Krugman's Model with Various Values of the Costs of Transport - Under Maple Software

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Abstract: The centrum-peripherical model due to Paul Krugman (1991) with all its merits for grounding the economics crowdings, also scored the appearance of a new field in the economic theory, known as “The New Economic Geography”. There were some criticism which were focused on this matter, such as the simetry of the transport costs for the industrial goods, and the ignoring the transport costs for the agricultural goods. These criticisms could be evaded by improving the model, so as this work tries to do. On the other hand, the achieving of a computational programme is very useful in numerical simulations, necessary for studying the model, taking into account that the model can not be analytically solved.

Keywords: core-peripherical model; transport costs; industrial products; agricultural products

JEL Classification: L91; L90; L99

1. Introduction

They suppose an economy composed from two sectors, named in this work as industry, and agriculture. It is assumed an economy formed by industry and agricultural, and with two regions: 1 and 2. These regions are supposed to be identical from the point of view of both the producers’ technologies and the consumers’ preferences, but different in the matter of transport costs scored both inside and also outside them.

The agricultural sector is one that scores a perfect competition and which produces homogeneous goods, with constant scale economy and which uses only the work of the farmers, as unique production factor, these workers being supposed to be immobile between regions.

The industrial sector is characterized by a monopolistic competition, which achieves an infinity of varieties from a good which is horizontally differentiated, and which uses the worker's labour as unique production factor, but which is mobile between the two regions.

They results from the presented assumption that only the existence of different transport costs inside and outside, and also between the regions differ from the framework of the initial centrum-peripherical model due to Paul Krugman (1991).

2. Preferencies

The man-power from the entire economy, formed from farmers and workers scored an utility function, due to the goods made of Cobb-Douglas type, supposed to be as follows:

$$U = C_M^{\mu} C_A^{1-\mu} \qquad (1)$$

In which C_A appointes the consumption of the agricultural good, and C_M the unit consumption from the industrial good. The parameter μ, represents the share of the expences from the total income allotted for the acquisition of the industrial goods, and obviously it will be placed in the interval of $0<\mu<1$.

The aggregate consumption from the industrial good is made of a variety of solutions, as this form:

$$C_M = [\int_0^N c_i^{(\sigma-1)/\sigma} di]^{\sigma/(\sigma-1)} \qquad (2)$$

in which N represents the number of varieties horizontally differentiated from an industrial good, c_i represents the consumption from a "i" variety, and $\boldsymbol{\sigma}$ represents the resilience of substitution between the varieties from the industrial good. So as the (2) relation shows, that means that a reduced size of the determines the fact that the varieties have a high level of differentiation, or that the consumer economic agents have a great preference for variety.

They suppose that a part of the population $0<\mu<1$ works in the industrial sector, and $(1-\mu)$ works in the agricultural sector. As they can notice, these are the same proportion as the income spent for buying the industrial and agricultural goods, and this determine as in equilibrium to score an equal level of the salaries in the two sectors. The farmers are uniformly distributed between the two regions, so as the population which is involved in this sector in each region is fixed and equal cu $(1-\mu)/2$.

The population engaged in the industrial sector in the 1 region is designated with L_1, and in the 2 region with $L_{2,}$ so as $L_1 + L_2 = \mu$. The share of the workers from the 1 region in the total weight of the workers from the economy stands for:

$$\lambda = \frac{L_1}{L_1 + L_2} \qquad (3)$$

and obviously the share of the workers from the 2 region from the total number of the workers from the economy is $(1-\boldsymbol{\lambda})$.

The industrial sector

The production of the each variety from the industrial good asks for a fix size $\boldsymbol{\alpha}>0$, and a variable size $\boldsymbol{\beta}>0$ from the production factor – labour, which is offered by the industrial workers, reminding that this factor is the only production factor considered in this model. Under these conditions, the function of unitary cost from the “j” region will be:

$$CT_j = w_j(\alpha + \beta x_i) \qquad (4)$$

In which: CT_j is the afferent cost for the producing of one unit, from a variety; w_j is the nominal wage of the workers from the region j; x_i represents the production of the firm.

