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Super (a, d)-edge antimagic total labeling of some classes of graphs

P. Roushini Leely Pushpam and A. Saibulla

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Abstract. A graph $G(V, E)$ is (a, d)-*edge antimagic total* if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ such that the edge-weights $\Lambda(uv) = f(u)+f(uv)+f(v)$, $uv \in E(G)$ form an arithmetic progression with first term a and common difference d. It is said to be a *super* (a, d)-*edge antimagic total* if $f(V(G)) = \{1, 2, \ldots, |V(G)|\}$. In this paper, we have obtained a relation between a super $(a, 0)$-edge antimagic total labeling and a super $(a, 2)$-edge antimagic total labeling of any graph. Also we study the super (a, d)-edge antimagic total labeling of fan graphs and two special classes of star graphs namely bi-star and extended bi-star.

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§1. Introduction

By a *graph* $G(V, E)$ we mean a finite, undirected, connected graph without loops or multiple edges. The order and size of $G(V, E)$ are denoted by p and q respectively. For graph theoretic terminologies we refer to Harary [7].

A *labeling* of a graph is an assignment of numbers (usually positive or non-negative integers) to the vertices (a *vertex labeling*) or to the edges (an *edge labeling*) or to the combined set of vertices and edges (a *total labeling*) of the graph. There are many types of labelings and a detailed survey of many of them can be found in the dynamic survey of graph labeling by J.A. Gallian [6].

The *edge weight* of an edge uv, denoted by $\Lambda(uv)$, is defined as the sum of labels of the graph elements associated with uv. That is, if f is an edge labeling, then $\Lambda(uv) = f(uv)$; if f is a vertex labeling, then $\Lambda(uv) = f(u) + f(v)$; and if f is a total labeling, then $\Lambda(uv) = f(u) + f(uv) + f(v)$. Similarly the *vertex weight* of a vertex v, denoted by $\Lambda(v)$, is defined as the sum of labels of the graph elements associated with v. That is, if f is a vertex labeling, then $\Lambda(v) = \sum_{u \in N(v)} f(u)$; if f is an edge labeling, then $\Lambda(v) = \sum_{uv \in E} f(uv)$; and if f is a total labeling, then $\Lambda(v) = f(v) + \sum_{uv \in E} f(uv)$.

In 1970 Kotzig and Rosa [9] defined an *edge-magic total labeling* of a graph $G(V, E)$ as a bijection f from $V \cup E$ to the set $\{1, 2, \ldots, |V| + |E|\}$ such that for each edge $uv \in E$, the edge weight $f(u) + f(uv) + f(v)$ is a constant.

Enomoto et al. [4] defined a *super edge magic labeling* as an edge-magic total labeling such that the vertex labels are $\{1, 2, \ldots, |V|\}$ and the edge labels are $\{|V| + 1, |V| + 2, \ldots, |V| + |E|\}$. They have proved that if a graph with p vertices and q edges is super edge-magic, then $q \leq 2p - 3$. They also conjectured that every tree is super edge-magic.

As a natural extension of the notion of edge-magic total labeling, Simanjuntak et al. [10] defined an (a, d)-*edge antimagic total labeling* of a graph $G(V, E)$ as an injective mapping f from $V \cup E$ onto the set $\{1, 2, \ldots, |V| + |E|\}$ such that the set $\{f(u)+f(uv)+f(v) | uv \in E\}$ is $\{a, a+d, a+2d, \ldots, a+(|E|-1)d\}$ for any two integers $a > 0$ and $d \geq 0$.

An (a, d)-edge antimagic total labeling of a graph $G(V, E)$ is called a *super* (a, d)-*edge antimagic total* if the vertex labels are $\{1, 2, \ldots, |V|\}$ and the edge labels are $\{|V| + 1, |V| + 2, \ldots, |V| + |E|\}$. The super $(a, 0)$-edge antimagic total labelings are usually called as super edge magic in the literature (see [4, 5]).

