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Orthogonal arrays of size 108 with six-level columns

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Abstract. A special structure of the two level orthogonal array of order 12 is given. By using this structure and the method of constructing mixed-level orthogonal arrays presented by Zhang, Lu and Pang (1999), a lot of mixed-level orthogonal arrays of run size 108 are obtained. Especially, we obtain an orthogonal array of size 108 with 11 6-level columns.

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§1. Introduction

An $n \times m$ matrix A, having k_i columns with p_i levels, $i = 1, 2, \ldots, r$, $m = \sum_{i=1}^{r} k_i$, $p_i \neq p_j$ for $i \neq j$, is called an orthogonal array(OA) of strength d and size n if each $n \times d$ submatrix of A contains all possible $1 \times d$ row vectors with the same frequency. Unless stated otherwise, we use the notation $L_n(p_1^{k_1} \cdots p_r^{k_r})$ for an OA of strength 2. An orthogonal array is said to have mixed-levels if $r \geq 2$. Orthogonal arrays have been used extensively in statistical design of experiments, computer science and cryptography. Constructions of mixed-level OA's have been studied in the literature. But our knowledge about the existence and nonexistence of arrays is rather limited. Hedayat, Sloane and Stufken (1999) stated that orthogonal arrays $L_{72}(6^7)$ and $L_{288}(6^{11})$ exist. How many 6-level columns can an orthogonal array of size 108 have? This paper presents a special structure of orthogonal array $L_{12}(2^{11})$. By using this structure and the method of constructing mixed-level orthogonal arrays presented by Zhang, Lu and Pang (1999), a lot of mixed-level orthogonal arrays of 108 are obtained. Especially, we obtain an orthogonal array of size 108 with 11 6-level columns.

§2. Construction method

The following definitions, notations and results are needed in the sequel.

Let $(r) = (0, \dots, r-1)^T$, 1_r be the $r \times 1$ vector of 1's and I_r the identity of order r. $P_r = \frac{1}{r} 1_r 1_r^T$ and $\tau_r = I_r - P_r$. Let $e_i(r) = (0 \cdots 0 \overset{i}{1} 0 \cdots 0)^T_{1\times r}$ be a base vector of R^r (r-dim vector space),

$$N_r = e_1(r)e_2^T(r) + \cdots + e_{r-1}(r)e_r^T(r) + e_r(r)e_1^T(r)$$

and

$$K(p,q) = \sum_{i=1}^{p}\sum_{j=1}^{q} e_i(p)e_j^T(q) \otimes e_j(q)e_i^T(p),$$

where $\otimes$ is the usual Kronecker product in the theory of matrices. Sometimes, it is necessary and easy to use the following properties of those permutation matrices N_r and $K(p,q)$ to obtain the orthogonal arrays needed:

$$N_r \cdot (r) = 1_r + (r), \bmod r$$

$$K(p,q)(1_q \otimes (p)) = (p) \otimes 1_q, K(p,q)((q) \otimes (p)) = (p) \otimes (q).$$

and

$$K(p,q)(P_q \otimes \tau_p)K^T(p,q) = \tau_p \otimes P_q, K(p,q)(\tau_q \otimes \tau_p)K^T(p,q) = \tau_p \otimes \tau_q.$$

Definition 1. Let A be an OA of strength 1, i.e.,

$$A = (a_1, \dots, a_m) = (S_1(1_{r_1} \otimes (p_1)), \dots, S_m(1_{r_m} \otimes (p_m))),$$

where $r_j p_j = n$ and S_i is a permutation matrix, for $i = 1, \dots, m$. Then the projection matrix $S_j(P_{r_j} \otimes \tau_{p_j})S_j^T$ is called the matrix image (MI) of the jth column a_j of A, denoted by $m(a_j)$. The MI of a subarray of A is defined as the sum of the MI's of all its columns. In particular, we denote the MI of A by $m(A)$.

