Automated Portfolio Optimization Based on a New Test for Structural Breaks

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Abstract: We present a completely automated optimization strategy which combines the classical Markowitz mean-variance portfolio theory with a recently proposed test for structural breaks in covariance matrices. With respect to equity portfolios, global minimum-variance optimizations, which base solely on the covariance matrix, yield considerable results in previous studies. However, financial assets cannot be assumed to have a constant covariance matrix over longer periods of time. Hence, we estimate the covariance matrix of the assets by respecting potential change points. The resulting approach resolves the issue of determining a sample for parameter estimation. Moreover, we investigate if this approach is also appropriate for timing the reoptimizations. Finally, we apply the approach to two datasets and compare the results to relevant benchmark techniques by means of an out-of-sample study. It is shown that the new approach outperforms equally weighted portfolios and plain minimum-variance portfolios on average.

Keywords: Fluctuation Test; Markowitz; Structural Break

JEL Classification: C12; C41; C61; G11

1. Introduction

The model by Markowitz (1952) represents a milestone in development of modern techniques concerning portfolio optimization. Nevertheless, it is well known that there are some serious challenges for the application of optimization techniques to portfolio management practice. In particular, the error-prone estimation of the expected returns is crucial for reasonable results of the optimization (Best & Grauer, 1991, Chopra & Ziemba, 1993). The global minimum-variance portfolio approach circumvents this problem. It determines the portfolio weights independently from expected returns. The optimization depends solely on the covariance matrix which can be estimated much more reliable than expected returns (Golosnoy et al., 2011). It leads to a minimum-variance portfolio that lies

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on the left-most tip of the efficient frontier. Considering equity portfolios, numerous historical backtests show that minimum-variance optimization provides higher returns and lower risk compared to capitalization-weighted portfolios (e.g. Haugen & Baker, 1991, Jagannathan & Ma, 2003, Clarke et al., 2006, Clarke et al., 2013).

However, some crucial challenges remain by this approach. In order to compose an efficient minimum-variance portfolio a precise estimation of the covariance matrix is essential. Surprisingly, in finance literature and practice the covariance matrix is often estimated on the basis of a constant historical (rolling) time-window of more or less arbitrary length (e.g. Haugen & Baker, 1991: 24 months; Jagannathan & Ma, 2003: 60 months and 1260 days; Pojarliev & Polasek, 2003: 800 days; Clarke et al., 2006: 60 months and 250 days; DeMiguel et al., 2012: 250 and 750 days; Behr et al., 2012: 120 months), although several studies show that variances and correlations of asset returns are not constant over time (e.g. Longin & Solnik, 1995). To this end, this common approach may suffer from serious sampling errors.

Besides parameter estimation, the question arises when a rebalancing or a reoptimization should be performed. In finance literature and in practice it is common to choose a fixed reoptimization frequency (e.g. Baltutis & Dockner, 2007: weekly; Lenoir & Tuchschmid, 2001, and Clarke et al., 2006: monthly; Haugen & Baker, 1991: quarterly; Chan et al., 1999, and Jagannathan & Ma, 2003: annually; MSCI Minimum Volatility World Index: semi-annually). Usually, previous studies fail to motivate the determination of the frequency in detail despite the fact that portfolio rebalancing is crucial for portfolio performance. Behr & Miebs (2008) showed that minimum-variance portfolios are highly sensitive to revision frequencies. Baltutis & Dockner (2007) found out that under high frequency revision the turnover of the portfolio increased undesirably not necessarily reducing its realized volatility significantly.

By improving on the naive approach of periodic rebalancing, the financial literature provides numerous paper dealing with the issue of (optimal) portfolio revisions. These works proposed rebalancing strategies based on different approaches like e.g. tolerance bands around the desired target allocation (e.g. Masters, 2003, and Donohue & Yip, 2003), dynamic programming (Sun et al., 2006), and quadratic heuristics (Markowitz & van Dijk, 2003, and Kritzman et al., 2009)[1].

To the best of our knowledge, there are just a few paper using explicitly changes in the covariance matrix as a trigger to perform a reoptimization. Baltutis (2009), Golosnoy & Schmid (2007) and Golosnoy et al. (2011) use control charts for monitoring changes in the the covariance matrix and global minimum variance

[1] See Sun et al. (2006) and Kritzman et al. (2009) for a discussion of these rebalancing strategies.

portfolio weights. In addition, Baltutis (2009) proposed a concept where an update of the portfolio weights is based on testing for statistically significant shifts in the covariance matrix which have already occurred in a realized sample.

In these contexts, we follow Baltutis (2009) by using a statistical test for structural breaks in the covariance matrix, but apply the recently proposed fluctuation test by Aue et al. (2009) for a constant covariance matrix to daily asset returns. Additionally, the break points detected by this test are used not only for automatically inducing dates for reoptimizations, but also for determining proper samples for parameter estimation. Wied et al. (2013b) introduce basic concepts of combining the minimum-variance approach with various fluctuation tests for volatility and dependence measures. Within the optimization context, they investigated a combination of the fluctuation tests for constant volatility and for constant correlations (Wied et al., 2012a; Wied et al., 2012b) as well as a fluctuation test for constancy of the entire covariance matrix (Aue et al., 2009). They find out that the usage of the test for constancy of the entire covariance matrix is the most promising approach.

However, despite the demonstrated potential of this approach they point out several serious drawbacks and challenges which have to be solved in further investigations in order to make this approach applicable for practitioners. In this paper, we take up these points and present useful methodological adjustments in order to develop algorithms and techniques for applications. Furthermore, we discuss the implementation of this new approach as an automated investment system for strategic asset allocations. Our empirical study shows that tests for structural breaks in the covariance matrix improve the results of a global minimum-variance optimization on average.

