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One-dimensional Fuzzy Poverty Measure from an Bootstrap Method Perspective

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Abstract. *This paper is a contribution to the analysis of deprivation seen as a one-dimensional condition. A most useful tool for such analysis is to view deprivation as a matter of degree, giving a quantitative expression to its intensity for individuals. Such 'fuzzy' conceptualisation has been increasingly utilised in poverty and deprivation research. This paper aims to further develop and refine this strand of research. The concern of the paper is primarily methodological rather than detailed numerical analysis from particular applications. We re-examine the two additional aspects introduced by the use of fuzzy (as distinct from the conventional poor/non-poor dichotomous) measures, namely: the choice of membership functions and the choice of rules for the manipulation of the resulting fuzzy sets, rules defining their intersection and averaging. The relationship of the proposed fuzzy monetary measure with the membership function and an estimate, by confidence interval, of the poverty line.*

Keywords: *strong poverty; medium poverty; weak poverty; membership function; fuzzy set operators*

Elasticity size corresponds to the elasticity of equivalence scale compared to the size of household

1 Introduction

Poverty measurement may be sensitive to how the poor are identified. The traditional approach supposes a rigid poor/non-poor dichotomy, whereas most of the literature on poverty measurement continues to be based upon the use of poverty thresholds. Yet it is taken for granted that such a clear-cut division causes a loss of information and removes the nuances that exist between the two extremes of substantial welfare on the one hand and the distinct material hardship on the other.

Nowadays many authors recognise that poverty should be considered as a matter of degree rather than as an attribute that is simply present or absent among individuals in the population. An early attempt to incorporate this concept at a methodological level (and in a multi-dimensional framework) was made by Cerioli and Zani (1990) who drew inspiration from the theory of Fuzzy Sets initiated by Zadeh (1965). Cerioli and Zani's original proposal was later developed by Cheli and Lemmi (1995) giving origin to the so called Totally Fuzzy and Relative (TFR) approach. Both methods have been applied by a number of authors subsequently, with a preference for the TFR version (Chiappero Martinetti, 2000; Clark & Qizilbash, 2002; Lelli, 2001], and in parallel the same TFR method was refined by Cheli (1995) who also used it to analyze poverty in fuzzy terms in the dynamic context represented by two consecutive panel waves. Ever since, the methodological implementation of this approach has evolved in two directions, with somewhat different emphasis despite their common orientation and framework. The first of these is typified by the contributions of Cheli and Betti (1999) and Betti, Cheli and Cambini (2004), focusing more on the time dimension, in particular utilizing the tool of transition matrices. The second, with the contributions of Betti and Verma (1999, 2008), Betti, Chelli and Verma (2006), has focused more on capturing the multidimensional aspects, developing the concepts of “manifest” and “latent” deprivation to reflect the intersection and union of different dimensions.

Although deprivation is widely recognised as a multidimensional phenomenon, we still believe that indicators based on monetary variables have a fundamental role, and therefore deserve a special analysis. For this reason, some recent contributions consider two different fuzzy measures: the first one is based only upon a monetary variable and is referred to as Fuzzy Monetary; the second measure is based on several non-monetary indicators related to housing conditions, durable goods, etc. and it is referred to as Fuzzy Supplementary.

When poverty is viewed as a matter of degree, i.e. as a fuzzy measure, two additional aspects are introduced into the analysis compared with the conventional poor/non-poor dichotomous approach: *(i)* the choice of membership functions i.e. quantitative specification of or households' degrees of poverty and deprivation; and *(ii)* the choice of rules for the manipulation of the resulting fuzzy sets, rules defining their complements, intersections, union and averaging. In this paper certain conceptual and theoretical aspects concerning fuzzy set logic and operations pertinent are utilised to measure three levels of poverty: strong poverty, medium poverty and low poverty. By referring to the overall population we propose a collective fuzzy monetary measure. Moreover we note the relationship of the proposed fuzzy monetary measure with the membership function and an estimate, by confidence interval, of the poverty line.

The methodology proposed in our research will be illustrated by the Tunisian case. The household survey data conducted by the INS (Tunisian Institute of Statistics) in 1990 and involving 7734 representing households from different parts of the country will be used in this study. Unfortunately, the household survey does not provide direct information on prices. Instead, it gives detailed information on expenditures, including consumption of consumed products, and quantities so that local prices can be estimated. Half of the sampled households were also included in another survey yielding information about the content of the goods.

This paper is structured as follows: Section 2 starts with an uncertainty of the poverty line. In section 3, we discuss the methodology of construction of a poverty fuzzy index. Section 4 is devoted to the case of Tunisia for the “rural-urban”, spatial comparison, a comparison by activity of the household chief and a comparison by educational level. Finally, a conclusion is given in section 5.

2 Uncertainty of the poverty line

Poverty analysis is based on the determination of poverty lines from which one then computes poverty indices such as the head count ratio or the more sophisticated ones (see e.g. Zheng, 1997). These indices can then be used by economists and policy- makers for temporal or spatial comparisons in a relatively-easy manner. Although the determination of the poverty line is an important and uncertain issue, we highlight in this section the computation of an interval of confidence for this line.

We suppose that the poverty line belongs to the interval $[\hat{z}-l, \hat{z}+l]=[z_1, z_2]$, where $\hat{z}$ represents an estimate of the poverty line. The determination of $\hat{z}$ is a delicate step because it is not independent of the socio-economic context in which the individual is established and must take into account of the particular characteristics of the choice of deprivation indicator.

In this paper, we consider the general approach for the assessment of poverty lines proposed by Ravallion and Bidani (1994). This approach consists of determining first the minimum income, to satisfy basic food needs, and second estimating the minimum income to satisfy non food needs. These minimum incomes constitute respectively the food and non food poverty lines. Basic food needs are computed on a regional basis depending on the local food consumer behaviour so that the typical consumption basket ensures a minimal calorific intake as determined by nutritionists. Then, this basket is evaluated using local prices so that the food poverty line can be calculated.

