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Parameter Identification of Wave Period by Superposition Method

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abstract

This paper presents a study of the parameter identification of period by the superposition of wave in the actuator model using the finite element method. In this research, the period of superposed wave is identified so as to minimize the performance function. As the state equation, the Navier-Stokes equations are used for the analysis of flow in the actuator model. The quasi-linear approximation of advection velocity is given by the Adams-Bashforth formula which has second order accuracy. To solve the state equation, the implicit time integration is applied to the temporal discretization. The bubble function element using the stabilized bubble function method is utilized for the spatial discretization.

Key words : Parameter Identification, Bubble Function, Finite Element Method, Sakawa-Shindo Method, Navier-Stokes Equations

1 INTRODUCTION

In this research, a numerical analysis is performed using an actuator model. The various actuators are used as an equipment that convert electric energy, thermal energy, chemical energy and pressure into a mechanical movement in the control system. For example, the actuator is important equipment which performs vibratory control of various motors, an oil pressure pump, air pressure pump, oil pressure cylinder and so on. In this research, attention is paid to the mechanism of actuator model that does the vibration control([1]). If this structure is used, and the actuator gives vibration to a flow, the wave of the periodic velocity is generated. Therefore, what wave can have significant influence on the flow can be examined. The parameter identification ([2]) is to obtain the optimal control parameter so as to minimize or maximize the objective variable. In this paper, the purpose is to identify the period of superposed wave so as to maximize the objective velocity at the objective point in the actuator model using the finite element method. To obtain the control variable, it is important to perform the stable computation and to know the phenomenon of forward analysis. The problem is observing the velocity influenced by the superposed wave at the objective point. The bubble function interpolation of the finite element method ([3]-[5]) is capable of eliminating the barycenter point by using the static condensation. The discretized form derived from the bubble function element is equivalent to those from the SUPG (Streamline-Upwind / Petrov Galerkin).

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Therefore, the stabilized parameter which is derived from the bubble function element is obtained and determines the magnitude of the streamline stabilized term. Generally, the stabilized parameter is not equal to the optimal parameter. Thus, the bubble function which gives the optimal viscosity is defined by using the stabilized control parameter. The fractional step projection scheme is a discretized method for the incompressible Navier-Stokes equations. It is possible to employ the same interpolation that a linear or bilinear homogeneous function is used to velocity and pressure fields. As for the numerical example, a flow analysis in the actuator model is carried out in this paper. The velocity can be simulated in case of the optimal control parameter. The superposed wave with a short period dose not necessarily enlarge the velocity at the objective point.

2 STATE EQUATIONS

The Navier-Stokes equation is used to calculate the fluid flow, which is:

$$\dot{u}_i + u_j u_{i,j} + p_{,i} - \nu(u_{i,j} + u_{j,i})_{,j} = 0, \tag{1}$$

$$u_{i,i} = 0, \tag{2}$$

where u_i, p and ν are the water velocity, the kinematic pressure and the inverse of Reynolds number, respectively. The boundary and initial conditions of the Navier Stokes equations is given as follows:

$$u_i = \hat{u}_i \qquad \text{on} \qquad \Gamma_1, \tag{3}$$

$$t_i = \{-p\delta_{ij} + \nu(u_{i,j} + u_{j,i})\} \cdot n_j = \hat{t}_i \qquad \text{on} \qquad \Gamma_2, \tag{4}$$

$$u_{i(0)} = \hat{u}_{i(0)} \qquad \text{on} \qquad \Omega, \tag{5}$$

where t_i is the flux on the boundary, in which δ_{ij} denotes the identity tensor and n_j is the outward normal on the boundary. The boundary Γ is divided into subset Γ_1 and Γ_2, which mean the Dirichlet and the Neumann boundaries, respectively. Ω means the computational domain.

