

Research Paper

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## Basic Considerations for the Optimal Allocation of Three Modes Intercontinental Transportation of Seasonal Products

Takashi AMEMIYA Katsuhisa OKITA

**[Summary]** There is a trend toward that world's manufacturing sites are moving to East Asia. After the production, these products are transported to the advanced nations for their consumption demands. Among such advanced nations, U.S.A. has the largest demand and then Japan and other European countries follow. It should be noted that these Asian districts used for the production sites are rather restricted. Therefore, the volume of products transported from these restricted districts of Asia to U.S.A. is becoming tremendously large. This situation is causing very serious traffic problems. The new products are required to be transported swiftly by air. Once the consumption and market demands are stable, the products should be sent rather slowly but in larger amount. However, the airports of China are quite restricted and while the amount of transportation is becoming large. As a result, transportation cost is increasing and also the time for transportation is increasing. Here, the third method is appearing. This is the so-called Sea and Air transportation. The cost and time in transit take mean positions between Air and Ocean flight services. There exists no reasonable strategy how allocate these three methods for the transportation. This paper is an attempt to theoretically describe this mechanism and to find out the optimal result.

**keyword** Transportation, Intercontinental, Multi-mode, Sea-and-Air, Optimal.

# 1 Introduction

It is the well known fact that many advanced nations have established many farms in far east for rather obtainable labor cost and transport their products to the many consuming countries. These consuming countries are usually developed countries and, therefore, the products are concentrated to rather small number of hub ports for their transportation. Among these products, there appear many seasonal products such as textile products and Xmas gifts. Every year such kinds of products appear seasonally and their movements resume similar styles. Although some products are not seasonal one, such as cellular phones, manufacturers produce new models rather cyclically and as a result their movements assume the same style. We may consider that the cycle is usually 6 months long from experiences, although the cycle has not special meanings.

Generally, the transportation of these products has particular characteristics. The new products should be sent as rapidly as possible. For that reason, the air transportation is usually used at this stage. However, when there are the more products to be sent by air, the transportation by air becomes the more difficult both economically and in terms of quantity of transportation. The location of the production is rather concentrated in small area, and as a result, the airports to be used are also restricted. After the first transportation, a slower but larger amount of transportation is needed. Here the transportation by sea is used. The transportation by sea is slow, which can make the cost of the transportation is very low and the amount to be sent at one time is extremely large. The difference of these two methods is extensively large and is still becoming larger. Here there arises the third method. This is the method so called sea and air transportation. The method means a combinational transportation. At first the products are sent by sea to some neighboring airports and then they are sent by air as a further transportation to the final destinations. The most popular route is the one through Japan. The detail of this situation are shown in the paper[3]. The cost for this new method is almost half and the time required is almost double of direct air service. Moreover, because of the sluggish condition of Japanese economy, obtaining air services is not difficult. And as a result, the obtainable transportation ability by air is larger than that by direct air services from China. However, introducing this method casts a difficult problem of how should this method is utilized and moreover how much should be sent by this method.

This also proposes an important economical problem of how to develop

the infrastructures of airport in Japan. In this article we consider the first problem mainly. On this problem there have been only few prior works[1][2]. The optimal calculation of the allocation of these methods is first considered in this report. This report is consisted as follows. In 3. a background is shown. 4 is devoted to the several assumptions and the calculation of the profit. Consideration on the obtained results is shown in 5. An example is shown in 6, where expected profit of several modes for a simple example are calculated and compared. The concluding remark is given in 7.

## 2 Motivation - Introduction of Sea and Air Mode

The motivation of this study is the present situation of world production area and transportation requirement. As is presented in the previous section, these several years many developed countries have constructed many firms in far east district, specially China. Among these Chinese districts the surrounding district of Shanghai is most popular. The products of these production sites are usually transported to U.S.A. or other advanced countries, namely consumer countries. The amount of the transportation to U.S.A. is tremendously large and it is not an exaggeration to say that it constitutes the world transportation circumstances. However, the transportation of their products to U.S.A. are concentrated to very few transportation eqmeans, that is the most products are gathered to Shanghai district and use Shanghai Airport for air transportation or Shanghai sea fright terminal for shipment. Specially the traffic condition of the airport is becoming the largest problem. Consequently, the air transportation is becoming to require much time and the cost for it is increasing.

On this point, the third transportation method is attracting world's attentions. This is the mode called Sea-and-Air transportation. In this mode the products are once sent, for instance, to Japan by sea and then transported by air to U.S.A. The time required for this mode is about a week to ten days and the cost for this is about half of direct air transportation from China to U.S.A. However, this method is not applied regularly and the application of this is quite optionally and in an inefficient way. The cheap cost for this is justifiable in many reason. If this method can be proved to be efficient, the usage of this mode would be more systematically performed. This would be

profitable for Japan. This is the motivation of this report. One of the most important reason of this is that the ship used from China is the returning one used for the transportation from Japan to China. Moreover the air cost from Japan to USA is comparatively low. Under these conditions we may say that this mode of transportation is fairly promising. These situations are to be explained in the accompanying paper[3].

However, the transportation strategy for using optimally these methods has not been developed until these days. This is also the motivation of this report. It is expected that, as the result of this new proposal, a direction of improving the infrastructure of the traffic system of Japanese ports or airports can be derived.

### 3 System Description

In this section, several characteristics of the system is presented first and then, by making some assumptions, the way to find the optimal transportation is studied.

