



Static and Dynamic Analysis of Spur and Bevel Gear

B.Malsoor^{1*}, B.Harish Raj¹, K.P.Reddy¹, J.Mallesh¹

¹Asst Professor, Dept of Mech Engg, Holy Mary Institute of Technology and science, Telanga, India.

*Corresponding author E-Mail ID: malsoormtech@gmail.com Mobile: 9963890891

ABSTRACT

Gear is the one of the important machine element in the mechanical power transmission system. Spur gear is most basic gear used to transmit power between parallel shafts. Spur gear generally fails by bending failure or contact failure. This paper analyses the bending stresses characteristics of an involute spur gear tooth under static loading conditions. The tooth profile is generated using CATIA and the analysis is carried out by Finite element method using ANSYS software. The stresses at the tooth root are evaluated analytically using existing theoretical models. The theoretical and FEM results are compared. The results obtained theoretically are in good agreement with those obtained from software. Also an attempt is made to introduce Stress and displacement characteristics of tooth under dynamic loading conditions.

1. INTRODUCTION

In the today's world of industrialization Gears are the major means for the mechanical power transmission system, and in most industrial rotating machinery. Because of the high degree of reliability and compactness gears dominates the field of mechanical power transmission. Gearbox is used to convert the input provided by a prime mover into an output required by end application. Due to increasing demand for quiet and long-term power transmission in machines, vehicles, elevators and generators, people are looking for a more precise analysis method of the gear systems. Spur gear is the most basic gear used to transmit power between two parallel shafts with almost 99% efficiency. It requires the better analysis methods for designing highly loaded spur gears for power transmission systems that are both strong and quiet. Due to development of computers people are using numerical approach for the analysis purpose as it can give more accurate analysis results. The finite element method is capable of providing information on contact and bending stresses in gears, along with transmission errors, which can be done easily in ANSYS software. Gear analysis in the past was done by using analytical methods which requires complicated calculations. Now with the use of FEA we can calculate the bending stresses in the gear tooth for given loading condition and we can compare the FEA results with existing models to decide the accuracy. Also static as well as dynamic, both loading conditions of gear can be easily analyzed in ANSYS which is not the case with Analytical method.

2. PROBLEM DEFINITION

B.Malsoor For this problem we are doing our calculations analytically and compare results with software results. Any problem can be solved by following same procedure.

2.1 Question

The Following data is given for a spur gear pair made of steel and transmitting 5KW power from an electric motor running at 720 rpm to a machine:

No. Of teeth on Pinion= 21, No. Of teeth on Gear= 40, Module= 5mm, Face width= 10m, Ultimate Tensile Strength for Pinion material= 600 N/mm², Ultimate Tensile Strength for Gear material= 400 N/mm², Tooth System =20 Degree Full-Depth Involute, Service Factor =1.25, Load Concentration Factor = 1.6, Tooth Form factor for pinion= .326, Tooth Form factor for gear= .389, Velocity factor= $6/(6+v)$.

2.1.1 Solution

Beam strength for pinion $\sigma_{\text{p}} = \sigma_{\text{ut}}/3 = 600/3 = 200 \text{ n/mm}^2$ and for gear $\sigma_{\text{g}} = \sigma_{\text{ut}}/3 = 400/3 = 133.34 \text{ n/mm}^2$. Now, $\sigma_{\text{p}} * y_{\text{p}} = 200 * .326 = 65.2 \text{ n/mm}^2$ and $\sigma_{\text{g}} * y_{\text{g}} = 133.34 * .389 = 51.87 \text{ n/mm}^2$.

As, Strength of gear < Strength of pinion, gear is weaker than pinion in bending. Hence it is necessary to design the gear for bending.

Pitch Line Velocity (V) = $\pi * D_P * N_p / (60000) = 3.9585 \text{ m/s}$

Theoretical Tangential Force Ft= $P/V = 5000/3.9585 = 1263.1047 \text{ N}$ (approx. 1200N)

3. STATIC ANALYSIS

3.1 The Lewis Formula (Stress Calculation)

The analysis of bending stress in gear tooth was done by Mr. Wilfred Lewis in his paper, 'The investigation of the strength of gear tooth' submitted at the Engineers club of Philadelphia in 1892. Even today, the Lewis equation is considered as the basic equation in the design of gears [1]. Wilfred Lewis was the first person to give the formula for bending stress in gear teeth using the bending of a cantilevered beam to simulate stresses acting on a gear tooth shown in Cross-section = $b * t$, height = h, Load=Ft uniform across the face.

