



# **Adaptive feedback feedforward compensation for disturbance rejection in a one DOF flexible structure: comparative analysis**

## **Compensación adaptativa feedback feedforward para el rechazo de perturbaciones en una estructura flexible de un grado de libertad: análisis comparativo**

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### **ABSTRACT**

In this paper an Active Vibrational Control (AVC) for a three-cart problem is studied. The Filtered-x Least Mean Square (FxLMS) and Recursive Least Square (RLS) algorithms are compared in terms of disturbance rejection, computational cost and control effort when a correlated measurement of the disturbance is available. The proposed RLS compensator considers a feedback coupling between the compensator and the disturbance. The secondary propagation path of the plant was estimated using normalized LMS (NLMS) algorithm. The internal positive coupling is modeled as a FIR filter estimated by the real plant parameters. Simulations showed a superior performance of RLS algorithm with a reasonable computer cost. The comparative analysis was performed comparing the tradeoff between the filter order and the magnitude of the rejection.

**KEYWORDS:** Index terms –active vibration control; fir adaptive filter; the filtered-x least mean square; recursive least square.

### **RESUMEN**

En este artículo se estudia un Control Vibratorio Activo (AVC) para un problema de tres carritos. Se comparan los algoritmos de mínimos cuadrados filtrados (FxLMS) y mínimos cuadrados recurrentes (RLS) en términos de rechazo de perturbaciones, costo computacional y esfuerzo de control cuando se dispone de una medición correlacionada de la perturbación. El compensador RLS propuesto considera un acoplamiento de retroalimentación entre el compensador y la perturbación. La ruta de propagación secundaria de la planta se estimó utilizando el algoritmo LMS normalizado (NLMS). El acoplamiento positivo interno se modela como un filtro FIR estimado por los parámetros reales de la planta. Las simulaciones mostraron un rendimiento superior del algoritmo RLS con un costo informático razonable. El análisis comparativo se realizó comparando la compensación entre el orden del filtro y la magnitud del rechazo.

**PALABRAS CLAVE:** Términos del índice -active vibration control; fir filtro adaptativo; el cuadrado medio menos filtrado de x; recursive least square.

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## 1. INTRODUCTION

Nowadays, the research in Active Vibration Control (AVC) is gaining importance because of the increase in the number of electro-mechanic devices working in high speed applications. AVC uses an electromechanical or electroacoustic system, which cancels out the unwanted vibrations based on the superposition of wave's principle.

The adaptive controller scheme used is the Self Tuning Regulator (STR), in which the estimate of the process parameters is updated and the controller parameters are obtained from the solution of a design problem using the estimated parameters. The Adaptive Controller can be thought of as being composed of two loops, the inner loop consists of the process and an ordinary feedback controller, the parameters of the controller are adjusted by the outer loop which is composed of a recursive parameter estimator and a design calculation.

Meanwhile, Active Disturbance Rejection Control (ADRC) is a robust control method based on extension of the system model with an additional fictitious state variable, representing the uncertainties present in the description of the plant.

This study was conducted through the "Young Researchers" program of Colciencias as part of a research project entitled "Comparative study of nonlinear techniques AVC infinite H-adaptive filter for a flexible structure whit one degree of freedom". The adaptive approach (STR), using RLS and LMS algorithms, are used both as a mechanism to estimate in real time the plant parameters and as a mechanism to adjust controller parameters. LMS (Least Mean Squares) algorithm represents the simplest and most easily applied adaptive algorithm while RLS (Recursive Least Squares) algorithm represents increased complexity, increased computational cost but faster convergence. Additionally, RLS algorithm approaches the Kalman filter performance in adaptive filtering applications at somewhat reduced throughput in the signal processor.

In the study of the additive feedback coupling between the compensator and the disturbance measurement, was concluded that the absence of this feedback propagation path causes an error due to lack of synchronization with the actual response simulation in the adaptive control [1].

On the other hand, this feedback coupling may destabilize the system because of the lag between the simulation feedback compensation and the disturbance in its additive correlation. The simultaneous use of both an adaptive feedback compensator and a feedforward compensation to reject the disturbance is proposed [2], where it is stated that the action of the feedback loop adds

a new design specification for the stability conditions to the adaptive feedforward compensation.

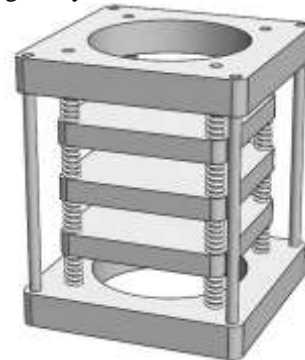
The differences in implementation between the Digital Signal Processing (DSP) broadband feedforward control and the adaptive feedback control and its application schemes in AVC are well studied in the literature [4-5], also the adaptive sinusoidal disturbance rejection in linear discrete time systems.