2. Portfolio Optimization

As the model by Markowitz (1952) is well known, we give only a very brief summary. It assumes the existence of *d* assets with normally distributed returns. Optimal selection of the portfolio weights $\omega = (\omega_1, \ldots, \omega_d)$ is intended, where ω_i is the fraction which is invested into asset *i*. For most applications it is required that $\omega_i \geq 0$, which avoids short selling, and $\sum_{i=1}^{d} \omega_i = 1$, which ensures an investor to be fully invested. The crucial parameter for a global minimum-variance optimization is the risk of the portfolio, which is defined by the variance σ_p^2. Hence, the portfolio weights are determined independently from expected returns and the optimization depends solely on the covariance matrix. The resulting portfolio lies on the left-most tip of the efficient frontier. These considerations result in the following optimization problem:

$\min \sigma_p^2$

$$\text{s.t. } \sum_{i=1}^{d} \omega_i = 1, \qquad (1)$$

where $\sigma_p^2 = \omega \sum \omega'$ and $\sum$ is the covariance matrix. Moreover, sometimes the additional constraint $\omega_i \geq 0, \forall\, i$, is imposed.

As mentioned above, the global minimum-variance optimization depends solely on the covariance matrix. In this context, however, the question arises which time window should be used in order to estimate the covariance matrix. In the following section, we present a new approch to tackle this issue.

3. Tests for Breaks in the Covariance Structure

Aue et al. (2009) present a nonparametric fluctuation test for a constant *d*-dimensional covariance matrix of the random vectors $X_1, \dots, X_T$ with $X_j = (X_{j,1}, \dots, X_{j,d})$. The basic idea of the procedure is to compare the empirical covariance matrix calculated from the first observations with the one from all observations and to reject the null hypothesis if this difference becomes too large over time. Denote *vech*(·) the operator which stacks the columns on and below the diagonal of a *d*×*d* matrix into a vector and *A*' the transpose of a matrix *A*. Then, we consider the term

$$S_k = \frac{k}{\sqrt{T}}\left(\frac{1}{k}\sum_{j=1}^{k} vech\left(X_j X_j'\right) - \frac{1}{T}\sum_{j=1}^{T} vech\left(X_j X_j'\right)\right) \qquad (2)$$

which measures the fluctuations of the estimated covariance matrices calculated by means of the first *k* observations and use the maximum of the results for $k = 1, \cdots, T$. Here, the factor $\frac{k}{\sqrt{T}}$ serves for standardization; intuitively it corrects for the fact that the covariance matrices cannot be well estimated with a small sample size. If the maximum is standardized correctly, the resulting test statistic converges against a well know distribution and the null of a constant covariance matrix is rejected, if the test statistic is larger than the respective critical value.

For sake of readability we will not describe the entire test statistic at this point and refer to the appendix or Aue et al. (2009). Nevertheless, the limit distribution under the null hypothesis is the distribution of

$$sup_{0 \leq t \leq 1} \sum_{l=1}^{d(d+1)/2} B_l^2(t), \qquad (3)$$

where $(B_l(t), t \in [0,1]), l = 1, \dots. d(d+1)/2$ are independent Brownian bridges.

The test basically works under mild conditions on the time series under consideration. One does not need to assume a particular distribution such as the normal distribution and the test allows for some serial dependence which makes it possible to consider e.g. GARCH models. Moreover, the test is consistent against fixed alternatives and has considerable power in finite samples. Regarding moments of the random variables, note that the correct application of the test needs constant expectations. The asymptotic result is derived under the assumption of zero expectation; if we had constant non-zero expectation, it would be necessary to subtract the arithmetic mean. While this assumption is sufficiently fulfilled for daily return series, the derivation of the asymptotic null distribution also needs the assumption of finite fourth moments. Theoretically, this assumption could be violated (Mandelbrot, 1963). However, in the following, we do not further consider this potential problem as this lies beyond our scope.

4. Empirical Study

The aim of this empirical study is to compare the out-of-sample performance of a global minimum-variance optimization combined with the test for a constant covariance matrix (hereinafter referred to as covariance-test optimization) to various relevant asset allocation strategies. First, we decide for a equally weighted asset allocation strategy as a natural benchmark.[1] For this, we obtain market values for each of the (sub)indices from *Thomson Reuters Datastream* and the portfolio weights are rebalanced each 21/63/252 traiding days, which corresponds approximately to monthly, quarterly and yearly rebalancings. The benchmark of most interest is the classical global minimum-variance portfolio where the optimization is based on constant rolling time-windows for calculation of the empirical covariance matrix (hereinafter referred to as plain optimization).

As this study is focused on strategic asset allocation, we use time series from indices or subindices rather than from single stocks. The pros and cons of active portfolio management are extensively discussed in numerous studies (e.g. Wermers, 2000, Jacobsen, 2011). However, we agree with Sharpe (1991) who pointed out that the return on the average actively managed dollar will equal the return on the average passively managed dollar. Including costs for the active management it will be even less. This statement is underpinned by Standard & Poor's (2012) who showed that 65% of all U.S. large cap equity funds do not outperform the S&P 500 index over the last five years. Moreover, indices are much more robust against unsystematic market risks and movements and can easily be replicated by means of ETFs. Note, as we deal with indices in a strategic asset allocation environment we can avoid questions arising from large investable sets (compare for example Michaud, 1989, Bai et al., 2009, Arnold et al., 2013).[2]

[1] We also investigated cap-weighted portfolios. Nevertheless, the results of the equally wighted portfolios were slightly better. The results for cap-weighted portfolios are available from the authors upon request.

[2] Furthermore, high-dimensional portfolios can be reduced to managable sizes for example by factor analysis (Krzanowski, 2000, Hui, 2005).

Hence, we apply each of these approaches to two samples consisting of five and ten indices, respectively. In detail, the empirical study is designed as follows:

4.1. Data

To carry out the out-of-sample study we compute log-returns from two different datasets. To avoid undesirable effects, both datasets have to fulfill the requirements of single currency and uniform time zone. For the first portfolio, we use daily total return quotes from five stock indices of main European countries that are founding members of the eurozone (AEX, CAC 40, DAX 30, FTSE MIB, IBEX 35). The quotes cover a period from the introduction of the Euro at January 1, 1999 to July 31, 2012 leading to 3481 trading days. For the second portfolio, we used daily total return quotes from the ten S&P 500 sector subindices (Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Telecommunication Services, Utilities). This quotes cover the total period provided by S&P starting at the initial publication on January 1, 1995 to July 31, 2012 leading to 4429 trading days. All quotes are obtained from *Thomson Reuters Datastream*.