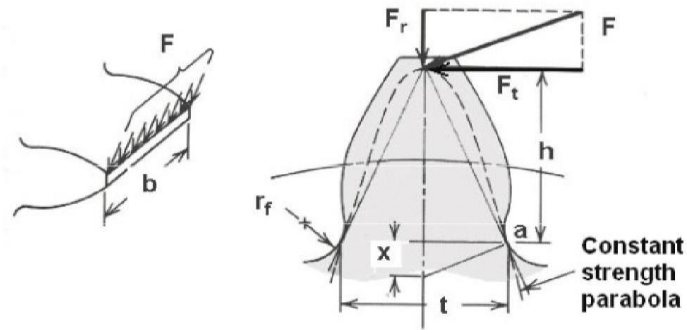


Fig.1 Lewis Equation

Lewis considered gear tooth as a cantilever beam with static normal force F applied at the tip. He took the critical section as parabola through point 'a' and tangent to tooth curves at the root as shown in fig.1. This parabola shown by dotted line is a beam of uniform strength.

Assumptions made in the derivation are: 1.The full load is applied to the tip of a single tooth in static condition, 2.the radial component is negligible, 3.the load is distributed uniformly across the full face width, 4.forces due to tooth sliding friction are negligible and 5.stress concentration in the tooth fillet is negligible. In the current analysis of spur gear we follow the Lewis assumptions and equation.

When the bending stress reaches the limiting value i.e. bending endurance strength or permissible bending stress, the corresponding tangential force is called the beam strength and given as Equation 3.1 is known as Lewis equation for beam strength of spur gear.

4. DERIVATION OF DEFLECTION FOR GEAR TOOTH USING CASTIGLIANO'S THEOREM

Lewis has assumed parabola in the gear tooth. So we directly find the deflection of the parabolic teeth with minor errors with actual deflection [2, 3]. We use the castigliano's theorem for the same.

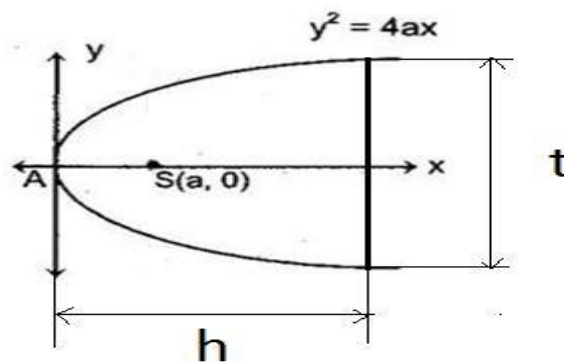


Fig.2 Parabolic Teeth

5. CASTIGLIANO'S THEOREM

Castigliano's theorem is one of the energy methods (based on strain energy) and it can be used for solving a wide range of deflection problems. Castigliano's theorem states that when a body is elastically deformed by a system of loads, the deflection at any point "P" in any direction "a" is equal to the partial derivative of the strain energy (U) with respect to a load at "F" in the direction "a"

The theory applies to both linear and rotational deflections

It should be clear that Castigliano's theorem finds the deflection at the point of application of the load in the direction of the load.

Strain energy is given by U_{uf}

Consider the parabolic teeth of height 'h' and tooth thickness 't'.

Equation of parabola is

Thus equation of parabola becomes $(t/2) = 4*a*h$.

This is the expression for the maximum deflection of spur gear teeth tip when tangential load is applied at the teeth tip.

6. ANALYTICAL RESULTS

Stress using flexure formula

The bending stress at the root of tooth can be given by flexure formula

Tangential load(Ft)= 1200N(approx.), Height of tooth(h)=11.25mm, Face Width(b)=50mm, Thickness(t)= 10.614mm(at root).....(Using basic values of various parameters).

$$\text{Therefore, Stress } (\sigma) = 6*1200*11.25 / (50*10.614^2)$$

$$\text{Theoretical Stress } (\sigma) = 14.3799 \text{ N/mm}^2.$$

7. DEFLECTION USING CASTIGLIANO'S THEOREM

$$\text{Deflection } (\delta) = \frac{A*4*}{6*} \%$$

Tangential load(Ft)= 1200N, Face Width(b)=50mm, Thickness(t)= 10.614mm(at root), Stiffness constant(E)= 206000 N/mm². (Using basic values for various parameters).

Height of tooth (h) calculated using Lewis constant, as we required height of parabola:-

8. ANALYSIS BY USING ANSYS

8.1 Ansys Procedure

The entire analysis is done on the single gear tooth in Ansys 14.0. For that purpose we use the following procedure,

8.2 Discretization of Continuum

Draw gear tooth in catia of given dimensions→ Divide the gear tooth in 25 sections→ calculate mean thickness for each section. Do not consider the fillet radius as assumed by Lewis.

