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Teacher as Researcher: A Two-Tiered Model

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TEACHER AS RESEARCHER:
A TWO-TIERED MODEL

A Thesis Presented
by
BARBARA B. NELSON

Submitted to the Office of Graduate Studies and Research of the
University of Massachusetts at Boston in partial
fulfillment of the requirements for the degree of

MASTER OF ARTS
December 1992
Critical and Creative Thinking Program

TEACHER AS RESEARCHER:


A TWO-TIERED MODEL


A Thesis Presented


by

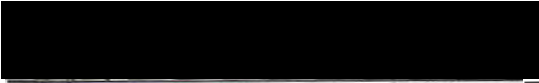
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ABSTRACT

TEACHER AS RESEARCHER: A TWO-TIERED MODEL

DECEMBER, 1992

BARBARA B. NELSON, B.S., NASSON COLLEGE

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This thesis focuses on an investigation I undertook to enhance my effectiveness to teach mathematics, a subject to which I was assigned, but for which I had not been formally trained. It describes my attempt to construct knowledge through the clinical interview method as to how middle school students construct knowledge about integers and think about the knowledge they are constructing. On one level, I was attempting to learn how students come to understand the concept of integers; on a second level, I was creating an understanding of how a teacher can construct knowledge about the construction of knowledge. This two-tiered model cast me in the roles of teacher, learner, and researcher; and my students in the roles of learner and teacher.

Six sixth-grade students, interviewed in groups of two each, for four or five sessions, used a model where yellow chips represent positive integers and blue chips represent negative integers. The investigation was concerned with how children construct knowledge about adding and subtracting integers, what they grasp easily or find difficult, what prior knowledge or misconceptions they bring, what connections they make to real-world applications, how they think about their thinking, how they create problems to solve, and how well they teach fourth graders.

The study allowed me to concentrate on aspects of teaching mathematics emphasized by the Standards (1989) of the National Council of Teachers of Mathematics: using manipulative models, problem solving, communicating, connecting, and reasoning. Three non-traditional techniques were used for evaluating children's understanding: reverse processing, metacognition, and the child as teacher.

As background, this thesis reviews relevant literature on Constructivism, meaning of knowledge, critical and creative thinking, the teacher's role, clinical interviewing, and representational models. Analyses of videotaped teacher/student scripts and other components of the interviewing process provided glimpses into the minds of children who learn in different ways (including interesting misconceptions held). Implications of this two-tiered model reinforce my belief that knowledge is not something passive to be given, but active to be created and re-created by both teacher and student on a day-to-day basis in the classroom.

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CHAPTER I
RATIONALE FOR A TWO-TIERED MODEL
OF THE TEACHER AS RESEARCHER

Introduction

The impetus for this thesis began when September brought with it a new teaching assignment at the middle school level (grades seven and eight) and my entry into the world of teaching mathematics. (Although I was an experienced teacher, I had not been formally trained in the teaching of mathematics.) Being a long-time student of human nature--a "people watcher"--I was fascinated by what I observed in my middle school mathematics classroom. Students had greater difficulty with addition facts than with multiplication facts. Students said they had studied a particular topic (such as fractions) but could not accurately complete simple computations or applications. The students displayed little tenacity and exhibited expectations of failure.

The students' repertoire seemed to consist primarily of four sentences: "I don't get this." "I hate math." "This is so boring!" "When are we going to learn something new?" Of course, from time to time, there was a fifth sentence: "My last year's teacher said to do it this way." (Gives a whole new meaning to "taking the Fifth," doesn't it? Students don't like to admit that they are wrong--and, after all, isn't the best defense a good offense?)

As I observed my students, I wondered what had happened to the child so full of curiosity and eagerness, who faces the world without any thought of success or failure but with only an overwhelming desire to understand. I had taught at the high school level and saw only limited numbers of students exhibiting the joy of learning. I had hoped to see joyful learning at the middle school, but such was not the case. As a teacher and parent, I had experienced first hand the exuberance and delight with which infants, toddlers, pre-schoolers, kindergarteners, and early elementary-aged children approach learning. Why and when does that curiosity turn into "CAN'T" and that anticipation

change into apathy? What happens in school to block that love of learning? Why and when does a child's self-concept become so damaged that mathematics becomes "undoable?"

I observed students in the middle-school mathematics classroom struggling with textbooks filled with what I call "Naked Numbers." Those "Naked Numbers," like $2 + 2$, $2x = 14$, $3y + x$, $5 - 3$, or $96/4$, devoid of any words of identification, provide no frame of reference. For example, $2 + 2$; the answer is always four, right? Not necessarily; it depends on the frame of reference. Two apples plus two oranges equals neither four apples nor four oranges; it equals four pieces of fruit. Two feet plus two yards does not equal four anything; the answer can be eight feet, 2 and $2/3$ yards or 96 inches, depending on the unit of measure preferred. Or even in the physical world, two drops of water plus two drops of water can be just one drop of water. The lack of any context surrounding the "Naked Numbers" prevents the student from placing those numbers in a real-world experience, interaction, or relationship. How can a student use knowledge previously constructed if there is no clue as to what knowledge base is to be accessed?

I heard students say "I don't understand how to do this" when faced with "Naked Numbers." Could working with "Naked Numbers" be a reason why students could have studied a topic yet not have remembered how to do the math when meeting it again at a later time? Was this a factor in why these students left their classrooms with what Skemp (1987) calls "instrumental understanding" (153)? The students did what the teacher said to do--the rules were followed--but no understanding of the WHY of the situation occurred?

The classroom that I inherited contained no alternatives to the paper-and-pencil manipulation of "Naked Numbers." Where were the teaching materials--the concrete objects--with which students learn? Was this one of the reasons for the "This is so boring" response?

Rationale for this Study

My approach to teaching is described by the saying "anything worth doing is worth doing well." I've always been willing to take that extra time or go that extra mile in the pursuit of excellence. My love of learning, my curiosity, and my desire to excel would not allow me to just "be" in that classroom; I needed to "be effective" in that classroom.

I've always believed that people learn in different ways and that new knowledge must "build" on previous knowledge. When introducing a new topic, I've always looked for that "hook," that piece of common knowledge that serves as the foundation for building new knowledge or as a key to open the doors to the previous knowledge locked in the mind, which allows me to connect with my students and establish real communication. I've also believed that I must go back as far as necessary to find out where the students are in their "building" process and then to start from that point. I've also always believed it was not only necessary but imperative that teaching begin with very concrete experiences and materials. "Doing" is an important part of my classroom approach; just hearing and seeing are not enough. My favorite questions are "Why?" and "What if?"

Despite my lack of formal training in teaching mathematics, I felt that my teaching philosophy and style were well suited to the teaching of middle school mathematics. Furthermore, I knew that "you can teach an old dog new tricks," particularly if the "dog" wants to learn; and I did want to learn.

I had now metamorphosed from teacher to teacher/learner. Since I was the teacher and the learner, I had to find my own "hook." What was my starting point going to be to ensure effectiveness in my mathematics classroom? Or, as David Cohen, a researcher at Michigan State University asks, "How can teachers teach a mathematics that they never learned, in ways they never experienced" (1990, 233)?

Coincidentally, while I was looking for my own "hook," I had the opportunity to enroll in Dr. Patricia Davidson's class, "Thinking Skills in Mathematics." An appreciation of a favorite old Chinese proverb "I hear and I forget; I see and I remember; I do and I understand" was apparent. While studying various math topics and models for teaching math, I was introduced to Labinowicz's (1985) book on children's numerical thinking. In that book were two statements very powerful for me: "Relationships are inventions of the mind" (23), and "*we see what we understand*" (italics in original) rather than understand what we see" (7). Davis (1984) asked the question: "Suppose you had a student who had never studied positive and negative numbers . . . what would you do?" (5). His question intrigued me because, as far as I knew, my students had never heard of integers. Here was the "something new" for my class.

Then, during Dr. John Murray's "Evaluation Practicum," an opportunity presented itself to investigate a form of qualitative or quantitative research. Because I had been reading Piaget (1971), Labinowicz (1985), and Ginsburg et al. (1983), I chose the qualitative clinical interview method. Piaget had learned so much from listening to children, why not use his model? My metamorphosis was complete. I was now Teacher and Researcher with an opportunity to construct knowledge about constructing knowledge. I had found my "hook"!

