An Empirical Study of Correlation and Volatility Changes of Stock Indices and their Impact on Risk Figures

Nicolai Bissantz[1]**, Daniel Ziggel**[2]**, Kathrin Bissantz**[3]

Abstract: During world financial crisis it became obvious that classical models of portfolio theory significantly under-estimated risks, especially with regard to stocks. Instabilities of correlations and volatilities, the relevant parameters characterizing risk, led to over-estimation of diversification effects and consequently to under-estimation of risks. In this article, we analyze the relevant risk parameters concerning stocks during different market periods of the previous decade. We show that parameters and risks significantly change with market periods and find that the impact of fluctuations and estimation errors is ten times larger for volatilities than for correlations. Moreover, it turns out that diversification between sectors is more efficient than diversification between countries.

Keywords: Model Evaluation; Portfolio Optimization; Risk Management

JEL Classification: C 52; G 11; G 32

1. Introduction

Efficient risk management and portfolio optimization are central tasks of the financial sector but are also important for private investors. In this context, asset allocation aims to share a given amount of money optimally between different assets, considering the crucial parameters of expected return and possible loss. Of particular importance is the diversification between different stock indices.

The model by Markowitz (1952) represents a milestone in development of modern theories in the area of risk management and portfolio optimization and was rewarded the Nobel prize in Economics in 1990. According to this model, any investor should put his money into efficient portfolios only, i.e. portfolios which have the smallest risk for a given return defined by the investor, or portfolios having a maximal return for a predefined acceptable risk. The risk of the portfolio is given by its volatility, i.e. the standard deviation of its returns. Correlations between the assets may decrease the risk for the overall portfolio significantly compared to investments into single assets.

[1] PhD, Ruhr-Universität Bochum, Department of Mathematics, Germany, Address: 44780 Bochum, Germany, tel. +49 (0)234 32-14201, fax: +49 (0)234 32-201, email: nicolai.bissantz@rub.de.
[2] PhD, Quasol GmbH, Marktallee 8, Germany, Address: 48165 Münster, Germany, Corresponding author: daniel.ziggel@quasol.de
[3] PhD, Schattbachstraße 48, Germany, Address: 44801 Bochum, Germany, email: nicolai.bissantz@rub.de.

As shown in numerous works strategic asset allocation makes up for the majority of performance of an investment. Brinson/Hood/Beebower (1986) and Brinson/ Singer/Beebower (1991) quantify the influence as 90% to 94%, while Ibbotson/Kaplan (2000) give values between 82% and 88%, both demonstrating significance of strategic asset allocation. Additional factors, such as timing and strategy realization, are only of minor importance.

Reliable estimation of the relevant parameters, i.e. return, volatility and correlation, is of major importance for optimal portfolio selection as well as risk management and therefore future success of the investment. Different studies show that return is the most important parameter in the Markowitz model. Chopra/Ziemba (1993) demonstrates that, for mean tolerated risk levels, wrong return estimators have an eleven times larger impact than wrong risk estimators. Analogously, Kallberg/Ziemba (1984) and Schäfer/Zimmermann (1998) demonstrate that estimation problems in the Markowitz model are mainly related to the return.

Nevertheless, the current situation at the financial markets shifts the focus on the risk perspective. Volatilities and correlations strongly increased during the financial crisis, as reflected by increased risk numbers. Zimmermann/Drobetz/Oertmann (2002) named this effect “Correlation Breakdown”. Campbell/Forbes/Koedijk/Kofman (2008) even described this phenomenon initially as “Diversification Meltdown”. Obvisiously, volatilities and correlations of different assets are positively correlated in times of crisis, and the diversification approach does not work - in particular when required to prevent losses.

In this paper, we empirically analyze the effects of changing parameters to risk figures of stock indices during different market periods. This is done by means of daily and monthly market data. To this end, the resulting risk numbers for different market periods are compared. From the results we draw conclusions on stability of diversification effects and risk estimators in classical portfolio theory. We are able to show that risks are significantly under-estimated, especially if historical mean values are used as parameter estimators. Besides, we find that the impact of fluctuations and estimation errors is ten times larger for volatilities than for correlations. Additionally, we determine how these effects influence diversification between countries and between sectors, demonstrating that diversification effects are more stable between the latter.

2. Classical Portfolio Theory

The model by Markowitz (1952) represents a milestone in development of modern theories in the areas of risk management and portfolio optimization. It assumes the existence of N assets with normally distributed return r_i for the i-th asset. Optimal selection of the portfolio weights ($\omega_1, \omega_2, \ldots, \omega_N$) is intended where ω_i is the fraction which is invested into asset i.

