

The Kalman Filter Approach for Estimating the Natural Unemployment Rate in Romania

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Abstract: The aim of this research is to determine the monthly natural rate of unemployment during the third quarter of 2013 in Romania. The Phillips curve approach is not valid for the Romanian economy, but Kalman filter is a suitable approach for computing the natural rate of unemployment. We make the assumption that the cyclical component follows a random walk. Predictions were made for the unemployment rate in Romania using Kalman approach during July-September 2013 and on this horizon an insignificant decrease was observed from a month to another. A value of 5.85% is expected for unemployment rate in Romania in September 2013.

Keywords: Kalman filter; natural rate of unemployment; forecasts; random walk

JEL Classification: E21; E27; C51; C53

1. Introduction

This Kalman approach is usually applied in determining the natural unemployment rate, the value for each we have a reasonable level or a stability of inflation rate and wages. The Phillips curve used to describe the relationship between inflation and unemployment rate is not checked in Romania, but the state space models are valid.

The objective of this research is to determine the monthly natural unemployment rate in Romania and to make predictions using Kalman filter. There are not relevant studies till now for the Romanian economy.

The organisation of this research is clear: after a brief literature presentation of the quantitative methods used in predicting unemployment rate, we explained the used methodology. One-step-ahead predictions are made for unemployment rate in Romania during the third quarter of 2013 using Kalman filter.

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2 Recent Results in Literature

A complete study related to the Measurement of the natural rates, gaps, and deviation cycles is provided by Murasawa (2013). Claar (2005) estimated the natural rate of unemployment using the Kalman filter for the civilian unemployment rate in USA during 1977-2002. The author also studies the relationship between the natural rate of unemployment and other macroeconomic variables of the labour market. Moreover, Groenewold and Hagger (2002) pointed out before that the natural rate of unemployment is model dependent. Garlach-Kristen (2004) estimated the natural unemployment rate assuming that it follows a random walk, being a determinant of Beveridge curve. Valletta (2006) used the same approach of Beveridge curve, but utilizing regional data. Basistha and Startz (2008) reduced the uncertainty that affects the NAIRU natural rate of unemployment by using multiple indicators.

King and Morley (2003) estimated the natural rate of unemployment without the utilization of the Phillips curve, considering that the natural rate that varies in time is endogenous. Schreiber (2011) estimated the natural rate of unemployment for euro countries by using the integrated systems. Greenslade, Pierse, and Saleheen (2003) applied Kalman filter technique to England Phillips curve models for the NAIRU unemployment during 1973-2000. Meļihovs and Zasova (2009) determined the natural unemployment rate for Latvia using Phillips curve for quarterly data.

Two parallel disturbances are presented for unemployment: a permanent effect and a temporary one. The permanent component is represented by supply shocks that modify the full-employment level while the temporary effect does not modify this full-employment level of output as in the approach of King, Stock and Watson (1995), Staiger, Stock and Watson (1997) and Gordon (1998). According to Apel and Jansson (1999) the cyclical component of unemployment presents serial correlation. Proietti (2003) compared the accuracy of several predictions based on linear unobserved components models for monthly US unemployment rate, drawing the conclusion that the shocks are not persistent during the business cycle.

Camba-Mendez (2012) built conditional forecasts for unemployment rate using VAR models and Kalman filter techniques. Sermpinis, Stasinakis and Karathanasopoulos (2013) made predictions for US unemployment rate, using Neural Networks and compared the utility of Support Vector Regression (SVR) and Kalman Filter in combining these forecasts.

3. Methodology

The Kalman filter is an econometric method for predicting the endogenous variables and for adjusting the estimated parameters in forecast equations. There are two systems of equations: a system of prediction equations and a system of update equations.

The stages for applying the Kalman filter are:

1. The estimation of endogenous variables values using available prior information;
2. The adjustment of estimated parameters using adjustment equations and the computation of prediction errors.