Shaping My Focus

I had indeed found my "hook"! To learn how to teach my class more effectively, I would embark on an investigation with children. The purpose of this two-tiered investigation would be for me to construct knowledge about how children construct knowledge about integers. I chose the topic of integers because I wanted to increase my knowledge of integers and my knowledge of how children learn about integers. Having been introduced to several models for teaching integers in Dr. Patricia Davidson's class, I developed a special affinity for the Colored Chip Model (see Appendices A and B),

which I decided to incorporate into my research. The Colored Chip Model, presented by Dr. Davidson (1989), uses yellow chips to represent positive integers (based on the psychological connections of feeling positive when the weather is bright and sunny) and blue chips to represent negative numbers (based on the emotional reaction of feeling blue when feeling negative). Bennett and Musser (1976) and Greenes (1990) also use chips to demonstrate the addition and subtraction of integers. Influenced by a model first developed by the Chinese, Bennett and Musser (1976) and Greenes (1990) employ black chips to represent positive numbers and red chips to represent negative numbers. No name is attached to their models.

My investigation with the Colored Chip Model would be concerned with the following: how children construct knowledge about adding and subtracting integers; how children construct knowledge about those integer operations using the Colored Chip Model; the intricacies of the Colored Chip Model for adding and subtracting; what children pick up easily; what they have difficulty with; what prior knowledge exists; what connections they make; how they think about their thinking; how they create problems to solve; and how they would teach someone else.

This study would also allow me to concentrate on aspects of teaching mathematics emphasized by the National Council of Teachers of Mathematics (NCTM) Standards (1989): the use of manipulative materials, problem solving, communication, connections, and reasoning.

Besides directing and facilitating student learning, the teacher has the additional task of evaluating the solidity and the depth of a student's knowledge. How does the teacher know when a student's construction of knowledge has resulted in what Skemp (1987) calls "relational understanding: knowing what to do and why" (153)? Of course, there is the paper-and-pencil test; but it provides no opportunity to question the student. All teachers have experienced the student who gets a good grade on a test or the student who can "parrot back the words," but who has no understanding of the "why."

Of course, the teacher can always ask the student, "Do you understand?" or "Do you have any questions?" The usual answer to the former is "yes" and to the latter "no." Unfortunately in school, students may respond with what they think the teacher wants to hear or with what will take them out of the spotlight. Students are no less anxious than adults to call attention to their deficits and cause themselves embarrassment.

Goals of the Thesis

Since this was a personal quest for the purpose of enhancing my effectiveness in the classroom, the questions I would investigate would be: How do students construct knowledge about integers using the Colored Chip Model? How do I assess the strength of the knowledge constructed about integers using the Colored Chip Model? and What are the implications for my teaching?

The focus of this thesis is my two-tiered research model: an attempt to construct knowledge through the clinical interview process about how students construct knowledge about integers through the Colored Chip Model. Emphasis is also on what I learned and its transfer to my classroom so that my teaching effectiveness would be enhanced.

As a student in the Critical and Creative Thinking Program, I was interested in the role of metacognition in this two-tiered construction process. Although the generic definition of metacognition tends to be "thinking about thinking" (Costa 1984, 57), I chose to use Swartz and Perkins' (1989) definition that to metacogitate is "to . . . monitor and direct . . . thinking by a deliberate, conscious effort" (178). This monitoring and/or directing of one's thinking can be in general or can be for a specific purpose such as problem solving or decision making. In addition to metacognition, the critical thinking skills of inferences, generalizations, and predictions were utilized with the students during the clinical interview sessions. Students' creative thinking abilities were exercised during problem solving activities, story telling, and the "child as teacher" sequences.

Overview of Chapters

The remainder of this chapter will discuss the role of Teacher as Researcher, introduce the participants involved in the clinical interviews, and describe the three ways I pursued the viewing of understanding.

In Chapter II, I will review literature on the topics of Constructivism, knowledge, critical and creative thinking, the teacher's role, clinical interviewing, manipulative models, and representational models.

Chapter III will discuss reverse processing, metacognition, and the child as teacher as ways of viewing children's understanding. As mentioned before, one of the important questions that my investigation sought to answer was "How do I assess the strength of the knowledge constructed about integers using the Colored Chip Model?" Three different techniques were used to evaluate the children's understandings about integers: reverse processing, metacognition, and "the child as teacher." Reverse processing was a term I encountered in Dr. Patricia Davidson's "Thinking Skills in Mathematics" class. She defines reverse processing as the ability to start with an answer and to work backwards to reconstruct questions that would result in that answer (1989). The second technique examines the child's ability to take charge of the problem solving and learning taking place. As for the third technique, teaching someone else almost always highlights the strengths and deficits in one's knowledge.

Chapter IV will focus on the clinical interviews I conducted in my role of Teacher as Researcher. The script and other components of the interviewing process will be reviewed.

The implications of the two-tiered model on the teacher will be the focus of Chapter V. Discussion will center on implications for me in my role of Teacher as Researcher, in answer to the question as to how teachers can "teach a mathematics that they never learned, in ways they never experienced" (Cohen 1990, 233), and in my role of classroom teacher.

The primary concern of Chapter VI will be the implication of the two-tiered model for the student and a specific "significant error" that deserves attention. In Appendices A and B the Colored Chip Model will be discussed in detail, and the operations of addition and subtraction with the model will be presented.

Why did I choose the Colored Chip Model? Simply stated, it was the model I liked best. My own "aha!" experience upon seeing the model for the first time convinced me that this was a useful and powerful tool. The colors are bright and have an inherent logic. The round shape of the chips makes them very easy to handle, the "balance" of the chips is good--neither too heavy nor too light. They make a pleasant "clinking" sound as they are placed on the table.

The more experience I have with the model, the more convinced I am of its effectiveness. It works equally well with children (as young as third grade), with teenagers, and with adults. (The adults often remark that this model helps them understand integers for the first time and causes them to ask where this model was when they were in school.)

This model also exemplifies what I believe a manipulative model should be. An effective model should **ARM** you against the unknown. The model should be **A**esthetically pleasing, **R**epresentationally rich (providing for extensions of the "basic idea" being conveyed), and **M**athematically sound. The more concrete a model is, the more likely a student is to construct good solid knowledge about the topic at hand. Students should not only be exposed to the most concrete model possible, but they should also be able to work with the model for as long as they find it necessary. They themselves will know when it is time to stop using the model.

The Role of Teacher as Researcher

"Teacher as Researcher" is a role I have only recently come to appreciate. A symposium held at the Kennedy Library provided my first introduction to the formal

linking of these two ideas at the elementary, middle, and secondary levels. I was familiar, of course, with the idea of "Teacher as Researcher" at the college level--that was a given--and Dr. Patricia Davidson's research was known to me through the Critical and Creative Thinking Program. Research at my level, however, was a new idea.

The phenomenon of raised consciousness was at work. (You're introduced to "something new"--something you were unaware of before--and suddenly you're finding this "something new" repeatedly!) I next found an article by Eleanor Duckworth (1986) entitled "Teaching As Research." Duckworth says that "through teaching, one is in a position to pursue questions about the development of understanding that one could not pursue in any other way" (490). These experiences allowed me to look at "Teacher as Researcher" in a new and more meaningful way. It also enabled me to see that, as a teacher, I had already been engaged in this role, but had not labeled my behavior as "research."

Teachers wear many "hats." One of my favorite "hats" has always been that of detective--I love a good mystery! Not that I've envisioned myself as any Sherlock Holmes, mind you; but as a teacher, I must determine what the students know, how well the students know it, how complete their knowledge is, if the students can apply the knowledge, if the students can transfer the knowledge to a similar situation, if the students hold any misconceptions, if there are any inaccuracies or flaws in the students' knowledge, if the students know that they know, if the students know what is known and what is not known, and if the students know how and when to ask for help. (These, of course, are only a few of the questions for which a teacher must find answers.)

