*LAUTECH Journal of Engineering and Technology 12(2) 2018: 21-32*

# ANALYSES OF THE MOTION OF A PIPE CONVEYING A FLUID WITH AN INVISCID FLOW

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**ABSTRACT**

***The transverse motion of a fluid-conveying pipe with an inviscid flow was considered in this work. A comprehensive governing differential equation for the transverse vibration of a fluid-conveying pipe system was derived according to the principles of conservation of mass and momentum of fluid in a pipe. The derived equation was then analyzed and solved for an inviscid flow using the integral Fourier-Laplace Transformations. Effect of some flow parameters like mass ratios, damping coefficient and velocity of flow were investigated. Results revealed that the first natural frequency increased with increase in mass ratio for low velocities and reverse was the case for relatively high velocities. First natural frequency also increased with damping coefficients. Similar profiles can be observed about the second natural frequency. The response increased with increase in velocity. The study concluded that damping coefficient and mass ratio played significant roles in the motion of a fluid-conveying pipe with an inviscid flow.***

**Keywords:** Inviscid, Flow parameters, Damping coefficient, Natural frequency

## INTRODUCTION

Several reports have been made by authors in the area of dynamics and stability of pipes conveying fluid, among which is Païdoussis and his team (Abid Al-Sahib *et al.*, 2010). According to Abid Al-Sahib *et al.* (2010), internal fluid speed of pipes have often been neglected because of perceived small magnitude but Païdoussis (2004) in his work revealed the possibility of large riser deflections due to such small fluid speed.

The theory of linear mechanics of pipes conveying fluids is essentially based on the interaction of inertia, elastic, centrifugal, and Coriolis forces. Both centrifugal and Coriolis forces arise from the fluid flow, while the inertia force is a combination of the fluid and pipe inertias. The analysis of the linear mechanics constitutes the solution of the pipe boundary value problem for the natural frequencies and the onset of instability. The instability of a pipe conveying fluid is mainly due to a decrease in the effective pipe stiffness with the flow speed. At a critical flow speed the stiffness vanishes (Ibrahim, 2010).

Païdoussis (1998) in his work reported that a tensioned pipe conveying fluid and having symmetrical boundary conditions behaves like a conservative gyroscopic system having a total energy varying periodically in time. Small velocities of conveyance in such pipes do not cause instability but this continues until a particular velocity known as the critical velocity where the system loses instability. The dynamics of systems with mixed support conditions are said to be more complicated and result in contradictions (Païdoussis, 1998). Most literatures made use of the Euler-Bernoullis equation for fluid-pipe structure which does not consider the stress tensor of the conveyed fluid.

Equation of motion of a fluid-pipe interactive system under the influence of a magnetic field is derived in this work using the conservation principles of fluid along with the Newtonian method of force interactions with consideration given to the stress tensor of the conveyed fluid.

Underlying assumptions are:

(i) pipe's cross-section is small relative to the length

(ii) the flow is a fully developed incompressible viscous fluid

(iii) the dynamic system is under the influence of both internal and external loads

