

Original article

The effect of flow resistance on water saturation profile for transient two-phase flow in fractal porous media

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Abstract:

Due to the rapid development of Micro-Electro-Mechanical System (MEMS), more and more attention has been paid to the fluid properties of porous media, which is significant for petroleum engineering. However, most of surfaces of pores and capillaries in porous media are rough. On the approximation that porous medium consists of a bundle of tortuous and rough capillaries, a Buckley-Leverett conceptual model with considering flow resistance is developed based on the fractal geometry theory, which is beneficial to predict water saturation profile in porous medium. The proposed Buckley-Leverett solution is a function of fractal structural parameters (such as pore fractal dimension, tortuosity fractal dimension, maximum and minimum diameters of capillaries), fluid properties (such as viscosity, contact angle and interfacial tension) and pore structure parameter (relative roughness) in fractal porous medium. Besides, the relationship between water saturation and distance is presented according to Buckley-Leverett solution. The impact of flow resistance on water saturation profile is discussed.

1. Introduction

In petroleum industry, water displacement has been used as an effective enhanced oil recovery (EOR) process for a long time. Nonlinearity of two-phase flow makes solving multiphase flow equations very complex until Buckley and Leverett (Buckley, 1942) established the fundamental principle for displacement of two-phase immiscible fluids in homogeneous system without considering capillary pressure based on fractional flow. Then more parameters including capillary effects (Yortsos et al., 1983; Spanos et al., 1986; Chen, 1988; Chang et al., 1992) were added in Buckley-Leverett solution in petroleum reservoirs engineering (Welge, 1952; Snyder et al., 1967; Larsen, et al., 1990). And Buckley-Leverett model has been applied in heterogeneous porous media (Langtangen et al., 1992; Wu et al., 1993; Wu et al., 2010).

It has been shown that the fractal geometry theory (Mandelbrot et al., 1984) can be used as a tool to characterize roughness of surfaces (Majumdar et al., 1990; Warren et al., 1996) and sandstone pores (Katz et al., 1985; Krohn et al., 1986). Thus, reservoirs heterogeneity can be described by fractal

geometry very well. The pore spaces of sandstones are fractal geometries by using the scanning electron microscopy and optical data (Katz et al., 1985). The work of some researchers (Chang and Yortsos, 1990; Acuna and Yortsos, 1995; Xu, 2015; Xu et al., 2017) contains the basic theoretical formalism as it pertains to porous media and oil reservoirs. Tree-shaped fractal structures (Lorente et al., 2006; Xu and Sasmito, 2016) and spontaneous imbibition with variably shaped apertures (Cai et al., 2014) help to model fluid flowing in multi-scale configurations porous medium.

From above brief introduction, the importance of predicting water saturation profile and the wide applications of the fractal geometry theory in heterogeneous reservoir with various seepage problem including flow resistance can be clearly seen. So, it may be possible to develop an analytical model for water saturation profile with considering flow resistance in fractal porous media. This paper presents a Buckley-Leverett analytical solution of water and oil two-phase immiscible fluids with flow resistance through porous media based on fractal geometry theory in section 2 and section 3. In section 4,



a procedure is provided for calculation water saturation profile for immiscible displacement in the fractal porous media with flow resistance. Moreover, a curve is drawn to show the impact of flow resistance on water saturation profile.

2. Mathematical model

It has been shown that rough unit on rough surface is a fractal system with statistic self-similarity (Li et al., 2001; Poljacek et al., 2008). In this literature, we assume that rough unit on the rough surface is cone as shown in Fig. 1.

According Fig. 1, volume of single cone can be calculated as follows

$$V_i = \frac{\pi d_i^2 h_i}{12} = \frac{\pi d_i^3}{12} \beta \quad (1)$$

where β is the ratio of height to diameter of a cone.

In this article, cones are assumed as to be not overlapping and cones distribution follows statistic self-similarity fractal theory. So the distribution of cones basal diameter agrees with fractal scale law which can be expressed as follows (Yu et al., 2001).

$$N(l \geq d) = \left(\frac{d_{\max}}{d} \right)^{D_f} \quad (2)$$

where d_{\max} is the maximum diameter of cone, and D_f is pore fractal dimension. Generally, $0 < D_f < 2$ denotes two dimensional space, and $0 < D_f < 3$ refers to three dimensional space.

Eq. (2) can be approximately considered to be a continuous and differentiable equation because there are numerous pores in a porous medium. Then, taking a derivative with respect to diameter in Eq. (2) yields the number of pores within the infinitesimal rang from d to $d + dd$.

$$-dN = D_f d_{\max}^{D_f} d^{-(D_f+1)} dd \quad (3)$$

Combining Eqs. (1) and (3), the total volume for all cones in a fractal unit can be calculated by integrating from minimum to maximum diameter:

$$V_t = - \int_{d_{\min}}^{d_{\max}} V_i dN = \frac{\pi \beta}{12} \frac{D_f}{3 - D_f} d_{\max}^3 \left[1 - \left(\frac{d_{\min}}{d_{\max}} \right)^{3-D_f} \right] \quad (4)$$

The total bottom area for all cones in a fractal unit can be expressed as follows:

$$S_t = - \int_{d_{\min}}^{d_{\max}} \frac{\pi}{4} d_i^2 dN = \frac{\pi}{4} \frac{D_f}{2 - D_f} d_{\max}^2 \left[1 - \left(\frac{d_{\min}}{d_{\max}} \right)^{2-D_f} \right] \quad (5)$$

So the total area for a fractal unit is as follows:

$$S_0 = \frac{S_t}{\phi} = \frac{\pi}{4\phi} \frac{D_f}{2 - D_f} d_{\max}^2 \left[1 - \left(\frac{d_{\min}}{d_{\max}} \right)^{2-D_f} \right] \quad (6)$$

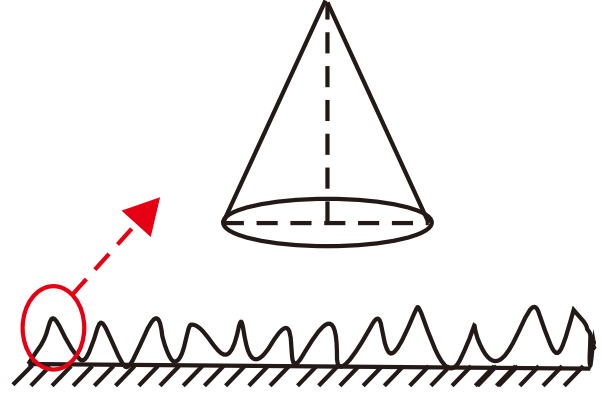


Fig. 1. Schematic diagram for tapered rough unit.

