Journal of Academic Writing
The Journal of the European Association for the Teaching of Academic Writing

Journal of Academic Writing
Vol. 6 No 1 Autumn 2016, pages 84-97
http://dx.doi.org/10.18552/joaw.v6i1.285

Writing, Calculating and Peer Feedback in a Mathematically-oriented Course for Process Engineers: Raising Motivation and Initiating Processes of Thinking and Learning

Nadine Stahlberg
Hamburg University of Technology, Germany

Stefan Mosler
Hamburg University of Technology, Germany

Michael Schlüter
Hamburg University of Technology, Germany

Abstract

Writing assignments can be seen as an important component of learning processes. Especially in the fields of engineering and sciences, writing assignments have the potential to consolidate subject-specific skills and to enhance motivation for solving technical problems. This paper introduces readers to a revised course structure that aims to strengthen motivation and mathematical understanding through written peer feedback based on mathematical exercises with written elements. The assignment was developed for the course *Computational Fluid Dynamics in Process Engineering*, a mathematically-oriented course for Master students of theoretical mechanical engineering and process engineering. Since the learning content was perceived as complex, students seemed to lack motivation in preparing for the course with the provided exercises. This paper suggests – based on the collected data, consisting of answers to mathematical problems, feedback texts, evaluation results, teachers' observation, and examination results – that the introduced assignment enhances students' understanding and has a positive impact on students' motivation to solve the mathematical exercises.[1]

Introduction

The idea that writing can help support learning is not new. Early approaches of 'writing to learn' can be traced back to the 1970s (e.g. Arapoff 1967, Britton et al. 1975, Emig 1977, Herrington 1981). Recent research indicates a close relationship between writing and deep learning. Drawing on data from the National Survey of Student Engagement (NSSE), a survey that annually collects information on students learning and engagement in the United States and in Canada, Gonyea and Anderson (2009) found that writing enhances learning processes that did not only aim for knowledge acquisition, but also referred to application and evaluation of knowledge (for further information on *surface and deep approaches to learning* see Biggs and Tang 2007). Other research studies show that writing possibly enhances conceptual knowledge and metacognitive awareness for processes of conceptual change in the sciences

[1] The majority of students gave permission for the anonymous use of their data. However, we were not able to contact all students from the course because some of them had already left the university.

(e.g. Mason and Boscolo 2000, Fellows 1994). Furthermore, a survey of students conducted by Light (2001) at Harvard University in the United States indicates that writing induces engagement. Students reported that the type and extent of the writing requested was directly related to their engagement in the coursework.

But what do we know about writing and learning mathematics? How can we apply these assumptions to maths and mathematically-oriented classes? By now, there are several studies that argue for 'writing to learn' mathematics. A study by Slaten (2013) focusses on students' self-reported growth of mathematical understanding, resulting from writing about the historical development of a mathematical concept. The results indicate that writing encourages students to authenticate their mathematical understanding and to develop new understanding. Another study, conducted by Teuscher, Hodges-Kulinna and Crookes (2015), suggests that writing has a positive effect on student achievement in mathematics at secondary schools. The authors state that the majority of teachers involved in the study reported significant or, at least, some effect on student achievement in mathematics when using 'writing to learn'. Other studies have been less conclusive. For instance, Porter and Masingila (2000) and Goss (1998) could not identify significant differences in studies that compare writing and verbalization. This is why we conclude that results are still somewhat vague. Hence, we argue that further research is essential to address the question of how and under which circumstances writing can enhance learning in mathematically-oriented courses.

Apart from writing, peer feedback is considered to be a promising way to enhance learning. A large number of research studies on peer feedback support the positive effects of feedback from peers (e.g. Liu and Carless 2006, Hyland 2000, Berg 1999, Villamil and De Guerrero 1998). Recent research focussing on the effects of giving feedback showed that not just the person receiving feedback but also the person giving feedback can benefit from the feedback process (e.g. Berggren 2014, Nicol and Macfarlane-Dick 2006, Van den Berg, Admiraal and Pilot 2006). Cho and MacArthur (2011) examined the effects of giving feedback on written assignments. They found that students improve their own writing by reviewing peer writing. A research study by Nicol, Thomson and Breslin (2014) shows that by giving feedback, students are engaged in various acts of evaluative judgement concerning the work of peers as well as concerning their own work. Moreover, giving feedback motivates the students to invoke and apply criteria to explain their judgements. According to Fallows and Chandramohan (2001), too, the feedback process encourages the student to get involved with the learning content more deeply than he or she would have otherwise done. These general findings are expected to be also true for mathematics, yet they need to be substantiated by further research.

Consequently, it is worthwhile to intensify research on how 'writing to learn' and written peer feedback work in mathematically-oriented courses – and to establish its possible benefit. Before attempting to convince instructors to engage in writing activities and higher education, it is important for developers to make changes in curricula and examination practice. Failing this, existing reluctance will be hard to overcome. The research study by Teuscher, Hodges-Kulinna and Crookes (2015) indicates that though many maths teachers consider writing to be worthwhile, time pressure is an obstacle for implementing writing to learn in maths at school. We assume that these obstacles apply to university instructors as well.

