Estimation of Gas Holdup in Three-phase Fluidized Bed Containing Small or Low Density Particles

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Abstract

A mechanistic model based on the drift line created by a spherical bubble passing through a liquid is developed to predict the gas holdup in gas-liquid-solid three-phase fluidized beds containing small or low density particles. In the model development, the drift line calculated from stream function for the three-dimensional case is used to predict the mean liquid rise path in the bubble street. The gas holdup can be estimated from the mean bubble rise velocity obtained by the sum of the following: the single bubble rise velocity, the mean liquid velocity calculated from the mean liquid rise path, the gas velocity in the bubble street, and the liquid velocity. Agreement between the calculated and measured values of ε_g is fairly good using the correction factor. Also agreement of the calculated values of ε_g with the measurements in the bubble column is good using a constant correction factor of around 0.7.

Introduction

Gas-liquid-solid fluidized bed systems have been widely applied to many biotechnological processes such as fermentation and aerobic wastewater treatment in which very small particles and /or light particles whose densities are very close to those of the liquied media are contained. Previously, however, most research was concerned with three-phase fluidized beds of glass beads, alumina particles, etc., of which the densities were more than 2500 kg/m^3 (Muroyama and Fan, 1984). One of the most important hydrodynamic characteristics of a three-phase fluidized bed as a design parameter is the gas holdup necessary to predict the interfacial area. Bhatia and Epstein (1974) developed the generalized wake model. To estimate the gas holdup using this model requires two unknown parameters which are quite difficult to obtain experimentally: the ratio of the solids holdup in the wake to that in the liquid-solid fluidized bed region and the ratio of the wake volume to the bubble volume for a multibubble system. In calculations using the wake model, a potential difficulty exists in the estimation of the gas haldup for the model. Therefore, many empirical correlations for the gas holdup were proposed and the effect of particle size on the gas holdup was described (Muroyama and Fan, 1984).

In the present study, apart from the generalized wake model, we will propose a model based on the drift line created by a bubble passing through a liquid; we call it the drift line model. From the drift line model, the mean bubble rise velocity can be calculated. Concerning this mean bubble rise velochty, a new approach for the calculation of the gas holdup is demonstrated for the study of the hydrodynamic characteristics of gas-liquid-solid fluidized bed systems containing small or low density particles.

1. Theoretical Model

1. 1 Mean liquid rise path created by a bubble passing through a liquid

As shown in **Fig. 1** (Joshi and Shah, 1981), in a bubble column, the upward liquid flow at the center of the column and the downward flow near the wall, namely the liquid circulation flow, can be found. According to previous experimental observations, liquid circulation was found in a three-phase fluidized bed. **Fig. 2** shows the radial liquid velocity profile in a three-phase fluidized bed (Kato *et al.*, 1983). We can observe the liquid circulation. Therefore, a stable and axial symmetrical bubble street created by

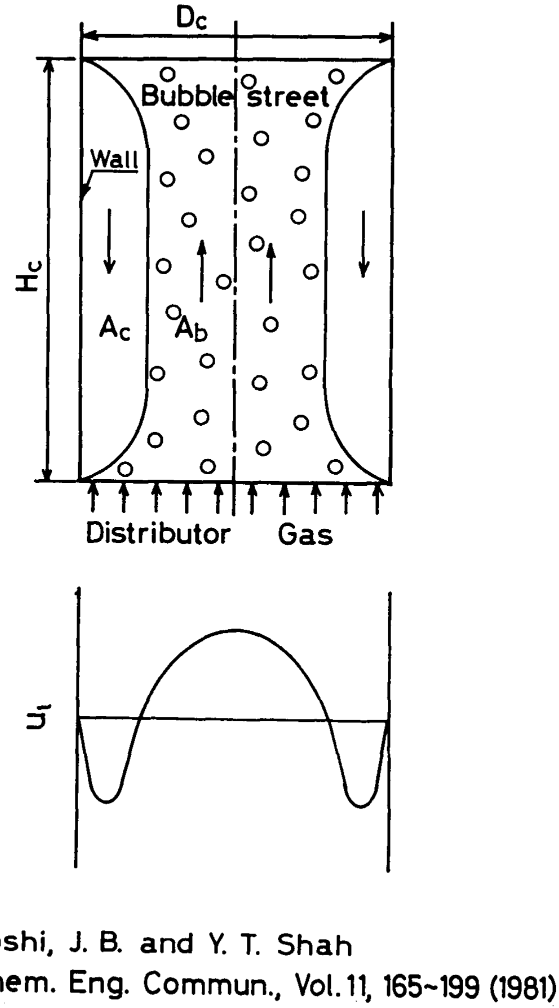

Fig. 1 Liquid flow pattern in a bubble column.

rising bubbles can also be anticipated in three-phase fluidized beds. We will then try to estimate the upward fluid flow using the drift created by rising bubbles.

