BOOTSTRAP AND JACKKNIFE METHODS IN TWO-PHASE SAMPLING

Mijanur Rahman[1], Dulal Chandra Roy[2] and M.Nasser[3]

ABSTRACT

In this paper, we have defined the biases and mean square errors of the two-phase sampling ratio and regression estimators in bootstrap and jackknife methods. From an empirical investigation, in the line of Reich et al. (1993), the ratio and regression estimators are compared using the classical, bootstrap and jackknife techniques.

KEY WORDS: Two-phase Sampling, AuxiliaryVariable, Regression and Ratio Estimators, Bootstrap and Jackknife Methods.

1 Introduction

Consider a finite population of N units for estimating the population mean $\overline{Y}$ of a study variable y. When information on an auxiliary variable, say, x which is highly correlated with the study variable y is readily available on all the units of the population, it is well known that the ratio and the regression estimators could be used for increased efficiency by incorporating the knowledge of $\overline{X}$, the population mean of the auxiliary variable x. However, in certain situation when $\overline{X}$ is not known a priori, the technique of two-phase sampling (double sampling) is effectively exploited with a view to furthering the precision of the estimators under consideration. This sampling procedure requires collection of information on x for the first-phase sample s' of size n' ($n' <$ N) and on y for the second-phase sample s of size n ($n < n'$) selected from the first-phase sample s'. Simple random sampling without replacement (SRSWOR) design is considered in both the phases. Let $\overline{y}$ and $\overline{x}$ be the sample means of y and x based on the sample s of size n, and let $\overline{x}'$ be the sample mean of variable x based on the sample s' of size n'.

The traditional two-phase sampling ratio and regression estimators using data on y and x are given by

$$\overline{y}_{rd} = \overline{y}\frac{\overline{x}'}{\overline{x}},$$

$$\overline{y}_{ld} = \overline{y} + b_{yx}(\overline{x}' - \overline{x})$$

where b_{yx} is the sample regression coefficient of y on x based on the sample s of size n.

In survey sampling literature, the problems relating to the classical method of estimation of population parameters have been dwelt upon extensively. In this paper, in addition to the classical method, we have exploited the idea of bootstrap and jackknife methods. The outline of the remaining

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[1] Research Student, Department of Statistics, University of Rajshahi. [2] Associate Professor, Department of Statistics, University of Rajshahi. [3] Associate Professor, Department of Statistics, University of Rajshahi.

sections is as follows. Section 2 contains a brief discussion on bootstrap and jackknife techniques in finite population sampling for estimating the population parameter. Biases and mean square errors of the two estimators are defined using three different methods in Section 3. Section 4 provides the analysis and discussions from an empirical investigation. Finally, Section 5 upholds the summary of the paper.

2. Bootstrap and Jackknife Methods

2.1 The Bootstrap Method

The bootstrap technique was introduced outside survey sampling as a means of obtaining approximate variance estimates and confidence intervals. The originator was Efron (1979, 1981, 1982). The bootstrap technique was originally designed for use with independent observations, the standard assumption of traditional statistical theory. One basic problem, not yet definitely answered, is how the techique should be correctly modified to accommodate the special features of survey sampling, including the non-independence arising in sampling without replacement and other complexities of designs and estimators. For further reading on bootstrap in survey sampling, the reader is referred to Bickel and Freedman (1984), Bondesson and Holm (1985), Kovar, Rao, and Wu (1988), McCarthy and Snowden (1985), Rao and Wu (1984, 1987) and Rao and Katzoff (1996).

In bootstrap resampling, B new samples, each of the same size as the observed data, are drawn with replacement from the observed data. The statistics is first calculated using the observed data, and then re-calculated using each of the new samples, yielding a bootstrap distribution. The resulting replicates are used to calculate the bootstrap estimates of bias, mean, and standard error for the statistic.

