Operations Research; Statistical Decision Theory

Linear Optimization Techniques for Product-Mix of Paints Production in Nigeria

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Abstract: Many paint producers in Nigeria do not lend themselves to flexible production process which is important for them to manage the use of resources for effective optimal production. These goals can be achieved through the application of optimization models in their resources allocation and utilisation. This research focuses on linear optimization for achieving product- mix optimization in terms of the product identification and the right quantity in paint production in Nigeria for better profit and optimum firm performance. The computational experiments in this research contains data and information on the units item costs, unit contribution margin, maximum resources capacity, individual products absorption rate and other constraints that are particular to each of the five products produced in the company employed as case study. In data analysis, linear programming model was employed with the aid LINDO 11 software to analyse the data. The result has showed that only two out of the five products under consideration are profitable. It also revealed the rate to which the company needs to reduce cost incurred on the three other products before making them profitable for production.

Keywords: Linear programming; LINDO; paints; optimization; operations research; models; cost

JEL Classification: C44; C61; M11

1. Introduction

The task of making effective decision in a modern day business environments has become so intricate given the rising complexities in the economic and the different socio-political factors. Though, technology changes constantly with new knowledge, new and unprecedented problems also surface by the day. Therefore, operational efficiency must extend beyond optimising the routine problems to

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addressing wider and strategic issues. To achieve this, decision makers must look beyond personal experience, intuitive knowledge to avert the serious and costly consequences often associated with wrong decisions. All these and many more have made the application of operational research techniques is of fundamental importance to decision- maker (Sharma, 2008).

An essential decision typical of a manufacturing concern is determination of profit maximising product mix without compromising quality Thus, in the present competitive environment, the efforts of manufacturing organizations should be towards optimization of all production variables for profitability and better financial performances. In Nigeria, where many organisational entities are managed by nonprofessional but experienced managers, the use of modern managerial tools is limited. However, non-application of modern tools of management in resource allocation, despite the growing number of modern scientific and tested methods of decision making in a global world can be very costly. Resource scarcity is a major constraint in any firm's decision regarding optimal products mix as well as profit maximisation (see Albright & Wayne, 2009: Arogundade & Adebiyi, 2011). According to Albright & Wayne (2009), these and many other optimisation decisions can be well handled by modern linear programming modelling procedures via appropriate problem formulation and equation fittings.

According to Savsar and Aldaihan (2011), the decision-making in product mix selection is difficult due to large number of possibilities involved. Limitation of resource capacities makes it difficult to meet customer demand and hence reduces the opportunity to earn profits. Several optimization models, including linear programming, have been applied to the product mix selection problem. Since its development in 1947, linear programming (LP) theory and related solution techniques have been widely extended and used as an optimization tool in a variety of problem areas including the production planning and product mix selection areas. However, most of the work involves either theoretical development or hypothetical applications. There is a definite need to apply these theoretical formulations to real life problems. Thus, this is one of those few applications to real life situation of a manufacturing company in Nigeria. Data were sought from the records of the day-to-day activities of the organization for modeling and analysis.

The technique needs the formulation of the problem and fitting it into a mathematical model which could be run with commercial packages like LINDO to get result that will aid optimum decision. Decision-makers are often faced with the problem of determining optimal allocation of limited resources. The problem of determining the best combination of activity levels which does not use more resources than are actually available and at the same time maximize output, revenue, service level,..., or to minimize costs. However, this research effort explored the benefits of using the Linear Programming techniques for product-mix optimization of paint production in Nigeria. This research tries to answer three basic questions. The basic research questions that prompted this work are:

(i) How can we model paints product mix problem with linear programming?;

(ii) Are managers of paint industry in Nigeria aware of the existence and the usefulness linear programming for paints product mix optimization?;

(iii) How can we recommend the use of linear programming software packages for accommodating large product mix data to Nigeria managers of resources?.

To this end, the present paper try to explore the application of basic linear programming procedure for product mix optimisation in manufacturing organisation in Nigeria with the view to establishing how linear programming techniques and the use of LINDO software could contributes to optimal product mix of paints production. While we are aware that both the method and the software have been applied in analysing the optimisation problems across several industries locally and beyond, there is no study on record that has applied same to the study of optimal product mix in paints production particularly in Nigeria.

