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Measurement points selection strategy to update models in frequency domain

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In model updating procedures, the experimental data provide information for a configuration error modelling. The selection of the measurement DOFs in the experimental frequency response function should make the updating procedure as capable as possible of detecting configuration mismodelling in the initial model. The selection of the measurement DOFs should be sensitive to the effects of the possible configuration mismodelling in the model. For the model updating procedure, the experimental data should be measured in such a way that the modified model resulting from the updating procedure is the most likely to be reckoned as an updated model. This paper proposes a technique for selecting measurement points that should provide the best structural information for updating. The selected measurement points would define the defects that can be detected. The proposed method is based on the derivative of the frequency response function which materializes the participation of each element on the variation of the measured frequency response.

1 Introduction

Several model updating methods currently exist; these often work well when the distance between the analytical model and the experimental structure is not very large. Nevertheless, their robustness depends on their sensitivity to noise and the number and location of exploited measurements. Indeed, this matter of sensors placement is of a paramount importance as one cannot measure all degrees of freedom; in practice, some DOFs are not accessible and often the number of DOFs of the structure is by far higher than the number of sensors that are usually used in the laboratory. Also, if the number of measurement points is insufficient or badly distributed, the updating process may be impossible. The effect of limited degrees of freedom is a parameter which should be considered [1]).

The question that still arises today in the field of model updating and structural damage detection is how to choose measurement points and their corresponding degrees of freedom to better update structural model. This is the objective we

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attempt to attain in the proposed procedure in the frequency domain. The main idea is to avoid measuring at degrees of freedom that would give the same information appreciably.

Many authors have contributed on this subject. Among all the procedures we may find in the literature, we will describe in the following just the fundamental procedures ones.

Kammer [2], presents a method called Effective Independence (EfI) to place a small number of sensors for modal identification of large space structure; this method is also used by Yao et al. [3] for comparison with genetic algorithm method (GA) to conclude that the sensor location obtained by GA correspond to larger fitness than the sensor locations estimated by the EfI method. To overcome the problem of selecting the measurement DOFs with low response, Imamovic [4] proposed a modified method, called the Average Driving DOF - Effective Independence (ADDOF-EI) method for selecting as few measurement DOFs as possible to identify the measured modes as linearly independent as possible. This algorithm does not differ very much from the algorithm of the basic EfI method. Reynier and Abou-Kandil [5] proposed two methods : the first one is based on minimisation of noise effect in least squares sense and the second one is based on the observability gramian, knowing that the smallest eigenvalues of the observability gramian represents the poorest case of information. Xia and Hao [6] introduced a new concept of damage measurability in terms of two sensitivity factors, namely the sensitivity of a residual vector to the structural damage, and the sensitivity of the damage to the measurement noise. He et al. [7] proposed a statistical method named tolerance domain, combined with modal independence method. Trendafilova [8],[9] used mutual information to find the optimal distance between measurement points so that no information is lost nor information is doubled; this method is a technique for determining a measurement point configuration based on the optimal distance between the sensors. The technique used by Skelton and Li [10], called economic design ED, minimizes the total required precision while satisfying the system performance constraints. The majority of these methods are applied in the modal domain and are all compared to the EfI technique which remains the reference method.

The technique of sensor placement proposed in this paper is for frequency domain updating method.

Based on a predefined number of sensors, the proposed method is a technique of measurement points selection which should provide the information of the structure that is sensitive to the updating parameters Asma and Bouazzouni [11].

2 Updating Method

The choice of the measurement points for updating finite element model depends mainly on the mathematical form of the updating method used. The one we consider here may be based on either modal data or frequency response measurement. The proposed sensors placement strategy is intended for frequency responses measurements (FRF) based updating methods. This family of methods employs the measured FRF data and optimizes an objective function which is a difference between the initial values and those to be determined.