Taking into account the fact that this sector acts in a monopolistic competition, the behavior of the firms which are watching to utmost the profit will determine that the price of each industrial products from the “j” region to be as follows:

$$p_j = \frac{\sigma}{\sigma-1}\beta w_j \qquad (5)$$

The condition of free entrance of the firms into the industrial sector determine the profits to be zero, so as all the firms will have the same production:

$$x_i = \frac{\alpha(\sigma-1)}{\beta} \quad (6)$$

And as result, each firm will engage the same number of workers, which for the 1 and 2 regions will mean:

$$\frac{L_1}{L_2} = \frac{n_1}{n_2} \quad (7)$$

The agricultural sector

The agricultural sector is one which registers a perfect competition, and also constant scale economy. A labour unit offered by the farmers is used for producing an unit of agricultural good.

The transport costs will be also considered in this sector, so as the agricultural good will be different in the two regions, and the wages from the agricultural will be inclusively different, and will exist a relation like this:

$$\frac{w_1^A}{w_2^A} = T \quad (8)$$

in which w_1^A is the nominal wage of the farmers from the 1 region, and w_2^A is the nominal wage of the farmers from the 2 region, and T represents the size of the transport cost for the agricultural good.

The transport costs

Up to now, the above model does not differ from the one exposed by Paul Krugman (1991).We further suppose the existing transport costs, or generally speaking of some commercial costs, both between regions, and also inside the regions; we also introduce commercial costs to the agricultural goods. These transport costs in this model, as otherwise in the majority of the models from this field are supposed to be “iceberg” type. This means that only a part from an unit of an industrial good sent from the “i” region to the “j” region, that means $0 < \tau_{i\,j} < 1$ effectively reaches to the destination, the rest is “melting” on the way, lost which is equivalent with the size of the transport costs. Consequently, a bigger size of the $\tau_{i\,j}$ will suit to the smaller transport costs. They details these transport costs, as follows:

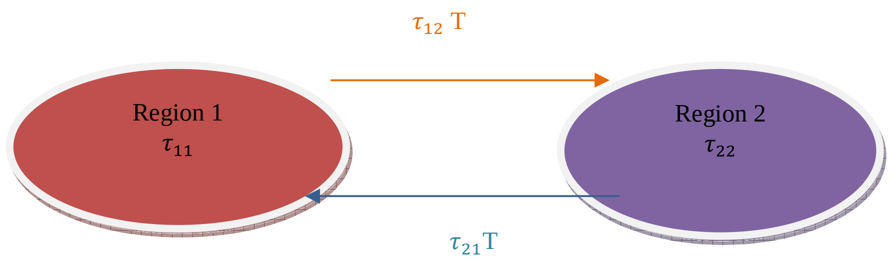

Figure 1. Lop-sided internal and external commercial costs

$\tau_{1\,2}$ represents the transport costs from the 1 region to the 2 region;

$\tau_{2\,1}$ represents the transport costs from the 2 region to the 1 region.

As these refer to the transport of the goods between regions, they could be associated to the external transport costs;

$\tau_{1\,1}$ represents the transport costs of the industrial goods from the producers to the consumers in the 1 region;

$\tau_{2\,2}$ represents the transport costs of the industrial goods from the producers to the consumers in the 2 region. As these refer to the transport of the goods inside a single region, they could be associated to the internal transport costs.

Otherwise, the transport costs represents the elements of a square matrix with all elements, differing between them.

T represents the transport costs for the agricultural goods, and it is symmetrical between the two regions, that is T_{12}=T_{21}=T.

The introduction of the transport costs to the agricultural goods determine the nominal prices and wages in this sector to be equal between the two regions, the difference being the commercial costs afferent to this sector.

The equilibrium on the short run

The special distribution of the workers on the short run is a given size, so that they do not score a migration of the industrial labour force.

Consumption of the regions

They appoint the consumption from the "i" region for a representative product of the "j" region, with $C_{j\,i}$. In the 1 region, the price or a local product is $p_1/\tau_{1\,1}$, in which $\tau_{1\,1} < 1$, meanwhile the price of a product made in the 2 region, that means an imported one is $p_2/\tau_{2\,1}$. The consumption from the two goods inside the region 1 will be:

$$C_{1\,1} = (\frac{p_1}{\tau_{1\,1}})^{-\sigma} \quad \text{and} \quad C_{2\,1} = (\frac{p_2}{\tau_{2\,1}})^{-\sigma} \tag{9}$$

The afferent expence for buying the local industrial goods $E_{1\,1}$ and with the foreign industrial goods $E_{2\,1}$ will be:

$$E_{1\,1} = (\frac{p_1}{\tau_{1\,1}})^{1-\sigma} n_1 \text{si} E_{2\,1} = (\frac{p_2}{\tau_{2\,1}})^{1-\sigma} n_2 \tag{10}$$