Suppose a probability sample s is drawn from a population U by an arbitrary sampling design without replacement. The population parameter θ is estimated by $\hat{\theta}$, and we seek an estimate of $V(\hat{\theta})$. The following is a brief description of how the bootstrap technique works.

i. Using the sample data, construct an artificial population U^*, assumed to mimic the real, but unknown, population U.

ii. Draw a series of independent samples, "resamples" or "bootstrap samples," from U^* by a design identical to the one by which s was drawn from U. Independence implies that each bootstrap sample must be replaced into U^* before the next one is drawn. For each bootstrap sample, calculate an estimate $\hat{\theta}_b^*$ $(b = 1,\ldots,B)$ in the same way as $\hat{\theta}$ was calculated.

iii. The observed distribution of $\theta_1^*,\ldots,\theta_B^*$ is considered an "estimate" of the sampling distribution of the estimator $\hat{\theta}$, $V(\hat{\theta})$ is estimated by

$$\hat{V}_{BS} = \frac{1}{B-1}\sum_{b=1}^{B}(\hat{\theta}_b^* - \hat{\theta}^*)^2 \tag{2.1}$$

where $\hat{\theta}^* = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}_b^*$

iv. The estimator of the bias of θ is given by

$bias_b = \hat{\theta}^* - \hat{\theta}$

and the corresponding bootstrap estimator is given by

$$\hat{\theta}_b = \hat{\theta} - bias_b = 2\hat{\theta} - \hat{\theta}^*.$$

2.2 The Jackknife Method

Like the bootstrap technique, the jackknife technique originated outside the field of survey sampling. For finite populations, the jackknife technique was first considered by Durbin (1959). The jackknife technique is commonly used to estimate a variance in survey sampling. A more detailed account is given in Wolter (1985).

The jackknife procedure begins by removing one of the observation (i.e., plot) from the sample data. The desired statistic is then computed each time, with one of the observation eliminated (Smith and Van Belle 1984). Standard error is then computed for the variability among these estimated values.

Suppose we have a sample $x = (x_1, x_2, \ldots, x_n)$ and an estimator $\hat{\theta} = s(x)$. We wish to estimate the bias and the standard error of $\hat{\theta}$. The jackknife focuses on the samples that leave one observation at a time

$$x_{(i)} = (x_1, x_2, \ldots x_{i-1}, x_{i+1}, \ldots x_n)$$

for $i = 1, 2, \ldots, n$, called jackknife samples. The ith jackknife sample consists of the data set with i th observation removed.

Let $\hat{\theta}_{(i)} = s(x_{(i)})$ be the ith jackknife replication of $\hat{\theta}$.

The jackknife estimate of bias is defined by

$$bias_{jack} = (n-1)(\hat{\theta}_{(.)} - \hat{\theta})$$

where $\hat{\theta}_{(.)} = \sum_{i=1}^{n} \hat{\theta}_{(i)}/n$.

Hence the jackknife estimator of θ is given by

$$\hat{\theta}_{jack} = \hat{\theta} - bias_{jack}.$$

The jacknife estimate of standard error is defined by

$$\hat{se}_{jack} = [\frac{n-1}{n} \sum (\hat{\theta}_{(i)} - \theta_{(.)})^2]^{1/2}. \tag{2.2}$$

3 Biases and Mean Square Errors of different Methods

3.1 Classical Method

In SRSWOR (N, n), it is well known that the estimate of biases of the ratio and regression estimators of the population mean are approximately given by

$$b(\bar{y}_{rd}) = \bar{Y}\left(\frac{1}{n} - \frac{1}{n'}\right)(c_x^2 - r c_x c_y),$$

$$b(\bar{y}_{ld}) = -b_{yx}\left(\frac{1}{n} - \frac{1}{n'}\right)\left[\frac{\hat{\mu}_{21}}{s_{xy}} - \frac{\hat{\mu}_{30}}{s_x^2}\right]$$

where $c_x = \frac{s_x}{\bar{x}}$, $c_y = \frac{s_y}{\bar{y}}$, $r = \frac{s_{xy}}{s_x s_y}$, $b_{yx} = \frac{s_{xy}}{s_x^2}$,

$$\hat{\mu}_{21} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - x)^2(y_i - y) \text{ and } \hat{\mu}_{30} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^3.$$

