

FORECAST PERFORMANCE OF UNIVARIATE TIME SERIES AND ARTIFICIAL NEURAL NETWORK MODELS

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ABSTRACT

In this paper, the better model for forecasting Nigeria monthly Precipitation time series data that exhibit seasonal, periodic variations and non-linearity is determined. The models considered are Seasonal Autoregressive Integrated Moving Average (SARIMA), Fourier Autoregressive (FAR) and Artificial Neural Networks (ANN) models. The accuracy of the out-sample forecast of the model considered was measured based on the following forecast evaluations sum of square error (SSE), Mean square error (MSE) and Root mean square error RMSE. From the results, the FAR model forecast was better than that of SARIMA model based on the values of the forecast evaluations when seasonal and period in the series is considered and ANN model forecast was better than both FAR and SARIMA when the non-linear nature of the precipitation is considered. In conclusion, the FAR model is the most appropriate model for forecasting seasonal and periodic variations while the ANN model is the most suitable model for forecasting non-linearity in Nigeria monthly precipitation time series data

Keyword: Time series model, Artificial Neural Network, Periodic variation, Seasonal variation, Forecasting, Forecasting evaluation.

INTRODUCTION

Time series analysis has been used to model and obtain forecast across several fields (Wei, 2006). It is a dynamic research area which has attracted the attention of researchers' over the last few decades (Bloomfield, 2000). Time series modelling involves a careful collection and rigorous study of past observation observed at equal space and time. The primary aim involved developing or using an appropriate model which can describe the inherent structure of the series. Time series forecasting thus can be termed as the act of predicting the future by understanding the past (Weigend and Gershenfeld, 1993). Due to the indispensable importance of time series forecasting in numerous practical fields such as climate, business, economics, finance, science and engineering and many more, proper care should be taken to fit an adequate model to the underlying time series and it is obvious that a successful time series forecasting depends on an appropriate model fitting (Prabodh *et al.*, 2017).

A lot of efforts have been done by many researchers over the years to develop efficient models to improve the forecasting accuracy using time series models see Ojo and Olatayo (2009), Aboagye-Sarfo *et al.*, (2015), Asemota *et al.*, (2015), Tularam and Saeed (2016), Jere *et al.*, (2017), Phan *et al.*, (2018). Despite this fact, a vivid look must be placed to determine the variation of the series to be model exhibited since the characteristics of the series usually determine the model to be used. In time

series analysis, Autoregressive Integrated Moving Average (ARIMA) model is usually used for modelling a series that exhibits secular or trend variation (Box and Jenkins, 1970). Seasonal Autoregressive Moving Average (SARIMA) model is often used to model series that exhibits seasonal variation (Box *et al.*, 2015). But the severe limitation of these models is the pre-assumed linear form of the associated time series which becomes inadequate in many practical situations. In essence, a Fourier-Autoregressive based model which has the capabilities of handling the seasonal and periodic variations in a given time series (Taiwo, 2017). As well as artificial Neural Networks (ANN) model based on multilayer perceptron that has attracted increasing attentions in the domain of time series forecasting because of its capabilities to handle non-linearity in time series data. These models will use used to forecast Nigerian precipitation time series data. This will be carried out with particular interest to determine the better model for forecasting Nigerian precipitation time series data that has been shown to exhibit seasonal variation, periodic variation and non-linearity characteristics (Olatayo *et al.*, 2014).

MATERIAL AND METHODS

Artificial Neural Network (Multilayer Perceptron Model)

ANN model that is based on Multilayer perceptron is considered in this research. The model is given thus:

The weight of the link from i^{th} neuron in the l^{th} layer to the j^{th} neuron in the $(l + 1^{th})$ layer $w_{li,(l+1)j}$, then the i^{th} neuron in the l^{th} layer is given by

$$y_{li} = f_{li}(z_{li}); z_{li} = \sum_{j=1}^{n_{l-1}} w_{(l-1)j,i} y_{(l-1)j} + b_{li} \quad (1)$$

where y_{li} , f_{li} a b_{li} are respectively the output, activation function and bias (Olatayo and Taiwo, 2016).

Seasonal Autoregressive Integrated moving average (SARIMA) Model.

Seasonal autoregressive integrated moving average model ($S_t(p, d, q)(P, D, Q)s$) is defined as

$$\phi_p(B)\Phi_P(B^S)^d(1 - B^S)X_t = \theta_q(B)\Theta_Q(B^S)\varepsilon_t \quad (2)$$

where $\phi(B)$ and $\theta(B)$ are polynomials of order p and q , respectively; $\Phi_P(B^S)$ and $\Theta_Q(B)$ are polynomial in B of degrees P and Q , respectively; p is the order of non-seasonal autoregression; d is the number of regular differences; q is the order of non-seasonal moving average; P is the order of seasonal autoregression; D is the number of seasonal differences; Q is the order of seasonal moving average; and S is the length of season.

