THERMODYNAMICS OF THE SCHWINGER MODEL IN A TWO-DIMENSIONAL de SITTER SPACE-TIME

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ABSTRACT

The finite temperature Schwinger model in a background 1-1 de Sitter space is studied. We take account of the curvature effect on the two point Green's function in a short distance. It is shown that the massless and massive boson excitations can not be decoupled and have the interactions through the de Sitter curvature.

1. INTRODUCTION

Recently there has been much interest in the study of the Schwinger model[1] in curved space-time. This interest originates primarily from an exact solvability of the model in flat space-time which might render help to attack almost unsolvable problems in curved space-time.

The purpose of this work is to discuss thermodynamical properties of the Schwinger model in a background 1-1 de Sitter space. Love[2] has calculated the finite temperature fermion Green's function in flat space-time and showed that the theory is equivalent to an ensemble of noninteracting, neutral, massive, Bose particles, and thus the particle content is identical to that obtained at zero temperature[3-4]. It is believed that in curved space-time, curvature affects thermodynamical properties of an ensemble of particles. This is very interesting but difficult enough to get something concrete in general case.

We take the Schwinger model in a 1-1 de Sitter space, because this space-time structure has the wellknown simple global symmetry very similar to the Lorentz symmetry. This space is asymptotically flat near the origin and all world points are equivalent because of the de Sitter symmetry. Therefore, a short distance two-point function may be evaluated in an approximately flat region around the de Sitter origin.

Gass[5] set up and solved the equations for the Green's functions in curved 1-1 space which follows closely Brown's flat space-time ones[6]. Unfortunately, the solutions are too complicated to be analytically managed. In this work, the finite-temperature Green's function at the points near the de Sitter origin will be evaluated by making use of Gass' solutions, and so the ensemble average of the Hamiltonian density will be given in 2nd order approximation of a^{-1} (a=de Sitter radius) expansion.

In Sec. 2 we outline the basic model. In Sec. 3 we present the finite-temperature Green's function (two-point) and discuss the ensemble average of the Hamiltonian density.

2. THE MODEL

At first, we review briefly the geometry of a two-dimensional de Sitter space. This will also allow us to introduce some notations.

The metric is given by

$$ds^2=(1-\frac{r^2}{a^2})dt^2-(1-\frac{r^2}{a^2})^{-1}dr^2, \tag{1}$$

where $-\infty\leqq t\leqq\infty$, $-\infty\leqq r\leqq\infty$ ($0\leqq r\leqq\infty$ in the 2-dim. case reduced from 4-dim.). This metric has a singularity at the de Sitter event horizon $|r|=a$. In the limit of $a\to\infty$, the space becomes a two-dimensional Minkowski space.

By using the "Kruskal type" (U, V) coordinates specified in Appendix A, we show the Kruskal diagram of 1-1 de Sitter space in Fig. 1. In this representation the apparent singularity at $|r|=a$ is removed. In Fig. 1, null geodesics are at $\pm 45°$ to the vertical. An emitted photon from any world point in the interior region (I) ($|r|<a$) takes infinite time to reach the future horizon which is defined by the straight line $U=V$. Note that the metric tensor does not satisfy the Einstein field equation $R^{\mu\nu}=3a^2g^{\mu\nu}$ in contrast with the four dimensional case, since the scalar curvature is $R=2/a^2$ as shown in Appendix B.

Because of the event horizon, we consider the equations only in the interior region (I). The Lagrangian density of the Schwinger model in a 1-1 de Sitter space is given by

$$\mathcal{L}=i\bar{\psi}\gamma^\mu(x)\nabla_\mu\psi-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+e\bar{\psi}\gamma^\mu(x)\psi A_\mu(x)-\frac{1}{2\beta}(\nabla_\mu A^\mu)^2, \tag{2}$$

where ∇_μ is the covariant derivative, $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu$, and the $1/\beta$ term is a

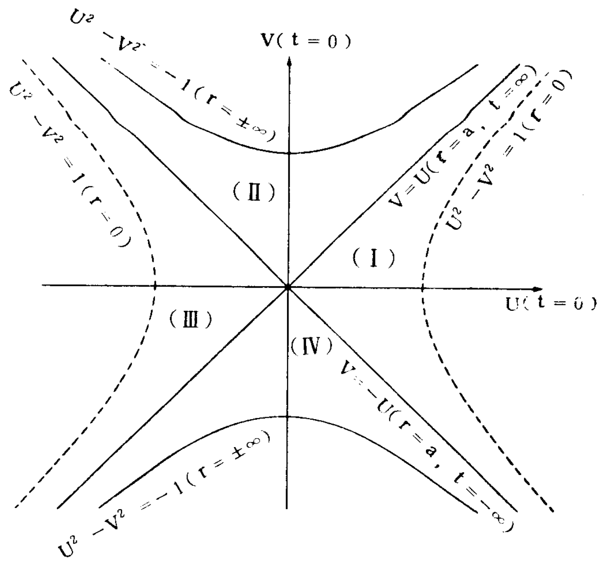

Fig. 1.: Kruskal diagram
Region (I) and (II) are interior regions.
Region (III) and (IV) are exterior regions.
The dashed curves are hyperbolas of $r=0$.
The solid curves are hyperbolas of $r\to\pm\infty$.
The solid lines $V=U$ and $V=-U$ are the future and past event horizon respectively.