**PROBLEM FORMULATION**

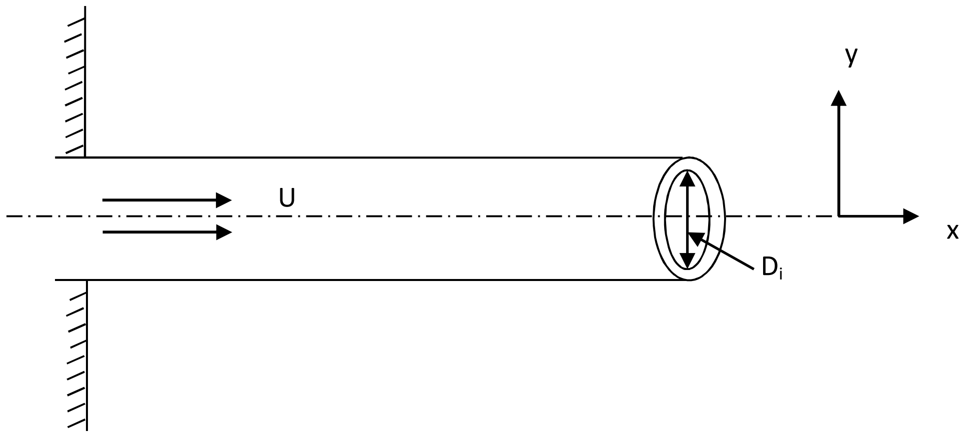


Fig. 1: A Model of the Fluid-Pipe System

**Derivation of Governing Differential Equation**

Force per unit volume in the fluid is given by

$$\frac{D\rho_f}{Dt} + \rho_f \nabla . v = 0 \tag{1}$$

$$\rho_f \frac{Dv}{Dt} = F_s + F_b \tag{2}$$

where $\rho_f$ is the density of the fluid, $v$ is the flow velocity, $\frac{D}{Dt} = \frac{\partial}{\partial t} + v.\nabla$ is a material derivative operator. $F_s$ is the surface force which is the sum of the pressure gradient, $\nabla p$, the viscosity, $\nabla.\tau$ representing the divergence of a stress tensor and $F_b$ which is the sum of the body forces stemming from the action at a distance, such as gravitational and electromagnetic forces acting on the system (Anderson, 2003; Montreal, 2016).

Eqs. (1) and (2) can be written for a Newtonian fluid under magnetic field as

$$\frac{\partial \rho_f}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho_f r v_r\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\rho_f v_\theta\right) + \frac{\partial}{\partial x}\left(\rho_f v_x\right) = 0 \tag{3}$$

$$\rho_f \frac{Dv}{Dt} = -\nabla p + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_x}{\partial \theta^2} + \frac{\partial^2 v_x}{\partial x^2}\right) + \mu_o\left(M.v_s\right)H + \rho_f g_x \tag{4}$$

where, $\mu$ is viscosity of the fluid, $\mu_o$ is permeability, M is magnetization, H is magnetic field and $v_s$ is specific volume.

Considering the assumptions of incompressible fluid, and taking symmetry into consideration Eq. (1) becomes

$$\frac{\partial v_x}{\partial x} = 0 \tag{5}$$

making,

$$\frac{\partial^2 v_x}{\partial x^2} = 0 \tag{6}$$

Then, Eq. (4) can be rewritten as

$$\rho_f \frac{Dv}{Dt} = -\nabla p + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right)\right) + \mu_o\left(\vec{M}.v_s\right)\vec{H} + \rho_f g_x \tag{7}$$

As a result of deflection, with $\varphi$ being the angle between the pipe position and the x-axis,

$$\left(T\cos\varphi\right)' dx - \left(Q\sin\varphi\right)' dx - \rho_p g dx - c dx \dot{w} = \rho_p dx \ddot{u} \tag{8}$$

$$\left(T\sin\varphi\right)' dx - \left(Q\cos\varphi\right)' dx - \rho_p g dx - c dx \dot{w} = \rho_p dx \ddot{w} \tag{9}$$

where T is the axial pre-tension in pipe (probably as a result of welding), Q is the shear force, c is the coefficient of structural damping of the pipe. $w(x,t)$ and $u(x,t)$ are transverse and lateral displacement of the system.

Due to the deflection of the pipe and the fact that a fluid will assume the shape of its containing vessel, Eq. (7) becomes

$$\rho_f \frac{Dv}{Dt} = -\left(p\sin\varphi\right)' + \left(\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right)\right)\sin\varphi\right)' + \mu_o\left(\vec{M}.v_s\right)\vec{H} + \rho_f g_x \tag{10}$$

Eq. (10) is a modified type of Navier-Stokes equation for fluid deformation in a pipe.