At our university, we have observed that, due to tight curricula and the importance of numerals and computations in engineering, writing plays a rather minor role in courses for students of engineering. Reluctance to integrate writing in class is often linked to time issues – either referring to the course itself, or to the time spent on reading students' texts. Especially in the basic subjects of engineering, like mechanics, thermodynamics, and mathematics, writing activities have sparsely been put into practice, as formulas and computations are considered more important. Two recent surveys at our university, conducted by the Center for Teaching and Learning (ZLL), support this observation. In a survey on writing, around 80% of undergraduates and more than 70% of graduate students said that writing reports or papers are rarely or not at all required (see Survey on writing ZLL 2016). A survey on different examination formats demonstrates that only 3.4% of all examinations taken throughout all disciplines are written texts (such as papers, reports) (see Survey on examination formats ZLL 2015). Research on the effects of writing can possibly contribute to introducing writing activities into the course design.

Organizational and educational framework of the course *Computational Fluid Dynamics in Process Engineering*

Our study is based on the course *Computational Fluid Dynamics in Process Engineering,* which is delivered in English in the form of a weekly two-hour lecture. The course is compulsory for students of theoretical mechanical engineering and optional for students of process engineering. All students are in their second year of their master's program. The course gives two credit points in the European Credit Transfer and Accumulation System.

The course used to be teacher-centred and was originally not accompanied by a tutorial. In Germany, engineering courses are occasionally accompanied by a tutorial, allowing students to apply the theoretical knowledge they learned in the course. In order to avoid a purely theoretical lecture and to support students' practical skills, we had some time ago introduced two practically-oriented sessions during the term: two lecture sessions were transformed into two tutorial sessions (see Fig. 1).

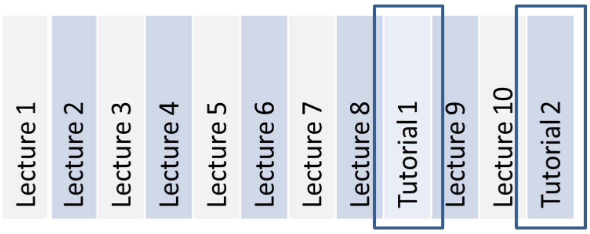

Figure 1: Procedure – one session per week

In the tutorial, exercises that illustrated the theoretical concepts used to be presented by the lecturer. Since preparation for and attendance in the tutorial were not compulsory – either for theoretical mechanical engineering or for process engineering students –, the motivation to do the exercises before class used to be minimal. In addition, the learning content and exercises were perceived as quite difficult.

This is why we innovated the course by introducing modified maths exercises and written peer feedback. During the first six weeks of the course, four exercises were given to the students. The exercises related to the following topics: boundary conditions, discretization of the Laplace's equation and the Poisson´s equation, diffusion equation and CFL-condition (*Courant-Friedrichs-Lewy* condition). As the exercises were rather sophisticated, students formed groups of four[2] in the first session of the course. Overall, 46 students attended class and formed 12 groups. Each group of students was supposed to do the four exercises – though it was not explicitly a requirement to pass the course. The solutions had to be submitted to the tutor within two weeks after the topic had been discussed in the course. The tutor anonymized the solutions and distributed them to another group. Then, each group had to write feedback on the solutions from the other groups (see Fig. 2).

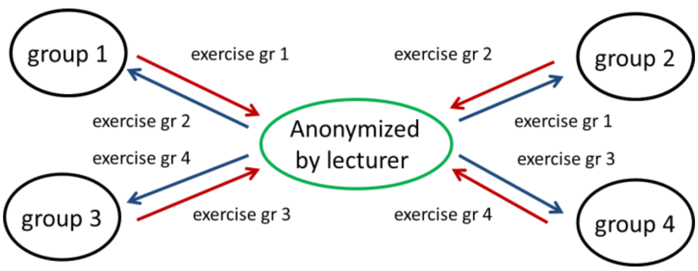

Figure 2: Exchange of anonymized exercises

[2] Two groups consisted of three students.

The feedback text had to be submitted to the tutor who passed it on to the respective group. After that, a tutorial was held in order to discuss the solutions. Here, one group initiated the discussion by presenting its solution to the entire class. The purpose of the tutorial was to ensure that everyone could prepare for the oral examination by having the correct answers.

A major aim of the written feedback assignment was that the students addressed the learning content continuously over the course of the study, and not just before the oral exam. Furthermore, the feedback assignment was intended to enhance critical thinking and to encourage the students to question their own solutions as well as those of others.

