Using penetration depth for phase matching in photonic crystal waveguides

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A new method of design for the phase-matching in waveguides is suggested. The approach is based on utilizing the concept of the penetration depth of light into the waveguide walls. The lateral components of wavevectors are employed to adjust the phase-matching condition in the propagation direction. The method is demonstrated in two systems: one using single and the other using double photonic-crystal mirrors. [DOI: 10.2971/jeos.2011.11018]

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1 INTRODUCTION

Non-linear optics is a very important sector of optics and physics. Generally, various non-linear effects are based on two main cornerstones. The first one, of course, is the presence of a non-linear material capable of mixing different frequencies. However, it is likewise important to fulfill the phase-matching condition, otherwise, the output is usually close to zero [1]. This condition is simply the conservation of the photon's momentum (wavevector) along with its frequency.

Among the most known methods to achieve phase-matching is the usage of birefringence to compensate for material dispersion [2, 3]. Another technique [1], [4]-[6], called quasi-phase-matching, employs periodically modulated media which adds its reciprocal lattice vector to the conservation of momentum. In waveguides and optical fibers, the modal (waveguide) dispersion has also been used [7]-[18] (see also [19]), along with self-phase modulation (see [19]) and Cerenkov phase-matching [20]. However, the analysis, if done, was numerical. Usually, the dependence of the phase-mismatch on the frequency or the waveguide width was calculated.

In this work, we suggest to design the waveguide by making different effective widths for different waves. The concept of a properly defined penetration depth is then very useful. It yields a clearer interpretation of the underlying physics and may simplify the consideration of phase-matching in waveguides considerably. For instance, we notice that the phase-matching itself may actually not depend on the width of the waveguide. The latter must be adjusted only to confine all the waves inside.

To be specific, we focus on 1D photonic crystal (multilayer) waveguides. Two photonic crystal structures are suggested for phase-matching: one using single-crystal and the other using double-crystal mirrors. As a numerical example accounting for both waveguide and material dispersion, we consider a ZnS/SiO2 system. ZnS can be used for second harmonic generation [21, 22], while ZnS/SiO2 multilayer films have been fabricated [23].

2 THE MAIN IDEA

The phase-matching condition generally follows from the translational invariance of the medium. In a waveguide, translational invariance holds only in the propagation direction, let us call it x. In order to illustrate the idea in the simplest case, let us consider the second harmonic generation. The phase-matching equations then are

$$\begin{aligned} 2k_{1,x} &= k_{2,x}, \\ 2\omega_1 &= \omega_2. \end{aligned} \tag{1}$$

The lateral components of the photon momentum are not conserved but instead, have to fulfill the confinement conditions. In this work we exploit these lateral components to match between the two Eqs. (1).

In a simple metallic waveguide the confinement conditions would be

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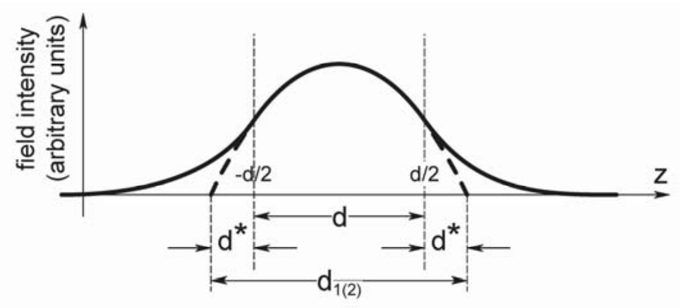

FIG. 1 Schematic explanation of the penetration depth concept in a generic waveguide. The field profile is shown as a function of the transverse coordinate z. The core has a width d and is located between $z = -d/2$ and $z = d/2$. The cladding is at $z < -d/2$ and $z > d/2$. The penetration depth is denoted as d^*.

$$k_{1,z} = \frac{\pi m_1}{d_1}, \quad k_{2,z} = \frac{\pi m_2}{d_2} \tag{2}$$

where $d_1 = d_2$ is the width of the waveguide and m is the mode number. It is easy to see that Eqs. (1) and (2) can be simultaneously satisfied only in the absence of dispersion (provided $m_2 = 2m_1$ and $d_1 = d_2$). Our main idea is to employ the penetration of the waves into the walls of the waveguide in order to make the effective widths of the waveguide different for various waves, i.e. $d_1 \neq d_2$. Hereafter it is shown how to realize this concept using two configurations of photonic crystals (PhCs).