But,

$$\sin\varphi = \left( w' - w'u' - \frac{w'^2}{2} \right)$$

Eqs. (8) and (9) can be combined together for the transverse motion of the fluid-pipe system as

$$\left( T\left( w' - w'u' - \frac{w'^2}{2} \right)\right)' dx - \left( Q\left( 1 - \frac{w'^2}{2} \right)\cos\varphi \right)' dx - \rho_p g dx - c dx \dot{w}$$

$$-\left( p\left( w' - w'u' - \frac{w'^2}{2} \right)\right)' + \left( \mu\left( \frac{1}{r}\frac{\partial}{\partial r}\left( r\frac{\partial v_x}{\partial r} \right)\right)\left( w' - w'u' - \frac{w'^2}{2} \right)\right)' \tag{11}$$

$$+ \mu_o\left( \vec{M}.v_s \right)\vec{H} + \rho_f g_x = \rho_p dx \ddot{w} + \rho_f \frac{Dv}{Dt}$$

$\frac{Dv}{Dt}$ can be written as $$\frac{D^2 w}{Dt^2} = \left( \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} \right)\left( \frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x} \right) \tag{12}$$

and

$$\frac{D^2 w}{Dt^2} = \ddot{w} + \dot{U}w' + 2U\dot{w}' + UU'w' + U^2 w'' \tag{13}$$

The moment of the pipe-fluid system is given by

$$M = EI\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial x} = \frac{w''(1+u') - w'u''}{(1+\varepsilon)^2} w''(1+u') - w'u''$$

$$Q = -\frac{\partial M}{\partial x}\frac{1}{1+\varepsilon} = -\frac{EI}{1+\varepsilon}\frac{\partial^2\varphi}{\partial x^2} \tag{14}$$

For small strain, $\varepsilon = u' + \frac{w'^2}{2}$

So,

$$\frac{\partial \varphi}{\partial x} = \left(w''(1+u') - w'u''\right)\left(1 - 2u' - w'^2\right) \tag{15}$$

and

$$\frac{\partial^2 \varphi}{\partial x^2} = \left(w''' - w'''u' - 2w''u'' - w'u''' - w'^2 w''' - 2w'w''^2\right) \tag{16}$$

then

$$\frac{EI}{1+\varepsilon}\frac{\partial^2 \varphi}{\partial x^2} = -EI\left(w''' - w'''u' - 2w''u'' - w'u''' - w'^2 w''' - 2w'w''^2\right)\left(1 - u' - \frac{w'^2}{2}\right) \tag{17}$$

Therefore,

$$Q = -EI\left(w''' - w'''u' - 2w''u'' - w'u''' - \frac{3}{2}w'^2 w''' - 2w'w''^2\right) \tag{18}$$

$$T = To + EA\varepsilon \tag{19}$$

Substituting Eqs. (18) and (19) into (11) gives

$$\begin{aligned}
&\left(\left(To + E\left(u' + \frac{w'^2}{2}\right)\right)\left(w' - w'u' - \frac{w'^2}{2}\right)\right)' - \left(Q\left(1 - \frac{w'^2}{2}\right)\right)' \\
&+ \left(\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right)\right)\left(w' - w'u' - \frac{w'^2}{2}\right)\right)' - \left(p\left(w' - w'u' - \frac{w'^2}{2}\right)\right)' \\
&+ \mu_o\left(\vec{M}.v_s\right)\vec{H} + \rho_f g_x = \rho_p dx\ddot{w} + \rho_f \ddot{w} + \rho_f \dot{U}w' + 2\rho_f U\dot{w}' \\
&+ \rho_f UU'w' + \rho_f U^2 w''
\end{aligned} \tag{20}$$

Eq. (20) can also be written as

$$EI\begin{pmatrix} w'^{v} - w'^{v}u' - w'''u'' - 2w'''u'' \\ -2w''u''' - w''u''' - w'u'^{v} - 3w'w''w''' \\ -\frac{3}{2}w'^{2}w'^{v} - 2w''w''^{2} - 4w'w''w''' \end{pmatrix} + \left(\rho_p + \rho_f\right)\ddot{w}$$

$$-\left(\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right)\right)\left(w' - w'u' - \frac{w'^{2}}{2}\right)\right)' + \left(p + p'\right)\begin{pmatrix} w'' - w''u' \\ -w'u'' - w'w'' \end{pmatrix} \tag{21}$$

$$-\mu_o\left(\vec{M}.v_s\right)\vec{H} - \rho_f g_x + \rho_f \dot{U}w'$$

$$+2\rho_f U\dot{w}' + \rho_f UU'w' + \rho_f U^2 w'' - \left(T_o w''\right) = 0$$

Eq. 21 is the transverse equation for the motion of a fluid-conveying pipe.

