

MATHEMATICAL MODEL TO DETERMINATION OF THE RESHARPENING TERRITORY OF CONICAL HOB

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ABSTRACT

Based on the general mathematical model of Illés Dudás which is appropriate for mathematical modelling of production technology methods we have worked out a model for resharpening analysis of conical hob. After the hob resharpening using numerical calculations the determination of the tooth surface of face gear by cutting edges is necessary for the analysis. Based on this methods we could calculate the permissible critical angle of the hob and the profiles of the hob and the face gear in axial section. The permissible critical angle of the hob is the critical angle the hob cutting edge of which manufactured face gear profile is situated in the permissible profile error tolerance. We have worked out a new geometric conical worm gear drive that is the conical worm gear drive having arched profile. Using this mathematical model we have done resharpening analysis for the hob having arched profile and determined the permissible critical angle.

Keywords: hob, face gear, mathematical model, resharpening

1. INTRODUCTION

The teething of a face gear is determined by the teething of the worm with contact surfaces mutually meshing with each other.

The teething of the worm, such a tool should be applied so that its cutting edges are situated on the surface of a substituting worm (Fig. 1) [3]. It is necessary so that the production tolerances could be provided.

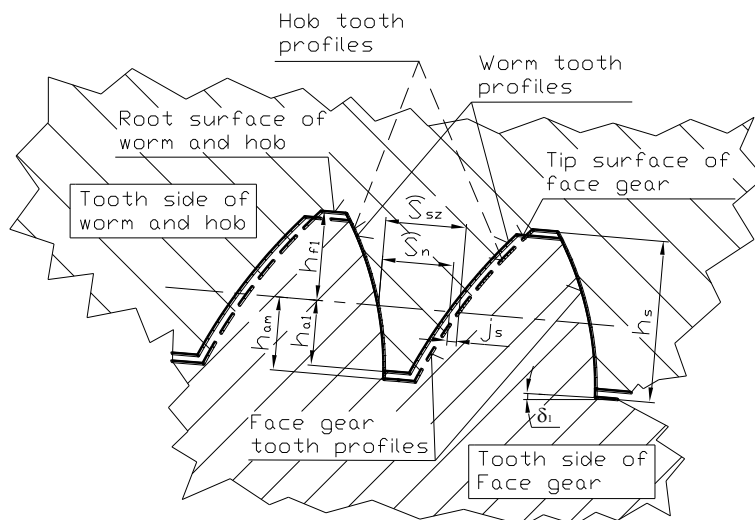


Figure 1. The basic profile of the face gear hob

The elements of the face gear hob is equal with the elements of the worm co-operating as a gear drive with the face gear by direct motion mapping [3]. Tip diameter and tooth thickness of the hob is larger than the parameters of the real worm because of the assurance of resharpening territory and backlash between the worm and the face gear (Fig. 1).

Based on these it could be determinable the equation of the hob cutting edge has to be known to determination of the tooth surface points of the spiroid face gear [2].

2. DEFINING OF THE HOB CUTTING EDGE AND THE FACE SURFACE OF THE HOB

Given the $\vec{r}_g(\xi, \eta, \zeta)$ generator curve of the B helicoidal surface having parameters p_a (axial) and p_r (radial thread) in the form (Fig. 2)

$$\vec{r}_{1F}(\eta, \vartheta) = M_{1F, sz} \cdot \vec{r}_g \tag{1}$$

$$\left. \begin{aligned} x_{1F} &= \xi(\eta) \cdot \cos \vartheta - \eta \cdot \sin \vartheta \\ y_{1F} &= \xi(\eta) \cdot \sin \vartheta + \eta \cdot \cos \vartheta + p_r \cdot \vartheta \\ z_{1F} &= \zeta(\eta) + p_a \cdot \vartheta \end{aligned} \right\} \tag{2}$$

which could be random profile (line, circle, ellipse, parabola, etc.).