Least Square and regression models commonly used in Active Noise Control (ANC) to model without using conventional simplifying assumption regarding the physical plant to be controlled are the Filtered-X LMS (FxLMS) and RLS due to its simplicity in calculation and Digital Signal Processing (DSP) implementation to adaptive filtering in contrast to the result in system identification [6-11].

In this work a feedforward adaptive compensator is proposed, considering the feedback coupling in the disturbance rejection problem for a three-cart model. First, the propagation paths are fully identified as a group of transfer functions in series. The propagation paths are NLMS-based estimated as FIR filters. The adaptive compensation filters with RLS and FxLMS algorithms are applied for disturbance rejection in the studied plant.

## 2. THREE CART DYNAMICS WITH INERTIAL ACTUATOR

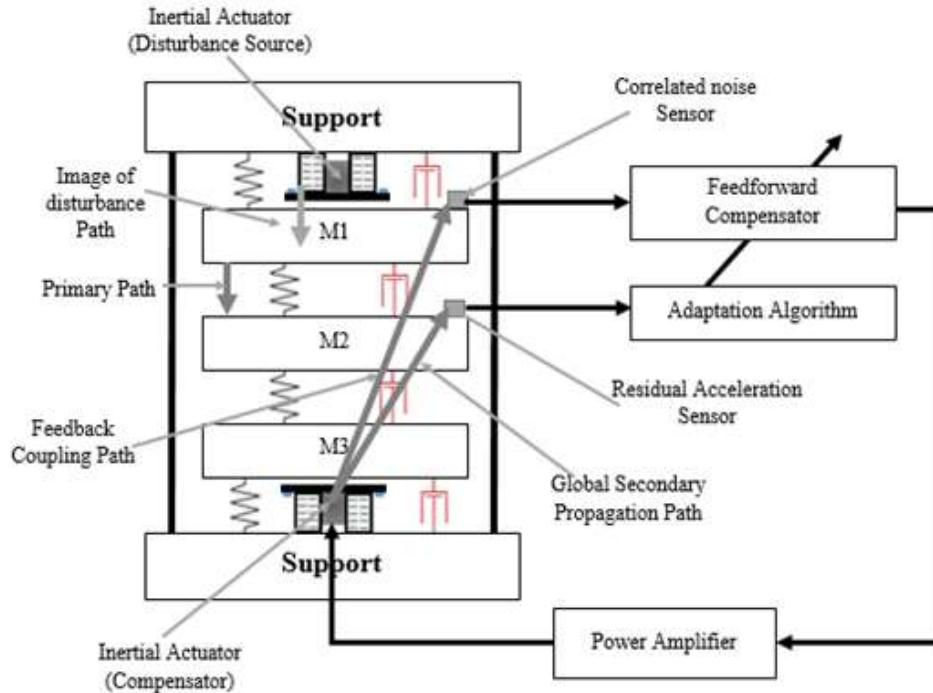
Figures 1 and 2 represent an AVC test bed in which the vibration measurement is correlated with the disturbance and an inertial actuator is used for reducing the residual acceleration. The system consists of five metallic plates connected by springs. The plates  $M_1$  and  $M_3$  are equipped with inertial actuators.  $M_1$  serves as disturbance generator (inertial actuator 1 in Figure 2) and  $M_3$  serves for disturbance compensation (inertial actuator 2 in Figure 2). The system is equipped with a measure of the residual acceleration (on plate  $M_2$ ) and a measure of the disturbance signal by an accelerometer on plate  $M_1$ .



**Figure 1.** Studied plant model, AVC system. **Source:** authors.

In the described scheme, the path between the disturbance (in this case, generated by the inertial actuator on the top of the structure), and the residual acceleration is called the *global primary path*. The path between the position of  $M_1$  (an image of the disturbance) and the residual acceleration (in open loop) is called the *primary path* and the path between the inertial actuator