**2.2 Analysis of the Governing Differential Equation**

For an inviscid flow where there is no body force Eq. (21) reduces to

$$EI\begin{pmatrix} w'^{v} - w'^{v}u' - w'''u'' - 2w'''u'' \\ -2w''u''' - w''u''' - w'u'^{v} - 3w'w''w''' \\ -\frac{3}{2}w'^{2}w'^{v} - 2w''w''^{2} - 4w'w''w''' \end{pmatrix} + \left(\rho_p + \rho_f\right)\ddot{w} + \rho_f \dot{U}w'$$

$$+\left(p - \frac{\Delta p}{L}\right)\begin{pmatrix} w'' - w''u' \\ -w'u'' - w'w'' \end{pmatrix} + 2\rho_f U\dot{w}' + \rho_f UU'w' + \rho_f U^2 w''$$

$$-\left(T_o w''\right) = 0 \tag{2 2}$$

This can be seen to be the same with the well known Euler-Bernoullis Equation for motion of a fluid-pipe structure.

**2.3 Method of Solution**

Linearizing and non-dimensionalizing Eq. (22) for a pipe with damping and under harmonic motion leads to

$$\left(\begin{array}{l}\frac{\partial^4 \overline{w}}{\partial \overline{x}^4}+\frac{\partial^2 \overline{w}}{\partial \bar{t}^2}+\left(\delta \overline{U}^2+\overline{p}_1-\overline{p}_2-\beta\right)\frac{\partial^2 \overline{w}}{\partial \overline{x}^2}+2\delta \overline{U}\,\frac{\partial^2 \overline{w}}{\partial \bar{t}\partial \overline{x}}+\overline{c}\,\frac{\partial \overline{w}}{\partial \bar{t}} \\ =\beta_1 F_o e^{i\beta_2 \overline{\omega}\bar{t}}\end{array}\right) \tag{23}$$

where,

$$\overline{x}=\frac{x}{L},\overline{w}=\frac{w}{L},\delta=\frac{m_f}{m},\overline{p}_1=\frac{pL^2}{EI},p_2=\frac{\Delta pL^2}{EI}\,\overline{c}=\frac{cL^2}{\sqrt{E\,\mathrm{Im}}},$$
$$\bar{t}=\frac{t}{L^2}\sqrt{\frac{EI}{m}},\overline{U}=UL\sqrt{\frac{m}{EI}},\beta=\frac{T_oL^2}{EI},\beta_1=\frac{L^3}{EI},$$
$$\beta_2=\omega_o L^2\sqrt{\frac{m}{EI}} \tag{24}$$

Applying Fourier complex integral transforms according to Wrede and Spiegel (2002), Jeffrey (2002), Olayiwola (2016) and Olunloyo *et al.* (2017) namely:

$$F\{\overline{w}(\overline{x},\bar{t})\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\overline{w}(\overline{x},\bar{t})e^{-i\bar{\lambda}\overline{x}}d\overline{x}=\overline{w}^F(\overline{\lambda},\bar{t})$$
$$F^{-1}\{\overline{w}(\overline{\lambda},\bar{t})\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\overline{w}^F(\overline{\lambda},\bar{t})e^{i\bar{\lambda}\overline{x}}d\overline{\lambda}=\overline{w}(\overline{x},\bar{t})$$

$$F(\overline{x},\bar{t})=\begin{cases}0 & when\ -\infty\le\overline{x}<0\\ \overline{w}(\overline{x},\bar{t}) & when\ 0\le\overline{x}\le 1\\ 0 & when\ 1<\overline{x}\le\infty\end{cases} \tag{25}$$