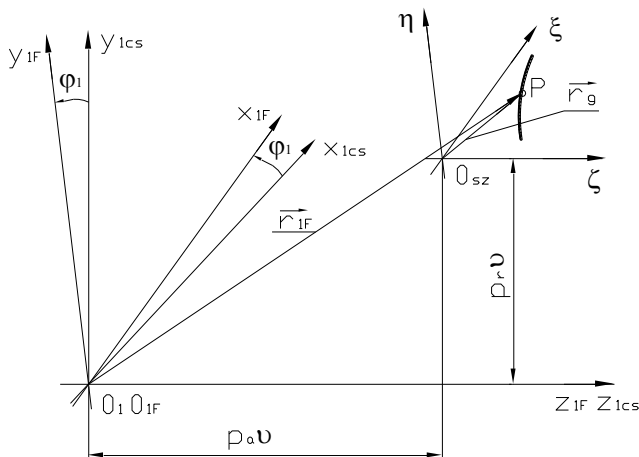


Figure 2. Defining of conical helical surface

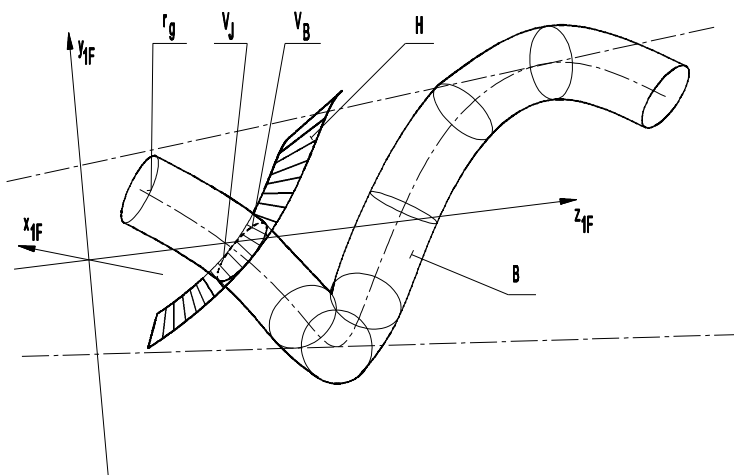


Figure 3. The generation of face surface H and cutting edges V [3]

To face surface H is a ruled (closed and flat) helicoidal surface with a generator line perpendicular to the axis of surface B showing a lead direction perpendicular to the lead direction measured on the pitch cone of helicoid B (Fig. 3)

$$\vec{r}_{1F}^H = \vec{r}_{1F}^H(\eta, \vartheta) . \tag{3}$$

The common solution of the equations of the worm surface (B) and the face surface (H) result the profile points of the cutting edge of the hob (V) (Fig. 3) [3], that is

$$\left. \begin{aligned} \vec{r}_{1F} &= \vec{r}_{1F}(\eta, \vartheta) \\ \vec{r}_{1F}^H &= \vec{r}_{1F}^H(\eta, \vartheta) \end{aligned} \right\} \vec{r}_{1F}^V(\eta', \vartheta) \quad (4)$$

Let the parameter of the relief thread surface be p'_a based on (2) the equations of the back surface of hob are:

$$\left. \begin{aligned} x_{hr} &= \xi(\eta) \cdot \cos \vartheta - \eta \cdot \sin \vartheta \\ y_{hr} &= \xi(\eta) \cdot \sin \vartheta + \eta \cdot \cos \vartheta + p_r \cdot \vartheta \\ z_{hr} &= \zeta(\eta) + p'_a \cdot \vartheta \end{aligned} \right\} \quad (5)$$

3. MATHEMATICAL MODEL TO DETERMINATION OF THE TOOTH SURFACE POINTS OF FACE GEAR DURING HOB RESHARPENING

Knowing the $\vec{r}_{1F}^V = \vec{r}_{1F}^V(\eta', \vartheta_1)$ two parametric vector scalar function of the hob cutting edge (meshing surface) our objective is to generate the tooth surface points of the face gear during resharpening along the face surface of the hob.