The natural approach is to construct a consumer's basket of non food goods associated to a poor household and then calculate its value by means of local prices. There are however two serious impediments to this approach. The first one is due to the fact that usually one doesn't have data on non-food products and the second is that it is almost impossible to elaborate a homogenous measure for the quantities of non food products and deduce representative unit values. We therefore choose to approximate the non food budget share of the poverty line by looking at the behaviour of the household with income equal to the food poverty line. The share, they are ready to sacrifice in order to satisfy their basic needs on non food products, will serve to estimate the non food part of the poverty line.

The valuation of the non food component is carried out by using a method presented in Ravallion (1994). This approach, based on the intuitive argument that the definition of "basic non food needs", requires the valuation to the willingness to give up a necessary food product in order to purchase the required item. Ravallion estimates the value of the food component by an AIDS class of functions:

$$\omega_{ij} = \alpha_j^0 + \beta_j \log\left(\frac{Y_{ij}}{z_j{}^f n_{ij}^{\theta}}\right) + \sum_k \delta_j^k d_{ij}^k + \varepsilon_{ij} \quad (1)$$

Where ω_{ij} is the food share of household i belonging to the region and/or area j, Y_{ij} is its total per capita expenditure, z_j^f is the already established food poverty line for area j, n_{ij}^{θ} is the equivalent size of household i belonging to the region and/or area j, θ is elasticity size[1], d_{ij}^k are socioeconomic variables such as the age of household head, the number of children , the number of working women, etc…, and ε_{ij} is a disturbance term. The value of $\alpha_j^{'} = \alpha_j^0 + \sum_k \delta_j^k \overline{d}_j^k$ estimates the expected non food shares of households with per capita expenditure that reaches the food poverty line, i.e. $Y_{ij} = z_j^f$. The evaluation of $\overline{d}_j^k$ is made by means of the sub sample with per capita expenditure around the poverty line. The poverty line is then given by $\hat{z}_j = \left(2 - \alpha_j^{'}\right) z_j^f$ and includes de minimum expenditure to satisfy basic food and non food needs. This is actually the so-called lower poverty line. To calculate a confidence interval for the poverty line the bootstrap method typically provides a better approximation to the asymptotic approximation than standard error. The bootstrap method (see e.g. Efron & Tibshirani, 1993) is computationally intensive but conceptually very simple. We take M random samples of size n, with replacement, from our original sample. The larger the value of M the better the approximation. Values of M between 100 and 200 are commonly used. Each of the M samples is called a bootstrap sample.

We calculate the $\hat{z}$ for every bootstrap sample. Let $\hat{z}_m$ be the value of the poverty line for the $m-th$ bootstrap sample. Then, the bootstrap standard error of $\hat{z}$ is just the standard deviation of the bootstrap poverty lines. That is,

$$se(\hat{z}) = \sqrt{\frac{1}{M}\sum_{m=1}^{M}(\hat{z}_m - \hat{z})^2}$$

The standard bootstrap confidence interval for the poverty line defined as following:

$$\hat{z} \pm \Phi(1-\alpha/2)\, se(\hat{z})$$

were $\Phi(1-\alpha/2)$ is the percentile of The Gauss Distribution.

[1] Elasticity size corresponds to the elasticity of equivalence scale compared to the size of household

3 Methodology of construction of the poverty fuzzy index

It is useful to begin by a brief clarification of the concept of treating poverty (or more generally, various forms of deprivation) as a matter of degree replacing the conventional classification of the population into a simple dichotomy. Basically, all individuals in a population are subject to poverty, but to varying degrees. We say that each individual has a certain propensity to be poor, the population covering the whole range [0,1]. The conventional approach is a special case of this, with the population dichotomised as {0,1}: those with an income below a certain threshold are deemed to be poor (i.e. are all assigned a constant propensity=1); others with an income at or above that threshold are considered to be non-poor (i.e. are all assigned a constant propensity=0).

As to the fuzzy sets, the basic idea is as follows. Given a set H of elements $x \in H$, any fuzzy subset A of H is defined as: $A = \{x, \mu_A(x) = \mu\}$ where $\mu_A(x)$: $H \rightarrow$ [0,1] is called the membership function in fuzzy subset A. The value $\mu_A(x)$ indicates the degree of membership of x in A. Thus $\mu_A(x) = 0$ means that x does not belong at all to A, whereas $\mu(x) = 1$ means that x belongs to A completely. When on the other hand $0 < \mu_A(x) < 1$, then x partially belongs to A and its degree of membership of A increases in proportion to the proximity of $\mu_A(x)$ to 1(Zadeh, 1975; Dubois & Prade, 1980; Kaufmann & Gupta, 1991).

3.1 Internal configuration by the fuzzy logic

In our internal configuration by the fuzzy logic, we propose four steps: In a first step, we model our input variable the “poverty line” in the form of triangular membership function (TMF), and we propose three states: a low level of the poverty line, a medium level and a high level, and consequently three fuzzy input subsets: Weak, Medium and High (Figure1). We define also the “poverty” as output variable. Three TMF of fuzzy subsets are proposed as shown in figure2: “Strong Privation (SP)”, “Medium Privation (MP)” and “Weak Privation (WP)”.

▪ A “Strong Privation” corresponds to a poverty line lower or equal to its minimal value $z \leq z_1$, and consequently, the privation is a very relevant poverty- indicator.

▪ A "Medium Privation”, corresponds to a poverty line belonging to the selected interval $[z_1 - z_2]$.

▪ A "Weak Privation” corresponds to a poverty line beyond its maximum value $z \geq z_2$; and the privation cannot be considered alone as a reliable index of poverty.