As a detective, you're finding answers for yourself and your own use within your own classroom. It never occurred to me that anyone outside my own classroom would have any interest in my students, my questions, or my findings. (Teachers work in such isolation that they do not always realize that there are others undertaking similar investigations with similar students and similar questions.) However, after all these

experiences, I could see that other teachers at the elementary and middle school levels might be interested.

How does one begin to undertake the role of researcher? Why, with a question of course; and, I had plenty of questions. Where was I to begin? Piaget, the famous Swiss psychologist and constructivist, provided the answer as he quoted Rousseau: "Begin by studying your pupils, for assuredly you do not know them at all. . . (for) . . . the child has its own peculiar ways of seeing, of thinking, and of feeling" (Rousseau in Piaget 1971, 140). Piaget spent a lifetime listening to and studying children; so, I also decided to take Rousseau's advice.

Participants in this Study

It was important to me that the results of this research be useful in my teaching and, therefore, I wanted to interview students who had not been formally introduced to the topic of integers. This would most closely approximate the students I would see in my math classes. Accepting the offer of assistance from a sixth-grade teacher in my system and receiving the necessary authorization from the school administration, I asked the sixth-grade teacher to draw nine names at random from the twenty-four homogeneously grouped students enrolled in her sixth-grade math class. I spoke briefly with the group of nine students to explain what I wished to do and to ascertain their interest in participation. All nine students were interested, so letters describing the project and requesting written permission for student participation were hand-carried to the parents. Eight of the nine parents responded positively; and, from that pool of eight students, I selected six names at random. Imagine my delight when the random drawing generated the names of three girls and three boys--not that gender was a factor in my research, but it was just a nice coincidence.

Why six, you might ask! There was no "magic" in the number. A time constraint was the overriding consideration as school would end in just five weeks. I wanted the

students to meet in groups of two to take advantage of any interaction that might occur between them, and three groups of two seemed like a manageable number.

Pseudonyms of the same gender are given here to the six participants who, for the purposes of this thesis, will be called Bill, Carol, Karen, Neil, Paul, and Susan. All six were 12 years of age. They had the usual variety of interests and hobbies and were actively involved in music and sports. Grouping of the students was accomplished on the basis of their availability: Susan and Neil met on Mondays, Carol and Bill met on Wednesdays, and Karen and Paul met on Thursdays.

These sixth graders were delightful. Their initial shyness dissipated quickly, and they were very open about describing their thinking. Despite the rapidly approaching summer vacation, they stayed focused and on task and seemed to enjoy the learning experience.

Differences, of course, were apparent. Karen quickly grasped the structure of integers and developed her own analogy. Carol kept looking for "the rule" that would drive the concept of integers. Paul relied on the model longer than any of the others; he was reluctant to accept his own answer until he had verified it with the chips. Bill's frustration level was lower than any of the others; he would withdraw within himself and appear uninterested until he had internalized the information. Susan appeared to understand adding with integers but had difficulty talking about her thinking. Neil had some misunderstandings about decimals and place value and kept trying to integrate the concepts of decimals and integers.

This research, of course, was of personal benefit. It provided glimpses into the minds of children who learn in different ways and emphasized for me again that education is not the stuffing of empty minds but an opportunity for the child to interact with the environment and to construct a world.

CHAPTER II
THEORIES OF THINKING AND KNOWING FOR
THE TEACHER AS RESEARCHER

Chapter Overview

In an attempt to enhance my effectiveness in the classroom, learn content I had never taught before, learn how children construct knowledge about integers, and learn first hand how the Constructivist theory of creating knowledge works, I embarked on an investigation using the Clinical Interview process and the Colored Chip Model as methods for constructing knowledge. This two-tiered model, whereby I constructed knowledge by having the children construct knowledge, has been a very interesting journey for me. It has in many unanticipated ways made me examine and re-examine what I believed, what I knew, what I believed I knew, and what I had been taught. My basic beliefs about teaching and how children learn, of course, have been shaped in large part by my actual experience with children of all ages in and out of the classroom. I was amazed to find that my teaching experience had given me the beginnings of a Constructivist viewpoint and vocabulary even though I had studied neither Piaget nor any other Constructivist philosopher or psychologist. (My educational philosophy courses had concentrated on Dewey and my educational psychology courses had concentrated on Behaviorism. As a high school teacher, I had never encountered Piaget or his research in my professional journals.)

Since I had the beginnings of a Constructivist viewpoint, I decided to learn more about Constructivism not only through reading, but also through experiencing the construction of knowledge myself--the knowledge of how children construct knowledge about integers. This would be the model for my answer to the gnawing question asked by Cohen and already mentioned in Chapter One: "How can teachers teach a mathematics that they never learned, in ways they never experienced?" (1990, 233). My model would be to take the role of Teacher as Researcher and learn not only the mathematics of

integers (content) but also how to learn integers in a way I had never experienced before (Constructivism) and at the same time construct knowledge about how children construct knowledge about integers using a representational model. By representational model, I mean diagrams, sketches, symbols or signs used to convey a complex idea.

This chapter reviews the literature on the critical aspects of teacher as researcher and outlines theories of Constructivism, knowledge, critical and creative thinking, the teacher's role, the clinical interview, manipulative models, and representational models.

Studying Your Pupils: The Teacher as Researcher

Teacher as Researcher is not a new role for professors at the university level. However, it is a relatively new concept for teachers at the pre-school, elementary, middle, and secondary levels. The literature recognizes three models of Teacher as Researcher in general. One model involves the teacher in a classroom who works in conjunction with a professor at a university. In this formal research model, the professor defines the topic and method of research, and the teacher assists in a specific manner under the direction of the professor. The research product is usually intended for publication.

A second model involves "outside" researchers who come into a teacher's classroom to do research. In this model, the teacher is a member of a formal research team; however, the topic and method of research are not teacher-defined or directed. Again, the research product is intended for publication.

The third model--the newest of the three--involves research conceived, directed, and evaluated by an individual teacher. The research, conducted in the teacher's own classroom or school, is not intended for publication. The research is informal and personal, and the research product will be used by the specific teacher to gain understandings which will enhance the teacher's effectiveness in the classroom.

The literature also uses the terms "action research" (Burton 1986, 718), and "reflective practitioner" (Schön 1983, 1) synonymously with "teacher-researcher."

Model for Teacher as Researcher.

I have chosen to use the informal, personal research model defined by McIntosh (1984):

Teacher-researcher refers specifically to public . . . school teachers who instigate research in their own classrooms or schools. . . .The teacher-researcher is at least partially, if not 100% responsible for the idea for the research and for conducting the research in his/her classroom or school. (2)

Patterson and Stansell (1987) describe the teacher-researcher as:

. . . a new breed, combining theoretical grounding, methodological training, and classroom experience in a non-traditional way, in order to make theory-based instructional choices and in order to observe, document, and interpret the responses of students within each classroom context. (719)

Wann (1952) tells us that teachers feel "a fundamental difference between . . . action research and other methods of study. . . . The research began with first-hand contact with children . . . and with real problems teachers face in teaching children" (491).

Filby (1991) posits that "Teachers engage in inquiry not so much for the value of formal knowledge but so as to understand and improve their own teaching (1). McIntosh (1984) believes "The teacher-researchers' expanded knowledge increases their effectiveness and, therefore, their self-concept as teachers" (3). This, of course, is one of my beliefs about my two-tiered model and one of my reasons for using this version of the Teacher-as-Researcher model. As Burton (1986) points out, "problems are best solved by those who own them" (718).

Teacher-Directed Research.

Kutz (1992) maintains that some of the most valuable classroom research begins with small questions, with the wonderings of individual teachers" (193). McIntosh (1984) believes that "In order to conceive of a research project, the teacher must first be curious about some phenomenon concerning children or learning" (3). Kutz (1992) emphasizes that teacher research is "work that directly

supports our teaching and our students' learning" (197). And, Filby (1991) tells us that the teacher's "questions are practical and rooted in their own context and experience" (1).

The statement made by Kutz is particularly apropos to my wonderings about the effects of all the "Naked Numbers" (numbers not set in any context) in the typical "Drill and Kill" exercises in mathematics textbooks and my wonderings about the effects of the seemingly contradictory vocabulary of integers upon my students. (The juxtaposition of the term "difference"--a word typically used with subtraction--within the concept of addition of integers may cause some consternation among the students.)