Effective height of roughness can be expressed as follows:

$$h_{eff} = \frac{V_t}{S_0} = \frac{\beta \phi d_{\max}}{3} \frac{2 - D_f}{3 - D_f} \frac{1 - (d_{\min}/d_{\max})^{3-D_f}}{1 - (d_{\min}/d_{\max})^{2-D_f}} \quad (7)$$

Here we assume that $\beta = 1$, so Eq. (7) can be rewritten as:

$$h_{eff} = \frac{\phi d_{\max}}{3} \frac{2 - D_f}{3 - D_f} \frac{1 - (d_{\min}/d_{\max})^{3-D_f}}{1 - (d_{\min}/d_{\max})^{2-D_f}} \quad (8)$$

This article emphatically apply Tans physical model to analyze water saturation profile which can enhance oil recovery. Whats more, roughness on the surface of porous media is also a big issue for two-phase flow. Based on Tans two-phase flow model in a fractal porous medium (Tan et al., 2014), we propose a hypothesis that porous media is comprised of a bundle of tortuous capillaries with rough surfaces. According to this assumption, a single fractal capillary with transient two-phase flow is proposed as shown in Fig. 2. The fractal capillary is only saturated with oil (red) initially, but later water (blue) intrudes into the capillary and displaces oil with a constant pressure difference, Δp , between points a and b. Therefore, the fractal capillary is separated by a two-phase flow interface at point c. The fractal capillary diameter, the straight distance of the capillary, and the straight distance between points a and c are λ , L , and X respectively.

$$v_w(\lambda) = \frac{\lambda^2 (p_a - p_w)}{32\mu_w X_T} \quad (9)$$

where p_a is pressure for inlet side, p_w is water pressure at interface, μ_w is water viscosity, X_T is the actual length of the capillary between points a and b.

Effective diameter of capillary would decrease if the capillary surface is rough (Wu et al., 2008; Yang, 2015), so we need to modify the effective diameter which is equal to $\lambda(1 - \varepsilon)$.

$$v_w(\lambda) = \frac{(\lambda - 2h_{eff})^2 (p_a - p_w)}{32\mu_w X_T} = \frac{\lambda^2 (1 - \varepsilon)^2 (p_a - p_w)}{32\mu_w X_T} \quad (10)$$

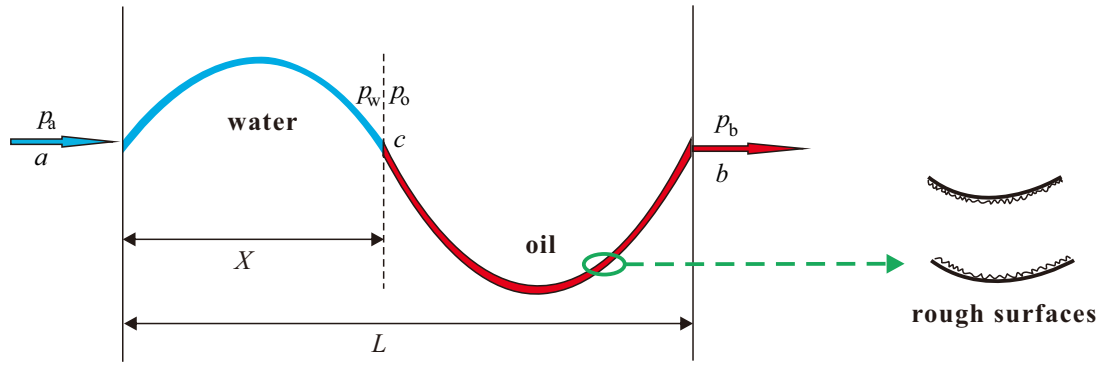


Fig. 2. Transient two-phase flow through a single fractal capillary with rough surfaces.

$$\varepsilon = \frac{2h_{eff}}{\lambda} = \frac{2\phi_s(h_{max})_{\lambda_{min}}}{3\lambda_{min}} \frac{2 - D_f}{3 - D_f} \frac{1 - (d_{min}/d_{max})^{3-D_f}}{1 - (d_{min}/d_{max})^{2-D_f}} \quad (11)$$

where ε is relative roughness with the definition of the ratio of roughness effective height to the capillary diameter, which can be understood as rough surfaces with some height can reject fluid to flow. Similarly, we can get the velocity of oil as follows:

$$v_o(\lambda) = \frac{\lambda^2(1 - \varepsilon)^2(p_o - p_b)}{32\mu_o(L_T - X_T)} \quad (12)$$

where p_o is oil pressure at interface, p_b is pressure for outlet side, μ_o is oil viscosity and L_T is the actual length of the capillary.

Capillary pressure can be expressed as:

$$p_c = \frac{4\sigma \cos \theta}{\lambda} \quad (13)$$

where σ is surface tension and θ is contact angle.

The pressure difference along a single capillary is expressed as follows:

$$\Delta p = p_a - p_b \quad (14)$$

Due to flow continuity, water velocity and oil velocity are the same.

$$v_w = v_o \quad (15)$$

Substituting Eqs. (10)-(14) into Eq. (15), we can get the analytical expression for two-phase flow velocity in a single capillary:

$$v = \frac{\lambda(1 - \varepsilon)^2(\lambda\Delta p + 4\sigma \cos \theta)}{32[(\mu_w - \mu_o)X_T + \mu_o L_T]} \quad (16)$$

The fractal scaling law for tortuous capillaries in porous medium is given by Yu et al., 2002.

$$L_T = L^{D_T} \lambda^{1-D_T} \quad (17)$$

and

$$X_T = X^{D_T} \lambda^{1-D_T} \quad (18)$$

where D_T is the tortuosity fractal dimension.