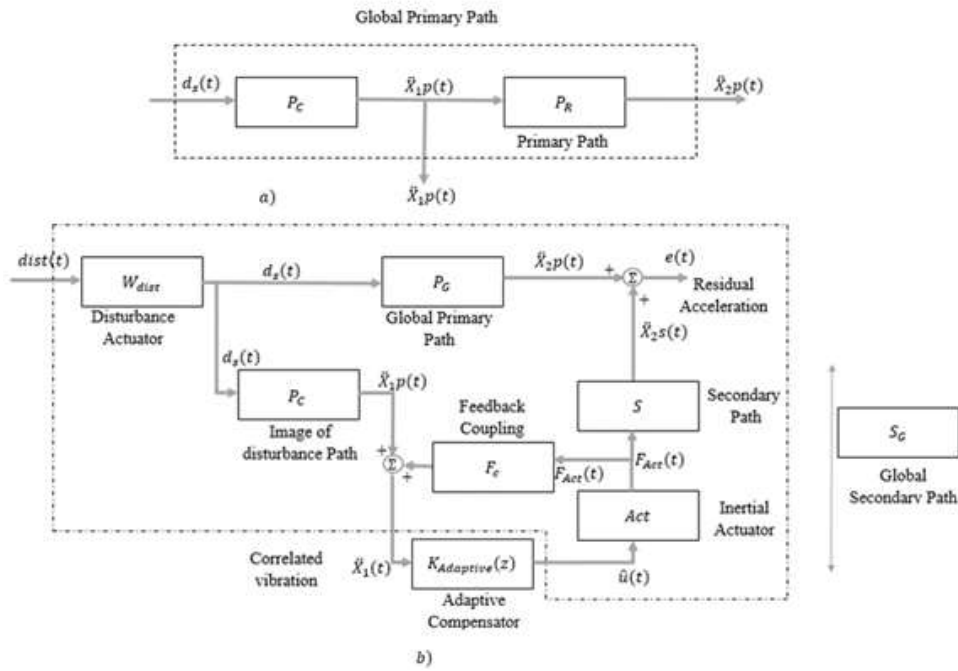
used for compensation and the residual acceleration is called the *secondary path*. When the compensator system is active, the actuator not only acts upon the residual acceleration, but also upon the measurement of the disturbance image (a positive feedback).



**Figure 2.** Scheme of the plant, AVC system. **Source:** authors.

The disturbance is the pressure wave of the inertial actuator (see Figures 1 and 2) located on top of the structure. The output of the compensator system is the pressure wave of the inertial actuator located on the bottom of the structure. The parameters of the filter are estimated to minimize the measurement of the residual acceleration. In Figure 3 it can be observed the block diagram of the AVC system. The  $W_{dist}(z)$  filter emulates the band limiter filter and the speaker. The disturbance source is white noise filtered by  $W_{dist}(z)$  to obtain  $ds(t)$ . The filter  $P_G$  emulates the global primary path which contains the disturbance and the mechanical path between the pressure wave and the residual acceleration. The filter  $P_C$  characterizes the dynamics of the disturbance source and the image of the disturbance (inertial actuator + dynamics of the mechanical system). The compensation actuator is modeled by the transfer function  $Act$  with the control signal as input and the pressure wave as output (power amplifier + the compensation inertial actuator).

The secondary path is represented by the  $S$  block (see Figure 3(a)), which models the dynamics of the pressure wave traveling from the inertial compensator actuator to the residual acceleration in the absence of the disturbance. The  $Fc$  block emulates the mechanical path between the compensation inertial actuator and the correlated disturbance. The feedforward compensator is the  $K_{Adaptive}$  block with  $\ddot{X}_1(t)$  as the correlated noise and the residual acceleration (the desired signal) as inputs and the output  $\hat{u}(t)$  as the control signal. The value of  $\ddot{X}_1(t)$  is the sum of the correlated disturbance measurement  $\ddot{X}_1p(t)$  obtained in the absence of the feedforward compensation (see Figure 3(a)) and the effect of the actuator used for compensation.

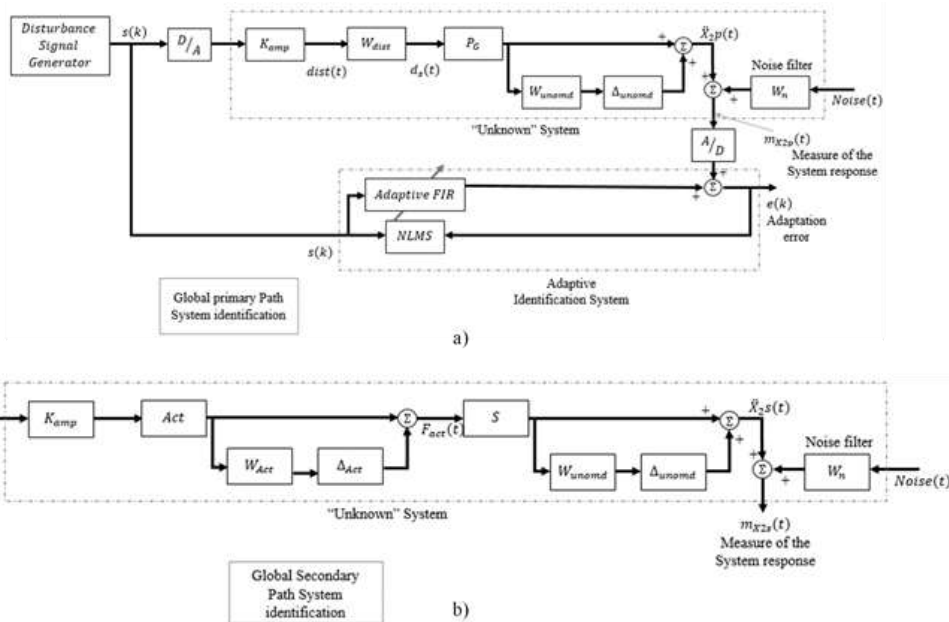


**Figure 3.** Block diagram of the plant of the AVC system a) open loop b) with the feedforward compensator. **Source:** authors.