The updating procedure used here consists of a least squares frequency domain minimization procedure [12], which uses the following output error:

$$\delta = Trace\left([E]^H [E]\right) = \sum_{t=1}^{s} \{e(\omega_t)\}^H \{e(\omega_t)\} \tag{1}$$

where [E] is the matrix containing columns of errors $e(\omega)$, Euclidean difference between measured and analytical response vectors respectively $\{y(\omega)\}^{(s)}$ and $\{y(\omega)\}$ of any frequency ω_t.

$$\{e(\omega_t)\}=\{y(\omega_t)\}-\{y(\omega_t)\}^{(s)} \tag{2}$$

Assuming the well known parameterization of the structure, the global mass and stiffness matrices are expanded into a linear sum of submatrices such as:

$$[M] = \sum_{i=1}^{Ne} m_i [M]_i^{(e)} \quad \text{and} \quad [K] = \sum_{i=1}^{Ne} k_i [K]_i^{(e)} \tag{3}$$

where $[M]_i^{(e)}$ and $[K]_i^{(e)}$ are the mass and the stiffness elementary matrices of the i^{th} element expanded to the dimension of the global structure.

m_i, k_i are the unknown mass and stiffness correction parameters of i^{th} element; the case where $m_i = k_i = 1$ corresponds to well modelised i^{th} element.

The damping matrix is assumed to be proportional to the mass and stiffness matrices: $[B] = \alpha[M] + \beta[K]$.

The solutions of the optimization method are given by:

$$\frac{\partial\delta}{\partial m_i} = 0 \text{ and } \frac{\partial\delta}{\partial k_i} = 0 \ \ (i=1...,N_e) \tag{4}$$

This led to a system with $2N_e$ unknowns with $2N_e$ equations.

$$\begin{cases} \sum_{t=1}^{s}(-\omega_t^2 + j\alpha\omega_t)\left(\{e(\omega_t)\}^H\left([Z(\omega_t)]^{-1}.[M]_i^{(e)}.\{y(\omega_t)\}\right)+\left([Z(\omega_t)]^{-1}.[M]_i^{(e)}.\{y(\omega_t)\}\right)^H\{e(\omega_t)\}\right)=0 \\ \sum_{t=1}^{s}(1+j\beta\omega_t)\left(\{e(\omega_t)\}^H\left([Z(\omega_t)]^{-1}.[K]_i^{(e)}.\{y(\omega_t)\}\right)+\left([Z(\omega_t)]^{-1}.[K]_i^{(e)}.\{y(\omega_t)\}\right)^H\{e(\omega_t)\}\right)=0 \end{cases}, \ (i=1...,N_e) \tag{5}$$

The Newton-Raphson resolution leads to an iterative system of the form:

$$\underbrace{\begin{pmatrix} m_1 \\ m_2 \\ ... \\ m_{Ne} \\ k_1 \\ k_2 \\ ... \\ k_{Ne} \end{pmatrix}^{(v+1)}}_{p^{(v+1)}} = \underbrace{\begin{pmatrix} m_1 \\ m_2 \\ ... \\ m_{Ne} \\ k_1 \\ k_2 \\ ... \\ k_{Ne} \end{pmatrix}^{(v)}}_{p^{(v)}} - \left(J^{(v)}\right)^{-1}.\begin{pmatrix} \frac{\partial\delta}{\partial m_1} \\ \frac{\partial\delta}{\partial m_2} \\ ... \\ \frac{\partial\delta}{\partial m_{Ne}} \\ \frac{\partial\delta}{\partial k_1} \\ \frac{\partial\delta}{\partial k_2} \\ ... \\ \frac{\partial\delta}{\partial k_{Ne}} \end{pmatrix}^{(v)} \tag{6}$$

where $p^{(0)}$ is the initial estimation and J is the Jacobian matrix.

$\bullet^{(v)}$ representing the iteration step v.