Then, they note with $Z_{1\,1}$ the ratio between the expenses for buying the goods for the 1 region and with the most important from the 2 region:

$$Z_{1\,1} = \frac{E_{1\,1}}{E_{2\,1}} = (\frac{w_1\tau_{2\,1}}{w_2\tau_{1\,1}})^{1-\sigma}\frac{n_1}{n_2} = (\frac{w_1\tau_{2\,1}}{w_2\tau_{1\,1}})^{1-\sigma}\frac{L_1}{L_2} \tag{11}$$

Similarly, they can get $Z_{1\,2}$,which will represent the ratio between the expenses of the 2 region for the products of the 1 region, and also for those local ones, that means those of the 2 region:

$$Z_{1\,2} = \frac{E_{1\,2}}{E_{2\,2}} = (\frac{w_1\tau_{2\,2}}{w_2\tau_{1\,2}})^{1-\sigma}\frac{L_1}{L_2} \tag{12}$$

The nominal wage

They appoint with Y_1 si Y_2 the nominal income of the 1 region, respectively of the 2 region, which will be equal with the sum of the incomes from the agricultural and industry from each region. So, they will be:

$$Y_1 = \frac{1-\mu}{2} w_!^A + w_1 L_1 \text{si} Y_2 = \frac{1-\mu}{2} w_2^A + w_2 L_2 \tag{13}$$

As for the nominal wage of the workers from the 1 region, it is equal with the expense due to the industrial goods of the 1 region, that is:

$$L_1 w_1 = \frac{Z_{1\,1}}{1+Z_{1\,1}}\mu Y_1 + \frac{Z_{1\,2}}{1+Z_{1\,2}}\mu Y_2 \Rightarrow w_1 = \frac{\mu}{L_1}[\frac{Z_{1\,1}}{1+Z_{1\,1}}Y_1 + \frac{Z_{1\,2}}{1+Z_{1\,2}}Y_2] \tag{14}$$

Similarly, they determine the nominal wage of the workers from the 2 regions:

$$w_2 = \frac{\mu}{L_2}[\frac{1}{1+Z_{1\,1}}Y_1 + \frac{1}{1+Z_{1\,2}}Y_2] \quad (15)$$

The (13) – (15) relations involves the fact that the sum of the nominal wages is a constant one, that means equal with:

$$L_1 w_1 + L_2 w_2 = \mu \quad (16)$$

By replacements, they get the following relations referring to the nominal wages from the two regions:

$$w_1 = \frac{\mu}{L_1}[\frac{\left(\frac{w_1\tau_{2\,1}}{w_2\tau_{1\,1}}\right)^{1-\sigma}\frac{L_1}{L_2}\left(\frac{1-\mu}{2}+w_1L_1\right)}{1+\left(\frac{w_1\tau_{2\,1}}{w_2\tau_{1\,1}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{w_1\tau_{2\,2}}{w_2\tau_{1\,2}}\right)^{1-\sigma}\frac{L_1}{L_2}\left(\frac{1-\mu}{2}+w_2L_2\right)}{1+\left(\frac{w_1\tau_{2\,2}}{w_2\tau_{1\,2}}\right)^{1-\sigma}\frac{L_1}{L_2}}] \quad (17)$$

$$w_2 = \frac{\mu}{L_2}[\frac{\left(\frac{1-\mu}{2}+w_1L_1\right)}{1+\left(\frac{w_1\tau_{2\,1}}{w_2\tau_{1\,1}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{1-\mu}{2}+w_2L_2\right)}{1+\left(\frac{w_1\tau_{2\,2}}{w_2\tau_{1\,2}}\right)^{1-\sigma}\frac{L_1}{L_2}}] \quad (18)$$

The (17) – (18) equations forms a system with w_1 si w_2 the unknowns under the conditions of a given distributions of the workers between the two regions. If they also take into account the (16) relation, they obtain an equation depending on $w_1, or\ on\ w_2$.

The indexes of price and real wages

The workers are not interested about the nominal wages, but they are really interested about the real wages in the each region, as these depend on the life cost of each region.