3 SPATIAL DISCRETIZATION

3.1 Bubble function

As for the spatial discretization, the bubble function element for the velocity and the linear element for the kinematic pressure are applied. The bubble function interpolation is expressed as follows:

$$u_i = \Phi_1 u_{i1} + \Phi_2 u_{i2} + \Phi_3 u_{i3} + \Phi_4 \tilde{u}_{i4}, \qquad \tilde{u}_{i4} = u_{i4} - \frac{1}{3}(u_{i1} + u_{i2} + u_{i3}),$$

$$\Phi_1 = \mathrm{L}_1, \quad \Phi_2 = \mathrm{L}_2, \quad \Phi_3 = \mathrm{L}_3, \quad \Phi_4 = 27\mathrm{L}_1\mathrm{L}_2\mathrm{L}_3,$$

and the linear interpolation is:

$$p = \Psi_1 p_1 + \Psi_2 p_2 + \Psi_3 p_3,$$

$$\Psi_1 = \mathrm{L}_1, \quad \Psi_2 = \mathrm{L}_2, \quad \Psi_3 = \mathrm{L}_3,$$

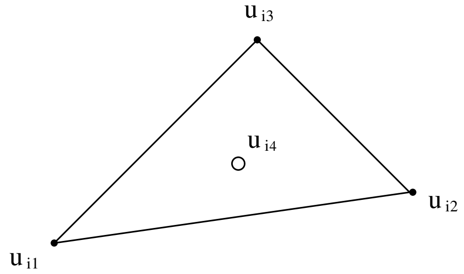


Fig. 1 Bubble function element

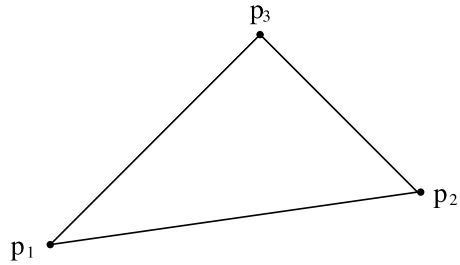


Fig. 2 Linear element

where $L_\lambda(\lambda = 1,2,3)$ is the area coordinate, $\Phi_\alpha(\alpha = 1,2,3,4)$ is the bubble function element for the velocity, respectively. The bubble function element is shown in Fig. 1. The bubble function of C^0 continuous can be considered, and $\Psi_\lambda(\lambda = 1,2,3)$ is the linear element for the kinematic pressure. The linear element is shown in Fig. 2.

The criteria for the steady problem is used, in which the discretized form of the bubble function element is equivalent to those of SUPG (Streamline-Upwind / Petrov-Galerkin). Therefore, in the bubble function element for the steady problem, the stabilized parameter τ_{eB} which determines the magnitude of the streamline stabilized term. The stabilized parameter τ_{eB} is expressed as follows:

$$\tau_{eB} = \frac{< \phi_e, 1 >^2_{\Omega_e}}{\nu \|\phi_{e,j}\|^2_{\Omega_e} A_e}, \tag{6}$$

where Ω_e is element domain and

$$< u, v >_{\Omega_e} = \int_{\Omega_e} uv d\Omega, \quad \|u\|^2_{\Omega_e} = \int_{\Omega_e} uu d\Omega, \quad A_e = \int_{\Omega_e} d\Omega,$$

the integral of bubble function is expressed as follows:

$$< \phi_e, 1 >_{\Omega_e} = \frac{A_e}{6}, \quad \|\phi_{e,j}\|^2_{\Omega_e} = 2A_e g, \quad g = \sum_{i=1}^{2} |\Psi_{\alpha,i}|^2.$$

From the criteria for the stabilized parameter in the SUPG, an optimal parameter τ_{eS} can be given as follows:

$$\tau_{eS} = \left[\left(\frac{2|u_i|}{h_e} \right)^2 + \left(\frac{4\nu}{h_e^2} \right)^2 \right]^{-\frac{1}{2}}, \tag{7}$$

where h_e is an element size.
Generally, the stabilized parameter in equation (6) is not equal to the optimal parameter in equation (7). Thus, the bubble function which gives the optimal viscosity satisfies the following equation expressed by the stabilized operator control parameter:

$$\frac{< \phi_e, 1 >^2_{\Omega_e}}{(\nu + \nu') \|\phi_{e,j}\|^2_{\Omega_e} A_e} = \tau_{eS}. \tag{8}$$

It is shown that equation (8) adds the stabilized operator control term only of the barycenter point to the equation of motion:

$$\sum_{e=1}^{Ne} \nu' \|\phi_{e,j}\|^2_{\Omega_e} b_e, \tag{9}$$

where N_e and b_e are the total number of elements and barycenter point, respectively.