Many researchers investigated different forms of antimagic labelings [8]. Bača et al. [1, 2] proved several results on antimagic labelings. Also in [3] Bača and Barrientos presented some relationships between (a, d)-edge antimagic vertex labelings and super (a, d)-edge antimagic total labelings.

In this paper, we prove that a graph is super $(a_1, 0)$-edge antimagic total, then it is super $(a_2, 2)$-edge antimagic total. Also we study the super (a, d)-edge antimagic total labeling of fan graphs and two special classes of star graphs namely bi-star and extended bi-star.

§2. Super (a, d)-edge antimagic total labeling

The following theorem gives a relation between a super $(a_1, 0)$-edge antimagic total labeling and a super $(a_2, 2)$-edge antimagic total labeling of any graph.

Theorem 1. *If a graph $G(V, E)$ is super $(a_1, 0)$-edge antimagic total, then it is super $(a_2, 2)$-edge antimagic total.*

Proof. Suppose the graph $G(V,E)$ is super $(a_1,0)$-edge antimagic total, then by definition, there exists a bijection $f : V \cup E \to \{1,2,\ldots,p+q\}$ such that

(i) $\{f(v)|v \in V\} = \{1,2,\ldots,p\}$

(ii) $\{f(uv)|uv \in E\} = \{p+1,p+2,\ldots,p+q\}$ and

(iii) for all $uv \in E, f(u)+f(uv)+f(v) = a_1$.

In order to prove $G(V,E)$ has a super $(a_2,2)$-edge antimagic total labeling, we define an induced map g_f as follows:
Let $g_f : V \cup E \to \{1,2,\ldots,p+q\}$ such that

(i) for all $u \in V$, $g_f(u) = f(u)$ and

(ii) for all $uv \in E$, $g_f(uv) = 2p+q+1-f(uv)$.

Then we see that $\{g_f(v)|v \in V\} = \{1,2,\ldots,p\}$ and $\{g_f(uv)|uv \in E\} = \{p+1,p+2,\ldots,p+q\}$.

Also for all $uv \in E$ we have

$$\begin{aligned} g_f(u)+g_f(uv)+g_f(v) &= f(u)+2p+q+1-f(uv)+f(v) \\ &= 2p+q+1+a_1-2f(uv) \\ &= a_1-q+1+2(p+q)-2f(uv). \end{aligned}$$

Thus the set of edge-weights is in arithmetic progression with first term $(a_1 - q+1)$ and common difference 2.

Hence $G(V,E)$ is super $(a_2,2)$-edge antimagic total with $a_2 = (a_1-q+1)$. □

§3. Fan graph

A *fan graph* $F_{m,2}$ is defined as the graph join $\bar{K}_m + P_2$, where $\bar{K}_m$ is an empty graph with m vertices and P_2 is a path with 2 vertices. Let the vertices be $u_1,u_2,\ldots,u_m$, v_1,v_2 and the edges be v_1v_2 and u_iv_j, $1 \le i \le m$, $1 \le j \le 2$.

Theorem 2. *If the fan graph $F_{m,2}$, $m \ge 2$, is super (a,d)-edge antimagic total, then $d \le 2$.*

Proof. Assume that $F_{m,2}$, $m \ge 2$ has a super (a,d)-edge antimagic total labeling $f : V(F_{m,2}) \cup E(F_{m,2}) \to \{1,2,\ldots,3m+3\}$ such that the set of edge-weights is given by $\{a, a+d,\ldots,a+2md\}$.

It is easy to see that the minimum possible edge-weight in a super (a,d)-edge antimagic total labeling is at least $|V|+4$. On the other hand, the maximum edge weight is no more than $3|V| + |E| - 1$.