For the analysis, the following conditions are assumed.

1) The bubble is spherical.

2) The energy of mixing and diffusion between media are neglected.

3) Bubbles do not coalesce with each other and there is no bubble-bubble interaction.

4) The bublle rises in an infinite fluid medium.

Suppose that the fluid is a perfect fluid. As shown in **Fig. 3**, a drift line is created at the rear of a bubble. For the three-dimensional case, the drift line in an infinite fluid medium can be calculated numerically from the following stream function.

$$\psi = \frac{U_s}{2} x^2 \left\{1 - \frac{a^3}{(x^2+y^2)^{3/2}}\right\} \tag{1}$$

When we have an infinite fluid medium, the fluid volume surrounded by the drift line is equivalent to half the volume of a spherical bubble with a radius of a (Darwin, 1953). The drift line, however, can be obtaind not analytically but numerically. At first, we will try to normalize the drift line calculated from Eq. (1) by dividing x and y by the bubble radius. The normalized drift line is shown in **Fig. 4** as the solid line (Toei *et al.*, 1966), where $Y = y/a$ and $X = x/a$.

Let us approximate the calculated drift line by the following equation

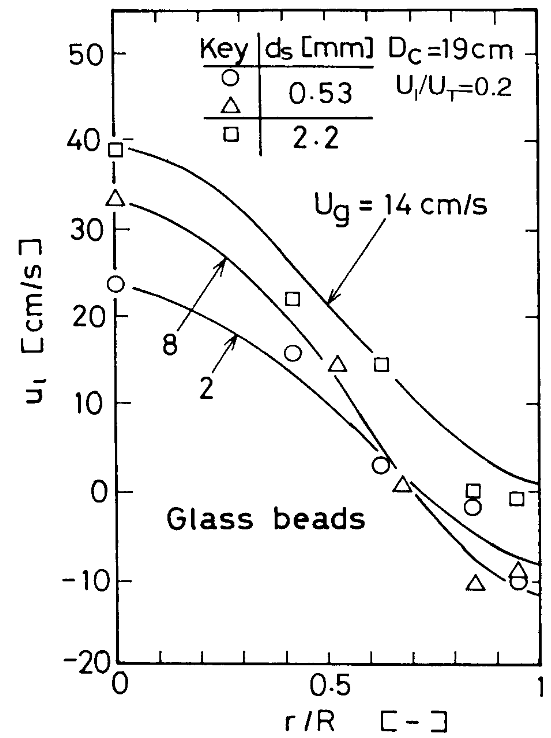

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Fig. 2 Radial distribution of local liquid velocity in a gas-liquid-solid fluidized bed.

$$Y = \frac{1.03}{(X+0.716)^4} \tag{2}$$

based on the fact that the fluid volume surrounded by the drift line is equivalent to half the volume of a spherical bubble when there is an infinite fluid medium (Darwin, 1953) (See **Appendix**). For comparison, the calculated values from Eq. (2) are plotted in Fig. 4. Agreement of calculated values with the numerical solution is very good. As expected from **Fig. 5**, the boundary of the bubble street X_1 mentioned above can be defined as

$$C = c/a = \frac{1.03}{(X_1+0.716)^4} \tag{3}$$

Note that X_R in the figure is the column radius normalized by the bubble radius. The ratio, β, of the bublle street sectional area to the column sectional area is defined by

$$\beta = \left(\frac{A_b}{A_c}\right) = \left(\frac{X_1}{X_R}\right)^2 \tag{4}$$

The dimenisionless mean fluid rise path after the bubble has passed through is expressed as

$$W = w/a = \frac{\int_0^{X_1} 2\pi X(Y-C)dX}{\pi X_1^2} \tag{5}$$

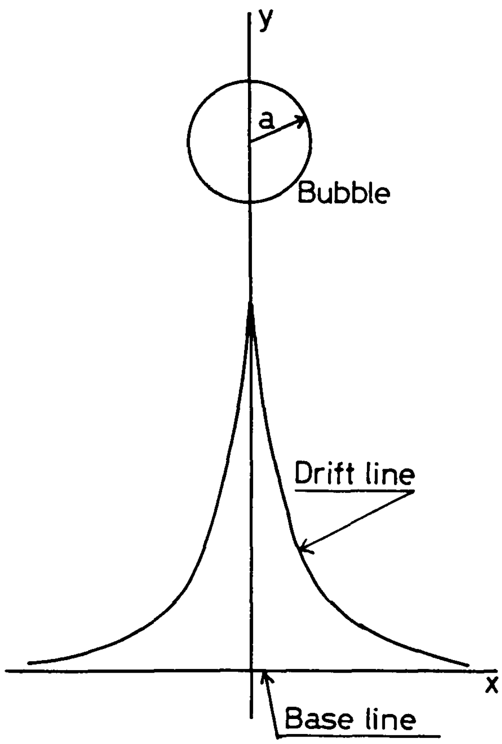

Fig. 3 Drift line after the bubble has passed through fluid thoroughly.