Fourier Autoregressive Model

Fourier autoregressive model is an extension of the class of autoregressive (AR) model. This is defined as

$$y_{k+v} = \varphi_0 + \sum_{i=1}^{p(v)} \left[\varphi_i(v) \cos \frac{2\pi i v}{\omega} + \varphi_i^*(v) \sin \frac{2\pi i v}{\omega} \right] y_{k+v-i} + \varepsilon_{k+v} \quad (3)$$

where v is the period index ($v = 1, 2, \dots, \omega$), k is the year index ($k = 0 \pm 1, \pm 2, \dots$), $\varphi_i(v)$ is the

periodic autoregressive coefficient, ω is the number of seasons and ε_{k+v} is normally identically distributed with mean zero(0) and periodic variance $\sigma_{\varepsilon}^2(v)$ (Taiwo, 2017).

Forecast Accuracy Measure

The measurement parameters for the accuracy of forecasts that will be used in this research are Mean square error (MAE), Mean Absolute percentage error (MAPE) and Root mean square error RMSE. These are given as

$$S_s = \frac{1}{h} \sum_{t=s}^{h+s} (X_t - \bar{X}_t)^2 \quad (4)$$

$$MSE = \frac{1}{N} \sum_{t=1}^N (\hat{X}_t - X_t)^2 \quad (5)$$

$$R = \sqrt{\frac{1}{N} \sum_{t=1}^N (\hat{X}_t - X_t)^2} \quad (6)$$

The actual and predicted values for corresponding t values are denoted by \hat{X}_t a X_t respectively. The smaller the values of RMSE SSE and MSE, the better the forecasting performance of the model. (Olatayo et al., 2014). Therefore, the model with minimum parameters of measurements is the most efficient model for the predictive or forecasting purpose.

RESULTS AND DISCUSSION

Nigerian monthly precipitation time series data from 1993 to 2017 obtained from the database of World Bank portal will be analysed and the forecast evaluation measures obtained from ANN, SARIMA and FAR models will be used to determine the better model for Nigerian precipitation series. From the time plot in figure 1, the precipitation series exhibits seasonal and periodic variations while as well there is present of nonlinear characteristics. This informed the use of ANN, SARIMA and FAR models.

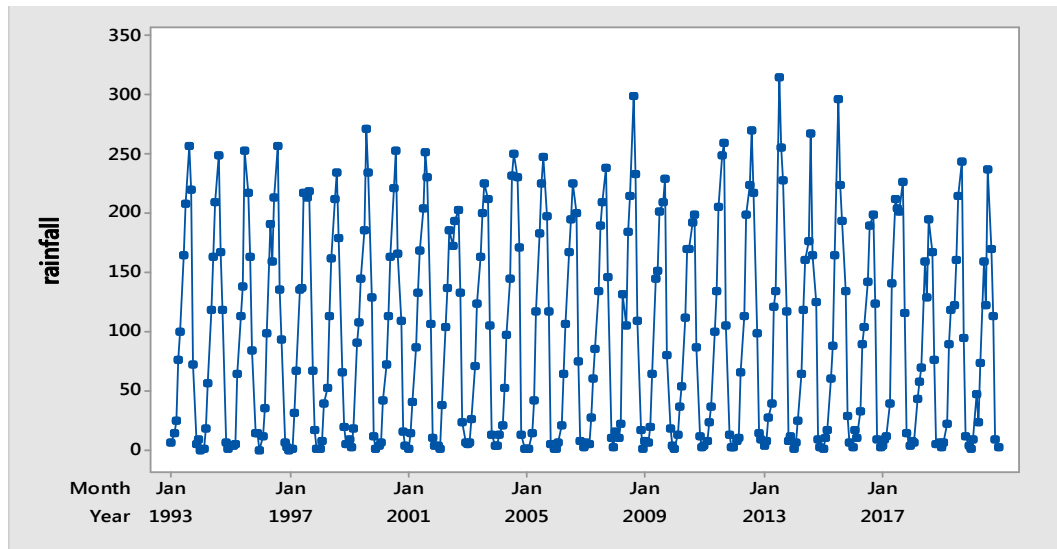


Figure 1: Time plot of Monthly Nigerian Precipitation from 1993 to 2017

Based on the Autocorrelation function and partial autocorrelation function used to select tentative models, SARIMA (1, 0, 1) × (1, 0, 0)₁₂ turns out to be the most suitable model based on the values of the information criterion. The fitted SARIMA model is shown to be the better model as the residual error followed a normal distributed based on the histogram which has a bell-shaped distribution with a p-value of 0.000. The fitted SARIMA model is

$$y_t + 0.556y_{t-1} + 0.951y_{t-3} + 0.095y_{t-1-3} = 1.942 + e_t + 0.409e_{t-1} \quad (7)$$

The forecast values from the fitted model in equation (7) deviate slightly from the original series with forecast performance measurement: SSE = 94.7077, MSE = 16458.7346 and RMSE = 128.2916.

