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Analysis of flow dynamics on Buslaev contour networks

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ABSTRACT

Traffic flow models on networks are relevant because of exponential growth of road transport in megalopolises worldwide. But there are not enough adequate approaches to describe such processes. Buslaev A.P. with co-authors had introduced networks of contours with common nodes, local particles movement on each contour in given direction and competition resolution rules in common nodes. The problem is to study limit system behavior in dependence on initial conditions. In mathematical models of communication systems and computer networks, particles correspond to messages or information blocks (message packages). Behavior of particles on a contour chain with two cells on each contour is similar to behavior of Ising model used in quantum mechanics. In particular, Ising model is used for modeling of behavior of experimental computers based on principles of quantum mechanics. Contour network models can be used for study of spectral quantization. We study a behavior contour networks of Buslaev type for regular cases depending on rules of competition resolutions in common nodes. Exact results were obtained for closed chain with opposite and odd-even resolution rules, and stochastic version of left-priority rule with non-zero probability of indecisive movement.

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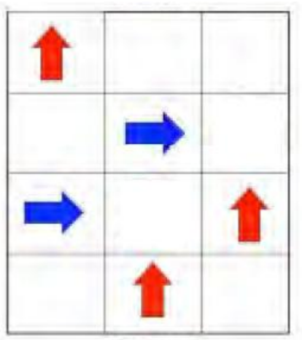
1. Introduction

Currently it is clear that classical traffic flow model of the mid-20th century have exhausted its possibilities in modeling of large number of subjects (agents) on complex networks. Agent-based approach has at least one more disadvantage as it suggests in belief of unlimited possibilities of computer. Agent-based models are unstable with respect to the approximate data and, if the set of agents is large, then errors are increasing uncontrollably. Therefore mid-level model development is relevant. The set of parameters of these models is limited. Not only simulation, but also analytical study of these models is possible. An initial example of such approach is study of movement on a ring. This study is related to cellular automata (Wolfram), dynamical systems (Blank, Gray, Griffeath, et. al.), and simulation (Nagel, Schreckenberg), [3]. Network versions of this

approach make modern branches of mathematics relevant on the one hand, and these versions allow to predict often observed states of suggested flows on networks on the other hand, [20].
We have studied dynamical systems containing contours and particles (clusters) moving along contours in a given direction with a given velocity. Discrete and continuous versions of systems are considered. Approaches to the modeling of traffic on networks were formed in the works of A. P. Buslaev [1]. The spectral properties of such models were first noted in [2].
Another well-known transport model is the two-dimensional BML model [6] (Biham, Middleton., Levine, 1992), in which two types of particles move in the perpendicular directions on a toroidal lattice, (Fig. 1). In this model, the conditions of self-organization (getting the system into a state of free movement from any initial state) and collapse (full stop of motion), [7] (D'Souza, 2005), [8] (Angel, Horloyd, Martin, 2005), [9], (Austin, Benjamini, 2006), were obtained by computer simulation method.



$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

Figure 1: BML – model and matrix of states

As results of these researches by simulation hypothesis about limit states of the system were formulated without exact statements.
We consider an approach being developed by research team of Buslaev A.P. which allows to get results in concrete formulation. The concept of contour networks was introduced in [12] (Kozlov, Buslaev, Tatashev, 2013). The system carrier is a system of closed contours with common points (nodes). In the discrete version, the contour is a closed sequence of cells. There is a certain number of particles on the contour moving along the contour at a constant speed in a given direction. Particles move at discrete points in time. An individual particle movement is considered, in which the particles are not grouped into clusters and a movement is called total-connected, in which particles are combined into clusters. Particles of the same cluster move simultaneously. The merged clusters of the same contour are combined into larger clusters. At the same time in the cell can be no more than one particle. Neighboring contours have common points (nodes). Particles (clusters) of different contours cannot cross the same node at the same time. With individual movement, delays in clusters movement occur when a particle attempts to move into an occupied cell and when particles pass through nodes. With cluster movement, delays occur only when passing through nodes. Particles (clusters) move along their contours. In the more general case, particles of contours can pass through a node to another contour.
In the continuous version, the contours can be considered as circles, on which there are clusters, representing segments with a constant length, moving at a constant speed in a given direction. Delays in clusters movement occur due to limitations due to the fact that more than one cluster cannot cross a node at the same time.
In [1] (Buslaev, Yashina, 2016), a generalized contour network with a continuous state space and a time was introduced, (Fig. 2).