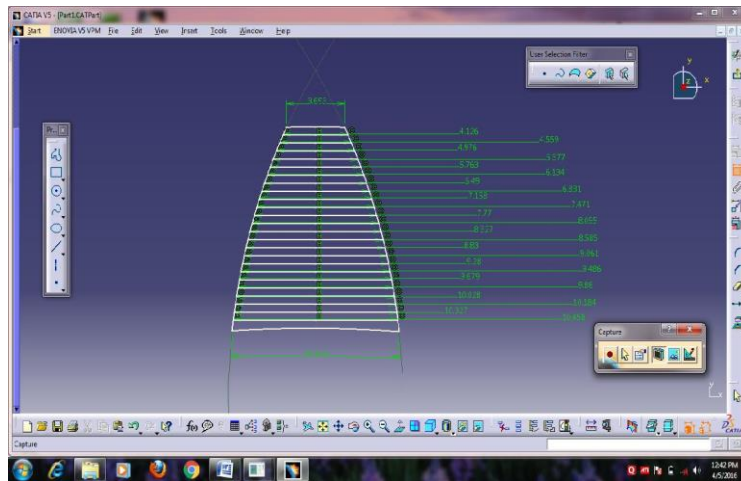


Fig.3 Gear Tooth in Catia

8.3 ANSYS Modelling and Boundary Conditions.

Import the IGS file of gear tooth in Ansys. Take element type as beam. Take nodes equidistantly on length equals to tooth height and create different sections of calculated dimensions (length and breadth). Then create elements at nodes of corresponding element attribute. One end of tooth is fixed and at other end tangential load of 1200N (approx.) is applied at the tip. All boundary conditions are applied on nodes.

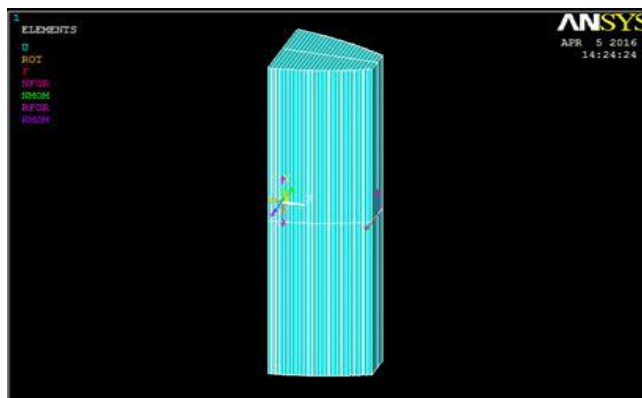


Fig.4 Oblique View of Teeth

9. PLOTTING STRESS AND DEFLECTION GRAPHS

For end conditions.

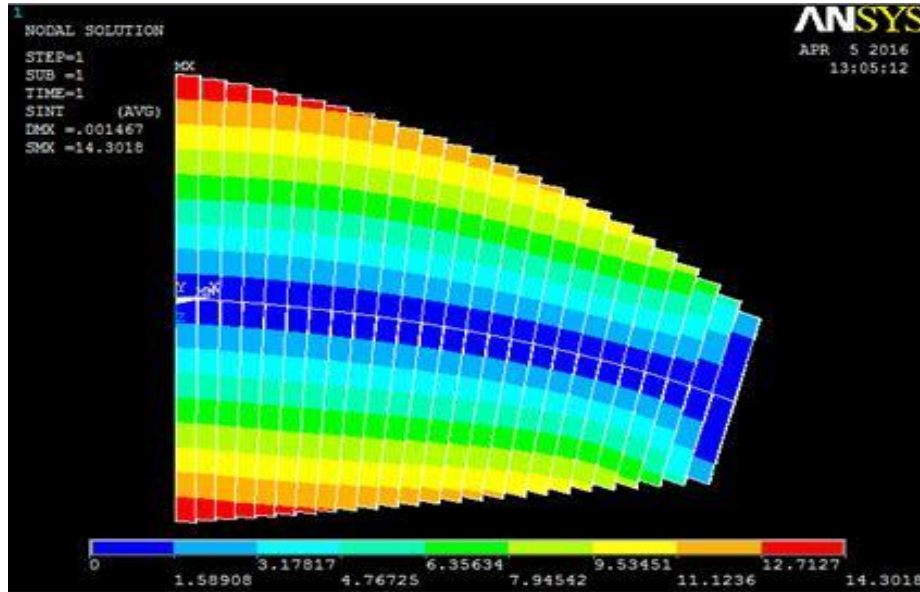


Fig.5 Von Mises Stress Plot (Top View)

10. ANSYS RESULTS

1. **Stress:** As it can be seen from above images Maximum Von Mises Stress= 14.3018 N/mm²
2. **Deflection:-** From above images Maximum Deflection = .0014 mm

11. COMPARISON OF RESULTS

Table 1 Stress

Sr. No.	Ansysis Stress(N/ mm ²)	Analytical Stress(N/mm ²)	% Accuracy	% Error
1	14.301 8	14.3799	99.45%	.543%

Deflection

Sr. No.	Ansysis Deflection(mm)	Analytical De flection(mm)
1	.0014	.0014

As it can be seen from comparison that % error in stress is negligible and no error in deflection of gear tooth calculated using element type as beam. So these FEA models are good enough for stress analysis of spur gear teeth in static condition.