Methods

When first introducing the course *Computational Fluid Dynamics in Process Engineering,* we faced two major challenges. We observed that students had difficulties in understanding the course because the learning content was rather sophisticated and complex. This observation was affirmed by the final oral examination as students' performances lagged behind expectations. In addition, we noticed that the students rarely prepared the exercises that were given as an optional homework. Instead, they rather waited for the tutor to present the solutions in the following session. In order to resolve these challenges, we altered the teacher-centered course, in which the teacher presents and the students listen, by introducing both modified math exercises containing writing and written peer feedback. The implemented changes are based on two hypotheses. First, we assume that peer feedback increases students' motivation to prepare the exercises. Second, we expect that writing intensifies the acquisition of complex concepts and enhances students' understanding. This paper closely examines the outcomes of the revised course and presents how the findings support these assumptions. The investigation draws on data obtained by collecting answers to mathematical problems and feedback texts, as well as data collected via an evaluative survey. Moreover, we shall refer to the results of the final oral examination as well as to the observations made by the engineering lecturer in the course to support our findings.

Mathematical problems and answers

Each of the four exercises that were distributed to the students focused on understanding basic topics of the course as well as on questions which were supposed to motivate the students to write short texts. As a consequence, each exercise included one writing-oriented question. In the first exercise, one question was about the interpretation of a formula. In exercise two, one question dealt with different kind of grids. In the third exercise, the differences between an explicit and implicit algorithm had to be explained based on an example. In the fourth exercise, a question served to show the differences between numerical algorithms for the transport equation. All questions were intended to prepare the students for the oral exam by raising their awareness of possible exam questions and by encouraging them to see and understand connections.

In the following, we will present one of the exercises in detail as an example. The topic deals with appropriate boundary conditions for the Navier-Stokes equations. The Navier-Stokes equations in two dimensions are given by:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),$$
$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right).$$

First, the velocity in *x*-direction, *u*, is zero due to the no-slip condition at the wall. It follows that the velocity in *y*-direction, *v*, is also zero. The question of what is a suitable boundary condition for the pressure *p* (Fig. 3) arises.

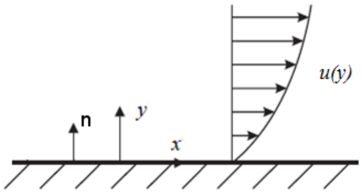

Figure 3: Velocity profile next to the wall. The no-slip condition is valid at the wall *u* (*t,x,0*)=0. Which is the right condition for the pressure *p*?

To answer this question, certain simplifications were given in the exercise. These simplifications were a fully developed flow and a steady state condition. In this way, with the help of the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

the students needed to find the boundary condition for the Navier-Stokes equations. In addition, the students were asked if the simplifications were necessary to find the boundary condition.

We expected the students to deduce the following formula: $\vec{n} \cdot \nabla p = 0$. This formula states that the pressure drop (pressure gradient ∇p) at the wall only occurs in a tangential direction. This behaviour should be explained by using the properties of the dot product. In this case the dot product is zero. This means that the normal vector and the pressure drop stand orthogonal to each other. It follows that the pressure drop shows tangential direction. Regarding the question of whether the simplifications were necessary, the students should explain that both assumptions were not needed because the boundary conditions do not change in time or space (Fig. 4). More precisely, the students would ideally say that for any x_1 and x_2 it follows

$$u(t, x_1, 0) = u(t, x_2, 0) = 0 \;\; \forall t \geq 0.$$

In addition to these steps, the students were expected to conclude that the last equation already describes a fully developed and steady state flow at the wall. Thus, the partial derivatives in time, $\frac{\partial u}{\partial t}$ and $\frac{\partial v}{\partial t}$, and in *x* direction, $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$, drop out in the Navier-Stokes equations.

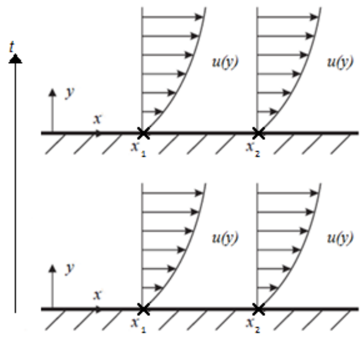

Figure 4: No-slip condition does not change in space and time.

As mentioned earlier, the learning content of the course is very sophisticated and complex. For that reason, we considered it worthwhile to design exercises that allowed students to apply what they had learned in the course. Usually, it is easier to memorize knowledge than to explain and interpret what it actually means because the latter involves a higher level of learning objectives (see e.g. Bloom et al. 1956, Anderson and Krathwohl 2001). Hence, it should be easier to memorize how to establish a formula than to interpret and explain its meaning. Therefore, we developed writing tasks that asked for interpretation and explanation of meaning and, hence, made students think about the problem and possible relations. Ten of twelve groups submitted solutions to the problems. The solution texts are shown by the following examples:

Example 1
'…the assumption of a fully developed flow is indeed necessary. Without this assumption we could not simplify the continuity equation to (iii) [*v*(*y*)=0] and the equation (2) would look like this (assuming a steady state):
$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)= -\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$…'