Thus we have,

$$a \geq m + 6 \tag{3.1}$$

and

$$a + 2md \leq 5m + 6. \tag{3.2}$$

From the inequalities (3.1) and (3.2), we get $d \leq 2$. □

Theorem 3. *Every fan graph $F_{m,2}$, $m \geq 2$ has a super $(a, 0)$-edge antimagic total labeling.*

Proof. Let us define the vertex labeling $f_1 : V(F_{m,2}) \to \{1, 2, \ldots, m+2\}$ and the edge labeling $f_2 : E(F_{m,2}) \to \{m+3, m+4, \ldots, 3m+3\}$ as follows:

$$f_1(v_1) = 1; \quad f_1(v_2) = m + 2 \quad \text{and} \quad f_2(v_1v_2) = 2m + 3.$$

For $1 \leq i \leq m$,

$$f_1(u_i) = i + 1; \quad f_2(u_iv_1) = 3m + 4 - i \quad \text{and} \quad f_2(u_iv_2) = 2m + 3 - i.$$

It is easy to verify that $\{\Lambda(uv) | uv \in E(F_{m,2})\} = 3(m + 2)$.
Thus the labelings f_1 and f_2 are super $(a, 0)$-edge antimagic total labeling of $F_{m,2}$, $m \geq 2$ with $a = 3(m + 2)$. □

In view of Theorem 1, it is clear that the fan graph $F_{m,2}$, $m \geq 2$ has a super $(a, 2)$-edge antimagic total labeling with $a = m + 6$.

Theorem 4. *Every fan graph $F_{m,2}$, $m \geq 2$ has a super $(a, 1)$-edge antimagic total labeling.*

Proof. Let the vertex labeling f_1 be defined as in Theorem 3.

We define the edge labeling $f_3 : E(F_{m,2}) \to \{m+3, m+4, \ldots, 3m+3\}$ as follows:

Case (i) m is odd:

$$f_3(v_1v_2) = 2(m + 1) - \left(\frac{m - 1}{2}\right)$$

For $1 \leq i \leq m$

$$f_3(u_iv_1) = \begin{cases} 3(m+1) - \frac{i-1}{2}, & \text{if } i \text{ is odd} \\ 2m + 3 - \frac{i}{2}, & \text{if } i \text{ is even} \end{cases}$$

and

$$f_3(u_iv_2) = \begin{cases} 3(m+1) - \frac{m+i}{2}, & \text{if } i \text{ is odd} \\ 2(m+1) - \frac{m-1+i}{2}, & \text{if } i \text{ is even.} \end{cases}$$

Case (ii) m **is even**:

$$f_3(v_1v_2) = 3(m+1) - \frac{m}{2}$$

For $1 \le i \le m$

$$f_3(u_iv_1) = \begin{cases} 3(m+1) - \frac{i-1}{2}, & \text{if } i \text{ is odd} \\ 2m+3 - \frac{i}{2}, & \text{if } i \text{ is even} \end{cases}$$

and

$$f_3(u_iv_2) = \begin{cases} 2m+3 - \frac{m+i+1}{2}, & \text{if } i \text{ is odd} \\ 3(m+1) - \frac{m+i}{2}, & \text{if } i \text{ is even.} \end{cases}$$

In both the cases, it is easy to see that $\{\Lambda(uv)|uv \in E(F_{m,2})\} = \{(2m+6), (2m+7), \ldots, (4m+6)\}$.

Thus the labelings f_1 and f_3 are super $(a,1)$-edge antimagic total labeling of $F_{m,2}$, $m \ge 2$ with $a = 2m+6$. □

§4. Bistar

A *bistar* $B_{m,n}$ is defined as the graph obtained by attaching an edge with the center vertices of two stars $K_{1,m}$ and $K_{1,n}$. Let the vertices be $c_1, c_2, u_1, u_2, \ldots,$ u_m, v_1, $v_2, \ldots, v_n$ and the edges be c_1c_2, c_1u_i, $1 \le i \le m$ and c_2v_j, $1 \le j \le n$.