Combination of Eqs. (2), (3) and (5) gives the mean fluid rise path in the bubble street as follows,

$$W = \frac{1}{\beta X_R^2}\left(0.672 - \frac{1.03\sqrt{\beta}X_R + 0.247}{(\sqrt{\beta}X_R + 0.716)^3}\right) - C \tag{6}$$

1. 2 Mean rise velocity of bubbles

In the above section, the mean fluid rise path was obtained using the drift line model. The mean upward fluid velocity in the bubble street caused by the bubbles then is

$$U_{lb} = wV_g/\left(\frac{4}{3}\pi a^3\right) = \frac{3}{4}(w/a)V_g/(\pi a^2) = \frac{3}{4}WU_gX_R^2 \tag{7}$$

Therefore, the mean rise velocity of bubbles in a gas-liquid-solid fluidized bed is as follows,

$$U_B = U_g/\varepsilon_g = U_s + \frac{3}{4}WU_gX_R^2 + \frac{U_g}{\beta} + U_l \tag{8}$$

where U_s is the bubble rise velocity relative to the fluid velocity, i. e., the rise velocity of a single bubble in a still fluid. Rearrangement of Eq. (8) gives the gas holdup including the correction factor K which represents the deviation the gas holdup from that in a perfect fluid.

$$\varepsilon_g = \frac{K\beta U_g}{\{U_s + (3/4)WU_gX_R^2 + U_l\}\beta + U_g} \tag{9}$$

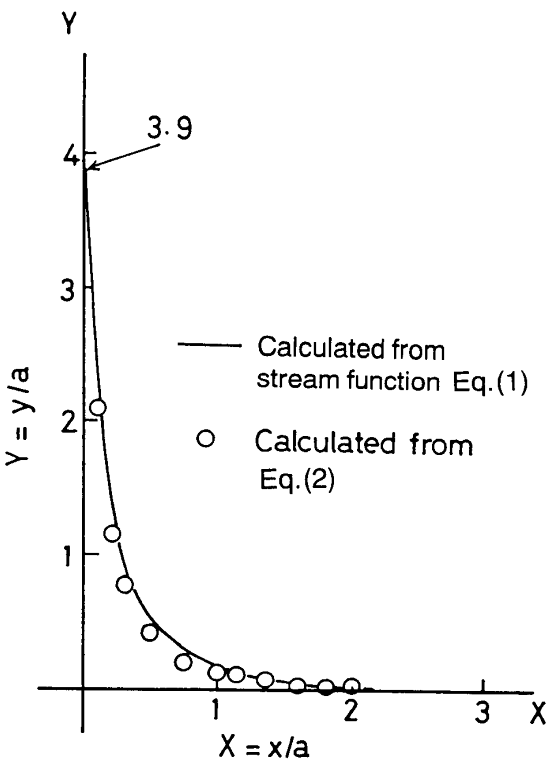

Fig. 4 Approximation of drift line.

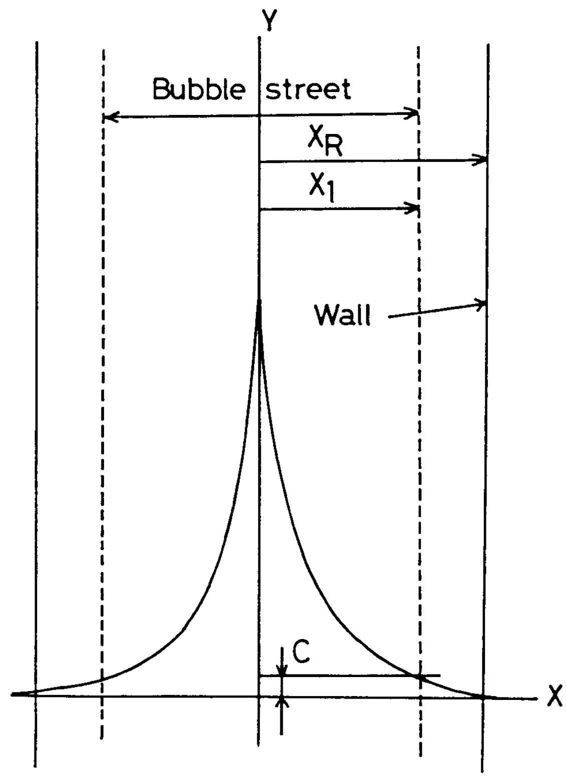

Fig. 5 Bubble street in the bed.