From the definition of matrix image, we have

$$m(1_r) = P_r, m((r)) = \tau_r.$$

Definition 2. Suppose that L_n is an OA with entries from a finite additive group G, and that a and b are two columns of L_n. i.e., $a = L_n(p) = (a_1, \dots, a_n)^T$ and $b = L_n(q) = (b_1, \dots, b_n)^T$. The generalized Hadamard product of a and b, denoted $a\Box b$, is defined as

$$a\Box b = (h(a_1,b_1), \dots, h(a_n,b_n))^T = (a_1 q + b_1, \dots, a_n q + b_n)^T = L_n(t),$$

where $t = pq$ and $h(a_i, b_i) = a_i q + b_i$ for $i = 1, \dots, n$.

Definitions 1 and 2 can be found in Zhang, et al (2001).

Lemma 1. *For any permutation matrix* S *and any array* L , $m(S(L \otimes 1_r)) = S(m(L) \otimes P_r)S^T$, *and* $m(S(1_r \otimes L)) = S(P_r \otimes m(L))S^T$.

Lemma 2. *Let* A *be an orthogonal array of strength 1, i.e.,*

$$A = (a_1, \dots, a_m) = (S_1(1_{r_1} \otimes (p_1)), \dots, S_m(1_{r_m} \otimes (p_m))),$$

where $r_j p_j = n$ *and* S_i *is a permutation matrix, for* $i = 1, \dots, m$. *The following statements are equivalent.*

(1) A *is an orthogonal array of strength* 2.

(2) $m(A)$ *is a projection matrix.*

(3) $m(a_i)m(a_j) = 0$ $(i \neq j)$.

(4) *The projection matrix* τ_n *can be decomposed as* $\tau_n = m(a_1) + \cdots + m(a_m) + \Delta$, *where* $rk(\Delta) = n - 1 - \sum_{j=1}^{m}(p_j - 1)$ *is the rank of the matrix* $\triangle$.

Lemma 3. *Suppose* $\tau_{n_1} = \sum_j S_j A S_j^T$ *and* $\tau_{n_2} = \sum_j T_j B T_j^T$ *are orthogonal decompositions of* τ_{n_1} *and* τ_{n_2}, *respectively, where the* S_j*'s and* T_j*'s are permutation matrices and* $n = n_1 n_2$. *Then* $\tau_{n_1 n_2}$ *can be orthogonally decomposed into*

$$\tau_{n_1 n_2} = \sum_j (S_j \otimes T_j)(A \otimes P_{n_2} + I_{n_1} \otimes B)(S_j^T \otimes T_j^T).$$

If there exists an orthogonal array H *such that* $m(H) \leq I_{n_1} \otimes B + A \otimes P_{n_2}$, *then*

$$L = ((S_1 \otimes T_1)H, (S_2 \otimes T_2)H, \dots)$$

is also an orthogonal array.

Lemmas 1,2 and 3 can be found in Zhang, Lu and Pang (1999).

§3. A special structure of orthogonal array $L_{12}(2^{11})$

Consider the following orthogonal array $L_{12}(2^{11})$

$$L_{12}(2^{11}) = [a_1, \dots, a_{11}] = \begin{pmatrix} 00000000111 \\ 00000111000 \\ 00111000000 \\ 01011011110 \\ 01101101101 \\ 01110110011 \\ 10110011101 \\ 10101110110 \\ 10011101011 \\ 11100001010 \\ 11010100100 \\ 11001010001 \end{pmatrix}.$$

Let $A_1 = (a_3, a_4, a_5)$, $A_2 = (a_6, a_7, a_8)$ and $A_3 = (a_9, a_{10}, a_{11})$. It is easy to see that $A_2 = TA_1$ and $A_3 = QA_1$, where $T = diag(T_1, T_2, T_3, T_4)$, $Q = diag(Q_1, Q_2, Q_3, Q_4)$,

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, T_2 = I_3, T_3 = T_4 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

and

$$Q_1 = Q_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, Q_3 = Q_4 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