2. Literature Review

Developed in 1939 by Kantorovich and extended by Dantzig (1947), linear programming is a typical mathematical method for resolving multiple constrained optimisation (MCO) problems particularly where both the objective and the constraints are linear. Unlike a single constraint (SCO) problem that can be resolve with basic traditional approach, the MCO problems are better handled by modern approach. Interestingly, the usage of this mathematical technique has been widely accepted and facilitated by the advent of computers that can ease complex calculations.

In typical SCO problems involving how to reach a particular isoquant in the face of a defined isocost and vice versa, output maximisation and cost minimisation can be traditionally attained at the point of tangency between isocost and isoquant. The real world scenarios require much more with firms facing several constraints either in terms of input sourcing (material adequacy, relevant skilled labour and cutting-edge technology) in the short run or in the form of output specification (quantity and quality requirements). The scope of MCO problems require the use of linear programming since their solutions cannot be obtained from the application of basic traditional approach. Accordingly, we can describe linear programming as a unique way of solving optimisation problems with linear and multi variable objective function where the constraints are linearly equal or unequal.

Linear programming has helped decision makers to set broad objectives and optimise schedules to meet set goals, linear programming and its various extensions like mathematical programming have demonstrated the capacity to solve advanced optimisation problems involving both linear and integer constraints. Simply put, the role linear programming in broad mathematical programme is typical of that of derivatives of functions in calculus. According to Dantzig and Thapa (1997), LP viewed from the methodical point seeks to identify the extreme of functions further satisfying set of constraints in a bid to rationalise several technological and managerial decisions.

Hillier and Lieberman (2005) describe linear (mathematical) programming as a planning instruments that produces outcomes which best approximates specified objectives amidst all feasible alternatives. They added that linear programming application is not limited to allocating resources to undertakings alone but extend to cover all other firm's issues whose mathematical model fits the very general format for the linear programming model particularly those problems that are multi-faceted. Studies on product mix has been considered in literature, Byrd and Moore, (1978) applied LP model to corporate policy making; Lee and Plenert (1996) worked on what is the best tool for analyzing maximizing product mix Profitability. Also Chung, Lee and Pearn, (2005) studied product mix optimization for semiconductor manufacturing based on AHP and ANP analysis while Savsar and Aldaihan (2011) presented an application of linear programming to product mix selection problem in an oil company. But, as far the researchers are aware; none of the previous studies on LP application to product mix was applied to paint production and more specifically in Nigeria where manufacturing sectors are facing some peculiar challenges. Thus, this study will fill the gap in literature.

3. Data and Sources

The major source of data for this study is the personal interview and records of production and operational plan of Paint product companies in Nigeria. However, in other to have an idea of what is obtainable in a particular company for the purpose of generalization of idea; one of such paint companies was chosen as a case study. The choice of this organisation is hinged on the readiness of this organization to release relevant information for the purpose of this study.

4. Data Analysis

This section of the paper analyses the data collected from personal interview at the company by consulting various operational reports and publication of the paint manufacturing company. The interview provided information on the production planning and product mix process techniques used in the organisation. This information in addition with the sales and other operating data was analysed to provide estimates for Linear Programming model parameter. The product mix optimisation was obtained by evaluating the mathematical model Programming model using LINDO computer software, the solution of which included the sensitivity analysis.

Model Nomenclature

The product of typical Paints and company in Nigeria:

Let Xi represent the number of products to be produced at Paint Company. Where **i** range from 1 to 5 represent the following products.

Product Labelling

X_1 represent Intra emulsion paint
X_2 represent Vinlymatt emulsion paint
X_3 represent Sandtex gloss paint
X_4 represent Intra gloss paint
X_5 represent Texture paint

The company produces 5 types of paints (Intra emulsion paint, Vinlymatt emulsion paint, Sandtex gloss paint, Intra gloss paint and textured paint). Ten inputs are needed here to make five different paints product. The ten inputs are; Water (in Litres), Titamin dioxide (kg), Calcium carbonate (kg), Binder resin (kg), Additives (kg), Colouring (kg) Sand (kg), Kerosene (Litres), Pigment-Colouring material (kg), Labour hours per product (hours) and Plant capacity.