12. DYNAMIC ANALYSIS

12.1 Introduction to Dynamic Analysis

Dynamic load is defined as the load which varies in magnitude, direction or point of application with respect to time. Dynamic load in the mechanical components causes the generation of fluctuating stresses. In case of spur gear the load acting on gear tooth is constant in magnitude as well as in direction but varies in point of application of load. Thus spur gear teeth are subjected to fluctuating stresses and lead to fatigue failure. Gears are the important power transmitting elements in mechanical system and hence sudden failure of gear tooth may lead to danger. Therefore it is necessary to perform the dynamic analysis of spur gear.

12.2 LOAD DISTRIBUTION

Load distribution on gear tooth In order to conduct a dynamic stress analysis the loads have to be evaluated. The load on the tooth of the finite element model, which produces the largest bending stress, is the full load acting at the highest point of single tooth contact (HPSTC). The magnitude of load at any point of contact on profile of gear tooth as the load moves from root to tip of tooth depends on the contact ratio. Fig. shows the contact path, the contact ratio CR is defined as the ratio of length of path of contact AB to base circle pitch P_b [4, 5].

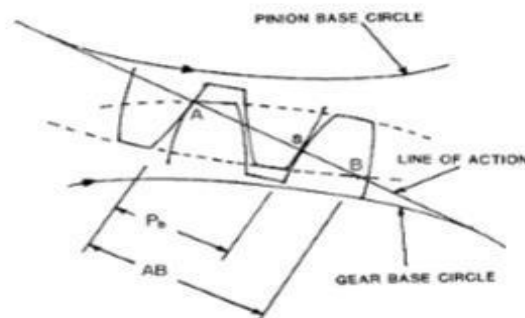


Fig.6 Path of Contact

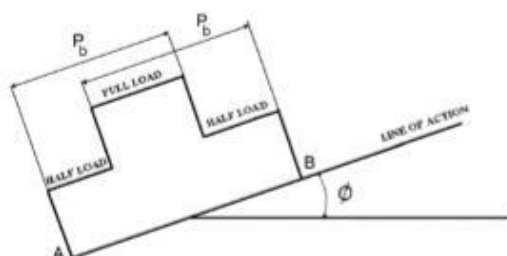


Fig.7 Load distribution

Contact ratio calculated as 1.64 which is approximately equal to 2. Now while applying load we have considered 39 nodes on left edge of tooth numbered from 6 to 44. Bottom nodes left to consider the clearance and the load variation is assumed symmetric variation from bottom to top.

14. ANSYS PROCEDURE

Draw gear tooth in catia→ Import IGS file in Ansys→ Choose element type as solid-plane 182(Quad 4 n ode 182) with element behaviour of plane stress with thickness input as Lewis has assumed the load acting on tooth as uniformly distributed load and this will generate plain stress condition at each cross section in off teeth of shown orientation→ Give its real constant (thickness) and material properties (density, young's modulus and poisson ratio) →Create area→ Mesh (Fine meshing).. 2. Loads→ new analysis→ transient analyse→ reduced methods→ Master degree of freedom→ user defined method→ select all nodes except nodes which are on zero displacement base line→ apply boundary conditions on last line.

Load Step opts→ Time/frequency→ Time/Time-step option→ Put Initial conditions and load type- stepped→ Write this LS file as 1→ start applying forces at various nodes for a particular time→ follow same procedure as mentioned a above→ write this file as 2→ Similarly y apply forces at various nodes for different time and write there file as 3, 4, 5.etc. → Solve- From LS files- LS .

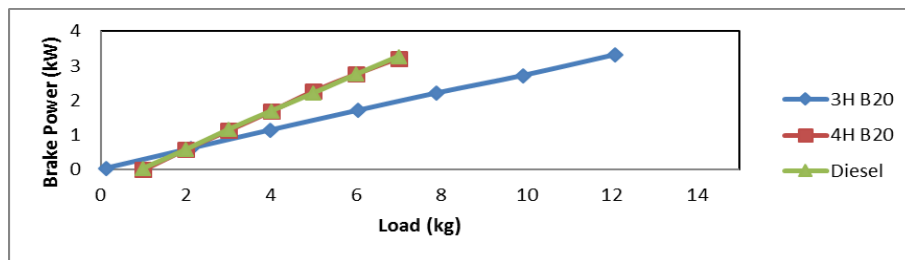


Fig: 2 Brake Power Vs Load

3.2 Brake Mean Effective Pressure

BMEP (Brake Mean Effective Pressure) is defined as the average pressure which is imposed on the pistons uniformly from the top to the bottom of each power stroke, would produce the measured power output. The brake mean effective pressure tends to increase as per the load applied. The BMEP of the 3H, 4H and Diesel are almost same shown in fig 3.