Example 2
'*Zero gradient condition* means no change of pressure at the wall, where no-slip-condition occurs for real fluids. With no flow through the wall in $\pm\vec{n}$-direction and no velocity in *x*-direction by no-slip-condition, no changes in pressure caused by velocity changes are possible ($\nabla p = 0$). In the special case of laminar flow, the pressure all over the boundary layer is constant for any *y* and set by the pressure at the boundary layer ($y = \delta_b$), where potential flow can be assumed.
To evaluate if the assumption of steady state and/or fully developed flow are necessary for *zero gradient*, we chose the way to derive the momentum equations once again for those assumptions. If eq. (5) is not fulfilled – respectively $\nabla p \neq \vec{0}$ we can state which assumptions are essential.
Without (a), (b) and with (d) momentum in *y*-direction is
$\frac{\partial p}{\partial x}=\eta\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}\right)+\rho\frac{\partial v_x}{\partial t}.$
For *y*-direction it is
$\frac{\partial p}{\partial y}=-\eta\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}\right)+\rho\frac{\partial v_y}{\partial t}.$
For a change of velocity *y* other than *y* = 0 cannot be expulsed anymore. Therefore the pressure gradient is
$\nabla p\cdot\vec{n}=\begin{pmatrix}\neq 0\\ \neq 0\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\neq 0.$
For the specific case of laminar flow, where $v_y = 0$, steady state would not be necessary if the flow is assumed to be fully developed because $\frac{\partial v_y}{\partial t}$ should also be 0. But without knowing the flow we have to assume both steady state and developed flow to achieve *zero gradient condition* in pressure at the wall.'

Feedback texts

Students were asked to provide feedback to the solutions written by their peers. Concerning the feedback process, it was important for us that the students did not only pay attention to the correctness of the solution, but also considered the way the solution was presented, that is, the style and structure of the solution. In our opinion, this kind of instruction makes students think through the solutions more carefully and, hence, intensifies students' involvement in the solution process. If they found mistakes, students were expected not only to name them but also to describe the error in reasoning and to give clues on how to find the correct answer. As the students were not used to doing written assignments, we offered support in the form of guiding questions. The questions were aimed at helping the students structure their feedback and focus on the important points:

- Are the steps taken in the solution comprehensible and documented in an appropriate way?
- Is the solution 'complete' - which means: Does it really answer the question? If not, try to give hints about how the appropriate answer could have been obtained.
- If you find mistakes, please do not just name them, but also try to give clues about how to solve the exercise correctly.
- Please acknowledge the work of your class mates. Do not forget to name positive aspects as well as points for improvement.

The scholarly literature also suggests that feedback needs to be practiced (e.g. Min 2006, Nicol 2011). Therefore, we consider it crucial to support students by giving clear guidance.

Ten of the twelve groups did the feedback assignment and submitted a text providing feedback on the solutions of a peer group.

Evaluation

Furthermore, we asked students to evaluate the course *Computational Fluid Dynamics in Process Engineering* and the implemented peer feedback assignment in particular at the end of term. The evaluation involved 13 questions and used the 5-point Likert scale, ranging from 'strongly agree' to 'strongly disagree'. Four questions generally dealt with the students' satisfaction with the course, its structure and the implemented teaching innovation. Three questions were about the course and its syllabus. A further three questions concerned the introduced assignment (see below Figures 5 - 7). In addition, three open questions were included. They asked for comment on aspects of the course the students liked in particular as well as for suggestions for improvement (in general and concerning the feedback process). Of the 46 students, 26 participated in the evaluation. Admittedly, it seems slightly problematic that we do not know if all of those 26 students also participated in the writing task and feedback process. However, the number of participating students taken together was quite high: ten of the twelve groups participated in the assignment, which adds up to about 38 to 40 students. Hence, there was only a small number of students who were not involved in the exercises and feedback process at all.

Oral examination and the teacher's observation

In order to make detailed observations and draw precise conclusions concerning our hypotheses, we will also take into account the results of the final oral examinations of the years 2013 and 2014. Furthermore, we will look at the number of oral exams taken in the years 2013, 2014 and 2015 to find out if there has been a change in the numbers of students taking the exam. In addition, a comparison of the number of solutions submitted in 2015 and the engagement in the math exercises in previous terms serves as supporting data.

Results

This section presents findings that relate to the two hypotheses we addressed in the introduction: We expect that the peer feedback assignment increases students' motivation to prepare the exercises and that writing helps to understand mathematical learning content.

Hypothesis 1: Motivation

Both the evaluation results and the teacher's observation indicate an increase in students' motivation as a consequence of integrating the new assignment. The evaluation includes one question that refers to the introduced assignment and that can directly be related to our assumption that written peer feedback supported students' motivation. As Fig. 5 shows, 50% of the students said that the feedback process had contributed to an increase in their motivation to do the maths exercises. Further 23% rated the motivation question neutrally. Only 27% stated that they had not been motivated.

