SUT Journal of Mathematics
Vol. 48, No. 2 (2012), 231–240

3-product cordial labeling

P. Jeyanthi and A. Maheswari

(Received December 8, 2010; Revised December 29, 2012)

Abstract. A mapping $f : V(G) \to \{0, 1, 2\}$ is called a 3-product cordial labeling if $|v_f(i)-v_f(j)| \leq 1$ and $|e_f(i)-e_f(j)| \leq 1$ for any $i, j \in \{0, 1, 2\}$, where $v_f(i)$ denotes the number of vertices labeled with i, $e_f(i)$ denotes the number of edges xy with $f(x)f(y) \equiv i$ $(mod\ 3)$. A graph with a 3-product cordial labeling is called a 3-product cordial graph. In this paper, we investigate the 3-product cordial behaviour for some standard graphs.

AMS 2010 *Mathematics Subject Classification.* 05C78.

Key words and phrases. cordial labeling, product cordial labeling, 3-product cordial labeling, 3-product cordial graph.

§1. Introduction

By a graph we mean finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ so that the order and size of G are $|V(G)|$ and $|E(G)|$ respectively. The union $G_1 \cup G_2$ of two graphs G_1 and G_2 has the vertex set $V(G_1) \cup V(G_2)$ and the edge set $E(G_1) \cup E(G_2)$. A corona $G_1 \circ G_2$ is a graph obtained from two graphs G_1 and G_2 by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining by an edge the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The graph $\langle B_{n,n} : w\rangle$ is obtained by joining the center vertices of two $K_{1,n}$ stars to a new vertex w. The flower graph Fl_n is the graph obtained from a helm graph H_n by joining each pendant vertex to the apex vertex of the helm. The complete survey of graph labeling is in [2].

Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by I. Cahit in [1]. Let f be a function from the vertices of G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. f is called a cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. M. Sundaram et al. introduced the concept of product cordial labeling of a graph in [3]. A product cordial labeling of a graph G with the vertex set V is a function f from V to $\{0,1\}$ such that if each edge uv is assigned the label $f(u)f(v)$, $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$. The concept of EP-cordial labeling was introduced in [4]. A vertex labeling $f : V(G) \to \{-1,0,1\}$ is said to be an EP-cordial labeling if it induces the edge labeling f^* defined by $f^*(uv) = f(u)f(v)$, for each $uv \in E(G)$ and if $|v_f(i) - v_f(j)| \leq 1$ and $|e_{f^*}(i) - e_{f^*}(j)| \leq 1, i \neq j,\ i,j \in \{-1,0,1\}$, where $v_f(x)$ and $e_{f^*}(x)$ denote the number of vertices and edges of G having the label $x \in \{-1,0,1\}$. In [5] it is remarked that any EP-cordial labeling is a 3-product cordial labeling. A mapping $f : V(G) \to \{0,1,2\}$ is called a 3-product cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for any, $i,j \in \{0,1,2\}$, where $v_f(i)$ denotes the number of vertices labeled with $i, e_f(i)$ denotes the number of edges xy with $f(x)f(y) \equiv i\ (mod\ 3)$. A graph with a 3-product cordial labeling is called a 3-product cordial graph.

§2. Main Results

Theorem 2.1. *If G_1 is a 3-product cordial graph with $3m$ vertices and $3n$ edges and G_2 is any 3-product cordial graph, then $G_1 \cup G_2$ is also 3-product cordial graph.*

Proof. Let f, g be a 3-product cordial labeling of G_1 and G_2 respectively. Since G_1 has $3n$ edges and $3m$ vertices we have $e_f(0) = e_f(1) = e_f(2) = n$ and $v_f(0) = v_f(1) = v_f(2) = m$. Define a labeling $h : V(G_1 \cup G_2) \to \{0,1,2\}$ by

$$h(u) = \begin{cases} f(u) & \text{if } u \in V(G_1) \\ g(u) & \text{if } u \in V(G_2) \end{cases}$$

Hence, $v_h(i) = v_f(i) + v_g(i)$ and $e_h(i) = e_f(i) + e_g(i)$ for $i = 0,1,2$. Therefore $|v_h(i) - v_h(j)| \leq 1$ and $|e_h(i) - e_h(j)| \leq 1$ for $i,j \in \{0,1,2\}$. Hence h is a 3-product cordial labeling of $G_1 \cup G_2$. □

In [4], it is proved that if G is 3-product cordial graph of order p and size q then $q \leq 1 + \frac{p}{3} + \frac{p^2}{3}$ and also proved that K_n is 3-product cordial if and only if $n \leq 3$. In this paper we give an improved upper bound for q in Theorems 2.2, 2.4 and 2.6 and prove that K_n is 3-product cordial if and only if $n \leq 2$.