3.2 Finite Element Equation

To solve the state equations, the finite element method that bases on the stabilized bubble function element is employed for the spatial discretization. Those finite element equations can be expressed as follows:

$$M\dot{u}_i + u_j^* S_j u_i - Bp + \nu H_{jj} u_i + \nu H_{ji} u_j = 0, \tag{10}$$

$$B^T u_i = 0, \tag{11}$$

where

$$u_i^* = \frac{1}{2}(3u_i^n - u_i^{n-1}), \tag{12}$$

u_i^* which is the linear approximation of particle velocity is given by the Adams-Bashforth formula which has second order accuracy.

4 PARAMETER IDENTIFICATION

4.1 Performance Function

The parameter identification is formulated on the calculation of variation in this research. The formulation is to find the optimal control parameter so as to minimize the performance function, which J is expressed as follows:

$$J = \frac{1}{2}\int_{t_0}^{t_f} \{(u - u_{req})^T Q(u - u_{req})\}dt + \frac{1}{2}\int_{t_0}^{t_f} \bar{u}^T R\bar{u}dt, \tag{13}$$

$$\bar{u} = \sum_{k=1}^{n} asin\omega_k t, \tag{14}$$

where u_{req} , $\bar{u}$ and a are the required velocity, the control velocity and the amplitude, respectively, and Q and R are the weighting diagonal matrix. The superscript T denotes transpose, respectively.

4.2 Minimization Technique

4.2.1 Sakawa-Shindo Method

The Sakawa-Shindo method is applied to the minimization technique. In this method, a modified performance function which is added to a penalty term to the performance function is introduced. The modified performance function $K^{(\ell)}$ is expressed as follows:

$$K^{(\ell)} = J^{(\ell)} + \frac{1}{2}\{\omega_k^{(\ell+1)} - \omega_k^{(\ell)}\}^T W^{(\ell)}\{\omega_k^{(\ell+1)} - \omega_k^{(\ell)}\}, \tag{15}$$

$$\left[\frac{\partial K^{(\ell)}}{\partial \omega_k}\right] = \int_{t_0}^{t_f} \left\{\left[\frac{\partial u}{\partial \omega_k}\right]^T Q(u - u_{req})\right\}^{(\ell)} dt - W^{(\ell)}(\omega_k^{(\ell+1)} - \omega_k^{(\ell)}). \tag{16}$$

where ℓ is the iteration number for the minimization, ω_k and $W^{(\ell)}$ are the control value and the weighting parameter. Applying the stationary condition $\left[\frac{\partial K^{(\ell)}}{\partial \omega_k}\right] = 0$, the following equation can be obtained. The control parameter is renewed by the following equation:

$$W^{(\ell)}\omega_k^{(\ell+1)} = W^{(\ell)}\omega_k^{(\ell)} + \int_{t_0}^{t_f} \left\{ \left[\frac{\partial u}{\partial \omega_k}\right]^T Q(u - u_{req}) \right\}^{(\ell)} dt. \tag{17}$$

4.2.2 Sensitivity Equation

The sensitivity matrix is denoted by $\left[\frac{\partial u}{\partial \omega_k}\right]$, which is obtained from equations (1) and (2) differentiating with respect to ω_k as follows:

$$\frac{\partial}{\partial \omega_k}\{\dot{u}_i + u_j u_{i,j} + p_{,i} - \nu(u_{i,j} + u_{j,i})_{,j}\} = 0, \tag{18}$$

$$\frac{\partial}{\partial \omega_k}\{u_{i,i}\} = 0, \tag{19}$$

In this research, the sensitivity equations derived from the finite element equations of governing equations are calculated. The boundary conditions and initial conditions should be defined to solve the sensitivity equations (18) and (19). Therefore, those are differentiated with respect to ω_k. The boundary conditions for the sensitivity equations are expressed as follows:

$$\frac{\partial u_i}{\partial \omega_k} = \frac{\partial \hat{u}_i}{\partial \omega_k} \qquad \text{on} \qquad \Gamma_1, \tag{20}$$

$$\frac{\partial u_n}{\partial \omega_k} = \frac{\partial u_i}{\partial \omega_k} n_i = \frac{\partial \hat{u}_i}{\partial \omega_k} \qquad \text{on} \qquad \Gamma_2. \tag{21}$$

Similarly, the initial conditions for the sensitivity equations are expressed as follows:

$$\frac{\partial u_{i(0)}}{\partial \omega_k} = \frac{\partial \hat{u}_{i(0)}}{\partial \omega_k} \qquad \text{in} \qquad \Omega. \tag{22}$$

Usually, a lot of computational storage is required to treat the inverse problem. However, it is not necessary for the calculation of sensitivity equations to store the much computational memory because the boundary and initial conditions can be defined. Therefore, the parameter identification technique using the sensitivity matrix can be applied to the problems that long calculation time is required.