A state space model includes two equations:

Measurement equation (the relationship between the observed and the unobserved variables): $y_t = H_t\beta_t + A_zt + e_t$

Transition equation (the dynamic of state (unobserved)): $\beta_t = \mu + F\beta_{t-1} + v_t$

y_t – data series

z_t –observed explanatory variables

H_t – variable coefficients of unobserved series

β_t , A , F and F' – constant coefficients

R and Q - state space parameters (matrix of covariance)

e_t and v_t – shocks

Assumptions

$e_t \sim \text{iid. } N(0, R)$

$v_t \sim \text{iid. } N(0, Q)$

$E(e_t, v_t) = 0$

The objectives are:

1. The estimation of state space model parameters;

$y_t = H_t\beta_t + A_zt + e_t$

$\beta_t = \mu + F\beta_{t-1} + v_t$

$e_t \sim \text{iid. } N(0, R)$

$v_t \sim \text{iid. } N(0, Q)$

2. Restoration of the unobserved state;

$$y_t = H_t \beta_t + A_z t + e_t$$

$$\beta_t = \mu + F \beta_{t-1} + v_t$$

$$e_t \sim \text{iid. } N(0, R)$$

$$v_t \sim \text{iid. } N(0, Q)$$

$\beta_{t/t-1}$ – the estimation of β_t latent state according to the information till t-1 moment

$\beta_{t/t}$ – the estimation of β_t state according to the information till t moment

$P_{t/t-1}$ - the β_t covariance according to the information till t-1 moment

$P_{t/t}$ C- the β_t covariance according to the information till t moment

$y_{t/t-1}$ P- the prediction of y using the information till t-1 moment

$$\eta_{t/t-1} = y_t - y_{t/t-1} \text{ - error prediction}$$

$f_{t/t-1}$ -the variance of prediction error

The Kalman filter offers an optimal estimation for β_t , conditioned by the information related to the H_t state space parameters: A, μ, F, R, Q .

We suppose that μ, F, R, Q are known. The recursive Kalman filters implies 3 stages:

1. We start with the supposed values at the initial moment 0: $\beta_0/0$ si $P_0/0$;
2. The prediction: the optimal prediction $y_1/0$ at moment 1, using $\beta_1/0$;
3. The update: the calculation of the prediction error, using the observed value for y at moment 1.

$$\eta_{1/0} = y_1 - y_{1/0}$$

The information included in the prediction error has data that can be recovered for redefining our assumption regarding the value that β could have

$$\beta_{1/1} = \beta_{1/0} + K_t \eta_{1/0}$$

K_t - the Kalman gain (the importance accorded to the new information).

The predicted values

$$\beta_{t/t-1} = \mu + F \beta_{t-1/t-1}$$

$$P_{t/t-1} = F P_{t-1/t-1} F' + Q$$

The prognosis for y and the error prediction

$$\eta_{t/t-1} = y_t - y_{t/t-1} = y_t - z_t \beta_{t/t-1}$$

$$f_{t/t-1} = x_t P_{t/t-1} z_t' + R$$

The update

$$\beta_{t/t} = \beta_{t/t-1} + K_t \eta_{t/t-1}$$

$$P_{t/t} = P_{t/t-1} - K_t Z_t P_{t/t-1}$$

Kalman gain: $K_t = P_{t/t-1} z_t' (f_{t/t-1})^{-1}$.

The actual observed unemployment rate is the sum of two components: the natural unemployment rate quantifying the persistent shocks from the supply side (we assume it follows a random walk) and the cyclical unemployment that refers to the shocks from the demand side which are limited as persistence (this component exhibits the serial correlation).

$$u_t = u_t^{nat} + \alpha_t$$

$$u_t^{nat} = u_{t-1}^{nat} + \varepsilon_t$$

$$\alpha_t = \rho \alpha_{t-1} + \omega_t$$

$$\varepsilon_t \sim N(0; \sigma_\varepsilon^2)$$

$$\omega_t \sim N(0; \sigma_\omega^2)$$

$$E(\varepsilon_t, \omega_t) = 0$$

A state space model for the natural unemployment can have the following form:

$$u_t = Z \beta_t, \quad t=1,2,\dots,T \text{ (measurement equation)}$$

$$Z = [1 \ 1], \quad \beta_t = \begin{bmatrix} u_t^{nat} \\ \alpha_t \end{bmatrix}$$

$$\beta_t = T \beta_{t-1} + R \vartheta_t \text{ (transition equation)}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}, \quad \vartheta_t = \begin{bmatrix} \varepsilon_t \\ \omega_t \end{bmatrix}$$