From the definition of orthogonal array, there exists a permutation matrix S such that

$$S(a_3, a_4) = (1_6 \otimes (2), 1_3 \otimes ((2) \oplus (2)))$$

where $(2) \oplus (2) = (0, 1, 1, 0)^T$, where $\oplus$ is the usual Kronecker sum in the theory of matrix. Let

$$S = \begin{pmatrix} 100000000000 \\ 001000000000 \\ 000000000010 \\ 000010000000 \\ 010000000000 \\ 000001000000 \\ 000000001000 \\ 000000010000 \\ 000000000001 \\ 000000100000 \\ 000100000000 \\ 000000000100 \end{pmatrix}$$

and $L^1_{12}(2^{11}) = S(a_1, a_2, \ldots, a_{11}) = (b_1, b_2, \ldots, b_{11})$, then we have $b_1 = (001000111101)^T$, $b_2 = (001101001011)^T$, $b_3 = 1_6 \otimes (2)$, $b_4 = 1_3 \otimes ((2) \oplus (2))$ and $b_5 = (010100111010)^T$. Let $B_1 = (b_3, b_4, b_5)$, $B_2 = (b_6, b_7, b_8)$ and $B_3 = (b_9, b_{10}, b_{11})$, then $B_2 = STS^{-1}B_1$ and $B_3 = SQS^{-1}B_1$. It follows that $L^1_{12}(2^{11}) = (M_1B_1, M_2B_1, M_3B_1, b_1, b_2)$, where $M_1 = I_{12}$,$M_2 = STS^{-1}$,$M_3 =$

SQS^{-1}. If set

$$M_4 = \begin{pmatrix} 100000000000 \\ 000010000000 \\ 010000000000 \\ 001000000000 \\ 000000001000 \\ 000000100000 \\ 000100000000 \\ 000000010000 \\ 000001000000 \\ 000000000001 \\ 000000000010 \\ 000000000100 \end{pmatrix},$$

then

$$L^1_{12}(2^{11}) = (M_1B_1, M_2B_1, M_3B_1, M_4(b_3, b_4)).$$
$$L^1_{12}(2^9) = (b_5, M_2B_1, M_3B_1, M_4(b_3, b_4)).$$
$$L^1_{12}(2^8) = (M_2B_1, M_3B_1, M_4(b_3, b_4)).$$

The structure of orthogonal array $L_{12}(2^{11})$ is useful for construction of other arrays.

We first give some smaller orthogonal arrays and their matrix images which will be used.

Theorem 1. *There exist orthogonal arrays $L_{36}(6^3)$ and $L_{36}(6^2 3^4)$ such that $m(L_{36}(6^3)) \leq I_{12} \otimes \tau_3 + m(B_1) \otimes P_3$ and $m(L_{36}(6^2 3^4)) \leq I_{12} \otimes \tau_3 + (P_6 \otimes \tau_2 + P_3 \otimes \tau_2 \otimes \tau_2) \otimes P_3$.*

Proof. An orthogonal array $L_{36}(6^3 \cdot 2^8)$ can be obtained by Zhang, et al (2001) from the following formula:

$$\begin{aligned} L_{36}(6^3 \cdot 2^8) &= [1_3 \otimes L_{12}(2^8), (((3) \oplus (3)) \otimes 1_4) \square (1_{18} \otimes (2)), \\ &\quad (((3) \oplus (3)^{+2}) \otimes 1_4) \square (1_9 \otimes ((2) \oplus (2))), ((3) \oplus a) \square (1_3 \otimes b)], \end{aligned}$$

where the operation $\square$ is defined in Definition 2 and OA $L_{12}(2^8)$ satisfies the condition:

$$L^1_{12}(2^{11}) = [1_6 \otimes (2), 1_3 \otimes ((2) \oplus (2)), b_5, L_{12}(2^8)] = [B_1, L_{12}(2^8)].$$

Let $L^1_{36}(6^3 \cdot 2^8) = K(12, 3)L_{36}(6^3 \cdot 2^8)$, then $L^1_{36}(6^3 \cdot 2^8)$ contains a subarray $L_{12}(2^8) \otimes 1_3$. On the other hand, we have

$$m(L^1_{36}(6^3 \cdot 2^8)) \leq \tau_{36} = I_{12} \otimes \tau_3 + \tau_{12} \otimes P_3 = I_{12} \otimes \tau_3 + (m(B_1) + m(L_{12}(2^8))) \otimes P_3.$$