The price indexes P_1 si P_2 the prices indexes establish a link between the expense and the utility of the economic agents from each region. These are depending on the price of the agricultural good, as for the price indexes of the industrial goods from the two regions, respectively P_{M1} and P_{M2} , which are:

$$P_{M1} = \gamma[\lambda\,(\frac{w_1}{\tau_{1\,1}})^{1-\sigma} + (1-\lambda)\left(\frac{w_2}{\tau_{2\,1}}\right)^{1-\sigma}]^{\frac{1}{1-\sigma}} \quad (19)$$

$$P_{M2} = \gamma[\lambda\,(\frac{w_1}{\tau_{1\,2}})^{1-\sigma} + (1-\lambda)\left(\frac{w_2}{\tau_{2\,2}}\right)^{1-\sigma}]^{\frac{1}{1-\sigma}} \quad (20)$$

in which $\lambda = \frac{L_1}{L_1+L_2}$ and $\gamma = \frac{\sigma\beta}{\sigma-1}[\frac{\mu}{\frac{\alpha}{\beta}(\sigma-1)}]^{\frac{1}{1-\sigma}}$.

The real wages of the industrial workers from the two regions will be as follows:

$\omega_1 = w_1 P_{M1}^{-\mu} (w_1^A)^{\mu-1}$(21)

$\omega_2 = w_2 P_{M2}^{-\mu} (w_{2A}^A)^{\mu-1}$(22)

The real wage, respectively ω_1/ω_2 will be as follows:

$$\Omega = \frac{\omega_1}{\omega_2} \quad (23)$$

The long run equilibrium

In a short time equilibrium, all the variables are determined by assuming as datum a certain distribution of the workers, that is of the $\boldsymbol{\lambda}$ variable. As for a long run equilibrium, the migration of the workers does not score. It represents a steady equilibrium if it resists to the small changes of the distribution between regions. A scattered equilibrium is that long run equilibrium if the regions score the same real wage. The concentrated equilibrium is that one in which all the workers are grouped in a single region, as that region scores the highest real wage.

3. Simulations on the Base of the Transport Costs

In order to make simulations, there was achieve a computational program on the base of the Maple 14 soft , as it could be seen in the figure nr. 2, which scores the described model.

They could approach the following cases:

a) Internal and external equal costs: $\tau_{1\,2} = \tau_{2\,1} = \tau_e$ and $\tau_{1\,1} = \tau_{2\,2} = \tau_i$;

b) Different internal costs: $\tau_{1\,2} = \tau_{2\,1} = \tau_e$ and $\tau_{1\,1} \neq \tau_{2\,2}$;

c) Different external costs : $\tau_{1\,2} \neq \tau_{2\,1}$ and $\tau_{1\,1} = \tau_{2\,2} = \tau_i$;

d) Transport costs for the agricultural goods: $T \neq 1$.

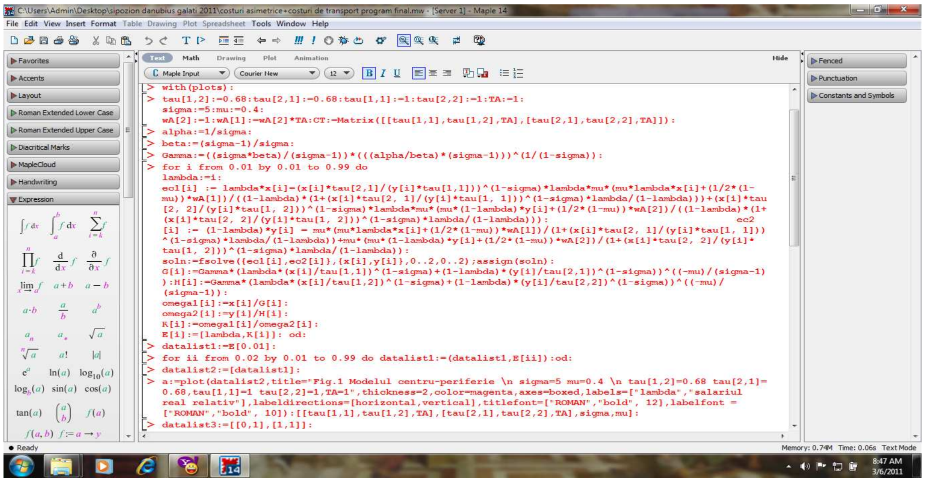

Figure 2 The computational program for simulations

By ignoring the internal transport costs of the industrial goods and of the transport costs for the agricultural products they obtain the same results al for the initial model of Paul Krugman. But this generalization of the transport costs proves useful both from a theoretical and also practical point of view.

4. Bibliography

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