Eq. (23) becomes

$$\left(\begin{array}{l}\frac{d^2\overline{w}^F}{d\bar{t}^2}+\overline{c}\,\frac{d\overline{w}}{d\bar{t}}^F+2in\delta\overline{U}\,\frac{d\overline{w}}{d\bar{t}}^F+n^4\overline{w}^F-\left(\delta\overline{U}^2+\overline{p}_1-\overline{p}_2-\beta\right)n^2\overline{w}^F\\ +k_b\overline{w}^F=\beta_1F_oe^{i\beta_2\omega\bar{t}}1^F-\Pi\end{array}\right) \tag{26}$$

where, for a cantilever pipe,

$$\Pi = \frac{mg}{EI}\left(\begin{array}{l}\frac{1}{2}+\frac{in}{2}-\frac{n^2}{12}e^{-in\pi}-\frac{9in^3}{24}e^{-in\pi}+\left(\delta\overline{U}^2+p_1-p_2-\beta\right)\frac{e^{-in\pi}}{12}\\+\frac{9in}{24}e^{-in\pi}+\frac{3\delta\overline{U}}{4}e^{-in\pi}\end{array}\right) \tag{27}$$

Applying Laplace Transform to Eq. (26) gives

$$\left(\tilde{\tilde{w}}^F = \frac{\beta_1 F_o}{(s-i\beta_2\varpi)\left(s^2+\hbar_1 s+\hbar^2\right)}-\frac{\Pi}{s\left(s^2+\hbar_1 s+\hbar^2\right)}\right) \tag{28}$$

where,

$$\begin{aligned}\hbar_1 &= \bar{c}+2in\delta\overline{U},\\ \hbar^2 &= n^4\pi^4-\left(\delta\overline{U}^2+\bar{p}_1-\bar{p}_2-\beta\right)n^2\pi^2+k_b\end{aligned} \tag{29}$$

**2.4 Analysis of the natural frequency**

The natural frequencies of the system can be obtained from Eq. (28), viz

$$s^2+\hbar_1 s+\hbar^2 = 0 \tag{30}$$

Let $s = i\omega_n$ (31)

Eq. (30) can be rewritten with Eq. (31) as

$$\omega_n^{\ 2}-i\omega_n\hbar_1-\hbar^2 = 0 \tag{32}$$

**2.5 Analysis of the Dynamic Response**

The response of the system is given by applying Fourier-Laplace Inversion to Eq.(28) leading to

$$\overline{w}(\bar{x},\bar{t}) = \left(\begin{array}{l}\beta_1 F_o\left(\begin{array}{l}\frac{1}{\eta_1\eta_2}-\frac{1}{(\eta_2-\eta_1)}\left(\frac{e^{-\eta_1\bar{t}}}{\eta_1}-\frac{e^{-\eta_2\bar{t}}}{\eta_2}\right)\\-\frac{1}{i\beta_2\varpi(\eta_2-\eta_1)}\left(\frac{e^{-\eta_1\bar{t}}}{\eta_1}-\frac{e^{-\eta_2\bar{t}}}{\eta_2}\right)\end{array}\right)\left(-\frac{\bar{x}^4}{24}+\frac{\bar{x}^3}{6}+\frac{\bar{x}^2}{4}\right)\\-\sum_{-\infty}^{\infty}\left(\frac{1}{\eta_1\eta_2}-\frac{1}{(\eta_2-\eta_1)}\left(\frac{e^{-\eta_1\bar{t}}}{\eta_1}-\frac{e^{-\eta_2\bar{t}}}{\eta_2}\right)\right)e^{i\lambda\bar{x}}\end{array}\right) \tag{33}$$