gauge-fixing term. With our conventions $x^0=t$, $x^1=r$, the natural unit $\hbar=c=1$ is used. The curved space-time $\underline{\gamma}_\mu(x)$ matices satisfy the commutation relation

$$\{\underline{\gamma}^\mu(x),\ \underline{\gamma}^\nu(x)\}=2g^{\mu\nu}(x). \tag{3}$$

The explicit representations of $\underline{\gamma}_\mu(x)$'s, 1-1 de Sitter space "zweibein", and other relevant quantities are given in Appendix C.

From the Lagrangian (2) follow the field equations

$$i\underline{\gamma}^\mu(x)\{\nabla_\mu-ieA_\mu(x)\}\psi(x)=0, \tag{4}$$

and

$$\frac{1}{\sqrt{-g}}\ \frac{\partial}{\partial x^\nu}[\sqrt{-g}\,F^{\mu\nu}]=ej^\mu(x), \tag{5}$$

in which

$$j^\mu(x)=\bar{\psi}(x)\underline{\gamma}^\mu(x)\psi(x), \tag{6}$$

is the fermion current with $\bar{\psi}=\psi^\dagger\gamma^0$.

The current conservation equation is

$$\nabla_\mu j^\mu(x)=0. \tag{7}$$

The anticommutation relation between ψ and $\bar{\psi}$ is defined by

$$\delta(n\cdot\{x-x'\})\{\psi(x),\ \bar{\psi}(x')\}=\frac{\gamma^0\delta^2(x-x')}{\sqrt{-g}}, \tag{8}$$

where n is a time like unit vector lying in the forward light cone.

The equation (5) is reduced to

$$\frac{\partial F^{01}}{\partial r}=ej^0(r,t). \tag{9}$$

The solution is given by

$$F^{01}=-e\partial_r\int_{-a}^{a}dr'\ D(r,r',t)j^0(r',t), \tag{10}$$

where $D(r, r', t)$ satisfies

$$-\frac{d}{dr}\{g^{11}(r)\frac{d}{dr}\}D(r,r',t)=\delta(r-r'). \tag{11}$$

The solution to the equation (11) is

$$D(r,r')=\begin{cases}-\dfrac{a}{4}\ln\dfrac{(a-r)(a+r')}{(a+r)(a-r')}, & r<r'\\[2ex] -\dfrac{a}{4}\ln\dfrac{(a+r)(a-r')}{(a-r)(a+r')}, & r>r'\end{cases} \tag{12}$$

Note that for $a\to\infty$ and $|r|$, $|r'|\to 0$, $D(r,r')\to -\frac{1}{2}|r-r'|$ which is the correct flat space-time result.

By usual definition, the Hamiltonian is

$$H=\int_{-a}^{a}dr\,\mathcal{H}, \tag{13}$$

with the Hamiltonian density

$$\mathcal{H}=\frac{\partial\mathcal{L}}{\partial\nabla_0\psi}\nabla_0\psi-\mathcal{L}=\frac{i}{2}\bar{\psi}\underline{\tilde{\gamma}}^{\mu}\nabla_{\mu}\psi, \tag{14}$$

in which $\underline{\tilde{\gamma}}^{\mu}=(\underline{\gamma}^0,-\underline{\gamma}^1)$.

3. THE FINITE-TEMPERATURE GREEN'S FUNCTIONS AND THE ENSEMBLE AVERAGE OF THE HAMILTONIAN DENSITY

Here, we follow Gass' formal expressions[5]. The two-points Green's function:

$$G(x_1,x_2)=\langle 0|T[\psi(x_1),\bar{\psi}(x_2)]|0\rangle \tag{15}$$

satisfies the equation

$$\underline{\gamma}^{\mu}(x_1)\nabla_{\mu}^{x_1}G(x_1,x_2)=\delta^2(x_1-x_2)-\underline{\gamma}^{\mu}(x_1)\{\nabla_{\mu}^{x_1}H(x_1,x_2)\}G(x_1,x_2)$$
$$+\underline{\gamma}^{\mu}(x_1)\{\nabla_{\mu}^{x_1}F(x_1,x_2)\}G(x_1,x_2), \tag{16}$$