#### 3.1 Characteristics of The System

It should be noticed that that these seasonal products should be transported to the consuming districts as fast as possible once they are produced. They are new products and the swift appearance gives the products great competitive advantages. After the first phase, the products are sent in large quantities to respond to the steady demand of the consuming districts and therefore the value of them decreases. This is the first characteristic.

*(Assumption I)* The value of the product monotonically decreases as time elapses, eventually approaching to a fixed value. How rapidly the value decreases and how high the first value is varies much depending on the characteristics of the products.

This is the basic assumption and there can be many variations depending on the products. The most important of them is the situation of the consuming amount. It may change dynamically. For the advanced analysis, these situations should be considered.

At the same time the swift transportation costs much and this is the second characteristic. Here the following assumption is introduced.

*(Assumption II)* The transportation cost decreases monotonically as time

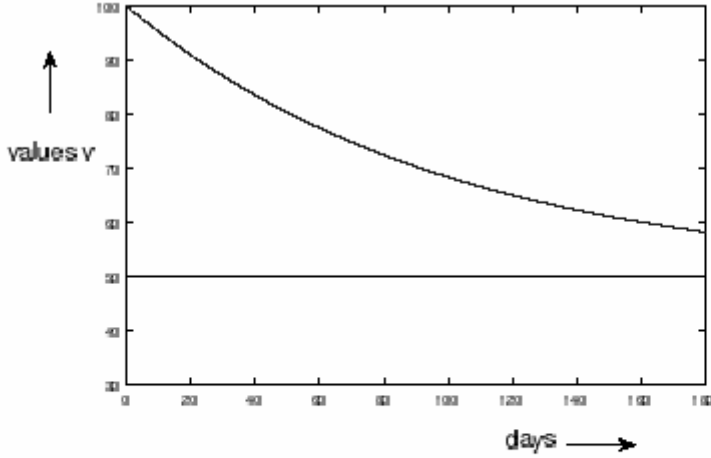


Figure 1: Schematic View of Value Function  $f_v$

pasts. The cost by air is about hundred times higher than that by sea. And the cost by Sea-and-Air is about half to one third of the cost by air.

On the top of the cost of transportation, the cost for inventory should be considered. This is rather small comparing to the transportation cost by air or by sea and air.

(*Assumption III*) Inventory cost is a linear function of time and the amount of goods.

## 3.2 Modeling and Unit Optimal Transportation

For the analysis of this system, several parameters and functions are necessary. For the simplicity the transportation by air, by Sea-and-Sir, by sea are called *A*-mode, *SA*-mode, and *S*-mode transportation respectively. Here the following parameters are introduced.

### 3.2.1 parameters

- $s$ : Amount of the products sent by one time of transportation by *A*-mode.
- $d$ : Time to produce the products of amount  $s$ . Here, this is considered as a unit time.

$\mu s$ : Amount of the products should be sent by one time transportation by SA-mode.

$\nu s$ : Amount of the products should be sent by one time transportation by S-mode.

$T$ : Total production time.

(note) The above parameters indicate that  $\mu$  times of  $s$  are sent at one time by SA-mode and  $\nu$  times of  $s$  are sent at one time by S-mode. These situations are caused by adopting containers for the transportation by sea and not for air. For SA-mode, CFS (container freight station ) cargo are used.

### 3.2.2 functions

On the basis of these assumptions several nonlinear functions are needed for the mathematical analysis of the problem. First of all, value curve  $f_v(t)$  or transportation cost curve  $f_{tc}(t)$  are necessary to be assumed. They are defined as follows.

$f_v(t)$ : The value of one unit product at time  $t$ , which is produced at  $t = 0$ . That means this function shows how rapidly the value of the product decreases.

$f_{tc}(t)$ : The cost of transportation of one unit product to be transported after  $t$  of its production.

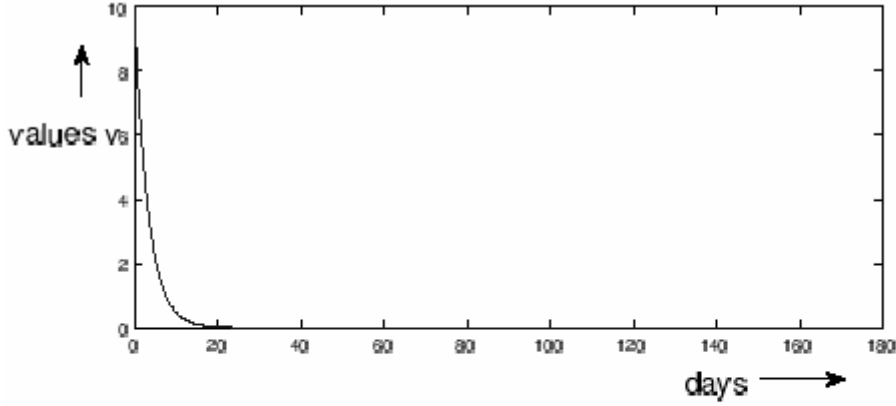
$T_{ic}(t)$ : The cost of inventory of one unit for  $t$  after its production.