Theorem 5. *If the bistar $B_{m,n}$, $m \ge 2$, $n \ge 2$ is super (a,d)-edge antimagic total, then $d \le 3$.*

Proof. Assume that $B_{m,n}$, $m \ge 2$, $n \ge 2$ has a super (a,d)-edge antimagic total labeling $f : V(B_{m,n}) \cup E(B_{m,n}) \to \{1, 2, \ldots, 2m+2n+3\}$ such that the set of edge-weights is given by $\{a, a+d, \ldots, a+(m+n)d\}$.

Clearly the maximum edge-weight is no more than

$$(m+n+1) + (m+n+2) + (2m+2n+3).$$

Thus,

$$a + (m+n)d \le 4m + 4n + 6. \tag{4.1}$$

On the other hand, the minimum possible edge-weight is at least $1+2+(m+n+3)$.

Thus,

$$a \geq m+n+6. \tag{4.2}$$

From the inequalities (4.1) and (4.2), we get $d \leq 3$. □

Theorem 6. *Every bistar $B_{m,n}$, $m \geq 2$, $n \geq 2$ has a super $(a,0)$-edge antimagic total labeling.*

Proof. Let us define the vertex labeling $f_4 : V(B_{m,n}) \to \{1, 2, \ldots, m+n+2\}$ and the edge labeling $f_5 : E(B_{m,n}) \to \{m+n+3, m+n+4, \ldots, 2m+2n+3\}$ as follows:

$$f_4(c_1) = 1; \qquad f_4(c_2) = m+n+2 \qquad \text{and} \qquad f_5(c_1c_2) = m+2n+3.$$

For $1 \leq i \leq m$

$$f_4(u_i) = n+i+1; \qquad f_5(c_1u_i) = 2m+2n+4-i$$

and for $1 \leq j \leq n$

$$f_4(v_j) = j+1; \qquad f_5(c_2v_j) = m+2n+3-j.$$

By direct computation we obtain that $\{\Lambda(uv) | uv \in E(B_{m,n})\} = 2m+3n+6$.

Thus the labelings f_4 and f_5 are super $(a,0)$-edge antimagic total labeling of $B_{m,n}$, $m \geq 2$, $n \geq 2$ with $a = 2m+3n+6$. □

In view of Theorem 1, it is clear that the bistar $B_{m,n}$, $m \geq 2$, $n \geq 2$ has a super $(a,2)$-edge antimagic total labeling with $a = m+2n+6$.

Theorem 7. *For $n \in \{m-1, m, m+1\}$ or $(m+n) \equiv 0 (mod\ 2)$, the bistar $B_{m,n}$, $m \geq 2$, $n \geq 2$ has a super $(a,1)$-edge antimagic total labeling.*

In order to prove the theorem, we prove the following lemmas.

Lemma 1. *For $n \in \{m-1, m, m+1\}$, $m \geq 2$, the bistar $B_{m,n}$, has a super $(a,1)$-edge antimagic total labeling.*

Proof. Let us define the vertex labeling $g_1 : V(B_{m,n}) \to \{1, 2, \ldots, m+n+2\}$ and the edge labeling $g_2 : E(B_{m,n}) \to \{m+n+3, m+n+4, \ldots, 2m+2n+3\}$ as follows:

Case (i) $n = m - 1$:

$g_1(c_1) = 2,$
$g_1(c_2) = m + n + 2,$
$g_1(u_i) = 2i - 1, \quad 1 \leq i \leq m,$
$g_1(v_j) = 2(j + 1), \quad 1 \leq j \leq n,$
$g_2(c_1c_2) = m + 2n + 3,$
$g_2(c_1u_i) = 2(m + n + 2) - i, \quad 1 \leq i \leq m,$
$g_2(c_2v_j) = m + 2n + 3 - j, \quad 1 \leq j \leq n.$

Case (ii) $n = m$:

$g_1(c_1) = 2,$
$g_1(c_2) = m + n + 1,$
$g_1(u_i) = 2i - 1, \quad 1 \leq i \leq m,$
$g_1(v_j) = 2(j + 1), \quad 1 \leq j \leq n$ and g_2 same as in Case (i).