where

$$F(x_1,x_2)=ie^2[\underline{\gamma}^{\lambda}(x_1)]_1\nabla_{\lambda}^{x_1}\nabla_{\nu}^{x_2}\int d^2z\triangle_+(x_1-z\,;0)\underline{\gamma}_0(z)$$
$$\times\int_{-a}^{a}dy^1[\underline{\gamma}^{\nu}(y)\underline{\gamma}^0(y)]_2D(z^1,y^1)\triangle_+(y^1-x_2^1\,;z^0-x_2^0\,;\frac{e^2}{\pi})$$
$$-\text{the same term with } x_1=x_2, \tag{17}$$

and

$$H(x_1, x_2) = ie^2\gamma^\lambda(x_1)\nabla_\lambda^{x_1}\nabla_\nu^{x_2}\int d^2z\triangle_+(x_1-z\,;\,0)\int_{-a}^{a} dy^1 D(z^1,\ y^1)$$
$$\times[\gamma_0(z)]_1[\gamma^0(y)\gamma^\nu(y)]_1\triangle_+(y^1-z^1,\ 0\,;\,\frac{e^2}{\pi})$$
$$-\text{the same term with } x_1=x_2\,. \tag{18}$$

In $F(x_1,x_2)$ and $H(x_1,x_2)$, the short-hand notation $\Gamma_1\langle 0|T(\psi\bar\psi)|0\rangle=\langle 0|T(\Gamma\psi)\bar\psi|0\rangle$ and $\Gamma_2\langle 0|T(\psi\bar\psi)|0\rangle=\langle 0|T(\psi(\Gamma\bar\psi))|0\rangle$, where Γ is a combination of γ matrices, is used.

$\triangle_+(x-y\,;\,\mu^2)$ is defined by

$$(\Box^x+\mu^2)\triangle_+(x-y\,;\,\mu^2)=\delta^2(x-y), \tag{19}$$

with the curved space D'Alembertian

$$\Box^x=g^{\mu\nu}\nabla_\mu^x\nabla_\nu^x. \tag{20}$$

The solution to the equation (16) solved by Gass is

$$G(x_1,\ x_2)=\exp\{F(x_1,\ x_2)-H(x_1,\ x_2)\}G_0(x_1,\ x_2), \tag{21}$$

where

$$G_0(x_1,\ x_2)=(1-\frac{(r_1)^2}{a^2})^{-\frac{1}{4}}(1-\frac{(r_2)^2}{a^2})^{-\frac{1}{4}}G_0^{flat}(x_1,\ x_2). \tag{22}$$

Setting $x_2=0$ and $x_1=x$, we have

$$\lim_{x\to 0} G_0(x,\ 0)=\lim_{x\to 0} G_0^{flat}(x,\ 0)=\frac{i\gamma^\mu x_\mu}{2\pi x^2}. \tag{23}$$

At finite temperature, the prescription is to replace the vacuum expectation value with the ensemble average. For instance we replace $G(x_1,\ x_2)$ with the ensemble average $G(x_1,\ x_2)_{(T)}=\langle T(\psi(x_1)\bar\psi(x_2))\rangle_{(T)}$. For any operator A, the canonical ensemble average is defined by

$$\langle A\rangle_{(T)}=(Tr\ e^{-\beta H})^{-1}Tr\ e^{-\beta H}A, \tag{24}$$

where $\beta=1/(kT)$, and H is the Hamiltonian. Tr in this expression dictates a sum over a complete set of states of the syetem.

Noticing the time-translational invariance in a short distance near the de Sitter origin, we assume the Heisenberg equation of motion

$$A(r,\ t)=e^{iHt}A(r,\ 0)e^{-iHt}. \tag{25}$$

The Heisenberg equation of motion and the cyclic property of the trace yields the symmetry condition

$$\langle A(r,\ t)B(r',\ t')\rangle_{(T)}=\langle B(r',\ t')A(r,\ t+i\beta)\rangle_{(T)} \tag{26}$$

which holds for all operators A and B. With the aid of this periodic condition, the ensemble average of the operator product is obtained as

$$\langle A(x)B(x')\rangle_{(T)}=\int\frac{d^2k}{(2\pi)^2}e^{ik(x-x')}\frac{g(k)}{1\mp\exp\{-\beta k^0\}}, \tag{27}$$

where the minus (plus) sign in the denominator on the right-hand side corresponds to the Fourier transfom $g(k)$ which associates with the commutator (anticommutator)

$$<[A(x), B(x')]_{\mp}>_{(T)} = \int \frac{d^2k}{(2\pi)^2} e^{ik(x-x')} g(k). \tag{28}$$

In the equations (27), (28) and the following, the notation $\int dk_1/2\pi$ means $a/\pi \sum_{k_1(n)}$ with $k_1(n)=\pi n/a$ $(n=0, \pm 1, \pm 2, \cdots)$, because the de Sitter space is bounded by the event horizon at $|r|=a$.