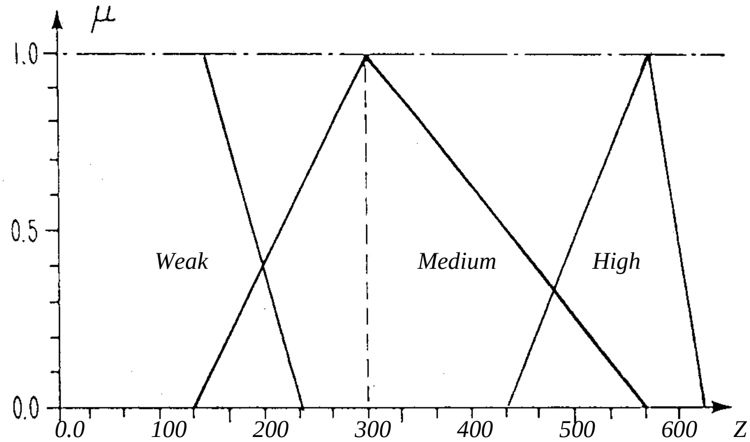

Figure 1 Poverty line

The logic of these scales of privation intensity is as follows: in the case of the monetary variables such as the income or the expenditure, one is confronted in general with situations where the living conditions improve with an increase of the indicator.

Fuzzy methodology translates these ordinal ranks into fuzzy membership scores or degrees that are capable of reflecting the content of the ordinal categories in line with our conceptual understanding of the phenomenon that we want to describe.

This leads us to the second step, i.e. how to assign membership degrees or scores and to calibrate appropriate membership functions.

Again, this step is neither automatic nor univocal as it would be in the case of an ordinal scale.

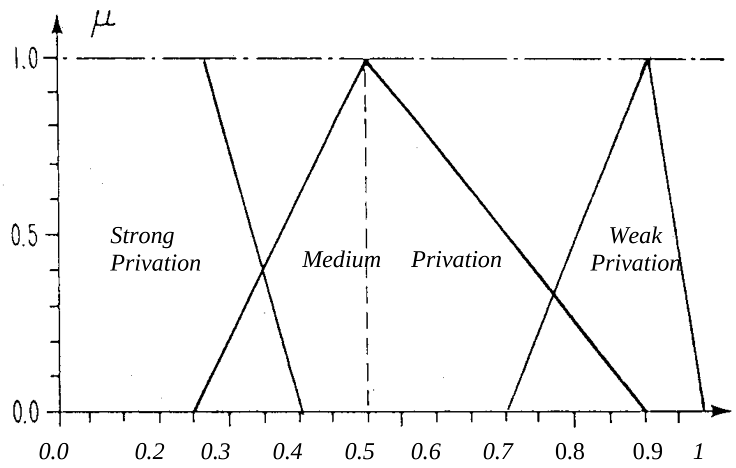

Figure 2 Poverty

Different methods can be adopted for constructing membership functions (Chiappero Martinetti, 2006). They can be chosen arbitrarily by the investigator, according to her or his common sense and experience, or the value judgements underlying the theoretical concept that she or he wishes to describe. For instance, a simple decreasing or increasing linear membership function can be adequate in order to depict variables or concepts distributed along a linear continuum between 0 and 1 (inclusive), where any value is proportional to its distance in the value axis . Triangular or Trapezoidal-shaped membership functions make it possible to preserve linearity and at the same time to incorporate minimum and/or maximum thresholds (section 3.2).

In the third step, we aggregate through fuzzy operators across dimensions or domains of poverty for each unit of analysis, whether they are individuals or households.

Similarly to what happens with conventional or crisp sets, complement, intersection and union operations make it possible to manipulate and combine elementary fuzzy sets. However, since fuzzy sets are not crisply partitioned as are conventional sets, the operators apply on the membership functions, determining membership degrees that, once again, will not be restricted simply to 0 and 1 (section 3.3).

Finally, the fourth step refers to the possibility of applying fuzzy logic rules and fuzzy approximate reasoning in order to infer a logical conclusion starting from premises that are known or assumed to be true. In this step, a fuzzy collective poverty index based only upon a monetary variable is proposed (section 3.4).

3.2 The membership functions

To measure poverty, we proceed as follows: We propose, in a first step, an mathematical expression of the membership function of each fuzzy subset. We choose, in the second step, a membership function for each level of poverty. In the third step, we put forward while using some fuzzy operations, a fuzzy poverty index based upon the membership function chosen and confidence interval for the poverty line estimed.

The mathematical expression of the membership function of each fuzzy subset is as follows:

$$\mu_{SP}(x)=\begin{cases}0 & x<0\\ 1 & 0\le x<z_1\\ \dfrac{-1}{P_1-z_1}x+\dfrac{P_1}{P_1-z_1} & z_1\le x<P_1\\ 0 & x\ge P_1\end{cases} \quad (2)$$

$$\mu_{MP}(x)=\begin{cases}0 & x<z_1\\ \dfrac{2}{z_2-z_1}x-2\dfrac{z_1}{z_2-z_1} & z_1\le x<z_B\\ \dfrac{-2}{z_2-z_1}x+\dfrac{2z_2}{z_2-z_1} & z_B\le x<z_2\\ 0 & x\ge z_2\end{cases} \quad (3)$$

$$\mu_{WP}(x)=\begin{cases}0 & x<P_2\\ \dfrac{2}{P_3-P_2}x-\dfrac{2P_2}{P_3-P_2} & P_2\le x<P_B\\ \dfrac{-2}{P_3-P_2}x+\dfrac{2P_3}{P_3-P_2} & P_B\le x<P_3\\ 0 & x\ge P_3\end{cases} \quad (4)$$

x being the income (the expenditure), P_1 is an unspecified income value higher than z_1, P_2 is an unspecified income value lower than z_2, P_3 is an unspecified income value higher than z_2, z_B is the barycentre of the interval [z_1, z_2] and P_B is the barycentre (Figure 3) of the interval [P_2, P_3] .