$$\varepsilon_t \sim N(0; \sigma_\varepsilon^2)$$

$$\omega_t \sim N(0; \sigma_\omega^2)$$

$$E(\varepsilon_t, \omega_t) = 0$$

Under these conditions the Kalman filter generates optimal predictions and updates of the state variables. The Kalman filter determines the estimator of the minimum square error of the state variables vector. There are two approaches in literature regarding the estimation of a variable using this filter. The first one assumes that the initial value of the non-stationary state variable can be fixed and unknown. On

the other hand, the second approach considers that the initial value is random. The diffuse prior is specified. If we analyse the first observations, the approach is better even if it can generate numerical instability. If m is the number of state variables we utilize the approach with diffuse prior of Koopman, Shepard and Doornik (1998) and m predictions are provided. The unknown parameters that will be estimated are ε_t , ω_t and ρ . However, some authors give these parameters some reasonable values from the start. For we have to establish the value from the start and the log-likelihood function is computed. The variance of the shocks coming from the demand side (σ_ω^2) is always greater than the variance of supply shocks (σ_ε^2).

4. The Computation of Natural Unemployment Rate and of the Predicted Unemployment

In this research the data set is represented by the unemployment rate in Romania (denoted by u) registered in the period 1992: January- 2013: June. The unemployment rate is an indicator used to measure the unemployment intensity, being computed as a ratio of number of registered unemployed people and the active population. One-step-ahead predictions are made on the horizon 2013: July-2013: September. The data series are provided by the National Institute of Statistics.

The natural unemployment rate is determined for diffuse prior and different values of ρ . ρ represents the starting value of the state space model.

$\alpha_t = \rho\alpha_{t-1} + \omega_t$, where α_t is the error term of the model that explains the evolution of the unemployment rate using the natural unemployment rate

$$u_t = u_t^{nat} + \alpha_t$$

The estimations based on Kalman filter are made in EViews:

@ signal ur= sv1+ sv2

@ state sv1= sv1(-1) + [var=exp(c(2))]

@ state sv2= c(4)* sv2(-1) + [var=exp(c(3))]

The state space models for different values of starting value of ρ are presented in **Appendix 1**. The proposed models in literature are also valid for Romania.

Table 1. The Natural Unemployment rate for Different Values of Starting Values (July 2013-September 2013)

Month	Unemployment rate (%) (dynamic forecasts)					
	$\rho = 1$	$\rho = 0.9$	$\rho = 0.8$	$\rho = 0.7$	$\rho = 0.5$	$\rho = 0.3$
July 2013	5.52	5.516	5.516	5.517	5.5177	5.518
August 2013	5.517	5.515	5.515	5.515	5.518	5.517
September 2013	5.518	5.515	5.516	5.5166	5.517	5.517

Dynamic forecasts are made for different values of ρ (July 2013-September 2013). These values include not only the natural unemployment rate, but also the cyclical component. For July 2013 the Kalman filter approach predicts a rate of 5.88% for the unemployment rate, followed by an insignificant decrease till 5.87% in August 2013 and 5.85% in September 2013.

Table 2. Dynamic Forecasts of the Unemployment Rate for Different Values of Starting Values ρ (July 2013-September 2013)

Month	Unemployment rate (%) (dynamic forecasts)					
	$\rho = 1$	$\rho = 0.9$	$\rho = 0.8$	$\rho = 0.7$	$\rho = 0.5$	$\rho = 0.3$
July 2013	5.8862	5.8862	5.88621	5.886239	5.886226	5.886235
August 2013	5.87249	5.87253	5.87246	5.87251	5.87248	5.87250
September 2013	5.85878	5.85885	5.85874	5.85881	5.85877	5.85880

The differences between the forecasts corresponding to a certain month are insignificant. The increase in the value of ρ does not imply necessary an increase in the value of the unemployment rate. For July 2013, the most accurate unemployment rate forecast was registered for the case of $\rho = 0.5$ (with an absolute error of 0.59622 percentage points).

The one-step-ahead forecasts based on Kalman filter and the actual values of unemployment rate are represented in the following graph.