Before minimizing the cost functions (1), a frequency parameterization is used in order to assign analytical frequency values to the chosen experimental frequency ones. We use a frequency parameterization presented in [13] which minimizes the error vector (2). This leads to:

$$\{e(\omega_t)\}= \{e(\omega_{At},\omega_{Xt})\}=\{y(\omega_{At})\}-\{y(\omega_{Xt})\}^{(s)}$$

$$\omega_{At}^{(k)} = \omega_{At}^{(k+1)} - \left\{\frac{\partial e(\omega_{At},\omega_{Xt})}{\partial\omega_{At}}\right\}\{e(\omega_{At},\omega_{Xt})\} \tag{7}$$

with

$$\omega_{At}^{(0)} = \omega_{Xt}$$

Knowing that ω_{Xt} is the frequency at which the measurement is taken and ω_{At} is the corresponding analytical one.

As the unmeasured degrees of freedom are replaced by their analytical counterparts in (2), this is thus a reduction technique. The efficiency of the method depends on the number and the positioning of the measurements points and experimental noise. The number and location of the measurement points must be chosen so that the measured response functions are sensitive to the type and location of structure's errors.

The choice of the parameters to be updated is an important task when updating models of complex structures. While choosing too many parameters increases the computing time, working with a small number of parameters limits updating possibility and may not help find an optimal solution. The way to come about this problem of selecting the optimal parameters is to carry out a sensitivity analysis.

3 Choice of the measurement points

Let us consider a structure with n degrees of freedom whose linear dynamic behaviour in the frequency measurement band $\omega_i \in \{\omega_1, \omega_2 .., \omega_s\}$, "s" is the number of working frequencies is described by the equations of motions.

$$\{y(\omega_i)\} = \left(-[M]\omega_i^2 + j\omega_i[B] + [K]\right)^{-1}\{f(\omega_i)\} \tag{8}$$

where [M], [K] and [B] are respectively n×n mass, stiffness and damping matrices.

$\{y(\omega_i)\} = \{y_1(\omega_i), y_2(\omega_i), \ldots, y_n(\omega_i)\}^T$ is the response vector, $\{f(\omega_i)\} = \{f_1, f_2, \ldots, f_n\}^T$ is the input vector. ω_i is the frequency measurement with $\omega_i \in \{\omega_1, \omega_2 .., \omega_s\}$, "s" is the number of excitation frequencies.

Let "r" be the number of sensors to be optimized. If initially all DOFs are assumed to be measured, we will thus have a matrix Y constituted by the columns $y(\omega_i)$

$$[Y] = [y(\omega_1) \quad y(\omega_2) \quad \ldots \quad \ldots \quad y(\omega_s)] \tag{9}$$

Let us then calculate $\frac{\partial [Y]}{\partial p_i}$; this matrix represents the influence on measurements [Y] of either mass (p_i=m_i) or stiffness (p_i=k_i) modelling defect of the i^{th} element. The sensitivity matrix should be capable of taking into account the effects on dynamic behaviour of the structure induced by a unit variation of the updating parameters [1]. In our case, consider the information matrix I defined by Asma and Bouazzouni [11]:

$$[I(p_i)] = \left(\frac{\partial Y}{\partial p_i}\right)\left(\frac{\partial Y}{\partial p_i}\right)^H, \ [I] \in R^{n\times n}, \ i=1,2N_e \tag{10}$$

where $(\ )^H$ denotes complex conjugate transpose.

The fact that the EfI method considers the trace of $[\Phi^T\Phi]$ in modal domain is very interesting for modal data based updating methods. In the case of frequency response based updating methods we may consider the diagonal of the information matrix which represents the global influence of either mass or stiffness variation of the i^{th} element on measurements:

$$\{v\}_i = Diag([I(p_i)]), \ \{v\}_i \in R^n, \ i=1,2N_e \tag{11}$$

This represents the sensitivity of the response vector to changes in structural parameter (i.e. damage in structures). The larger components of the vector $\{v\}_i$ correspond to the DOF of greater sensitivity to i^{th} element damage.

Having determined $\{v\}_{i,}$, the updating procedure can then be performed. These vectors are all sensitive to damage in one or another structural element.