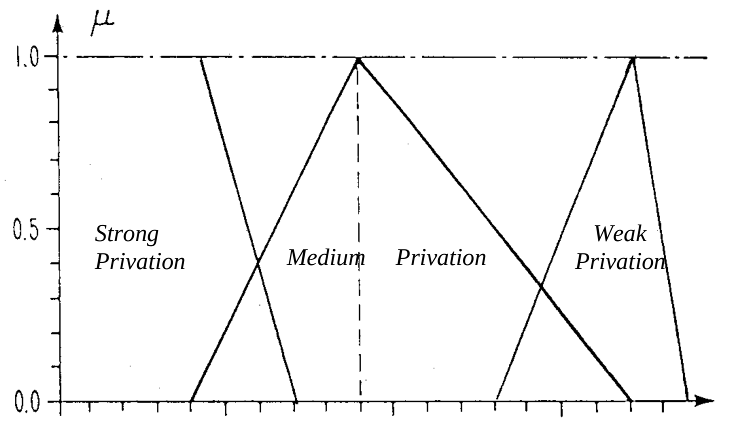

Figure 3 Poverty line values

$\mu_{SP}(x)$, $\mu_{MP}(x)$ and $\mu_{WP}(x)$ respectively indicate the membership- function of the fuzzy subsets "Strong Privation", "Medium Privation" and "Weak Privation".

It is worth underlining that for the different values of the "poverty line", the membership function which takes its values in the interval [*0*, *1*] indicates the degree to which a household is considered as poor.

3.3 Basic rules of the fuzzy set operations

There are three types of fuzzy set operations on membership functions which are relevant to our application to one-dimensional poverty measurement: fuzzy addition, aggregation (or averaging) over fuzzy sets.

Addition of fuzzy numbers

The addition of fuzzy numbers follows the same process to add two confidence intervals (Kaufmann & Gupta, 1991), but on a level -by –level basis. For example, let A and B be two fuzzy numbers and A_α and B_α their intervals of confidence for the level of presumption α, $\alpha \in [0,1]$. We can then write:

$$A_\alpha(+)B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right](+)\left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right]$$

Let us now consider another method for the addition of fuzzy numbers. Let $A, \ B \subset IR$,

$$\forall\, x, y, z \in IR : \mu_{A(+)B}(z) = \bigvee_{z = x + y} (\mu_A(x) \wedge \mu_B(y))$$

Fuzzy aggregation and averaging

Aggregation of membership functions over different sets is related to the concept of fuzzy partitions. More generally, if for each unit in the population, its membership function μ in a certain set is fractioned into components μ_j such that $\mu = \sum_i \mu_j$, then the μ_j values constitute membership functions corresponding to fuzzy partitions of the original set.

This concept of fuzzy partitions is relevant in the specification of marginal constraints which the fuzzy set operations must satisfy.

3.4 The Fuzzy unidimensional poverty

The measurement of the total poverty is the sum of strong, medium and weak poverties. Let SP, MP and WP be three fuzzy numbers belonging respectively to fuzzy subsets "Strong Privation", "Medium Deprivation" and "Weak Deprivation" and SP_α, MP_α and WP_α their intervals of confidence for the level of presumption $\alpha, \alpha \in [0,1]$. We can then write

$$\begin{aligned} SP_\alpha(+)MP_\alpha(+)WP_\alpha &= \left[0^{(\alpha)}, P_1^{(\alpha)}\right](+)\left[z_1^{(\alpha)}, z_2^{(\alpha)}\right](+)\left[P_2^{(\alpha)}, P_3^{(\alpha)}\right] \\ &= \left[z_1^{(\alpha)} + P_2^{(\alpha)}, \mathrm{P}_1^{(\alpha)} + z_2^{(\alpha)} + P_3^{(\alpha)}\right] \\ SP_\alpha = \left[0^{(\alpha)}, P_1^{(\alpha)}\right] \quad MP_\alpha &= \left[z_1^{(\alpha)}, z_2^{(\alpha)}\right] \; WP_\alpha = \left[P_2^{(\alpha)}, P_3^{(\alpha)}\right] \end{aligned} \tag{5}$$

Let us now consider the membership function of fuzzy set $Q = SP\ (+)MP\ (+)WP$:

$$\mu_Q(x) = \mu_{SP_\alpha(+)MP_\alpha(+)WP_\alpha}(x_i) = \bigvee_{x = u + v + k} \left(\mu_{SP\alpha}(u) \wedge \mu_{MP\alpha}(v) \wedge \mu_{WP\alpha}(k)\right) \tag{6}$$

$(\vee),(\wedge)$ is maximum (minimum) of fuzzy numbers by max-min convolution.

To compute the intervals of confidence for each level α the triangular shapes will be described by functions of α as follows:

From (3), $\alpha = \frac{2a_1}{z_2 - z_1} - \frac{2z_1}{z_2 - z_1}$

and $\alpha = \frac{-2a_2}{z_2 - z_1} + \frac{2z_2}{z_2 - z_1}$

Hence, the interval of confidence at level α is given by

$$MP_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] = \left[\frac{z_2 - z_1}{2}\alpha + z_1, -\frac{z_2 - z_1}{2}\alpha + z_2\right] \quad (7)$$

From (4), $\alpha = \frac{2b_1}{P_3 - P_2} - \frac{2P_2}{P_3 - P_2}$

and $\alpha = \frac{-2b_2}{P_3 - P_2} + \frac{2P_3}{P_3 - P_2}$

Therefore

$$WP_\alpha = \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[\frac{P_3 - P_2}{2}\alpha + P_2, -\frac{P_3 - P_2}{2}\alpha + P_3\right] \quad (8)$$

$$J_\alpha = MP_\alpha (+) WP_\alpha = \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right] = \left[\frac{(z_2 - z_1) + (P_3 - P_2)}{2}\alpha + z_1 + P_2, -\frac{(z_2 - z_1) + (P_3 - P_2)}{2}\alpha + z_2 + P_3\right] \quad (9)$$

From

$$a_1^{(\alpha)} + b_1^{(\alpha)} = \frac{(z_2 - z_1) + (P_3 - P_2)}{2}\alpha + z_1 + P_2$$

$$a_2^{(\alpha)} + b_2^{(\alpha)} = -\frac{(z_2 - z_1) + (P_3 - P_2)}{2}\alpha + z_2 + P_3$$