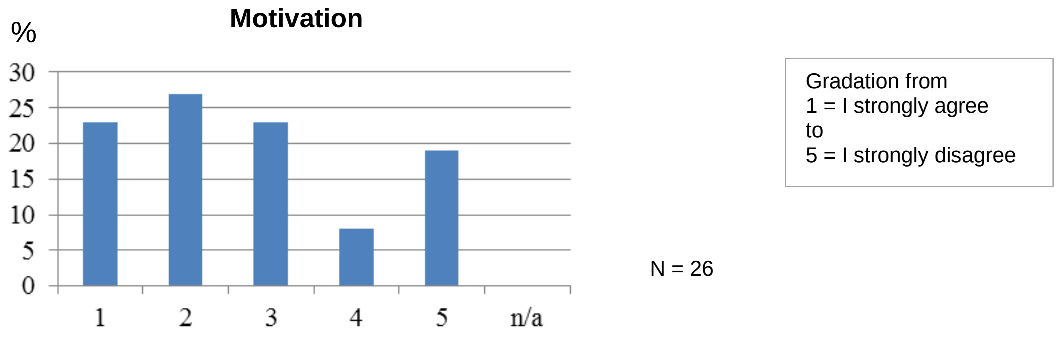

Figure 5: Students' motivation to solve the exercise ('Knowing that I would get a review was a motivation for me to solve the exercise.')

Referring to the open questions, we found various support for the idea of integrating written peer feedback into engineering courses. Several students suggested implementing the written feedback assignment in other courses in a similar fashion, inter alia, in the courses Mechanics I-V, Electrical Engineering Fundamentals (EEF), and Finite Element Method (FEM). Other students would like written peer feedback to become an obligatory assignment in the course so that every student receives helpful feedback text on each exercise. These responses show that implementing the feedback assignment appears to be an effective method that was valued by most students.

Furthermore, acknowledgement from the students shows that the innovation is generally appreciated. The final question, which left room for open comments, returned statements like:

- 'Very good idea, helps a lot for motivation and understanding.'
- 'The teaching innovation including the exchange of answers was very good.'
- '[The feedback] increases liability/obligation/commitment to really solve the maths exercises.'

It seems that the assignment made the students more engaged and active. Most students actively dealt with the learning content during term and many of them stated that they felt quite well-prepared for the examination. Their positive perception was reinforced by the examination results (see following section).

The teacher's observation provides support for the positive impact on students' motivation. An increase is indicated by the number of solutions submitted. In comparison with previous terms, when the exercises and their solutions were presented in class, far more students were generally engaged with the exercises when the peer feedback procedure was introduced. Previously, students simply waited for the teacher to present the right solution in the next class. Only two or three students in the whole class usually tried to solve the exercises at home. When implementing the peer feedback assignment, many students submitted proposals for solutions, and ten of the twelve groups submitted their approaches. As in both the old and the revised course, the maths exercises were optional. This observation seems to support the motivational effect the assignment had.

Hypothesis 2: Learning and understanding

Our data seems to indicate that students dealt with the learning content more deeply and therefore seems to suggest that processes of learning were enhanced. In the following we will present our findings by referring to the solutions to the maths exercises, the feedback texts, the results of the oral exam, and the results of the evaluation.

First, we look at the solutions to the exercises. The solutions show that the students wrote interpretations and explanations and hence handed in short continuous texts (see examples 1 and 2 in the methods section). The texts illustrate in which way the students put their explanations down in writing to solve the exercises, e.g. 'for the specific case of laminar flow' (use of background knowledge) and 'without this assumption we could not simplify the continuity equation' (explanation concerning a certain assumption). Although the texts are not long, the writing assignment made the students write down their explanations and interpretations, and therefore provided opportunities for 'writing to learn'. The students were required to thoroughly think about their first assumptions and find logical arguments to support and explain their assertions. As a result of this, we assume that students' mathematical understanding was enhanced. Although an increase in understanding seems rather hypothetical, the exam results seem to support this assumption (see below). However, the relation to the exam results needs to be viewed with caution as we need to relate exam results to all changes made: to the different type of mathematical exercises as well as to the written peer feedback assignment.

Second, we refer to the feedback texts. The texts indicate an strong engagement with the solutions suggested by other students. In the feedback, the students describe, for instance, what they find unclear ('it is not very clear which equation…'), ask for explanation of statements ('Unfortunately, you give no further explanation…'), and make concrete suggestions for improvement ('I suggest replacing the sentence…'). Furthermore, some feedback texts illustrate that by responding to someone else's solution students think about what makes the solution text difficult to follow and to understand. This is illustrated by the following examples:

- 'The presented problem is only comprehensible for people who looked into the subject as well.'
- 'This is only comprehensible for people who exactly know the background of the problem.'
- 'The results which were drawn from the equations are only partially formed into full sentences, which leads to a lack of understanding on the side of the reader. The author should refrain from drawing conclusions by means of an arrow.'

All three excerpts indicate that the students giving the feedback deal with the comprehensibility of the text. The feedback givers reflect on the question of which information is important in order to enable the reader to understand the solution text. The examples refer to the background of the problem (examples 1 and 2) and the conclusion drawn by the author (example 3). The last example cited above especially illustrates how the writer becomes aware of the fact that full sentences – instead of using arrows (→) – contribute to clarity and help the reader to understand the solution. Thus, the feedback assignment seems to have encouraged the students to think about what makes a text that introduces a maths solution comprehensible and well-structured.