4.2. Parameter Estimation

The optimization of a global minimum-variance portfolio based solely on the covariance matrix. Consequently, the performance differences between plain optimizations and covariance-test optimizations are due to the varying length of time-windows for parameter estimation. For the plain optimizations we define constant rolling time-windows of 250, 500 and 1000 trading days. The time-window of the covariance-test optimization is determined by following procedure:

1. Initialize i=1 and k=1000.
2. Apply the test of a constant covariance matrix to the data $\{x_i, \ldots, x_k\}$.
3. If the test rejects the null, set p=k, otherwise set p=i.
4. Adjust the time-window by $i=min\{p,k-126+1\}$ in case of the five-dimensional portfolio or $i=min\{p,k-252+1\}$ in case of the ten-dimensional portfolio.
5. Use the data $\{x_i, \ldots, x_k\}$ for estimating the empirical covariance matrix.
6. Set k=k+n, where n is the number of trading days between two tests and optimizations and go back to step 2.

Note, a reliable estimation of the covariance matrix requires a sufficient sample size. To this end, the modifications $i=min\{p,k-126+1\}$ and $i=min\{p,k-252+1\}$ ensure that the estimation is based on data of the last (half) year, depending on the dimensionality of the portfolio. As before, we choose n=21,63 and 252.

The determination of critical values is a crucial issue for the application of the test for a constant covariance matrix. Aue et al. (2009) approximated critical values by simulating Brownian bridges on a fine grid. Wied et al. (2013b) showed that this approximation does not perform well if the sample size is small. In this case, the critical values are overestimated and hence lead to low numbers of rejections. We take up this point and propose an alternative approach which is suitable for a practical application of the test. To this end, we generate d-dimensional standard normal distributed random variables. Then, we apply the test for a constant covariance matrix to the sample. This procedure is carried out 10000 times. After that, we determine the $(1-\alpha)$-quantile of the resulting test statistics as the critical value. In line with Wied et al. (2013b), we compute the critical values for α=1% and α=5%. Depending on the chosen length of the sample, the critical value varies within a relatively wide range. Therefore, regarding the five-dimensional (ten-dimensional) portfolio, we estimate critical values for 18 (12) different sample sizes which are congruent to time-windows of 126 (250) to 1400 trading days (Table 1).

Table 1. Critical Values

Sample Size	five-dimensional Portfolio		ten-dimensional Portfolio	
	α=5%	α=1%	α=5%	α=1%
126	4.25	4.63	-	-
138	4.39	4.80	-	-
150	4.54	4.96	-	-
175	4.74	5.19	-	-
200	4.92	5.45	-	-
225	5.11	5.65	-	-
250	5.24	5.84	8.60	8.94
275	5.37	6.01	8.97	9.35
300	5.48	6.10	9.36	9.77
350	5.69	6.41	10.01	10.48
400	5.89	6.68	10.60	11.18
500	6.11	6.99	11.49	12.12
600	6.31	7.25	12.28	13.05
700	6.47	7.41	12.88	13.83
800	6.57	7.52	13.41	14.35
1000	6.76	7.76	14.26	15.27
1200	6.86	7.90	14.95	16.07
1400	6.99	8.12	15.47	16.61

Critical values for the five and the ten dimensional portfolio estimated by use of a Monte-Carlo-Simulation.

Using these critical values as grid points, we compute critical values for time-windows of any required length by linear interpolation. Although it seems only to be a small modification, it leads to a much more realistic determination of the dates where structural breaks in the covariance matrix occur. Moreover, it allows us to establish an automated investment strategy, which automatically determines dates for reoptimizations.

As we have just mentioned, the more precise estimation technique for critical values allows us to investigate an automated investment system, where the test is performed on a daily basis and the optimization is conducted only if the test rejects the null. Hence, an investor does not need to decide for a particular time-window in order to estimate the covariance matrix and reoptimization interval. Only the significance level has to be determined in advance. In more detail, we set n=1 and modify the last step of the previous procedure as follows:

7. If the test rejects the null, set k=k+63, otherwise set k=k+1. Then go back to step 2.

By conducting the fluctuation test at each day, clustered rejections are very likely due to the small changes in the sample. The condition k=k+63 in case of a null rejection assures that the sample for the subsequent test includes an adequate amount of new data.

4.3. Optimization Setup

The portfolio performance is strongly affected by the frequency of reoptimizations. In line with the test intervals of the previous section, we optimize every 21, 63, and 252 trading days in the first setting. In this case, the asset weights are reoptimized after each test, regardless whether the null is rejected or not. Because of the identical intervals, this procedure allows for a direct comparison between the plain optimization and the covariance test optimization. In contrast to that, if the constancy of the covariance is tested on a daily basis, optimizations will be conducted only when a structural break is detected. In this context, portfolio weights remain unchanged in the sense that no trading takes place until the test again rejects the null. Hence, the portfolio weights will drift from the initially determined portfolio weights due to the variation in asset returns. Note, however, the simulations for the equally weighted portfolios suggest that the rebalancing frequency is only of minor importance. Besides, we consider two different constraints concerning the portfolio weights. First, we assume $0 \leq \omega_i \leq 1, \forall\, i$, which in particular excludes short selling (hereinafter referred to as long portfolios). In addition to that, we assume $|\omega_i| \leq 1, \forall\, i$, throughout the second run (hereinafter referred to as short portfolios). The optimizations are performed by using the *fmincon*-function of *MATLAB R2012a*.[1]

4.4. Performance Measurement

The portfolio performance is analyzed from various perspectives. First of all, the measurement of the risk in terms of volatility takes a prominent part of the evaluation, as portfolio variances are optimized. Nevertheless, we investigate the impact on the resulting returns and the relationship between risk and return in terms of the Sharpe-ratio, too. For its computation we assume 1.1% as risk free return which corresponds to the average return of German goverment bonds with less than 3 years to maturity in 2011.