Owing to the assumption of monotone decreasing property, one candidate of the value curve and transportation cost curve are exponential functions of time such as

$$f_v(t) = a \exp(-\alpha t) + e, \quad (1)$$

where  $a, \alpha, e$  are constant, decided by the characteristics of the product. In the case of the new models of goods, it is extremely high usually. That means  $a$  is rather large. This is also the case of product in severe competitive condition. The speed to be brought into the market is strongly required to be high. The value decreases monotonically and it approaches to certain fixed value  $e$ . Also a candidate of the function for the transportation cost is given as

$$f_{tc}(t) = b \exp(-\beta t) + h, \quad (2)$$


 Figure 2: Schematic View of Cost Function  $f_{ic}$  (Transportation Cost)

where  $b$ ,  $\beta$ , and  $h$  are also constants, determined by the characteristics of the products as well as by the size and other characteristics of the product. Of course, this curve does not mean there exists every service on this curve. It should be noted that this  $f_{ic}(t)$  indicates the cost of one time service to transport unit product at  $t$ . It should also be noted that there exist only three services on this curve, that is  $A$ -mode,  $SA$ -mode and  $S$ -mode. In the usual case, the cost by air is about 100 times larger than that of by sea, and the time required for this transportation is 2 days for air, and 7 to 10 days for Sea-and-Air, and 20 to 30 days by sea. Comparison of the costs indicates  $h$  is comparatively small and almost  $1/100$  of  $b$ , although it may not be zero. This situation implies that the cost curve approaches very near the static value  $h$  after almost 30 days from the system started. Schematic view of these value functions and cost function are shown in Figure 1 and 2.

On the top of these values, the inventory cost should be subtracted. Naturally, the inventory cost is a linear function of time. Therefore it is given as

$$f_{ic}(t) = \gamma t. \quad (3)$$

Note that this is the cost of inventory for a unit product.

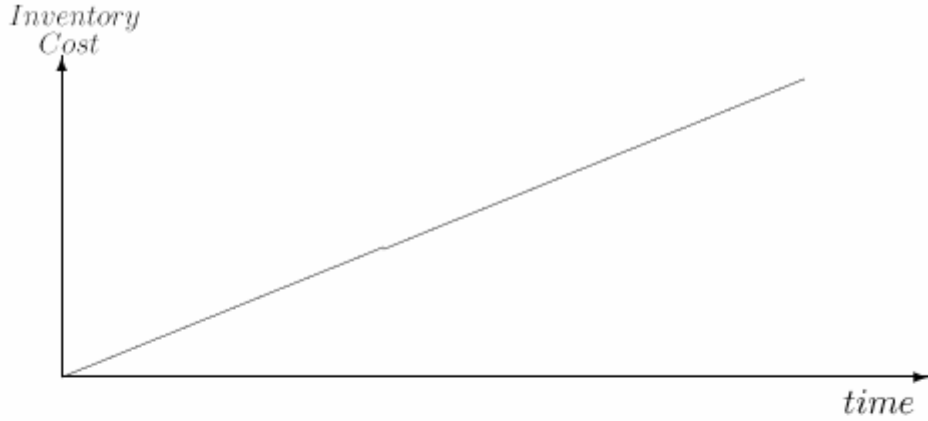


Figure 3: Schematic View of Inventory Cost Function  $f_{ic}$

### 3.2.3 Assumptin on the Modeling

This report is the first trial of the optimal allocation of the transportation. Therefore, the several assumptions for the modeling are to be introduced.

(*Assumption IV*) Production speed is equal to the transportation ability by air. That means the production speed is  $s/d$  throughout  $T$ .

(*Assumption V*) No lead time requirement exists.

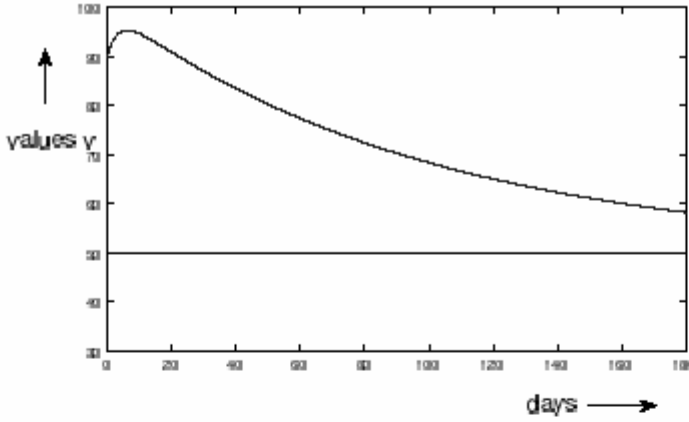
As is easily understood, this is the fairly restricted case and it is natural that there exists a demand from the destination region. It is also natural that the production speed is much higher than one days production and inventory amount increase as days past. These cases are to be considered on the basis of this result. Even for the present restricted cases the necessity of three mode transportation can be proven. It is shown in the following section.