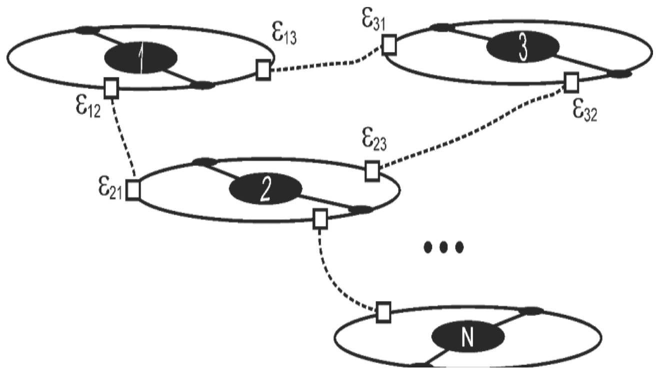


Figure 2: Contour network with continuous state space and time

In modern complex socio-technical systems the main processes of mass transfer are characterized by periodicity on a space and a time with natural boundaries on space location, i.e. in a point in fixed time there is no more one particle. In opposite case the collision can occur. Method of Buslaev contour networks gives us possibilities to generate models with necessary properties by various scenarios.

In the paper we consider a closed chain of equal contours with two symmetrical alternating nodes. The paper has the following structure. In section 2 we formulate the Buslaev contour networks concept. The spectrum definition as characterization of behavior for the dynamical system of this type is presented in the section.3. In the Section 4 we formulate the closed chain system for which the main results have been obtained. Also, we give definitions of deterministic rules of competition resolution in alternating nodes with left-(right)-priority, opposite and odd-even types. The Section 5 presents a study of system spectrum in the deterministic rule cases. Section 6 introduces the stochastic variation of left-priority rule when the winning particle moves with probability lesser than 1, i.e. "indecisive" movement. Section 7 summarize the research results and future works.

2. Structure of Buslaev contour networks

Works of Buslaev A.P. et al introduce the following concept of *contour networks.* We consider the discrete case when a space is quantizing and particles move on cells consistently.

(A) *Geometry:* we have a *system of contours*. The *coordinates system* is given on each contour. Neighboring contour connects in *common nodes*. The node can be a cell, where a particle may be located, and can be alternating node, throw which particles can pass without a stop. Data structure of system's geometry is Nodes Matrix, the values of which matches the nodes coordinates.

(B) *Particle movement mode*: On contours the particles move in given *direction*. The direction can be the same for each contour, can alternate or can be personally assigned for each contour. We differ the *movement mode*. Individual movement is a rule when a particle can move forward if a cell is vacant. In Wolfram classification we have CA 184. *Connected movement* is the process when, if in a time the several particles are located consistently in neighboring cells, then next time the head particle moves into vacant cell with the all other particles together. In Wolfram classification we have CA 240.

(C) *Competition in Nodes:* In the common nodes the competition can be located, if particles on neighboring contours simultaneously come to the node and want to enter to it next time moment. In these cases the competition resolution rule must be defined. It can be deterministic, stochastic or combined. In result of rule application, one of the particles wins the competition and can move to the common node or pass it in the altering node case. The lost particle stays in the same cell and has a delay.

(D) *Boundary conditions:* When a contour network is configuring, there are cases of *open and closed* network. In the case of a closed network, all contours have the same number of neighboring contours.

In the case of an open network, contours are divided by the number of neighbors into internal and boundary ones with fewer neighboring contours.

In [5], [7] canonical chains have been introduced.