Substituting Eqs. (17) and (18) into Eq. (16), the expression for two-phase flow velocity can be obtained as:

$$v = \frac{(\lambda^{1+D_T} \Delta p + 4\lambda^{D_T} \sigma \cos \theta)(1 - \varepsilon)^2}{32[(\mu_w - \mu_o)X^{D_T} + \mu_o L^{D_T}]} \quad (19)$$

Also, effective seepage area will be equal to $\pi\lambda^2(1 - \varepsilon)^2/4$ and the transient two-phase flow rate, q_t , in a single rough capillary can be obtained as follows:

$$q_t = \frac{\lambda^{3+D_T} \pi \Delta p + 4\lambda^{2+D_T} \pi \sigma \cos \theta}{128[(\mu_w - \mu_o)X^{D_T} + \mu_o L^{D_T}]} (1 - \varepsilon)^4 \quad (20)$$

In Eq. (20), when $\sigma = 0$ and $X = L$, the two-phase flow rate can be regarded as single-phase flow rate, q_s , which is expressed as:

$$q_s = \frac{\lambda^{3+D_T} \pi \Delta p}{128\mu_w L^{D_T}} (1 - \varepsilon)^4 \quad (21)$$

It is known that:

$$v = \frac{dX_T}{dt} \quad (22)$$

Combining Eq. (19) and Eq. (22), we get the expression for the two-phase flow velocity in a single capillary.

$$\frac{dX_T}{dt} = \frac{\lambda^{1+D_T} \Delta p + 4\lambda^{D_T} \sigma \cos \theta}{32[(\mu_w - \mu_o)X^{D_T} + \mu_o L^{D_T}]} (1 - \varepsilon)^2 \quad (23)$$

From Eq. (18):

$$dX_T = d(X^{D_T} \lambda^{1-D_T}) = D_T X^{D_T-1} \lambda^{1-D_T} dX \quad (24)$$

Substituting Eq. (24) into Eq. (23), we can yield:

$$32D_T X^{D_T-1} \lambda^{1-D_T} [(\mu_w - \mu_o)X^{D_T} + \mu_o L^{D_T}] dX = (\lambda^{1+D_T} \Delta p + 4\lambda^{D_T} \sigma \cos \theta)(1 - \varepsilon)^2 dt \quad (25)$$

Taking an integration of Eq. (25) with initial condition $t = 0$ and $X = 0$, and after rearranging, we get the relationship between the two-phase interface position, X , and corresponding displace time, t .

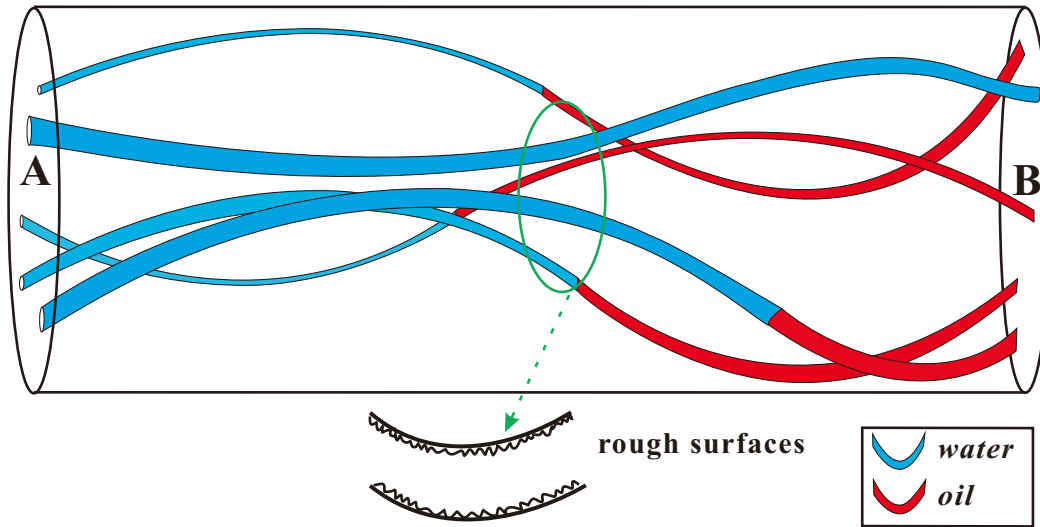


Fig. 3. Transient two-phase flow in a porous medium with tortuous and rough capillaries.

$$X^{2D_T} + \frac{2\mu_o L^{D_T}}{\mu_w - \mu_o} X^{D_T} - \frac{(\lambda^{2D_T} \Delta P + 4\lambda^{2D_T-1} \sigma \cos \theta) (1 - \varepsilon)^2 t}{16(\mu_w - \mu_o)} = 0 \quad (26)$$

From Eq. (26), we can see that when $X = 0$ and $t = 0$, the capillary is only saturated with oil. $0 < X < L$ represents the flow regime in capillary is transient two-phase flow. When $X = L$, oil is completely displaced by water. Substituting $X = L$ into Eq. (26), the expression for displace completion time, t_d , can be written as:

$$t_d = \frac{16(\mu_w + \mu_o)L^{2D_T}}{(\lambda^{2D_T} \Delta p + 4\lambda^{2D_T-1} \sigma \cos \theta) (1 - \varepsilon)^2} \quad (27)$$

Eq. (27) indicates that the completely displacing time is greatly affected by capillary diameter and relative roughness. Assuming that relative roughness value in every capillary is the same, a bigger capillary diameter corresponds to a smaller displace completion time, i.e. it will take less time for water to displace oil completely in a capillary with bigger diameter.