3 PHASE MATCHING USING A SINGLE PHOTONIC CRYSTAL

In this section we consider a slab waveguide constituted by two identical multilayer mirrors (1D PhCs). When the frequency is in the forbidden band of the crystals, the light still penetrates into the mirrors though its amplitude exponentially decreases with the distance from the surface. The concept of penetration depth is illustrated in Figure 1.

The field profile is cosine or sine in the core of the waveguide (between $z = -d/2$ and $z = d/2$). Importantly, it does not vanish at the core boundary. In the cladding ($z < -d/2$ or $z > d/2$) the field decays exponentially. We call "penetration depth" the distance between the first "would be" zero of the cosine (or sine) and the real core boundary (d^* in the figure). The field inside the core is equivalent to that of the metallic waveguide of a larger width: $d_{1(2)} = d + d^*_{1(2)}$. Our definition of penetration depth is different from the usual one (the length of the field decay into the cladding). However, it is more relevant for phase-matching and it is still connected to the penetration into the walls. The penetration depth is clearly related to the phase of refraction.
We use the transfer matrix method [24, 25] to calculate the field profile and then determine the penetration depth into a photonic crystal mirror. In Figure 2 it is shown as a function of the wavevector and the frequency.
Contrary to the intuition drawn from the simplified picture in Figure 1, the penetration depth can be negative. The field

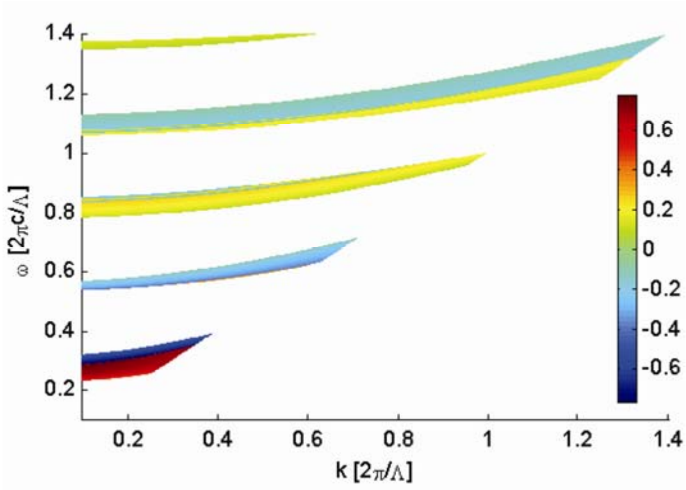

FIG. 2 The penetration depth of light into a PhC mirror (in units of the PhC lattice constant (Λ)) is shown by different colors as function of the dimensionless lateral wavevector $k_x \cdot \Lambda/2\pi$ (x-axis) and the dimensionless frequency $\omega \cdot \Lambda/2\pi c$ (y-axis). Penetration depth makes sense only in the bandgap regions of the PhC and within the incident light cone. The rest of the plot is left blank. The PhC is made of alternating layers of SiO_2 (with refractive indexes n(1064 nm) = 1.45, n(532 nm) = 1.46) and ZnS (with refractive indexes n(1064 nm) = 2.29, n(532 nm) = 2.40). The layer thicknesses are 0.542 Λ and 0.458 Λ.

profile inside the mirror is oscillating with an exponentially decaying envelope. This allows the derivative to be continuous even though the absolute value of the function inside the core is increasing as it approaches the boundary.
In the present configuration, the formulas (2) can be rewritten as

$$\frac{\pi m_1}{k_{1,z}} = d + 2d^*(\omega_1, k_{1,x}), \quad \frac{\pi m_2}{k_{2,z}} = d + 2d^*(\omega_2, k_{2,x}) \tag{3}$$

where $d^*(\omega, k_x)$ is the penetration depth and d is the real width of the waveguide. It is useful to substitute Eqs. (1) into Eqs. (3) and take the difference between the later ones. The resulting equation then does not contain d:

$$\frac{\pi}{2}\left(\frac{m_1}{\sqrt{n^2(\omega_1)\frac{\omega_1^2}{c^2} - k_{1,x}^2}} - \frac{m_2/2}{\sqrt{n^2(2\omega_1)\frac{\omega_1^2}{c^2} - k_{1,x}^2}}\right) = d^*(\omega_1, k_{1,x}) - d^*(2\omega_1, 2k_{1,x}) \tag{4}$$

Where $n(\omega)$ is the refractive index within the waveguide. Eq. (4) must be solved for ω_1 and $k_{1,x}$. The right-hand and the left-hand sides of this equation are shown in Figure 3 as a function of ω_1 for a particular $k_{1,x}$ (for $m_1 = 1$ and $m_2 = 1,2,3$).