Deleting $L_{12}(2^8) \otimes 1_3$ from $L^1_{36}(6^3 \cdot 2^8)$, we get an OA $L_{36}(6^3)$ satisfying

$$m(L_{36}(6^3)) \leq I_{12} \otimes \tau_3 + m(B_1) \otimes P_3.$$

Similarly and from the OA $L_{36}(6^2 \cdot 3^4 \cdot 2^9)$ in Zhang et al(2001), one can obtain an OA $L_{36}(6^2 3^4)$ such that $m(L_{36}(6^2 3^4)) \le I_{12} \otimes \tau_3 + (P_6 \otimes \tau_2 + P_3 \otimes \tau_2 \otimes \tau_2) \otimes P_3$.

This completes the proof.

Corollary 1. *There are orthogonal arrays* $L_{36}(3^1 2^{19})$*,* $L_{36}(3^2 2^{12})$*,* $L_{36}(3^3 2^5)$*,* L_{36} $(6^1 3^1 2^{10})$*,* $L_{36}(6^1 3^2 2^3)$*,* $L_{36}(6^2 3^1 2^1)$*,* $L_{36}(6^1 3^8 2^2)$*,* $L_{36}(6^2 3^4 2^1)$ *and* $L_{36}(3^{12} 2^3)$ *whose matrix images are less than or equal to* $I_{12} \otimes \tau_3 + m(B_1) \otimes P_3$*. There exist orthogonal arrays* $L_{36}(3^1 2^{19})$*,* $L_{36}(3^2 2^{11})$*,* $L_{36}(3^3 2^4)$*,* $L_{36}(6^1 3^1 2^9)$*,* $L_{36}(6^1 3^2 2^2)$*,* $L_{36}(6^1 3^8 2^1)$ *and* $L_{36}(3^{12} 2^2)$ *whose matrix images are less than or equal to* $I_{12} \otimes \tau_3 + (P_6 \otimes \tau_2 + P_3 \otimes \tau_2 \otimes \tau_2) \otimes P_3$*.*

Proof. The proof is similar to that of Theorem 1. Using those orthogonal arrays in Zhang, et al (1999) and Zhang, et al (2001), we can obtain the orthogonal arrays we needed.

Now we illustrate construction of orthogonal array $L_{108}(6^{11} 3^4)$.

Theorem 2. *There exist orthogonal arrays* $L_{108}(6^{11} 3^4)$*,* $L_{108}(6^{10} 3^8 2^1)$*, etc.*

Proof. There exists S_j $(j = 1, 2, 3, 4)$ such that

$$\tau_9 = \sum_{j=1}^{4} S_j (\tau_3 \otimes P_3) S_j^T,$$

where $S_1 = I_9$, $S_2 = K(3,3)$, $S_3 = K(3,3) diag(I_3, N_3, N_3^2) K(3,3)^T$, and $S_4 = K(3,3)$ diag$(I_3,\ N_3^2, N_3)$ $K(3,3)^T$. And from the OA $L_{12}^1(2^{11})$, we have the following decomposition of the projection matrix τ_{12}:

$$\tau_{12} = \sum_{j=1}^{3} (M_j m(B_1) M_j^T) + M_4 (P_6 \otimes \tau_2 + P_3 \otimes \tau_2 \otimes \tau_2) M_4^T.$$

Hence projection matrix τ_{108} can be orthogonally decomposed as

$$\begin{aligned}
\tau_{108} &= I_{12} \otimes \tau_9 + \tau_{12} \otimes P_9 \\
&= \sum_{j=1}^{3} (M_j \otimes S_j)(I_{12} \otimes \tau_3 \otimes P_3 + m(B_1) \otimes P_9)(M_j \otimes S_j)^T \\
&\quad + (M_4 \otimes S_4)(I_{12} \otimes \tau_3 \otimes P_3 + (P_6 \otimes \tau_2 + P_3 \otimes \tau_2 \otimes \tau_2) \otimes P_9)(M_4 \otimes S_4)^T.
\end{aligned}$$