Up to now, the Markowitz model is broadly used by investors to optimize portfolios and control risks. The crucial parameters for portfolio selection are the expected return of the portfolio (r_P) and the risk of the portfolio, which is defined by the standard deviation (σ_P). According to Markowitz theory efficient portfolios, which are attractive investments, should have a combination (r_P, σ_P), which is not dominated by a portfolio with smaller standard deviation for the same return or a portfolio with a larger return for the same standard deviation.

The consideration of correlation effects offers the advantage that investments into assets, which seem to be disadvantageous on the first sight, may decrease the overall risk of the portfolio. This is e.g. illustrated by a portfolio containing 80% of an asset with an expected return of 5% and a volatility of 3% and 20% of a more risky asset having an expected return of 10% and a standard deviation of 6%. This combination results in an expected portfolio return of 6%. If both assets are not correlated, the overall volatility of the portfolio is only 2.7%., i.e. the expected return of the overall portfolio is larger than the expected return of the more secure asset. Moreover, the risk is significantly smaller than for each single asset. Figure 1 illustrates the effect of different correlations and portfolio weights ($\omega_i \in [0,1]$) for both assets showing returns and volatilities of the portfolio.

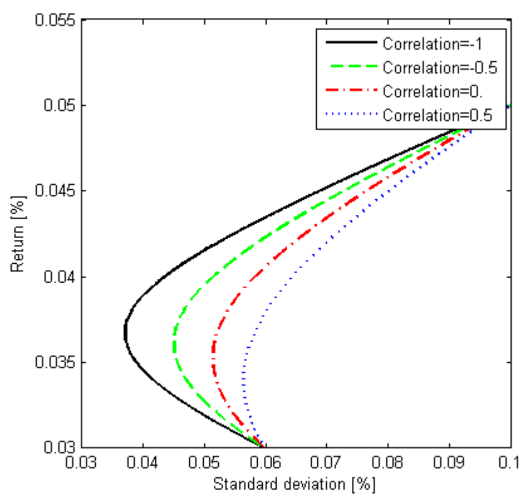

Figure 1. Efficiency frontiers for portfolios consisting of two stocks with returns and standard deviations of (5%, 3%) and (10%, 6%) for different correlations between the stocks

The assumptions within this model are that the returns are normally distributed and that the parameters of the assets, i.e. returns, correlations and volatilities, can be reliably estimated. Moreover, it is assumed that the parameters do not change during the investment period. In the previous years, the reliable estimation of parameters became significantly more difficult: On the one hand, it became obvious that correlations and volatilities depend on time so that both tend to increase when markets decrease and vice versa. On the other hand, there are strong indications that volatilities and correlations depend on each other as it is shown by Frennberg/Hansson (1993), Zimmermann/Drobetz/Oertmann (2002) and Andersen/Bollerslev/Diebold/Ebens (2001).

3. Correlation Breakdown

In recent discussions concerning correlations and volatilities in risk management and portfolio optimization, the terms "Correlation Breakdown" and "Diversification Meltdown" were introduced and describe the phenomenon that correlations and volatilities tend to increase, if the market decreases and also the other way round. Moreover, there is a strong positive relation between correlations and standard deviations. Thus, diversification effects are particularly overestimated during nervous market periods for which they are of high importance. Hence, the permanent changing pattern of market parameters complicates selection of an optimal risk strategy.

The stock market crash in October 1987 and the 2008 financial crisis revealed, that the structure of correlations reflects extreme situations on markets. In both cases correlations strongly increased to a high level remaining constantly high for a certain period.

Meric/Meric (1997) confirms this situation from a European perspective: Average correlations between 13 European stock markets increased from 0.37 before the crash in 1987 to a value of 0.5 afterwards. Rey (2000) describes similar events: Average correlations based on data from Switzerland, USA, UK, Canada, Germany, Italy, France and Japan increased from 0.40 measured from January 1973 to December 1986 to 0.55 between January 1988 and December 1999. During October 1987, the average correlation between international stock markets was, according to Rey (2000), even 0.68. A result by Longin/Solnik (1995) generally confirms that volatilities and correlations are stronger connected when volatility is on a high level.

These results make it necessary for investors to have a critical look on the idea of diversification: Assumptions, which should minimize the overall risk, collapse exactly when markets decrease. Hence, regarding the two great financial crisis of the last decade, it is questionable if classical portfolio theory is able to generate reliable risk estimators.