We obtain

$$\mu_J(x) = \quad (10)$$

$$\begin{cases} 0 & x < z_1 + P_2 \\ \frac{2}{(z_2 - z_1) + (P_3 - P_2)}x - \frac{2(z_1 + P_2)}{(z_2 - z_1) + (P_3 - P_2)} & z_1 + P_2 \le x < z_B + P_B \\ \frac{-2}{(z_2 - z_1) + (P_3 - P_2)}x + \frac{2(z_2 + P_3)}{(z_2 - z_1) + (P_3 - P_2)} & z_B + P_B \le x < z_2 + P_3 \\ 0 & x \ge z_2 + P_3 \end{cases}$$

From (4), $\alpha = 0$

and $\alpha = \frac{-c_2}{P_1 - z_1} + \frac{P_1}{P_1 - z_1}$

Therefore $SP_\alpha = [0,\ -(P_1 - z_1)\alpha + P_1]$ (11)

Adding (9) and (10) yields

$$Q_\alpha = SP_\alpha(+)\ MP_\alpha(+)\ WP_\alpha = \left[\frac{(z_2 - z_1) + (P_3 - P_2)}{2}\alpha + z_1 + P_2, -\frac{(z_2 - z_1) + (P_3 - P_2)}{2}\alpha + P_1 + z_2 + P_3\right] \quad (12)$$

From
$$c_1^{(\alpha)} + (a_1 + b_1)^{(\alpha)} = \frac{(z_2 - z_1) + (P_3 - P_2)}{2}\alpha + z_1 + P_2$$
$$c_2^{(\alpha)} + (a_2 + b_2)^{(\alpha)} = -\frac{(z_2 - z_1) + (P_3 - P_2) + 2(P_1 - z_1)}{2}\alpha + P_1 + z_2 + P_3$$

We obtain

$$\mu_Q(x\ ) = \quad (13)$$

$$\begin{cases} 0 & x < 0 \\ 1 & 0 \le x < z_1 + z_B + P_B \\ \dfrac{-2}{(z_2 - z_1) + (P_3 - P_2) + 2(P_1 - z_1)}x + \dfrac{2(P_1 + z_2 + P_3)}{(z_2 - z_1) + (P_3 - P_2) + 2(P_1 - z_1)} & z_1 + z_B + P_B \le x < P_1 + z_2 + P_3 \\ 0 & x\ \ge P_1 + z_2 + P_3 \end{cases}$$

We propose to retain like degree of deprivation of any household $i, i = 1,...,n$ its degree of membership in the fuzzy set of the poor and we define an underlying individual poverty function by attribute $j,\ j = 1,...,m$ as follows:

$$f_j(x_i) = [\mu_{Qj}(x_i)]^\beta\ ;\ \beta \ge 1 \quad (14)$$

The parameter β defines the concavity of the underlying individual poverty function and it is related to the extreme poverty aversion parameter involved in Foster, Greer, and Thorbecke's (1984) poverty measure.

In practice, it seems or reasonable to use the traditional values 1 and 2 for the unidimensional aspect or a higher dimensionality — 3 is sometimes used in the unidimensional context to get a measure that is more sensitive to transfers involving the poorest members of the population —for β. As noted in Atkinson (2003) "there is not necessarily any reason to change our views about the value of β simply because we have moved to a higher dimensionality."

By aggregating all these values we obtain a collective index referring to the overall population which is given by:

$$H\ (\mu, Q\ ) = \frac{1}{n}\sum_j\ w_j \sum_{i \in Q}\ n_i^\theta\ f_j(x_i) \quad (15)$$

where μ_{iQ} is the characteristic function of Q (12). Then, we adopt (15) as the definition of $H(\mu, Q)$ for a fuzzy Q by interpreting $\mu_{iQ}(x)$ as the degree of membership of a household i in Q.

$H(.)$ represents the "fuzzy proportion" of the poor according to the total per capita expenditure X. In any case, it must be said that this index must not be interpreted as the Head Count Ratio of the poor, but simply as the average degree of poverty in the surveyed population.

The measure $H(.)$ complies with Extended strong focus, Monotonicity, Restricted strong monotonicity, Restricted continuity, Non-decreasingness in poverty domain, Subgroup consistency, Anonymity and Population invariance. It can be shown that this measure satisfies the transfer and transfer sensitivity axioms. It is also continuous and even decomposable. It can be adapted equally well to any other multidimensional framework for the analysis of well-being and poverty. The membership's functions will have to be regarded in this case as deprivation indicators.

4 Empirical illustration

In this section, we suggest the application of (section 3) to evaluate strong, medium and weak poverties of different regions in Tunisia, also to compare poverty in rural versus urban regions, according to the activity of the household -chief bread-winner, and according to the educational level.

As stated in the introduction, the information used is supplied from the household survey data conducted by the INS in 1990 involving 7734 households. The sampling scheme and the results of the survey are explained in INS (1990). The survey also provides the demographic characteristics of households. In order to take into account the different geographical and socio-economic characteristics of the regions in Tunisia, we split the country in 5 different homogenous regions[2], three of which are urban areas.

We retained three indicators of deprivation:

- The economic area (Great Tunis (GT), urban littoral (LU), rural littoral (LR), urban interior (IU), and rural interior (IR))
- The activity of the household chief (Inactive (I), Farm labourer (FL), Farmer (F), Non-agricultural worker (NW), Independent agricultural (IA), Employer and Manager (EM), Others (O))
- The educational level of the household chief (Illiterate (IL), Primary (PE), Secondary 1st cycle (S1), Secondary 2-cycles (S2), Academics (A)). We initially calculated the interval $[z_1 - z_2]$ and then the poverty fuzzy index per economic area, occupation and educational level.