Remarkably, the right-hand side is nearly constant inside every bandgap. The expression on the left-hand side diverges at $\omega_1 \to ck_{1x}/n(\omega_1)$. and vanishes at large ω_1. So it spans the whole range $(0, \infty)$ when ω_1 changes; for $m_2 > 2m_1$ it can also be negative. Since $k_{1,x}$ can be adjusted to find an intersection of the curves, a solution of Eq. (4) can always be found. It can be further seen that a solution exists for any desired mode number m_1. In the example displayed in Figure 3, we chose

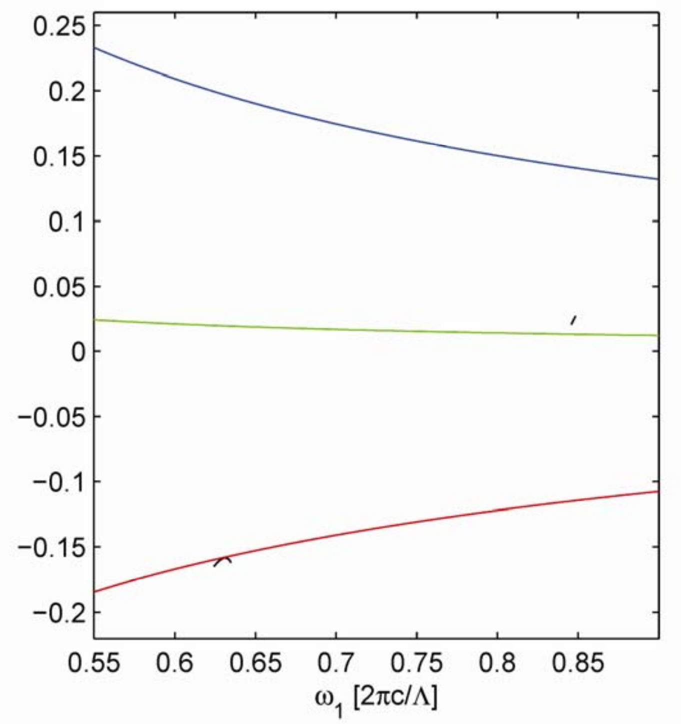

FIG. 3 Graphical solution of Eq. (4). The right-hand and the left-hand sides of the equation are plotted versus the dimensionless frequency $\omega_1 \cdot \Lambda/2\pi c$. The right-hand side is shown in black and only when both ω_1 and $\omega_2 = 2\omega_1$ are within the bandgaps of the photonic crystal. The left-hand side is drawn in blue (for $m_2 = 2m_1 - 1 = 1$), green (for $m_2 = 2m_1 = 2$) and red (for $m_2 = 2m_1 + 1 = 3$).
The wavevector is $k_{1x} = 0.55 \cdot 2\pi/\Lambda$. The left-hand side diverges at $\omega_1 \to ck_{1x}/n(\omega_1)$.

to match between the modes $m_1 = 1$ and $m_2 = 3$. Interestingly, the phase-matching has been achieved while the width of the waveguide d is not yet determined. It should now be chosen using one of the Eqs. (3). This means that the phase-matching is essentially a function of the material (the walls). The field profiles of the waves phase-matched using Figure 3 are shown in Figure 4.

4 PHASE MATCHING USING A DOUBLE PHOTONIC CRYSTAL

Phase-matching can be realized in a conceptually simpler way if two PhCs are involved. Indeed, every wave can be confined by an associated PhC, as shown in Figure 5.

Since every crystal confines one wave and is transparent for the other, the effective widths for the two waves are determined independently by the inner and the outer crystals. The thickness of the central region (i.e. between the mirrors) is chosen to match the standing wave conditions for the inner wave (second harmonic in Figure 5). Both the central region and the inner PhC of the mirrors serve as the propagation medium for the other wave. The number of layers of the inner PhC and the thickness of its outer layer may be adjusted to fulfill the standing wave conditions for that wave.