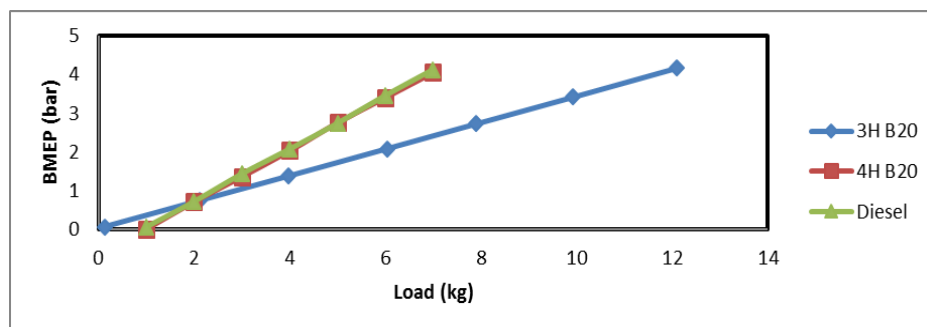


Fig: 3 Brake Mean Effective Pressure Vs Load

The variations are negligible. BMEP can further extended to the relation with torque in given displacement values.

3.3 Brake Thermal Efficiency

The Brake Thermal Efficiency is defined as brake power of a heat engine as a function of the thermal input from the fuel. It is used to evaluate how well an engine converts the heat from a fuel to mechanical energy. From the graph shows that initially all the test samples were same, on increasing the load the rate of increase of efficiency varied for each.

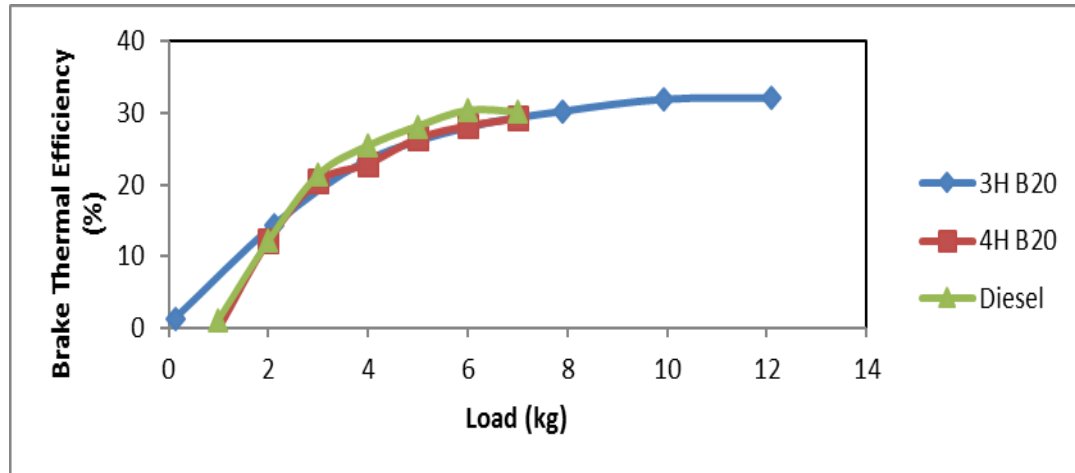


Fig: 4 Brake Thermal Efficiency Vs Load

Up to the load of 4 Kg both 4H and diesel performs same efficiency show in fig 4, the 4H nozzle tends to fall the rate after 4Kg and joins back with diesel after 12 Kg. overall B20 3H nozzle performs well at the load of 12 Kg it shows 32% while Diesel and 4H shows 30.15% and 29.35%. So, it is clearly understand that 3H nozzle B20 can convert heat energy produced to useful mechanical energy.

3.4 Specific Fuel Consumption

Thrust Specific Fuel Consumption or Specific Fuel Consumption is defined as the ratio of mass of fuel consumption per hour to the brake power in that particular hour. From the beginning of combustion of 3H, 4H and diesel the diesel consumes more fuel and gradually decreased to a stabilized value. On high load the SFC is more for the 4H nozzle shown in fig 5.