Case (iii) $n = m + 1$:

$g_1(c_1) = m + n + 2,$
$g_1(c_2) = 2,$
$g_1(u_i) = 2(i + 1), \quad 1 \leq i \leq m,$
$g_1(v_j) = 2j - 1, \quad 1 \leq j \leq n,$
$g_2(c_1c_2) = 2m + n + 3,$
$g_2(c_1u_i) = 2m + n + 3 - i, \quad 1 \leq i \leq m,$
$g_2(c_2v_j) = 2m + 2n + 4 - j, \quad 1 \leq j \leq n.$

In all the above three cases, it is easy to verify that $\{\Lambda(uv)|uv \in E(B_{m,n})\} = \{2m + 2n + 6, 2m + 2n + 7, \ldots, 3m + 3n + 6\}$.

Thus the labelings g_1 and g_2 are super $(a, 1)$-edge antimagic total labeling of $B_{m,n}$, $n \in \{m - 1, m, m + 1\}$, $m \geq 2$ with $a = 2m + 2n + 6$. □

Lemma 2. *For* $(m + n) \equiv 0(mod\ 2)$*, the bistar* $B_{m,n}$ *has a super* $(a, 1)$*-edge antimagic total labeling.*

Proof. Let the vertex labeling f_4 be defined as in Theorem 6.

We define the edge labeling $g_3 : E(B_{m,n}) \to \{m+n+3, m+n+4, \ldots, 2m+2n+3\}$ as follows:

Case (i) m **and** n **are even**:

$$g_3(c_1c_2) = 2m + 2n + 3 - \frac{m}{2},$$

For $1 \le i \le m$,

$$g_3(c_1u_i) = \begin{cases} 2m+2n+3-\frac{i-1}{2}, & \text{if } i \text{ is odd} \\ 2m+2n+3-\frac{m+n+i}{2}, & \text{if } i \text{ is even} \end{cases}$$

and for $1 \le j \le n$,

$$g_3(c_2v_j) = \begin{cases} 2m+2n+3-\frac{2m+n+j+1}{2}, & \text{if } j \text{ is odd} \\ 2m+2n+3-\frac{m+j}{2}, & \text{if } j \text{ is even.} \end{cases}$$

Case (ii) m **and** n **are odd**:

$$g_3(c_1c_2) = 2m+2n+3-\frac{2m+n+1}{2},$$

For $1 \le i \le m$,

$$g_3(c_1u_i) = \begin{cases} 2m+2n+3-\frac{i-1}{2}, & \text{if } i \text{ is odd} \\ 2m+2n+3-\frac{m+n+i}{2}, & \text{if } i \text{ is even} \end{cases}$$

and for $1 \le j \le n$,

$$g_3(c_2v_j) = \begin{cases} 2m+2n+3-\frac{m+j}{2}, & \text{if } j \text{ is odd} \\ 2m+2n+3-\frac{2m+n+j+1}{2}, & \text{if } j \text{ is even.} \end{cases}$$

In both the cases, we see that the bistar $B_{m,n}$ is super $(a,1)$-edge antimagic total with $a = 2m+3n-\frac{m+n}{2}+6$. □

Proof of Theorem 7, directly follows from Lemmas 1 and 2.

Theorem 8. *For $n \in \{m-1, m, m+1\}$, $m \ge 2$, the bistar $B_{m,n}$ has a super $(a,3)$-edge antimagic total labeling.*

Proof. Let the vertex labeling g_1 be defined as in Lemma 1.

We define the edge labeling $f_6 : E(B_{m,n}) \to \{m+n+3, m+n+4, \ldots, 2m+2n+3\}$ as follows:

Case (i) $n = m-1$ **or** $n = m$:
$f_6(c_1c_2) = 2m+n+3$,
$f_6(c_1u_i) = m+n+2+i, \quad 1 \le i \le m$,
$f_6(c_2v_j) = 2m+n+3+j, \quad 1 \le j \le n$.