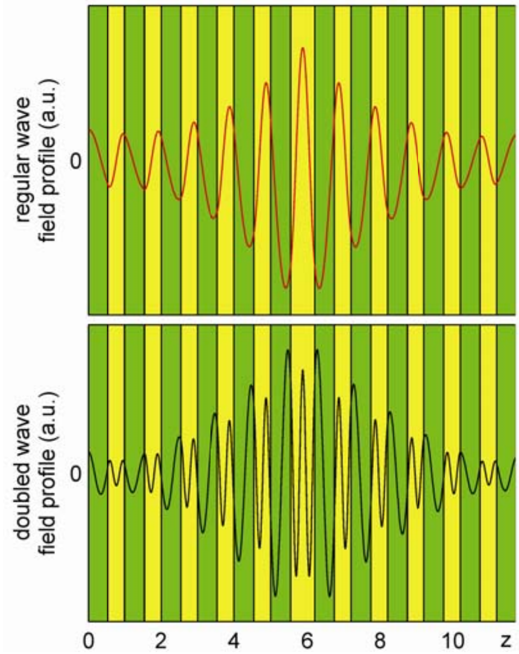

FIG. 4 Field profiles (in arbitrary units) of the regular (top) and the doubled (bottom) waves on the phase-matching conditions. The z-axis is in units of Λ, yellow color denotes ZnS layers, while green stands for SiO_2. The parameters are:
$k_{1,x} = 0.55 \cdot 2\pi/\Lambda$, $\omega_1 \approx 0.63 \cdot 2\pi c/\Lambda$, $k_{2,x} = 2k_{1,x}$, $\omega_2 = 2\omega_1$, $d \approx 0.66 \cdot \Lambda$. The PhC layer thicknesses are the same as in Figure 2.

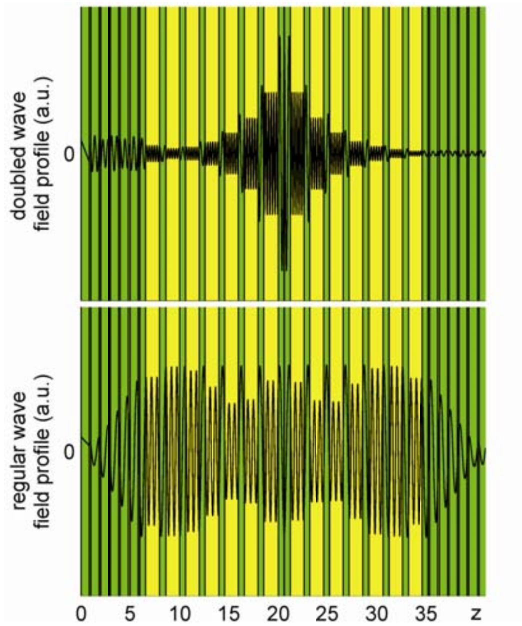

FIG. 5 Field profiles (in arbitrary units) of the doubled (top) and the regular (bottom) waves in a double PhC waveguide on the phase-matching conditions. The z-axis is in units of the lattice constant of the outer crystal (Λ_1), yellow color denotes ZnS layers, while green stands for SiO_2. The parameters are:
$k_{1,x} = 0.76 \cdot 2\pi/\Lambda_1$, $\omega_1 \approx 0.85 \cdot 2\pi c/\Lambda_1$, $k_{2,x} = 2k_{1,x}$, $\omega_2 = 2\omega_1$, $d \approx 0.03\ \Lambda_1$. The layer thicknesses for the outer crystal are $0.842\Lambda_1$ and $0.158\Lambda_1$; for the inner one they are $0.6\Lambda_1$ and $1.6\Lambda_1$. The last layer of the inner crystal is adjusted to properly confine the regular wave, so its thickness is $\approx 0.59\Lambda_1$.

5 CONCLUSIONS

A new technique for the design of phase-matching in waveguides has been proposed. In the present paper, it has been illustrated in the simplest example of the second harmonic generation. It can be applied, however, to any other nonlinear process. The method works for any given frequency and any desired waveguide mode number. Two different approaches have been shown. The first one (with a single type of photonic crystal) uses the penetration depth dependence on the parameters of the system. In the second approach (with double crystal mirrors), it is shown how to explicitly design the effective widths for every wave. Both ideas can be realized also in other waveguides, not only in photonic crystal ones. For instance, optical fibers and dielectric waveguides can be handily designed for this purpose. We notice that the waveguide should be further engineered in order to increase the modes overlap integral, thus improving the efficiency of the frequency conversion. However, this is a subject for a separate work.

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