Third, we compare the number of oral exams taken as well as the results. In Germany, the grades range from 1.0 (best) to 5.0 (failed). The number of students taking the oral exam increased from 10 out of 36 students (28%) in 2013 to 37 out of 54 students (69%) in 2014, the year in which the innovation was implemented. This trend continues in 2015, rising to 46 out of 67 students (69%). This shows that the motivation to take the exam increased. The increase in exams taken possibly indicates that the students felt better prepared for the exam by completing the new assignment.

A comparison of the exam results of 2013 and 2014 supports the assumption that students were better prepared. Comparing the two years, we notice that the average grade increased from 2.2 in 2013 to 1.4 in 2014 - even though the number of exams for 2013 (only 10) is low. In addition, the poorest grade in 2013 was 5.0 whereas it was 2.7 in 2014. The poorest ten exams in 2014 achieved a better average grade (1.8) than the ten exams in 2013 (2.2).

Hence, the exam results support the hypothesis that writing and peer feedback have a positive impact on motivation and on understanding. As mentioned, differentiation cannot be made between the effects of the new type of mathematical exercises asking for written explanations and the effects of the written peer feedback assignment because both assignments were implemented at the same time. Further, we cannot entirely rule out that other variables might have had an impact on the results, such as a very high or low achieving class, or a very high or low intrinsically motivated class.

Fourth, we analyse the results of the evaluation at the end of term. Two questions refer to the assumption that the feedback assignment supported students' learning. The results indicate a perceived beneficial effect of the written peer feedback assignment. The majority of the students confirmed that dealing with other students' answers to the maths exercises and writing a feedback text enhanced their own understanding of the learning content. 92% of the students surveyed stated that the assignment facilitated their understanding (38% of the students *strongly agreed*, 54% *agreed*) – as shown in Fig. 6.

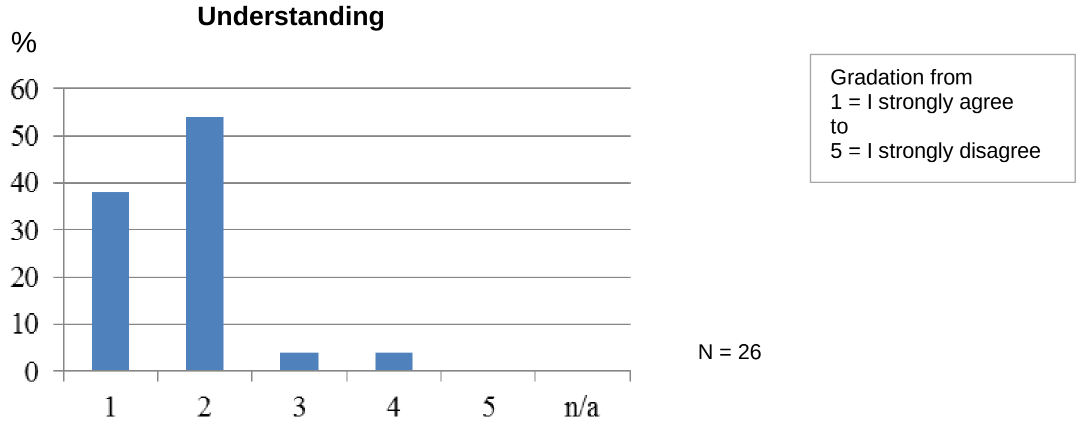

Figure 6: Understanding of the learning content ('Doing the assignment has helped me to better understand the content of the course.')

However, it is hard to differentiate precisely whether it is the engagement with the solutions of fellow students in general or writing a feedback text that was the determining factor. The item only asks for the peer feedback assignment in general.

Likewise, the assumption that dealing with the solution proposed by fellow students motivates the process of learning seems to be supported by many students (Fig. 7). The results are less clear than in Fig. 6 though. 42% of the students either *strongly agreed* or *agreed* with the statement, whereas only 23% of them disagreed with it. The additional 35% of the students gave a neutral response to the item.

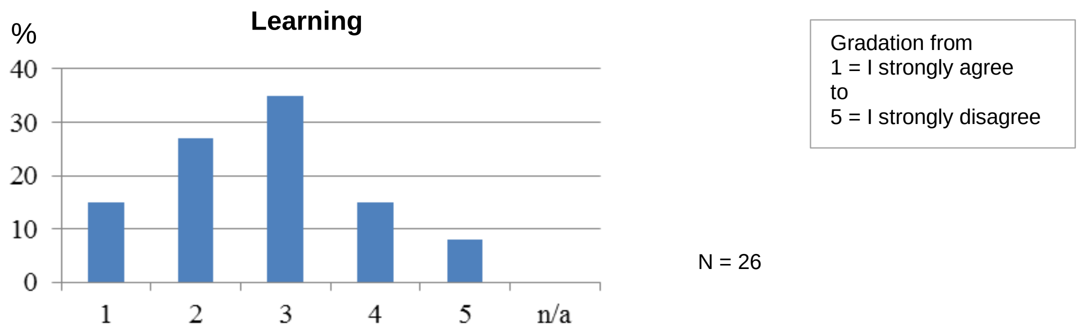

Figure 7: Learning via solutions of others ('I learned from the solutions of other students.')