### 3.2.4 Calculation of Optimal Unit Transportation

In this subsection, as a basis of the theory for finding the total optimal strategy of the transportation, the optimal method only for one unit product is calculated first. First, the profit which should be obtained by the transportation of a unit product at time  $t$ , which is calculated as follows.

$$\begin{aligned} f_{tp}(t) &= f_v(t) - f_{tc}(t) - f_{ic}(t) \\ &= a \exp(-\alpha t) + e - b \exp(-\beta t) - h - \gamma t \end{aligned} \quad (4)$$




 Figure 4: Value - Transportation Cost  $f_v - f_{tc}$  (Day 0)

The extremal point is the time when the profit reaches the highest point. This point, if any, is obtained by calculating,

$$\frac{df_{tp}}{dt} = -a\alpha \exp(-\alpha t) + b\beta \exp(-\beta t) - \gamma = 0$$

It is difficult to solve the equation. The schematic view of the function  $f_v(t) - f_{tc}(t)$  is shown in Figure 4. It is found that there exists a desired point in this case, where  $a = 10b$  and  $\beta = 30\alpha$ . It should be noted that this extremal point is for the product produced at  $t = 0$ . Also note that the value curves change for the products produced later. The value curve for the product produced at  $t = d$  is given as

$$f_{vd}(t) = a \exp(-\alpha(t + d)) + e. \quad (5)$$

Therefore, the total profit function for that product is given as

$$f_{tp}(t) = f_{vd}(t) - f_{tc}(t) - f_{ic}(t) = a \exp(-\alpha(t + d)) + e - b \exp(-\beta t) - h - \gamma t. \quad (6)$$

The view of these functions are shown as in Figure 5 and Figure 6 for  $d = 10(\text{days})$  and  $d = 30(\text{days})$ , respectively, both for  $\gamma = 0$ . These figures also show that the transportation by medium speed is best. It is clear that if  $a\alpha \geq b\beta$  then there exists a extremal point for the products of  $t = 0$ .

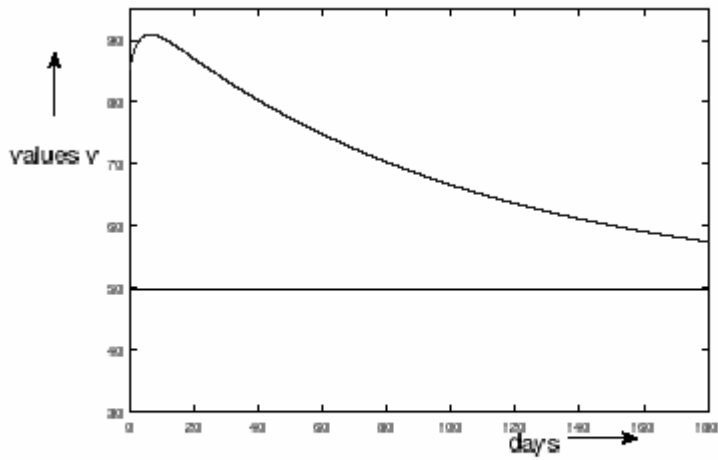


Figure 5: Value - Transportation Cost (10 Days later)  $f_{v10d} - f_{tc}$

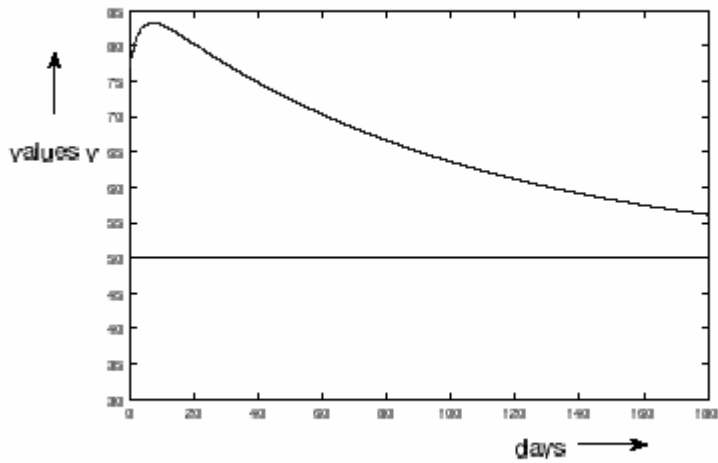


Figure 6: Value - Transportation Cost (30 Days Later)  $f_{v30d} - f_{tc}$

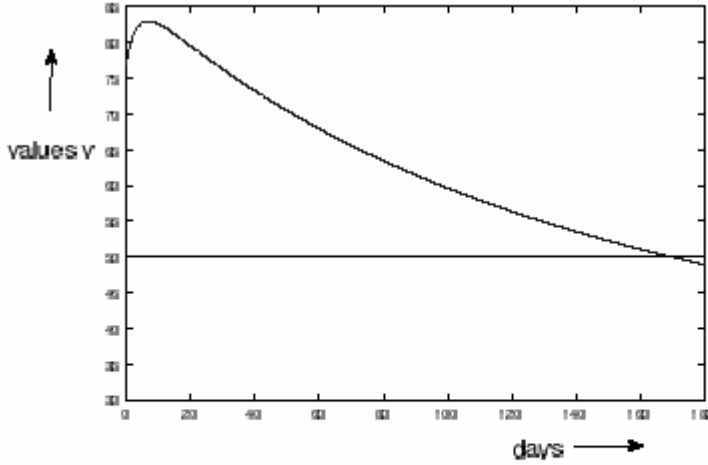


Figure 7: Value - Costs (30Days Later)  $f_{v30d} - f_{tc} - f_{ic}$ : (Inventory cost is considered)

Figures so far considered are all for cases such that no inventory costs are considered. We may assume the inventory costs are comparatively low since these products are transported rather soon after their production. However, in the case of  $S$ -mode it becomes comparatively large. For the comparative reason the case that the inventory cost is considered is shown for the same situation. Figure 7 shows this case. Here the cost of inventory for one day is assumed to be  $1/100$  of the transportation cost by air. This figure shows monotone decreasing properties of the value curve continues to even below the original value of the products. The rapid transportation is again efficient for rather late period.