To perform the strategy, the sum of all vectors $\{v\}_i$ called witness vector W is calculated as follow:

$$\{W\} = \sum_{i=1}^{2Ne} \{v\}_i \text{ , } \{W\} \in R^n \quad (12)$$

Since a large term in W will cause a larger residual vector in (2), to a change in parameters p (damage), the damage will be easily and accurately detected. It is then interesting to place a sensor on the DOF corresponding to the larger component of the witness vector.

In a second way, we should make measurement independent in the sense of information, none information is lost and none information is redundant. Thus, we should remove all DOFs which give identical or linearly dependent overall information, i.e. those which do not increase the rank of the measured matrix Y.

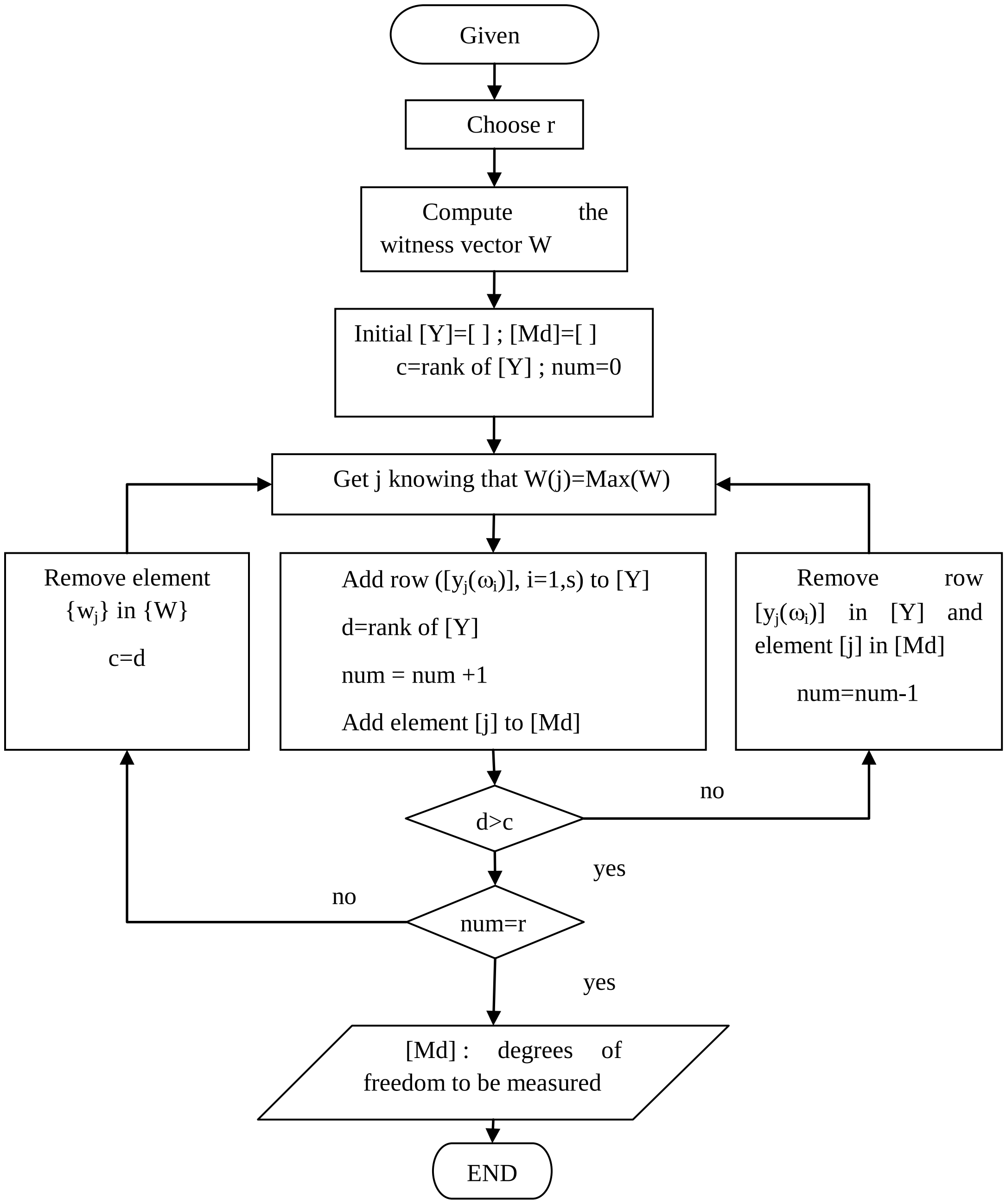

Fig. 1 – schematic of the proposed strategy

Finally, the proposed measurement point selection method consists of three steps:

compute the witness vector {W} using (12);

the maximum value of {W} corresponding to the first DOF to be measured, the measured response vector Y is constructed by the row of the corresponding DOF;

the next DOF to be selected is the one corresponding to the component of second largest value of {W} and increasing the rank of Y when the corresponding row is added to the Y matrix.

Repeat step c until the number of measurement points is equal to the predefined number “r”.

This is materialised by the corresponding organigram showed on figure 1.

4 Numerical Results

To simulate measurements, random noise is added according to the model Trendafilova and Heylen [8]:

$$\{y(i,\omega_{Xt})\}^{(s)} = (1+g.d.n)\{y(i,\omega_{Xt})\} \tag{13}$$

where g is equal randomly to 1 or –1,

d : random value between 0 and 1,

n : noise percentage.

$\{y(i,\omega_{Xt})\}^{(s)}$: Simulated response measurement

$\{y(i,\omega_{Xt})\}$: Calculated response measurement

Simulation was carried out up to a 5% rate noise.

To improve the efficiency of measurement point’s choice and taking into account the updating procedure, consider the plane doubly embedded lattice structure made up of 30 welded beams (figure 2). The structure is discretised into 30 finite elements with 39 degrees of freedom.

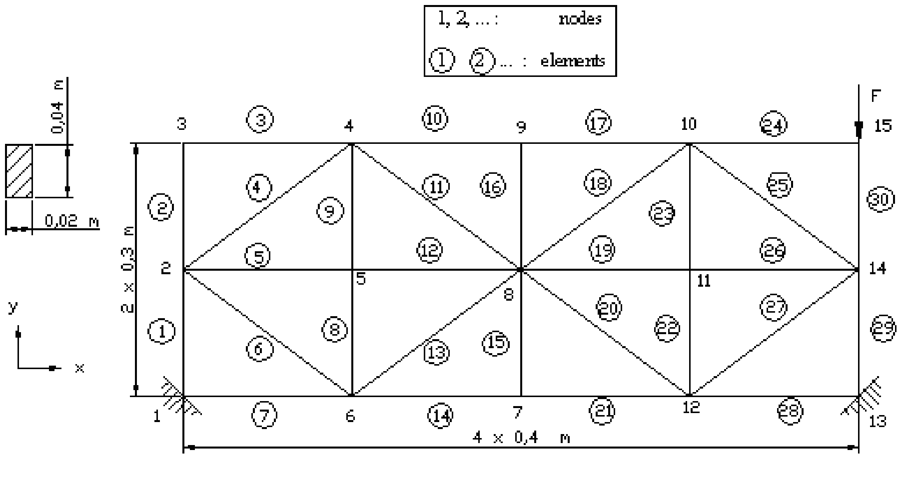

Fig. 2 – Structure out of lattice doubly embedded

Choice of the working frequencies and the force position:

As presented previously, it is interesting to maximize the witness vector. So, in the presented application the force position should be chosen on a DOF which maximizes this vector. For this purpose, we calculate the witness vector for all possible positions of the force and compare the modulus of the obtained vectors; Working frequencies are other parameters which can induce a great variation of this kind of vectors. The optimization must include those parameters.

Consider the above structure working in the frequency band [0, 520] Hz. In order to choose the working frequencies, we divide the frequency band into three parts, the first one represent low frequencies [0, 200], the second part, middle frequencies [150, 350] and the third part represents high frequencies [330, 520].