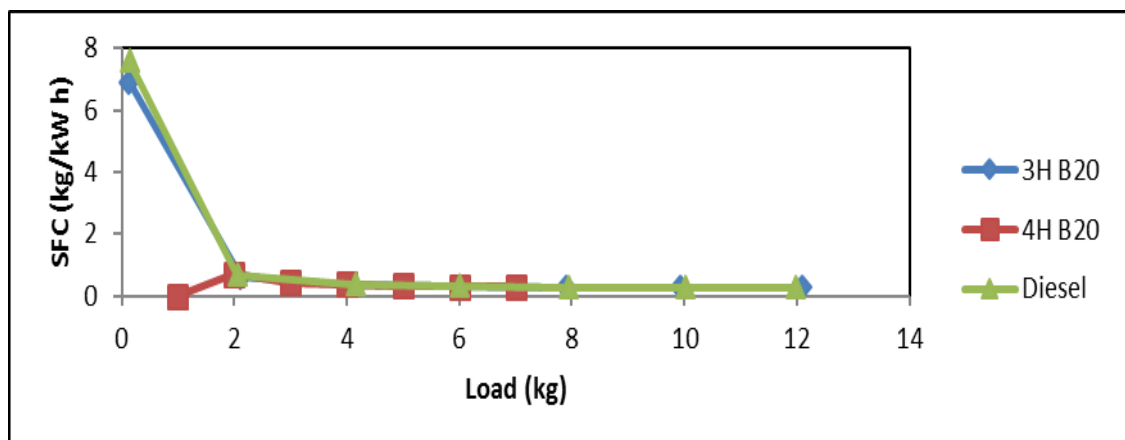


Fig: 5 Specific Fuel Consumption Vs Load

The performance of 3H B20 and Diesel are same at the higher loads. So 3H nozzle consumes less fuel per brake power produced in the engine while compared to 4H.

3.5 Mechanical Efficiency

Mechanical Efficiency is the parameter to find the effectiveness of an engine in transforming the input energy to the output energy. Mechanical energy also shows how much of the power developed by the expansion of the gases in the cylinder is actually delivered as useful power.

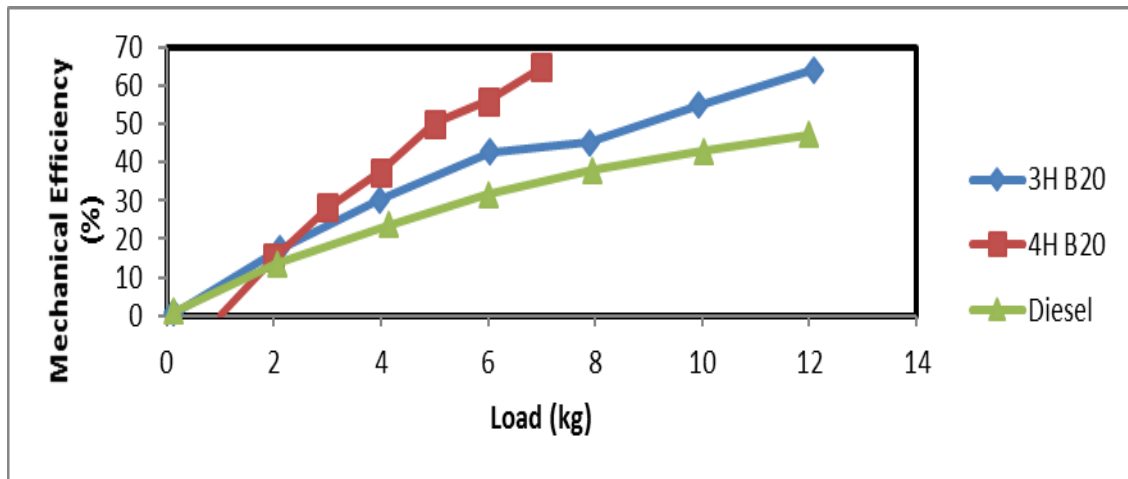


Fig. 6 Mechanical Efficiency Vs Load

Here the 3H nozzle shows in fig 6 better efficiency in medium range of loads and its rate decreases, where the 4H nozzle outperforms the 3H nozzle in those range. While the diesel has comparatively lower efficiency compared to B20 blend. On a load of 12 Kg 4H nozzle has efficiency more than 65% and diesel has only 47%.

3.6 Carbon Monoxide (CO)

Carbon monoxide is a colourless, odourless and toxic gas which is slightly denser than air. It is difficult to identify with the naked eye. Exposure to 100 ppm or more can be dangerous to human health. When carbon monoxide is inhaled into the body it gets dissolved into haemoglobin easily and causes severe damage to cells.