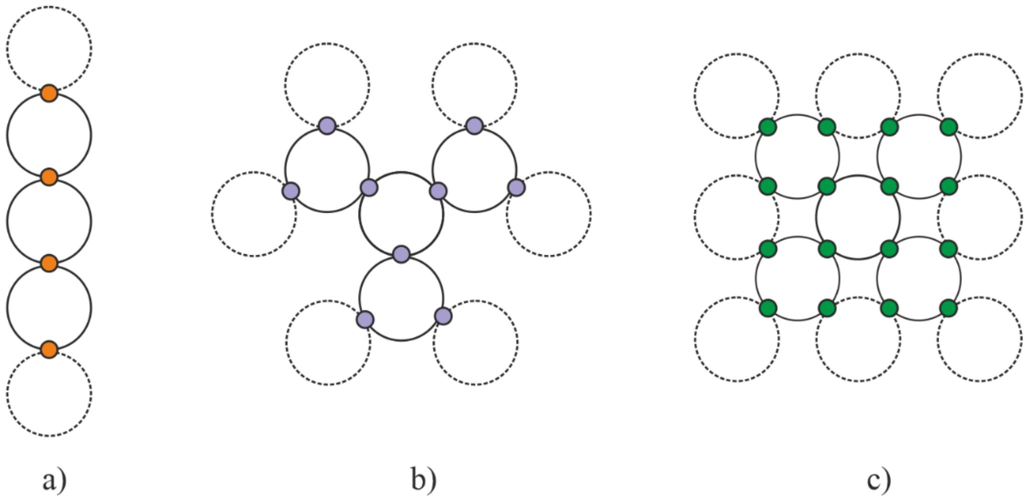


Figure 3: Closed contour: a) chain, b) honeycomb, c) chainmail

3. Spectrum

In [13], (Buslaev, Fomina, Tatashev, Yashina, 2018) the concept of a contour network spectrum and spectral cycles were introduced. In the discrete version, the set of possible states of the system is finite and, with a fully deterministic movement, the sequence of states periodically repeats from a certain point in time (spectral cycle). In the continuous versions studied by the system, it also turns out that the sequence of states is cyclically repeated from a certain point in time. Each spectral cycle corresponds to the average velocity of the particles (clusters). In the general case, it depends on the initial state, which spectral cycle is realized, and accordingly the average cluster velocities depend on the initial state. The spectral pair is the spectral cycle and the vector of cluster velocity values. The spectrum of a system is the set of spectral pairs corresponding to different initial states of the system.

Let a set X be a state space of the system. Then the dynamical system determinates a mapping A: X→ X

For every element *x* in X we have trajectory in state space X

$$x \rightarrow A(x) \rightarrow A(A(x)) \rightarrow \ldots \rightarrow A(A\ldots A(x)) \rightarrow \ldots$$

As X is finite set then for some state at moment t we have $X(t_0) = X(t_0+T^*)$, where T* is a period.

The cycle $\{X(t_0), X(t_{0+1}), \ldots, X(t_0+T^*-1)\}$ in the state space X is called the spectral contour (cycle) or wreath of the considered dynamical system. The tail of a cycle is the subset of system states such that from this state the system results in a state in the cycle over a limited time.

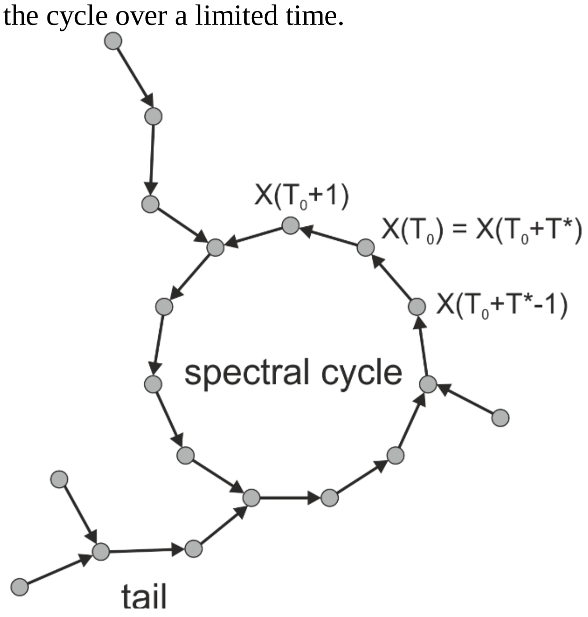


Figure 4: Spectral cycles with tails

The ***spectral eigenvalue of system spectrum*** (average velocity of system) λ is defined as