The product of teacher-directed research is not for publication; it is personal. As teacher-researcher, I am not searching for universal truths nor do I intend to revolutionize the teaching of integers. The products from my research are for me increased understanding of how children construct knowledge about integers and an enhanced effectiveness in the classroom as I teach the topic of integers.

The Constructivist viewpoint, with its emphasis on the construction process, knowledge, the teacher's role, and understanding, is central to the two-tiered research process I employed. The vehicle for constructing knowledge--the clinical interview--is discussed briefly here and more fully in Chapter IV.

Children Must Reinvent the Wheel: A Constructivist Viewpoint

I've always believed that, as a teacher, I must start from where the children are. I must find out what the children know, compare that to what they need to know, and then decide how best to move the children along. How many times have I said to myself that I didn't want to reinvent the wheel! Yet, as I study the pure Constructivist viewpoint, I find the belief is that the wheel must indeed be reinvented by each child.

Constructivism.

The major tenet of Constructivism is that children "invent" their knowledge rather than discover it. Knowledge is neither innate nor acquired by transmission; it is constructed internally in response to the senses, emotions, observations, and perceptions. The child constructs knowledge by physically interacting with the environment. The goal of Constructivism is to produce an intellectually, socially, and morally autonomous learner. Probably the name most commonly associated with Constructivism is Jean Piaget (1971).

In addition to Piaget, others who embrace a Constructivist viewpoint and speak of the active role of the learner in that process are Labinowicz (1985), Kamii (1985), Perkins (1986), Duckworth (1986), Skemp (1987), and von Glasersfeld (1987a). Each has focused on a particular area of individual concern that had relevance for me in my two-tiered model. Perkins (1986), Skemp (1987), and von Glasersfeld (1987a) have focused on the active role of the learner in constructing knowledge and understanding, and Labinowicz (1985), Kamii (1985), and Duckworth (1986) have all focused on the active role of the learner in the classroom and its implications for teaching and teachers.

I have chosen to focus on four aspects of the Constructivist viewpoint that are relevant to my work. The four are: the "construction" process, knowledge, the teacher's role, and understanding.

Since I am a "big picture" person, it was important to me to understand what was involved in the construction process, particularly since this was a new way for me of experiencing the learning process. I needed to know about the process of construction, how it "feels" to construct knowledge, how you know that construction is taking place, and what kind of knowledge is being constructed. Since Piaget is still considered the foremost Constructivist, I have drawn extensively from his work.

The Construction Process.

In Piaget's (1971) view, the child exhibits two characteristics of intelligence from birth. These characteristics consist of organization, "a structuring activity" (158), and adaptation, "a perpetual adjustment . . . to the data of experience" (158). "Adaptation" (Piaget 1971, 158) was the term used to describe the construction process, and "equilibration" (Piaget 1980, 101) was the term used for restructuring knowledge. According to Piaget's (1980) theory, the child experiences equilibrium as long as the interaction with the environment produces information that is consistent with the existing knowledge constructs. However, a perceived problem, a new physical reality, an error, or other catalyst produce the opposite state, which Piaget calls "disequilibrium" (1980, 83).

In order to precipitate the construction of knowledge, my task was to provoke disequilibrium. (A pure constructivist might argue against "provoking" the construction of knowledge; however, with the ever-growing body of knowledge that a student must confront today, the "pure" viewpoint is a luxury the teacher cannot afford.) I must present my students with a changed environment and confront them with "new objects or an experimental situation" (Piaget 1980, 103). If, by my actions, I could provoke a feeling of frustration or un-ease, then I would know the process of equilibration had begun. As I listened to my students trying to make sense out of the Colored Chip Model and the topic of integers, I would know that they were trying to assimilate or construct new concepts. Once my students had no need of the model and could accurately solve integer problems, then I would know accommodation had taken place and equilibrium had been restored.

Knowledge

I now knew what to look for to ascertain that construction was taking place. But what was being constructed?

Knowledge as Transformation.

According to the Constructivist viewpoint, knowledge can be described as mental structures. These mental structures are of two kinds: one set of structures consists of the assimilation of reality (the WHY), and the second set of structures consists of the mechanics of transformations (the HOW). Or as Piaget (1971) described it: "To know an object is to act upon it and to transform it, in order to grasp the mechanisms of that transformation as they function in connection with the transformative actions themselves" (29).

Piaget (1971) believed that knowledge was not innate, nor was it acquired by transmission; it was "derived from action" (28). He believed that interacting with objects in the environment caused the child to create knowledge in the mind. Piaget (1980) used the term "construct" (29) when speaking of knowledge because what was created was not static, but dynamic. New knowledge was "built" (89) upon or re-constructed from existing knowledge (89).

Constructivists classify knowledge as social, physical, or logico-mathematical. By social or conventional knowledge, the constructivist means the knowledge that is arbitrary or cultural in nature. Social knowledge includes language, customs, traditions, manners, and mores; anything we learn from other people. Physical knowledge comes to us through objects in our environment. The objects themselves provide the knowledge. Logico-mathematical knowledge is the knowledge that is constructed solely within us.

Knowledge as Design.

Another Constructivist viewpoint regarding knowledge is that of David Perkins, co-director of Harvard Project Zero. He defines knowledge as design and views "pieces of knowledge as structures adapted to a purpose" (1986, 3). Agreeing with Piaget, Perkins (1986) says that knowledge "is active, to be used, rather than passive to be stored" (18). He further states that knowledge as design speaks to "the critical and creative thinking behind knowledge emphasizing knowledge as constructed by human

inquiry rather than knowledge as 'just there'" (1986, 19). According to Piaget and Perkins then, the knowledge being constructed by my students would be social (for the vocabulary of integers and the Colored Chip Model), physical (using the chips), and logico-mathematical (the concepts of and the operations with integers). The logico-mathematical knowledge would be on two levels: recognition of applicable existing knowledge and construction of new knowledge. I found Perkins' (1986) statement that knowledge as design speaks to "the critical and creative thinking behind knowledge" (19) particularly appropriate. For purposes of this thesis, metacognition was the thinking skill that would be central to the research. The existence and use of specific critical and creative thinking skills were also important indicators of the students' progress in constructing their knowledge of integers.

Critical thinking. Theorists hold differing views of what critical thinking is or does. Ennis (1991) defines critical thinking as "reasonable, reflective thinking that is focused on what to believe or do" (68). Swartz and Perkins (1989) write "many educators and scholars prefer to include in critical thinking virtually all good thinking" (38), although they themselves indicate that when people think critically they:

- aim at critical judgment about what to accept as measurable and/or to do;
- use standards that themselves are the results of critical reflection in making these judgments;
- employ various organized strategies of reasoning and arguments in determining and applying these standards, and
- seek and gather reliable information to use as evidence or reasons in supporting these judgments. (38)

There are a number of skills that can be called upon to assist the formation of beliefs and judgments to which critical thinking leads. Critical thinking skills are not central to my thesis; however, it is my contention that they were used to whatever degree they were possessed by the students during the process of constructing knowledge about integers. The specific critical thinking skills involved were making inferences, making predictions, and making generalizations.

Creative thinking. Creativity, according to Perkins (1991), is a "messy and myth-ridden subject" (85) but is not "a single distinctive ability and a matter of talent" (85). He defines creative thinking as "thinking patterned in a way that tends to lead to creative results" (85).

J. P. Guilford (1959), one of the fathers of the creativity movement, describes the creative person as "confident and tolerant of ambiguity and . . . [liking] . . . reflective and divergent thinking and esthetic expression" (177). He went on to hypothesize that "fluency of thinking would be an important aspect of creativity" (170). By fluency, Guilford (1959) meant "fertility of ideas" (170). He also posited that "creative thinkers are flexible thinkers" (172). By flexible, Guilford (1959) meant the "ability or disposition to produce a great variety of ideas, with freedom from inertia or perseveration" (172) or to "get away from the obvious, the ordinary, or the conventional" (173). The factors of fluency and flexibility were relevant to the knowledge of integers that my students constructed and served as "self-tests" relative to the depth and breadth of that knowledge.