Theorem 2.2. *If $G(p,q)$ is a 3-product cordial graph with $p \equiv 0\ (mod\ 3)$, then $q \leq \frac{p^2-3p+6}{3}$.*

Proof. Let f be a 3-product cordial labeling of the graph G. Take $p = 3n$. Then we have $v_f(0) = v_f(1) = v_f(2) = n$. Hence, $e_f(1) \leq n(n-1)$. Since f is a 3-product cordial labeling, $e_f(2) \leq n(n-1)+1$ and $e_f(0) \leq n(n-1)+1$. Hence, $q = e_f(0) + e_f(1) + e_f(2) \leq 3n(n-1) + 2 = \frac{p^2-3p+6}{3}$. □

Remark 2.3. *The graph given in Figure 1 is an example for a 3-product cordial graph with* $p = 6$ *and* $q = \frac{p^2-3p+6}{3} = 8$.

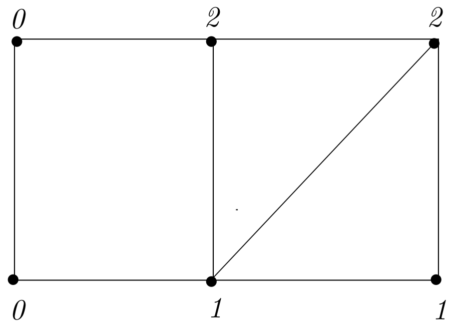

Figure 1

Theorem 2.4. *If* $G(p, q)$ *is a 3-product cordial graph with* $p \equiv 1 \pmod 3$, *then* $q \leq \frac{p^2-2p+7}{3}$.

Proof. Let f be a 3-product cordial labeling of the graph G. Take $p = 3n+1$. If $v_f(0) = v_f(2) = n$ and $v_f(1) = n+1$, or $v_f(0) = v_f(1) = n$ and $v_f(2) = n+1$. Then $e_f(1) \leq n^2$. If $v_f(0) = n+1$ and $v_f(1) = v_f(2) = n$. Then $e_f(1) \leq n(n-1)$. Thus in any case, we have $e_f(1) \leq n^2$. Since f is a 3-product cordial labeling, $e_f(2) \leq n^2+1$ and $e_f(0) \leq n^2+1$. Hence, $q = e_f(0)+e_f(1)+e_f(2) \leq 3n^2 + 2 = \frac{p^2-2p+7}{3}$. □

Remark 2.5. *The graph given in Figure 2 is an example for 3-product cordial graph with* $p = 4$ *and* $q = \frac{p^2-2p+7}{3} = 5$.

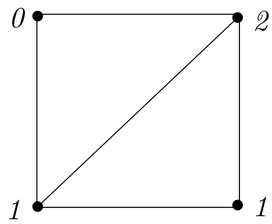

Figure 2

Theorem 2.6. *If* $G(p, q)$ *is a 3-product cordial graph with* $p \equiv 2 \pmod 3$, *then* $q \leq \frac{p^2-p+4}{3}$.

Proof. Let f be a 3-product cordial labeling of the graph G. Take $p = 3n+2$. If $v_f(0) = v_f(1) = n+1$ and $v_f(2) = n+1$. Hence $e_f(1) \leq n(n+1)$. If $v_f(0) = n+1, v_f(1) = n$ and $v_f(2) = n+1$ or $v_f(0) = n+1, v_f(1) = n+1, v_f(2) = n$. Then $e_f(1) \leq n^2$. Thus in any case, we have $e_f(1) \leq n(n+1)$. Since f is a 3-product cordial labeling, $e_f(2) \leq n(n+1)+1$ and $e_f(0) \leq n(n+1)+1$. Hence, $q = e_f(0) + e_f(1) + e_f(2) \leq 3n(n+1)+2 = \frac{p^2-p+4}{3}$. □

Remark 2.7. *The graph given in Figure 3 is an example for a 3-product cordial graph with $p = 5$ and $q = \frac{p^2-p+4}{3} = 8$.*