Reoptimization (and rebalancing) of portfolio asset weights naturally leads to increasing trading volume. Hence, we measure this turnover in absolute and relative Terms. Following DeMiguel (2009), we define the sum of absolute changes in the weights as

$$Turnover(A) = \sum_{i=1}^{RD-1} \sum_{j=1}^{d} \left| a_{i+1,j} - a_{i^+,j} \right|, \qquad (4)$$

where *RD* is the number of the reoptimization (rebalancing) days and *d* the number of assets. The portfolio weight of asset *j* before a rebalancing or reoptimization at time *i*+1 is defined as $a_{i^+,j}$. Besides, we call *Turnover*(*R*) the average amount of changes at each *RD*, that means $Turnover(R) = \frac{1}{RD-1} \cdot Turnover(A)$.

In order to attribute a financial impact to the trading volume, we transform turnover to transaction costs and analyzes the effects. In line with Wied et al. (2013b) we compute adjusted returns and Sharpe-ratios by subtracting transaction costs from the return *R*. These costs are defined by $Turnover(A) \cdot \frac{s_c}{2}$, where the constant relative bid-ask spread s_c represents the bid-ask spread divided by bid-ask midpoint. We quantify the spread on the basis of the average relative bid-ask spread of the stocks listed on the European indices (5 asset portfolio) and stocks listed on the S&P 500 (10 asset portfolio) for the time-span August 1, 2011 to July 31, 2012. The spread of the analyzed stocks amounts to about 0.15% (European indices) and about 0.05% (S&P 500). Moreover, we refine this methodology used in Wied et al. (2013b) and introduce critical relative bid-ask spreads. To this end, consider two portfolio selection methods where a superior method outperforms an

[1] Note, we checked the performance of the *fmincon*-function by means of several examples and comparison to the *quadprog*-function. All results indicate that there are no conversion problems within this optimization task. Nevertheless, to minimize the risk of detecting local minima, we use an adequate number of different starting points for the optimization. These starting points include the defined weighting boundaries as well as the equal weighted portfolio and random weights.

inferior method in terms of Sharpe-ratio (excluding transaction costs) and the absolute turnovers are different. Then, the critical relative bid-ask spread is defined as the spread at which for both portfolios the Sharpe-ratios adjusted by transaction costs are equal. In this context, we use the average Sharpe-ratio of the equally weighted portfolios as benchmark in order to calculate critical spreads for optimized portfolios.

5. Results

In the following, we present the results of the out-of-sample study.

5.1. European Stock Indices Portfolio

We start with the dataset including the five European stock indices. The results of the equally weighted portfolios are presented in Table 2. Volatilities, returns, and Sharpe-ratios remain in a narrow range and show only small variations due to the rebalancing interval. On average, an annualized return of 3.73% and an annualized volatility of 22.67% results to a Sharpe-ratio of 0.1161. The low turnover leads to neglectable transaction costs.

Table 2. Results for the Equally Weighted European Stock Indices Portfolio

Interval	Sharpe Ratio		Return p.a.		Volatility p.a.	Turnover (R)	Turnover (A)
21	0.1164	(0.1158)	3.74%	(3.73%)	22.70%	0.02	1.83
63	0.1162	(0.1159)	3.74%	(3.73%)	22.69%	0.03	1.06
252	0.1155	(0.1154)	3.71%	(3.71%)	22.61%	0.04	0.39
Average	0.1161	(0.1157)	3.73%	(3.72%)	22.67%	0.03	1.09

Results for the equally weighted portfolio consisting of the five European stock indices. Interval refers to the frequency at which a rebalancing is conducted. Values in parentheses refer to Sharpe-ratios and returns adjusted by transaction costs.

As expected, the volatility of the plain optimization portfolios (Tables 3 and 4, Panel A) is reduced significantly by averaged 1.08% for the long portfolios. Furthermore, the portfolio return is improved by 0.61% on average. Nevertheless, the reoptimizations generate a much higher trading volume and the related transaction costs decrease the returns by 0.02% to 0.15%. The allowance for short selling reduces volatilities even more. However, compared to the long portfolios, the returns and Sharpe-ratios tend to be lower and do not even achieve the level of the equally-weighted portfolios on average. Furthermore, the turnover increased by more than two times. Consequently, the average critical spread is negative. On average, the choice of the time-window length has a bigger impact to returns and Sharpe-ratios than the choice of the reoptimization interval. Conversely, the volatility is slightly more affected by the choice of the reoptimization interval.

From a theoretical point of view the allowance for short selling should lead to lower volatilities because it implies less stringent constraints for the optimization. As shown by Table 3 and 4 for example, applying the optimization to financial market data, a loosening of constraints could lead to a less efficient portfolio in some cases. This finding is in line with the empirical study of Jagannathan & Ma (2003) who argue that constraints for portfolio weights increase specification error, but can also reduce sampling error. The trade-off between both error types determines the gain or loss in efficiency.

Table 3. Results for the Optimized European Stock Indices Portfolio and $0 \leq \omega_i \leq 1$

# Data / α	Interval	Sharpe Ratio		Return p.a.		Volatility p.a.	Turnover (R)	Turnover (A)	Critical Spread
Panel A: Plain Optimizations									
250	21	0.1687	(0.1615)	4.66%	(4.51%)	21.11%	0.17	19.86	1.16%
	63	0.1958	(0.1901)	5.27%	(5.15%)	21.30%	0.41	15.96	2.24%
	252	0.1437	(0.1404)	4.27%	(4.20%)	22.09%	1.05	9.41	1.44%
500	21	0.1505	(0.1465)	4.29%	(4.20%)	21.18%	0.09	11.19	1.41%
	63	0.1664	(0.1633)	4.65%	(4.58%)	21.34%	0.22	8.71	2.75%
	252	0.1663	(0.1643)	4.70%	(4.66%)	21.68%	0.61	5.48	4.82%
1000	21	0.1192	(0.1170)	3.69%	(3.64%)	21.71%	0.05	6.19	0.26%
	63	0.1168	(0.1151)	3.65%	(3.61%)	21.80%	0.12	4.73	0.09%
	252	0.1261	(0.1251)	3.88%	(3.86%)	22.07%	0.33	2.98	2.28%
Average		0.1504	(0.1470)	4.34%	(4.27%)	21.59%	0.34	9.39	1.83%
Panel B: Optimization + Test for a Constant Covariance Matrix									
5%	21	0.2127	(0.2028)	5.52%	(5.32%)	20.79%	0.23	26.83	1.53%
	63	0.2447	(0.2378)	6.23%	(6.08%)	20.94%	0.49	19.10	2.93%
	252	0.1315	(0.1275)	4.01%	(3.92%)	22.13%	1.27	11.47	0.65%
1%	21	0.2167	(0.2074)	5.63%	(5.44%)	20.91%	0.21	25.34	1.70%
	63	0.2601	(0.2534)	6.59%	(6.45%)	21.12%	0.48	18.63	3.40%
	252	0.1555	(0.1522)	4.46%	(4.39%)	21.63%	1.03	9.31	2.03%
Average		0.2035	(0.1969)	5.41%	(5.27%)	21.25%	0.62	18.45	2.04%
Panel C: Optimization + Daily Test for a Constant Covariance Matrix									
5%	1	0.1946	(0.1882)	5.21%	(5.07%)	21.10%	0.69	17.82	1.94%
1%	1	0.1301	(0.1261)	3.95%	(3.86%)	21.91%	0.66	11.30	0.59%
Average		0.1623	(0.1572)	4.58%	(4.47%)	21.51%	0.68	14.56	1.27%