### 3. SYSTEM IDENTIFICATION OF THE PROPAGATION PATHS

A System identification process is implemented to estimate the impulse response of the four propagation paths in the AVC system. The obtained models consider the un-modeled dynamics inherent to the simplification

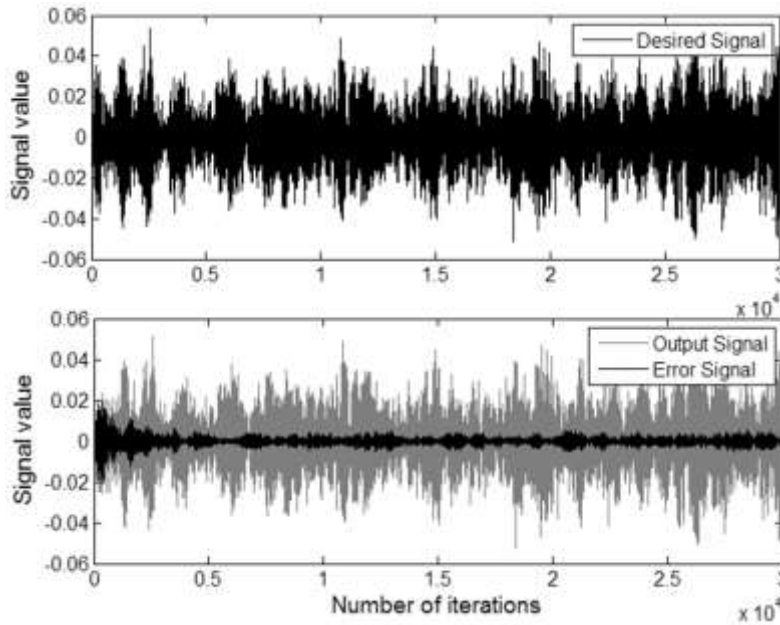
of the plant. The system is modeled using an *Adaptive Filter* with Normalized LMS algorithm (NLMS) to adapt the impulse response of the *Unknown System* (Nominal plant + uncertainties + the measurement error) injecting band limited noise to both i.e. the *Adaptive Identification System* and the *Unknown System* and comparing their response, see Figure.



**Figure 4.** Secondary path Identification Using the NLMS Adaptive Filter. **Source:** authors.

Each propagation path is fully identified and emulated as a FIR filter using the coefficients of the adaptive filters. The FIR filter obtained for the secondary path has a

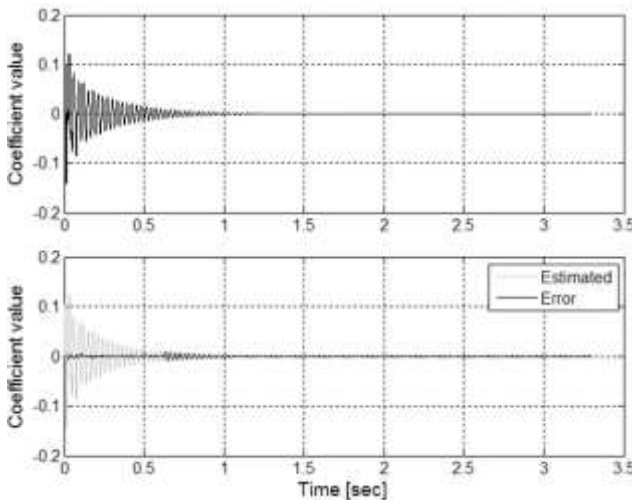
response time of 10 ms and its identification process is illustrated in Figure. 5.



**Figure 5.** Secondary path Identification Using the NLMS Adaptive Filter. **Source:** authors.

Figure 6 shows the behavior of the estimated secondary path impulse response and the comparison with the estimated path. The performance in the estimation of the impulse response of the adaptive filter in the tail is poor but does not affect the operation of the studied AVC system in a significant way.

*Primary Propagation and Feedback Coupling Propagation Paths identification*



**Figure 6.** Secondary path Impulse response identification. **Source:** authors.

The primary propagation path  $Pc$  is modeled by a linear filter. This filter is obtained in absence of compensation and observing the signal of the accelerometer, which measures the correlation signal, after an impulse disturbance is applied by the disturbance actuator. The coefficients of the FIR impulse response filter represent the response of the entire global primary path.

