Analysis of GARCH Modeling in Financial Markets: An Approach Based on Technical Analysis Strategies

Mircea Cristian Gherman[1]

Abstract: In this paper we performed an analysis in order the make an evidence of GARCH modeling on the performances of trading rules applied for a stock market index. Our study relays on the overlap between econometrical modeling, technical analysis and a simulation computing technique. The non-linear structures presented in the daily returns of the analyzed index and also in other financial series, together with the phenomenon of volatility clustering are premises for applying a GARCH model. In our approach the standardized GARCH innovations are resampled using the bootstrap method. On the simulated data are then applied technical analysis trading strategies. For all the simulated paths the "*p*-values" are computed in order to verify that the hypothesis concerning the goodness of fit for GARCH model on the BET index is accepted. The processed data with trading rules are showing evidence that GARCH model is a good choice for econometrical modeling of financial time series including the romanian exchange trade index.

Keywords: conditional heteroscedasticity; volatility clustering; conditional variance; bootstrap; trading strategies

JEL Classification: C 52; G 11; G 32

1 Introduction

In finance and especially in financial markets, one of biggest challenge is to find a best trade-off between the return and the risk associated to a certain traded asset. There are several approaches for measuring the risk with implications to transaction's profit, but none of them is working all the time. Thus, a non linear model is much closer to the real phenomena encountered in the financial markets. One good measure for the risk of an asset is the volatility. Volatility itself is a very complex measure and it is often hard to measure it with high precision. This is why a lot of investors and financial institution are using complex approaches, in order to model the volatility. Modeling and forecasting volatility or, in other words, the covariance structure of asset returns, is therefore important. The fact that volatility

[1]University of Orléans, Faculty of Law, Economic Sciences, France, Address: Château de La Source, Avenue du Parc Floral, BP 6749, 45067 Orléans cedex 2, France, Tel.: + 33(0)2.38.41.71.71, Corresponding author: mircea-cristian.gherman@univ-orleans.fr.

AUDŒ, Vol 7, no 4, pp. 158-171

in returns fluctuates over time has been known for a long time. Since the distributions of the return series were found to be leptokurtic and the returns were modeled as independent and identically distributed over time, the idea of modeling the variable volatility over time was not fully incorporated in models. As a matter of fact, in a classic work, (Mandelbrot & Taylor, 1967) applied the so-called stables Paretian distributions to characterize the distribution of returns. An informative discussion of stable *paretian* distributions and their use in finance and econometrics is presented in (Rachev & Mittnik, 2000).

The daily and intraday observations from a return series of financial assets are in fact not independent. While observations in these series are uncorrelated or nearly uncorrelated, the series contain higher order dependence. Thus, models having the form of the Autoregressive Conditional heteroscedasticity (ARCH) are some of the most popular way to parameterize this dependence. Hence, *GARCH* stands for Generalized Autoregressive Conditional Heteroscedasticity and it can be seen as a modified *ARCH* model. Generally speaking, one can think of heteroscedasticity as time-varying variance (i.e. volatility). Conditional implies a dependence on the observations of the immediate past, and autoregressive describes a feedback mechanism that incorporates past observations into the present. *GARCH* then is a mechanism that includes past variances in the explanation of future variances.

In the financial, statistical and econometrical literature, several procedures were developed for the characterization of financial data using *GARCH*. The critics of *GARCH* are saying that the Brock-Dechert-Scheinkman test (*BDS*) can be used as general test for nonlinearities in financial series without using too many specifications (Brocks & Heravi, 1999). Since the *BDS* test have a strong power against *GARCH* models, they were widely used like a diagnostic method for the back testing of *GARCH*. In case of non-linear structures when using an "adjusted" *GARCH* model the innovation processes are tested with *BDS*. If the *BDS* tests cannot reject the null hypothesis by using the outcome values from driven simulation, then the adjusted *GARCH* model is fitted well on the data.