The outputs are five types of paints produces at the company. Hence the optimal implementation of these products by the simplex algorithm will be using five (5) decisions variables.

Table 1. Quantity unit variable cost, resources absorption rate and contribution margin of each product

Variables Material resources	Products					Maximum Available
	X_1	X_2	X_3	X_4	X_5	
Water (in Litres)	135000	60000	0	0	105000	300,000Ltrs
Titamin dioxide	9810	8190	3192	1650	7140	30,000kg
Calcium carbonate	128304	59928	63360	2640	74976	264,000kg
Binder resin	10200	8400	7620	4200	29400	60000kg
Additives	357	535.5	36	36	535.6	1500kg
Colouring paste	200	100	15	10	175	500kg
Sand	0	0	0	0	11550	11550kg
Kerosene	0	0	4500	3000	0	7500Ltrs

Pigment-Colouring material	720	360	54	36	630	1800kg
Labour hours per product	X_1	X_2	X_3	X_4	X_5	36000 Hrs/month
Demand for product	X_1	X_2	X_3	X_4	X_5	600000Ltrs
Plant capacity	250,000	141700	15000	10000	85000	600000Ltrs
Warehouse capacity	X_1	X_2	X_3	X_4	X_5	2000000Ltrs
Unit contribution margin (#)	690.66	169.37	2188.74	2037.70	136.72	

Sources: 2012 Average monthly production plan of the paint company

Model Formulation

In the model formulation, the following steps are followed: with reference to the data in table 1;

$\mathbf{x_1}$ = units of product 1 (Intra emulsion paint) produced per month

$\mathbf{x_2}$ = units of product 2 (Vinlymatt emulsion paint) produced per month

$\mathbf{x_3}$ = units of product 3 (Sandtex gloss paint) produced per month

$\mathbf{x_4}$ = units of product 4 (Intra gloss paint) produced per month

$\mathbf{x_5}$ = units of product 5 (Texture paint) produced per month

Z = total profit per week (in thousands of naira) from producing these five products

Problem identification: The problem of this study is to maximize reward (Max Z: $CX_1 + CX_2 + ... + CX_5$)

In formulating the model properly, we have:

Maximise Z (₦ Profit): $690.66 X_1 + 169.37X_2 + 2188.74X_3 + 2037.70 X_4 + 136.72 X_5$

Subject to:

$135000X_1 + 60000X_2 + 105000X_5 \leq$ 300,000Ltrs (water constraints)

$9810X_1 + 8190X_2 + 3192X_3 + 1650X_4 + 7140X_5 \leq$ 30,000kg (Titamin dioxide constraints)

$128304 X_1 + 59928X_2 + 63360X_3 + 2640X_4 + 74976 X_5 \leq 264$, 000kg (Calcium carbonate)

$10200 X_1 + 8400X_2 + 7620X_3 + 4200X_4 + 29400X_5 \leq$ 60000kg (Binder resin constraints)

$357X_1 + 535.5X_2 + 36X_3 + 36X_4 + 535.6X_5 \leq$ 1500kg (Additives constraints)

$200 X_1 + 100X2 + 15X3 + 10X4 + 175X_5 \leq$ 500kg (Colouring paste constraints)

$X_5 \leq 11550$ (sand constraints)

$4500X_3 + 3000\,X_4 \leq 7500$ Ltrs (Kerosene constraints)

$720X_1 + 360\,X_2 + 54X3 + 36X4 + 630X_5 \leq 1800$kg (Pigment-Colouring material constraints)

$X_1 + X2 + X3 + X4 + X_5 \leq 36000$ Hrs/month (labour constraints)

$250{,}000X_1 + 141700X2 + 15000X3 + 10000X4 + 85000X_5 \leq 600{,}000$Ltrs (plants capacity constraints)

$X_1, X_2, X_3, X_4, X_5 \geq 0$ (Non negativity constraint: There is an assumption that no negative output can be produced).