$$\lambda = \frac{V(T*)}{NT*}$$

where *V(T*)* is the total distance which all particles of the system pass during the spectral cycle, *T** is the period, *N* is the number of particles.
Let *V(t)* be the total distance which all particles of the system pass during the time interval (0, T). Then

$$\lambda = \lim_{T\to\infty} \frac{V(T)}{NT}$$

The objectives of the research are to study the spectrum of the system, in particular, the dependence of the average velocities of particles (clusters) on the initial state of the system, finding the conditions for self-organization of the system (for any initial state, the system falls into a state of free movement, i.e., clusters move from a certain moment of time without delay) and collapse (since some moment of time no particle moves).
We consider systems containing two or three contours, a given number of contours with one common node and networks whose carriers represent a system of contours with a regular, periodic, symmetric structure (open and closed chains of contours, two-dimensional rectangular and toroidal structures).

4. Closed contours chain with different competition resolution rules

In [14], a dynamic system, called a binary closed chain of contours, was investigated.

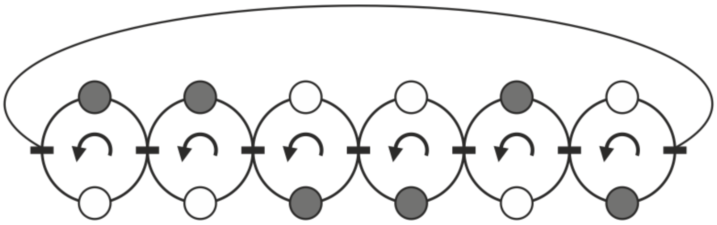


Figure 5: Binary closed contour chain with N=8 with state (1,1,0,0,1,0)

There are N *contours*, each of which has 2 *cells* and one *particle*, Figure 5. Each node has a common node with two adjacent contours located on each contour between the cells. At each discrete point in time, the contour is in the 0 or 1 state, depending on whether there is a particle in the upper or lower contour cell that moves counterclockwise along the contour. The condition of each contour is changed at each step, if no more than two particles simultaneously intersect the same node. If two particles try to simultaneously cross one common node, then a competition occurs. In the event of a competition, only one particle is moved, selected using the *competition resolution rule*.
The following competition resolution rules are considered in [14]. In the case of the *left-priority (right-priority) rule for competition resolution*, a priority is granted to the particle located to the left (right) of the node.
Under the *even-odd* rule, the particle on the even-numbered contour has the advantage.
A *fair* rule is a stochastic rule, in which particles in competition are given an advantage with equal probability.
Let us define conflict resolution rules, (Fig. 6-8)