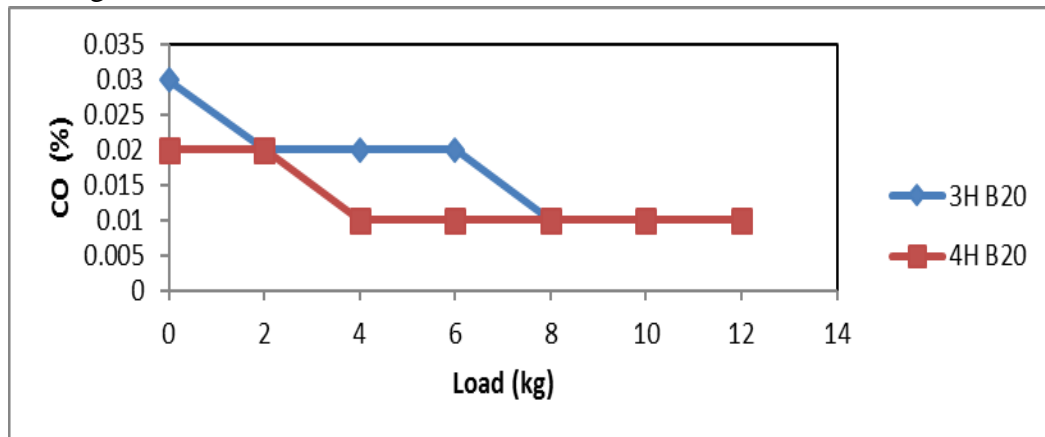


Fig. 7 Carbon Monoxide Vs Load

In the idling of the engine 4H nozzle produced least CO gas shown in fig 7. After a load of 2 Kg onwards the CO gas stabilized to 0.01%, while the 3H nozzle stabilized on 8 Kg. Carbon monoxide is produced when incomplete combustion happens in the combustion chamber and as per increase of load the carbon monoxide tend to reduce.

3.7 Hydro Carbon (HC)

Hydro carbon present in the exhaust gas is the waste unburned fuel from the combustion chamber. It shows the amount of effective combustion happened inside the engine. For greater efficiency the HC presence must be nearly zero, but it's very difficult to achieve.

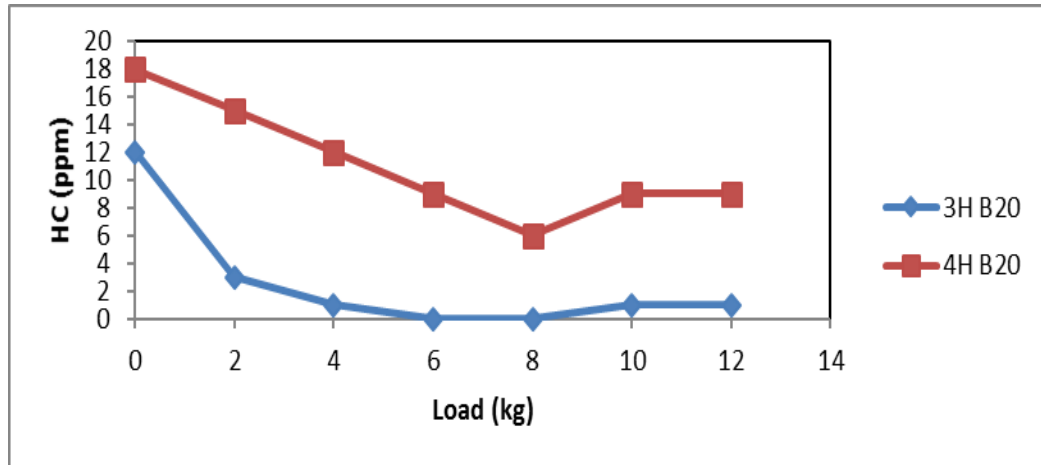


Fig: 8 Hydro Carbon Vs Load

While analyzing the observation data we can see that the HC composition reduces as per the load shown in fig 8. In the ideal condition 4H nozzle produced more HC while it gradually reduced and the 3H nozzle have least value throughout all the load values shown in fig 8. So we can say that combustion is more in 3H nozzle.

3.8 Carbon Dioxide (CO₂)

Carbon dioxide is the colourless and odourless gas which is vital to life on earth. Carbon dioxide is produced by all aerobic organisms when they metabolize carbohydrates and lipids to produce energy by respiration.