4.2.3 Algorithm of Sakawa-Shindo Method

The algorithm of the Sakawa-Shindo method is shown as follows:

♯1. Select initial identified vector $\omega_k^{(\ell)}$, set weighting parameter $W^{(\ell)} = 1.3$ and set $\ell = 0$.
♯2. Solve ${u_i}^{(\ell)}$ and $p^{(\ell)}$ by using the finite element equations.
♯3. Compute the initial performance function $J^{(\ell)}$.
♯4. Solve the sensitivity matrix $\left[\frac{\partial K^{(\ell)}}{\partial \omega_k}\right] = 0$ by using the sensitivity equations.
♯5. Compute the identified vector $\omega_k^{(\ell+1)}$.
♯6. Check the convergence: Compute $\epsilon = \|\omega_k^{(\ell+1)} - \omega_k^{(\ell)}\|$,
and if $\epsilon < \varepsilon$ then stop, else go to step ♯7.
♯7. Solve ${u_i}^{(\ell+1)}$ and $p^{(\ell+1)}$ by using the finite element equations.

♯8. Compute the performance function $J^{(\ell+1)}$.
♯9. Renew a weighting parameter $W^{(\ell)}$:
if $J^{(\ell+1)} \leq J^{(\ell)}$, then set $W^{(\ell+1)} = 0.85W^{(\ell)}$ and go to step ♯4,
else $W^{(\ell+1)} = 2.0W^{(\ell)}$ and go to step ♯5.

5 TEMPORAL DISCRETIZATION

In this section,the fractional step projection finite element scheme is employed for the Navier-Stokes equations. To solve the state equations, the implicit time integration is used for the temporal discretization. This method is capable of taking the long time increment and superior in stability. Therefore, many time cycles can be taken in the computation.

The Navier-Stokes equations can be discretized as:

$$M\frac{\tilde{u}_i^{n+1} - u_i^n}{\Delta t} + u_j^* S_i \tilde{u}_i^{n+1} + \nu H_{jj}\tilde{u}_i^{n+1} + \nu H_{ji}\tilde{u}_j^{n+1} = 0, \tag{23}$$

$$M\frac{u_i^{n+1} - \tilde{u}_i^{n+1}}{\Delta t} - B^T p^{n+1} = 0, \tag{24}$$

$$B^T u_i^{n+1} = 0. \tag{25}$$

Equation (23) shows that the intermediate velocity $\tilde{u}^{n+1}$ is computed as solution of the discretized momentum equation without the pressure term. On the other hand, equations (24) and (25) show that the intermediate velocity $\tilde{u}$ is decomposed into velocity u_i^{n+1} and the pressure p^{n+1}.

6 NUMERICAL EXAMPLE

In this section, an analysis of flow in the actuator model is carried out using the Navier-Stokes equations. The bubble function using the stabilized function method is employed for the comparative purpose. The computational model is shown in Fig. 3. The finite element mesh is illustrated in Fig. 4. The total number of nodes and elements are 2351 and 4400, respectively. The total number of control points and objective point are 7 and 1, respectively. On the left side, u is given as the Poiseuille flow as the boundary condition.

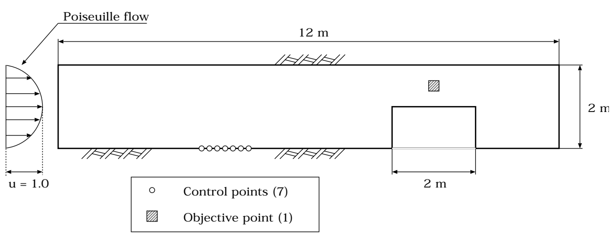
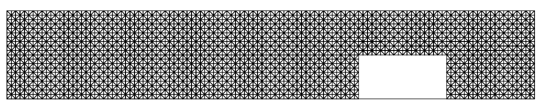