The estimate of MSE of the twin estimators $\bar{y}_{rd}$ and $\bar{y}_{ld}$ are approximately given by

$$mse\,(\bar{y}_{rd}) = \left(\frac{1}{n} - \frac{1}{N}\right)s_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)(s_y^2 + r^2 s_x^2 - 2r s_{yx}),$$

$$mse\,(\bar{y}_{ld}) = \left(\frac{1}{n} - \frac{1}{N}\right)(1 - r^2)s_y^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)r^2 s_y^2$$

where $s_y^2 = \sum_{i=1}^{n}\frac{(y_i - \bar{y})^2}{n-1}$, $s_x^2 = \sum_{i=1}^{n}\frac{(x_i - \bar{x})^2}{n-1}$,

$$s_{yx} = \sum_{i=1}^{n}\frac{(y_i - \bar{y})(x_i - \bar{x})}{n-1},\quad r = \frac{\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})}{(n-1)s_x s_y}.$$

3.2 Bootstrap Method

In bootstrap method, discussed in Section 2, the biases of $\bar{y}_{rd}$ and $\bar{y}_{ld}$ can be obtainable as

$$bias\,(\bar{y}_{rd})_B = \bar{y}^*_{rd}(\cdot) - \bar{y}_{rd},$$

$$bias\,(\bar{y}_{ld}) = \bar{y}^*_{ld}(\cdot) - \bar{y}_{ld}$$

where $\bar{y}^*_{rd}(\cdot) = \sum_{b=1}^{B}\frac{\bar{y}^*_{rd}(b)}{B}$ and $\bar{y}^*_{ld}(\cdot) = \sum_{b=1}^{B}\frac{\bar{y}^*_{ld}(b)}{B}$.

Using (2.1), the bootstrap estimators for the MSE's of the ratio and regression estimators ($\bar{y}_{rd}$ and $\bar{y}_{ld}$) are expressible as

$$mse\,(\bar{y}_{rd})_B = \sum_{b=1}^{B}\frac{\left[\bar{y}^*_{rd}(b) - \bar{y}^*_{rd}(\cdot)\right]^2}{B-1},$$

$$mse\,(\bar{y}_{ld})_B = \sum_{b=1}^{B}\frac{\left[\bar{y}^*_{ld}(b) - \bar{y}^*_{ld}(\cdot)\right]^2}{B-1}$$

where $\bar{y}_{rd} = \bar{y}\frac{\bar{x}'}{\bar{x}}$ ratio estimator of population mean of bth bootstrap sample and

$\bar{y}^*_{ld}(b) =$ regression estimator of population mean of bth bootstrap sample,

$$\bar{y}^*_{rd}(\cdot) = \sum_{b=1}^{B}\frac{\bar{y}^*_{rd}(b)}{B} \text{ and } \bar{y}^*_{ld}(\cdot) = \sum_{b=1}^{B}\frac{\bar{y}^*_{ld}(b)}{B}.$$

3.3 Jackknife Method

In this method, the bias of $\bar{y}_{rd}$ and $\bar{y}_{ld}$ are given by

$$bias\ (\bar{y}_{rd})_{jack} = (n-1)\left(\bar{y}_{rd(\cdot)} - \bar{y}_{rd}\right),$$
$$bias\ (\bar{y}_{ld})_{jack} = (n-1)\left(\bar{y}_{ld(\cdot)} - \bar{y}_{ld}\right)$$

where $\bar{y}_{rd(\cdot)} = \sum_{i=1}^{n} \bar{y}_{rd(i)}/n$, $\bar{y}_{ld(\cdot)} = \sum_{i=1}^{n} \bar{y}_{ld(i)}/n$,

$\bar{y}_{rd(i)}$ = the ith jackknife replication of $\bar{y}_{rd}$,

and $\bar{y}_{ld(i)}$ = the ith jaccknife replication of $\bar{y}_{ld}$.