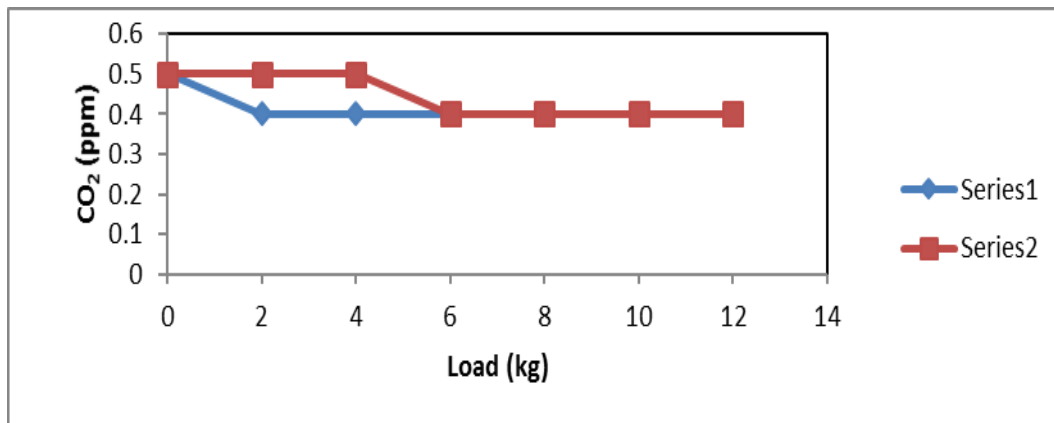


Fig: 9 Carbon dioxide Vs Load

Nowadays carbon dioxide emission is more due to combustion of fossil fuel. In the idling stage of engine both 3H and 4H produced same amount of CO₂ shown in fig 9. On increasing the load 3H started to stabilize first followed by the 4H nozzle. The CO₂ increases as per the increase of combustion.

3.9. Nitrogen Oxides (NO_x)

Nitrogen oxides are the group of gases that are composed of nitrogen and oxygen. The most common forms are nitric oxide and nitrogen dioxide. Among it nitrous gas is the greenhouse gas and it contributes to climate changes.

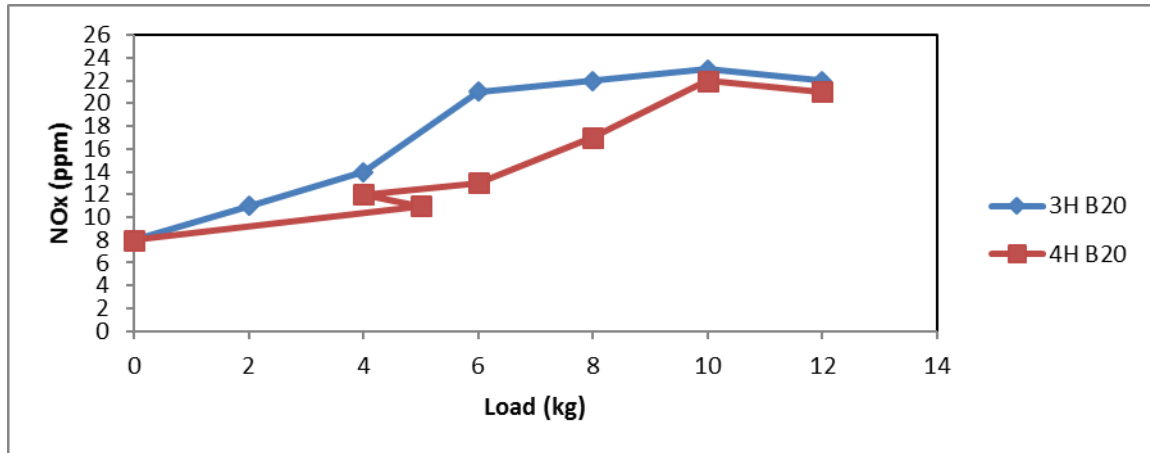


Fig: 10 Nitrogen oxides Vs Load

The NO_x on proper combustion the by-products of the reaction are water and carbon dioxide. Since the atmosphere consists of nearly 80% of nitrogen NO_x gases are produced. On idealizing of engine both 3H and 4H has same NO_x composition shown in fig 10. As per load the NO_x gases increased, 3H nozzle produced more NO_x gases while compared to 4H.

4. CONCLUSION

In the biodiesel characterization, sulphuric acid is used as catalyst for esterification process followed by usage of sodium hydroxide as catalyst for transesterification of Pongamia and Jatropha oil. 3H nozzle shows 32% of Brake thermal efficiency at a load of 12 kg, the BTHE increases with the load. The SFC of 3H B20 and Diesel are same at the higher loads.

So 3H nozzle consumes less fuel per brake power produced in the engine while compared to 4H. As number of holes decreases SFC decreases. The Mechanical efficiency of the 3H nozzle and 4H Nozzle has varying results. 3H nozzle is better for medium loads, while 4H nozzle has higher load application benefits and Diesel have the least efficiency. CO gas tends to reduce to lower values as the load increases. On ideal state 4H releases lower CO values ie, better combustion. HC of 3H nozzle have least value throughout all the load values and it is found that combustion is more in 3H nozzle. When no load is applied in the engine, both 3H and 4H have same NO_x composition. As per load, the NO_x gases increased, 3H nozzle produced more NO_x gases while compared to 4H nozzle. Hence, it is concluded that 3H nozzle is recommended to all Biodiesel usage and testing in the engine, except in the terms of emission of NO_x gases.

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