Here again, since the question focusses on the peer feedback process as a whole, we cannot clearly say to what extent the written form of the feedback was important and in which way it contributed to the process of learning.

Discussion

This study examined the extent to which written peer feedback and mathematical exercises including writing tasks can enhance students' motivation and mathematical understanding in a process engineering context. The data presented suggests that the implementation of the assignments has affected students' motivation and mathematical understanding in a positive way. However, we are aware that some results require further investigation, especially concerning the differentiation of written maths tasks and written peer feedback, and the accuracy of conclusions on the feedback process.

Similar to the findings by Slaten (2013), a large majority of students in our course self-reported that writing a feedback text had a positive effect on their mathematical understanding. Whereas Slaten focusses on a writing assignment, we applied a feedback assignment consisting of writing a text providing feedback. Because our study addresses the peer feedback process as a whole, we are not able to draw conclusions on the effects of writing and feedback separately but rather only on the whole assignment.

A reliable result of our study is the positive effect of peer feedback in general. Peer feedback appears to have a positive impact on students' motivation and seems to affect understanding in a positive way. As we did not focus on the distinction between written and oral feedback (e.g. by introducing a control group), we are not able to draw conclusions as to whether the effects of written peer feedback differ from the effects of oral peer feedback. We hypothesize that they do because the written form requires a greater commitment on the part of the student. However, our assumption should be tested by subsequent research.

Concerning research on peer feedback, it seems that our results particularly support the assumption that the person who gives feedback can benefit from the feedback process (e.g. Berggren 2014, Nicol and Macfarlane-Dick 2006, Van den Berg, Admiraal and Pilot 2006). A comparison of Fig. 6, which refers to students' views on writing feedback, and Fig. 7, which implicitly refers to students' views on receiving feedback, indicates a more positive appraisal of writing feedback for learning and understanding mathematics. Overall, general findings on peer feedback that were presented in the theoretical section are likely to be true also for mathematics.

Although many findings still require further investigation, especially to rule out other variables that might have influenced the research results, they indicate an affirmative trend concerning the hypotheses stated in the introduction.

Conclusion

To sum up, the implementation of both mathematical exercises that include writing tasks and written peer feedback in mathematically-oriented courses for engineers seems to be a beneficial approach. Therefore, we suggest implementing writing assignments in mathematically-oriented courses frequently. In addition, written peer feedback is beneficial for teachers in that that students receive feedback on their answers while teachers do not have to read all pieces of work that have been submitted. This addresses reservations instructors might have concerning the increase in workload that would otherwise arise from integrating writing in class.

When looking back on the implementation process and the findings of our evaluation, we faced a few challenges. Considering these challenges, we provide some suggestions for successful implementation of the feedback procedure in mathematics and engineering courses. One challenge concerns the matching of feedback groups. If two groups are randomly matched for peer feedback, it might happen that groups that have made the same mistake might give each other feedback. Thus, this procedure runs the risk of reinforcing students' misperceptions. However, it is difficult to optimize the matching process. A similar problem occurs when students do not know the correct answer. Some students noted that it was very difficult for them to write a feedback text without knowing the correct solutions of the exercises. Students' struggle to evaluate the solutions of others cannot entirely be solved when implementing written peer feedback in the course, unless instructors circulate the right answers before the feedback process. However, this is not in accordance with the purpose of feedback assignments. Experiencing difficulties in the learning process is not necessarily detrimental, as long as students ultimately understand the problem and reach the correct answer. Yet, the students' comments emphasise the importance of the final tutorial to ensure that everyone has the right answer and is well-prepared for the examination at the end of term. Moreover, the tutorial might support students' engagement with their feedback texts – a condition for effective feedback. To intensify engagement with feedback texts, instructors could introduce an alternative procedure for the tutorial. Instead of presenting the right answers to the rest of the class on the blackboard, students could work out the right solutions via discussion groups. By discussing their approaches to solutions, students would engage with their feedback texts again and thereby with the learning content. Learning from each other would therefore be intensified.

A third challenge we faced was the preparation of the students for the feedback procedure. Although they were supported by guiding questions, many students felt uncertain about how to write a feedback text. This can be explained by the fact that the students were not used to do writing assignments in their engineering classes. This observation supports the claim that feedback needs to be practiced. In addition to guiding questions, the teacher needs to explicitly introduce students to writing feedback texts. Moreover, he or she needs to thoroughly explain the aims of the assignment aims. As writing a feedback text is not natural to students, they have to be guided by a detailed description of requirements.

Lastly, we propose implementing written peer feedback as an obligatory assignment in class, as favoured by many students in their evaluation, so that each group gets a feedback text on each exercise.