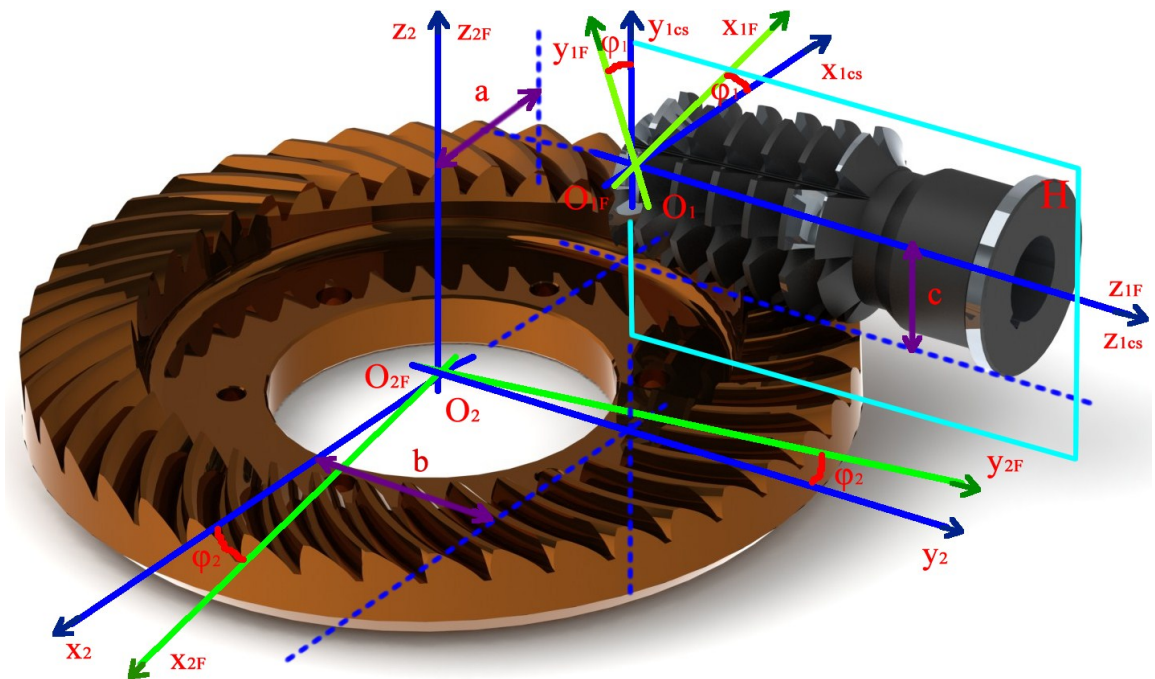


Figure 4. The position of spatial coordinate systems to determination of the tooth surface points of the face gear

For the description of the motion relations we define the own motion of each coordinate system (Fig. 4). Thus $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ coordinate system rotates with

$$\bar{\omega}_1 = \frac{d\varphi_1}{dt} = const. \quad (6)$$

angular velocity in $K_{1cs}(x_{1cs}, y_{1cs}, z_{1cs})$ stationary coordinate system [3, 4, 5].

The $K_{2F}(x_{2F}, y_{2F}, z_{2F})$ coordinate system in the $K_2(x_2, y_2, z_2)$ coordinate system rotates with

$$\bar{\omega}_2 = \frac{d\varphi_2}{dt} = const. \quad (7)$$

angular velocity [3, 4, 5].

Transformation matrix between the $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ rotational coordinate system fixed to member 1 and the $K_{2F}(x_{2F}, y_{2F}, z_{2F})$ rotational coordinate system fixed to member 2 (Fig. 4) is:

$$M_{2F,1F} = M_{2F,2} \cdot M_{2,1cs} \cdot M_{1cs,1F} =$$

$$= \begin{bmatrix} -\cos \varphi_2 \cdot \cos \varphi_1 & \cos \varphi_2 \cdot \sin \varphi_1 & \sin \varphi_2 & -a \cdot \cos \varphi_2 + b \cdot \sin \varphi_2 \\ \sin \varphi_2 \cdot \cos \varphi_1 & -\sin \varphi_2 \cdot \sin \varphi_1 & \cos \varphi_2 & a \cdot \sin \varphi_2 + b \cdot \cos \varphi_2 \\ \sin \varphi_1 & \cos \varphi_1 & 0 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (8)$$

3.1. Application of the direct kinematical method to generation of the tooth surface points of the face gear

Given the $\vec{r}_{1F}^V = \vec{r}_{1F}^V(\eta', \vartheta_1)$ two parametric vector – scalar function of the hob cutting edge in the $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ rotational coordinate system of the hob [2].

In the K_{1F} coordinate system the normal vector could be calculated [3, 5]:

$$\vec{n}_{1F}^v = \frac{\partial \vec{r}_{1F}^V}{\partial \eta'} \times \frac{\partial \vec{r}_{1F}^V}{\partial \vartheta_1} \quad (9)$$

The relative velocity between the two surfaces can be defined by the transformation between the K_{1F} rotational coordinate system of the hob and the K_{2F} rotational coordinate system of the face gear in the K_{1F} system:

$$\vec{v}_{1F}^{(12)} = M_{1F,2F} \cdot \frac{dM_{2F,1F}}{dt} \cdot \vec{r}_{1F} \quad (10)$$

where

$$P_{1k} = M_{1F,2F} \cdot \frac{d}{dt}(M_{2F,1F}) \quad (11)$$

matrix of the kinematic motion mapping:

$$P_1 = \begin{bmatrix} 0 & -1 & -i_{21} \cdot \cos \varphi_1 & -b \cdot i_{21} \cdot \cos \varphi_1 \\ 1 & 0 & i_{21} \cdot \sin \varphi_1 & b \cdot i_{21} \cdot \sin \varphi_1 \\ i_{21} \cdot \cos \varphi_1 & -i_{21} \cdot \sin \varphi_1 & 0 & a \cdot i_{21} \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (12)$$