Here we will use the values of $\beta=0.5$ reported by Kojima *et al.* (1980) and Miyauchi and Shyu (1970) for bubble columns and take the results of the anthors as an approximation of U_s (Miyahara and Takahashi, 1983) and the result of Tanaka *et al.* (1988) as d_B in the three-phase fluidization regime. Also, we will use the results of the authors (Miyahara *et al.*, 1982; Miyahara and Takahashi, 1983) for d_B in the bubble columns and suspension regimes mentioned later.

2. Experimental

The experimental apparatus used is shown schematically in **Fig. 6.** The experiments were carried out in transparent cylindrical column of 0.065 m i. d. and 1.6 m height. A cylindrical stainless steel screen with a opening of about 0.5 mm was placed at the column top to prevent the elutriation of particles. Air and liquid were introduced co-currently through a distributor where the two phases were separated. At the column base, a packed bed of glass particles of about 15 mm in diameter was placed to ensure homogeneous liquid flow. The properties of the particles and the physical properties of the liquids employed are summarized in **Table 1** and **Table 2,** respectively.

Measurements of the axial pressure profile were made by means of manometers from 17 taps fitted normal to the column wall ranging from the column base to the column top. To reduce the fluctuations in readings which are caused by bubbling, orifices with holes 1 mm in diameter were placed at the mouth of each tap. This method for measuring the axial pressure profile has been sometimes employed. Deionized water and aqueous glycerol solutions were used as the liquid phases. Most of the tests were carried out over a range of superficial gas velocity of 0.1-7 cm/s and over a range of superficial liquid velocity of 0.1-7 cm/s.

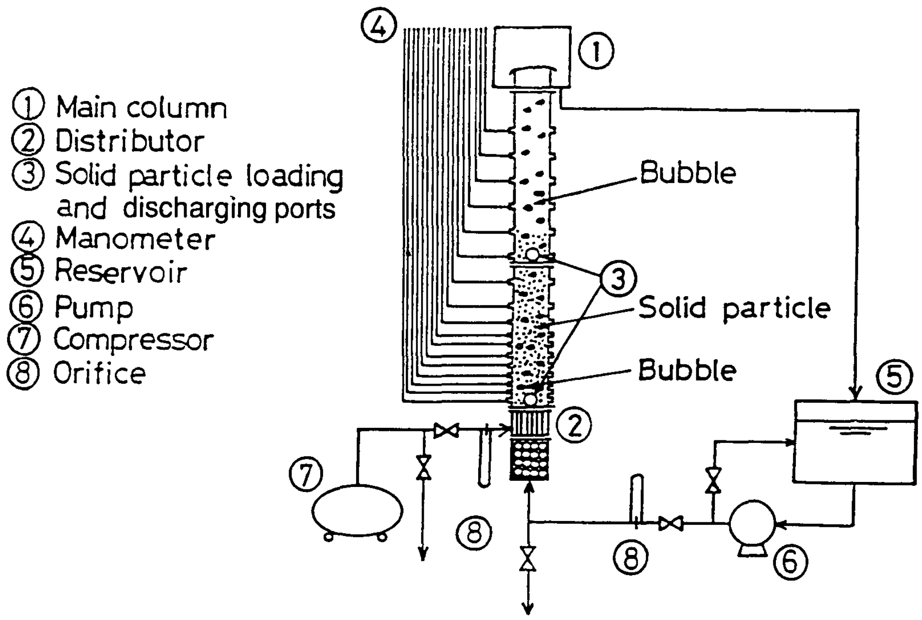

Fig. 6 Experimental apparatus.

3. Results and Discussion

3. 1 Operation regime

The hydrodynamic behavior of gas-liquid solid fluidized beds can be classified into three basic operation regimes on the basis of static pressure profile along the axial direction shown in **Fig. 7**: the complete three-phase fluidized bed regime, the partial suspension regime and the suspension regime (Miyahara *et al.*, 1990). In the complete fluidized bed regime, one notices the distinct boundary between the bed and the freeboard. On the other hand, the partial suspension regime shows axial non-homogeneity of the solids holdup. The suspension regime consists of a homogeneous axial solids holdup.

Table 1 Properties of particles employed

Particle	$d_s \times 10^3$ [m]	ρ_s [kg/m^3]
Nylon-6	6.315	1115.6
	7.837	1133.7
	9.520	1127.4
	12.682	1135.4
Glass beads	2.19	2489.4
	0.464	2490.0
	0.551	2490.0
Active carbon	0.65	1428.5*
Polystyrene	3.095	1043.0