Results for the portfolio consisting of five European stock indices under the constraint $0 \leq \omega_i \leq 1$. For Panel A, # Data refers to the sample size used for the optimization. For

Panel B and C, α refers to the significance level for the test for a constant covariance matrix. The interval refers to the frequency at which optimizations and tests are conducted. Values in parentheses refer to Sharpe-ratios and returns adjusted by transaction costs.

Table 4. Results for the Optimized European Stock Indices Portfolio and $|\omega_i| \leq 1$

# Data / α	Interval	Sharpe Ratio		Return p.a.		Volatility p.a.	Turnover (R)	Turnover (A)	Critical Spread
Panel A: Plain Optimizations									
250	21	0.0603	(0.0443)	2.33%	(2.00%)	20.37%	0.36	42.67	-0.54%
	63	0.0766	(0.0647)	2.69%	(2.44%)	20.74%	0.83	32.38	-0.51%
	252	0.1468	(0.1399)	4.30%	(4.15%)	21.79%	2.17	19.54	0.71%
500	21	0.1315	(0.1217)	3.85%	(3.65%)	20.92%	0.23	26.98	0.25%
	63	0.1399	(0.1325)	4.07%	(3.91%)	21.24%	0.53	20.75	0.51%
	252	0.1839	(0.1792)	5.11%	(5.01%)	21.80%	1.49	13.40	2.36%
1000	21	0.0570	(0.0515)	2.33%	(2.21%)	21.51%	0.13	15.38	-1.74%
	63	0.0616	(0.0572)	2.44%	(2.35%)	21.81%	0.32	12.41	-2.06%
	252	0.0870	(0.0841)	3.05%	(2.98%)	22.42%	0.96	8.65	-1.69%
Average		0.1050	(0.0972)	3.35%	(3.19%)	21.40%	0.78	21.35	-0.30%
Panel B: Optimization + Test for a Constant Covariance Matrix									
5%	21	0.1466	(0.1226)	4.03%	(3.55%)	20.00%	0.53	62.86	0.19%
	63	0.1337	(0.1167)	3.83%	(3.49%)	20.45%	1.17	45.64	0.16%
	252	0.1360	(0.1284)	4.07%	(3.91%)	21.87%	2.40	21.57	0.42%
1%	21	0.1634	(0.1405)	4.37%	(3.91%)	20.02%	0.51	59.90	0.32%
	63	0.1363	(0.1210)	3.88%	(3.56%)	20.36%	1.05	40.82	0.20%
	252	0.1497	(0.1436)	4.26%	(4.13%)	21.11%	1.88	16.94	0.88%
Average		0.1443	(0.1288)	4.07%	(3.76%)	20.64%	1.26	41.29	0.36%
Panel C: Optimization + Daily Test for a Constant Covariance Matrix									
5%	1	0.0928	(0.0793)	3.01%	(2.73%)	20.55%	1.40	36.40	-0.27%
1%	1	-0.0192	-(0.0295)	0.67%	(0.45%)	22.16%	1.76	29.95	-2.04%
Average		0.0368	(0.0249)	1.84%	(1.59%)	21.35%	1.58	33.17	-1.15%

Results for the portfolio consisting of five European stock indices under the constraint $|\omega_i| \leq 1$. For Panel A, # Data refers to the sample size used for the optimization. For Panel B and C, α refers to the significance level for the test for a constant covariance matrix. The interval refers to the frequency at which optimizations and tests are conducted. Values in parentheses refer to Sharpe-ratios and returns adjusted by transaction costs.

The results of the covariance-test optimizations are presented in Panel B of the Tables 3 and 4. Considering the long (short) portfolios, the returns increase by 1.07% (0.72%) while the volatility decrease by 0.34% (0.76%) on average compared to the plain optimization portfolios. This leads to an improvement of the average Sharpe-ratio by 0.0531 (0.0393). For both, long and short portfolios, the application of the tests for structural breaks leads to almost a doubling of the average turnover. Nevertheless, the average critical spreads are higher compared to the plain optimization. The significance level of 1% leads to superior returns, whereas the impact of the significance level on the volatility is inconsistent.

Panel C of the Tables 3 and 4 presents the results for the covariance-test optimizations where the test is performed on a daily basis. It is remarkable that the significance level of 5% leads to much better results compared to a level of 1%. Using 5%, long portfolios are comparable to the corresponding covariance-test optimizations. With respect to the short portfolio, this applies also for the volatility, whereas returns and Sharpe-ratios are worse.

5.2. S&P500 Subindices Portfolio

Below, we continue with the results for the portfolio consisting of ten Standard & Poor’s 500 subindices. The results of the equally weighted portfolios are presented in Table 5. On average, a annualized return of 4.99% and an annualized volatility of 20.15% results to a Sharpe-ratio of 0.1933. As before, the low turnover leads to neglectable transaction costs.