The main reason of using the other tests for heteroscedasticity is due to the easy access to a large numbers of software and computers utilities which implement them. On the other hand the *GARCH* software has been intensively used only in the last decade. Using other classical tests can raise a multiple of issues since their asymptotical distribution cannot do an accurate approximation of the applied statistics by those tests in respect to *ARCH*, *GARCH* and *Exponential GARCH* residuals (Hsieh, 1991).

The *GARCH* model can be simulated for every resampled dataset. Different statistics can be then computed on the standard innovations of the model, the results showing that the unspecified filter effect of *GARCH* is more present when the resampling process of the data is non-linear (Durbin & Koopman, 2000).

Therefore, we used as a statistic for GARCH model – the "*p*-value" indicator related to technical analysis trading rules applied on the resampled data. In the following sections of this paper will be presented the completed methodology which performs these tests, but first a brief presentation of the characteristics of *GARCH* model will be made.

2 Model Specifications

2.1 Modeling Financial Time Series with GARCH Model

The *GARCH* models are based at the same time on the previous autoregressive models (the *ARMAX/ARIMA* models) and on the conditional heteroskedasticty models (the ARCH models). Bollerslev in his work (Bollerslev, 1986) developed the *GARCH* like a more general model of the original *ARCH* model (Engle, 1982). Both of them are modeling the volatility but the *GARCH* model is using a reduced number of parameters which also decrease the computational effort time.

Hence, in order to express some of the characteristics that are commonly associated with statistical characteristics of financial time series like *fat tails* and *volatility clustering*, a good choice for modeling is the to use the *GARCH* specifications.

Probability distributions for the asset returns often exhibit fatter tails than in the case of standard normal, distribution. The fat tail phenomenon is known as excess kurtosis. Time series that exhibit a fat tail distribution are often referred to as leptokurtic. A part of the fat tail effect can also result from the presence of non-Gaussian asset return distributions that just happen to have fat tails. Heteroscedasticity explains some of the fat tail behavior, but typically not all of it. Fat tail distributions, such as *Student-t*, have been applied in *GARCH* modeling with good results, but often the choice of distribution is a matter of trial and error.

GARCH models are parametric specifications that operate best under relatively stable market conditions (Gourieroux, 1997). These stable conditions could not be present on every market and in practice it is well known that errors made in predicting markets are not of a constant magnitude. There are periods when unpredictable market fluctuations are larger and periods when they are smaller. This behavior, known as heteroscedasticity, refers to the fact that the size of market volatility tends to cluster in periods of high volatility and periods of low volatility. This phenomenon is called volatility clustering, which means that the large changes tend to follow large changes, and small changes tend to follow small changes. In either case, the changes from one period to the next will typically have an unpredictable sign. Volatility clustering, or persistence, suggests a time series model in which successive disturbances, although uncorrelated, are nonetheless serially dependent.

2.2 Correlation in Financial Time Series

If a financial time series are treated as a sequence of random observations, this random sequence, or stochastic process, may exhibit some degree of correlation from one observation to the next. This correlation structure can be used to predict future values of the process based on the past history of observations. Exploiting the correlation structure, if any, allows the decomposition of the time series into a deterministic component (i.e., the forecast), and a random component (i.e., the error, or uncertainty, associated with the forecast). There are used these components in order to represent a univariate model of an observed time series y_t:

$y_t = f(t-1, X) + \varepsilon_t$ where :

- $f(t-1, X)$ represents the deterministic component of the current return as a function of any information known at time $t-1$, including past innovations (residuals) $\varepsilon_t\{\varepsilon_{t-1}, \varepsilon_{t-2}, ...\}$, past observations $\{y_{t-1}, y_{t-2}, ...\}$, and any other relevant explanatory time series data, *X*.

- ε_t is the random component. It represents the residuals in the mean of y_t. These can be also an interpretation of the random disturbance, or shock, ε_t, as the single-period-ahead forecast error.

Usually the returns at time *t* are less correlated with return at time $t\text{-}_1$. That means the close past observations cannot be used to predict future returns. If on a market the financial assets are less correlated then this market is characterized by a weak informational efficiency – one cannot use the past information to make future profits.