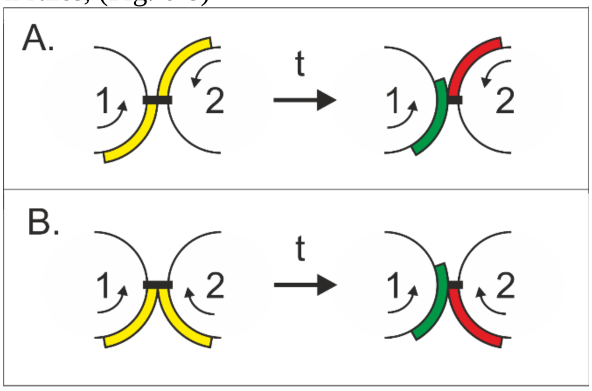


Figure 6: Left – (right -) priority rule

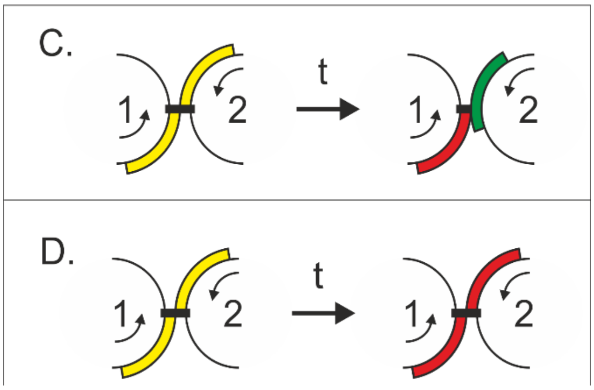


Figure 7: Deterministic rules

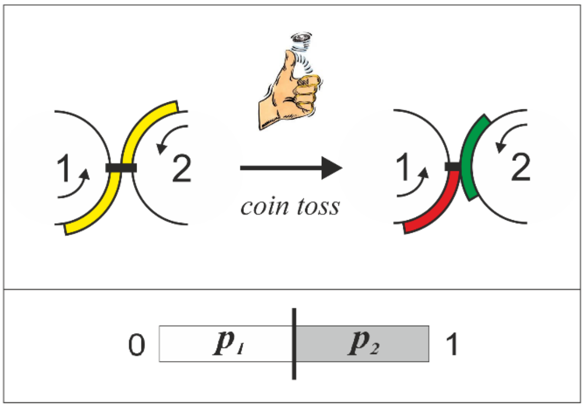


Figure 8: Stochastic resolution rules

Definition. *Opposition rule* - if particles simultaneously approach a node, then they both stop.
The rule of *opposition* is also can be considered, in which particles in competition do not move.

5. Spectrum 0f closed chain in the case of deterministic competition resolution rules

Suppose particles move counterclockwise. Then a binary closed chain with a left-priority rule is equivalent to the CA 063 elementary cellular automaton (CA) in the Wolfram classification, [15], and a binary closed chain with a rule of opposition is equivalent to CA 029.
On the spectral cycle of a binary closed chain of contours, (Fig. 5), with a left-priority competition resolution rule, either all particles move (*free movement*) at each step, or *k* particles do not move at each step, where *k* is an arbitrary natural number not exceeding the integer part of *n/3*, at the same time, in two steps, the state vector is cyclically shifted one position to the right. The time averages of the particles velocities are the same. The spectrum of time-average particle velocities at different initial values consists of

$$v = 1 - \frac{k}{N},\ k = 0,1, \dots, \left[\frac{N}{3}\right],$$

v — average particle velocity. Under the even-odd rule, the priority particles move at each step, and any non-priority particle moves at each step, if adjacent right and left contours are in each time point in the same state and the non-priority particle moves once every 2 steps, if adjacent priority particles are in various states.
The following statement is proved.
Theorem 1. Under the rule of opposition, the spectrum of average particle velocities over time consists of the values

$$v = 1 - \frac{2k}{N},\ k = 1, \dots, \left[\frac{N}{2}\right].$$

With a fair rule, the system enters a state of free movement over time with a finite mathematical expectation.
In [16], a continuous system was studied, similar to the system considered in [14].
Theorem 2. Under the even-odd rule, there are $\left[\frac{N}{\mathrm{T}}\right] + 1$ different values of average velocities

$$v = 1 - \frac{k}{N}, k = 0,1, \dots, \left[\frac{N}{4}\right];$$

N is an even number. The number of different average velocities grows linearly with increasing *N*. The number of different spectral cycles grows exponentially with increasing *N*.