Consider now the five sets of twenty excitation frequencies shown on table 1 and figure 3:

1. frequencies close to the Eigen-frequencies {ω1}
2. frequencies covering the first part of the band {ω2}
3. frequencies covering the second part of the band {ω3}
4. frequencies covering the third part of the band {ω4}
5. frequencies covering all the considered band {ω5}

Table 1: Considered working frequencies sets

{ω1}	{ω 2}	{ω 3}	{ω 4}	{ω 5}
12	14	160	339	14
37	22	108	348	39
84	29	186	361	62
111	39	195	370	94
124	49	208	385	120
141	54	224	391	139
170	62	232	396	160
205	70	240	407	186
220	78	245	415	208
248	94	257	426	232
266	100	268	434	257
325	109	276	442	284
340	120	284	450	318
361	139	302	456	339
384	144	310	466	370
405	152	318	474	391
426	160	324	482	426
457	168	332	490	450
466	186	339	498	474
511	195	350	510	498

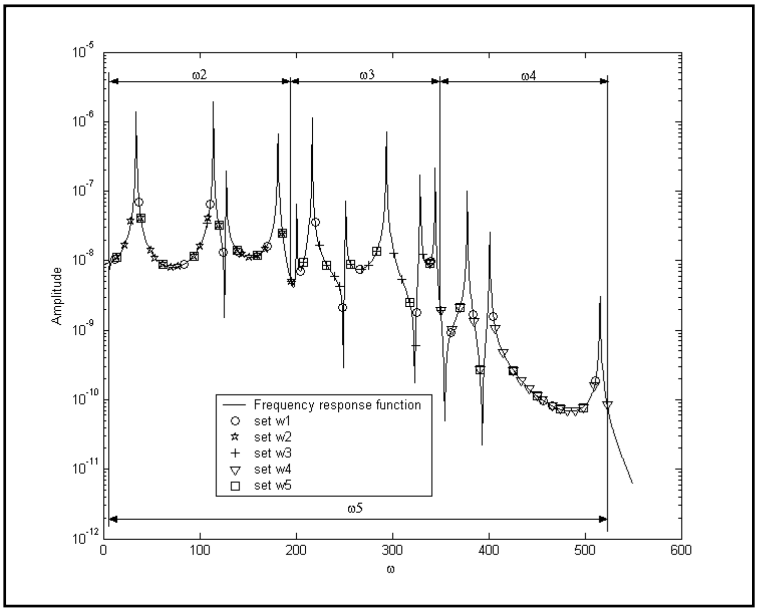

Fig. 3 – repartition of the working frequencies sets in the frequency band considered

We should choose the set which maximizes the modulus of the witness vector W. In other way, we should choose either the position of the excitation force which maximizes this witness vector. This is calculated for different positions of the excitation force. Figure 4 represents the witness vector modulus calculated for every sets of working frequencies, and for different positions of the excitation force (rotational degrees of freedom are eliminated on the figure).