2.3. Conditional Variances

The key insight of GARCH lies in the distinction between conditional and unconditional variances of the innovations process $\{\varepsilon_t\}$. The term conditional implies explicit dependence on a past sequence of observations. The term unconditional is more concerned with long-term behavior of a time series and assumes no explicit knowledge of the past.

If the values for returns have random values then a daily distribution can be used. The unconditioned distribution refers to asymptotic repartition – the repartition to which the daily return tends. Thus, the unconditioned mean is the simple mathematic moving average of the returns. It is called unconditioned because supposing that all the possible value can be realized, and an infinite number of data is available then a single return distribution for periods of time can be computed. This distribution assumes that return process is an i.i.d. process.

GARCH models characterize the conditional distribution of ε_t by imposing serial dependence on the conditional variance of the innovations. Specifically, the variance model imposed by *GARCH*, conditional on the past, is given by

Equation 1 **Conditional Variance of a time series**

$$Var_{t-1}^2(y_t) = E_{t-1}(\varepsilon_t^2) = \sigma_t^2$$

Equation 2 **Variance of a time series described by *GARCH*(*P*,*Q*) parameters**

$$\sigma_t^2 = K + \sum_{i=1}^{P} G_i \sigma_{t-1}^2 + \sum_{j=1}^{Q} A_j \varepsilon_{t-1}^2 \; K > 0,\; G_i \geq 0,\; A_j \geq 0$$

The σ_t^2 is the forecast of the next period's variance, given the past sequence of variance forecasts, σ_{t-i}^2, and past realizations of the variance itself, ε_{t-j}^2.

When $P = 0$, the *GARCH*(*0*,*Q*) the model becomes the original *ARCH*(*Q*) model (Engle, 1982).

Equation 3 **Variance of a time series described by ARCH parameters**

$$\sigma_t^2 = K + \sum_{j=1}^{Q} A_j \varepsilon_{t-j}^2$$

When $P = Q = 0$, the variance of the process is simply white a noise with variance *K*.

Since in practice, is needed a large lag Q for *ARCH* modeling, and estimation for a large number of parameters. Bollerslev (Bollerslev, 1986) extended Engle's *ARCH* model by including past conditional variances. This results in a more parsimonious representation of the conditional variance process.

Large disturbances, positive or negative, become part of the information set used to construct the variance forecast of the next period's disturbance. In this manner, large shocks of either sign are allowed to persist, and can influence the volatility forecasts for several periods. The lag lengths *P* and *Q*, as well the magnitudes of the coefficients G_i and A_j, determine the degree of persistence.

2.4 Serial Dependence in Innovations

A common assumption when modeling financial time series is that the forecast errors (i.e., the innovations) are zero-mean random disturbances uncorrelated from one period to the next. In fact, an explicit generating mechanism for a

GARCH(*P*,*Q*) innovations process, $\{\varepsilon_t\}$, is: $\varepsilon_t = \sigma_t z_t$, where σ_t is the conditional standard deviation, and z_t is a standardized, independent, identically distributed (i. e., i.i.d.) random draw from some specified probability distribution. The *GARCH* literature (Nelson, 1998; Bollerslev, 1986) uses several distributions to model GARCH processes, but the vast majority of research assumes the standard normal density such that $\varepsilon_t \sim N(0, \sigma_t^2)$. The *GARCH* innovations process $\{\varepsilon_t\}$ simply rescales an i.i.d. process $\{z_t\}$ such that the conditional standard deviation incorporates the serial dependence.

The GARCH models are consistent with various forms of efficient market theory, which state that asset returns observed in the past cannot improve the forecasts of asset returns in the future. Since *GARCH* innovations $\{\varepsilon_t\}$ are serially uncorrelated, *GARCH* modeling does not violate efficient market theory.

2.4 Homoskedasticity of the Unconditional Variance

The GARCH model is strictly related to the conditional variance as a standard process with Gaussian innovations. It can be used a general *GARCH*(*P*,*Q*) form with Gaussian innovations for the conditional variance. The model conditional variance is described by the Equation 2 presented above.