Case (ii) $n = m+1$:
$f_6(c_1c_2) = m+2n+3$,

$f_6(c_1u_i) = m + 2n + 3 + i, \quad 1 \leq i \leq m,$
$f_6(c_2v_j) = m + n + 2 + j, \quad 1 \leq j \leq n.$

In both the cases, we see that $\{\Lambda(uv)|uv \in E(B_{m,n})\} = \{m + n + 6, m + n + 6 + 3, \ldots, 4m + 4n + 6\}$.

Thus the bistar $B_{m,n}$, $n \in \{m - 1, m, m + 1\}$, $m \geq 2$ is super $(a, 3)$-edge antimagic total with $a = m + n + 6$. □

§5. Extended Bistar

An *extended bistar* $< K_{1,m} : n >$ is defined as the graph obtained by attaching a path of length n with the centre vertices of two copies of the star graph $K_{1,m}$. Let the vertices be $c_1, c_2, \ldots, c_{n+1}, u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_m$ and the edges be $c_1u_i, c_{n+1}v_i$, $1 \leq i \leq m$ and c_jc_{j+1}, $1 \leq j \leq n$.

Theorem 9. *If the extended bistar* $< K_{1,m} : n >$, $m, n \geq 2$ *is super* (a, d)*-edge antimagic total, then* $d \leq 3$.

Proof. Assume that $< K_{1,m} : n >$, $m, n \geq 2$ has a super (a, d)-edge antimagic total labeling $f : V(< K_{1,m} : n >) \cup E(< K_{1,m} : n >) \rightarrow \{1, 2, \ldots, 4m+2n+1\}$ such that the set of edge-weights is given by $\{a, a + d, \ldots, a + (2m + n - 1)d\}$.

Clearly the maximum edge-weight is no more than

$$(2m + n) + (2m + n + 1) + (4m + 2n + 1).$$

Thus,

$$a + (2m + n - 1)d \leq 8m + 4n + 2. \tag{5.1}$$

On the other hand, the minimum possible edge-weight is at least $1 + 2 + (2m + n + 2)$.
Thus,

$$a \geq 2m + n + 5. \tag{5.2}$$

From the inequalities (5.1) and (5.2), we get $d \leq 3$. □

Theorem 10. *Every extended bistar* $< K_{1,m} : n >$, $m, n \geq 2$ *has a super* $(a, 0)$*-edge antimagic total labeling.*

Proof. Let us define the vertex labeling $f_7 : V(< K_{1,m} : n >) \rightarrow \{1, 2, \ldots, 2m + n + 1\}$ and the edge labeling $f_8 : E(< K_{1,m} : n >) \rightarrow \{2m + n + 2, 2m + n + 3, \ldots, 4m + 2n + 1\}$ as follows:

Case (i) n is odd:
For $1 \leq i \leq m$,

$$f_7(u_i) = \left(\frac{n+1}{2}\right) + m + i; \qquad f_7(v_i) = \left(\frac{n+1}{2}\right) + i$$

and for $1 \leq j \leq n+1$,

$$f_7(c_j) = \begin{cases} \left(\frac{j+1}{2}\right), & \text{if } j \text{ is odd} \\ 2m + \left(\frac{n+j+1}{2}\right), & \text{if } j \text{ is even.} \end{cases}$$

Case (ii) n is even:
For $1 \leq i \leq m$,

$$f_7(u_i) = \frac{n}{2} + i + 1; \qquad f_7(v_i) = m + n + i + 1$$

and for $1 \leq j \leq n+1$,

$$f_7(c_j) = \begin{cases} \left(\frac{j+1}{2}\right), & \text{if } j \text{ is odd} \\ m + 1 + \left(\frac{n+j}{2}\right), & \text{if } j \text{ is even.} \end{cases}$$

For any n, we define
$f_8(c_1u_i) \quad = 4m + 2(n+1) - i, \quad 1 \leq i \leq m,$
$f_8(c_jc_{j+1}) = 3m + 2(n+1) - j, \quad 1 \leq j \leq n,$
$f_8(c_{n+1}v_i) = 3m + (n+2) - i, \quad 1 \leq i \leq m.$