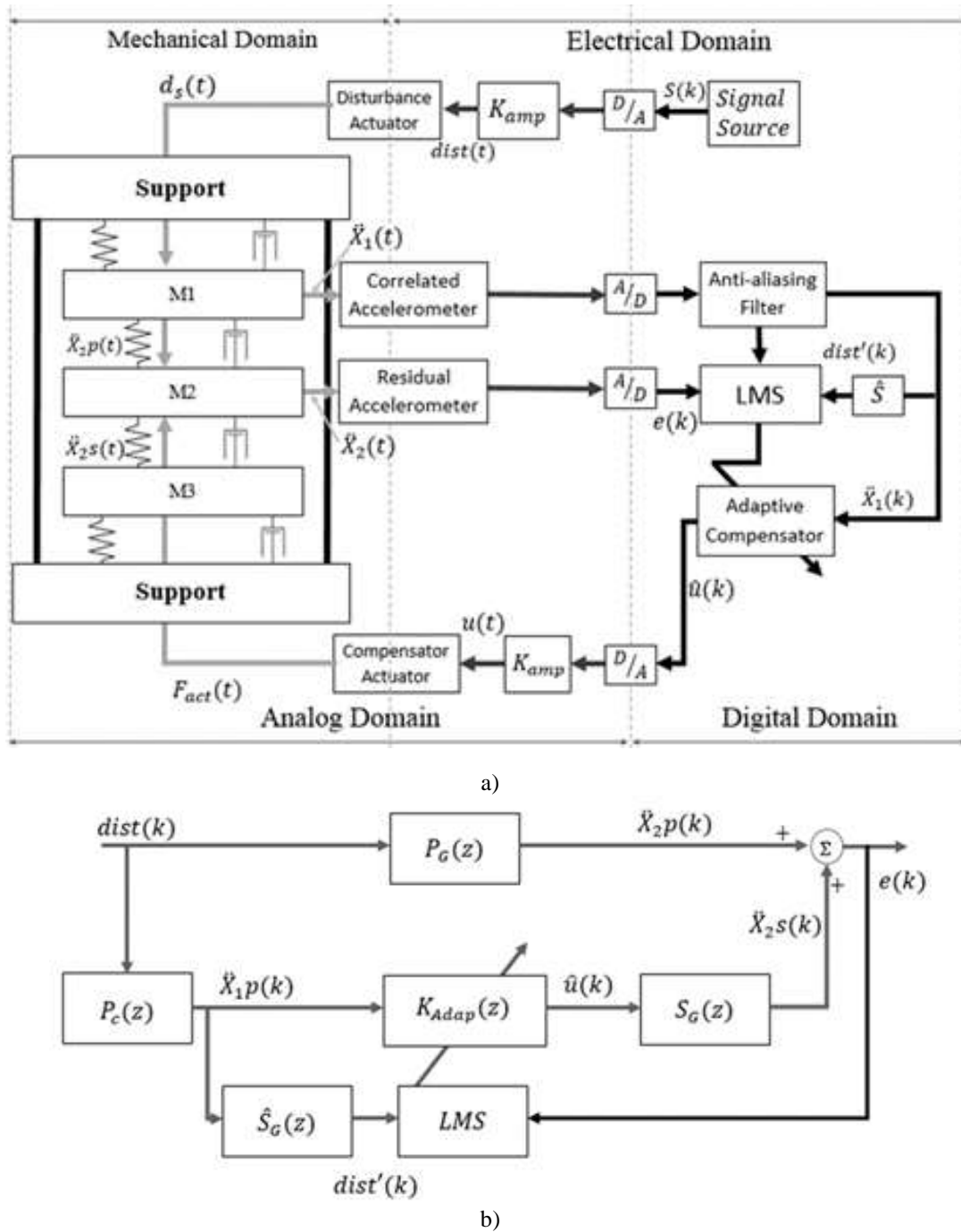
The system identification of the propagation path of the “Additive” Feedback Coupling is the measure of the effect of the inertial actuator compensation over the correlated accelerometers in the absence of disturbance.

**4. AVC USING FILTERED-X LMS FIR ADAPTIVE FILTER**

In the design of the FIR adaptive filter using the filtered-x-LMS algorithm the additive feedback coupling was not considered. In Figure. 7 a) the scheme of the proposed plant and in 7 b) the block diagram of the same plant, it can be observed that the correlated noise is the measure of the image of the perturbation,  $\hat{X}_1(t)$  and the desired signal is  $\hat{X}_2(t)$ . The digital adaptive compensator is a feedforward controller of 500<sup>th</sup> order and step size of 0.01.