* : wet density for water

Table 2 Physical propertes of liquids employed

Solution	ρ_l [kg/m^3]	$\mu \times 10^3$ [Pa.s]	$\sigma \times 10^3$ [N/m]
Water	998	1.00	72.8
Glycerol aq. soln.			
40%	1112	3.8	69.0
35%	1090	2.9	69.5
30%	1083	2.4	69.9
20%	1053	1.65	7.09

at 20℃

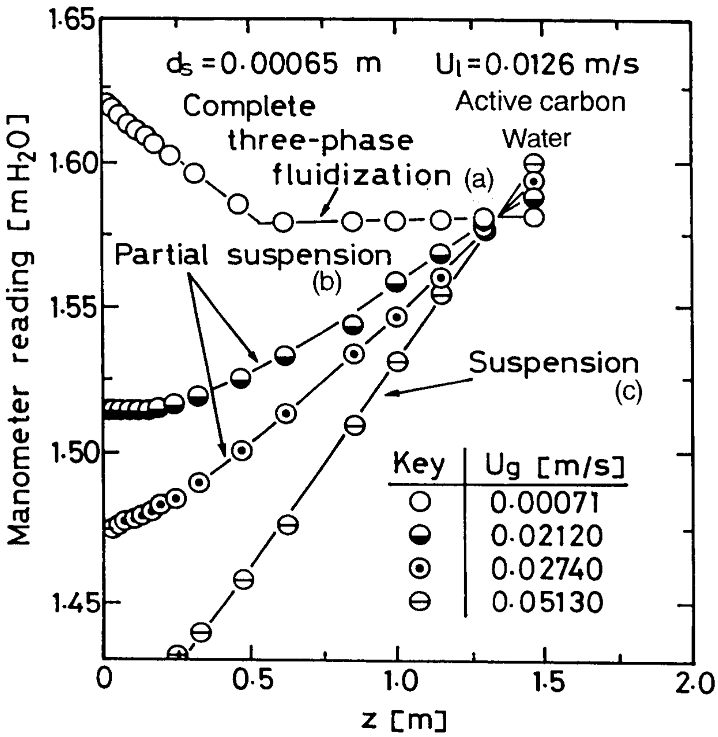

Fig. 7 Pressure profile along the axis.

In this study, the gas holdup was measured in the complete three-phase fluidized bed and the suspension regime which are shown by the lines (a) and (c) in Fig. 7 .The gas holdup in the complete three-phase fluidized bed and the suspension regime was determined directly from the measurement of the weight of particles within the bed and the pressure profile. The following relationships were used to evaluate the gas holdup:

$$\varepsilon_s = \frac{W_s}{A_c H \rho_s} \tag{10}$$

$$-\frac{dp}{dz} = g(\varepsilon_g \rho_g + \varepsilon_l \rho_l + \varepsilon_s \rho_s) \tag{11}$$

$$\varepsilon_g + \varepsilon_l + \varepsilon_s = 1 \tag{12}$$

3. 2 Gas holdup in bubble column

Fig. 8 shows the gas holdup in a bubble column. The solid line is the result of Akita and Yoshida (1973, 1974) for a bubble column without liquid flow. In general, we cannot observe the remarkable effect of liquid velocity on gas holdup. However. as the liquid velocity becomes larger, the gas holdup becomes slightly smaller. The parity ploy for gas holdup using Eq. (9) with $K = 0.707$ is shown in **Fig. 9**. The excellent agreement between them attests to the cogency of the drift line model in gas-liquid bubble columns.

3. 3 Gas holdup in three-phase fluidized bed

The gas holdup in the complete three-phase fluidized bed regime and the suspenion regime is shown in **Fig. 10**, which also includes the data of El-Temtamy and Epstein (1980) for the partial suspension regime and that of Kato *et al,* (1983) for the complete

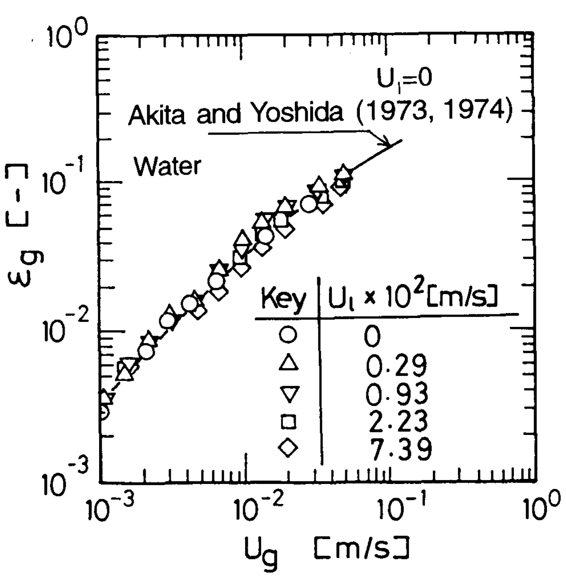

Fig. 8 Gas holdup in bubble columns.