By direct computation, we get

$$\{\Lambda(uv) | uv \in E\} = \begin{cases} 5\left(m + \frac{n+1}{2}\right) + 1, & \text{if } n \text{ is odd} \\ 4(m+1) + \frac{5n}{2}, & \text{if } n \text{ is even.} \end{cases}$$

Thus the labelings f_7 and f_8 are super $(a, 0)$-edge antimagic total labeling of $< K_{1,m} : n >$, $m, n \geq 2$ with

$$a = \begin{cases} 5\left(m + \frac{n+1}{2}\right) + 1, & \text{if } n \text{ is odd} \\ 4(m+1) + \frac{5n}{2}, & \text{if } n \text{ is even.} \end{cases}$$

□

In view of Theorem 1, it is clear that the extended bistar $< K_{1,m} : n >$, $m, n \geq 2$ has a super $(a, 2)$-edge antimagic total labeling with

$$a = \begin{cases} 3\left(m + \frac{n+3}{2}\right), & \text{if } n \text{ is odd} \\ 2(m+2) + \frac{3n}{2} + 1, & \text{if } n \text{ is even.} \end{cases}$$

Theorem 11. *For odd n, the extended bistar $< K_{1,m} : n >$, $m, n \geq 2$ has a super $(a, 1)$-edge antimagic total labeling.*

Proof. Let us define the vertex labeling $f_9 : V(< K_{1,m} : n >) \to \{1, 2, \dots, 2m+n+1\}$ as follows:

$f_9(u_i) = 2i, \quad 1 \leq i \leq m,$

$f_9(v_i) = n + 2i, \quad 1 \leq i \leq m,$ (since n is odd, f_9 is bijective)

and for $1 \leq j \leq n + 1$,

$$f_9(c_j) = \begin{cases} j, & \text{if } j \text{ is odd} \\ 2m + j, & \text{if } j \text{ is even.} \end{cases}$$

Let the edge labeling f_8 be as defined in Theorem 10.

Then we see that $\{\Lambda(uv) | uv \in E\} = \{4m+2n+4, 4m+2n+5, \dots, 6m+3n+3\}$.

Hence, when n is odd, the extended bistar $< K_{1,m} : n >$, $m, n \geq 2$ is super $(a, 1)$-edge antimagic total with $a = 4m + 2n + 4$. □

Theorem 12. *For odd n, the extended bistar $< K_{1,m} : n >$, $m, n \geq 2$ has a super $(a, 3)$-edge antimagic total labeling.*

Proof. Let the vertex labeling f_9 be as defined in Theorem 11.

We define the edge labeling $f_{10} : E(< K_{1,m} : n >) \to \{2m + n + 2, 2m + n + 3, \dots, 4m + 2n + 1\}$ as follows:

$f_{10}(c_1 u_i) \quad = 2m + n + 1 + i, \quad 1 \leq i \leq m,$

$f_{10}(c_j c_{j+1}) = 3m + n + 1 + j, \quad 1 \leq j \leq n,$

$f_{10}(c_{n+1} v_i) = 3m + 2n + 1 + i, \quad 1 \leq i \leq m.$

Then we see that $\{\Lambda(uv) | uv \in E\} = \{2m + n + 5, 2m + n + 5 + 3, \dots, 2m + n + 5 + (2m + n - 1)3\}$.

Hence, when n is odd, the extended bistar $< K_{1,m} : n >$, $m, n \geq 2$ is super $(a, 3)$-edge antimagic total with $a = 2m + n + 5$. □

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P. Roushini Leely Pushpam
Department of Mathematics, D.B. Jain College
Chennai - 600097, Tamil Nadu, India
E-mail: roushinip@yahoo.com

A. Saibulla
Department of Mathematics, B.S. Abdur Rahman University
Chennai - 600048, Tamil Nadu, India
E-mail: saibulla.a@gmail.com