From Lemma 3, we obtain an orthogonal array $L_{108}(6^{11} 3^4)$ as follows(see Table 1):

$$L_{108}(6^{11} 3^4) = [(M_1 \otimes S_1)(L_{36}(6^3) \otimes 1_3),$$
$$(M_2 \otimes S_2)(L_{36}(6^3) \otimes 1_3), (M_3 \otimes S_3)(L_{36}(6^3) \otimes 1_3), (M_4 \otimes S_4)(L_{36}(6^2 3^4) \otimes 1_3)].$$

Using the arrays in Corollary 1 to replace the arrays $L_{36}(6^3)$ and $L_{36}(6^2 3^4)$ in $L_{108}(6^{11} 3^4)$ respectively, we can construct 1760 (220×8) orthogonal arrays such as $L_{108}(6^{10} 3^8 2^1)$, $L_{108}(6^{10} 3^2 2^2)$, $L_{108}(6^{10} 3^1 2^9)$, etc.

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Table 1. The orthogonal array $L_{108}(6^{11}3^4) = s_1 - s_{11}, t_1 - t_4$

No.	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	$t_1 - t_4$
1	0	0	2	0	0	2	1	1	3	0	0	1122
2	0	0	2	2	2	4	3	3	5	4	4	0011
3	0	0	2	4	4	0	5	5	1	2	2	2200
4	2	2	4	0	0	2	3	3	5	2	2	2200
5	2	2	4	2	2	4	5	5	1	0	0	1122
6	2	2	4	4	4	0	1	1	3	4	4	0011
7	4	4	0	0	0	2	5	5	1	4	4	0011
8	4	4	0	2	2	4	1	1	3	2	2	2200
9	4	4	0	4	4	0	3	3	5	0	0	1122
10	1	1	3	2	4	2	0	0	2	2	4	1212
11	1	1	3	4	0	4	2	2	4	0	2	0101
12	1	1	3	0	2	0	4	4	0	4	0	2020
13	3	3	5	2	4	2	2	2	4	4	0	2020
14	3	3	5	4	0	4	4	4	0	2	4	1212
15	3	3	5	0	2	0	0	0	2	0	2	0101
16	5	5	1	2	4	2	4	4	0	0	2	0101
17	5	5	1	4	0	4	0	0	2	4	0	2020
18	5	5	1	0	2	0	2	2	4	2	4	1212
19	0	1	4	5	2	2	5	2	2	1	1	1122
20	0	1	4	1	4	4	1	4	4	5	5	0011
21	0	1	4	3	0	0	3	0	0	3	3	2200
22	2	3	0	5	2	2	1	4	4	3	3	2200
23	2	3	0	1	4	4	3	0	0	1	1	1122
24	2	3	0	3	0	0	5	2	2	5	5	0011
25	4	5	2	5	2	2	3	0	0	5	5	0011
26	4	5	2	1	4	4	5	2	2	3	3	2200
27	4	5	2	3	0	0	1	4	4	1	1	1122
28	1	0	1	1	0	1	1	0	1	0	1	2211
29	1	0	1	3	2	3	3	2	3	4	5	1100
30	1	0	1	5	4	5	5	4	5	2	3	0022
31	3	2	3	1	0	1	3	2	3	2	3	0022
32	3	2	3	3	2	3	5	4	5	0	1	2211
33	3	2	3	5	4	5	1	0	1	4	5	1100
34	5	4	5	1	0	1	5	4	5	4	5	1100
35	5	4	5	3	2	3	1	0	1	2	3	0022
36	5	4	5	5	4	5	3	2	3	0	1	2211
37	2	4	2	1	1	3	2	4	2	4	2	1221
38	2	4	2	3	3	5	4	0	4	2	0	0110
39	2	4	2	5	5	1	0	2	0	0	4	2002
40	4	0	4	1	1	3	4	0	4	0	4	2002