Fig. 3 Computational model



nodes : 2351 elements : 4400

Fig.4 Finite element mesh

In this numerical example, five waves are imposed changing the velocity given at the control points. They are referred to as the fixed wave and the superposed waves. These waves are given as the entrance velocity at the control points, respectively. The fixed wave which is employed as the wave of 1.8(s) period is shown in Fig. 5 as in case-[1]. In this case, the time increment Δt is set to 0.03(s), the Reynolds number is set to 400 ($\nu = 0.0025$), the amplitude is set to 0.1, the weighting diagonal matrix Q and R are set to $1.0I$ and $0.0I$ and the required velocity u_{req} is set to 2.3(m/s). As results of this research, the optimal control parameter is obtained. The component wave of superposed waves is shown in Fig. 9. This wave is given from the control points. In case of Fig. 9, the velocity becomes the largest at the objective point. The velocity is obtained at the objective point in Fig. 11. Fig. 10 shows the variation of periods of wave-[1], [2], [3] and [4], respectively. Fig. 12 shows the variation of performance function. The performance function is almost decreased and converged in Fig. 12. The velocity vector is shown in Fig. 13. The velocity can be simulated in the optimal case.
As in case-[2], the initial waves are changed. These waves are longer periods than that in case-[1] like Fig. 6. The fixed wave is employed the same wave of case-[1]. In case of Fig. 16, the velocity is obtained at the objective point in Fig. 18. Fig. 17 shows the variation of periods of wave-[1], [2], [3], and [4], respectively in case-[2]. Then, the performance function and the velocity vector are Fig. 19 and Fig. 20, respectively.

Table 1 Initial value of period

	Case-[1]	Case-[2]
Wave-[1]	9.000[s]	18.00[s]
Wave-[2]	8.100[s]	12.00[s]
Wave-[3]	7.200[s]	9.000[s]
Wave-[4]	6.300[s]	7.200[s]

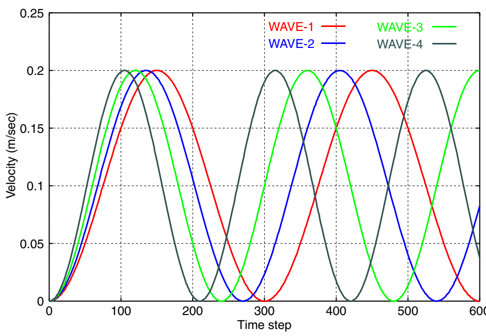


Fig. 5 Initial wave in Case-[1]

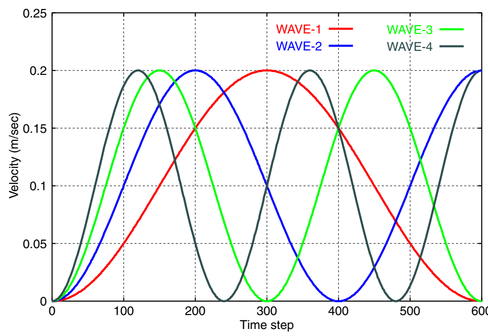


Fig. 6 Initial wave in Case-[2]