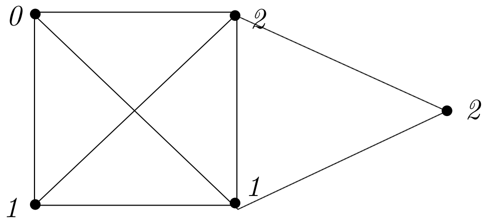

Figure 3

Theorem 2.8. *$\langle B_{n,n} : w\rangle$ is a 3-product cordial graph.*

Proof. Let $v_1^{(1)}, v_2^{(1)}, \ldots, v_n^{(1)}$ be the pendant vertices of the star $K_{1,n}^{(1)}$ and $v_1^{(2)}, v_2^{(2)}, \ldots, v_n^{(2)}$ be the pendant vertices of the star $K_{1,n}^{(2)}$. Let c_1, c_2 be the center vertices of $K_{1,n}^{(1)}, K_{1,n}^{(2)}$ respectively and they are adjacent to a new common vertex w. Let $G = \langle B_{n,n} : w\rangle$.

Case i. $n \equiv 0 \ (mod\ 3)$. Let $n = 3k$.

Define $f : V(G) \to \{0, 1, 2\}$ by

$$f(w) = 1,$$
$$f(c_1) = f(c_2) = 1,$$
$$f(v_i^{(1)}) = \begin{cases} 0 & \text{for } 1 \le i \le 2k+1 \\ 1 & \text{for } i > 2k+1 \end{cases}$$

and

$$f(v_i^{(2)}) = \begin{cases} 2 & \text{for } 1 \le i \le 2k+1 \\ 1 & \text{for } i > 2k+1. \end{cases}$$

Since $v_f(0) = v_f(1) = v_f(2) = 2k+1$, $e_f(0) = e_f(2) = 2k+1$ and $e_f(1) = 2k$, we have $|e_f(i) - e_f(j)| \le 1$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$. Thus f is a 3-product cordial labeling.

Case ii. $n \equiv 1 \ (mod\ 3)$. Let $n = 3k+1$.

Define $f : V(G) \to \{0, 1, 2\}$ by

$$f(w) = 1,$$
$$f(c_1) = f(c_2) = 1,$$
$$f(v_i^{(1)}) = \begin{cases} 0 & \text{for } 1 \le i \le 2k+2 \\ 1 & \text{for } i > 2k+2 \end{cases}$$

and

$$f(v_i^{(2)}) = \begin{cases} 2 & \text{for } 1 \le i \le 2k+1 \\ 1 & \text{for } i > 2k+1. \end{cases}$$

Hence $e_f(1) = e_f(2) = 2k+1$, $e_f(0) = 2k+2$, $v_f(0) = v_f(1) = 2k+2$, and $v_f(2) = 2k+1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Case iii. $n \equiv 2$ $(mod\ 3)$ and let $n = 3k+2$.

Define $f : V(G) \to \{0, 1, 2\}$ by

$$f(w) = 1,$$
$$f(c_1) = f(c_2) = 1,$$
$$f(v_i^{(1)}) = \begin{cases} 0 & \text{for } 1 \leq i \leq 2k+2 \\ 1 & \text{for } i > 2k+2 \end{cases}$$

and

$$f(v_i^{(2)}) = \begin{cases} 2 & \text{for } 1 \leq i \leq 2k+2 \\ 1 & \text{for } i > 2k+2. \end{cases}$$

Since $e_f(0) = e_f(1) = e_f(2) = 2k+2$, $v_f(0) = v_f(2) = 2k+2$ and $v_f(1) = 2k+3$, we have $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| \leq 1$ for all, $i, j = 0, 1, 2$. Thus $\langle B_{n,n} : w \rangle$ is 3-product cordial. □

§3. 3-product cordial labeling of cycle related graphs

Theorem 3.1. *K_n is a 3-product cordial if and only if $n \leq 2$.*

Proof. K_n has n vertices and $\frac{n(n-1)}{2}$ edges. Clearly K_1 and K_2 are 3-product cordial graphs. Conversely assume that K_n is a 3-product cordial. We consider the following three cases.

If $n \equiv 0(mod\ 3)$ then by Theorem 2.2, $\frac{n(n-1)}{2} \leq \frac{n^2-3n+6}{3}$, which implies that $n^2 + 3n - 12 \leq 0$. This is true only for $n \leq 2$.