4. Empirical Analysis

This section analyzes the development of correlation structures and volatilities of stock markets during four different phases of the last decade. The complete period of analysis covers March, 31st 1999 to February, 26th 2010 for sectors and January, 1st 2001 to February, 26th 2010 for countries. The differentiation concerning these periods is due to data availability. Additionally, two bear markets (dot-com crisis, 31.03.2000 (sectors) respective 01.01.2001 (countries) to 31.03.2003 and financial crisis, 30.04.2008 to 31.03.2009) and a bull market (30.04.2003 to 31.03.2008) are analyzed separately. Figure 2 clarifies the temporal sequence of these periods.

Figure 2: Schematic illustration of the analyzed periods at the example of the development of EURO STOXX 50.

4.1 Data base

Monthly and daily final quotes of selected important stock indices are used to determine the relevant parameters of each asset class differing between subsectors (10) and country indices (5). Especially the following stock indices are taken into account for analysis:

- EURO STOXX OIL & GAS;
- EURO STOXX BASIC MATERIALS;
- EURO STOXX INDUSTRIALS;
- EURO STOXX CONSUMER GOODS;
- EURO STOXX HEALTH CARE;
- EURO STOXX CONSUMER SERVICES;
- EURO STOXX TELECOM;
- EURO STOXX UTILITIES;
- EURO STOXX FINANCIALS;
- EURO STOXX TECHNOLOGY;
- MSCI EMERGING MARKETS;
- MSCI USA;
- MSCI JAPAN;
- STOXX EUROPE 50;
- MSCI WORLD.

Time series were obtained from Thomson Reuters Datastream and collected in Excel, which was used for all computations and analyses.

4.2 Calculation of Relevant Parameters

In this section, we describe calculation of the relevant parameters based on monthly data. We investigate for each index a ($a \in \{1, \dots, m\}$) continuous returns. These are determined as:

$$r_a(j) = \ln\left(\frac{index\ at\ the\ end\ of\ the\ j-th\ month}{index\ at\ the\ end\ of\ the\ (j-1)-month}\right).$$

For sake of simplicity, the following characteristic numbers, especially volatilities and Value-at-Risks, are given for an one-year investment period. The expected average annual return is $\widehat{R}_a(J) = 12 * \overline{r_a(J)}$, where $\overline{r_a(J)}$ represents the average monthly return in the respective period. The corresponding months are summarized by the index set J.

From the returns $r_a(j)$ for asset a follows the estimator for the variance of returns:

$$\hat{\sigma}_a^2(J) = 12 * [\frac{1}{n-1}\sum_{j \in J}(r_a(j) - \overline{r_a(J)})^2],$$

where n is the number of months in the respective period. Volatility is calculated as the square root of the variance. Analogously, we determine estimators for the correlation between two assets a and b ($a, b \in \{1, \dots, m\}$):

$$\hat{\rho}_{a,b}(J) = \frac{12}{n-1}\sum_{j \in J}\left(\frac{r_a(j) - \overline{r_a(J)}}{\sqrt{\hat{\sigma}_a^2(J)}}\right)\left(\frac{r_b(j) - \overline{r_b(J)}}{\sqrt{\hat{\sigma}_b^2(J)}}\right).$$

And the estimator for the corresponding covariance:

$$\hat{\sigma}^2_{a,b}(J) = \sqrt{\hat{\sigma}^2_a(J)\hat{\sigma}^2_b(J)} * \hat{\rho}_{a,b}(J).$$

The estimated return $\hat{R}_a(J)$ and variance $\hat{\sigma}^2_a(J)$ yield a parametric estimation of the 99%-Value-at-Risk of an asset with the 1%-quantile of the standard normal distribution $q_{0.01} = -2.326$ as:

$$VaR_{a,99\%}(J) = \hat{R}_a(J) - 2.326 * \sqrt{\sigma^2_a(J)}.$$

The $VaR_{a,99\%}$ can be split into a component $VaR^{ex}_{a,99\%}$, which is given by the expected return (respectively the corresponding estimator) and a "stochastic" component, $VaR^{st}_{a,99\%} = -2.326 * \sqrt{\sigma^2_a(J)}$ which is calculated from the (estimated) volatilities. As shown below, correlations also influence the Value-at-Risk of a portfolio because they are required to calculate the overall volatility of a portfolio. Based on the estimated parameters of the different assets it is possible to calculate return and risk of a portfolio using the portfolio weights $(\omega_1, \dots, \omega_m)$. For a period J, the expected return is given by:

$$\hat{r}_P(J) = \sum_{a=1}^{m} \omega_a \hat{R}_a(J).$$

And its variance is:

$$\hat{\sigma}^2_P(J) = \sum_{a,b=1}^{m} \omega_a \omega_b \hat{\sigma}^2_{a,b}(J).$$