The temperature Green's function satisfies the equation being the finite-temperature version of the equation (16)

$$\underline{\gamma}^{\mu}(x)\nabla_{\mu}G(x, 0)_{(T)} = \delta^2(x) - \underline{\gamma}^{\mu}(x)\{\nabla_{\mu}H(x, 0)_{(T)}\}G(x, 0)_{(T)} + \underline{\gamma}^{\mu}(x)\{\nabla_{\mu}F(x, 0)_{(T)}\}G(x, 0)_{(T)}. \tag{29}$$

The solution to the equation (29) is expressed as

$$G(x, 0)_{(T)} = \exp\{F(x, 0)_{(T)} - H(x, 0)_{(T)}\}G_0(x, 0)_{(T)}. \tag{30}$$

Here, $G_0(x, 0)_{(T)}$ is a free massless fermion Green's function satisfying

$$\underline{\gamma}^{\mu}(x)\nabla_{\mu}G_0(x, 0)_{(T)} = \delta^2(x), \tag{31}$$

with finite-temperature boundary condition[5].

$F(x, 0)_{(T)}$ and $H(x, 0)_{(T)}$ are defined by (17) and (18) except that the zero temperature function $\triangle_+(x; \mu^2)$ is replaced by its finite-temperature version $\triangle_+(x; \mu^2)_{(T)}$, which satisfies

$$(\Box + \mu^2)\triangle_+(x; \mu^2)_{(T)} = (\Box + \mu^2)\int \frac{d^2k}{(2\pi)^2} e^{ikx}\triangle_+(k; \mu^2)_{(T)} = \delta^2(x).$$

In the limit $x \to 0$, we have

$$\triangle_+(k; \mu^2)_{(T)} = \frac{1}{k^2 - \mu^2 - i\varepsilon} + \frac{2\pi i\delta(k^2 - \mu^2)}{\exp\{\beta(k^2+\mu^2)^{\frac{1}{2}}\} - 1}, \tag{32}$$

and also

$$G_0(x, 0)_{(T)} = -i\int \frac{d^2k}{(2\pi)^2} \exp(ikx)\gamma^a k_a G_0(k)_{(T)}, \tag{33}$$

in which

$$G_0(k)_{(T)} = \frac{1}{k^2 - i\varepsilon} - \frac{2\pi i\delta(k^2)}{\exp\{\beta|k_1|\} + 1}. \tag{34}$$

Using the equation (14) and the definition of the two-point temperature Green's function, the ensemble average of $\mathcal{H}$ is given by

$$<\mathcal{H}>_{(T)} = \frac{i}{2} \lim_{x \to +0} Tr\, \tilde{\underline{\gamma}}^{\mu}\nabla_{\mu}G(x, 0)_{(T)}. \tag{35}$$

Here the limiting procedure $x \to +0$ dictates taking the limit $t \to 0$ following by $r \to 0$. We expand $\exp\{F(x, 0)_{(T)} - H(x, 0)_{(T)}\}$ in the solution $G(x, 0)_{(T)}$ for x. Taking into account of the limiting property of $\lim_{x\to+0} F(x,0)_{(T)}$ and $\lim_{x\to+0} H(x,0)_{(T)}$ shown later on, we find

$$<\mathcal{H}>_{(T)} = \frac{i}{2} \lim_{x\to+0} Tr\, \tilde{\underline{\gamma}}^{\mu} \nabla_{\mu} G_0(x,0)_{(T)}$$
$$+ \frac{i}{2} \lim_{x\to+0} Tr\, \tilde{\underline{\gamma}}^{\mu} \nabla_{\mu} \{[F(x,0)_{(T)} - H(x,0)_{(T)}] G_0(x,0)_{(T)}\}. \quad (36)$$

The first term is the ensemble average of the free fermion field Hamiltonian density $<\mathcal{H}^0>_{(T)}$.

The curved space-time $\underline{\gamma}^{\mu}(x)$ is defined by

$$\underline{\gamma}^{\mu}(x) = a^{\mu}{}_{a}(x)\gamma^{a},$$

where zweibein $a^{\mu}{}_{a}(x)$ and $b^{a}{}_{\mu}(x)$ are given in Appendix C.