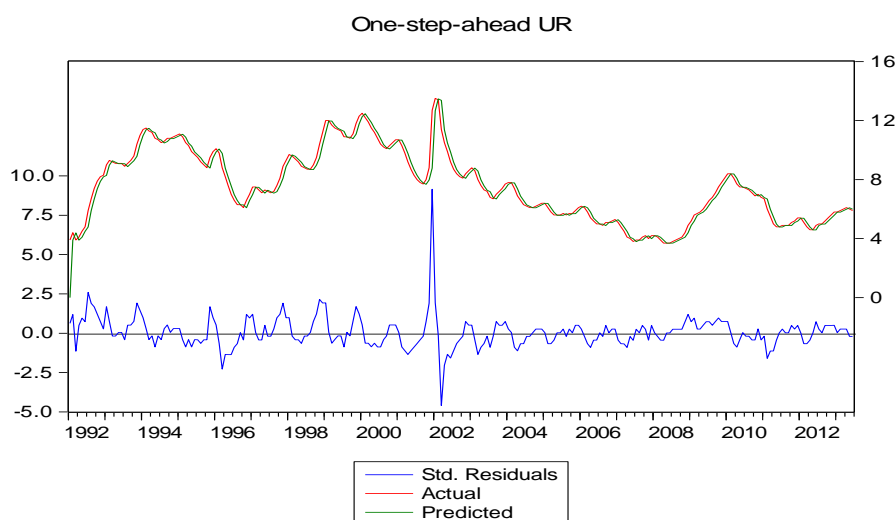


Figure 1. The Actual and Predicted Values of Monthly Unemployment rate in Romania (1992: January- June: 2013)

As we can observe, the differences between the actual values and the predicted ones are low. In 2002 the greatest unemployment rates were registered.

5. Conclusions

An important conclusion is that the classical state space model used in literature to determine the natural unemployment rate provided expected results for the Romanian economy. A very slow decrease in the monthly unemployment rate is observed during the third quarter of 2013 when Kalman approach is used. A value of 5.85% is predicted for September 2013.

This research provides pertinent results regarding the prediction of unemployment rate in Romania, but the study could be improved by assessing the forecasts accuracy and making the comparison with other predictive quantitative techniques.

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APPENDIX 1

$\rho = 1$

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-1.694572	0.025524	-66.39032	0.0000
C(2)	0.997666	0.003013	331.1242	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886231	0.428577	13.73437	0.0000
Log likelihood	-150.2963	Akaike info criterion		1.180591
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191666

Unknown ρ

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 15 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-1.695059	0.025506	-66.45642	0.0000
C(2)	0.997660	0.003009	331.5723	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886193	0.428472	13.73763	0.0000
Log likelihood	-150.2964	Akaike info criterion		1.180592
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191667

$\rho = 0.9$

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-1.694995	0.025518	-66.42405	0.0000
C(2)	0.997670	0.003014	330.9585	0.0000

	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886252	0.428486	13.73733	0.0000
Log likelihood	-150.2964	Akaike info criterion		1.180592
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191667

$\rho = 0.8$

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-1.694879	0.025515	-66.42797	0.0000
C(2)	0.997664	0.003011	331.3113	0.0000

	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886217	0.428511	13.73645	0.0000
Log likelihood	-150.2963	Akaike info criterion		1.180592
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191667

$\rho = 0.7$

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-1.694806	0.025520	-66.41028	0.0000
C(2)	0.997668	0.003014	331.0630	0.0000

	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886240	0.428526	13.73600	0.0000
Log likelihood	-150.2963	Akaike info criterion		1.180592
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191667

$\rho = 0.5$

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
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C(1)	-1.694716	0.025520	-66.40697	0.0000
C(2)	0.997666	0.003012	331.1881	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886227	0.428546	13.73536	0.0000
Log likelihood	-150.2963	Akaike info criterion		1.180592
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191666

$\rho = 0.3$

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-1.694646	0.025523	-66.39575	0.0000
C(2)	0.997667	0.003013	331.0847	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886236	0.428561	13.73489	0.0000
Log likelihood	-150.2963	Akaike info criterion		1.180591
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191666

$\rho = 0$

Sspace: SS01
Method: Maximum likelihood (Marquardt)

Included observations: 258
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-1.694508	0.025527	-66.38215	0.0000
C(2)	0.997667	0.003013	331.0771	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.886235	0.428590	13.73394	0.0000
Log likelihood	-150.2963	Akaike info criterion		1.180591
Parameters	2	Schwarz criterion		1.208134
Diffuse priors	0	Hannan-Quinn criter.		1.191666