In similar manner using (2.2), the jackknife estimators for MSE of the twin estimators $\bar{y}_{rd}$ and $\bar{y}_{ld}$ are obtainable as.

$$mse\ (\bar{y}_{rd})_J = \frac{n-1}{n}\sum_{i=1}^{n}\left(\bar{y}_{rd(i)} - \bar{y}_{rd(\cdot)}\right)^2,$$

$$mse\ (\bar{y}_{ld})_J = \frac{n-1}{n}\sum_{i=1}^{n}\left(\bar{y}_{ld(i)} - \bar{y}_{ld(\cdot)}\right)^2$$

where $\bar{y}_{rd(i)}$ = ratio estimator of population mean of ith jackknife sample,

$\bar{y}_{ld(i)}$ = regression estimator of population mean of ith jackknife sample,

$\bar{y}_{rd}(\cdot) = \sum_{i=1}^{n} \frac{\bar{y}_{rd(i)}}{n}$ and $\bar{y}_{ld}(\cdot) = \sum_{i=1}^{n} \frac{\bar{y}_{ld(i)}}{n}$.

4 Empirical Investigation and Discussions

4.1 Description of the Data

To examine the performance of estimators (mentioned in subsection 1) using classical, bootstrap and jackknife methods, we have considered the data on head length (y) and head circumference (x) of 1490 Japanese adult female students. The age range of the students was between 18 and 25 years. A single observer (Fumio Ohtsuki)* took the measurements from 1975 to 1979, using the technique of Martin and Saller (1957).

4.2 Analyses, Results and Discussions

In the empirical investigation, we are mainly interested to estimate some characteristics of head lengths of the Japanese Universities students between age group (18−25) years. Since the head length (y) is correlated with head circumference (x), we have used the double sampling technique to estimate the population characteristics of the main variable y (head length) exploiting the idea of auxiliary variable x (head circumference).

According to the principle of double sampling technique, first we have taken a larger sample (first-phase) of size n' (=500) by using SRSWOR design and a sub-sample of size n using the same design

* Dr.Fumio Ohtsuki, Emeritus Professor, Tokyo Metropolitan University, Japan.

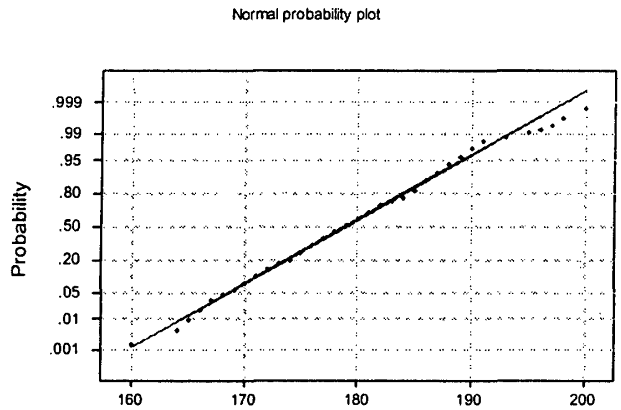

Fig.4.1 Normal probability plot of head length (y)

Fig.4.2 Normal probability plot of head Circumference (x)

Table 4.1 Descriptive statistics for y and x on the larger sample of size 500.

	Mean	Median	Min	Max.	Var.	Anderson-Darling test Statistic for normality	P value
Head length	178.86	179	160	200	38.253	1.064	.992
Head circumference	55.1852	55.15	51.1	59.5	1.6796	.638	.904

from selected larger sample (n').