As before, the application of the plain optimization improves the performance measures significantly (Tables 6 and 7, Panel A). Compared to the equally weighted portfolio, the volatility of the long-portfolio decreases by 4.83% whereas the return increases by 1.03% on average. Transaction costs vary between 0.007% and 0.035%. In contrast to the European indices portfolio, the allowance for short selling for the S&P500 portfolio leads to considerable improvements on the long portfolio with respect to volatility, return, and Sharpe-ratio. This goes along with a rise in averaged relative turnover from 0.21 to 0.56. The critical spreads reach considerably high values.

As presented in Tables 6 and 7 (Panel B), the application of the test for a constant covariance matrix yields to superior results on average. The long portfolio shows only slight improvements of the return whereas the return of the short portfolio increases by 0.52% on average. Moreover, the volatility decreases by 0.29% for the long and 0.25% for the short portfolio. Although the average trading volume rises by more than 40% compared to the plain optimizations, the improvements of the results are not offset by a loss of return due to transaction costs. However, the critical spreads are somewhat lower compared to the plain optimizations. The choice of the significance level has no substantial impact to both return and volatility.

Table 5. Results for the Equally Weighted Standard & Poor's 500 Subindices Portfolio

Interval	Sharpe Ratio		Return p.a.		Volatility p.a.	Turnover (R)	Turnover (A)
21	0.1916	(0.1912)	4.99%	(4.98%)	20.29%	0.03	4.75
63	0.1953	(0.1950)	5.04%	(5.03%)	20.16%	0.05	2.89
252	0.1929	(0.1928)	4.96%	(4.96%)	20.01%	0.11	1.37
Average	0.1933	(0.1930)	4.99%	(4.99%)	20.15%	0.06	3.00

Results for the equally weighted portfolio consisting of the ten Standard & Poor's 500 subindices. Interval refers to the frequency at which a rebalancing is conducted. Values in parentheses refer to Sharpe-ratios and returns adjusted by transaction costs.

Panel C of the Tables 6 and 7 shows the results for the covariance-test optimizations where the test is performed on a daily basis and the optimization is conducted only if the test rejects the null. On average, the results of this approach improve even on the covariance-test optimizations with a fixed test and reoptimization interval. Furthermore, the turnover is reduced considerably. In contrast to the first sample, the significance level has a minor impact on the results. Nevertheless, a level of 5% results in slightly superior results.

5.3. Rejection Dates

In this section we have a closer look at the rejection dates of the null. Considering the European indices dataset as an example, Figure 1 presents the dates at which the test for a constant covariance matrix rejects the null (63 days test interval / 1%-level) in connection with a trend of variances and covariances.

The chart illustrates that significant changes of variances and covariances are due to points in time at which the test rejects the null. Consequently, this procedure leads to considerably improved results with respect to volatility, return, and Sharpe-ratio compared to the optimizations with a fixed historical time-window. Figure 2 compares exemplary the performance of an equally weighted portfolio, a plain optimization portfolio, and a covariance-test optimization portfolio in connection with the dates at which the test for a constant covariance matrix rejects the null.

Table 6. Results for the Optimized Standard & Poor's 500 Subindices Portfolio and $0 \leq \omega_1 \leq 1$

# Data / α	Interval	Sharpe Ratio		Return p.a.		Volatility p.a.	Turnover (R)	Turnover (A)	Critical Spread
Panel A: Plain Optimizations									
250	21	0.3037	(0.3013)	5.63%	(5.60%)	14.93%	0.12	19.11	2.66%
	63	0.3219	(0.3204)	5.93%	(5.91%)	15.00%	0.22	11.71	5.53%
	252	0.3694	(0.3686)	6.71%	(6.70%)	15.19%	0.55	7.15	14.87%
500	21	0.3082	(0.3069)	5.75%	(5.73%)	15.09%	0.07	11.42	5.15%
	63	0.3138	(0.3128)	5.87%	(5.86%)	15.20%	0.15	7.89	8.85%
	252	0.3459	(0.3452)	6.46%	(6.45%)	15.49%	0.40	5.22	22.09%
1000	21	0.2935	(0.2927)	5.65%	(5.64%)	15.51%	0.04	6.65	9.75%
	63	0.3050	(0.3044)	5.86%	(5.85%)	15.61%	0.09	4.84	18.81%
	252	0.3299	(0.3295)	6.34%	(6.33%)	15.88%	0.29	3.75	42.64%
Average		0.3213	(0.3202)	6.02%	(6.01%)	15.32%	0.21	8.64	14.48%
Panel B: Optimization + Test for a Constant Covariance Matrix									
5%	21	0.3027	(0.3003)	5.62%	(5.58%)	14.93%	0.12	19.13	2.63%
	63	0.3349	(0.3336)	6.12%	(6.10%)	15.00%	0.21	11.13	6.50%
	252	0.3696	(0.3687)	6.71%	(6.70%)	15.19%	0.55	7.10	15.06%
1%	21	0.3088	(0.3066)	5.71%	(5.68%)	14.93%	0.11	17.89	2.99%
	63	0.3262	(0.3249)	5.99%	(5.97%)	14.99%	0.20	10.93	6.23%
	252	0.3655	(0.3647)	6.64%	(6.63%)	15.16%	0.51	6.69	16.01%
Average		0.3346	(0.3331)	6.13%	(6.11%)	15.03%	0.28	12.14	8.24%
Panel C: Optimization + Daily Test for a Constant Covariance Matrix									
5%	1	0.3519	(0.3506)	6.33%	(6.32%)	14.88%	0.24	10.16	8.08%
1%	1	0.3667	(0.3657)	6.63%	(6.61%)	15.07%	0.30	8.75	10.93%
Average		0.3593	(0.3581)	6.48%	(6.46%)	14.98%	0.27	9.45	9.51%

Results for the portfolio consisting of ten Standard & Poor's 500 subindices under the constraint $0 \leq \omega_1 \leq 1$. For Panel A, # Data refers to the sample size used for the optimization. For Panel B and C, α refers to the significance level for the test for a constant covariance matrix. The interval refers to the frequency at which optimizations and tests are conducted. Values in parentheses refer to Sharpe-ratios and returns adjusted by transaction costs.