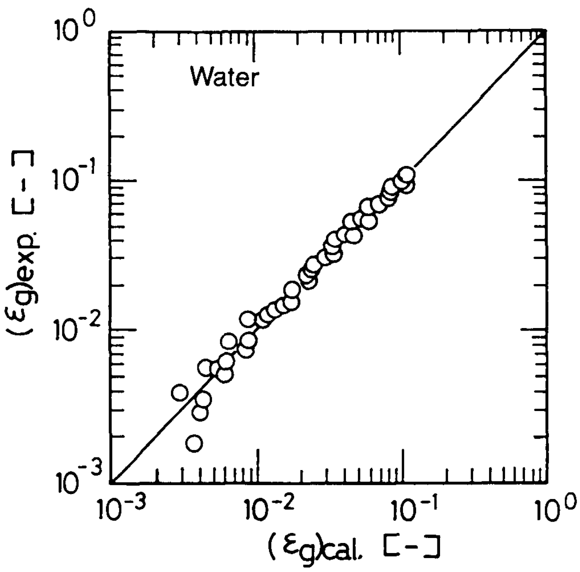

Fig. 9 Parity plot for gas holdup in bubble columns.

three-phase fluidized bed. In the same graph, the solid line is the result of Akita and Yoshida (1973, 1974) for a bubble column without liquid flow. As can be seen from the figure, the values of gas holdup for particles with Re_t' smaller than about 100 are larger than those for particles with Re_t' larger than about 100. This is most likely due to the fact that bubbles for the bed of particles with Re_t' smaller than about 100 may coalesce with each other. Usually, the larger the bubble diameter, the larger the rise velocity of it. Therefore, the gas holdup decreases. However, the reverse is true. We could not find the exact reason for this phenomenon, but it may be due to the wall effect based on a large bubble compared to the column diameter. Based on these results, the correction factor K can be determined from Eq. (9). **Fig. 11** shows the correction factor K versus Re_t' including the density difference between particle and liquid. K increases with increasing Re_t' , decreases suddenly at about $Re_t' = 100$, increases gradually with increasing Re_t' and is roughly constant.

The parity plot for gas holdup in the complete three-phase fluidization and the suspension regime for an air-water system using K values given in Fig. 11 is shown in **Fig. 12** where the data of El-Temtamy and Epstein (1980) are also included for an air-water system. **Fig. 13** shows the parity plot for gas holdup in air-aqueous glycerol solution systems using Nylon-6 beads (d_s = 6.315 mm) as a solid phase. The agreement of gas holdup measured with those calcuated from the drift line model is fairly good in both figures.

Concluding Remarks

A drift line model has been derived to estimate the gas holdup in bubble columns and in three-phase fluidized beds containing small or low density particles. The model in

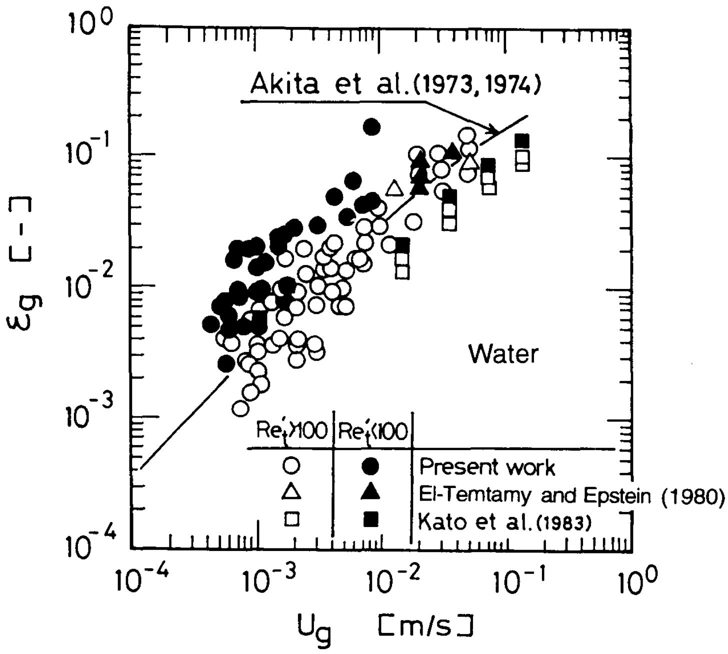

Fig. 10 Gas holdup in three-phase fluidized beds.

for a perfect fluid but is found to give a good repesentation of experimental data in bubble columns and in three-phase fluidized beds using the correction factor K.