First, to make an idea about the nature of the data, we have given normal probability plots and performed Anderson Darling test for normality.

Basic statistic for the data related to y and x are given in Table 4.1. The above table shows that the P value is very high and the value of the statistics is small. Even at α=.90, normality of the variables would be accepted. Thus the Anderson-Darling test statistic confirmed that both the variables significantly fit the normal distribution. It is also observed from normal probability plots that the data clearly match the criteria of normal distribution.

The decision to use the ratio or regression estimator depends on the relationship between head length (y) and head circumference (x). If the line of y on x does not pass through the origin, the regression estimator is appropriate. If the line of y on x passes through the origin, the ratio estimator is appropriate (Cochran 1977). The scatter diagram given in figure 4.3 shows that the relationship between the data on head length and head circumference is approximately linear and the line does not pass through the origin but near through the origin. So regression estimator may appropriate to use.

Now to compare regression and ratio methods of estimation in different procedure (classical, jackknife and bootstrap methods), subsample of sizes n =50, 75, 100, 125, 150, 175, 200 are drawn from the larger

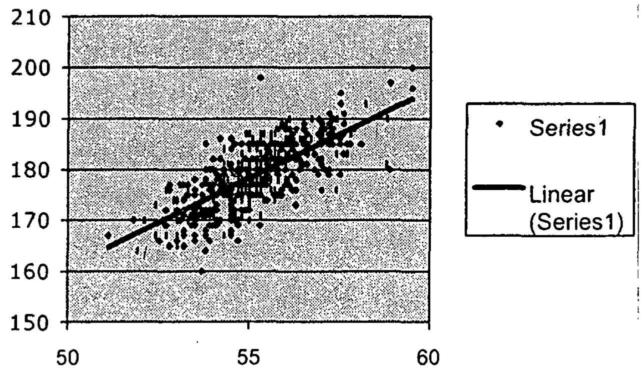

Fig.4.3 Scatter diagram of head length (y) and head circumference (x)

Table 4.2 Estimate of population mean by using different methods.

Method	Sample size (n) 50	75	100	125	150	175	200	Coefficient of variation
Ratio	179.19	178.17	178.99	179.17	178.49	178.3	178.92	0.23
Ratio(B)	179.14	178.18	179.00	179.20	178.40	178.70	178.50	0.22
Ratio(J)	179.20	178.20	179.00	179.20	178.40	178.70	178.50	0.22
Regression.	179.13	178.23	178.96	179.11	178.48	178.29	179.20	0.24
Regression(B)	179.11	178.22	178.90	179.20	178.40	178.70	178.50	0.21
Regression(J)	179.11	178.18	178.90	179.20	178.40	178.70	178.50	0.20

sample of size n'=500 by simple random sampling without replacement design. The estimate of population mean for different subsamples using different methods are given in the following table 4.2.

We have the population mean (for N=1490) of head length is 178.986 cm. Again, we observe from the above Table 4.2 that the sample mean given by ratio (classical, jackknife and bootstrap methods) or regression (classical, jackknife and bootstrap methods) method is very close to the population mean. Also the coefficient of variations in all the methods for different sample sizes are very small which implies precise estimation of population mean.

Table 4.3 Effects of sample size on percent bias of estimates of head length for double sampling ratio and regression estimators

Sample size	Ratio	Regression
50	1.20	1.86
75	1.24	1.69
100	1.02	1.38
125	1.07	1.38
150	1.02	1.37
175	1.04	1.37
200	0.84	1.18

Table 4.4 Estimation of mean square errors of $\bar{y}_{rd}$ and $\bar{y}_{ld}$ in different methods