Table 7. Results for the Optimized Standard & Poor's 500 Subindices Portfolio and $|\omega_1| \leq 1$

# Data / α	Interval	Sharpe Ratio		Return p.a.		Volatility p.a.	Turnover (R)	Turnover (A)	Critical Spread
Panel A: Plain Optimizations									
250	21	0.4034	(0.3967)	6.83%	(6.73%)	14.20%	0.32	51.84	1.63%
	63	0.4186	(0.4145)	7.15%	(7.09%)	14.45%	0.60	32.32	2.94%
	252	0.4960	(0.4935)	8.44%	(8.40%)	14.79%	1.53	19.95	6.86%
500	21	0.3952	(0.3911)	6.75%	(6.70%)	14.31%	0.20	31.84	2.64%
	63	0.3996	(0.3969)	6.92%	(6.88%)	14.56%	0.39	21.13	4.31%
	252	0.4569	(0.4552)	8.01%	(7.99%)	15.13%	1.06	13.75	9.44%
1000	21	0.2944	(0.2921)	5.44%	(5.41%)	14.74%	0.11	18.37	2.51%
	63	0.3228	(0.3213)	5.92%	(5.90%)	14.93%	0.22	11.85	5.46%
	252	0.3614	(0.3603)	6.67%	(6.66%)	15.42%	0.65	8.45	11.44%
Average		0.3942	(0.3913)	6.90%	(6.86%)	14.73%	0.56	23.28	5.25%
Panel B: Optimization + Test for a Constant Covariance Matrix									
5%	21	0.4045	(0.3978)	6.84%	(6.75%)	14.20%	0.31	51.26	1.66%
	63	0.4169	(0.4130)	7.12%	(7.07%)	14.45%	0.57	30.68	3.08%
	252	0.4953	(0.4929)	8.43%	(8.40%)	14.80%	1.54	19.96	6.85%
1%	21	0.3968	(0.3906)	6.74%	(6.65%)	14.20%	0.30	48.26	1.70%
	63	0.3989	(0.3951)	6.87%	(6.81%)	14.46%	0.56	30.16	2.89%
	252	0.5013	(0.4989)	8.51%	(8.48%)	14.79%	1.47	19.06	7.35%
Average		0.4356	(0.4314)	7.42%	(7.36%)	14.48%	0.79	33.23	3.92%
Panel C: Optimization + Daily Test for a Constant Covariance Matrix									
5%	1	0.4763	(0.4727)	7.83%	(7.78%)	14.14%	0.67	28.19	4.17%
1%	1	0.4580	(0.4547)	7.70%	(7.65%)	14.40%	0.88	25.44	4.45%
Average		0.4672	(0.4637)	7.76%	(7.72%)	14.27%	0.77	26.82	4.31%

Results for the portfolio consisting of ten Standard & Poor's 500 subindices under the constraint $|\omega_1| \leq 1$. For Panel A, # Data refers to the sample size used for the optimization. For Panel B and C, α refers to the significance level for the test for a constant covariance matrix. The interval refers to the frequency at which optimizations and tests are conducted. Values in parentheses refer to Sharpe-ratios and returns adjusted by transaction costs.

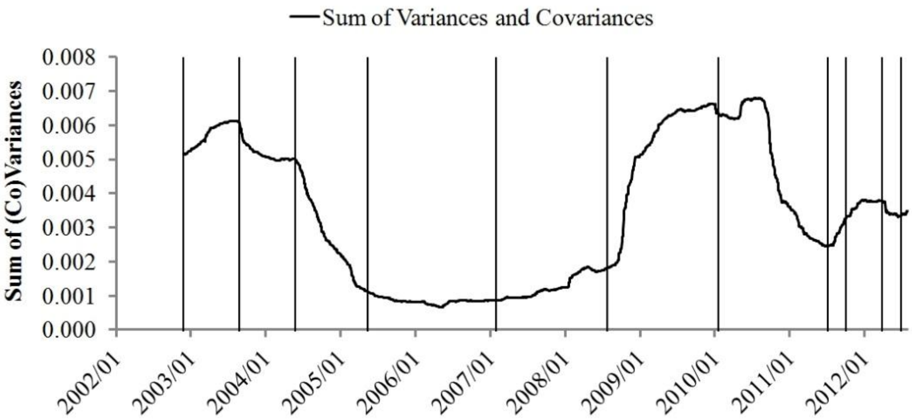

Figure 1. Trend of Variances and Covariances and Dates of Structural Breaks

The Figure shows the trend of the sum of variances and covariances for the European indices dataset over the time span November 26, 2002 to July 31, 2012 (2481 trading days). For each trading day, the sum results by adding up the entries on and below the diagonal of a covariance matrix. The matrix is computed on the basis of a rolling 500 trading day time-window. In addition, the points in time at which the test for a constant covariance matrix rejects the null (structural break) are marked by vertical bars. The tests are conducted under a setup of a 63 trading days test interval and a 1% significance level.

The chart reveals that the covariance-test optimization outperforms the equally weighted portfolio and/or the plain optimization throughout most of the time. In particular during the late phase of the bull market 2006/2007 and the European sovereign-debt crisis beginning in the fall 2009, this new method outperforms the remaining portfolio selection approaches.

The results of the covariance-test optimization indicate that they are quite sensitive to the choice of the test and reoptimization interval, whereas the selected significance level plays only a minor role. This finding leads to a strategy, where we apply the test on a daily basis and conduct a reoptimization only if the test rejects the null. However, this strategy does not improve upon the covariance-test optimizations for fixed intervals in most settings. Moreover, the results are even worse for the European indices.