To have a stationary process are imposed the following parameter constraints on the conditional variance parameters.

Equation 4 **Constraint inequality for the *GARCH* parameters parameters**

$$\sum_{i=1}^{P} G_i + \sum_{j=1}^{Q} A_j < 1; \;\; G_i \geq 0, \;\; A_j \geq 0$$

The first constraint, a stationarity constraint, is necessary and sufficient for the existence of a finite, time-independent variance of the innovations process $\{\varepsilon_t\}$. The remaining constraints are sufficient to ensure that the conditional variance $\{\sigma_t\}$ is strictly positive.

The GARCH model used in this study is the simple conditional mean model with *GARCH(1,1)* normal innovations. It is completely described by two equations, the first one called the mean equation and the second one called the variance

Equation 5 **The GARCH model equations**

$$y_t = C + \varepsilon_t$$

$$\sigma_t^2 = K + G_1\sigma_{t-1}^2 + A_1\varepsilon_{t-1}^2$$

In the conditional mean equation, the returns y_t, consist of a simple drift, plus an uncorrelated, white noise disturbance, ε_t. In the conditional variance equation, the variance forecast, σ_t^2, consists of a constant plus a weighted average of last period's forecast, σ_{t-1}^2, and last period's squared disturbance, ε_{t-1}^2. Although financial return series, typically exhibit little correlation, the squared returns often indicate significant correlation and persistence. This implies correlation in the variance process and it could be an indication that the data is a candidate for *GARCH* modeling. Although simplistic, the default model, on which the current study is focused on, has the benefit of representing a parsimonious model that requires you to estimate only four parameters (*C*, *K*, G_1 and A_1). According to (Box & Jenkins, 1994) the fewer parameters to estimate, the less that can go wrong. Some researchers are stating that elaborate models often fail to offer real benefits when forecasting (Hamilton, 1994).

The simple *GARCH*(1,1) model captures most of the variability in most return series. Small lags for *P* and *Q* are common in empirical applications. Typically, *GARCH*(1,1), *GARCH*(2,1), or *GARCH*(1,2) models are adequate for modeling volatilities of different assets even over long sample periods (Bollerslev & Chou, 1992).

3 Methodology

The objective of determining the parameters for the underlying process applied to the index price evolution is to allow the development of better stock pricing (index) models. In this study, the parameters of the model are all calculated on data basis before performing the simulations. In order to see the effects of applying the *GARCH* model on the data, the bootstrap simulation technique is used. It is straightforward to apply the bootstrap to derive some estimates of standard errors and confidence intervals for the complex estimators of the distribution parameters. One standard choice for an approximating distribution is the empirical distribution of the observed data. In the case of a dataset which is assumed to be an independent and an identically distributed process, the bootstrapped distribution can be simulated by constructing a number of samples from the observed dataset (and of equal size to those related to original data). Each of them is obtained by random sampling with replacement from the original dataset.

Hence, for testing the benefits of using the *GARCH* model, we used two of the technical analysis strategies: the filter strategy and the moving average strategy.

The filter strategy takes implies the usage of percentage value (called filter), which is then compared with the change in the current stock (asset) price. If the increase in the stock' price is bigger than the filter, then a buy signal is generated. Usually this kind of behavior is associated by the investors with the bullish market. If the decrease in the stock' price is bigger than the filter, then a sell signal is generated. The decreasing price in a stock market is associated with the concept of bearish market.

The second strategy is using two moving averages and it is called the Moving Average strategy. One of those moving averages is called the short moving average (SMA) and it uses a small number of past observation (e.g. from 1 to 10) and the other is called the long moving average (LMA) and it uses a bigger number of past observations (e.g. from 20 to 200). When the short moving average line is crossing from the downside the long moving average then a buy signal is generated, otherwise when the short moving average line is crossing from the upside the long moving average then a sell signal is generated.