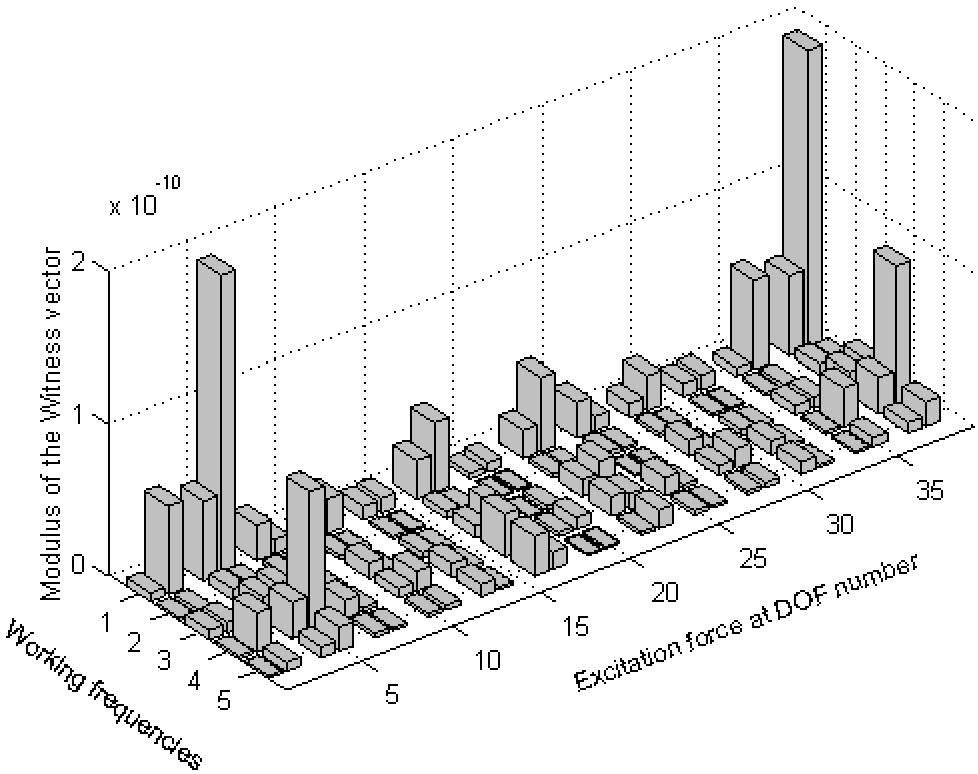

Figure 4: Modulus of the witness vector for different position of the excitation force and various working frequencies

We can see that excitation frequencies close to the eigenfrequencies maximize the witness vector modulus and consequently are more useful for updating purpose. For these working frequencies, we see from figure 4 that for maximizing the witness vector modulus the excitation point should be at 38th DOF, i.e. the y-displacement of the 15th node.

Application of the method

To simulate the real structure, defaults of +20%, +30%, and +20% to the stiffness are introduced respectively in the elements 2, 14, and 17 with 5% of random noise.

Figure 5a shows a comparison of the elements of the witness vector using barplot. This comparison shows that the most important values are for rotational degrees of freedom, this indicate that they are most useful than translation degrees of freedom. We deduce from this figure that the four DOFs to be selected are {6, 35, 38, 39}. i.e. y-displacement of nodes 14, 15, and rotational DOF of nodes 2, 15. Considering that only translational degrees of freedom can be measured and avoiding to measure at the excitation node 15, the comparison of the witness vector elements without rotational and excitation node DOFs (Figure 5b) shows that the four DOFs to be selected are {2, 5, 16, 35}, i.e. the y-displacement of nodes 3, 4, 14, and the x-displacement of the node 6.

The proposed sensor placement method is then compared to the EfI one. The latter technique gives the selection {26, 19, 14, 22} as sensor positions.

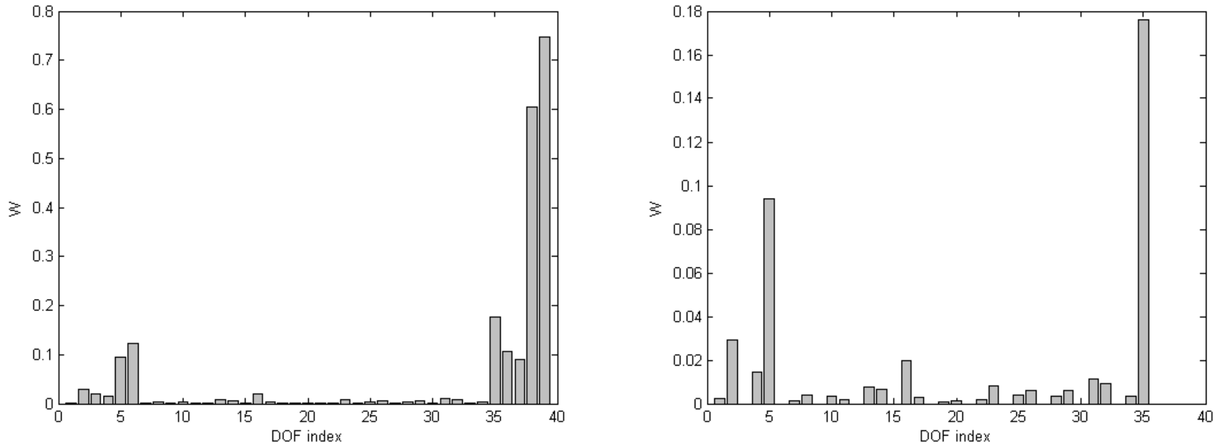

Figure 5: (a) Witness vector elements (b) Witness vector elements without rotational and node excitation DOFs