Results and Interpretation

Taken in order of the result obtained, the result tells us that LINDO took (2) iteration to solve the product mix optimization problem of the case company. Second that the maximum profit attainable from the five products as it was constrained in the above model specified is ₦ 6, 479, 833 (Nigeria naira)

Third, the quantities of each Paints product to be produced: Intra emulsion paint (X_1), Vinlymatt emulsion paint (X_2)**,** Sandtex gloss paint (X_3), Intra gloss paint (X_4) and Texture paint (X_5), are 2, 006, 173, 0, 0, 2,500, 000 and 0 litres respectively. What is interesting in this result is the fact that only two out of the five products are profitable, while others were not profitable for production. Thus product X_1 (Intra emulsion paint) and X_4 (Intra gloss paint) are responsible for the maximum objective of ₦ 6, 479, 833 per month (Nigeria naira) while other three products still required considerable improvement (reduction in the cost resources) before they can be considered for production.

However, from the result of data analysis, it was found out that for the organization to operate optimally in the background of inherent constraints, it must produce 2.006,173 litres of product X_1,**-** 2.500,000 litres of product X_4 and zero litres of other three products for optimum production.

The essence of operation research therefore is to minimize waste, one of the beauties of the computer software used is that it went further to postulate possible reduced cost incurred the other three unprofitable products if the company wants to produce still achieve maximum profit desired. This includes:

- ₦ 153, 222 (Nigeria naira) on product X_2;
- ₦ 118, 756 (Nigeria naira) on product X_3;
- ₦ 266, 875 (Nigeria naira) on product X_5.

From the above premise, it could be inferred that, though un-utilised capacities abound, but if optimality is allowed, the overall objective of reward maximization of the organization is achievable. It is important to state that solution was optimal at second iteration of the Linear programming problem.

However, it was of greater interest to note also that, organisations should not only crave for maximisation, but optimality. The implication of this is that, reward should be maximized at the lowest cost possible. This is evident in the analysis of this research work using the Linear and Algorithm solution pack called LINDO, where optimum quantities were not only arrived at, but also with the possible cost reduction per unit/litre.

The surplus/slack units were also very interesting in this research work, since it creates opportunity for organisations to see the need for non-idleness of resources. To this end, organisations, having idea of the expected idle resources a priori, it can plan for optimum materials to be ordered for.

The Slack or Surplus column in a LINDO solution report tells you how close you are to satisfying a constraint as equality. This quantity, on less-than-or-equal-to (≤) constraints, is generally referred to as slack. On greater-than-or-equal-to (≥) constraints, this quantity is called a surplus.

If a constraint is exactly satisfied as equality, the slack or surplus value will be zero. If a constraint is violated, as in an infeasible solution, the slack or surplus value will be negative.

5. Conclusion

In the course of this research, a number of findings were deduced. These include the fact that managers of the organisation of case study do not make use of modern scientific tools like LINGO to plan their operations and production.

This optimization model can be used in a production environment to allow managers to choose the production schedule that maximizes profit. From the results of LINDO software use in analysing product mix of company five products (Intra emulsion paint, Vinlymatt emulsion paint, Sandtex gloss paint, Intra gloss paint and Textures paint), we are able to see the effects of changing resources on a profit and how it can help making informed decisions about resources used for production; water, titamin dioxide, calcium carbonate, binder resin, colour paste, additives, sand, kerosene, Pigment-colouring material, plant capacity and labour hours used. It also allows managers to make decisions regarding overtime for employees, temporary worker hiring, and raw materials usage and ordering. This model can be reused on an on-going basis providing management with the ability to do what-if scenarios and give them real insight into what effect changes in their resources might have on profitability.

Considering the challenges of today's world economy at the threshold of the 21st century and technological advancement and other competitive factors which interact in a complicated fashion to give rise to complex decision making problem. It is not easy to make up schedule that is both realistic and economical. Hence, the acquisition of analytical method like linear programming and its software for analysing and manipulating large organisation operations and production data gives edges to the firms.

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