### 3.3 Problems Description

To start the problem the following restrictions must be introduced.

- (H1) The production speed is constant throughout all the production time.
- (H2) Value curves and cost curve are fixed.
- (H3) Inventory cost is a linear function of time.
- (H4) The transportation should be restricted to three methods.
- (H5) Each transportation method has fixed transportation amount and until the amount of the products satisfy this restriction, the

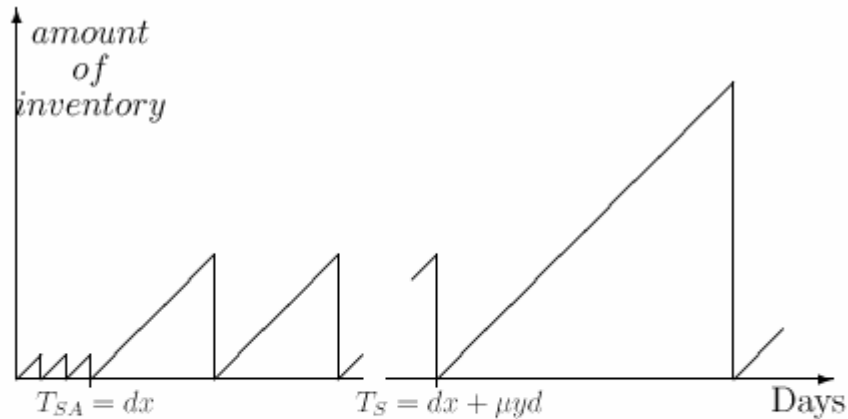


Figure 8: Sample view of Change of Inventory Amount case for production speed =  $s/d$

- (H6) transportation should not be performed, except the last one by sea. The transportation amount by  $SA$ -mode is  $\mu$ -times larger than that by  $A$ -mode.
- (H7) Transportation amount by  $S$ -mode is again  $\nu$ -times larger than that by  $A$ -mode.
- (H8) Only one mode of transportation is selected at one time. This means the products should be decided to be transported by only one mode at each decision time.
- (H9) Transportation by  $S$ -mode is not zero for all cases.

Reviewing these properties, the following considerations are derived.

(Lemma1) We may consider the transportation interval of  $A$ -mode consists of a unit time and may assume every decision is made at each unit time.

(Lemma2) At each unit time the choice of the three mode is accomplished.

(Lemma3) By the properties of monotone increasing characteristics of cost function and monotone decreasing value function, the decision times are divided into three periods, That is the first period is assigned to  $A$ -mode and then to  $SA$ -mode and the final period is to the  $S$ -mode transportation.

As the results, the problem considered is that:

**Problem** Whether is the period of  $SA$ -mode efficient to be introduced? And if so, where are the point  $t = T_{SA}$  and  $t = T_S$  to make the profit maximum? Where

$t = T_{SA}$  is the time at which  $SA$ -mode begins,  
 $t = T_S$  is the time at which  $S$ -mode begins.

This indicates that the  $A$ -mode transportation is performed from  $t = 0$  to  $t = T_{SA}$ . Then from  $t = T_{SA}$  to  $t = T_S$  the  $SA$ -mode transportation and after  $t = T_S$  only  $S$ -mode transportations are performed.

To mathematically define the problem, variables  $x$  and  $y$  are to be defined.

**Definition**

$x$  denotes the number of transportation by  $A$ -mode.

$y$  denotes the number of transportation by  $SA$ -mode.

Thus the following relations hold.

$$\begin{aligned} T_{SA} &= xd \\ T_S &= xd + y\mu d \end{aligned}$$

Then, mathematically the problem can be shown as follows.

**Mathematical description of the problem** Let the total profit obtained by the transportation using three modes be denoted as  $TP_{all}(x, y)$  as a function of  $x, y$ . Then the problem should be as follows.

Find  $x, y$  which satisfy

$$Max TP_{all}(x, y),$$

Although this is the final object, the first part of the problem is only considered here, because the problem becomes fairly complicated and is out of the range of this paper.

## 4 Calculation of the Profit

Here, the total expected profit by the allocation of the three modes transportation is calculated. It is assumed that  $x$  times  $A$ -mode transportation and  $y$  times  $SA$ -mode transportation are performed. It is assumed that although all costs remain constant the value function changes as time elapses.

### 4.1 Profit by the $A$ -mode Transportation

By definition of unit time, at  $t = d, \dots, t = dx$   $A$ -mode transportations are performed. Let  $V_{air}$  denote the total value obtained by one transportation by air. Note that the total value is also depends on the amount to be sent, which

is same at each time by the restriction H5. Since the value of the products changes as time  $t$ , the total value transported by air is given as

$$V_{air} = \sum_{i=1}^x s f_{v(i-1)d}(d)$$

Since  $f_{v(i-1)d}(d) = f_v(id)$ , we may write it as

$$V_{air} = \sum_{i=1}^x s f_v(id). \quad (7)$$

Let  $TC_{air}$  denote the transportation cost by air. Since the amount to be sent at one time is same and  $TC_{air}$  is a linear function of the transportation amount, it is given as

$$TC_{air} = x s f_{tc}(d). \quad (8)$$

The inventory const is the linear function of time and the amount to be stored and, therefore, it is given as

$$IC_{air} = \frac{1}{2} \gamma x d s. \quad (9)$$

The total profit by air is given as

$$TP_{air} = V_{air} - TC_{air} - IC_{air}. \quad (10)$$

Substituting (7),(8),(9) into (10), the following equation is obtained.

$$TP_{air} = \sum_{i=1}^x s f_v(id) - x s f_{tc}(d) - \frac{1}{2} \gamma x d s. \quad (11)$$

This is the total profit obtained by air.