We now consider the 2nd order approximations of a^{-1} expansion of $<\mathcal{H}>_{(T)}$. In this approximation, we assume

$$a^{\mu}{}_{a} \simeq \begin{pmatrix} 1+\frac{1}{2}\frac{r^2}{a^2} & 0 \\ 0 & 1-\frac{1}{2}\frac{r^2}{a^2} \end{pmatrix}, \quad b^{a}{}_{\mu} \simeq \begin{pmatrix} 1-\frac{1}{2}\frac{r^2}{a^2} & 0 \\ 0 & 1+\frac{1}{2}\frac{r^2}{a^2} \end{pmatrix}$$

and

$$g_{\mu\nu} \simeq \begin{pmatrix} 1-\frac{r^2}{a^2} & 0 \\ 0 & -(1+\frac{r^2}{a^2}) \end{pmatrix}, \quad g^{\mu\nu} \simeq \begin{pmatrix} 1+\frac{r^2}{a^2} & 0 \\ 0 & -(1-\frac{r^2}{a^2}) \end{pmatrix}.$$

The Fourier transforms of $b^0{}_0(r)$ and $g^{00}(r)$ which are relevant to $<\mathcal{H}>_{(T)}$ are represented as

$$b^0{}_0(k_n) = \sqrt{2a}\,\delta_{k_n,0} - \frac{1}{2}\sigma(k_n), \quad (37)$$

and

$$g^{00}(k_n) = \sqrt{2a}\,\delta_{k_n,0} + \sigma(k_n), \quad (38)$$

with

$$\sigma(k_n) = \frac{4a(-1)^n}{\sqrt{2a}(ak_n)^2}. \quad (39)$$

The $D(r)$ defined by (12) can be relaced by the flat space-time expression $D(r) = -\frac{1}{2}|r|$. The Fourier transform of $D(r)$ is

$$D(k_n) = \frac{\{1-(-1)^n\}a^2}{\sqrt{2a}(ak_n)^2}. \quad (40)$$

In term of Fourier transforms, we obtain

$$F(x,0)_{(T)} = ie^2 a^{\mu}{}_{a}(x) \int \frac{d^2k d^2k'}{(2\pi)^3} \tilde{\alpha}^{a} k_{\mu} \triangle_{+}(k;0)_{(T)} \triangle_{+}(k';\mu^2)_{(T)}$$
$$\times \delta(k_0 - k'_0) \exp(ikx)$$

$$\times \{\hat{\alpha}^1 k'_1 b^0{}_0(k_1-k'_1) D(k'_1) + \hat{\alpha}^0 k_0 \int \frac{dl}{2\pi} b^0{}_0(k_1-l) D(l) g^{00}(l-k'_1)\}$$

$$-\text{the same term with } x=0, \tag{41}$$

and

$$H(x, 0)_{(T)} = ie^2 a^0{}_0(x) \int \frac{dk_1 dk'_1}{(2\pi)^2} \triangle_+(0, k_1; 0)_{(T)} D(k'_1) b^0{}_0(k_1)$$

$$\times \triangle_+(0, k'_1; \mu^2)_{(T)} k_1^2 \exp(ik_1 x^1)$$

$$-\text{the same term with } x=0, \tag{42}$$

where

$$\hat{\alpha}^a = \gamma^a \gamma^0.$$

The $x \rightarrow +0$ limit is given by

$$\lim_{x\rightarrow+0} F(x, 0)_{(T)} = ie^2 \int \frac{d^2k d^2k'}{(2\pi)^3} \hat{\alpha}^a k_a \triangle_+(k; 0)_{(T)} \triangle_+(k': \mu^2)_{(T)}$$

$$\times \{\alpha^1 k'_1 b^0_0(k_1-k'_1) D(k'_1) + \alpha^0 k'_0 \int \frac{dl}{2\pi} b^0_0(k_1-l) D(l) g^{00}(l-k'_1)\}$$

$$\times \delta(k_0-k'_0) \frac{(kx)^2}{2} \tag{41'}$$

and

$$\lim_{x\rightarrow+0} H(x, 0)_{(T)} = ie^2 \int \frac{dk_1 dk'_1}{(2\pi)^2} k_1^2 \triangle_+(0, k_1; 0)_{(T)} \triangle_+(0, k'_1; \mu^2)_{(T)}$$

$$\times D(k'_1) b^0{}_0(k_1) \frac{(kx)^2}{2} \tag{42'}$$

respectively.

If we replace $b^0{}_0(k_1)$ and $g^{00}(k_1)$ with their $a \rightarrow \infty$ limit $\delta(k_1)$, the equations (41′) and (42′) reduce to the flat space-time expressions.

We can separate $<\mathcal{H}>_{(T)}$ into two parts, i.e.,

$$<\mathcal{H}>_{(T)} = <\mathcal{H}>^{(a=\infty)}_{(T)} + <\mathcal{H}>^{(a)}_{(T)}, \tag{43}$$

here $<\mathcal{H}>^{(a=\infty)}_{(T)}$ is the curvature independent (flat space-time) ensemble average which has been obtained by Love[2].