Here we define the critical capillary diameter, λ_{cr} , to be the capillary diameter (Xu et al., 2013). Where oil is just displaced by water completely at a given time, t . Substituting $t_d = t$ into Eq. (27), we obtain the following expression for the critical diameter as:

$$\lambda_{cr}^{2D_T} + \frac{4\sigma \cos \theta}{\Delta p} \lambda_{cr}^{2D_T-1} - \frac{16(\mu_w + \mu_o)L^{2D_T}}{\Delta p t (1 - \varepsilon)^2} = 0 \quad (28)$$

From Eq. (28), we can see that the critical capillary diameter shows a decrease with the increase of displacement time. If $\lambda \geq \lambda_{cr}$, oil is displaced by water completely and water flows out point b of the capillary. Otherwise, flow

regime remains two-phase flow and oil flows out point b of the capillary.

Besides, we can build a model for transient two-phase flow in a fractal porous medium as shown in Fig. 3, which is based on the approximation that porous medium consist of a bundle of tortuous fractal capillaries with variable diameters in porous medium. The distribution of fractal capillaries follows fractal scale law which can be expressed as Eq. (29). Similarly, the number of capillaries in the fractal porous media can be calculated by taking integration within the infinitesimal rang from λ to $\lambda + d\lambda$ as Eq. (30).

$$N(l \geq \lambda) = \left(\frac{\lambda_{\max}}{\lambda} \right)^{D_f} \quad (29)$$

$$-dN = D_f d_{\max}^{D_f} \lambda^{-(D_f+1)} d\lambda \quad (30)$$

As is shown in Fig. 3, water flows out of lateral section B in fractal capillaries whose diameters are larger than the critical capillary diameter and it is single-phase flow. On the contrary, if capillary diameters are smaller than the critical capillary diameter, oil flows out of lateral section B (or water hasn't arrived at lateral section B) and it is transient two-phase flow.

By summing up the flow rates through relevant flow regime capillaries at lateral section B, we can get the total flow rate of a certain phase, Q_w or Q_o . The total flow rate of water, Q_w , at lateral section B can be obtained by integrating Eq. (21) from the critical capillary diameter to maximum diameter.

$$Q_w = - \int_{\lambda_{cr}}^{\lambda_{\max}} q_s dN = \frac{\pi D_f \Delta p \lambda_{\max}^{D_f} (1 - \varepsilon)^4}{128 \mu_w L^{D_T} (3 + D_T - D_f)} \left(\lambda_{\max}^{3+D_T-D_f} - \lambda_{cr}^{3+D_T-D_f} \right) \quad (31)$$

Oil flows out of lateral section B in these capillaries whose diameters are smaller than the critical capillary diameter and flow pattern is still two-phase flow. The total flow rate of oil, Q_o , at lateral section B can be obtained by integrating Eq. (20) from minimum diameter to the critical capillary diameter.

$$\begin{aligned}
 Q_o &= - \int_{\lambda_{\min}}^{\lambda_{cr}} q_t dN \\
 &= \frac{\pi D_f \Delta p \lambda_{\max}^{D_f} (1 - \varepsilon)^4}{128} \int_{\lambda_{\min}}^{\lambda_{cr}} \frac{\lambda^{2+D_T-D_f}}{(\mu_w - \mu_o) X^{D_T} + \mu_o L^{D_T}} d\lambda \\
 &+ \frac{\pi D_f \lambda_{\max}^{D_f} \sigma \cos \theta (1 - \varepsilon)^4}{32} \int_{\lambda_{\min}}^{\lambda_{cr}} \frac{\lambda^{1+D_T-D_f}}{(\mu_w - \mu_o) X^{D_T} + \mu_o L^{D_T}} d\lambda
 \end{aligned} \tag{32}$$

Based on the fractal theory, the total pore volume in the fractal porous medium can be expressed as:

$$\begin{aligned}
 V_p &= - \int_{\lambda_{\min}}^{\lambda_{\max}} \frac{\pi \lambda^2}{4} (1 - \varepsilon)^2 L_T dN \\
 &= \frac{\pi D_f L^{D_T} \lambda_{\max}^{D_f} (1 - \varepsilon)^2}{4(3 - D_T - D_f)} \left(\lambda_{\min}^{3-D_T-D_f} - \lambda_{\max}^{3-D_T-D_f} \right)
 \end{aligned} \tag{33}$$

As is seen from Fig. 3, the pore volume saturated with water consists two portions: the whole capillary volume whose flow regime is single-phase flow and the blue part of capillary whose flow regime is transient two-phase flow.

$$\begin{aligned}
 V_w &= - \int_{\lambda_{\min}}^{\lambda_{cr}} \frac{\pi \lambda^2}{4} (1 - \varepsilon)^2 X_T dN - \int_{\lambda_{cr}}^{\lambda_{\max}} \frac{\pi \lambda^2}{4} (1 - \varepsilon)^2 L_T dN \\
 &= \frac{\pi D_f \lambda_{\max}^{D_f} (1 - \varepsilon)^2}{4} \int_{\lambda_{\min}}^{\lambda_{cr}} X^{D_T} \lambda^{2-D_T-D_f} d\lambda \\
 &+ \frac{\pi D_f L^{D_T} \lambda_{\max}^{D_f}}{4(3 - D_T - D_f)} (1 - \varepsilon)^2 \left(\lambda_{cr}^{3-D_T-D_f} - \lambda_{\max}^{3-D_T-D_f} \right)
 \end{aligned} \tag{34}$$

The pore volume saturated with oil is only the red part of capillary whose flow regime is transient two-phase flow.

$$\begin{aligned}
 V_o &= - \int_{\lambda_{\min}}^{\lambda_{cr}} \frac{\pi \lambda^2}{4} (1 - \varepsilon)^2 (L_T - X_T) dN \\
 &= \frac{\pi D_f L^{D_T} \lambda_{\max}^{D_f}}{4(3 - D_T - D_f)} (1 - \varepsilon)^2 \left(\lambda_{cr}^{3-D_T-D_f} - \lambda_{\min}^{3-D_T-D_f} \right) \\
 &- \frac{\pi D_f \lambda_{\max}^{D_f}}{4} (1 - \varepsilon)^2 \int_{\lambda_{\min}}^{\lambda_{cr}} X^{D_T} \lambda^{2-D_T-D_f} d\lambda
 \end{aligned} \tag{35}$$