References

Anderson, L. W. and Krathwohl, D. (2001) *A Taxonomy for Learning, Teaching, and Assessing. A Revision of Bloom's Taxonomy of Educational Objectives*. New York: Addison Wesley

Arapoff, N. (1967) 'Writing: A Thinking Process'. *TESOL Quarterly* 1(2), 33-39

Berg, C. E. (1999) 'The Effects of Trained Peer Response on ESL Students' Revision Types and Writing Quality'. *Journal of Second Language Writing* 8(3), 215–41

Berggren, J. (2014) 'Learning from Giving Feedback: A Study of Secondary-level Students'. *ELT Journal* 69(1), 58-70

Biggs, J. and Tang, C. (2007) 'Teaching for Quality Learning at University.' 3rd edn. Open University Press. [online] available from <http://www.umweltbildung-noe.at/upload/files/OEKOLOG%202014/2_49657968-Teaching-for-Quality-Learning-at-University.pdf> [27 June 2016]

Bloom, B., Englehart, M., Furst, E., Hill, W. and Krathwohl, D. (1956) *Taxonomy of Educational Objectives: The Classification of Educational Goals. Handbook I: Cognitive Domain.* New York, Toronto: Longmans, Green

Britton, J., Burgess, T., Martin, N., McLeod, A. and Rosen, H. (1975) *The Development of Writing Abilities*. London: Macmillan Education

Cho, K. and MacArthur, C. (2011) 'Learning by Reviewing.' *Journal of Educational Psychology*, 103 (1), 73-84

Emig, J. (1977) 'Writing as a Mode of Learning.' *College Composition and Communication* 28(2), 122-128

Fallows, S. and Chandramohan, B. (2001) 'Multiple Approaches to Assessment: Reflections on the Use of Tutor, Peer, and Self-assessment.' *Teaching in Higher Education* 6(2), 229-246

Fellows, N. J. (1994) 'A window into Thinking: Using Student Writing to Understand Conceptual Change in Science Learning.' *Journal of Research in Science Teaching* 31, 985-1001

Gonyea, R. and Anderson, P. V. (2009) 'Writing, Engagement, and Successful Learning Outcomes in College.' Paper presented at the Annual Meeting of the American Educational Research Association. [online] available from <http://cpr.indiana.edu/uploads/Gonyea-Anderson%20AERA%202009.doc> [27 June 2016]

Goss, M. A. (1998) *Writing to Learn: An Experiment in Calculus.* Unpublished doctoral dissertation, University of Southern Mississippi, Hattiesburg, MS

Herrington, A. J. (1981) 'Writing to learn: writing across the disciplines.' in: *College English* (43) 4, 379-387

Hyland, F. (2000) 'ESL Writers and Feedback: Giving More Autonomy to Students'. *Language Teaching Research* 4(1), 33-54

Light, R. J. (2001) *Making the Most of College: Students Speak Their Minds*. Cambridge

Liu, N. and Carless, D. (2006) 'Peer Feedback: The Learning Element of Peer Assessment.' *Teaching in Higher Education*, 11(3), 279-290

Mason, L. and Boscolo, P. (2000) 'Writing and Conceptual Change. What Changes?' *Instructional Sciences* 28(3), 199-226

Min, H. T. (2006) 'The Effects of Trained Peer Review on EFL Students' Revision Types and Writing Quality. *Journal of Second Language Writing* 15, 118-141

Nicol, D. J. (2011) 'Developing Students' Ability to Construct Feedback.' [online] available from <http://www.enhancementthemes.ac.uk/docs/publications/developing-students-ability-to-construct-feedback.pdf?sfvrsn=30> [27 June 2016]

Nicol, D. J. and Macfarlane-Dick, D. (2006) 'Formative Assessment and Self-regulated Learning. A model and Seven Principles of Good Feedback Practice.' *Studies in Higher Education* 31(2), 199-218

Nicol, D., Thomson, A. and Breslin, C. (2014) 'Rethinking Feedback Practices in Higher Education: A Peer Review Perspective.' *Assessment & Evaluation in Higher Education* 39(1), 102–122

Porter, M. K. and Masingila, J. O. (2000) 'Examining the Effects of Writing on Conceptual and Procedural Knowledge in Calculus.' *Educational Studies in Mathematics* 42 (2), 165–177

Slaten, K. M. (2013) 'Writing about the History of Mathematics as a Means for Growth in Understanding.' *Investigations in Mathematics Learning* 5(3), 9-24

Teuscher, D., Hodges-Kulinna, P. and Crooker, C. (2015) 'Writing to Learn Mathematics: An Update.' in: *The Mathematics Educator* 24 (2), 56–78.

Van den Berg, I., Admiraal, W. and Pilot, A. (2006) 'Designing Principles and Outcomes of Peer Assessment in Higher Education.' *Studies in Higher Education* 31(3), 341-356

Villamil, O. A. and De Guerrero, M. C. M. (1998) 'Assessing the Impact of Peer Revision on L2 Writing.' *Applied Linguistics* 19(4), 491-514

Zentrum für Lehre und Lernen (ZLL) (2016) *Survey on Writing.* Hamburg University of Technology

Zentrum für Lehre und Lernen (ZLL) (2015) *Survey on Examination Formats*. Hamburg University of Technology