41	4	0	4	3	3	5	0	2	0	4	2	1221
42	4	0	4	5	5	1	2	4	2	2	0	0110
43	0	2	0	1	1	3	0	2	0	2	0	0110
44	0	2	0	3	3	5	2	4	2	0	4	2002
45	0	2	0	5	5	1	4	0	4	4	2	1221
46	3	5	4	3	5	4	4	3	3	2	5	2121
47	3	5	4	5	1	0	0	5	5	0	3	1010
48	3	5	4	1	3	2	2	1	1	4	1	0202
49	5	1	0	3	5	4	0	5	5	4	1	0202
50	5	1	0	5	1	0	2	1	1	2	5	2121
51	5	1	0	1	3	2	4	3	3	0	3	1010
52	1	3	2	3	5	4	2	1	1	0	3	1010
53	1	3	2	5	1	0	4	3	3	4	1	0202
54	1	3	2	1	3	2	0	5	5	2	5	2121
55	2	5	3	3	4	1	2	5	3	1	0	2211
56	2	5	3	5	0	3	4	1	5	5	4	1100
57	2	5	3	1	2	5	0	3	1	3	2	0022
58	4	1	5	3	4	1	4	1	5	3	2	0022
59	4	1	5	5	0	3	0	3	1	1	0	2211
60	4	1	5	1	2	5	2	5	3	5	4	1100
61	0	3	1	3	4	1	0	3	1	5	4	1100
62	0	3	1	5	0	3	2	5	3	3	2	0022
63	0	3	1	1	2	5	4	1	5	1	0	2211
64	3	4	1	5	3	4	5	3	4	3	4	2121
65	3	4	1	1	5	0	1	5	0	1	2	1010
66	3	4	1	3	1	2	3	1	2	5	0	0202
67	5	0	3	5	3	4	1	5	0	5	0	0202
68	5	0	3	1	5	0	3	1	2	3	4	2121
69	5	0	3	3	1	2	5	3	4	1	2	1010
70	1	2	5	5	3	4	3	1	2	1	2	1010
71	1	2	5	1	5	0	5	3	4	5	0	0202
72	1	2	5	3	1	2	1	5	0	3	4	2121
73	4	2	1	0	1	4	4	2	1	3	5	1212
74	4	2	1	2	3	0	0	4	3	1	3	0101
75	4	2	1	4	5	2	2	0	5	5	1	2020
76	0	4	3	0	1	4	0	4	3	5	1	2020
77	0	4	3	2	3	0	2	0	5	3	5	1212
78	0	4	3	4	5	2	4	2	1	1	3	0101
79	2	0	5	0	1	4	2	0	5	1	3	0101
80	2	0	5	2	3	0	4	2	1	5	1	2020

81	2	0	5	4	5	2	0	4	3	3	5	1212
82	5	3	4	2	5	3	3	4	1	5	2	2112
83	5	3	4	4	1	5	5	0	3	3	0	1001
84	5	3	4	0	3	1	1	2	5	1	4	0220
85	1	5	0	2	5	3	5	0	3	1	4	0220
86	1	5	0	4	1	5	1	2	5	5	2	2112
87	1	5	0	0	3	1	3	4	1	3	0	1001
88	3	1	2	2	5	3	1	2	5	3	0	1001
89	3	1	2	4	1	5	3	4	1	1	4	0220
90	3	1	2	0	3	1	5	0	3	5	2	2112
91	4	3	3	4	3	3	3	5	4	4	3	2112
92	4	3	3	0	5	5	5	1	0	2	1	1001
93	4	3	3	2	1	1	1	3	2	0	5	0220
94	0	5	5	4	3	3	5	1	0	0	5	0220
95	0	5	5	0	5	5	1	3	2	4	3	2112
96	0	5	5	2	1	1	3	5	4	2	1	1001
97	2	1	1	4	3	3	1	3	2	2	1	1001
98	2	1	1	0	5	5	3	5	4	0	5	0220
99	2	1	1	2	1	1	5	1	0	4	3	2112
100	5	2	2	4	2	1	0	1	4	5	3	1221
101	5	2	2	0	4	3	2	3	0	3	1	0110
102	5	2	2	2	0	5	4	5	2	1	5	2002
103	1	4	4	4	2	1	2	3	0	1	5	2002
104	1	4	4	0	4	3	4	5	2	5	3	1221
105	1	4	4	2	0	5	0	1	4	3	1	0110
106	3	0	0	4	2	1	4	5	2	3	1	0110
107	3	0	0	0	4	3	0	1	4	1	5	2002
108	3	0	0	2	0	5	2	3	0	5	3	1221

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