Guilford (1959), Perkins (1991), Swartz and Perkins (1989), and Amabile (1983) agree that current definitions of creative thinking also focus on "product" as an integral part of the definition. Amabile (1983) notes that "*product*" (358) (italics in original) is broadly defined to include "any observable outcome or response" (358). Swartz and Perkins (1989) tell us that "the classic challenge of creative thinking is 'breaking set' and seeing a situation in a new way" (40). They further state that "another challenge, more recently recognized, is 'problem finding'" (40). And, it is the problem finding aspect of creative thinking--asking questions and creating problems--that was most relevant to my students' construction of knowledge and to my work in evaluating the knowledge that the students constructed.

When the construction process is successful, knowledge has been constructed

Martin (1986), I knew that knowledge was often "fragile" (2). During my interviews with the children, I would have to be alert to fragile knowledge and to the remedies that could be employed to make it "robust" (2).

Fragile Knowledge.

It was not enough for knowledge to be just constructed; it was imperative that students' knowledge be "robust . . . [not] . . . fragile" (Perkins and Martin 1986, 2). When my students and I were constructing knowledge, it was important that our knowledge was solid and healthy, not shallow and filled with misconceptions. It was important to my students and to me that we both knew what to do and why.

Perkins and Martin (1986) delineate categories of fragile knowledge. The three most pertinent to this thesis are partial, inert, and misplaced knowledge. They are defined as follows.

Partial knowledge. "The simplest sort of fragile knowledge is partial knowledge" (9), according to Perkins and Martin (1986). "A student knows something about . . . [the topic] . . . but has minor gaps that impair the student's functioning" (9).

Inert knowledge. Perkins and Martin (1986) define this as "knowledge that a person has, but fails to muster when needed" (10). They see this "as a problem of transfer" (10). Knowledge that has been acquired cannot be applied. They say that "knowledge often tends to remain bound to the context of initial learning unless the learner deploys self-monitoring strategies" (10).

Misplaced knowledge. Misplaced knowledge, according to Perkins and Martin (1986), occurs when "knowledge suitable for some roles invades occasions where it does not fit" (14). "Overgeneralization--or underdifferentiation . . .--provides one obvious cause" (15).

Perkins and Martin (1986) also propose three remedies for fragile knowledge: prompts, hints, and provides.

Prompts. Prompts are initial questions. Perkins and Martin (1986) call them "questions that did not require any foreknowledge of the true nature of the difficulty" (7). The student could be his own prompter; however, the teacher usually takes the role of prompter and asks questions such as "What's the first thing to do?" or "How would you do . . .?", or "What does this do?" (7).

Hints. When a student does not respond to a prompt, the teacher moves to the next level. Hints nudge "the student toward a resolution with leading questions or bits of information" (Perkins and Martin 1986, 7).

Provides. Provides represent "an exact solution to the immediate dilemma" (Perkins and Martin 1986, 8), so that the student can keep moving on in his work. Perkins and Martin (1986) theorized that "the escalation from prompts to hints to provides not only helped the student but served as a probe of the student's level of mastery and understanding" (8). This information would be valuable in my assessment of the strength of knowledge constructed.

As teacher, I had the opportunity to facilitate the student's construction of knowledge. My role, as I saw it, was that of manager and facilitator. Theories of experts in the field also helped me examine the roles of the teacher in the construction process.

The Teacher's Role

Piaget believed the result of constructing knowledge and, therefore, the purpose of education, was to produce an intellectually, socially, and morally autonomous learner (Piaget 1971). His belief certainly supported my desire that my students become life-long learners. How then was that to be accomplished? In my view, it had a great deal to do with the role of the teacher.

According to Duckworth (1987), there are two aspects to teaching: putting students in contact with real phenomena and listening to students try to explain the knowledge they are constructing. By bringing students together with real phenomena

rather than books, pictures, or other representations, the student has the opportunity to interact actively with the environment. A student, then, needs the opportunity for thoughtful discussions with other students to test, modify, or expand the knowledge that has been created.

Teacher as Model.

Perkins (1986) sees the role of the teacher as a model and as a "generic scaffolder" (220) and sees the function of the teacher as one who fosters "overt thinking" (220). "Thinking," he says, "usually occurs within the silence of our own minds" (221). To foster overt thinking causes the student to be an active participant in social interaction. Through this social interaction, the student builds knowledge as attempts are made to explain, support arguments, and question.

Kamii's (1985) emphasis has been to help teachers understand how to structure the child's environment and how to apply the Constructivist viewpoint in the classroom. She believes that "social interaction, or more specifically. . . the mental activity that takes place in the context of social exchanges" (26) is critical in the student's construction of knowledge.

Teacher as Guide.

According to von Glasersfeld (1987a), "the teacher's role . . . [is not] . . . to dispense 'truth,' but rather to help and guide the student in the conceptual organization of certain areas of experience" (16).

The theorists cited here would seem to agree that it is the teacher's role to provide an active rather than a passive experience for the student, to act as interpreter and/or guide (as for instance in the case of which attributes should be considered when the physical objects are not present), and further to provide opportunities for students to interact with one another. The more interaction, the more likely that "disequilibrium" (Piaget 1980, 83) will occur, and the more likely that new knowledge will be constructed.

A Teaching Experiment: The Clinical Interview

The purpose of any teacher-directed research or experiment is to enable the teacher be a better teacher and to enhance effectiveness in the classroom. The research method that is employed should be chosen carefully and should match the topic being researched and the researcher. Since I prefer the verbal mode of communication and since interaction between researcher and subject was important to me, I chose the clinical interview as my research tool. This method had the advantage of being "user friendly" to me in my initial role of Teacher as Researcher and allowed me direct access to the thinking of the students involved in learning about integers.

Defining the Research Process.

The clinical interview is a qualitative research technique that seeks to discover through a personal interview how a person thinks, in the fullest possible sense of the word. The interviewer examines the knowledge, perceptions, perspectives, attitudes, emotions, misconceptions, judgment, and learning processes of the interviewee. According to Borg and Gall (1989), the uniqueness of the interview as a research method is "that it involves the collection of data through direct verbal interaction between individuals" (436). This research method was originated by Piaget (1952) during his studies of how knowledge originates, according to Ginsburg et al. (1983) and Labinowicz (1985). Piaget (1952) describes his method as "free conversation with the child, conversation that is governed by the questions put, but which is compelled to follow the direction indicated by the child's spontaneous answers" (vii).

Labinowicz (1985) tells us that "Piaget's clinical method is marked by its flexibility in adapting the interview to the individual child in an attempt to follow his thinking" (27). Ginsburg et al. (1983) tell us that Piaget's purpose in developing the clinical method was to study "cognitive development and specifically intellectual activity, cognitive processes, and cognitive competence" (11).

Piaget (1952) states that investigations have shown "the necessity for actual manipulation of objects" (vii) during these interviews and further that "conversation with the child is much more reliable and fruitful . . . when the child, instead of thinking in the void, is talking about actions he has just performed" (vii). This method, then, was appropriate for the research I wished to do. It would allow me the freedom to hear the students think and would also allow and encourage questions.

Limitations.

No research method is without shortcomings; and just as Borg and Gall (1983) see advantages in the clinical interview's "flexibility, adaptability, and human interaction" (437), they also cite "subjectivity and possible bias" (437) as disadvantages. One area of possible bias--that of bias in note taking and/or note transcribing--is always a concern.

Borg and Gall (1983) further state:

The interactions between the respondent and the interviewer are subject to bias from many sources. Eagerness of the respondent to please the interviewer, a vague antagonism that sometimes arises between interviewer and respondent, or the tendency of the interviewer to seek out answers that support his preconceived notions are . . . a few of the factors that may contribute to biasing of data. (437-438)

Borg and Gall (1989) caution the interviewer about prejudicing an answer "either by specific comment, tone of voice, or nonverbal cues" (443). The interviewer must take care to present a neutral, non-judgmental façade if the interviewer wishes to "encourage the child to consider further, think more specifically, or rethink the process used in arriving at a solution" (Labinowicz 1985, 27). Perhaps my rationale for choosing the clinical interview, in spite of its limitations, is best summed up by Labinowicz (1985) when he posits that "getting an intimate look at children in the process of thinking . . . offers a different perspective of teaching and learning" (36). Since my two-tiered model cast me in the roles of teacher, learner, and researcher, and my students in the roles of learner and teacher, the clinical interview was my choice.