6.1 Case-[1]

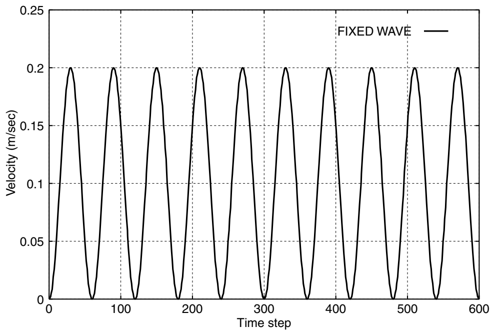


Fig. 7　Fixed wave

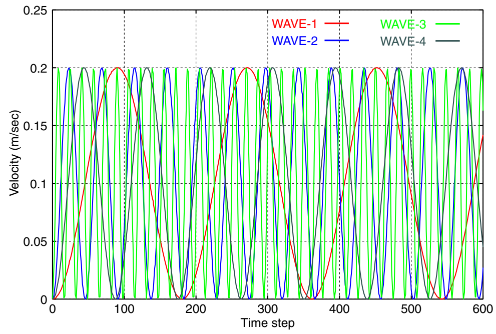


Fig. 8　Superposition of wave at control points

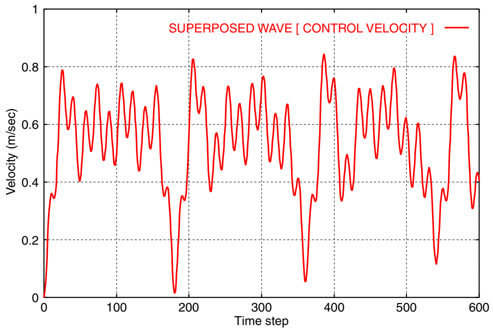


Fig. 9　Control velocity at control points

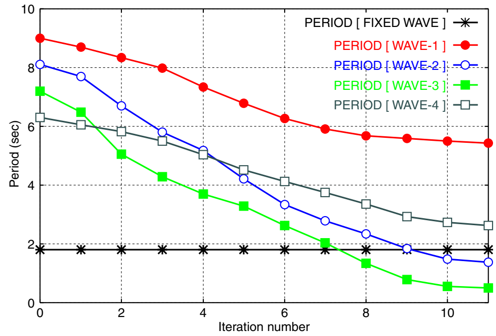


Fig. 10　Period of wave-[1][2][3][4]

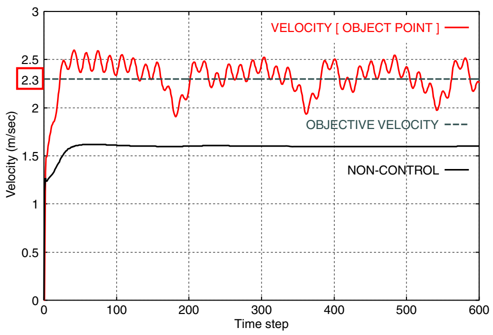


Fig. 11　Velocity at objective point

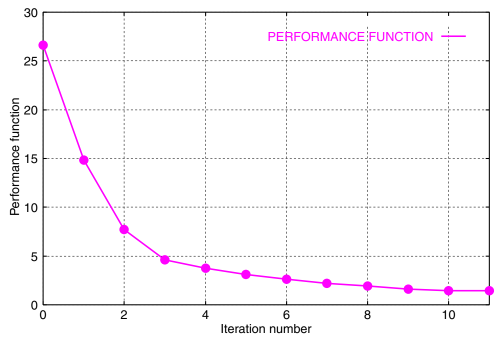
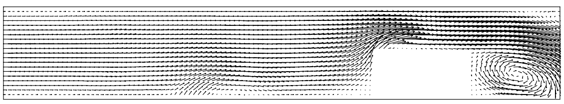
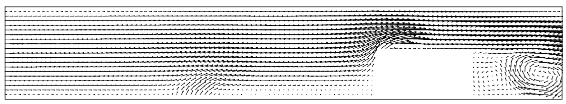