The main fact of the described strategies (filter and moving average strategy without using bootstrap) is that they don't assume the hypothesis which state that the returns are not normally distributed. Some of the results could suggest that even the average return of these strategies is statistically bigger than a result for a simple buy-hold strategy. From some points of view the presented strategies could be considered similar. One of these aspects is related to the fact that the excess of return obtained when using these trading strategies has close values in both cases. In order to have the certitude that the particularities of the return series doesn't modify the distribution of statistical tests we used the bootstrap methodology. The main idea of this is to simulate the empirical distribution and calculate the associated "*p*-values" for both applied strategies. In order to achieve this we considered the next steps:

1) First we estimated the GARCH parameters related to both equations.

2) In the second step we performed the simulation for empirical distributions of returns.

3) In the final step we computed the "*p*-values" associated with each trading strategy.

All of these steps and also the analysis of the BET index were performed in an econometrical computer program. The third step is described in more details in the results sections since it is involving some considerations about the number of buy or hold signals and their statistical distribution.

4 The Data

For the simulation and analysis using *GARCH* model the used data is the BET index between years 1997 and 2010 (end of December). BET (Bucharest Exchange Trading) is the official index for the Bucharest Stock Exchange. It is a compounded weighted index which includes the 10 most liquid stocks from the market (the „blue chips”). The BET index is a price index which does not contains dividends, meaning that it s not a performance index. Since the dividends in Romania are in many years equals to zero, this index is considered to be representative for the purpose of the *GARCH* analysis.

Before proceeding with forward data processing, a quick statistical analysis is performed on the BET index in order to highlight its properties. These properties are taking into account when the GARCH model is applied.

In the next figure is represented the empirical distribution of the daily returns. As already stated it has not a gaussian distribution and it presents fat tails, being an asymptotic distribution with a mean with a value slightly greater than zero and with a daily standard deviation close to 2%.

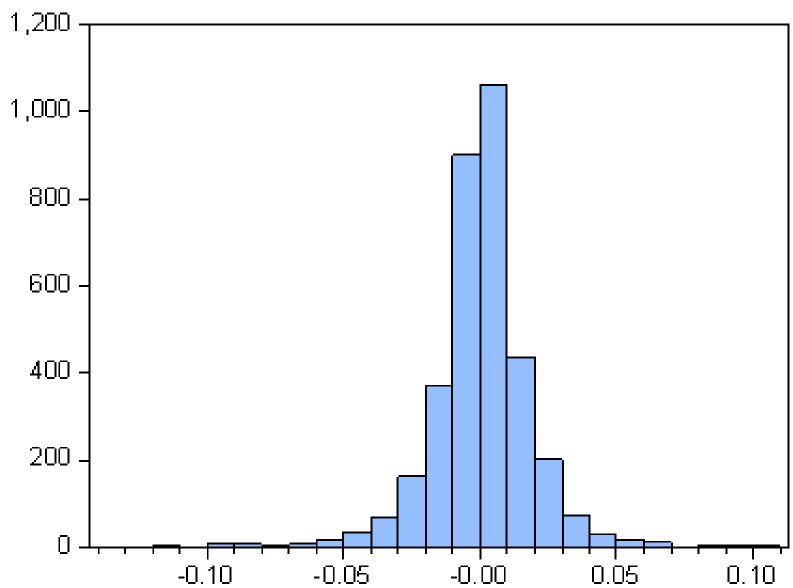

Figure 1 Distribution of daily returns for BET index

The next table presents some of the principal statistical characteristics of the BET return series, which are relevant for our study in order to apply the *GARCH* model.

Table 1 The principal statistical parameters of BET returns

Statistical characteristic	**Value**
Mean	0.000484
Median	0
Maximum	0.105645
Minimum	-0.13117
Std. Dev.	0.018848
Skewness	-0.3166
Kurtosis	9.083299

The Table 1 can be considered on of the starting point of our analysis and its reflect the described characteristics of financial time series, on which a heteroskedastic model can be applied. Therefore, the data presented here is used for estimation of *GARCH*(1,1) model.