Method	Sample size (n) 50	75	100	125	150	175	200	Coefficient of variation
Ratio	.4343	.2359	.2074	.1429	.1276	.1118	.1029	59.32
Ratio(B)	.4524	.2210	.1898	.1599	.1248	.1111	.1059	62.11
Ratio(J)	.4068	.2255	.1969	.1276	.1182	.1101	.1034	59.10
Regression.	.4062	.2349	.2066	.1428	.1276	.1129	.1028	59.16
Regression(B)	.4198	.2193	.1896	.1607	.1260	.1141	.1044	57.37
Regression(J)	.4064	.2259	.2075	.1298	.1196	.1153	.1027	57.89

Table 4.3 clearly points to the fact that both ratio and regression estimators are biased. The regression estimator consistently has a larger bias than the ratio estimator. However, the bias for the regression estimator decreases linearly as the sample size increases, while the bias for the ratio estimator is non-linear over the sample size.

Since both the ratio and regression estimators are biased, one way to compare the variance of the two estimators with different amount of bias is to use the mean square errror(MSE) and is defined as

Mean square error(MSE)=variance + bias^2.

Thus an estimator with a smaller MSE is considered more precise than one with a lareger MSE, even though the latter may have smaller variance.

In terms of MSE, the regression estimator is comparatively better than ratio estimators. The last column of the Table 4.4 clearly shows that the coefficient of variation of classical, bootstrap and jackknife regression estimatiors are always smaller than the respective classical, bootstrap and jackknife ratio estimators. So we can say that regression estimator is comparatively better than ratio estimator for the estimation of head length of university student of Japan.

Now we use simulation variances of double sampling ratio and regression estimators to compare. To find out the simulated result we use Monte-Carlo simulation of 10,000 samples of different sizes n =50, 75, 100, 125, 150, 175 and 200 which are drawn from the large sample size n'=500 using a Turbo C program (Rahman 2003).

Table 4.5 Simulation variance of double sampling ratio and regression estimator of population mean

Method	Sample size (n) 50	75	100	125	150	175	200
Ratio	.3743	.2541	.1941	.1583	.1345	.1173	.1045
Regression	.3653	.2499	.1917	.1568	.1334	.1165	.1039

In the above table we see that the simulation variance of double sampling regression estimator is always smaller than simulation variance of double sampling ratio estimators. So regression estimator provides better estimation of the average head length.

Ratio of classical, bootstrap and jackknife variance estimates to the simulation variance are given in Table 4.6.

Table 4.6 Ratio of variances of the classical, jackknife and bootstrap estimates to the simulation variance

Method	Sample size (n)							$\sum_{j=1}^{7} \mid 1-R_{ij} \mid$
	50	75	100	125	150	175	200	
Ratio	1.16	0.93	1.07	0.90	0.95	0.95	0.99	0.51
Ratio(B)	1.20	0.87	0.97	1.01	0.91	0.94	1.01	0.53
Ratio(J)	1.09	0.89	1.01	0.81	0.89	0.94	0.99	0.58
Regression.	1.11	0.94	1.10	0.91	0.96	0.97	0.99	0.44
Regression(B)	1.15	0.89	0.98	1.02	0.93	0.97	1.00	0.40
Regression(J)	1.11	0.90	1.08	0.83	0.89	0.99	0.99	0.59

(R_{ij} indicates that the value of the ratio of the ith method at the jth sample size)

A ratio less than 1 indicates an underestimation of the variance, while a ratio greater than 1 indicates an over estimation. Using this as a guideline, we observe from the last column of the Table 4.6 that the classical regression and the bootstrap regression estimators provide the best estimates of the variance across all sample size tested.

Boxplot (given in Fig 4.4) also shows the same result.

Boxplot of ratio of variances of classical, bootstrap and jackknife estimates to the simulation variances.