Finally, we determine the 99%-Value-at-Risk $VaR_{P,99\%}(J)$ of the portfolio over a period J as:

$$VaR_{P,99\%}(J) = \hat{r}_P(J) - 2.326 * \sqrt{\sigma^2_P(J)}.$$

Hence, the stochastic component is given by:

$$VaR^{st}_{P,99\%} = -2.326 * \sqrt{\sigma^2_P(J)}.$$

Calculations based on daily data are performed in a similar way. For the sake of simplicity, we assume a year to have 250 trading days.

4.3 Parameters during Different Market Periods

Tables 1 resp. 2 and Tables 3 resp. 4 summarize average correlations and volatilities for different market periods sorted by sectors respectively countries showing strong fluctuations of volatility over time. Comparison of parameters during the bull market and the financial crisis, which followed immediately afterwards, shows an alarming increase of volatilities by a factor of 1,5 to 3 for monthly data. Based on daily data, the fluctuations are even stronger. Only exemptions from this are the TELECOM and HEALTH CARE sectors calculated from monthly data. While volatilities in the TELECOM sector strongly increased during the dot-com crisis, they remained on a constant level for the HEALTH CARE sector over the complete observation period.

Moreover, it is quite interesting to compare values based on monthly and daily data. With the exception of the financial crisis, volatilities are rather similar for both frequencies. However, during the financial crisis, volatilities based on daily data are significantly larger than their counterparts determined from monthly data. This is caused by the fact that if monthly data is used, a significant fraction of the variability in the data is smoothed out. Hence, in order to capture the full amount of variability during a time of crisis, it is preferable to use daily data for computation of the volatility.

Surprisingly, it can be observed that the average correlation between all sectors remained constant over all periods, i.e. there was no "Correlation Breakdown" even not during the financial crisis. Thus, diversification between sectors appears to remain stable even during crisis. For country indices, this result does not turn out to be true. Average correlations on a monthly basis were on a constant level until upset of the crisis. More precise, the average correlation between countries increased by 0.21 during financial crisis. Even the smallest value was 0.84. This shows a clear "Correlation Breakdown".

Again, it is interesting to consider differences between results based on daily respectively those based on monthly data. Whereas these differences are rather negligible for sector based indices, they are much more significant for country based indices. Here, correlations based on daily data are much smaller than expected and in particular than those based on monthly data. This is due to the different time zones covered by the individual country based indices, which leads to differences in the point (and the subsequent interval) of time, during which they are traded. In consequence, different amounts of information are available to the investor during trading time and hence included in the final quote, which is a problem nearly non-existing for the sector based indices. From this, it is not recommendable to use daily data to determine correlations of country based indices, which do not belong to the same time zone. Nevertheless, even on a daily basis there was a significant increase of correlations during the financial crisis between country indices.

For correlations between single indices, even higher fluctuations can be observed. This turns out to be true for sectors as well as for countries, e.g. the correlation between TELECOM and HEALTH CARE decreased between bull market and financial crisis by 0.47, while correlation between Japan and the US increased by 0.44. Complete correlation matrices can be requested by the authors.

These surprising results demonstrate that diversification effects between sectors remain constant during crisis but not between countries, where structures of correlations change. Thus, diversification within the asset category "stocks" between countries seems to be impossible and the true risks are significantly larger than expected. Apart from that, correlations between stock indices are per se quite high. Hence, an asset allocation solely based on stocks is always risky.

Table 1. Volatilities during different market periods (sectors) – Monthly/Daily

Index/Period	Total	Dot-com	Bull Market	Financial Crisis
Oil & Gas	18,1% / 22,7%	18,2% / 28,2%	15,1% / 17,2%	26,8% / 49,5%
Basic Materials	21,5% / 21,7%	24,9% / 25,0%	16,0% / 17,7%	33,0% / 46,9%
Industrials	21,3% / 20,4%	22,8% / 21,5%	15,5% / 16,9%	34,2% / 46,9%
Consumer Goods	19,0% / 20,4%	21,5% / 24,1%	14,5% / 15,1%	23,7% / 45,8%
Health Care	16,2% / 21,2%	19,5% / 28,6%	14,1% / 16,2%	17,9% / 33,8%
Consumer Services	18,8% / 19,1%	25,3% / 28,2%	13,5% / 14,3%	20,3% / 32,4%
Telecom	26,4% / 25,8%	36,0% / 38,2%	14,4% / 14,8%	15,0% / 33,2%
Utilities	17,4% / 19,1%	16,8% / 22,2%	12,7% / 14,6%	24,4% / 43,1%
Financials	24,2% / 23,7%	28,2% / 29,2%	16,1% / 17,1%	42,3% / 55,6%
Technology	31,4% / 31,2%	48,7% / 51,9%	22,1% / 23,2%	36,2% / 42,3%