5 Empirical Results

In the financial markets the investors are applying different rules when they make trades. Probably, one of the simplest and at the same time not the worst strategy from the performance point of view is the buy-and-hold strategy. This strategy implies that the investors buy a stock and it sells it at a different time in the future. There no special assumptions made on what the investors take these actions. In the technical analysis field, the next level is to apply on the asset time series some rules, regarding the buy and the sell moments.

Using the described methodology the *GARCH* model was applied to the BET index data. The empirical distribution of the parameters (the daily average return) is simulated in order to generate of the returns process. The *GARCH* null model is estimated starting from the initial return series. The model parameters are estimated by minimizing the error terms and by applying a *Student-t* law for testing the parameter significance. In the next table are presented the estimation results for the *GARCH*(1,1) model:

Table 2 The estimated GARCH parameters

Parameter Name	**Value**	**Standard Erorr**	**T- Statistic**
C	0.0011904	0.00025809	4.6124
K	1.387	1.402	9.8978
G1	0.75757	0.0097676	77.5600
A1	0.21814	0.012456	17.5135

Hence, for the analyzed data set the model equations are:

Equation 6 **The estimated parameters for the GARCH model applied for BET returns**

$$y_t = 0.00119 + \varepsilon_t$$

$$\sigma^2{}_t = 1.387 \cdot 10^{-5} + 0.75757 \cdot \sigma_{t-1}^2 + 0.21814 \cdot \varepsilon_{t-1}^2$$

Then, the residuals from the first equation are resampled (randomly with replacement) for the purpose of obtaining the new returns and then the new price series. The empirical results are grouped in tables as it can be seen in the following sections. The first one is presenting the results for the filter strategy and the other the results related to moving average strategy.

The technical analysis strategies are applied to these resampled series and are calculated values for average daily returns for the buy sub periods and for the sell sub periods. Also the average return for the entire strategy is being computed. These steps are repeated for *N* times (number of resampling times) in order to obtain the empirical distribution of the average daily returns. Theses returns are then compared with the initial returns of the BET index in order to compute the "*p*-values". The "*p*-values" are presented in the next section and they represent the average percentage of the simulated index values which are greater than the initial values of the BET index. A value close to one for this "*p*-values" means that the null hypothesis which consists in the existence of the *GARCH* effect in the BET series is accepted. Otherwise, if "*p*-values" are close to zero the null hypothesis is rejected. When we performed this simulation we made usage of more parameters.

Thus, the used *GARCH* model is the "default" or the "basic" one: *GARCH*(1,1) and the number of bootstrapped sample paths is 1000. For both strategies we modified the parameters in order to show also the performance of strategy according for various values of the parameters. The results are grouped in two mail tables and depending on the investor position – long or short – the mean and the standard deviation of each simulated strategy are computed. When both positions are combined together, there is possible to have an overall result. Thus, this is presented in last two columns of each table.

In the next table are shown results for the simulation using *GARCH*(1,1) and the described parameters when a moving average strategy is used. The first column is by definition the *Moving Average* having the values on the short term – *S* and the value for the long term – *L*.

Table 3 Empirical results for the bootstrap analysis using the moving average strategy

MA(S,L) parameters	**μ(buy)**	**σ(buy)**	**μ(sell)**	**σ(sell)**	**μ(strategy)**	**σ(strategy)**
(1, 10)	0.09	0.188	0.965	0.095	0.024	0.145
(1, 20)	0.08	0.173	0.941	0.106	0.021	0.140
(1, 50)	0.04	0.173	0.928	0.119	0.086	0.126
(2, 50)	0.07	0.183	0.910	0.115	0.142	0.123
(1, 150)	0.11	0.165	0.960	0.121	0.075	0.124
(2, 150)	0.12	0.188	0.959	0.115	0.087	0.119
(1, 200)	0.11	0.169	0.930	0.112	0.231	0.120

From the above table it can be seen that both the long and short values have an impact on the strategy profitability. The influence of long and short values is specially shown in daily average returns for the moving average strategy.