$<\mathcal{H}>^{(a)}_{(T)}$ is the curvature dependent correction term which is expressed as

$$<\mathcal{H}>^{(a)}_{(T)} = \frac{\mu^2}{4} \int \frac{d^2k d^2k'}{(2\pi)^3} \triangle_+(k; 0)_{(T)} \triangle_+(k'; \mu^2)_{(T)} \delta(k_0-k'_0) k_0^2$$

$$\times \sigma(k_1-k'_1)\{D(k_1)k_1^2 - D(k'_1)k_1 k'_1)\} \tag{44}$$

with $\mu^2 = \frac{e^2}{\pi}$.

On substituting the explicit expression $\triangle_+(k; \mu^2)_{(T)}$ (Eq. (32)) in (44) and integrating for k_0 and k'_0, we find

$$<\mathcal{H}>^{(a)}_{(T)} = \frac{\mu^2}{4(2\pi)^3} \int dk\, dk' \Big[\frac{2\pi i}{k'^2 - k^2 - \mu^2 - i\varepsilon}$$

$$\times\left\{-\frac{\sqrt{k^2+\mu^2}}{\exp\{\beta\sqrt{k^2+\mu^2}\}-1}+\frac{|k'|}{\exp\{\beta|k'|\}-1}\right\}$$

$$+\frac{(2\pi i)^2\,\delta(k'-k^2-\mu^2)(k'^2)}{|k'|(\exp\{\beta|k'|\}-1)(\exp\{\beta\sqrt{k^2+\mu^2}\}-1)}\Big]$$

$$\times\sigma(k-k')\{D(k')k'^2-D(k)kk'\}$$

$$=-\frac{\mu^2}{8\pi}\int dk\,\frac{\sigma(\sqrt{k^2+\mu^2}-k)\{D(\sqrt{k^2+\mu^2})(k^2+\mu^2)-D(k)k\sqrt{k^2+\mu^2}\}}{\exp\{\beta\sqrt{k^2+\mu^2}\}-1} \tag{45}$$

$$\times\left(1+\frac{\sigma(\sqrt{k^2+\mu^2}-k)D(\sqrt{k^2+\mu^2})}{\exp\{\beta\sqrt{k^2+\mu^2}\}-1}\right)$$

Alternatively, changing a temporary notation of $\int\frac{dk}{2\pi}$, $\sigma(k)$ and $D(k)$ for $\frac{a}{\pi}\sum_{k_n}$, $\sigma(k_n)$ (Eq. (39)) and $D(k_n)$ (Eq. (40)),

$$<\mathcal{H}>^{(a)}_{(T)}=-\frac{1}{4\pi^2 a}\sum_{k_n}\frac{\{1-(-1)^n\}}{\exp\{\beta\sqrt{k_n{}^2+\mu^2}\}-1}\left(1+\frac{1}{\exp\{\beta\sqrt{k_n{}^2+\mu^2}\}-1}\right). \tag{46}$$

After combining all pieces the ensemble average is

$$<\mathcal{H}>_{(T)}=<\mathcal{H}>_{(T=0)}+\sum_{k_n}\frac{\sqrt{k_n{}^2+\mu^2}}{\exp\{\beta\sqrt{k_n{}^2+\mu^2}\}-1}-\sum_{k_n}\frac{1-(-1)^n}{4\pi^2 a(\exp\{\beta\sqrt{k_n{}^2+\mu^2}\}-1)}$$

$$-\sum_{k_n}\frac{1-(-1)^n}{4\pi^2 a(\exp\{\beta\sqrt{k_n{}^2+\mu^2}\}-1)^2}. \tag{47}$$

Here we have explicitly written only the finite-temperature modification. In this expression, the second term is the flat space-time average and the third term is the curvature correction, finally the fourth term represents the correlation between the massless and massive boson excitations.

If the universe has the de Sitter metric, "a" may be of the order of 2×10^{10} light-years at the present stage of the universe. Then the curvature correction term is completely negligible. In the early universe which could have the curvature of $\frac{1}{a}\simeq\mu$, however, we may expect that the curvature affects the thermodynamical ensemble average.

REFERNECES

1. J. Schwinger, Phys. Rev. **123**, 2425 (1962) ; and in *Proceedings of the Seminar on Theoretical Physics*, Trieste, 1962 (I.A.E.A., Vienna, 1963), p. 89.
2. S. T. Love, Phys Rev. D **23**, 420 (1981).
3. L. Dolan and R. Jackiw, Phys Rev. D **9**, 3320 (1974).
4. W. Fischler, J. Kogut, and L, Susskind, Phys. Rev. D **19**, 1188 (1979).
5. R. Gass, Rhys. Rev. D **27**, 2893 (1983).
6. L. S. Brown, Nuovo Cimento **29**, 617 (1963).