## 4.2 Profit by the SA-mode Transportation

By assumption, the unit time for SA-mode is  $\mu d$ . After  $x$  times A-mode transportations are preformed SA-mode transportations are performed at  $t = xd + \mu d, t = xd + 2\mu d, \dots, t = xd + y\mu d$ . As in the previous subsection, the total revenue by SA-mode is calculated in this subsection. Each time the value decreases as time  $t$ . Since the  $y$  times transportations are performed by this mode, the total value obtained by this transportation is given as,

$$V_{sea-air} = \sum_{i=1}^y \mu s f_v(xd + i\mu d). \quad (12)$$

For the calculation of the profit of the *SA*-mode the cost for this transportation should be subtracted from the calculated value in (12) as in the previous subsection. Therefore, the total profit for this mode is given as,

$$TP_{sea-air} = \sum_{i=1}^y \mu s f_v(xd + i\mu d) - y\mu s f_{tc}(\mu d) - \frac{1}{2}\gamma y(\mu d)(\mu s). \quad (13)$$

### 4.3 Total Profit of the Product

After *A*-mode and *SA*-mode transportation *S*-mode transportation is performed. The start times of this transportations are  $t = xd + \mu dy + \nu d, t = xd + \mu dy + 2\nu d, \dots, t = xd + \mu dy + r\nu d$  and  $T$ . Following the previous subsection the profit of the *S*-mode is calculated. First the total value obtained by this transportation is

$$V_{sea} = \sum_{i=1}^r \nu s f_v(xd + y\mu d + i\nu d) + \frac{s}{d}(T - xd - y\mu d - r\nu d)f_v(T), \quad (14)$$

where  $r$  is defined as

$$r = [(T - xd - y\mu d)/(\nu d)] \quad (15)$$

$[x]$  is the largest integer which does not exceed  $x$ . Subtracting the total costs for this mode from the above value, the total profit is obtained as follows.

$$\begin{aligned} TP_{sea} &= \sum_{i=1}^r \nu s f_v(xd + y\mu d + i\nu d) + \frac{s}{d}(T - xd - y\mu d - r\nu d)f_v(T) \\ &\quad - r\nu s f_{tc}(\nu d) - \frac{1}{2}r\gamma(\nu d)(\nu s) - \frac{s}{d}(T - xd - y\mu d - r\nu d)f_{tc}(\nu d) \\ &\quad - \frac{s}{2d}(T - xd - y\mu d - r\nu d)^2, \end{aligned} \quad (16)$$

where the second from last term denotes the shipment cost of the remaining amount of the inventory transported at  $T$  and the last term indicates the inventory cost of the same amount. Overall profits obtained by these three modes transportations are given as the summation of (11), (13) and (16). Therefore the following expression is obtained.

$$TP_{all} = \sum_{i=1}^x s f_v(id) - xs f_{ic}(d) - \frac{1}{2}\gamma xds$$

$$\begin{aligned}
& + \sum_{i=1}^y \mu s f_v(xd + i\mu d) - y\mu s f_{tc}(\mu d) - \frac{1}{2}y\gamma(\mu d)(\mu s) \\
& + \sum_{i=1}^r \nu s f_v(xd + y\mu d + i\nu d) + \frac{s}{d}(T - xd - y\mu d - r\nu d)f_v(T) \\
& - r\nu s f_{tc}(\nu d) - \frac{1}{2}r\gamma(\nu d)(\nu s) - \frac{s}{d}(T - xd - y\mu d - r\nu d)f_{tc}(\nu d) \\
& - \frac{s}{2d}(T - xd - y\mu d - r\nu d)^2. \tag{17}
\end{aligned}$$

The above equation is the function of  $x$  and  $y$ . To maximize this value and to find  $x$  and  $y$  to maximize this is the problem to be solved. This is a fairly complicated nonlinear estimation. For this purpose the application of genetic algorithms may be considerable. However, it is rather a complicated problem and is now out of range of this report. It is to be considered in the future.

## 5 Some Consideration

The above obtained equation (17) is a very special case of the expected profit obtained by allocating the three modes of transportations. On the basis of this consideration, several reasonable situations should be considered. First of all the production speed is usually much higher and therefore after several unit times a situation can arise such that several modes can be used at the same time. For example, one part of the products is sent by air and at the same time the remaining parts are sent by sea and air. These situations should be considered lately.

Next, as is shown before in this report, the demand from the consuming districts is not considered. This is the very large problem. However, the request from the consuming district is the base of the value of the product. In this sense we may say that this request is considered here in the most basic form. The restrictions derived from the assumed form of several curves are also problems to be considered. As is seen in the form of these adopted curves, they may not fully satisfy the present restrictions. Sometimes linear functions which are to be considered in the examples might be adequate. They should be considered in the following studies.