Table 4 Empirical results for the bootstrap analysis using the filter strategy

Filter value	**μ(bu**	**σ(buy)**	**μ(sell)**		**μ(strateg**	**σ(strateg**
1%	0.087	0.131	0.901	0.134	0.045	0.131
2%	0.092	0.141	0.927	0.123	0.033	0.139
5%	0.117	0.203	0.943	0.082	0.055	0.134
8%	0.155	0.183	0.930	0.089	0.094	0.128
12%	0.200	0.127	0.775	0.193	0.424	0.136
20%	0.132	0.172	0.937	0.160	0.082	0.132
25%	0.199	0.168	0.932	0.155	0.106	0.123

In the above table, the "Filter value" column represent the percentage value on based it is taken the decision of generating a buy or a sell signal and μ(buy), μ(sell), μ(strategy) are the percentages for number of simulated *GARCH* data on which the strategy return is bigger than for the initial index values.

The "*p*-values" computed for both strategies, using different parameters are showing which one is more proper to be used together with *GARCH* model. The results from μ columns are the daily returns, and those for σ columns represent the result for standard deviations.

6 Conclusions

In this his paper was examined the original GARCH model contribution to our understanding of the stochastic process underlying index stock markets. Our approach tried to determinate if the movement of research in the *GARCH* modeling field is warranted.

Overall, our results demonstrate that, although previous research indicates that volatility clustering plays a role in determining stock price changes, it is not the primary factor generating these changes. Hence, *GARCH* models with normality assumptions provide a description of stock prices dynamics. The returns distributions show independence in the data after removing the *GARCH* effects. Over these returns (often called residuals), the technical analysis strategies, are powerful tools used to find out if the model is fitting well or not on the data. The results are showing that for some parameters of the trading strategies, their profitability combined together with the *GARCH* modeling is higher that a classical buy-and-hold strategy.

Although *GARCH* is explicitly designed to model time-varying conditional variances, *GARCH* models can capture sometimes the highly irregular phenomena, including wild market fluctuations (e.g., crashes and subsequent rebounds), and other highly unanticipated events that can lead to significant structural changes.

Future research can examine if other forms of the *GARCH* process might be used for testing the serial independences of residuals (i.e., *EGARCH*, *FIGARCH*, *MGARCH*). These models should also be tested to determine if they are superior to mean variance standardization approach. Since all forms of the *GARCH* process are similar in form, focusing on volatility clustering, it would be interesting to see if they are important improvements.

7 Acknowledgement

This paper was supported by the project "Doctoral studies in engineering sciences for developing the knowledge based society-SIDOC” contract no. POSDRU/88/1.5/S/60078, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

8 References

Bollerslev, T., & Chou, R.Y. (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence. *Journal of Econometrics*, Vol. 52, pp. 5-59.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, Vol. 31, pp. 307-327.

Box, G.E.P., & Jenkins, G.M. (1994). *Time Series Analysis: Forecasting and Control, Third Edition.* Prentice Hall.

Brooks, C. & Heravi, S.M. (1999). The Effect of Mis-Specified GARCH Filters on the Finite Sample Distribution of the BDS Test. *Computational Economics*, Vol 13, pp. 147–162.

Durbin, J., & Koopman, S.J. (2000). Time series analysis of non-Gaussian observations based on state space models from both classical and Bayesian perspectives. *Journal of Royal Statistical Society Series*, Vol 62, pp. 3-56.

Engle, R. F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, Vol. 50, pp. 987-1007.

Gourieroux, C. (1997). *ARCH Models and Financial Applications*, Springer-Verlag.

Hamilton, J.D. (1994). *Time Series Analysis.* Princeton University Press.

Hsieh, D.A. (1991). Chaos and Nonlinear Dynamics: Application to Financial Markets. *Journal of Finance*, Vol 46, pp. 1839–1877.

Mandelbrot, B., & Taylor, H.(1967). On the distribution of stock price differences. *Operations Research 15*, pp. 1057-1062.

Nelson, D. (1991). Conditional Heteroscedasticity in Asset Returns: A New Approach. *Econometrica*, Vol. 59, No. 2, pp. 347-370.

Rachev, S., & Mittnik, S. (2000). *Stable Paretian Models in Finance*. Wiley, Chichester.