Combining Eqs. (33)-(35), saturation of water, S_w , and saturation of oil, S_o , can be expressed respectively:

$$S_w = \frac{V_w}{V_p} = \frac{(3 - D_T - D_f) \int_{\lambda_{\min}}^{\lambda_{cr}} X^{D_T} \lambda^{2-D_T-D_f} d\lambda + L^{D_T} \left(\lambda_{\max}^{3-D_T-D_f} - \lambda_{cr}^{3-D_T-D_f} \right)}{L^{D_T} \left(\lambda_{\max}^{3-D_T-D_f} - \lambda_{\min}^{3-D_T-D_f} \right)} \tag{36}$$

$$S_o = \frac{V_o}{V_p} = \frac{L^{D_T} \left(\lambda_{cr}^{3-D_T-D_f} - \lambda_{\min}^{3-D_T-D_f} \right) - (3 - D_T - D_f) \int_{\lambda_{\min}}^{\lambda_{cr}} X^{D_T} \lambda^{2-D_T-D_f} d\lambda}{L^{D_T} \left(\lambda_{\max}^{3-D_T-D_f} - \lambda_{\min}^{3-D_T-D_f} \right)} \tag{37}$$

In Eqs. (36) and (37), when $X = L$ and $\lambda_{cr} = \lambda_{max}$, oil is completely displaced by water, so $S_w = 1$ and $S_o = 0$. While $X = 0$ and $\lambda_{cr} = \lambda_{min}$, the fractal porous medium is only saturated with oil, thus $S_w = 0$ and $S_o = 1$.

Fractional flow of water, f_w , can be defined as the proportion of water flow rate in the total flow rate. So, fractional flow of water can be expressed as:

$$f_w = \frac{Q_w}{Q_w + Q_o} \tag{38}$$

Fractional flow of water, f_w , can be defined as the proportion of water flow rate in the total flow rate. Substituting Eqs. (31) and (32) into Eq. (38), fractional flow of water can be obtained:

From Eqs. (31) and (32), we can see that both water and oil flow rate is proportional to $(1 - \varepsilon)^4$, so fractional flow of water has no relationship with relative roughness, ε .

3. Analytical solution for fractal porous media with flow resistance

The extended Buckley-Leverett solution in this paper is derived based on the following assuming conditions: 1) both porous media and fluids are incompressible. 2) gravity segregation effect is ignored which insures stable displacement exists near the front.

The equation of water saturation profile for a one-dimension flow and displacement with a constant cross-sectional area, A , is expressed as follows:

$$f_w(S_w) = \frac{\frac{\Delta p}{\mu_w L^{D_T} (3+D_T-D_f)} \left(\lambda_{\max}^{3+D_T-D_f} - \lambda_{cr}^{3+D_T-D_f} \right)}{\frac{\Delta p}{\mu_w L^{D_T} (3+D_T-D_f)} \left(\lambda_{\max}^{3+D_T-D_f} - \lambda_{cr}^{3+D_T-D_f} \right) + \Delta p \int_{\lambda_{\min}}^{\lambda_{cr}} \frac{\lambda^{2+D_T-D_f}}{(\mu_w - \mu_o) X^{D_T} + \mu_o L^{D_T}} d\lambda + 4\sigma \cos \theta \int_{\lambda_{\min}}^{\lambda_{cr}} \frac{\lambda^{1+D_T-D_f}}{(\mu_w - \mu_o) X^{D_T} + \mu_o L^{D_T}} d\lambda} \tag{39}$$

$$\frac{dx}{dt} = \frac{q(t)}{\phi A} \frac{df_w}{dS_w} \tag{40}$$

Taking an integration of Eq. (40) and arranging:

$$x - x_0 = \frac{1}{\phi A} \frac{df_w}{dS_w} \int_0^t q(t) dt \tag{41}$$

where x_0 is the beginning location of transient two-phase flow, and x the location of equal saturation plane, A is cross-sectional area.

Based on the fractal theory, cross-sectional area of the fractal porous medium can be expressed with modifying diameter:

$$A = - \int_{\lambda_{min}}^{\lambda_{max}} \frac{\pi \lambda^2 (1 - \varepsilon)^2}{4} dN \tag{42}$$

$$= \frac{\pi D_f (1 - \varepsilon)^2}{4 (2 - D_f)} \lambda_{max}^2 \left[1 - \left(\frac{\lambda_{min}}{\lambda_{max}} \right)^{2 - D_f} \right]$$

Porosity can be expressed as follows (Yu et al., 2001):

$$\phi = \left(\frac{\lambda_{min}}{\lambda_{max}} \right)^{2 - D_f} \tag{43}$$

Substituting Eqs. (42) and (43) into Eq. (41), we get the expression for location of waterflood front, x_f :

$$\frac{df_w}{dS_w} \frac{4 (2 - D_f) \int_0^t q(t) dt}{\pi D_f (1 - \varepsilon)^2 \lambda_{max}^2 \left(\frac{\lambda_{min}}{\lambda_{max}} \right)^{2 - D_f} \left[1 - \left(\frac{\lambda_{min}}{\lambda_{max}} \right)^{2 - D_f} \right]} = x_f - x_0 \tag{44}$$

4. Results and discussions

Fig. 4 shows a typical plot of fractional flow based on the calculation of Eq. (39). Fig. 4 shows an increase in fractional flow of water, $f_w(S_w)$, with the increase of the saturation of water, S_w .

The determination of front water saturation is shown graphically in Fig. 4. By Eq. (44), water saturation vs. distance is plotted as follows.