Appendix A: TWO-DIMENSIONAL de SITTER SPACE IN "KRUSKAL TYPE" COORDINATES

<table>
<tr><th colspan="2">Region</th><th>Interior region (I) $|r|\leqq a$</th><th>Exterior region (II) $|r|\geqq a$</th><th>Interior region (III) $|r|\leqq a$</th><th>Exterior region (IV) $|r|\geqq a$</th></tr>
<tr><td rowspan="4">"Kruskal type" coordinates</td><td>U</td><td>$\sqrt{\frac{a-r}{a+r}}\cosh\frac{t}{a}$</td><td>$\sqrt{\frac{r-a}{r+a}}\sinh\frac{t}{a}$</td><td>$-\sqrt{\frac{a-r}{a+r}}\cosh\frac{t}{a}$</td><td>$-\sqrt{\frac{r-a}{r+a}}\sinh\frac{t}{a}$</td></tr>
<tr><td>V</td><td>$\sqrt{\frac{a-r}{a+r}}\sinh\frac{t}{a}$</td><td>$\sqrt{\frac{r-a}{r+a}}\cosh\frac{t}{a}$</td><td>$-\sqrt{\frac{a-r}{a+r}}\sinh\frac{t}{a}$</td><td>$-\sqrt{\frac{r-a}{r+a}}\cosh\frac{t}{a}$</td></tr>
<tr><td>$U+V$</td><td>$\exp(u/a)$</td><td>$\exp(u/a)$</td><td>$-\exp(u/a)$</td><td>$-\exp(u/a)$</td></tr>
<tr><td>$U-V$</td><td>$\exp(-v/a)$</td><td>$-\exp(-v/a)$</td><td>$-\exp(-v/a)$</td><td>$\exp(-v/a)$</td></tr>
<tr><td colspan="2">Equation for r=const.</td><td colspan="4">$U^2-V^2=\frac{a-r}{a+r}$</td></tr>
<tr><td colspan="2">Equation for t=const.</td><td>$V/U=\tanh\frac{t}{a}$</td><td>$V/U=\coth\frac{t}{a}$</td><td>$V/U=\tanh\frac{t}{a}$</td><td>$V/U=\coth\frac{t}{a}$</td></tr>
<tr><td colspan="2">"Tortoise" coordinates</td><td>$r^*=\frac{a}{2}\ln\frac{a-r}{a+r}$</td><td>$r^*=\frac{a}{2}\ln\frac{r-a}{r+a}$</td><td>$r^*=\frac{a}{2}\ln\frac{a-r}{a+r}$</td><td>$r^*=\frac{a}{2}\ln\frac{r-a}{r+a}$</td></tr>
<tr><td rowspan="2">Null coordinates</td><td>u</td><td colspan="4">$t+r^*$</td></tr>
<tr><td>v</td><td colspan="4">$t-r^*$</td></tr>
<tr><td rowspan="2">Line element of de Sitter space</td><td>Null</td><td colspan="4">$ds^2=(1-r^2/a^2)\,du\,dv$</td></tr>
<tr><td>Kruskal</td><td colspan="4">$ds^2=\left[\frac{2a}{1+U^2-V^2}\right]^2(dV^2-dU^2)$</td></tr>
</table>

Appendix B: AFFINE CONNECTIONS AND CURVATURE TENSORS OF TWO-DIMENSIONAL de SITTER SPACE

Coordinates		(r, t) coordinates		(U, V) coordinates
Metric	$g_{\mu\nu}$	$\begin{bmatrix} (1-r^2/a^2) & 0 \\ 0 & -(1-r^2/a^2)^{-1} \end{bmatrix}$	$g_{\mu\nu}$	$\left[\frac{2a}{1+U^2-V^2}\right]^2 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
	$g^{\mu\nu}$	$\begin{bmatrix} (1-r^2/a^2)^{-1} & 0 \\ 0 & -(1-r^2/a^2) \end{bmatrix}$	$g^{\mu\nu}$	$\left[\frac{1+U^2-V^2}{2a}\right]^2 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Affine connection	$\Gamma^{\lambda}_{\mu\nu}$	$\frac{1}{2} g^{\lambda\tau}(g_{\mu\tau,\nu}+g_{\nu\tau,\mu}-g_{\mu\nu,\tau})$		
Nonzero affine connections	$\Gamma^{0}_{01}=\Gamma^{0}_{10}$	$-\frac{r}{a^2}(1-r^2/a^2)^{-1}$	$\Gamma^{0}_{00}=\Gamma^{0}_{11}=\Gamma^{1}_{01}$	$\frac{2V}{1+U^2-V^2}$
	Γ^{1}_{00}	$-\frac{r}{a^2}(1-r^2/a^2)$		
	Γ^{1}_{11}	$-\frac{r}{a^2}(1-r^2/a^2)^{-1}$	$\Gamma^{0}_{01}=\Gamma^{1}_{00}=\Gamma^{1}_{11}$	$\frac{-2U}{1+U^2-V^2}$
Riemann-Christoffel curvature tensor	$R^{\rho}_{\mu\lambda\nu}$	$\Gamma^{\rho}_{\mu\lambda,\nu} - \Gamma^{\rho}_{\mu\nu,\lambda} + \Gamma^{\sigma}_{\mu\lambda}\Gamma^{\rho}_{\sigma\nu} - \Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\lambda}$		
Ricci tensor	$R_{\mu\nu}$	$R^{\rho}_{\mu\rho\nu}$		
Scalar curvature	R	$g^{\mu\nu}R_{\mu\nu}=2/a^2$		