Table 2. Volatilities during different market periods (countries) – Monthly/Daily

Index/Period	Total	Dot-com	Bull Market	Financial Crisis
Emerging Markets	21,1% / 17,7%	22,0% / 15,1%	15,8% / 14,3%	33,1% / 34,2%
USA	16,4% / 21,6%	18,8% / 23,5%	9,1% / 12,9%	27,1% / 44,8%
Japan	18,6% / 23,3%	15,6% / 22,2%	14,6% / 18,7%	31,7% / 42,8%
Europe	17,3% / 23,2%	20,7% / 29,5%	11,2% / 14,6%	21,2% / 40,3%
World	17,0% / 17,7%	17,4% / 18,5%	9,7% / 10,9%	28,8% / 36,1%

Table 3. Average correlations for different market periods (sectors) – Monthly/Daily

Index/Period	Total	Dot-com	Bull Market	Financial Crisis
Oil & Gas	0,53 / 0,61	0,52 / 0,57	0,45 / 0,63	0,53 / 0,74
Basic Materials	0,66 / 0,55	0,61 / 0,65	0,64 / 0,74	0,64 / 0,74
Industrials	0,73 / 0,61	0,68 / 0,72	0,71 / 0,75	0,71 / 0,77
Consumer Goods	0,68 / 0,64	0,69 / 0,76	0,69 / 0,76	0,48 / 0,40
Health Care	0,46 / 0,59	0,36 / 0,59	0,39 / 0,57	0,47 / 0,60
Consumer Services	0,69 / 0,49	0,67 / 0,74	0,69 / 0,75	0,66 / 0,76
Telecom	0,50 / 0,63	0,40 / 0,64	0,55 / 0,66	0,38 / 0,70
Utilities	0,63 / 0,52	0,52 / 0,66	0,65 / 0,65	0,70 / 0,71
Financials	0,69 / 0,59	0,71 / 0,76	0,69 / 0,77	0,68 / 0,71
Technology	0,63 / 0,65	0,61 / 0,65	0,54 / 0,66	0,67 / 0,71
Average	0,62 / 0,59	0,58 / 0,68	0,60 / 0,69	0,59 / 0,68

Table 4. Average correlations for different market periods (countries) – Monthly/Daily

Index/Period	Total	Dot-com	Bull Market	Financial Crisis
Emerging Markets	0,78 / 0,57	0,71 / 0,45	0,69 / 0,54	0,87 / 0,67
USA	0,82 / 0,50	0,77 / 0,49	0,71 / 0,43	0,92 / 0,53
Japan	0,66 / 0,35	0,47 / 0,28	0,54 / 0,36	0,89 / 0,42

Europe	0,79 / 0,54	0,77 / 0,47	0,70 / 0,51	0,84 / 0,64
World	0,85 / 0,68	0,80 / 0,62	0,77 / 0,65	0,93 / 0,73
Average	0,78 / 0,53	0,71 / 0,46	0,68 / 0,50	0,89 / 0,60

4.4 Effects of changing parameters on the VaR

To illustrate and quantify the effects of changing parameters, we consider risk numbers of five different portfolios for each period. Three portfolios reflect sectors whereas two are diversified by countries. A large variability of correlations and volatilities leads to a strongly varying stochastic component (VaR_{stoch}) of the overall Value-at-Risk (VaR). Since changes in the stochastic component are based on variability of correlations and volatilities, we restrict our analysis to this component as it also represents the effect of diversification which can be achieved for a portfolio. This component on its own leads to a strong change of the overall VaR.