Clearly, the plot shows that there are two values of water saturation at each x-position caused by multivalued solution of df_w/dS_w vs. S_w , which is an impossible physical situation. As shown in Fig. 5, Buckley-Leverett solution is used to modify the plot by getting the value of water saturation of front, S_{wf} . Fig. 6 reflects how to get the final water saturation profile. Furthermore, comparison water saturation profiles between the present model and standard Buckley-Leverett equation is made in Fig. 6. The result shows that our models water saturation is smaller at the same distance as flow resistance is considered in the new model.

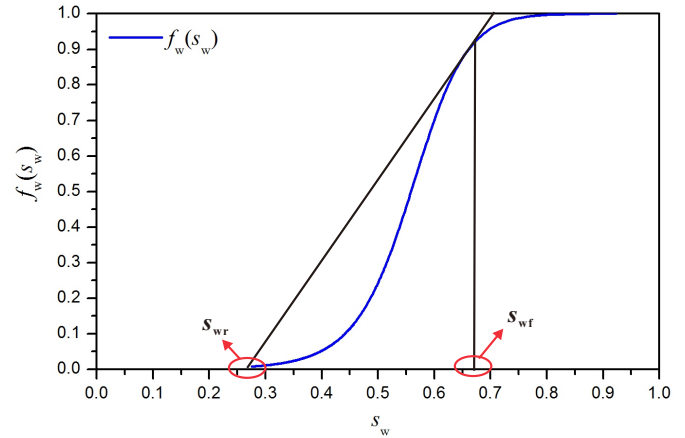


Fig. 4. Fractional flow curve.

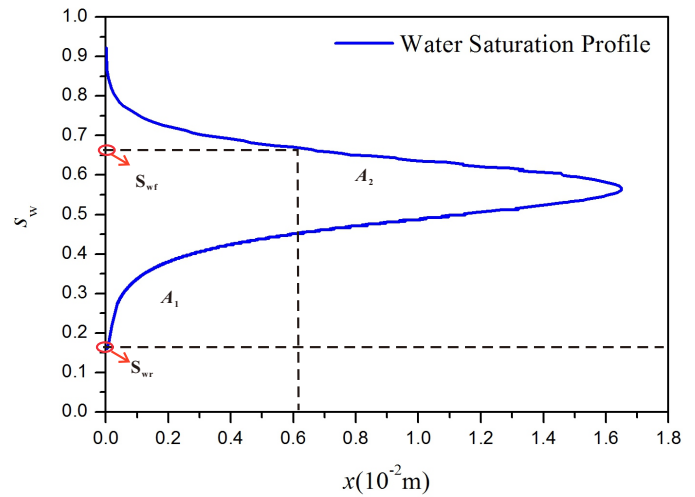


Fig. 5. Balancing of areas in water saturation profile.

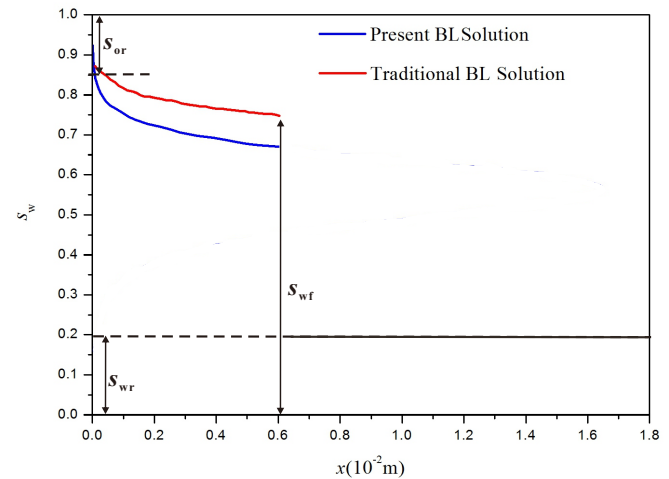


Fig. 6. Final water saturation profile and comparison between present and traditional BL solution.

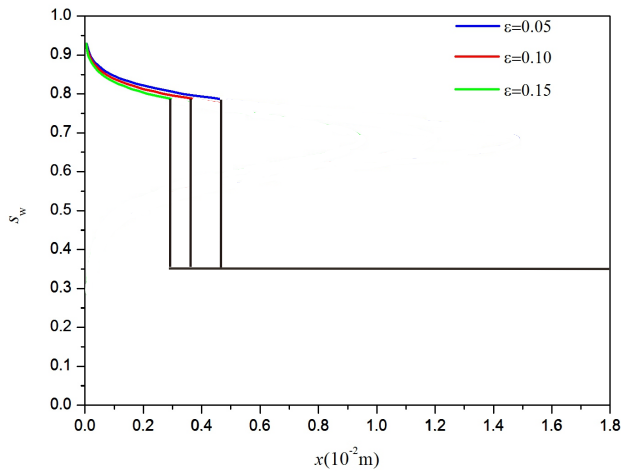


Fig. 7. Final water saturation profile affected by ε ($\mu_1 = 1 \times 10^{-3} \text{ Pa} \cdot \text{s}$, $\mu_2 = 0.2 \times 10^{-3} \text{ Pa} \cdot \text{s}$, $\sigma = 0.06 \text{ N/m}$, $\theta = 0.5$, $L = 1 \times 10^{-2} \text{ m}$, $\Delta p = 5 \times 10^5 \text{ Pa}$, $D_f = 1.60$, $D_T = 1.14$, $\lambda_{\max} = 1.5 \times 10^{-4} \text{ m}$, $\lambda_{\min} = 0.2 \times 10^{-7} \text{ m}$).