To improve and validate the proposed strategy, the previously presented updating algorithm is used to detect defaults considering that measurements are taken in the selected DOFs {2, 5, 16, 35} and for those obtained by the EfI method {26, 19, 14, 22}. Obtained results by each of these methods as for detected defaults are compared with all translational DOFs measured for the same chosen excitation frequencies (lightly closes to the eigenfrequencies, the first set on table 1).

Table 2 shows that the selection of measurement points that we obtained by the suggested method give better results than those obtained by the EfI method. Indeed the updating results obtained using the measurement points obtained by the EfI method show defects where they do not exist; this is due certainly to the position of the excitation force. The EfI algorithm is a very powerful technique for the choice of measurement points for modal data based updating and expansion methods; it may be very useful in the frequency domain too. Contrarily to our proposed method where the position of the excitation forces plays a dominating part, the EfI algorithm does not deal with it.

Table 2: Updating results for different sets of measurement points

Elts	Defect	Set I	Set II	Set III	Elts	Defect	Set I	Set II	Set III
1	0	-0,72	2,97	0,19	16	0	2,34	-2,12	1,27
2	**20**	**19,28**	3,48	**18,89**	17	**20**	**12,79**	**26,68**	**19,67**
3	0	2,26	**5,14**	-0,59	18	0	3,28	-1,99	1,38
4	0	0,01	**7,00**	0,50	19	0	-1,17	0,01	0,47
5	0	1,91	**8,96**	0,64	20	0	0,41	**-6,00**	-1,10
6	0	1,03	4,28	0,08	21	0	-0,62	-0,01	-0,98
7	0	-2,64	0,10	-1,27	22	0	-1,27	-0,02	0,76
8	0	-0,50	-0,37	1,23	23	0	0,55	-0,93	-0,85
9	0	2,07	-4,15	-0,68	24	0	-3,15	2,65	0,81
10	0	4,15	-4,02	0,84	25	0	-0,66	-3,65	-1,01
11	0	0,27	-1,46	-0,48	26	0	-1,74	3,91	-0,28
12	0	0,74	1,92	-1,19	27	0	-0,71	**8,06**	0,12
13	0	-1,32	-2,46	-0,04	28	0	-1,19	1,29	1,99
14	**30**	**31,16**	**22,60**	**30,76**	29	0	-0,04	4,18	0,64
15	0	-2,90	-0,03	-0,89	30	0	0,29	-0,05	-0,20

The results obtained with the limited number of DOFs selected {2, 5, 16, 35} (set I on table 2) are very close to those obtained when all translational DOFs are measured (set III on table 2). This confirms that the proposed strategy gives a limited number of measurements equivalent to measurements on all translational DOFs. The introduced stiffness defects are localized and nearly quantified.

The method is validated by many other simulated cases which show the effectiveness of this technique.

5 Conclusion

A method based on first order derivative of the response vector function has been presented for the choice of measurement points to successfully update models. Working frequencies and the position of the excitation force are optimized regarding to the maximisation of the witness vector. In order to estimate the performance of the suggested method, an updating procedure is applied with a limited number of measurements taken on the DOFs selected by the proposed method and those selected by the EfI method; the same updating procedure is used with measurements on all translational DOFs. The validation on numerical test cases of the proposed method has been carried out. Compared to the EfI method, the suggested method gave better results. Adding to that, it is well indicated for the frequency domain.

This technique can be also used for structural identification and damage detection purposes.

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