To me in the role of Teacher as Researcher, the clinical interview offered the advantage of allowing the students to interact with each other and with objects of my choosing. Objects are important to the child's construction of knowledge according to constructivists. The clinical interview was an excellent choice for me also because it not only allowed me to view the students as they constructed knowledge, it also allowed me to examine and better understand the way in which the Colored Chip Model worked.

The Colored Chip Model is a concrete manipulative model that I used as a representation in the teaching of integers. I chose this model because I believe that concrete models are most effective when learning something new, because I had seen the power of this model with children of different ages and with adults (who said they "understood" integers for the first time), because I was trying to follow a Constructivist approach and the philosophy demanded an active learning process, and because I personally preferred this model.

A number of theorists, whose work concerns representational models and their use in the construction of mathematical knowledge, will be reviewed next.

I Do and I Understand: Manipulative Models

It is important that the Constructivist viewpoint be reviewed if understanding of the rationale and value of manipulative models is to be achieved. According to Piaget's (1971) Constructivist philosophy, the knowledge that is being constructed through the use of a manipulative model is "knowledge . . . [that] . . . is in fact abstracted from the objects themselves . . . acting upon those objects in order to transform them, in order to dissociate and vary the factors they present . . . and not . . . simply extracting a figurative copy of them" (72).

In simpler terms, integers (the math content being modeled) are a mental construct. Since a mental construct is unseen, something must be used or created that is visible and which "stands in the place of" the mental structure so that interaction with the

environment and communication with others can take place. The term "representation," meaning symbol, sign, and/or model is used to describe that which stands in the place of the mental construct.

Piaget's (1952) theory of number describes number as a mental construct that each person must build and that proceeds from logic. According to Kamii (1985), "number is an idea that, when constructed. . . is *imposed on . . . objects* (italics in original) by the child" (53).

Therefore, Piaget's (1971) Constructivist viewpoint confirms that "experience is necessary to the development of knowledge . . . [and] . . . that it occurs in two very different forms . . . physical experience and logico-mathematical experience" (37). It is physical experience that supports the use of manipulative models. Piaget (1971) says, "Physical experience consists in acting upon objects and in discovering properties by abstraction from these objects . . ." (37). It is "discovering properties by abstraction" (37) and the resulting construction of logico-mathematical knowledge that form the rationale for manipulative models.

Representational Models.

The term "model" generally means a representation that uses symbols and signs to convey a complex idea. Hayes (1981) recognizes two types of representations: "internal" and "external." The internal representation is in the head; external representations are "sketches and diagrams or . . . symbols or equations" (5).

The National Council of Teachers of Mathematics (NCTM) (1989) talks about "physical, pictorial, graphic, symbolic, verbal, and mental representations" (26). NCTM sees "representing" (27) as an important communications skill which "encourages children to focus on the essential characteristics of a situation" (27).

There are three types of models: concrete, pictorial, and symbolic. A concrete model according to Fennema (1972) "represents a mathematical idea by means of three-dimensional objects" (635) or symbols. These objects, called manipulatives, appeal to

several senses and can be touched, moved about, rearranged, and otherwise handled by children" (Kennedy 1986, 6). These physical objects can be manipulated in a variety of ways; hence, the name.

The Colored Chip Model, which I used in my work, is an example of a concrete manipulative model for the study of integers (see Appendices A and B). This model uses round chips of yellow and blue. In and of themselves, they have nothing to do with integers. However, what can be done with the chips, such as adding chips of the same color, adding chips of different colors, taking away chips of the same color, taking away chips of different colors, and making sets containing one chip of each color mimics what can happen with integers. Since the classroom or working environment does not always have thermometers, elevators, football fields, stock markets, or other real-life integer situations in them, the mimicry or representations provided by the chips allows and facilitates an understanding of how integers work.

Vocabulary of Integers for the Colored Chip Model.

Any well-constructed model must be able to represent the precise vocabulary of the concept it is trying to teach, and the Colored Chip Model allows this to be done very effectively. Important vocabulary concepts such as absolute value, negative (as in opposite or inverse), equal to, greater than, and less than can be modeled very simply, yet very powerfully. One of the more valuable aspects of this model is the ease with which vocabulary can be explicated.

Absolute value. The absolute value of a number refers to its "manyness" when using the Colored Chip Model. (The typical mathematics textbook defines absolute value in terms of the number line model upon which most textbooks rely. In this case absolute value is defined in terms of its placement on the number line relative to zero. The question being asked then is "How far away from zero is x ?") Absolute value answers the question "how many?" in the Colored Chip Model; color of the chips representing an integer has no impact on the "manyness." Negative three has the same "manyness" as

positive three. The symbol for absolute value is $|x|$, as in $|-3|$ or $|3|$. In both cases, the absolute value is 3. The larger a positive integer, the larger its absolute value; for example, $+5$ is greater than $+3$. The absolute value of $+5$ is greater than the absolute value of $+3$.

The situation with negative integers is more difficult for the students to understand. The larger a negative integer, the smaller its absolute value. For example, -3 is greater than -5 , but the absolute value of -3 is less than the absolute value of -5 .

Students will benefit from using the Colored Chip Model for these types of problems as well as when a positive integer and a negative integer are compared. For example, -5 is less than $+3$, but the absolute value of -5 is greater than the absolute value of $+3$.

Negative. Not only do we have the term "negative number," we also have the term "negative" as in "opposite." The illustration of a photographic negative is often used for clarity. On a photographic negative, "light" values appear dark, and "dark" values appear light. The "negative" of a positive number is a negative number, and students readily accept this. However, they may exhibit some difficulty with the concept of the "negative" of a negative number being a positive number. It is not that the concept in and of itself is difficult; it is that students have not met this concept before.

Equal to. "Equal to" means representing the same quantity. It is a comparative term used in mathematical sentences called equations and is represented by "=", as in $2 + -2 = -5 + 5$.

Less than or greater than. "Less than" or "greater than" are expressions of inequality. The quantities being compared are not the same. Symbols used are $<$ for "less than" as in $-6 + -2 < 6 + 2$, and $>$ for "greater than," as in $7 + -2 > -6 + -6$.

Pictorial and Symbolic Models.

Pictorial models (semi-concrete models) are considered a "bridge or intermediate step between concrete and symbolic models" (Fennema 1972, 635). These models use a two-dimensional sketch or drawing. For example, a drawing of three blue chips is a pictorial model for the integer -3 .

A symbolic model uses culturally derived conventions as its representations. Fennema (1972) defines a symbolic model as one which "represents a mathematical idea by means of commonly accepted numerals and signs that denote mathematical operations or relationships" (635). "Signs bear no similarity to the thing represented and are parts of systems devised to communicate some message to others" (Kamii 1985, 52). Number names (seven), numerals (7), and mathematical notation ($=$, $+$) are examples of signs that are in the realm of social knowledge. Symbolic models often derive a sense of reality from active involvement with concrete manipulative materials.

Manipulative Models.

For many years, the use of concrete manipulative models had been relegated to the primary grades in school. Manipulatives were frowned upon as "toys" and were not given credence in the learning process. Pictorial and symbolic models were used extensively. However, if a Constructivist viewpoint is to be practiced, then the active nature of the learner must be recognized and provided for. Active learners need more than paper and pencil.

Rationale. Piaget (1973) spoke specifically of the necessity for working with objects: "in mathematics, many failures in school are due to . . . [the] . . . excessively rapid passage from the qualitative (logical) to the quantitative (numerical)" (14). The National Council of Teachers of Mathematics (NCTM) (1989) believes that children are by their natures active learners "who construct, modify, and integrate ideas by interacting with . . . materials . . ." (17). Kennedy (1986) says that children who have used

manipulatives successfully and extensively will be "more likely to bridge the gap between the world in which they live and the abstract world of mathematics" (6).