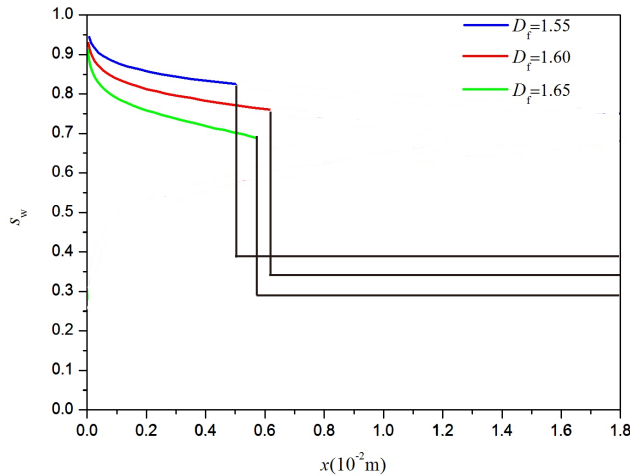


Fig. 8. Final water saturation profile affected by D_f ($\mu_1 = 1 \times 10^{-3} \text{ Pa} \cdot \text{s}$, $\mu_2 = 0.2 \times 10^{-3} \text{ Pa} \cdot \text{s}$, $\sigma = 0.06 \text{ N/m}$, $\theta = 0.5$, $L = 1 \times 10^{-2} \text{ m}$, $\Delta p = 5 \times 10^5 \text{ Pa}$, $\varepsilon = 0.1$, $D_T = 1.14$, $\lambda_{\max} = 1.5 \times 10^{-4} \text{ m}$, $\lambda_{\min} = 0.2 \times 10^{-7} \text{ m}$).

From Eq. (38), water saturation profile is affected by fractal structural parameters (such as pore fractal dimension, tortuosity fractal dimension, maximum and minimum diameters of capillaries) and fluid properties (such as viscosity, contact angle and interfacial tension) in fractal porous medium. We emphatically analysis the influence of relative roughness, pore fractal dimension, tortuosity fractal dimension, maximum and minimum diameters of capillaries on water saturation profile.

Combining Eqs. (36) and (39), relative roughness has no effect on the value of S_w and $f_w(S_w)$, so the plots in Fig. 7 have the same front water saturation ($S_{wf} = 0.78$) and irreducible water saturation ($S_{wr} = 0.35$). However, the water saturation curves become lower and two-phase flow distance is shorter with the increase of relative roughness. Thus, water displaces oil more easily when relative roughness is smaller. Its mainly because water saturation profile depends on the water

rate of injection and roughness is resistance for liquid to flow.

Fig. 8 shows how water saturation profile is affected by pore fractal dimensions, D_f . D_f represents the sectional distribution of pores in porous medium. As D_f increases, reservoir becomes more heterogeneous and the distributions of water and oil in fractal porous medium are more complex. It can be seen from Fig. 8 that the larger value of D_f is, the smaller values of front water saturation ($S_{wf} = 0.83$ ($D_f = 1.55$); 0.76 ($D_f = 1.60$); 0.70 ($D_f = 1.65$)) and irreducible water saturation ($S_{wr} = 0.40$ ($D_f = 1.55$); 0.35 ($D_f = 1.60$); 0.30 ($D_f = 1.65$)) are. A bigger value of front water saturation implies that water displaces oil more easily. And its also easy to understand that the value of water saturation is bigger at the same position implies that water can displace oil more easily. Overall, its harder for water to displace oil with a bigger value of D_f .

5. Conclusions

An extended Buckley-Leverett solution with considering roughness in fractal porous medium is established in this work. We emphatically exhibit the process to predict final water saturation profile by Buckley-Leveret solution, followed by analyzing flow resistances impact on water saturation profile. Finally, some conclusions can be summarized as follows:

(1) The proposed model connects the water saturation profile of two-phase flow with the fractal structural parameters (e.g. relative roughness, tortuosity fractal dimension, pore fractal dimension, and maximum and minimum capillary diameters) and fluid properties (e.g. interfacial tension, contact angle, viscosities and so on).

(2) Though relative roughness has no influence on the value of S_w and $f_w(S_w)$, it acts as resistance for displacement. The value of the flow distance at the same water saturation becomes smaller with the increase of relative roughness, which means its harder for water to displace oil.

(3) With the decrease of D_f , the homogeneity of porous media is better and the value of water saturation is bigger at the same two-phase position. It will take a shorter time for water to completely displace oil.

Nomenclature

- V_i = volume of single cone, m^3
- V_t = total volume for all cones in a fractal unit, m^3
- S_t = total bottom area for all cones in a fractal unit, m^2
- S_o = total area for a fractal unit, m^2
- d_i = diameter of single cone, m
- d_{max} = maximum diameter of single cone, m
- d_{min} = minimum diameter of single cone, m
- h_i = height of single cone, m
- h_{eff} = effective height of roughness, m
- β = the ratio of height to diameter of a cone, decimal
- ε = relative roughness, decimal
- X_T = actual length of the capillary, m
- L_T = actual length of the capillary, m
- Δp = pressure difference, Pa
- p_c = capillary pressure, Pa

p_a = pressure for inlet side, Pa
 p_b = pressure for outlet side, Pa
 p_w = water pressure at interface, Pa
 p_o = oil pressure at interface, Pa
 μ_w = water viscosity, Pa · s
 μ_o = oil viscosity, Pa · s
 σ = surface tension, N/m
 θ = contact angle, decimal
 v = velocity of two-phase flow, m/s
 v_w = water velocity, m/s
 v_o = oil velocity, m/s
 q_t = two-phase flow rate, m³/s
 q_s = single-phase flow rate, m³/s
 t_d = completely displacing time, s
 D_f = pore fractal dimension, decimal
 D_T = fractal dimension, decimal
 λ = diameter of capillary, m
 λ_{cr} = critical capillary diameter, m
 λ_{max} = maximum diameter of capillary, m
 λ_{min} = minimum diameter of capillary, m
 Q = total flow rate, m³/s
 Q_w = total flow rate of water, m³/s
 Q_o = total flow rate of oil, m³/s
 S_w = water saturation, decimal
 S_o = oil saturation, decimal
 S_{wr} = irreducible saturation of water, decimal
 V_P = total pore volume, m³
 V_w = pore volume of the fractal porous medium saturated with water, m³
 V_o = pore volume of the fractal porous medium saturated with oil, m³
 f_w = fractional flow of water, decimal
 A = cross-sectional area, m²
 x = location of equivalent saturation plane, m
 x_0 = beginning location of transient two-phase flow, m
 x_f = location of waterflood front, m
 ϕ = porosity of porous media, decimal

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References

Acuna, J.A., Yortsos, Y.C. Application of fractal geometry to the study of networks of fractures and their pressure transient. *Water Resour. Res.* 1995, 31(3): 527-540.