Two of the portfolios use a naive diversification and all indices hold the same share of the overall portfolio, one being diversified by sectors and the other one by countries. Also two funds, based on sectors (AriDeka CF, Deka-Institutionell Aktien Europa I (T)), and one fund, based on different countries (Deka-bav Fonds), are analyzed. Exact diversification of the portfolios is given in Tables 5 and 6. For sake of simplicity, we assume that the asset categories contained in the portfolio are perfectly reflected by the respective index. Hence, we obtain realistic estimations for the behavior of risk numbers of real portfolios although they are not exactly replicated, which is not in the scope of this work. Furthermore, we assume an investment of 100,000,000€ to provide the VaR in €.

Table 5. Portfolio weights (sectors)

Index/Portfolio	Naive	AriDeka CF	Deka-Institutionell
Oil & Gas	10,00%	12,32%	15,60%
Basic Materials	10,00%	10,89%	9,01%
Industrials	10,00%	8,49%	4,29%
Consumer Goods	10,00%	14,23%	10,88%
Health Care	10,00%	13,76%	16,92%
Consumer Services	10,00%	6,70%	1,98%
Telecom	10,00%	7,78%	9,45%
Utilities	10,00%	4,31%	5,60%
Financials	10,00%	19,02%	23,74%
Technology	10,00%	2,51%	2,53%

Table 6. Portfolio weights (countries)

Index/Portfolio	Naive	Deka-bav Fonds
Emerging Markets	20,00%	0,67%
USA	20,00%	44,70%
Japan	20,00%	6,20%
Europe	20,00%	35,30%
World	20,00%	13,80%

Tables 7 to 11 show the stochastic component VaR_{stoch} for all portfolios. Our results strongly indicate that by solely varying correlations and volatilities the VaR is dramatically fluctuating. Thus, the VaR increased by a factor of approximately $2 * VaR_{stoch}$ for all portfolios upon exchange of the bull market parameters by values holding for the financial crisis. Even during the dot-com crisis, the risk was significantly larger than during the bull market.

Comparing sector-based to country-based portfolios, fluctuations are marginally smaller for the first. During bull market, the risk for sector based portfolios was slightly larger whereas it was similar during the financial crisis. Comparing risk figures based on monthly and daily data, they are quite different for times of crisis. This is due to the fact that volatilities based on daily data are much larger and changes of volatilities are the main reason for fluctuations of the $\mathrm{VaR}_{\mathrm{stoch}}$ as shown in the next paragraph.

Table 7. $\mathbf{VaR_{stoch}}$ naive diversification (sectors) – Monthly/Daily

Period/VaR_{stoch}	In %	In €
Total	-40,64% / -43,41%	-33,4 Mio. € / -35,2 Mio. €
Dot-com	-48,60% / -57,97%	-38,5 Mio. € / -44,0 Mio. €
Financial Crisis	-52,56% / -84,58%	-40,9 Mio. € / -57,1 Mio. €
Bull market	-28,57% / -33,04%	-24,9 Mio. € / -28,1 Mio. €

Table 8. VaR_{stoch} AriDeka CF – Monthly/Daily

Period/VaR_{stoch}	In %	In €
Total	-39,66% / -42,97%	-32,7 Mio. € / -34,9 Mio. €
Dot-com	-45,71% / -55,39%	-36,7 Mio. € / -42,5 Mio. €
Financial Crisis	-53,62% / -86,50%	-41,5 Mio. € / -57,9 Mio. €
Bull market	-28,27% / -32,95%	-24,6 Mio. € / -28,1 Mio. €

Table 9. VaR_{stoch} Deka-Institutionell – Monthly/Daily

Period/VaR_{stoch}	In %	In €
Total	-39,21% / -43,41%	-32,4 Mio. € / -35,2 Mio. €
Dot-com	-44,89% / -56,28%	-36,2 Mio. € / -43,0 Mio. €
Financial Crisis	-53,98% / -88,09%	-41,7 Mio. € / -58,6 Mio. €
Bull market	-27,65% / -32,79%	-24,2 Mio. € / -28,0 Mio. €

Table 10. VaR_{stoch} naive diversification (countries) – Monthly/Daily

Period/VaR_{stoch}	In %	In €
Total	-38,16% / -37,51%	-31,7 Mio. € / -31,3 Mio. €
Dot-com	-38,80% / -38,18%	-32,2 Mio. € / -31,7 Mio. €
Financial Crisis	-63,16% / -75,26%	-46,8 Mio. € / -52,9 Mio. €
Bull market	-24,07% / -25,30%	-21,4 Mio. € / -22,4 Mio. €