Appendix C: ZWEIBEIN COMPONENTS AND COVARIANT DERIVATIVE

Coordinates		(r, t) coordinates		(U, V) coordinates
Two-dimensional Minkowski space time metric	η_{ab}	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$		
Relation between Minkowski space metric and curved space-time metric		$\eta_{ab}=g_{\mu\nu}a^{\mu}_{a}\ a^{\nu}_{b}$		
Two-dimensional de Sitter space Zweibein	a^{μ}_{a}	$\begin{bmatrix}(1-r^2/a^2)^{-1/2} & 0\\0 & (1-r^2/a^2)^{1/2}\end{bmatrix}$	a^{μ}_{a}	$\frac{1+U^2-V^2}{2a}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
Inverse matrix of a^{μ}_{a}	b^{a}_{μ}	$\begin{bmatrix}(1-r^2/a^2)^{1/2} & 0\\0 & (1-r^2/a^2)^{-1/2}\end{bmatrix}$	b^{a}_{μ}	$\frac{2a}{1+U^2-V^2}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
Flat space-time γ matrices	γ^0	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$		
	γ^1	$\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$		
	$\gamma^5=\gamma^0\gamma^1$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$		
Relation between flat space-time γ_a and curved space-time $\underline{\gamma}_\mu(x)$		$\underline{\gamma}_\mu(x)=b^{a}_{\mu}(x)\ \gamma_a$		
$\underline{\gamma}_\mu(x)$ matrix forms of two-dimensional de Sitter space	$\underline{\gamma}_0$	$\begin{bmatrix}(1\text{-}r^2/a^2)^{1/2} & 0\\0 & \text{-}(1\text{-}r^2/a^2)^{-1/2}\end{bmatrix}$	$\underline{\gamma}_0$	$\frac{2a}{1+U^2-V^2}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
	$\underline{\gamma}_1$	$\begin{bmatrix}0 & \text{-}(1\text{-}r^2/a^2)^{-1/2}\\(1\text{-}r^2/a^2)^{-1/2} & 0\end{bmatrix}$	$\underline{\gamma}_1$	$\frac{2a}{1+U^2+V^2}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$
	$\underline{\gamma}_5=\gamma_5$	$\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$	$\underline{\gamma}_5$	$\left[\frac{2a}{1+U^2-V^2}\right]^2\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$
Covariant derivative of $\underline{\gamma}_\mu(x)$	$\nabla_\mu\underline{\gamma}_\nu(x)$	$\partial_\mu\underline{\gamma}_\nu(x)-\Gamma^{\kappa}_{\mu\nu}\ \underline{\gamma}_\kappa(x)+[\underline{\gamma}_\nu(x),\ \Gamma_\mu(x)]\equiv 0$		
Deffinition of spinorial affine connection $\Gamma_\mu(x)$	$\Gamma_\mu(x)$	$\frac{1}{4}\gamma_a\gamma_b b^{b}_{\lambda}(x)g^{\lambda\sigma}(x)\nabla_\mu b^{a}_{\sigma}(x)$		
Covariant derivative of Zweibein b^{a}_{μ}	$\nabla_\mu b^{a}_{\sigma}(x)$	$\partial_\mu b^{a}_{\sigma}(x)-\Gamma^{\kappa}_{\mu\sigma}\ b^{a}_{\kappa}(x)$		
Spinorial affine connection components of two-dimensional de Sitter space	$\Gamma_0(x)$	$-\frac{\gamma_5 r}{2a^2}$		
	$\Gamma_1(x)$	0		
Covariant derivative of $\psi(x)$	$\nabla_\mu\psi(x)$	$\{\partial_\mu-\Gamma_\mu(x)\}\psi(x)$		