[2] To make use of the characteristics of different regions in Tunisia, we separated the households according to their location with respect to 5 different homogenous regions. Tunisia is traditionally subdivided into three natural regions: North, Center and South. This decomposition is motivated by the geographical characteristics of the country. However, from an economic point of views, it is more appropriate to divide Tunisia into three parts: The Greater Tunis and two homogenous sets namely the Littoral and the Interior. The Greater Tunis area, which involves almost 25% of the total population, is characterised by very special administrative, social and economic properties. The Tunisian Littoral (Bizerte, Cap-Bon, Sahel, Sfax and Gabes) have known since the independence an economic and social prosperity. This coastal fringe extending from North to South contains, together with the Greater Tunis area, the essential of the tourist, industrial and urban activity of the economy. Despite a certain economic progress, the Interior region has several acute social and economic problems which distinguish it from the other two regions. If one compares the per capita expenditure (during 1990), one sees that this subdivision is justified. In addition to this regional decomposition, it is necessary to take into account the rural-urban distinction. We also aggregated the rural part of the Greater Tunis and the littoral. Two reasons support this aggregation. First, the size of the rural Greater Tunis is very small, only 167 households and second, the rural of Greater Tunis and those of the rest of the littoral are very similar and can be lumped together to form a homogenous spatial set. This leads us to five homogenous regions, namely the urban Greater Tunis, the urban Littoral, the urban Interior, the rural Littoral and rural Interior.

4.1 The regional fuzzy poverty

The estimated intervals for the 5 regions of Tunisia are presented in the first column of Table 1. [3]We note that, in 1990, for the “Great Tunis”, for example, any household whose annual expenditure is lower than 263DT is considered poor and its degree of membership to the fuzzy sub-set "Strong Privation" is very high. On the other hand, any household whose annual total expenditure exceeds 277DT is considered as non-poor. Its degree of membership of fuzzy subset "Weak Privation" is high.

The poverty lines estimated are lower in the poorest regions (Interior Urban and Rural, Littoral Rural.

Table 1 Strong, medium and weak fuzzy poverties by area (1990)

	$\hat{z}$	$[z_1, z_2]$	ω_R	$\overline{\mu_{SP}(x_i)}$	$\overline{\mu_{MP}(x_i)}$	$\overline{\mu_{WP}(x_i)}$	$\mu_{Qj}(x_i)$
Great Tunis	270	[263 - 277]	0.25	0.057	0.131	0.099	0,099
Urban littoral	243	[235 - 251]	0.15	0.048	0.108	0.114	0,108
Urban interior	202	[193 - 211]	0.20	0.099	0.063	0.029	0,063
Rural littoral	162	[157 - 167]	0.22	0.092	0.059	0.027	0,059
Rural interior	159	[151 - 167]	0.10	0.139	0.039	0.019	0,039
Fuzzy poverty			**1**	**0.074**	**0.078**	**0.055**	**0.070**

$\overline{\mu_{SP}(x_i)}, \overline{\mu_{MP}(x_i)}, \overline{\mu_{WP}(x_i)}$, ω_R *indicates the average membership function of the respective fuzzy subsets “Strong Privation”, “Medium Privation” and “Weak Privation” and the weight of region R;* $\alpha = 0.05$

However, what attracts our attention is the difference between the urban and rural lines. The urban/rural ratio in the littoral region is equal to 1.5. On the other hand, the ratio urban/rural in the Interior is equal to 1.27. Indeed, the urban littoral has seen a rapid economic development compared to the interior which has led to an increase in living costs and which explains why the urban/rural difference in the littoral region should be greater than the one in the interior region.

To compute the sum of the interval of confidence at level α, we shall use (3) to obtain Table 2.

Table 2

	LU	*GT*	*LR*	*IU*	*IR*
1 . .					
0.139					1
0.099				1	1
0.092			1	1	1
0.057		1	1	1	1
0.048	1	1	1	1	1
	0.048	0.057	0.092	0.099	0.139

(+)

	IR	*LR*	*IU*	*LU*	*GT*
1 . .					
0.131					1
0.108				1	1
0.063			1	1	1
0.059		1	1	1	1
0.039	1	1	1	1	1
	0.039	0.059	0.063	0.108	0.131

(+)

[3] We grouped Greater Tunis and Littoral urban together to compute the consumer’s basket because the respective samples were relatively small. We however considered different unit values and therefore poverty lines for the two regions.

	IR	LR	IU	GT	LU
1 . .					
0.124					1
0.097				1	1
0.019			1	1	1
0.017		1	1	1	1
0.009	1	1	1	1	1
	0.009	0.017	0.019	0.097	0.124

It is worth emphasising that we could have computed the sum given in Table 2 by using (6). Using these computations, we obtain the following set of equations:

$$\mu_Q(GT) = (0.057 \wedge 0.131) \vee (0.099 \wedge 0.131) = 0.099$$
$$\mu_Q(LU) = (0.048 \wedge 0.108) \vee (0.114 \wedge 0.108) = 0.108$$
$$\mu_Q(IR) = (0.139 \wedge 0.039) \vee (0.019 \wedge 0.039) = 0.039$$
$$\mu_Q(IU) = (0.990 \wedge 0.063) \vee (0.029 \wedge 0.063) = 0.063$$
$$\mu_Q(LR) = (0.139 \wedge 0.039) \vee (0.019 \wedge 0.039) = 0.039$$

By examining third column of Table 1, one first remarks that strong poverty in Tunisia during the year 1990 is mainly a phenomenon that affects more severely the rural areas than the urban ones. In each region the rural poverty index exceeds that of the urban one. We can observe conspicuously that the ratio of the rural over the urban in the interior region amounts to 140%. Moreover, it reaches the peak of 192% for the littoral region. Medium poverty affects rather the areas of the Great Tunis and the littoral urban area.

The results show that total fuzzy poverty is about 0,07 and that the areas “urban littoral” and “Great Tunis” present on average living condition that are different from the others and better than the national average.

4.2 Fuzzy poverty by activity of the household chief

Based on Table 3 our survey reveals plainly that, on an average scale, the Tunisian farm labourers and non-agricultural workers are affected by strong poverty. On the other hand, independent agricultural workmen are affected by medium poverty. By taking into account these outcomes, we can note that any structural Tunisian socio-economic policy to reduce poverty must include a reform aiming at helping this socio-professional category.