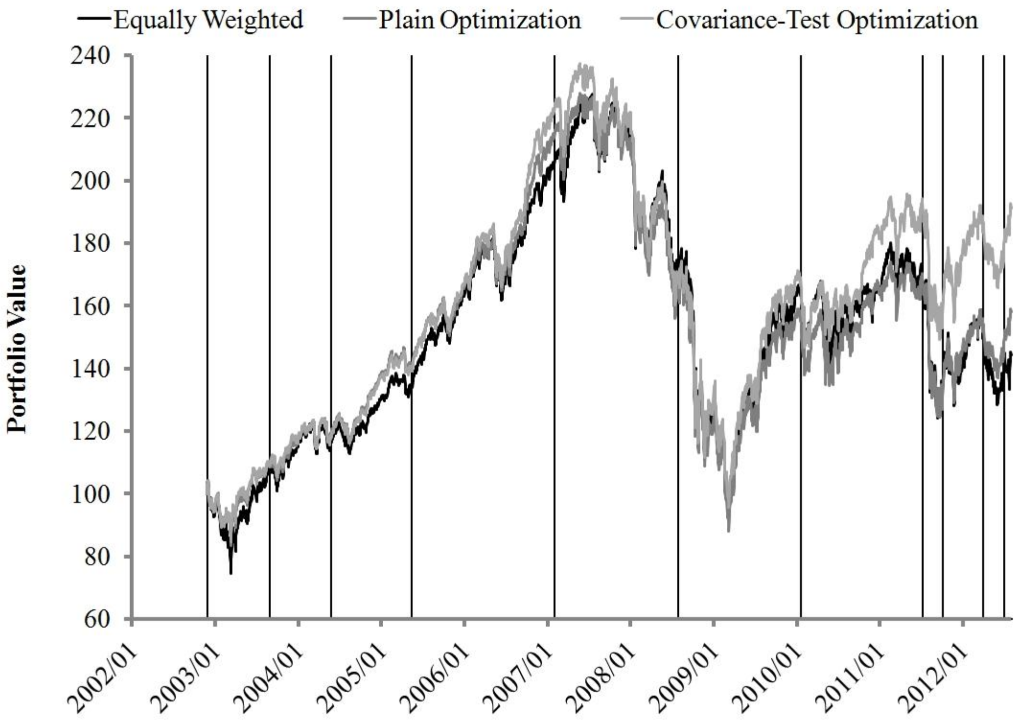

Figure 2. Portfolio Values

The Figure shows the portfolio values for the European indices dataset over the time span November 26, 2002 to July 31, 2012 (2481 trading days). The portfolio values are based on a rebalancing, reoptimization, and test interval of 63 trading days and a 500 trading day time-window with respect to the plain optimization. In addition, the points in time at which the test for a constant covariance matrix rejects the null are marked by vertical bars. The tests are conducted under a setup of a 63 trading days test interval and a 1% significance level.

This behaviour is explained by the unreliable high number of detected structural breaks. For the S&P indices there are 29 (1%-level) and 42 (5%-level) rejections, respectively. The same holds true for the European indices where 17 (1%-level) and 26 (5%-level) rejections occured. This phenomenon can plausible be explained with the effect of sequential testing. Wied et al. (2013a) investigated this issue for a test of constant correlations. Hence, additional adjustments have to be carried out in order to make this strategy applicable for practice. However, this modifications are not in the scope of the present paper.

6. Conclusion

Our empirical study shows that minimum-variance optimization significantly improves return, volatility, and Sharpe-ratio compared to equally weighted portfolios. Although the optimizations lead to considerably increased trading volumes, the turnover in connection with relatively low bid-ask spreads for heavily traded blue chips causes modest transaction costs. Furthermore, the computation of critical relative bid-ask spreads suggests that an optimization is preferable even under much higher transaction costs. However, the study also reveals the sore point of the optimization setup: The results are very sensitive to the chosen historical time-window and to the reoptimization interval.

To overcome the issue of determining appropriate time-windows, we use the test of Aue et al. (2009) for a constant covariance matrix to detect structural breaks which set the starting point of a sample. We implement a consistent and essential advancement of the promising approach introduced by Wied et al. (2013b) and apply the optimizations in combination with the test in two different ways. First, we conduct the test and the optimization after a fixed interval where the rejection of the null sets a new beginning point for the time-window. Second, we apply the test on a daily basis and conduct a reoptimization only if the test rejects the null. That means, the procedure determines the length of the time-windows as well as the point in time where the portfolio is reoptimized.

Finally, we can conclude that minimum-variance optimizations in combination with the test for a constant covariance matrix provides a usable approach to replace an arbitrary sample selection for parameter estimation by a procedure which is statistically justified. Therefore, it can be used as an automated investment system for strategic asset allocations. Besides, there are some more remarkable benefits. First, the system is completely automated and no expensive funds managers and analysts are required. Hence, costs could be decreased significantly. Moreover, the out-of-sample study shows that there is a good chance to outperform an equally distributed portfolio over longer periods of time. Consequently, the approach seems to be an appropriate alternative for an usage in practice and in order to overcome the already mentioned weak points of actively managed portfolios. Nevertheless, the new approach is not suited so resolve the timing issue yet. To this end, some modifications considering sequential testing have to be performed. We will use the results achieved so far as a starting point and take up this topic in our future research.

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Appendix

For $l=0,\ldots,[\log(T)]$, let $\sigma_{l,1}$ and $\sigma_{l,2}$ be matrices with $d(d+1)/2$ columns and $T-l$ rows such that the columns contain certain products (component by component) of the one-dimensional marginal time series. Concretely, if the entries on and below the diagonal of a $d\times d$ matrix are numbered from $c=1,\ldots,d(d+1)/2$ such that c corresponds to one pair $(i,j),1\leq i,j\leq d$, it holds that the c-th column of $\sigma_{l,1}$ is equal to the vector

$$\left(X_{l+1,i}\cdot X_{l+1,j},\ldots,X_{T,i}\cdot X_{T,j}\right)$$

and that the c-th column of $\sigma_{l,2}$ is equal to the vector

$$\left(X_{1,i}\cdot X_{1,j},\ldots,X_{T-l,i}\cdot X_{T-l,j}\right).$$

Define $\hat{\Sigma}_l$ as the empirical covariance matrix of $\sigma_{l,1}$ and $\sigma_{l,2}$. Then, we introduce the quantity

$$\hat{\Sigma}=\hat{\Sigma}_0+2\sum_{l=1}^{[\log(T)]}\left(1-\frac{l}{[\log(T)]}\right)\hat{\Sigma}_l$$

which is an estimator for the covariance matrix of S_k that captures fluctuations in higher moments and serial dependence and thus also serves for standardization. The test statistic is then the maximum over quadratic forms, i.e.

$$\Lambda_T=\max_{1\leq k\leq T} S_k{}'\,\hat{\Sigma}^{-1}S_k\,.$$