The first stage in ANN based on multilayer perception the partitioning of the dataset and randomly assign cases based on the relative number of cases and specify relative number (ratio) of cases to each sample that is, training, testing, and holdout. Here we specify 7, 3, and 0 as the relative numbers for training, testing, and holdout samples corresponding to 70%, 30% and 0% respectively. In the architecture stage, two hidden layers and the activation function was based on sigmoid given as; $y(v_i) = (1 + e^{-v_i})^{-1}$ and it takes real-valued arguments and transforms them into the range (0, 1). The training stage was used to determine how the network processes the records. The batch training method was used since it can update the synaptic weights only after passing all training data records; that is, batch training uses information from all records in the training dataset and it is preferred

because it directly minimizes the total error and the optimization algorithm, Olatayo and Taiwo (2016).

Table 1. Case Processing Summary of Artificial Neural Network

Case Processing Summary			
		N	Percent
Sample	Training	233	69.3%
	Testing	103	30.7%
Valid		336	100.0%
Excluded		0	
Total		336	

The ANN result table 1 showed that 233 observation are trained, 103 observation was tested and this implies that 233 observations are valid. This showed that 69.3% of the observations inputted was trained and 30.7% is tested. The precipitation prediction from the ANN model with respect to multilayer perception showed a slight perfect match to the original data. The forecast performance measurement showed that SSE = 14.13, MSE = 0.301 and RMSE = 0.55.

Periodic autocorrelation and periodic partial autocorrelation function was used to choose FAR(1), FAR(2) and FAR(3) as tentative models. After the estimation, FAR(1) model was chosen as the most suitable for forecasting Nigerian monthly precipitation based on the values of the information criterion. Periodic residual autocorrelation for FAR(1) was used to show that the residual were white noise. The fitted January to December Fourier Autoregressive models is given as

$$\left. \begin{aligned}
 y_{j_1} &= 3.163271 - 1.133662 c_1(t) y_{t-1} - 0.742311 s_1(t) y_{t-1} \\
 y_f &= 9.345211 + 2.342522 c_1(t) y_{t-1} + 0.644311 s_1(t) y_{t-1} \\
 y_{m_r} &= 25.343171 - 2.192561 c_1(t) y_{t-1} + 0.184311 s_1(t) y_{t-1} \\
 y_a &= 63.153379 + 6.193441 c_1(t) y_{t-1} - 0.742001 s_1(t) y_{t-1} \\
 y_m &= 121.365321 + 5.340560 c_1(t) y_{t-1} + 1.582314 s_1(t) y_{t-1} \\
 y_{j_2} &= 167.563271 - 8.127563 c_1(t) y_{t-1} - 0.342311 s_1(t) y_{t-1} \\
 y_j &= 321.153101 - 10.053568 c_1(t) y_{t-1} - 0.043311 s_1(t) y_{t-1} \\
 y_a &= 325.165211 - 12.135567 c_1(t) y_{t-1} - 0.152311 s_1(t) y_{t-1} \\
 y_s &= 202.261171 + 3.147566 c_1(t) y_{t-1} + 0.900311 s_1(t) y_{t-1} \\
 y_o &= 102.263471 - 6.123561 c_1(t) y_{t-1} - 0.812001 s_1(t) y_{t-1} \\
 y_n &= 98.165221 - 7.143561 c_1(t) y_{t-1} - 0.122901 \sin(t) y_{t-1} \\
 y_d &= 2.963171 + 0.823900 \cos(t) y_{t-1} + 0.542311 \sin(t) y_{t-1}
 \end{aligned} \right\} (8)$$

where $t = \frac{2 \cdot k}{\omega}$. The forecasted values from equation (8) exhibited a very close match in periodic form to the original series with forecast performance measurement SSE = 25.09, MSE = 7535.7 and RMSE = 86.81.

For comparative evaluation purpose, the forecast performance measures values of FAR, SARIMA, and ANN showed that FAR model has the capabilities of forecasting Nigerian monthly precipitation better when seasonal and periodic variations are considered. But when the nonlinear characteristics of the Nigerian monthly precipitation is considered, ANN based on multilayer perception is the most suitable. Overall, based on the forecast evaluation measures that is, SSE, MSE and RMSE, FAR and ANN models outperformed SARIMA model when analysing and forecasting Nigerian monthly precipitation.

CONCLUSION

In this research work, Seasonal Autoregressive Integrated Moving Average model, Artificial Neural Network model based on multilayer perception and Fourier autoregressive model methods were used to forecast Nigerian monthly precipitation time series data from January 1993 to December 2017. The FAR model was shown to have the capabilities of forecasting Nigerian monthly precipitation better when seasonal and periodic variations are considered. But when the nonlinear characteristics of the Nigerian monthly precipitation is considered, ANN based on multilayer perception is the most suitable. Overall, based on the forecast evaluation measures that is, SSE, MSE and RMSE, FAR and ANN models outperformed SARIMA model when analysing and forecasting Nigerian monthly precipitation. Conclusively, the complexity of the nature of precipitation as a factor for climatic change with evident of seasonality, periodicity and non-linearity has been studied using SARIMA, ANN and FAR models. Therefore, any of the three models can be used to obtain forecast based on the characteristics of the time series data under evaluation.

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