Table 3 Strong, medium and weak fuzzy poverties distributed according to the occupation (1990)

	$[z_1 - z_2]$	ω_F	$\overline{\mu_{SP}(x_i)}$	$\overline{\mu_{MP}(x_i)}$	$\overline{\mu_{WP}(x_i)}$	$\mu_{Qj}(x_i)$
Inactive	[167 - 175]	0.094	0.060	0.045	0.025	0.045
Farm labourers	[156 - 168]	0.180	0.110	0.068	0.038	0.068
Farmers	[178 - 188]	0.059	0.090	0.053	0.029	0.053
Nonagricultural Worker	[152 - 166]	0.457	0.120	0.072	0.023	0.072

Independent Agricultural	[179 - 188]	0.118	0.080	0.042	0.027	0.042
Employers and Managers	[325 - 349]	0.015	0.005	0.010	0.052	0.010
Others	[185 - 195]	0.077	0.040	0.064	0.075	0.064
Fuzzy poverty		**1**	**0.096**	**0.062**	**0.031**	**0.062**

$\overline{\mu_{SP}(x_i)}, \overline{\mu_{MP}(x_i)}, \overline{\mu_{WP}(x_i)}$, ω_F *indicates the average membership function of the respective fuzzy subsets "Strong Privation", "Medium Privation" and "Weak Privation" and the weight by profession;* $\alpha = 0.05$

To compute the sum of the interval of confidence at level α , we shall use (3) to obtain Table 4.

Table 4

	EM	O	I	IA	F	FL	NW
1 . . .							
0.12							1
0.11						1	1
0.09					1	1	1
0.08				1	1	1	1
0.06			1	1	1	1	1
0.04		1	1	1	1	1	1
0.005	1	1	1	1	1	1	1
	0.005	0.04	0.06	0.08	0.09	0.11	0.12

(+)

	EM	IA	I	F	O	FL	NW
1 . . .							
0.072							1
0.068						1	1
0.064					1	1	1
0.053				1	1	1	1
0.045			1	1	1	1	1
0.042		1	1	1	1	1	1
0.010	1	1	1	1	1	1	1
	0.010	0.042	0.045	0.053	0.064	0.068	0.072

(+)

	NW	I	IA	F	FL	EM	O
1 . . .							
0.075							1
0.052						1	1
0.038					1	1	1
0.023				1	1	1	1
0.025			1	1	1	1	1
0.027		1	1	1	1	1	1
0.029	1	1	1	1	1	1	1
	0.075	0.052	0.038	0.023	0.025	0.027	0.029

Note that we could have computed the sum given in Table 4 by using (6). Using these computations, we obtain the following set of equations:

$$\mu_Q(I) = (0.060 \wedge 0.045) \vee (0.045 \wedge 0.025) = 0.045$$
$$\mu_Q(FL) = (0.110 \wedge 0.068) \vee (0.068 \wedge 0.038) = 0.068$$
$$\mu_Q(F) = (0.090 \wedge 0.053) \vee (0.053 \wedge 0.029) = 0.053$$
$$\mu_Q(NW) = (0.120 \wedge 0.072) \vee (0.072 \wedge 0.023) = 0.072$$
$$\mu_Q(IA) = (0.080 \wedge 0.042) \vee (0.042 \wedge 0.027) = 0.042$$
$$\mu_Q(EM) = (0.052 \wedge 0.010) \vee (0.010 \wedge 0.005) = 0.010$$
$$\mu_Q(O) = (0.040 \wedge 0.064) \vee (0.064 \wedge 0.075) = 0.064$$

4.3 Fuzzy poverty by educational level

According to the educational level of the chief bread- winner of the household, poverty is more significant among the illiterate and those having a primary education level. Non agricultural workers and farm labourers are affected by medium poverty. The intensity of poverty is low at the households whose head is university-degree-holder (Table 5).

Table 5 Strong, medium and weak fuzzy poverties according to the educational level (1990)

	$[z_1 - z_2]$	ω_N	$\overline{\mu_{SP}(x_i)}$	$\overline{\mu_{MP}(x_i)}$	$\overline{\mu_{WP}(x_i)}$	$\mu_{Qj}(x_i)$
Illiterate	[152-164]	0.464	0.113	0.055	0.003	0.055
Primary education	[179-193]	0.344	0.071	0.121	0.005	0.071
Secondary 1st cycle	[185-195]	0.100	0.046	0.097	0.009	0.046
Secondary 2er cycles	[265-277]	0.071	0.032	0.043	0.033	0.033
Academic	[325-339]	0.021	0.005	0.008	0.022	0.008
Fuzzy poverty		**1**	**0.083**	**0.079**	**0.007**	**0.057**

$\overline{\mu_{SP}(x_i)}, \overline{\mu_{MP}(x_i)}, \overline{\mu_{WP}(x_i)}, \omega_N$ *indicates the average membership function of the respective fuzzy subsets "Strong Privation", "Medium Privation" and "Weak Privation" and the weight by education level;* $\alpha = 0.05$

Eventually, if the targeting is carried out according to the educational level of the chief-bread winner of the household, this reveals that we must focus attention on the illiterate category. As this fringe of the population occupies the greatest contribution to the measurement of poverty, we can conclude that targeting this group can involve a noticeable improvement of the welfare of the poor population.

To compute the sum of the confidence -interval at level α, we shall use (3) to obtain Table 6.

Table 6

	A	S2	S1	PE	IL
1 . . .					
0.113					1
0.071				1	1
0.046			1	1	1
0.032		1	1	1	1
0.005	1	1	1	1	1
	0.005	0.032	0.046	0.071	0.113

(+)

	A	S2	IL	S1	PE
1 . . .					
0.121					1
0.097				1	1
0.055			1	1	1
0.043		1	1	1	1
0.008	1	1	1	1	1
	0.008	0.043	0.055	0.097	0.121

(+)

	IL	PE	S1	A	S2
1 . . .					
0.033					1
0.022				1	1
0.009			1	1	1
0.005		1	1	1	1
0.003	1	1	1	1	1
	0.003	0.005	0.009	0.022	0.033