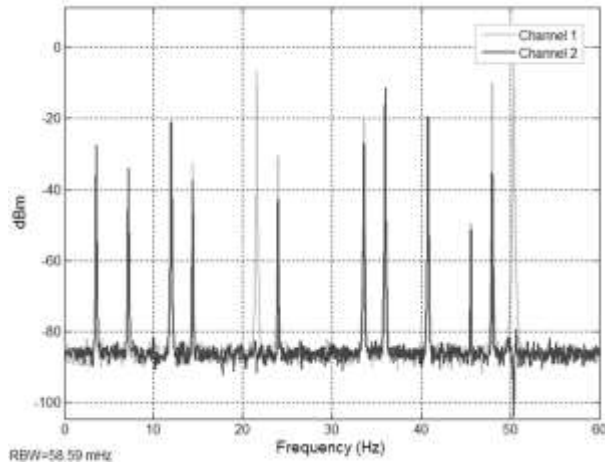
The experiments were carried out by first applying the disturbance in open loop during 30 s and then closing the loop with the adaptive feedforward-feedback algorithms. The band limited disturbance source emulates the

bandwidth attribute to the vibration of rotating machinery, that are generally the primary source of vibrations in industry.



**Figure 7.** Schematic arrangement of feedforward AVC system with FxLMS b) Block Diagram. **Source:** authors.

Figure 8 shows the resulting power spectral density of the residual acceleration, where Channel 1 line corresponds to the residual accelerometer without compensation and Channel 2 line is the residual acceleration with compensation.



**Figure 8.** Power spectral of AVC of the Filtered-x LMS. **Source:** authors.

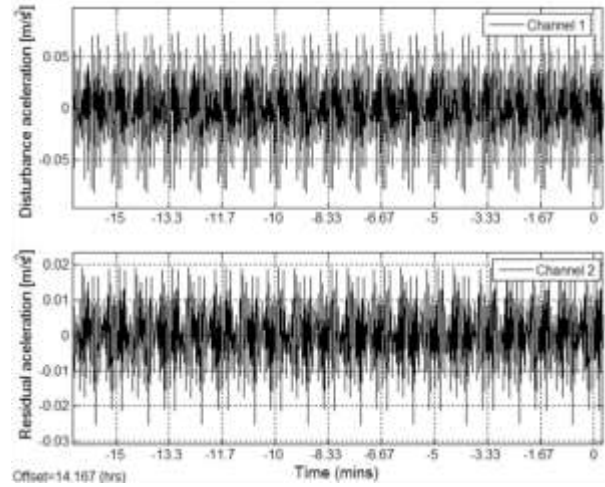
In the same Figure, Channel 2 exhibits frequencies of 21 Hz and 50 Hz (the higher components of the disturbance) which are attenuated below -40 dB. On the other hand, it is also observed a decrease in the compensation performance at components 34 Hz and 36 Hz, which are partially ignored by the compensator generating the maximum amplitude value of the error signal. Time domain signal obtained in open loop and with the compensator (using adaptive feedforward compensation algorithm FxLMS) on the AVC system are shown in Figure 9. The residual acceleration, in channel 2, is quantified as the variance of the residual force (error) on the mass 2. The compensator provides a mean reduction of 26.40 dB in the acceleration of the controlled mass.

**5. ADAPTIVE FEEDFORWARD AVC USING RLS ALGORITHM WITH FEEDBACK COUPLING**

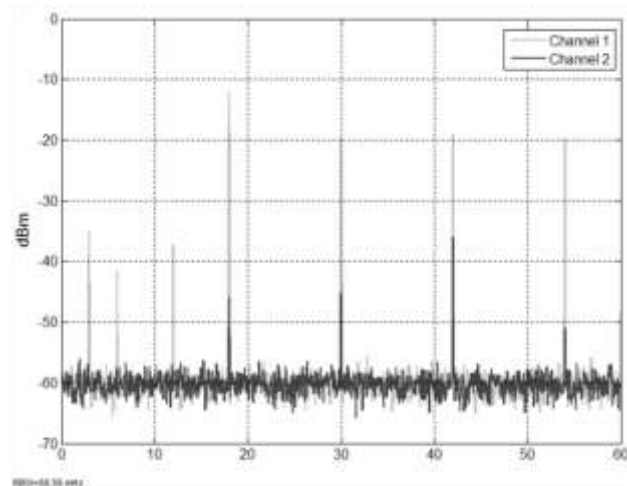
A new control scheme is proposed to accomplish disturbance rejection in order to improve the performance observed in the feedforward FxLMS. This new compensator considers the feedback coupling caused by the compensator actuator affecting the correlated disturbance shown in Figure 3.