Bird, R.B. Transport phenomena. *Appl. Mech. Rev.* 2002, 55(1): R1-R4.

Buckley, S.E., Leverett, M.C. Mechanism of fluid displacement in sands. *SPE J.* 1942, 146(1): 107116.

Cai, J.C., Perfect, E., Cheng, C.L., et al. Generalized modeling of spontaneous imbibition based on Hagen-Poiseuille flow in tortuous capillaries with variably shaped apertures. *Langmuir* 2014, 30(18): 51425151.

Chang, J., Yortsos, Y.C. Effect of capillary heterogeneity in Buckley-Leverett displacement. *SPE Reserv. Eng.* 1992, 7(2): 3-5.

Chen, Z.X. Some invariant solutions to two-phase fluid displacement problems including capillary effect. *SPE Reserv. Eng.* 1988, 3(2): 691-700.

Chang, J., Yortsos, Y. C. Pressure transient analysis of fractal reservoirs. *SPE Reserv. Eng.* 1990, 5(1): 31.

Guo, L.X., Jiao, L.C., Wu, Z.S. Electromagnetic scattering from two-dimensional rough surface using the Kirchhoff approximation. *Chinese Phys. Lett.* 2001, 18(2): 214.

Katz, A.J., Thompson, A.H. Fractal sandstone pores: implications for conductivity and pore formation. *Phys. Rev. Lett.* 1985, 54(12): 1325-1328.

Krohn, C.E., Thompson, A.H. Fractal sandstone pores: automated measurements using scanning-electron-microscope images. *Phys. Rev. B* 1986, 33 (9): 63666374.

Langtangen, H.P., Tveito, A. Instability of Buckley-Leverett flow in a heterogeneous medium. *Transp. Poros. Med.* 1992, 9(3): 165-185.

Larsen, L., Kviljo, K., Litlehamar, T. Estimating skin decline and relative permeabilities from clean up effects in well-test data with Buckley-Leverett method. *SPE Form. Eval.* 1990, 5(4): 5-7.

Lorente, S., Bejan, A. Heterogeneous porous medium as multiscale structures for maximum flow access. *J. appl. phys.* 2006, 100(11): 114909-114916.

Mandelbrot, B.B., Passoja, D.E., Paullay, A.J., Fractal character of fracture surfaces of metals. *Nature* 1984, 308(5961): 721722.

Majumdar, A., Bhushan, B., Role of fractal geometry in roughness characterization and contact mechanics of surfaces. *J. Tribol.* 1990, 112(2): 205216.

Poljacek, S.M., Risovic, D., Furic, K., et al. Comparison of fractal and profilometric methods for surface topography characterization. *Appl. Surf. Sci.* 2008, 254(11): 3449-3458.

Snyder, R.W., Ramey, Jr.H.J. Application of Buckley-Leverett displacement theory to noncommunicating layered systems. *J. Petrol. Tech.* 1967, 19(11): 3-5.

Spanos, T.J.T., De, L.C.V., Hube, J. An analysis of Buckley-Leverett theory. *J. Can. Petr. Technol.* 1986, 25(1): 71-75.

Tan, X.H., Li, X.P., Liu, J.Y. et al. Analysis of permeability for transient two-phase flow in fractal porous medium. *J. of Appl. Phys.* 2014, 115(11): 113502.

Welge, H.G. A simplified method for computing oil recovery by gas or water drive. *J. Pet. Technol.* 1952, 4(4): 9198.

Wu, J.S., Yu, B.M. A fractal resistance model for flow through porous media. *Transp. Poros. Med.* 2007, 71(3): 331-343.

- Warren, T.L., Krajcinovic, D. Random Cantor set models for the elastic-perfectly plastic contact of roughness surfaces. *Wear* 1996, 196(1-2): 1-15.
- Wu, Y.S., Pruess, K., Chen, Z.X. Buckley-Leverett flow in composite porous media. *SPE Adv. Technol. Ser.* 1993, 1(2): 36-42.
- Wu, Y.S., Fakcharoenphol, P., Zhang, R.L. et al. Non-Darcy displacement in linear composite and radial flow porous media. Paper SPE130343 presented at the SPE EUROPEC/EAGE Annual conference and exhibition held in Barcelona, Spain, 14-17 June, 2010.
- Xu, P., Qiu, S., Yu, B.M. et al. Prediction of relative permeability in unsaturated porous medium with a fractal approach. *Int. J. Heat Mass Transf.* 2013, 64: 829837.
- Xu, P. A discussion on fractal models for transport of porous media. *Fractals* 2015, 23(3): 1530001.
- Xu, P., Sasmito, A.P., Yu, B.M., et al. Transport phenomena and properties in treelike networks. *Appl. Mech. Rev.* 2016, 68(4): 040802.
- Xu, P., Qiu, S., Cai, J., et al. A novel analytical solution for gas diffusion in multi-scale fuel cell porous media. *J. Power Sources.* 2017, 362: 73-79.
- Yang, S.S., Yu, B.M. A fractal analysis of flow properties in roughened microchannels. *Huazhong University of Science and Technology.* 2015: 42-44.
- Yortsos, Y.C., Fokas, A.S. An analytical solution for linear waterflood including the effects of capillary pressure. *SPE J.* 1983, 23(1): 115-124.
- Yu, B.M., Chen, P. A fractal permeability model for bi-dispersed porous media. *Int. J. Heat Mass Tran.* 2002, 45(14): 2983-2993.
- Yu, B.M., Li, J. H. Some fractal characters of porous media. *Fractals* 2001, 9(3): 365-372.