Table 11. VaR_{stoch} Deka-bav Fonds – Monthly/Daily

Period/VaR_{stoch}	In %	In €
Total	-37,47% / -42,88%	-31,3 Mio. € / -34,9 Mio. €
Dot-com	-42,40% / -48,99%	-34,6 Mio. € / -38,7 Mio. €
Financial Crisis	-57,79% / -84,41%	-43,9 Mio. € / -57,0 Mio. €
Bull market	-21,72% / -25,72%	-19,5 Mio. € / -22,7 Mio. €

We performed another data analysis to investigate whether the changes in risk are caused by changing correlations or by changing volatilities (or to find out which are their respective contributions). Here, we assumed for all market periods the average volatilities solely changing correlation matrices. Hence, changes of the covariance matrix result from changing correlations. Based on these covariance matrices, volatilities of the naively diversified portfolios were determined for all market periods. Results are given in Table 12.

For sector indices the increased risk is completely explained by increased volatilities. If the portfolio volatility only changed due to changes of the correlation matrix, it would remain constant over different market periods being consistent to results in the prior section, showing that the average correlation did not change. This holds true for both monthly and daily data.

Considering country based indices, it turned out that the volatility increased by approximately 1,5% during the financial crisis due to increased correlations. If we also took the changes of volatilities into account, the increase would be about 18% (daily data) respectively 13% (monthly data) between countries, i.e. the effect of changing volatilities on the risk is about ten times larger than that of changing correlations. To draw a conclusion, fluctuations in volatilities have a significantly stronger impact on diversification effects and risk figures than changes in correlations, whose impact is negligible. Hence, the terms "Correlation Breakdown" and "Diversification Meltdown" seem to be very deceptive for portfolios solely consisting of stocks. In contrast to that, "Volatility Burst" would be a much more reasonable term.

Table 12. Resulting volatilities using average volatilities for single indices and changing correlation matrices by period – Monthly/Daily

Period/Volatility	Sectors	Countries
Total	17,47% / 18,66%	16,41% / 16,13%
Dot-com	17,03% / 18,92%	15,75% / 15,46%
Financial Crisis	17,21% / 19,10%	17,26% / 16,88%
Bull market	17,14% / 19,12%	15,58% / 15,82%

5. How to Deal with Changing Parameters

Results of the last sections show that risks of the individual portfolios differ strongly in dependence of the time period which is used for parameter estimation. These differences in risk are important to consider for institutional as well as for private investors. Thus, it is of great interest to analyze how these risks can be minimized or at least be appropriately measured. It is demonstrated that correlations and volatilities cannot be estimated simply from historical data due to large estimation errors in some cases.

To illustrate this effect, Figure 3 represents the different temporal evolution of estimators of correlation between the sectors TELECOM and FINANCIALS. Here, the correlation was estimated by means of historical data using different moving averages.

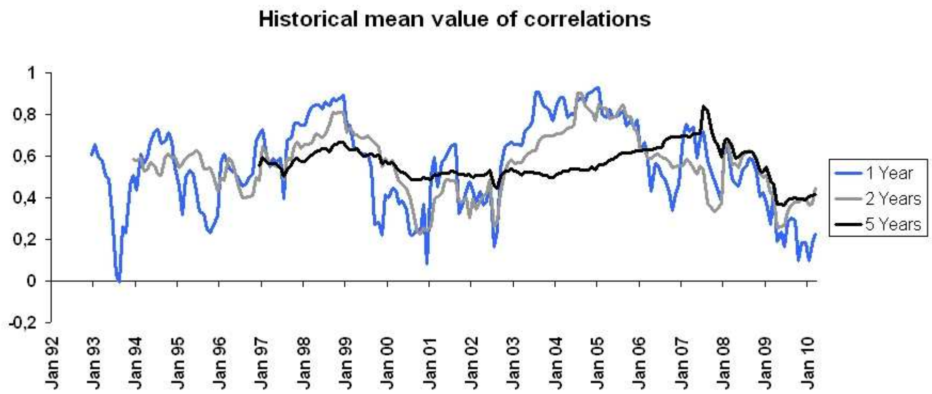

Figure 3: Moving averages of the correlation between TELECOM and FINANCIALS for a history of 1-year, 2-years and 5-years