Proof. Let *N* be an even number. Suppose 0,1, …,*N*-1 are numbers of contours. The contours *i*-1, *i*+1 (addition by modulo *N*) are neighboring contours to the left and to the right of contour *i*. The index of any priority contour is even, and the index of any non-priority contour is odd. At any step, each priority contour changes its state if there is no delay. There is a delay if a non-priority contour is in the state 0 and the neighboring contour on the right is in the state 1, or a non-priority contour is in the state 1 and the neighboring contour on the left is in the state 0.
Assume that *N* is divided by 4. Then there exists a state such that *N*/2 non-priority contours are located between two priority contours being in different states. The number of non-priority contours, located between two priority contours being in different states, is even. Denote this number by 2*k*. The number *k* can be any integer number from 0 to *N*/4. Suppose *k* is a fixed number. Then the velocity of particles equals 1 excluding 2*k* non-priority particles such that their velocity equals 1/2. Thus the average velocity of particles is equal to

$$v = 1 - \frac{k}{N}, k = 0,1 \dots, \frac{N}{4} = \left[\frac{N}{4}\right]$$

Suppose *N* is odd but this number is not divided by 4. The proof is such as in the case of *N* divided by 4 excluding the maximum number of non-priority contours, locating between priority contours that are in different states, is equal to (*N*-2)/2. Suppose

$$2k = \frac{N-2}{2}$$

Then we have

$$k = \frac{N}{4} - \frac{1}{2} = \left[\frac{N}{4}\right]$$

Therefore the number of spectral cycles is equal to $2^{(N-2)/2}$. Indeed, if the state of one of contours is fixed, then the states of the other contours can be chosen by $2^{(N-2)/2}$ ways.

6. Closed contour chains with stochastic version of left-priority competition resolution rule

In [17] stochastic binary closed chains of contours with a left-priority rule and an even-odd rule were considered, differing from deterministic chains in that if the corresponding deterministic chain is in a state such that the given particle moves, then in the stochastic chain the particle moves with probability $1 - \varepsilon$, where где $\varepsilon > 0$ is a given small number. In [16], as examples of the system behavior, there are cases when the number of contours is *N* = 3 or *N* = 4.
The following can be proved.
Theorem 3. Let number of contours *N* be arbitrary.
(1) Suppose that the competition resolution rule is a left priority. Then, as $\varepsilon \to 0$, it turns out that at each moment with a probability tending to 1, in the steady state, all the contours are in the same state, and in the next step all the contours are in the opposite state.
(2) Under the rule of an even-odd system, over long time intervals it cycles through a sequence of states corresponding to one of the spectral cycles, and all spectral cycles are equiprobable. The average speed of non-priority particles is equal to 3/4.
(3) With the rule of counteraction with even *N*, the system gets into a state of collapse with finite mathematical expectation, and with odd *N* into a state in which one particle moves at each step, and the remaining particles remain in place.
Suppose that the number of spectral cycles in a deterministic closed chain is equal to M. We enumerate these spectral cycles in an arbitrary way. Let G_i be the set of states of the stochastic chain, such that in the deterministic chain the corresponding states belong to the i-th spectral cycle, $i = 1, \dots, M$; $G_0$is the set of states of the stochastic chain corresponding to the states of the deterministic chain that do not belong to any spectral cycle. We define a random process $X(t)$ as follows: $\xi(t) = i$, if at the time t the stochastic chain is in the state i, $i = 0,1, \dots, M$. The task of studying the characteristics of the random process $\xi(t)$ can be included to future works.

7. Conclusions

The basic concepts of Buslaev contour networks are considered. The influence of the types of competition resolution rules on the behavior of a dynamic system is investigated. For opposition and odd-even rules the exact results are obtained.

The future works will include:

- research of the behavior of non-canonical open and closed chains of contours (call a chain of contours non-canonical, if the nodes located on the contour divide the contours into unequal parts).
- research of the behavior of canonical closed chains of contours with an arbitrary number of contours.
- research of the behavior of open and closed chains of contours with the same length contours and unequal clusters.
- research of two-dimensional open and closed contour networks.
- research of a two-circuit system with the possibility of transition of particles (clusters) to another circuit.
- research of stochastic closed binary chains of contours.

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