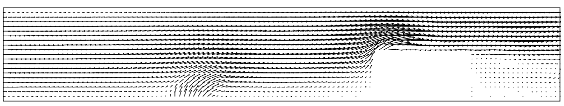
Fig. 12　Performance function



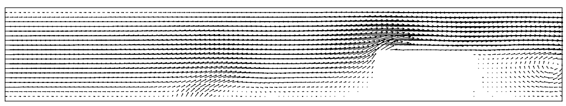
Time step = 50



Time step = 100



Time step = 300



Time step = 600

Fig. 13　Velocity vector

6.2 Case-[2]

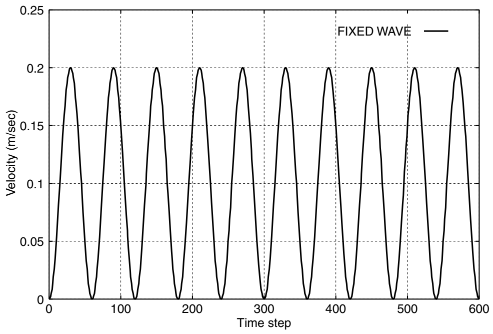


Fig. 14 Fixed wave

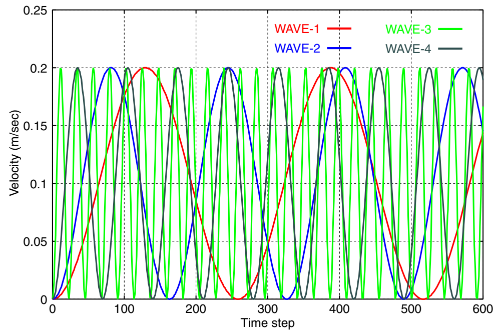


Fig. 15 Superposition of wave at control points

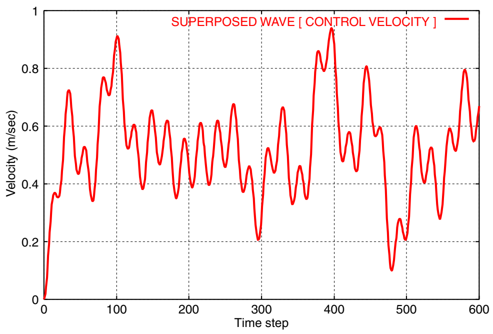


Fig. 16 Control velocity at control points

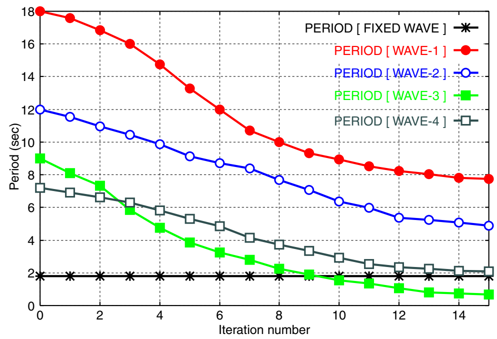


Fig. 17 Period of wave-[1][2][3][4]

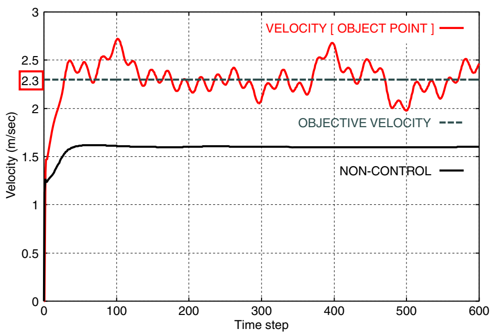


Fig. 18 Velocity at objective point

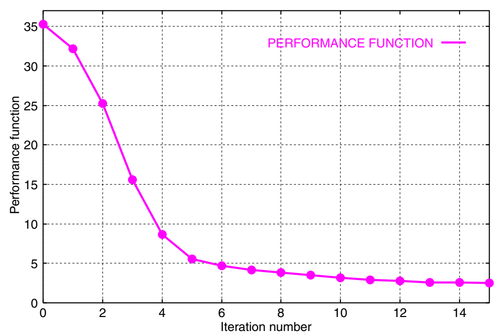
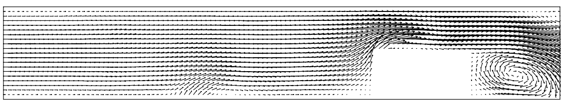
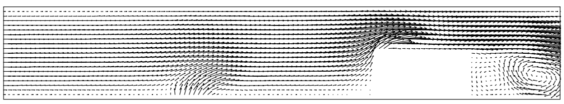


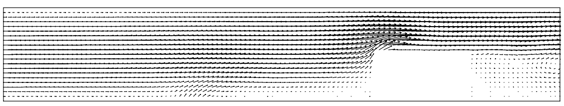
Fig. 19 Performance function



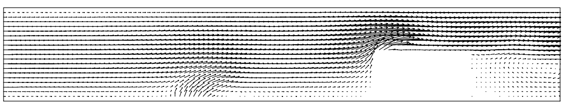
Time step = 50



Time step = 100



Time step = 300



Time step = 600

Fig. 20 Velocity vector

Table 2 Periods in Case-[1]

	INITIAL WAVE	IDENTIFIED WAVE
Wave-[1]	9.000[s]	5.423[s]
Wave-[2]	8.100[s]	1.372[s]
Wave-[3]	7.200[s]	0.491[s]
Wave-[4]	6.300[s]	2.630[s]

Table 3 Periods in Case-[2]

	INITIAL WAVE	IDENTIFIED WAVE
Wave-[1]	18.00[s]	7.750[s]
Wave-[2]	12.00[s]	4.897[s]
Wave-[3]	9.000[s]	0.683[s]
Wave-[4]	7.200[s]	2.099[s]

7 CONCLUSION

In this paper, the parameter identification of period by the superposition of wave in the channel of actuator shape. The bubble function using the stabilized bubble function method was applied to the Navier-Stokes equations. As the numerical example, a flow in the channel of actuator shape was employed. In this research, the optimal periods of the superposed waves are identified and the velocity can be simulated. It is shown that the superposed wave with a short period dose not necessarily enlarge the velocity at the objective point. By the parameter identification using sensitivity equation, the optimal wave will surely be able to be found. It is important to superpose a lot of waves and what kind of wave is fixed. To determine the appropriate position of the objective point is very important.

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