## 3.0 RESULTS AND DISCUSSION OF RESULTS

### 3.1 Results

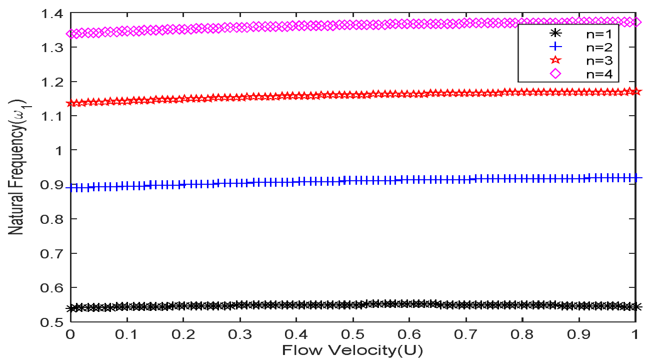


Figure 2: Natural frequency ($\omega_1$) profile as a function of L for the case

$\delta = 0.2$, L = 10 m, n = 1

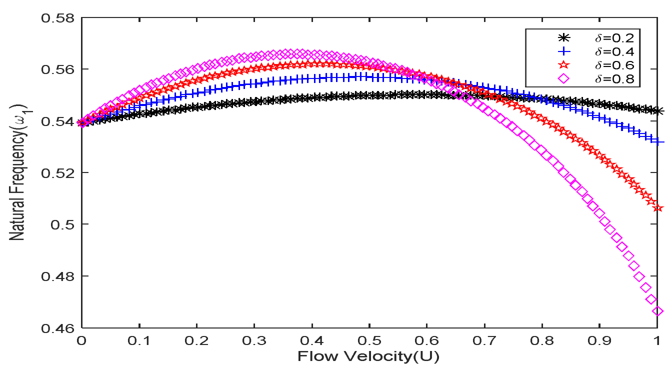


Figure 3: Natural frequency ($\omega_1$) profile as a function of delta for the case

L = 10 m, n = 1

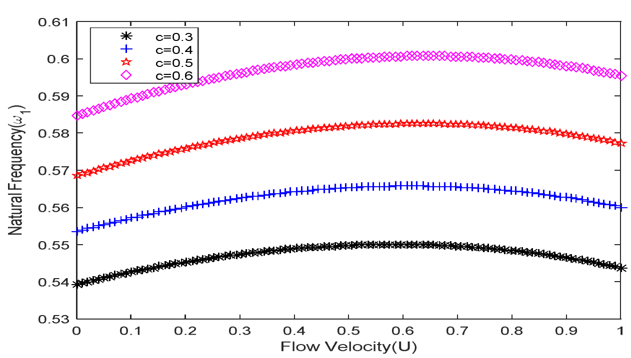


Figure 4: Natural frequency ($\omega_1$) profile as a function of damping coefficient for the case

$\delta = 0.2$, L = 10 m, n = 1

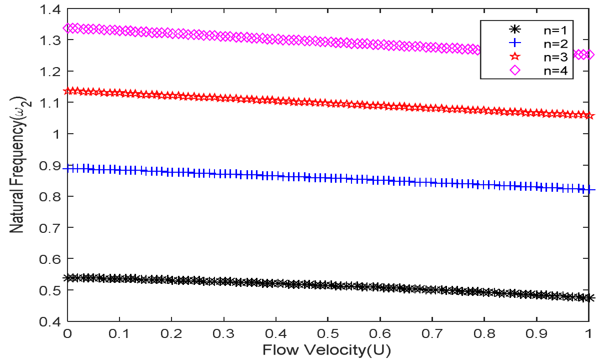


Figure 5: Natural frequency ($\omega_2$) profile as a function of n for the case

$\delta = 0.2$, L = 10 m

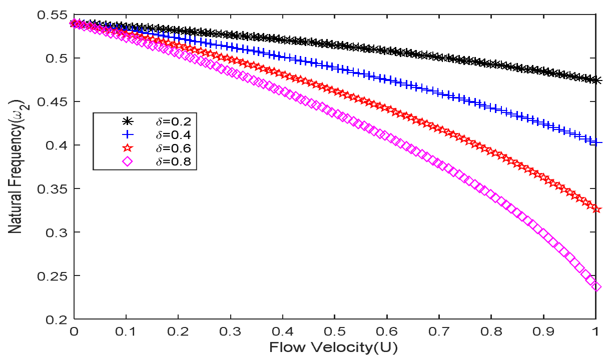


Figure 6: Natural frequency ($\omega_2$) profile as a function of delta for the case

n=1, L = 10 m

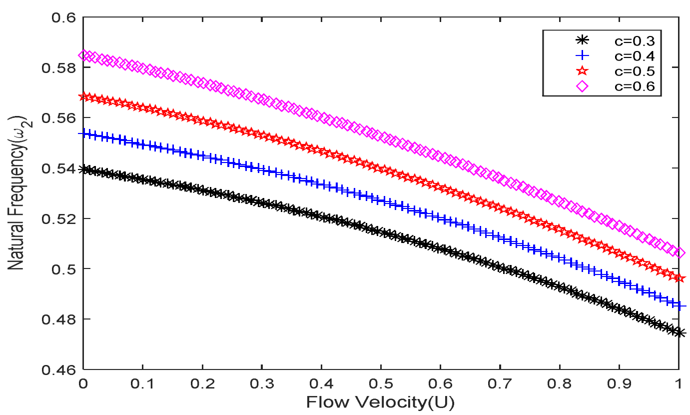


Figure 7: Natural frequency ($\omega_2$) profile as a function of damping coefficient for the case