Figure 3 shows that fluctuations of the estimators decrease with an increase of the time period used for analysis. This implies the problem that a long time period leads to very inflexible estimators, due to its strong smoothing of the results. If only short time periods are used for estimation this may lead to drastic estimation errors because of strong variability of the estimators. In current research, there exist different approaches to deal with changing parameters. Although this is not in focus of the present analysis, we will briefly describe two methods and refer to comprehensive sources. A promising approach for the timely recognition of parameter changes is testing for structural breaks, i.e. changes in parameters which define a time series. Aue/Hörmann/Horváth/Reimherr (2009) proposed a test to detect changes in the covariance structure, while Wied/Krämer/Dehling (2011) and Wied/Arnold/Bissantz/Ziggel (2011) present methods to test for changes in the correlations structure and of variances, respectively. These tests can be used in various ways. First, it is possible to determine appropriate subsets of data which are used to estimate the different parameters. Second, the tests can be used as an alert system in order to recognize unfavorable parameter changes. Finally, optimal points in time for a re-optimization can be determined, because an optimal solution for the portfolio is no longer valid if input parameters have changed. Apart from fluctuation tests, time series models (e.g. GARCH models) are of special interest with regard to parameter estimation, because they adapt to changing data structures in a very flexible way (McNeil/Frey/Embrechts (2005)).

6. Acknowledgement

The work of the first author was supported by DFG (Sonderforschungsbereich 823, Project C4) and the BMBF (Project 03BIPAH4 im Verbund INVERS).

7. References

Andersen, T.G., Bollerslev, T., Diebold, F.X., Ebens, H. (2001). The distribution of realized stock return volatility. *Journal of Financial Economics*, Vol. 61, No. 1, 43-76.

Aue, A., Hörmann, S., Horváth, L., Reimherr, M. (2009). Break detection in the covariance structure of multivariate time series models. *Annals of Statistics*, 37(6B), 4046-4087.

Brinson, G. P., Hood, R., Beebower, G. L. (1986). Determinants of Portfolio Performance. *Financial Analysts Journal*, Vol. 42, No. 4, 39-48.

Brinson, G. P., Singer, B. D., Beebower, G. L. (1991). Determinants of Portfolio Performance II: An Update. *Financial Analysts Journal*, Vol. 47, No. 3, 40-48.

Campbell, R.A.J., Forbes, C.S., Koedijk, K.G., Kofman, P. (2008). Increasing correlations or just fat tails? *Journal of empirical finance*, 15, 287-309

Chopra, V. K., Ziemba, W. T. (1993). The Effects of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice. *The Journal of Portfolio Management*, 6-11

Frennberg, P., B. Hansson (1993). Testing the Random Walk Hypothesis on Swedish Stock Prices 1919-1990. *Journal of Banking and Finance*, Vol. 17, No. 1, 175-191.

Ibbotson, R.G., Kaplan, P.D. (2000). Does Asset Allocation Policy Explain 40, 90 or 100 Percent of Performance? *Financial Analysts Journal*, 26-33.

Kallberg, J.G., Ziemba, W.T. (1984). *Mis-Specifications in Portfolio Selection Problems. Risk and Capital*. New York: Hrsg. G. Bamberg und K. Spremann, 74-87.

Longin, F., Solnik B. (1995). Is the correlation in international equity returns constant: 1960-1990? *Journal of International Money and Finance*, 14, 3-26.

Markowitz, H.M. (1952): Portfolio Selection. *Journal of Finance*, 7, No. 1, 77-91.

McNeil, A. J., Frey, R., Embrechts, P. (2005). Quantitative Risk Management: Concepts, Techniques and Tools. *Princeton Series in Finance.*

Meric, I., Meric, G. (1997): Co-Movement of European Equity Markets Before and After the 1987 Crash, in: Multinational Finance Journal, Vol. 1, No. 2, 137-152

Otto, K. (2008). Strategische Asset Allocation auf Gesamtbankebene. in: Akademie Deutscher Genossenschaften (ADG), 27./28. November 2008.

Rey, D.M. (2000): Time-varying Stock Market Correlations and Correlation Breakdown. *Schweizerische Gesellschaft für Finanzmarktforschung*, No. 4, 387-412.

Schäfer, K., Zimmermann, P. (1998): Portfolio Selection und Schätzfehler bei den erwarteten Renditen - Ergebnisse für den deutschen Aktienmarkt. *Finanzmarkt und Portfolio Management*, 12. Jg., No. 2, 131-149.

Wied, D., Arnold, M., Bissantz, N., Ziggel D. (2011). *A new fluctuation test for constant variances with applications to finance*. Preprint, Ruhr-University Bochum.

Wied, D., Krämer, W., Dehling, H. (2011). Testing for a change in correlation at an unknown point in time, to appear in *Econometric Theory*.

Zimmermann, H., Drobetz, W., Oertmann, P. (2002). *Global asset allocation*. New Jersey: John Wiley & Sons.