Appendix

Derivation of Eq. (2)

The drift line formed after the bubble has passed through an infinite three-dimensional bed can be calculated from the stream function Eq. (1) not analytically but numerically. Here we will try to approximate the drift line by the following equation.

$$Y = \frac{A}{(X+B)^n} \tag{A-1}$$

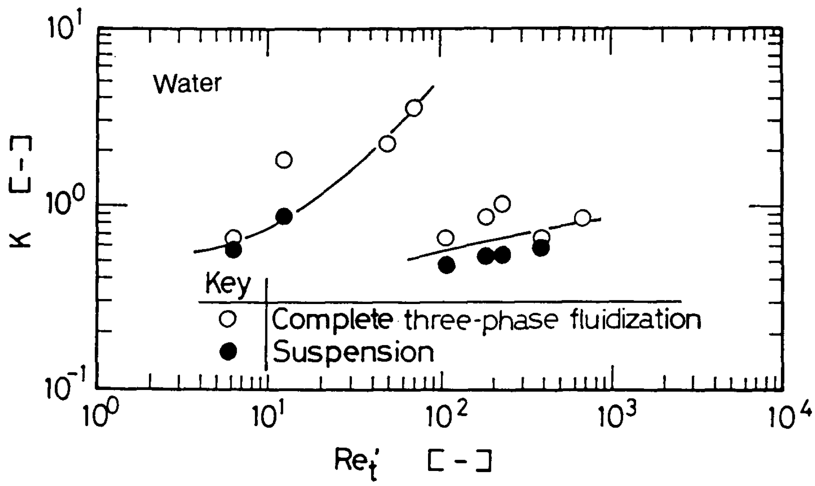

Fig. 11 Variation of K with respect to Re_t'.

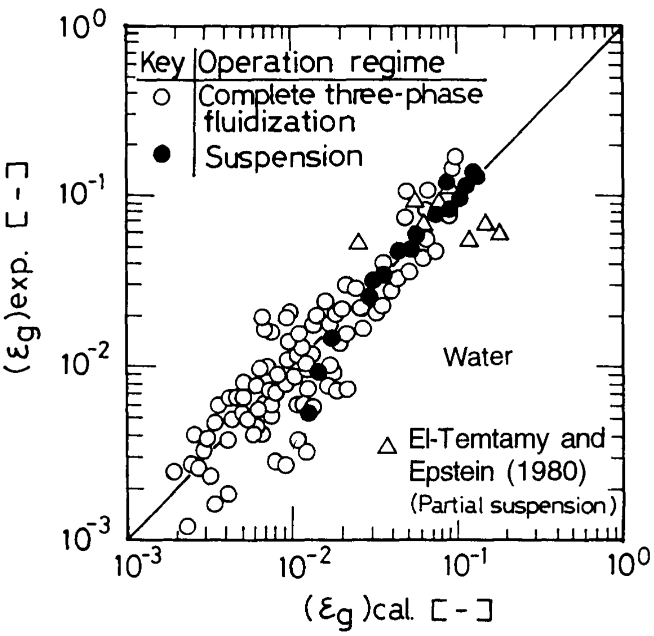

Fig. 12 Parity plot for gas holdup in three-phase fluidized beds with water as the liquid.