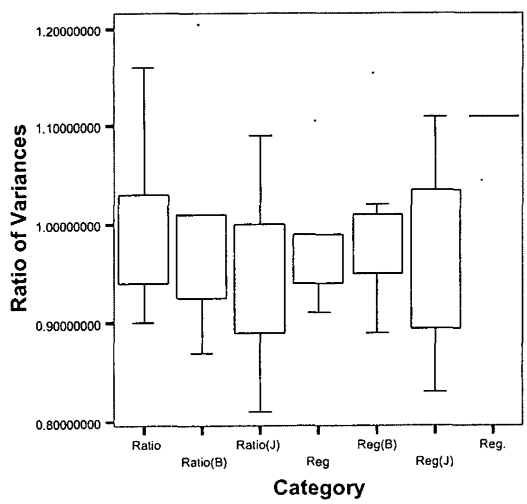

Fig. 4.4 Boxplot

Another method of evaluating the effect of bias on the statistical properties of the ratio and regression estimators is to examine the relative error which is defined as the bias divided by the root mean square error (bias/$\sqrt{MSE}$). A large relative error has the effect of distorting the confidence probabilities (Cochran 1977). For example, with a bias/$\sqrt{MSE}$=0.2, the actual confidence level associate with a nominal 95% confidence interval is about 0.9454.

Table 4.7 The relative error of the ratio and regression estimators in different methods

Method	Sample size (n)							Mean absolute relative error
	50	75	100	125	150	175	200	
Ratio	.0009	-.00014	.00015	-.00005	.00006	-.00006	.00004	.0002
Ratio(B)	-.0850	.01700	-.08120	.06580	-.12470	.06940	.05280	.0651
Ratio(J)	-.0009	-.00018	.00018	.00006	.00003	-.00020	-.00019	.00024
Regression.	.0620	-.01700	.00780	-.0053	.00940	.00634	.00660	.0163
Regression(B)	-.0325	-.00920	-.0690	.07120	-.10830	.09110	.07210	.0648
Regression(J)	-.0580	-.04490	.01580	-.00720	.01670	.01150	.01030	.0234

The Table 4.7 reflects that all the relative errors are very small for two estimators in different techniques (classical, jackknife and bootstrap). These small relative errors do not effect to distort the confidence probabilities.

5. Summary

The Anderson-Darling test statistic for normality indicate that the data related to the head length (y) and head circumference (x) follow normal distribution. It is also shown that the relationship between y and x is linear and the line of y on x does not pass through the origin and hence double sampling regression estimator may appropriate to use as an estimator of the population mean of y.

The estimate of the population mean of head length (y) in ratio and regression methods (using classical, jackknife and bootstrap techniques) are very nearer and they are very close to the population mean. Hence one can use any method to estimate the population mean.

The regression estimator has the larger bias than the ratio estimator. However, the bias for the regression estimator decreased linearly as the sample size increased, while the bias for the ratio estimator is non-linear over the sample size.

In terms of the MSE, the regression estimator provides the best estimation in all three methods (classical, jackknife and bootstrap). For large sample size (n=175,200), the MSE of the ratio and regression estimators for all the above techniques are very close. We also observe from simulation variance of double sampling ratio and regression estimators (Table 4.6) and the ratio of the variance of different methods to the simulation variance, the double sampling regression method provids the best method of estimation.

The biases in different methods are very small and negligible and hence do not provide any significant effect on the estimators. Consequently, the mean absolute relative errors are very small and they do not have effect to distort the confidence probabilities of the estimators.

The standard theory tells us that the bootstrap and jackknife method will provide better estimation if the data is non-normal. Reich et al. (1993) showed that the ratio method of estimation provide better estimation as the regression line passed through the origin and their relevant data did not follow the normal distribution.

It is clear from the above discussion that we are unable to get positive findings using bootstrap and jackknife technique. Finally, we can conclude that the classical regression method provides the best method of estimation for estimating the population mean of the study variable (y).

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Corresponding Address :

Dr. Dulal Chandra Roy

Associate Professor, Dept. of Statistics
Univerisity of Rajshahi
Rajshahi-6205
Bangladesh
E-mail : dulalroystat@yahoo.com