If $n \equiv 1(mod\ 3)$ then by Theorem 2.4, $\frac{n(n-1)}{2} \leq \frac{n^2-2n+7}{3}$, which implies that $n^2 + n - 14 \leq 0$. This is true only for $n \leq 3$.

If $n \equiv 2(mod\ 3)$ then by Theorem 2.6, $\frac{n(n-1)}{2} \leq \frac{n^2-n+4}{3}$, which implies that $n^2 - n - 8 \leq 0$. This is true only for $n \leq 3$.

When $n = 3, K_3 = C_3$ which is not a 3-product cordial graph. Hence K_n is not a 3-product cordial graph for $n \geq 3$. □

Theorem 3.2. *$C_n \cup P_n$ is 3-product cordial for all $n \geq 3$.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n and $u_1, u_2, \ldots, u_n$ be the vertices of P_n. Let $G = C_n \cup P_n$.

Define $f : V(G) \to \{0, 1, 2\}$ for the following three cases.

Case i. $n \equiv 0$ $(mod\ 3)$ and let $n = 3k$.

Define f by

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq 2k \\ 1 & \text{otherwise} \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{for } i = 2m-1, 1 \leq m \leq k \\ 2 & \text{otherwise.} \end{cases}$$

Hence, $v_f(0) = v_f(1) = v_f(2) = 2k, e_f(0) = e_f(2) = 2k$ and $e_f(1) = 2k-1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Case ii. $n \equiv 1$ $(mod\ 3)$ and let $n = 3k+1$.

Define f by

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq 2k \\ 1 & \text{otherwise} \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{for } i = 2m-1, 1 \leq m \leq k \\ 2 & \text{otherwise.} \end{cases}$$

Here $e_f(0) = e_f(2) = 2k$, $e_f(1) = 2k+1, v_f(0) = 2k$ and $v_f(1) = v_f(2) = 2k+1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Case iii. $n \equiv 2$ $(mod\ 3)$ and let $n = 3k+2$.

Define f by

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq 2k+1 \\ 1 & \text{for } 2k+1 < i \leq 3k+1 \\ 2 & \text{for } i = 3k+2 \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{for } i = 3k+2, 2m-1, 1 \leq m \leq k \\ 2 & \text{otherwise.} \end{cases}$$

Hence, $v_f(0) = v_f(1) = 2k+1, v_f(2) = 2k+2$ and $e_f(0) = e_f(1) = e_f(2) = 2k+1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, $C_n \cup P_n$ is 3-product cordial. □

Theorem 3.3. *$C_m \circ \overline{K_n}$ is 3-product cordial for $m \geq 3$ and $n \geq 1$.*

Proof. Let $V(C_m \circ \overline{K_n}) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_{mn}\}$ where $u_1, u_2, \ldots, u_m$ be the vertices of the cycle and $v_1, v_2, \ldots, v_{mn}$ be the vertices of m copies of $\overline{K_n}$. Hence, we have $|V(C_m \circ \overline{K_n})| = |E(C_m \circ \overline{K_n})| = m(n+1)$.

Case i. $n \equiv 2$ $(mod\ 3)$. Take $n = 3k-1$, $m \geq 1$.

Define $f : V(C_m \circ \overline{K_n}) \to \{0,1,2\}$ by $f(u_i) = 1$ for $1 \le i \le m$;

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \le j \le km \\ 2 & \text{for } km < j \le 2km \\ 1 & \text{for } j > 2km. \end{cases}$$

Here we have, $v_f(0) = v_f(1) = v_f(2) = km, e_f(0) = e_f(1) = e_f(2) = km$. Therefore, $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$.

Case ii. $n \equiv 0$ $(mod\ 3)$ and let $n = 3k$. We consider the following subcases:

Subcase i. $m = 3c$.

Define f by $f(u_i) = 1$ for $1 \le i \le m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \le j \le c(3k+1) \\ 2 & \text{for } c(3k+1) < j \le 2c(3k+1) \\ 1 & \text{for } j > 2c(3k+1). \end{cases}$$

Here we have, $v_f(0) = v_f(1) = v_f(2) = c(3k+1)$ and $e_f(0) = e_f(1) = e_f(2) = c(3k+1)$. Therefore, $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$.