However, it is possible to see some interesting features of transportations even in the present form. All figures presented here show peaks of expected profits as  $t$  passes. This shows that the medium transportation time is most



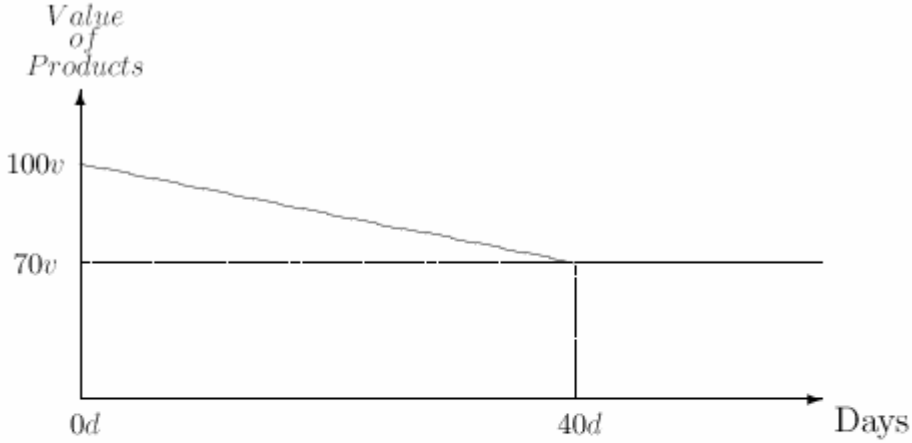


Figure 9: Value Curve of Example

efficient for the given case of value and cost functions. How much should be sent is calculated by the obtained equations. This depends much on the characteristics of the products. However, it is clear that this method is efficient and is profitable for many cases and ought to be adopted more systematically.

## 6 Example

Although the ultimate object of this research is to find the optimal allocation of the three modes of transportations theoretically, it is fairly complicated and is beyond the range of this paper. Here, a numerical example showing the efficiency of the introduction of the *SA*-mode is presented.

### 6.1 Assumptions and 5 Cases of Examples

Some parameters should be assumed first. Assume

1.  $T = 150d$ .
2.  $\mu = 5$ .
3.  $\nu = 4$ .

To make the calculation easy for this problem, here it is assumed that the value curve and transportation cost curve are both linear as in Figure 9 and Figure 10, respectively. This Figure 9 indicates that at first the value of the product is  $100v$  and it decrease linearly to  $70v$  for  $30d$ . After  $30d$  the value of it stays unchanged until  $t = 150d$ . The transportation cost curve of Figure 10 indicates that the transportation cost also decreases linearly for  $20d$ . The

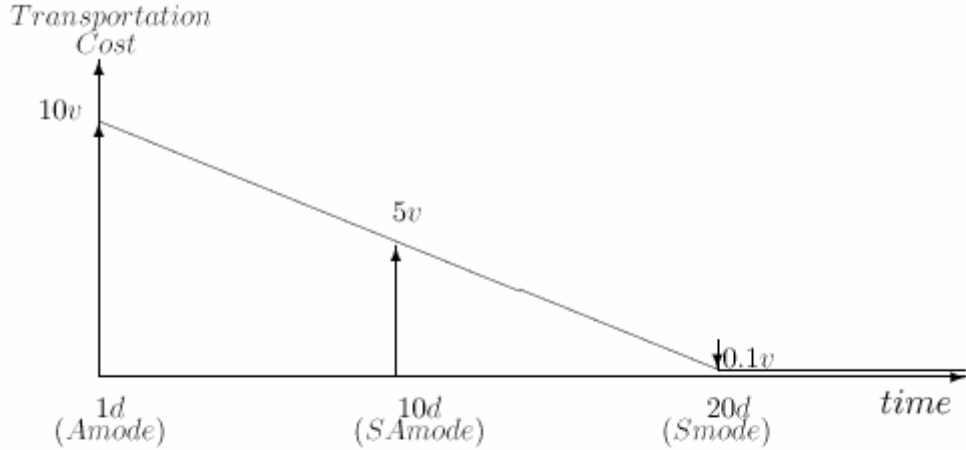


Figure 10: Transportaton Cost Curve of Example

transportation cost for  $t = 1d$  is  $10v$ . This is the cost for  $A$ -mode. The cost for  $SA$ -mode is  $5v$  and the time to be required is  $10d$ . The cost for  $S$ -mode is  $0.1v$  and the time to be required is  $20d$ . After  $20d$  this cost also remains unchanged. On this problem 5 cases are considered.

**(Case 1)** (5  $A$ -modes and  $S$ -mode)

In this case 5 times of  $A$ -modes are performed and the remaining products are transported by  $S$ -mode. Therefore,  $TP_{air}$  and  $TP_{sea}$  are given as

$$\begin{aligned} TP_{air} &= (99 - 10)v + (98 - 10)v + (97 - 10)v + (96 - 10)v + (95 - 10)v, \\ &= 435v. \end{aligned}$$

$$TP_{sea} = 145(70 - 0.1)v = (10150 - 14.5)v = 10135.5v.$$

By  $A$ -mode transportation only one unit of product is transported at each time. The first ( $\cdot$ ) in the RHS of the first equation means the profit 99 obtained by the first transportation by air costs  $10v$ . The remaining terms are calculated as before. Remaining 145 units of products are transported by sea. Since the cost for the transportation by sea is only  $0.1v$  per unit. The total profit  $TP_{all}$  in this case is given as

$$TP_{all} = TP_{air} + TP_{sea} = 435v + 10135.5v = 10570.5v.$$

**(Case 2)** (5  $A$ -modes 1 $SA$ -mode and  $S$ -mode)

In this case 5 times of  $A$ -modes and 1 time of  $SA$ -modes are used. The remaining products are transported by  $S$ -mode. Therefore, 5 units, 5 units and