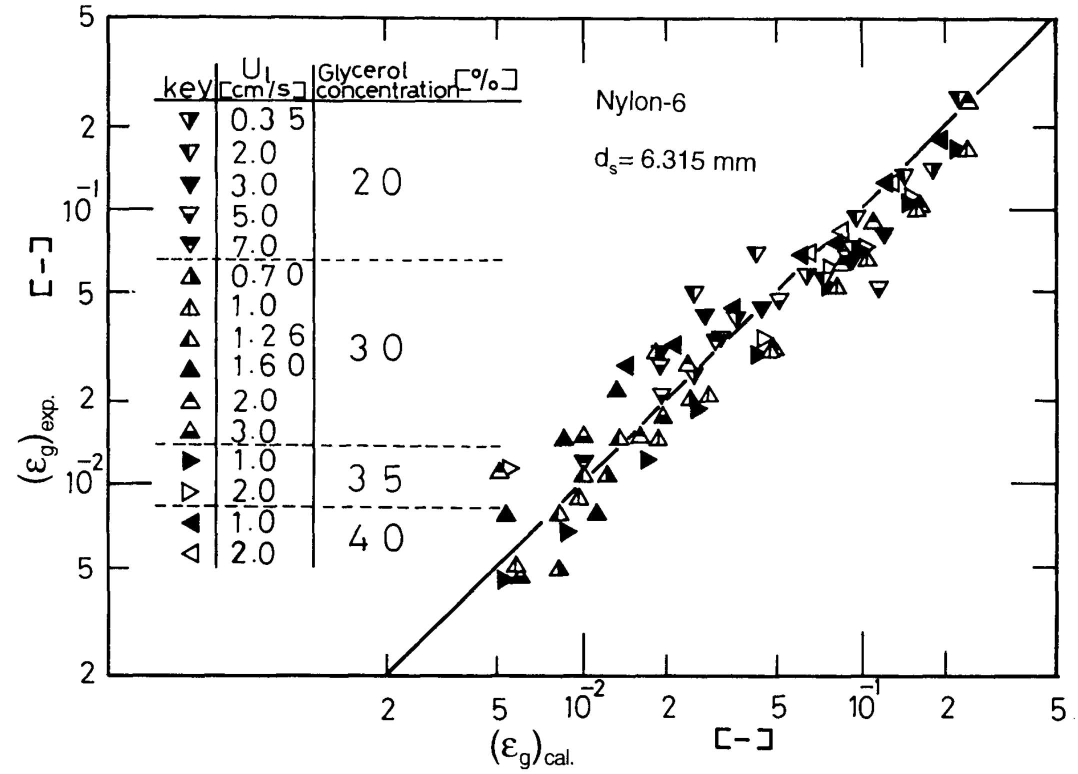

Fig. 13 Parity plot for gas holdup in three-phase fluidiued beds with aqueous glycerol solutions as liquids.

When we have an infinite fluid medium, the fluid volume surrounded by the drift line is equivalent to half the volume of a spherical bubble in a perfect fluid (Darwin, 1953). Then we obtain as

$$2\pi\int_0^\infty XYdX = \frac{2}{3}\pi \tag{A-2}$$

$$2\pi\int_0^\infty X\frac{A}{(X+B)^n}\,dX = \frac{2}{3}\pi \tag{A-3}$$

$$A\int_0^\infty \frac{X}{(X+B)^n}\,dX = \frac{1}{3} \tag{A-4}$$

Now, Eq. (A-4) may be rearranged using $Z = X+B$ as follows

$$\frac{1}{3A} = \int_B^\infty \frac{Z-B}{Z^n}\,dZ = \int_B^\infty\left(\frac{1}{Z^{n-1}} - \frac{B}{Z^n}\right)dZ$$

$$= \left[\frac{1}{(2-n)Z^{n-2}} - \frac{B}{(1-n)Z^{n-1}}\right]_B^\infty$$

$$= \frac{1}{B^{n-2}}\cdot\frac{1}{(1-n)(2-n)} \qquad n \neq 1, 2 \tag{A-5}$$

Therefore,

$$3A = B^{n-2}(1-n)(2-n) \tag{A-6}$$

From Fig. 4, we obtain $Y = 3.9$ at $X = 0$. Substitution of this relationship into Eq. (A-1) gives

$$\frac{A}{B^n} = 3.9 \tag{A-7}$$

Then the following equation can be obtained,

$$3.9B^n = B^{n-2}(1-n)(2-n)/3 \tag{A-8}$$

Therefore, the following equation can be obtained with $A = 1.03$ and $B = 0.716$ at $n = 4$.

$$Y = \frac{1.03}{(X+0.716)^4} \tag{2}$$

Nomenclature

A	= constant defined by Eq. (A-1)	[−]
A_b	= bubble street area	[m^2]
A_c	= column area	[m^2]
a	= bubblle radius	[m]
B	= constant defined by Eq. (A-1)	[−]
C	= defined by Eq. (3)	[m]
D_c	= column diameter	[m]
d_B	= bubble diameter in three-phase fluidized bed	[m]
d_s	= particle diameter	[m]
g	= acceleration of gravity	[m/s^2]
H	= bed height	[m]
H_c	= column height	[m]
K	= correction factor	[−]
p	= static pressure	[Pa]
R	= column radius	[m]
Re'_t	= modified particle Reynolds number ($= d_s \lvert\rho_s - \rho_l\rvert U_T/\mu$)	[−]
r	= radial distance	[m]
U_B	= mean bubble rise velocity	[m/s]
U_g	= superficial gas velocity	[m/s]
U_l	= superficial liquid velocity	[m/s]
U_{lb}	= mean fluid velocity in bubble street	[m/s]
U_s	= velocity of single bubble	[m/s]
U_T	= terminal velocity of particle	[m/s]
u_l	= local liquid velocity	[m/s]
V_g	= gas flow rate	[m^3/s]
W	= dimensionless liquid upward rise path	[−]
w	= liquid upward rise path	[m]
W_s	= weight of particles	[kg]
X	= dimensionless axis ($= x/a$)	[−]
X_1	= dimensionless radius of bubble street ($= x_1/a$)	[−]

x	= axis	[m]
x_1	= radius of bubble street	[m]
X_R	= dimensionless column radius ($= x_R/a$)	[−]
x_R	= column radius	[m]
Y	= dimensionless axis ($= y/a$)	[−]
y	= axis	[m]
Z	$= X+B$	[−]
z	= axial distance	[m]
ψ	= stream function	[m^3/s]
β	= ratio of bubble street area to column area	[−]
ε_g	= gas holdup	[−]
ε_l	= liquid holdup	[−]
ε_s	= solids holdup	[−]
μ	= viscosity of liquid	[Pa.s]
ρ_g	= density of gas	[kg/m_3]
ρ_l	= density of liquid	[kg/m^3]
ρ_s	= density of solids	[kg/m^3]

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