Subcase ii. $m = 3c + 1$.

Define f by $f(u_i) = 1$ for $1 \le i \le m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \le j \le c(3k+1) + k \\ 2 & \text{for } c(3k+1) + k < j \le 2c(3k+1) + 2k \\ 1 & \text{for } j > 2c(3k+1) + 2k. \end{cases}$$

Hence we have, $v_f(0) = v_f(2) = c(3k+1) + k, v_f(1) = c(3k+1) + k + 1$, $e_f(0) = e_f(2) = c(3k+1) + k$ and $e_f(1) = c(3k+1) + k + 1$. Therefore, $|e_f(i) - e_f(j)| \le 1$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j = 0, 1, 2$.

Subcase iii. $m = 3c + 2$.

Define f by $f(u_i) = 1$ for $1 \le i \le m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \le j \le c(3k+1) + 2k + 1 \\ 2 & \text{for } c(3k+1) + 2k + 1 < j \le 2c(3k+1) + 4k + 1 \\ 1 & \text{for } j > 2c(3k+1) + 4k + 1. \end{cases}$$

Hence we have, $v_f(0) = v_f(1) = c(3k+1) + 2k + 1, v_f(2) = c(3k+1) + 2k$, $e_f(0) = e_f(1) = c(3k+1) + 2k + 1$ and $e_f(2) = c(3k+1) + 2k$. Therefore, $|e_f(i) - e_f(j)| \le 1$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j = 0, 1, 2$.

Case iii. $n \equiv 1$ $(mod\ 3)$ and let $n = 3k + 1$. We consider the following subcases:

Subcase i. $m = 3c$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+2) \\ 2 & \text{for } c(3k+2) < j \leq 2c(3k+2) \\ 1 & \text{for } j > 2c(3k+2). \end{cases}$$

Hence we have, $v_f(0) = v_f(1) = v_f(2) = c(3k+2)$ and $e_f(0) = e_f(1) = e_f(2) = c(3k+2)$. Therefore, $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$.

Subcase ii. $m = 3c + 1$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+2) + k + 1 \\ 2 & \text{for } c(3k+2) + k + 1 < j \leq 2c(3k+2) + 2k + 1 \\ 1 & \text{for } j > 2c(3k+2) + 2k + 1. \end{cases}$$

Hence we have, $v_f(0) = v_f(1) = c(3k+2) + k + 1, v_f(2) = c(3k+2) + k$, $e_f(0) = e_f(1) = c(3k+2) + k + 1$ and $e_f(2) = c(3k+2) + k$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Subcase iii. $m = 3c + 2$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+2) + 2k + 1 \\ 2 & \text{for } c(3k+2) + 2k + 1 < j \leq 2c(3k+2) + 4k + 2 \\ 1 & \text{for } j > 2c(3k+2) + 4k + 2. \end{cases}$$

Hence, $v_f(0) = v_f(2) = c(3k+2) + 2k + 1, v_f(1) = c(3k+2) + 2k + 2$, $e_f(0) = e_f(2) = c(3k+2) + 2k + 1$ and $e_f(1) = c(3k+2) + 2k + 2$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Thus f is a 3-product cordial labeling. □

If we delete an edge from a cycle C_m of $C_m \circ \overline{K_n}$, we get a graph $P_m \circ \overline{K_n}$. Hence, we have the following corollary.

Corollary 3.4. *$P_m \circ \overline{K_n}$ is a 3-product cordial graph for $m, n \geq 1$.*

Theorem 3.5. *Fl_n is a 3-product cordial.*

Proof. Let H_n be a helm with v as the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of cycle and $u_1, u_2, \ldots, u_n$ be the pendant vertices for $n \geq 3$. Let Fl_n be the flower graph obtained from helm H_n. Then $|V(Fl_n)| = 2n + 1$ and $|E(Fl_n)| = 4n$. We define $f : V(Fl_n) \to \{0, 1, 2\}$ as follows:

Case (i) $n \equiv 0(mod\ 3)$.