The design process of the compensator using the RLS algorithm shown in Figure 3, resulted in 70<sup>th</sup> order FIR Filter. The performance of the new control scheme can

be appreciated in Figure 10-11. In Figure 10 the frequency response of the residual acceleration with and without compensation in Channel 2 and Channel 1 is presented, respectively. In Figure 11, Channel 1 signal is the disturbance applied to the plant and Channel 2 signal corresponds to the residual acceleration on mass 2.



**Figure 9.** Disturbance rejection of the feedback Filtered-x LMS. **Source:** authors.



**Figure 10.** Spectrum of the frequency response of AVC using RLS algorithm with feedback coupling. **Source:** authors.

When using only adaptive feedforward compensation RLS the mean disturbance reduction is 39.2 dB. Clearly, RLS scheme brings a significant improvement in performance with respect to the other schemes offering in addition adaptation capabilities with respect to the disturbance characteristics.

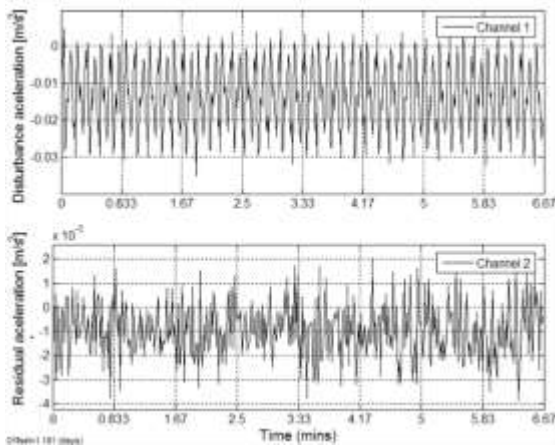


Figure 11. Disturbance rejection of the AVC using RLS algorithm with feedback coupling. Source: authors.

### 6. ADAPTIVE FEEDBACK AVC USING FXLMS ALGORITHM

The Figure 12 a) shows the schema of the Adaptive Feedback AVC System using the FxLMS algorithm and 12 b) the block diagram. The system produces its own reference signal using an estimated path, the adaptive filter output and the error signal. The main advantage of this scheme is the use of only one accelerometer. The reference signal or primary noise is expressed in Z-Domain as:

$$dist'(k) = e(k) + \hat{S}_G(z)\hat{u}(z) \quad (1)$$

Where  $\hat{S}_G(z)$  is the estimated secondary propagation path,  $e(k)$  is the error signal and  $\hat{u}(k)$  is the secondary signal produce by the adaptive filter.

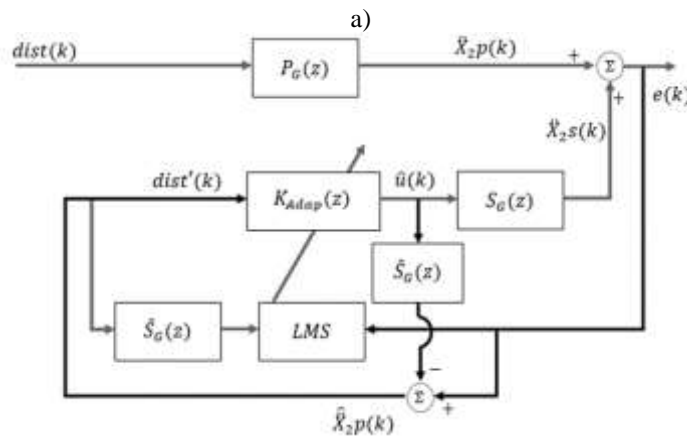
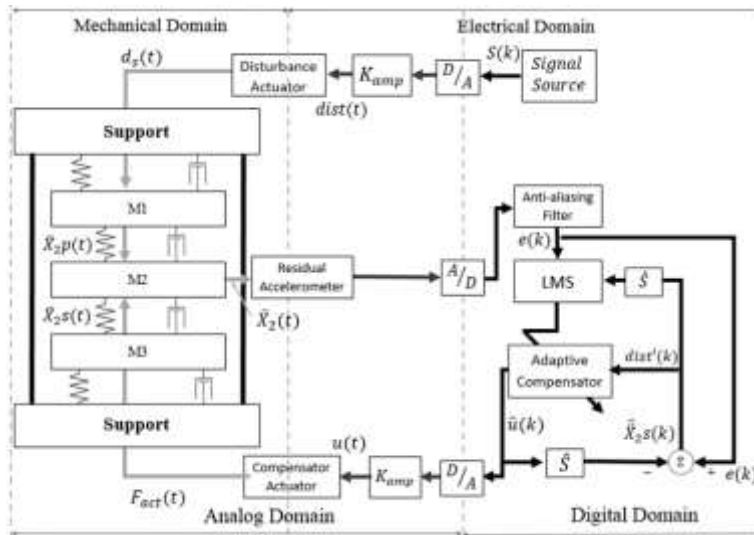
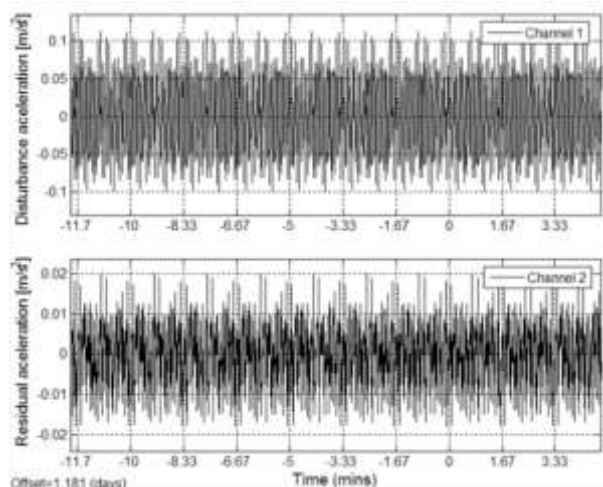