Belsky (1990) defines manipulatives as "materials that the student physically handles to explore and/or demonstrate a mathematical concept" (23). According to Belsky, "the use of manipulatives opens windows . . . [and] . . . allows students to grow at their own rate" (23).

Greenes (1990) believes that the "ability to observe, to compare, to classify, to sequence, and to make inferences . . . [are] . . . fundamental abilities . . . best developed and enhanced through the use of manipulative materials" (4). Manipulatives are not only for the child; they are for all ages according to Greenes (1990). She states, "Materials are also useful at all levels to develop and enhance mathematical concepts" (5). In problem-solving situations Greenes (1990) says, "Manipulative materials may be used by students to model situations, collect data, visualize relationships and solve problems" (10).

Using manipulatives. Moser (1986) indicates that manipulatives are "for all students, regardless of their ability and developmental level" (8). Greenes (1990) states manipulatives "are often best used by students working in pairs or in small groups" (11), although manipulatives may also be used successfully by an individual child. Belsky (1990) believes that manipulatives are successful in "group work . . . [because they encourage] . . . active learning and discussion" (22).

Once the decision to use manipulatives has been made, "time to fiddle" (Belsky 1990, 26) must be provided. Once the "get acquainted time" has been provided, Belsky (1990) says, "State the problem, give a brief demonstration of the intended use of the manipulatives, then let . . . [the students] . . . go to work" (26). "The beauty of manipulatives," according to Belsky (1990), "is that the students find a solution by themselves" (26). The question of overuse or over-reliance on manipulatives is a concern for some. However, this is unlikely to happen because children naturally "let go" of manipulatives just as soon as knowledge has been constructed. There is a natural

"curtailment" (51) according to Krutetskii (1976); the child has the ability to know when manipulatives are no longer needed because sufficient input has been received with which to "forge or construct a relationship with existing knowledge" (84).

The National Council of Teachers of Mathematics (NCTM) (1989) believes that the use of manipulative models is critical and the Standards assume that "Every classroom will be equipped with ample sets of manipulative materials and supplies . . ." (67).

For all of their advantages, manipulative models are not without disadvantages. As with any representational model, problems of appropriateness, interpretation, and ambiguity can arise.

Problems with Representations.

Problems of appropriateness and ambiguity may arise with any manipulative model. While it is not my intent to discuss the role of the teacher in detail here, suffice it to say that one of the roles of the teacher is to anticipate and deal with problems of representation.

Appropriateness. There are instances where more than one manipulative model exists for a particular concept. Care must be taken that the model selected is the best one for that student in terms of developmental levels (including motor development) and particular learning style. Has the child constructed the prior knowledge that is required to understand this model? Is the model age appropriate, and would the child be motivated to use the model?

A great deal of research is being done on the brain and how it works with regard to learning. Davidson (1983) has looked at the learning of mathematics from a neurobiological perspective and identifies two separate and distinct styles of learning which she characterized as Learning Style I and Learning Style II. These learning styles correlate with the cognitive styles associated with left and right cortical hemispheric preference (Davidson 1989). My intent here is not to focus on brain research but to

discuss appropriateness of a manipulative model as it relates to the learning style of the child.

Davidson (1983) believes that "distinct mathematical learning styles warrant distinct and differentiated approaches to specific math topics" (151). "In general terms," according to Davidson (1983) "a variety of models . . . must be available" (152). Further, Davidson's (1983) research shows that "By appealing to a preferred learning style, the opportunity to maximize mathematical learning is present" (152-153).

According to Davidson (1983), Learning Style I children prefer a "recipe" (24) approach to math; they are interested in understanding a step-by-step sequence of operations that lead to a solution and tend to remember parts rather than wholes. Their perception of a model, then, is colored by how they learn. A child's "part" preference may cause the child to selectively view the model being presented. If he "sees what he understands" (71) to use Labinowicz's (1985) words, the model he perceives is much different from that of a Learning Style II child.

A Learning Style II child, according to Davidson (1983), recognizes patterns, pictures, geometric situations, and visualizes three-dimensional configurations easily. A Learning Style II child's perception of a model includes the logic that gives the model its meaning. The Learning Style II child has a much more global perspective than the Learning Style I child.

While no manipulative model will change a Learning Style I child into a Learning Style II child, appropriate models may make the knowledge constructed more nearly similar and will afford the child an opportunity to work from strength rather than weakness. Learning Style I and Learning Style II just require two different means of getting to the same result. In Davidson's (1983) view:

. . . the recognition of differences in learning style mandates that a variety of models (discrete/continuous, verbal/perceptual and a variety of approaches (carefully sequenced deductive procedures or highly inductive presentations) must be available in regular classrooms. (152)

Interpretation. In addition to the problem of appropriateness, a problem of interpretation has also been noted. According to von Glasersfeld (1987b), "A representation does not represent by itself--it needs interpreting and, to be interpreted, it needs an interpreter" (216). The interpreter may be the viewer of the representation, or it may be the teacher. From von Glasersfeld's (1987b) viewpoint, the interpretation has to do with comparing the representation to previously constructed knowledge. Does the perception match previously constructed knowledge, does it integrate parts of previously constructed knowledges to create a new presentation of that knowledge, or is it input from which new knowledge will be created?

Ambiguity. Moser (1986) asks if the manipulative model "unambiguously depicts the mathematical context to be learned" (9). Ambiguity of the representation is another problem for the child. Perkins (1986) says, "In a sense, models are chronically ambiguous" (130). He goes on to say that any model has a "multitude of properties that might . . . [be representational] . . . ; the question becomes, which ones count in . . . [what] . . . circumstances. Nothing about the model itself indicates which properties are important" (130).

Driscoll (1981) tells us, "As a general rule, teachers need to be aware that children . . . have a tendency to be distracted by irrelevant detail" (23). Although this is more pronounced in younger children, it is a problem area. How are these problems of appropriateness, interpretation, and ambiguity to be surmounted? The answer lies in the role of the teacher. While I recognize that a "teacher" is not always confined to an academic setting, for purposes of this thesis I will elect the restricted viewpoint.

Role of the Teacher.

The teacher must function effectively as judge, facilitator, interpreter, communicator, and coach if students are to benefit from representational models. The teacher who understands how knowledge is being constructed can evaluate and choose carefully the most effective model for the student.

To counteract ambiguity, the teacher may, by words, symbols, or signs help the child uncover which of the attributes or properties of the representation are important in this particular circumstance. The teacher can provide a rich interactive environment and encourage communication between the students in the classroom.

Fennema (1972) tells us that "the best indicator of which representational model should be used . . . [is] . . . the performance of the child" (639). The teacher, therefore, will constantly evaluate the performance of the student to determine that the model selected adequately matches the student's learning style.

According to Driscoll (1981), "the teacher stands at the very center of the child's experience with manipulatives, and . . . is critical for the child's success" (22). It is the teacher who can clear away ambiguity; it is the teacher who can question or act as a sounding board.

It is the teacher who can facilitate communication between children, for as Piaget (1971) confirms "the cooperation among the children themselves has an importance as great as that of adult action" (180). Piaget (1971) believes "it is such cooperation that is most apt to encourage real exchange of thought and discussion" (180).

The teacher's role is also to facilitate communication between children so that they may actively test the knowledge they have constructed. Too often the only voice heard in the classroom is that of the teacher. Children need to talk with other children about the knowledge that is being constructed. Above all, the role of the teacher is to assist the student in becoming an independent, life-long learner. Instead of being an authority figure and telling the child what is to be believed, the teacher can lead the way toward "reflection and the critical discussion that help to constitute reason" (Piaget 1971, 179).

Driscoll (1981) tells us that "the role of the teacher appears . . . pivotal . . ." (23). Just as the teacher must facilitate communication between the children, so too it is the teacher who must determine the means of validating the construction that has taken place.

In Chapter III, reverse processing, metacognition, and the child as teacher will be examined as vehicles to validate the children's understanding of the knowledge that has been constructed through the Colored Chip Model.

