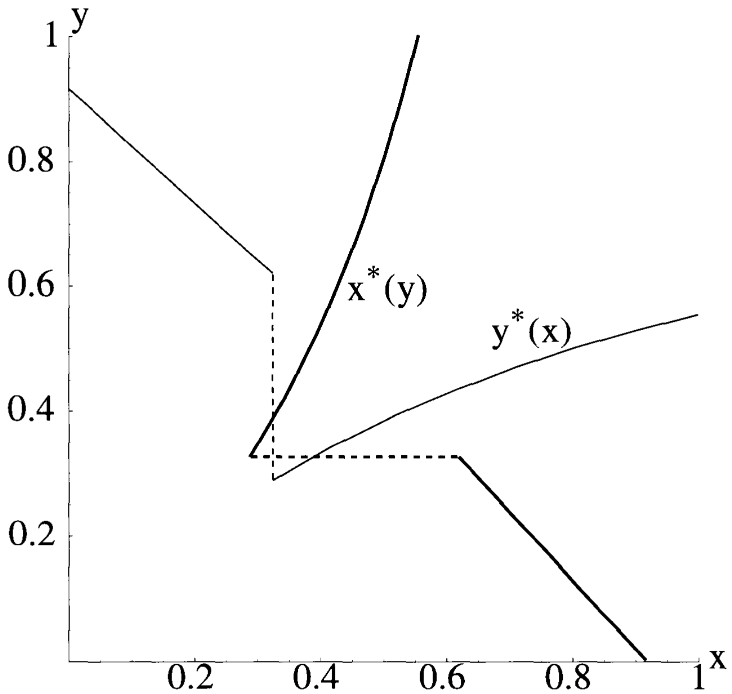


Figure 9. Advertising Best-response Correspondences: A = 12/11

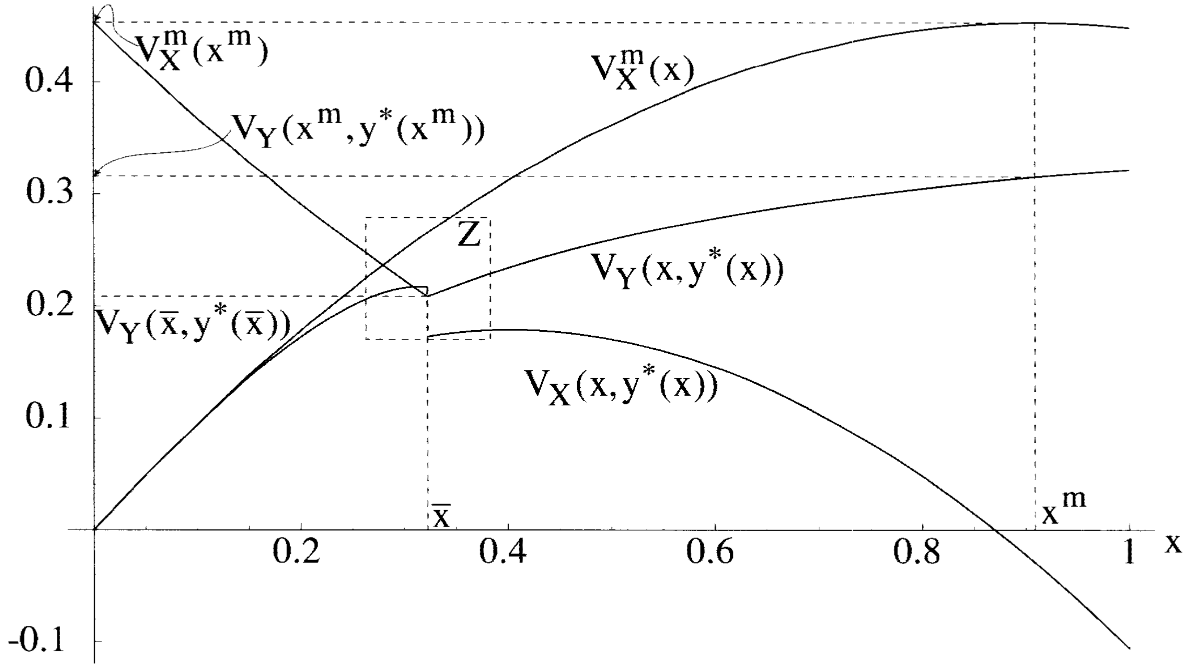


Figure 10a. Entry Deterrence: Variable Quadratic Cost with A = 12/11

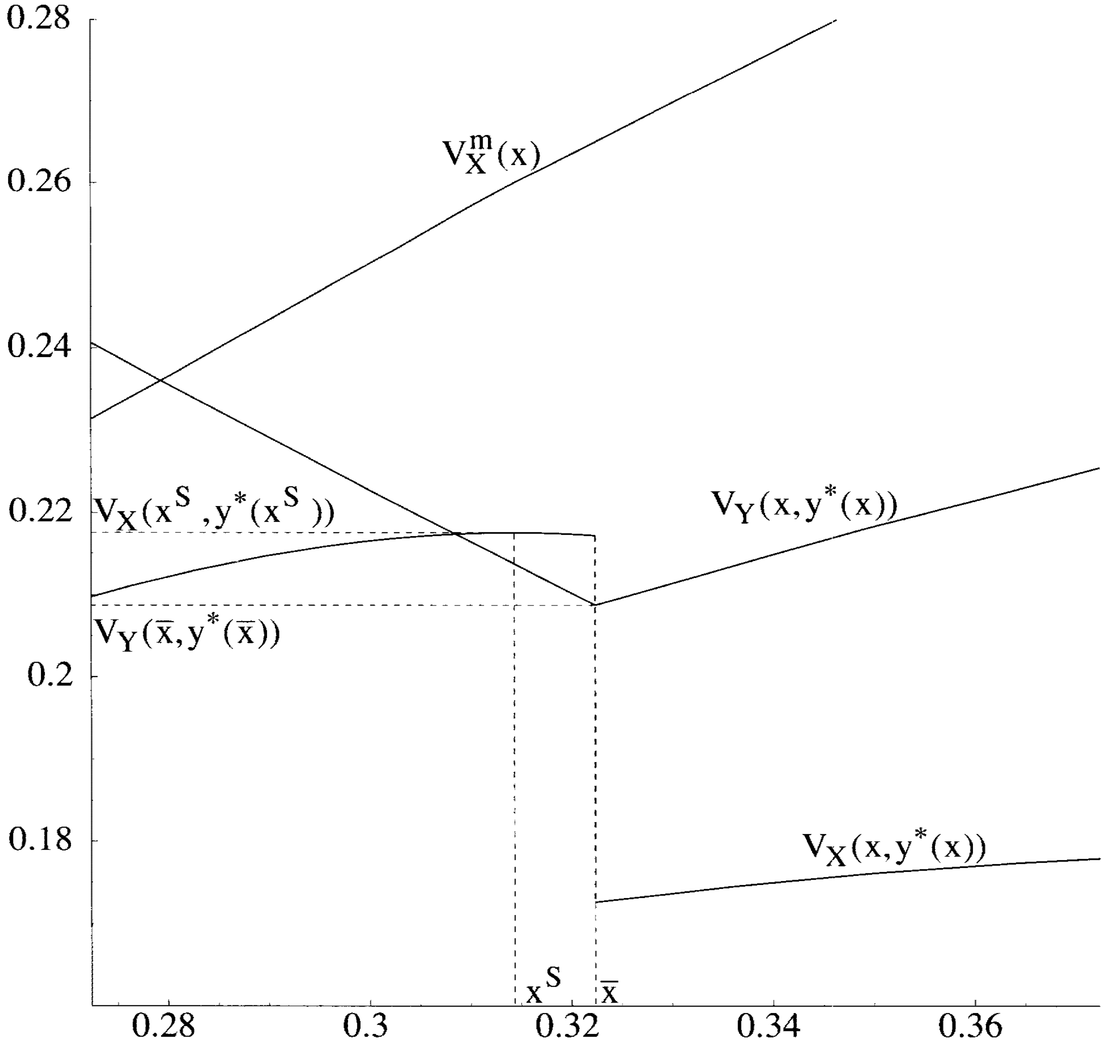


Figure 10b. Magnified part Z of Figure 16a

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