Figure 12. a) Schematic arrangement of feedback AVC system with FxLMS b) Block Diagram. Source: authors.



The resulting 500<sup>th</sup> order filtered-XLMS uses the conventional LMS algorithm. Comparing Figure 11 and Figure 13 it can be noted a decrease in performance of the compensation when the adaptive feedback using FxLMS is implemented. The lack of compensation



**Figure 13.** Disturbance rejection of the AVC using feedback FxLMS. **Source:** authors.

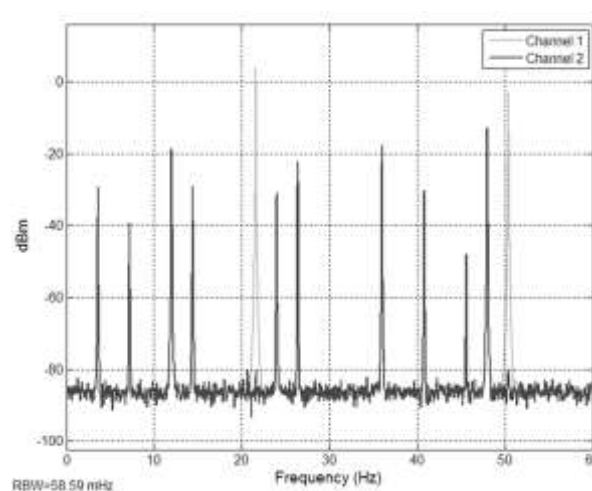
Figure 14 shows the frequency spectrum of the closed-loop system, the peaks with the high amplitude to be attenuated at 21 Hz and 50 Hz can be observed in the Channel 1. Comparing Figure. 10 with Figure 14 it can be observed the lower performance in disturbance rejection of the feedback FxLMS AVC system compared with the feedforward compensators. This lack of performance is the result of an additional computational cost associated to the calculation of the reference disturbance signal. Additionally, in the generation of the reference signal some frequencies of the desired signal are attenuated by the filters that emulate the propagation path so the compensator misses out attenuated signals in the calculation. The mean disturbance reduction is (20dB), when the feedback controller is active.

**Table 1.** Comparative performance board

AVC Filter	Filter order	Attenuation (dB)
FxLMS in feedforward	500	26,40
RLS in feedforward	70	39,2
FxLMS in feedback	500	20

**Source:** Own elaboration.

performance is attributed to the system identification process because although the correlation between the desired signal and the correlated signal is 1 the system was incapable to fully identified specific values of frequencies.



**Figure 14.** Power spectral of the AVC system using feedback FxLMS. **Source:** authors.

## 7. CONCLUSIONS

In this paper three different designs of the adaptive filter with FxLMS and RLS as adaptive algorithm were applied to periodical disturbance rejection in an AVC system. Simulations show that the implementation of the FxLMS feedforward AVC System uses a reasonable amount of effort to find the opposite form of the disturbance compared with feedback FxLMS. The compensation using the FxLMS feedforward scheme was unable to fully identify all frequency components to be attenuated in the residual acceleration. This poor performance can be attributed to the high computational cost associated with the adaptation algorithm.

The implementation of the feedforward AVC system with RLS algorithm considering the feedback coupling shows better results in the attenuation of the disturbance, with a moderate size for the adaptive filter. Tests reveal instability when the unappropriated adaptation step is chosen. The instability is attributed to the positive feedback.

The feedback coupling provides a best estimation of the plant model compared with estimated secondary path filters obtained by NLMS. Simulations demonstrate a better disturbance rejection and low computational cost using the feedforward AVC system with RLS algorithm.

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