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \frac{2n}{3} \\ 2 & \text{for } i = \frac{2n}{3} + j \text{ where } j \equiv 1,3(mod\ 4), 1 \leq j < \frac{n}{3} \\ 1 & \text{for } i = \frac{2n}{3} + j \text{ where } j \equiv 0,2(mod\ 4), 1 \leq j < \frac{n}{3} \end{cases}$$

$$f(u_n) = \begin{cases} 1 & \text{for } n \equiv 0(mod\ 4) \\ 2 & \text{Otherwise} \end{cases}$$

For $1 \leq i \leq n,\ f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1,2(mod\ 4) \\ 2 & \text{if } i \equiv 0,3(mod\ 4) \end{cases}$

$$f(c) = 1.$$

We have $v_f(0) = v_f(1) - 1 = v_f(2) = \left\lfloor \frac{2n+1}{3} \right\rfloor$ and $e_f(0) = e_f(1) = e_f(2) = \frac{4n}{3}$.

Case (ii) $n \equiv 1(mod\ 3)$.

For $1 \leq i \leq n,\ f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1,2(mod\ 4) \\ 2 & \text{if } i \equiv 0,3(mod\ 4) \end{cases}, f(c) = \begin{cases} 1 & \text{if } n \text{ is even} \\ 2 & \text{if } n \text{ is odd.} \end{cases}$

If n is even, $f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \frac{2n+1}{3} \\ 2 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 1,2(mod\ 4), 1 \leq j < \frac{n-1}{3} \\ 1 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 0,3(mod\ 4), 1 \leq j < \frac{n-1}{3}. \end{cases}$

$$f(u_n) = 2.$$

If n is odd, $f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \frac{2n+1}{3} \\ 1 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 1,3(mod\ 4), 1 \leq j \leq \frac{n-1}{3} \\ 2 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 0,2(mod\ 4), 1 \leq j \leq \frac{n-1}{3}. \end{cases}$

We have $v_f(0) = v_f(1) = v_f(2) = \frac{2n+1}{3}$ and $e_f(0) - 1 = e_f(1) = e_f(2) = \left\lfloor \frac{4n}{3} \right\rfloor$.

Case (iii) $n \equiv 2(mod\ 3)$.

$$f(c) = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even.} \end{cases}$$

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \left\lfloor \frac{2n}{3} \right\rfloor \\ 1 & \text{for } i = \left\lfloor \frac{2n}{3} \right\rfloor + j \text{ where } j \equiv 1,2(mod\ 4), 1 \leq j \leq \frac{n}{3} \\ 2 & \text{for } i = \left\lfloor \frac{2n}{3} \right\rfloor + j \text{ where } j \equiv 0,3(mod\ 4), 1 \leq j \leq \frac{n}{3}. \end{cases}$$

$$f(u_n) = 2.$$

For $1 \leq i \leq n$, if n is even, $f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1,2(mod\ 4) \\ 2 & \text{if } i \equiv 0,3(mod\ 4). \end{cases}$

For $1 \leq i \leq n-2$, if n is odd, $f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1,2(mod\ 4) \\ 2 & \text{if } i \equiv 0,3(mod\ 4). \end{cases}$

$$f(v_{n-1}) = f(v_n) = 2.$$

We have $v_f(0)+1 = v_f(1) = v_f(2) = \left\lceil \frac{2n+1}{3} \right\rceil$ and $e_f(0)+1 = e_f(1) = e_f(2) = \left\lceil \frac{4n}{3} \right\rceil$.

Thus is each case we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Hence the graph Fl_n is a 3-product cordial graph. □

References

[1] I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, *Ars Combin.*, **23** (1987), 201-207.

[2] J.A. Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **16** # DS6 (2010).

[3] M. Sundaram, R. Ponraj and S. Somasundaram, Product cordial labeling of graphs, *Bulletin of Pure and Applied Sciences*, **23E**(1)(2004), 155-163.

[4] M. Sundaram, R. Ponraj and S. Somasundaram, EP-cordial labelings of graphs, *Varahmihir Journal of Mathematical Sciences*, **7**(1) (2007), 183-194.

[5] R. Ponraj, M. Sivakumar and M. Sundaram, k-product cordial labeling of graphs, *Int. J. Contemp. Math. Sciences*, **7**(15) (2012), 733-742.

P. Jeyanthi
Department of Mathematics
Govindammal Aditanar College for Women
Tiruchendur-628 215, Tamil Nadu, India.

E-mail: jeyajeyanthi@rediffmail.com

A. Maheswari
Department of Mathematics
Kamaraj College of Engineering and Technology
Virudhunagar- 626 001, Tamil Nadu, India.

E-mail: bala_nithin@yahoo.co.in