Knowing the Connection I. Statement and the equation of the hob cutting edge (4) the tooth surface points of the face gear by cutting edges are the common solution of the below equations:

$$\left. \begin{aligned} \vec{r}_{1F}^V &= \vec{r}_{1F}^V(\eta', \vartheta_1) \\ \vec{n}_{1F}^V \cdot \vec{v}_{1F}^{(12)} &= 0 \\ \vec{r}_{2F} &= M_{2F,1F} \cdot \vec{r}_{1F}^V \end{aligned} \right\}. \quad (13)$$

Using the received equations (13) for example we have done resharpener analysis for the our worked out, new geometric conical worm gear drive having arched profile in axial section [1]. On Fig. 5 it could be seen the hob face surface is situated in the axial section.

We have done computer program to the analysis (Fig. 6). As a result considering of the profile preciseness of the face gear we have received the permissible critical angle of the hob $\nu_{\text{hat}}=5^\circ$ (Fig. 5 and 6).

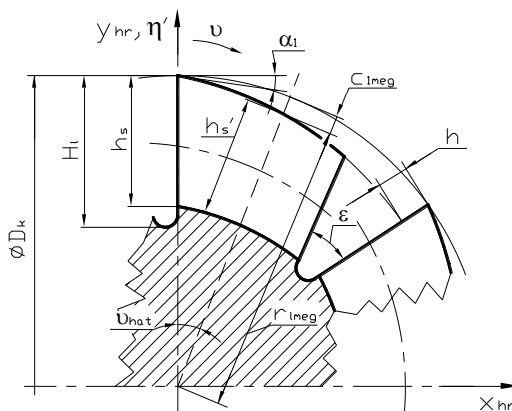


Figure 5. Definition of the permissible resharpener provision and the resharpener critical angle of the hob

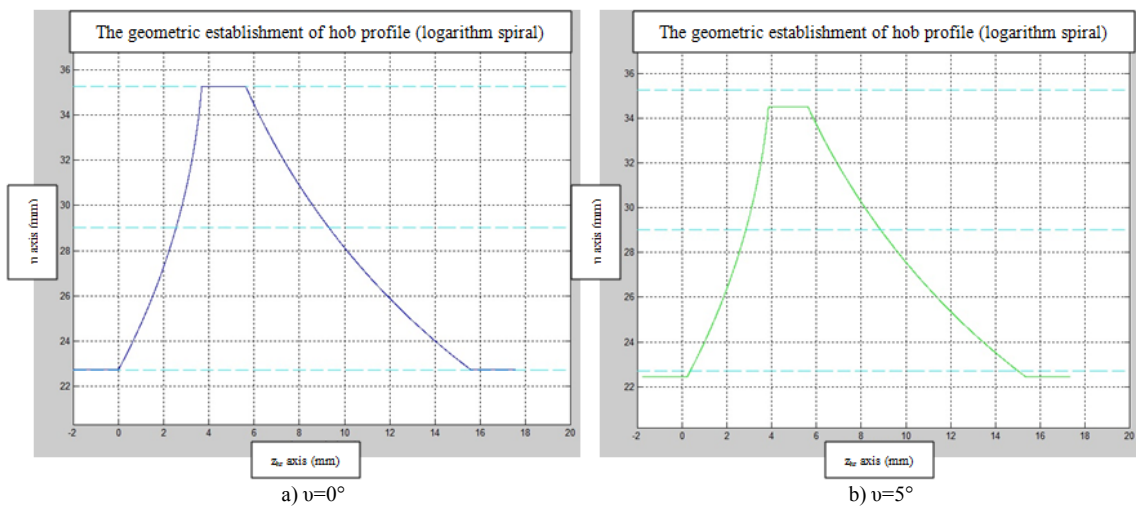


Figure 6. The geometric establishments of hob cutting edges

4. CONCLUSION

Based on the general mathematical model of Illés Dudás [3] we have worked out a model to determination of the tooth surface points of the face gear which are generated by the hob cutting edges in discreet resharpener angle position.

Based on the permissible profile tolerances of the face gear and the hob the resharpener critical angle of the hob could be determined in the hob axial section.

This is the critical angle the hob cutting edge of which manufactured face gear profile is situated in the permissible profile error tolerance. Consequently it is correct in case of operating. For example we have shown in our results for the hob cutting edge on Fig. 6.

ACKNOWLEDGEMENT

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Figure 7. The hob backward grinding could be seen on this figure (DifiCAD Engineering Office Ltd.)

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