140 units are transported by *A*-mode, *SA*-mode and *S*-mode, respectively. The  $TP_{air}$ ,  $TP_{sea-air}$  and  $TP_{sea}$  are calculated as

$$\begin{aligned} TP_{air} &= (99 - 10)v + (98 - 10)v + (97 - 10)v + (96 - 10)v + (96 - 10) \\ &= 435v. \\ TP_{sea-air} &= 5(80 - 5)v = 375v. \\ TP_{sea} &= 140(70 - 0.1)v = 9800v - 14 = 9786v. \end{aligned}$$

The total profit  $TP_{all}$  in this case is given as

$$TP_{all} = TP_{air} + TP_{sea} = 435v + 375v + 9786v = 10590v.$$

**(Case 3)** (5 *A*-modes 2*SA*-modes and *S*-mode)

In this case 5 times of *A*-modes and 2 times of *SA*-modes are used. The remaining products are transported by *S*-mode. The  $TP_{air}$ ,  $TP_{sea-air}$  and  $TP_{sea}$  are calculated as

$$\begin{aligned} TP_{air} &= (99 - 10)v + (98 - 10)v + (97 - 10)v + (96 - 10)v + (96 - 10) \\ &= 435v. \\ TP_{sea-air} &= 5(80 - 5)v + 5(75 - 5)v = 375v + 350v = 725v. \\ TP_{sea} &= 135(70 - 0.1)v = 9450v - 13.5v = 9436.5v. \end{aligned}$$

The total profit  $TP_{all}$  in this case is given as

$$TP_{all} = TP_{air} + TP_{sea-air} + TP_{sea} = 435v + 725v + 9436.5v = 10596.5v.$$

**(Case 4)** (4 *A*-modes 2*SA*-modes and *S*-mode)

In this case 4 times of *A*-modes and 2 times of *SA*-modes are used. The remaining products are transported by *S*-mode. The  $TP_{air}$ ,  $TP_{sea-air}$  and  $TP_{sea}$  are calculated as

$$\begin{aligned} TP_{air} &= (99 - 10)v + (98 - 10)v + (97 - 10)v + (96 - 10)v = 350v. \\ TP_{sea-air} &= 5(81 - 5)v + 5(76 - 5)v = 380v + 355v = 735v. \\ TP_{sea} &= 136(70 - 0.1)v = 9520v - 13.6 = 9506.4v. \end{aligned}$$

The total profit  $TP_{all}$  in this case is given as

$$TP_{all} = TP_{air} + TP_{sea-air} + TP_{sea} = 350v + 735v + 9506.4v = 10591.4v.$$

**(Case 5) (All S-mode)**

As a comparison here the case that all the products are sent by sea is considered. In this case  $TP_{all} = TP_{sea}$  and it is obtained as

$$TP_{all} = TP_{sea} = 150(70 - 0.1)v = (10500 - 15)v = 10485v.$$

As the result the following table is obtained, where  $TC_{all}$  indicates the total of the all transportation costs.

<i>modes</i>	5A-modes S-modes	5A-modes 1SA-modes S-modes	5A-modes 2SA-modes S-modes	4A-modes 2SA-modes S-modes	S-modes
$TP_{air}$	435v	435v	435v	350v	0
$TP_{sea-air}$	0	375v	725v	735v	0
$TP_{sea}$	10135.5v	978v	9436.5v	9506.4v	10485v
$TC_{all}$	64.5v	89v	113.5v	103.6v	15v
$TP_{all}$	10570.5v	10590v	10596.5v	10591.4v	10485v

Table 1 Obtained Values

## 6.2 Considerations on the Results

In this example, the best result is obtained in Case 3, that means the best result is obtained by introducing 2SA-modes and 5A-modes. At the same time in this case the transportation cost is highest. It may said that although the transportation cost becomes highest, the final gain is the largest in this case. It depends on cases whether it is more profitable to increase the number of SA-mode by decreasing the number of A-mode. In this example it is not rather profitable. Although this example is a very simple one we may safely suppose that the introduction of SEA and AIR transportation is profitable and moreover, if it is possible to make this service a little more speedy, then this tendency will increase. This example is a very restricted one such that the production speed is just the same as one day's transportation ability. This is not the usual case and usually transportation begins after the amount of the products become sufficiently large. Optimal allocation of the transportation methods of such cases are to be considered in the future.

## 7 Conclusion

It was shown that the importance and efficiency of the third method of intercontinental transportation of products are fairly large. This method is called Sea-and-Air because it uses these two methods. The most popular Sea-and-Air transportation is performed through Japan. It is rather a favorable situation for Japan both for her situation on sea and air transportation. A theoretical analysis for the optimal allocation of these three methods was first considered in this report. Although the conditions or situations considered in this report is very primitive, the efficiency of this method was explained theoretically. At present, this method is utilized quite optionally. It is desirable to perform this method more systematically. In the application to the actual problem, the nonlinear functions for the value or cost functions must be estimated through theoretical and experimental considerations more precisely. These functions are much more complicated in reality and different to each of products and the amount to be transported. Actually they should be constructed and polished by each product individually. Though it seems fairly difficult task, it might be needed in the future.

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