$\delta = 0.2$, L = 10 m, n=1

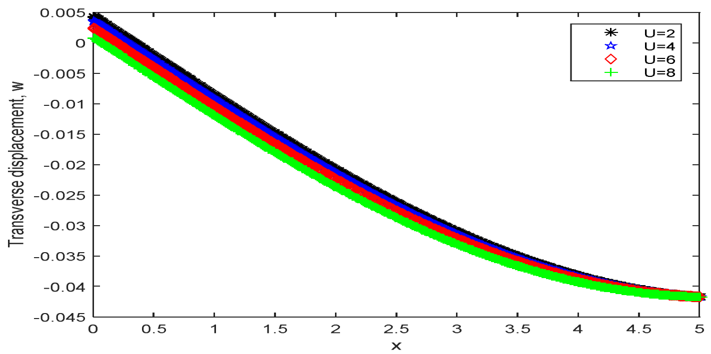


Figure 8: Transverse displacement (w) profile as a function of U for the case δ = 0.2, L = 10 m, n=1

**Discussion of Results**

The transverse vibration of a cantilever fluid-conveying pipe with inviscid flow has been investigated in this work.

A steel pipe of length 10 m with diameter 1 m, thickness 10 mm, density 7850 kg/m$^3$ conveying a fluid of density 977 kg/m$^3$ was used for the computations.

The linearized version of this problem revealed there are two basic frequencies for any solution, this is in agreement with Olunloyo *et al*. (2007). The study shows that the behaviours of the natural frequencies, $\omega_1$ and $\omega_2$, reproduce the pattern of Thurman and Mote (1969) and Olunloyo *et al.* (2007). Furthermore the natural frequencies of the system are influenced by factors like the velocity of flow. Fig.1 shows the pipe-fluid system while Figures 2-8 show the effects of varying important flow parameters like the fluid mass ratios, the damping coefficients, the modes etc on the performance of the fluid-pipe structure.

The first natural frequency is plotted against the dimensionless fluid flow in Figures 2-4. Modes can be seen to influence the natural frequency in Fig.2, as an increase in mode leads to an increase in the natural frequency. In Fig.3, two regions are observed, initially, the same value of natural frequency is noted for the mass ratios considered which later increased with increase in velocity and mass ratios until a particular value of the dimensionless velocity is reached and then a reverse is observed.

Figure 4 is the plot of the first natural frequency against dimensionless flow velocity as a function of damping coefficients. It is seen that increase in damping coefficient is associated with increase in the natural frequency, but natural frequency is generally observed to increase with dimensionless flow velocity until the value of 0.6 and then decreased.

Figs.5-7 present the graphs of the complimentary frequency against the dimensionless flow velocity for the system under consideration. Fig.5 has a similar profile with Fig.2 where natural frequency increases with increase in mode. Unlike in Fig.3 where two regions are observed for mass ratios, only one can be seen in Fig.6, here, natural frequency decreases with mass ratios. In Fig.7, whereas, the natural frequency generally increases with damping coefficient, it also decreases with increase in fluid velocity.

The response of the system is shown in Fig.8 which increases negatively with increase in velocity.

## CONCLUSIONS

The motion of a fluid-conveying pipe with an inviscid flow was investigated in this work. A compressive equation for the transverse vibration of a pipeline conveying fluid was derived using the conservation principles of continuity and momentum. The equation was analysed and solved for an inviscid flow with the aids of Fourier-Laplace Transformations and important parameters like mass ratio, damping coefficient and velocity were shown to influence the performance of the system. This study concluded that damping coefficient and mass ratio played important roles in the motion of the system.

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