We notice that we could have computed the sum given in Table 6 by using (6). Using these computations, we obtain the following set of equations:

$$\mu_Q(IL) = (0.113 \wedge 0.055) \vee (0.055 \wedge 0.003) = 0.055$$
$$\mu_Q(PE) = (0.072 \wedge 0.121) \vee (0.121 \wedge 0.005) = 0.072$$
$$\mu_Q(S1) = (0.046 \wedge 0.097) \vee (0.097 \wedge 0.009) = 0.046$$
$$\mu_Q(S2) = (0.032 \wedge 0.043) \vee (0.043 \wedge 0.033) = 0.033$$
$$\mu_Q(A) = (0.005 \wedge 0.008) \vee (0.008 \wedge 0.022) = 0.008$$

5 Conclusions

When poverty is viewed as a matter of degree in contrast to the conventional poor/non-poor dichotomy, that is, as a fuzzy state, two additional aspects are introduced into the analysis: *(i)* The choice of membership functions i.e. quantitative specification of individuals' or households' degrees of poverty and deprivation. *(ii)* And the choice of rules for the manipulation of the resulting fuzzy sets, rules defining their complements, intersections, union and aggregation. Specifically, for longitudinal analysis of poverty using the fuzzy set approach, we need joint membership functions covering more than one time period, which have to be constructed on the basis of the series of cross-sectional membership functions over those time periods. This paper has a measure of monetary deprivation using the fuzzy set approach.

In fact, procedures for combining fuzzy measures in multiple dimensions were applied in the literature. We have proposed a general rule for the construction of fuzzy set intersections, that is, for the construction of a total poverty measure from a sequence of different individual measures under fuzzy conceptualization. This general rule is meant to be applicable to any sequence of "poor" and "non-poor" sets. On the basis of the results obtained, a fuzzy unidimensional poverty measure is constructed. It satisfies many desirable properties.

Numerical results of these procedures applied to measures of unidimensional poverty and deprivation and to combinations of such measures. The results showed that in 1990 strong poverty in Tunisia was clearly a rural phenomenon and this contradicts the findings of governmental institutions. In 1990, Medium poverty affects rather the areas of the Great Tunis and Coastal Urban. This is true even if we adopt the fuzzy approach which makes it possible to break up poverty into several levels. We also noted that strong poverty affects more severely the interior regions, the farm labourers, the non-agricultural and the illiterate people.

6. References

Atkinson, A. (2003). Multidimensional Deprivation: Contrasting Social Welfare and Counting Approaches, Journal of Economic Inequality,1 (1), 51–65.
Betti, G. & Verma, V.K. (1999). Measuring the degree of poverty in a dynamic and comparative context: A multidimensional approach using fuzzy set theory, Proceedings ICCS-VI, (11), 289-301, Lahore, Pakistan
Betti, G. & Verma, V.K. (2008). Fuzzy measures of the incidence of relative poverty and deprivation: a multi-dimensional perspective, *Statistical Methods and Applications*, 12 (2), 225-250.
Betti, G., Cheli, B. & Cambini, R. (2004). A Statistical Model for The Dynamics Between Two Fuzzy States, *Theory and Application to Poverty Analysis*, Metron 62(3), 391-411.
Betti G, Cheli, B. & Verma, V. (2006). On longitudinal analysis of poverty conceptualised as a fuzzy state, *Working Paper Series*, ECINEQ WP32 .
Cerioli, A. & Zani, S. (1990). A Fuzzy Approach to the Measurement of poverty. In C.Dagum and M.Zenga, editors, Income and Wealth Distribution, Inequality and Poverty, *Studies in Contemporary Economics* 272-284. Spinger Verlag, Berlin.
Cheli, B. & Betti, G. (1999). Fuzzy Analysis of Poverty Dynamics on an Italian Pseudo Panel, 1985-1994. Metron (57), 83-103
Cheli, B. & Lemmi, A. (1995) "Totally" Fuzzy and Relative Approach to the Multidimensional Analysis of Poverty, *Economic Notes*, 24, 115-134.
Cheli, B. (1995). Totally Fuzzy and Relative Measures in Dynamics Context, Metron, 53 (3/4), 83-205.
Chiappero Martinetti, E. (2000). A multidimensional assessment of well-being based on Sen's functioning approach, *Rivista Internazionale di Scienze Sociali*, 108, 207-239.
Chiappero Martinetti, E. (2006). Capability Approach and Fuzzy Set Theory: Description, Aggregation and inference Issues, *in Fuzzy Set Approach to Multidimensional Poverty Measurement*, Lemmi A and Betti G. (ed), Springer + Business Media, LLC, New-York, 139-153.
Clark, D. A. & Qizilbash, M. (2002). Core Poverty and Extreme Vulnerability in South Africa, Presented to the IARIW Conference, Stockholm.
Dubois, D. & Prade, H. (1980). *Fuzzy Sets and Systems*, Academic Press, Boston.
Efron, B. & Tibshirani, R.J. (1993). *An introduction to the bootstrap*. New York: Chapman and Hall.
Foster, J., Greer, J. & Thorbecke, E. (1984). A class of decomposable poverty measures, *Econometrica*, 52,761-765.
INS, (1990). *Enquête sur le budget et la consommation des ménages en Tunisie*, Ministère du plan, Tunis.
Kaufmann, A. & Gupta, M.M. (1991). *Introduction to Fuzzy Arithmetic*, International Thomson Computer Press.
Lelli, S. (2001). Factor Analysis vs. Fuzzy Sets Theory: Assessing the Influence of Different Techniques on Sen's Functioning Approach, Discussion Paper Series DPS 01.21, Center for Economic Studies, Catholic University of Leuven, Belgium, November .
Ravallion, M. (1994). *Poverty Comparisons. Fundamentals of Pure and Applied Economics Series*, Harwood Academic Press, Newyork,.
Ravallion, M. & Bidani, B. (1994). How Robust is a Poverty Profile? *The word Bank Economic Review*.
Zadeh, L. (1965). Probability Theory and Fuzzy Logic are Complementary rather than Competitive, *Technometrics*, 37,271-276.
Zadeh, L. (1975). The Concept of a Linguistic Variable and its Application to Approximate Reasoning, *Information Sciences* 9, 43-80.
Zheng, B. (1997). Aggregate poverty measures, *Journal of Economic Surveys*, 11, 123–162.