CHAPTER III

THREE WAYS OF VIEWING CHILDREN'S UNDERSTANDING

Chapter Overview

This chapter will focus on sources and types of understanding, as well as the three ways of viewing understanding that I used in my two-tiered Teacher-as-Researcher model: reverse processing, metacognition, and "the child as teacher." By understanding, I mean the ability to know what to do, how and when to do it, and why it is being done. This is what Skemp (1987) calls "relational understanding" (153). It is knowing the WHY that is the critical distinction between knowledge and understanding for me. Without the WHY there can be no transfer; one piece of knowledge is not seen as being related to another.

Because I believe a paper-and-pencil test is not always an adequate measure of a student's knowledge, I deliberately chose other means of viewing the WHY's that the students were constructing. Although these three ways of viewing understanding may be considered unorthodox, their efficacy for the Teacher as Researcher will be shown in this chapter.

Sources and Types of Understanding

Labinowicz (1985) tells us that "each person's understanding is like a personal painting resulting from one's own interpretation and synthesis of reality" (5). He says that what is learned by a student is not an exact duplicate of what is taught for "*we see what we understand* rather than understand what we see" (7). Further, it is the way that the child organizes and interprets his reality in his mind that is the source of understanding. Labinowicz (1985) says that "relationships are inventions of the mind" (23).

Duckworth (1987), who studied and worked with Jean Piaget, believes that "Meaning is not given *to* (italics in original) us in our encounters, but it is given *by*

(italics in original) us--constructed by us, each in our own way, according to how our understanding is currently organized" (112).

Skemp (1987) posits that "to understand something means to assimilate it into an appropriate schema" (29). By "schema," Skemp means a conceptual structure; for example, the integer number system. According to Skemp, a schema "integrates existing knowledge, it acts as a tool for future learning, and it makes possible understanding" (24).

Skemp (1987) recognizes two types of understanding: instrumental and relational. Instrumental understanding is used by Skemp to mean "rules without reason" (153) or rote learning--knowing what to do, but not knowing why. Relational understanding, however, is used by Skemp to mean that which has been constructed by the learner--"knowing both what to do and why" (153).

Understanding takes knowledge a step beyond construction. Understanding makes knowledge usable and interactive. The teacher, in the role of researcher, examines not only knowledge but also understanding. Understanding--knowing WHY--leads to application and transfer; without understanding, knowledge is only a HOW trapped in memory.

Viewing Understanding

"How does the teacher know when a child's construction of knowledge has resulted in what Skemp (1987) calls 'relational understanding'?" Being mindful of knowledge (to assimilate reality into structures of transformation) as defined by Piaget (1971) and the possibility of imperfectly constructed knowledge as defined in Chapter II by Perkins and Martin (1986), I chose three methods that were meaningful to me as a student of critical and creative thinking and which would allow me to examine the individual understanding--the WHY--achieved by each child. The three methods chosen were reverse processing, metacognition, and "the child as teacher."

Reverse Processing

Piaget (1971), Krutetskii (1976), and Davidson (1989) discuss well-constructed knowledge in terms of the child's ability to engage in what Piaget (1971) describes as "reversibility" or "backward thinking" (30); Krutetskii (1976) describes as "reverse associations" or going from a subsequent to a previous stimulus (85); or Davidson (1989) describes as "reverse processing" or the ability to move from an answer back to reconstruction of a question. The premise is that the student's ability to use reverse processing speaks to the understanding that has been constructed. If the student can move from an answer backwards to an initial problem with fluency, flexibility, and accuracy, then it can be assumed that the student has relational understanding.

I was introduced to the concept of "reverse processing" in Dr. Patricia Davidson's "Thinking Skills in Mathematics" class and was intrigued with its possibilities. The idea of reverse processing as an evaluation tool was so powerful for me that I decided to incorporate it as a feature of my Teacher-as-Researcher model. In this way, I could test it as an evaluation tool and as a creative thinking tool for my regular classroom.

When Dr. Davidson's model of reverse processing is applied to understanding constructed about integers using the Colored Chip Model, then it would have three components: (1) From the problem to the chips, (2) From the chips to the problem, and (3) From an answer to a problem.

From the Problem to the Chips.

Reverse processing is activated by a question. In the first component, a problem such as $+5 + -2$ would be written on paper or on a flash card. The question would be "If this is the problem, what would the chips look like?" The purpose of this facet of reverse processing would be to verify an understanding of the model for the addition of integers. Reverse processing is not examining the effectiveness of The Colored Chip Model *per se*; it is examining the constructs created by the students. Are the students able easily to

recreate how the chips were used? Can they verbalize the process and the meaning--the how and the why?

From the Chips to the Problem.

The second component of reverse processing involves going from the chips to the problem. A problem is selected (in writing or on a flash card) but is not shown to the students. Then the chips are laid out in front of the students as though the model was being introduced. The students must then deduce the problem and write it down. It is not necessary for the students to write the solution; however, they may do so. (Some children find it difficult to accept the ambiguity of an unfinished problem; they must achieve closure. If this is the case, then the child should be allowed to solve the problem.) The focus or purpose of this component is to ascertain if the child has found the pattern and assimilated the structure of integers.

From an Answer to a Problem.

The third component of this reverse processing model is the most sophisticated and creative. In this component, the teacher places a number of chips before the student. It is the task of the student to create as many problems as possible with that number as the answer. This component allows the teacher to examine not only a student's accuracy but also to examine the students' creative thinking skills, specifically fluency and flexibility.

If the students were successful in the three modes of reverse processing, then this would indicate mastery of the material presented. Therefore, knowledge about integers had been constructed and understanding existed.

Performance with Reverse Processing.

Of the six students who began this project, only four were involved in reverse processing. (Two students were ill and unable to complete the sequence.) All four of the children were successful in all three components of the reverse processing. Although all the students were equally successful in components one and two, differences among the

four were apparent in the third component. Bill and Carol showed more flexibility. They started out with the operation of addition and wrote quite a few problems, as did Paul and Kathy. However, Bill and Carol asked if they could use subtraction; the use of subtraction had to be suggested to Kathy and Paul. Three of the children began with a first term that was positive. Carol and Kathy used a negative second term more quickly than Bill and Paul. Only Bill began with a negative term easily, but none of the students used zero as a term until they were asked to do so. All complied quickly, accurately, and fluently when asked. Reverse processing in all three modes was so non-threatening to the students that I had to ask them to stop or to set a limit on the number of problems they were to write.

Another way in which the teacher can view the student's progress is by examining metacognitive ability. Metacognitive ability, the thinking skill necessary for autonomy, was the second non-traditional component for viewing understanding in the two-tiered model used in my role of Teacher as Researcher.

Metacognition

Considerable attention has been paid in recent years to the mind and in particular to that body of knowledge called thinking skills. The designation of "skill" is used because current research (Costa 1991) in the field shows that the ability to think can be improved by correct and directed practice. "Thinking" is generally classified as Critical Thinking, Creative Thinking, or Metacognition.

Metacognitive Thought.

Marzano et al. (1988) defines metacognition as "being aware of our thinking as we perform specific tasks and then using this awareness to control what we are doing" (9). McTighe (1987) says:

Research has shown that effective learners and thinkers monitor their own learning and thinking. They are aware of what they know, what they don't know, and what they need to know in order to solve a problem or

comprehend a difficult concept. Many of these capable reasoners engage in 'self-talk' during which they ask questions, maintain concentration, try new problem-solving strategies, and check performance. (29)

According to Costa and Lowery (1989), metacognition occurs in the neocortex of the brain and is thought by some to be uniquely human. They define metacognition as:

. . . our ability to know what we know and what we don't know . . . our ability to plan a strategy . . . be conscious of our own steps and strategies during the act of problem solving, and to reflect on and evaluate the productiveness of our own thinking. (64)

In their model of thinking, "metacognition is an overarching cognitive ability that monitors our other thinking processes" (64).

Swartz and Perkins (1989) label metacognition as a "superordinate kind of thinking" (51). They recognize metacognition as being of a higher order than other thinking skills and distinguish four levels of metacognitive thought as a gauge of sophistication of thinking.

The two levels that are important for my purposes here are "strategic use . . . [and] . . . reflective use" (52). In strategic use, "the individual organizes . . . thinking by way of particular conscious strategies" (52).

In reflective use, "the individual reflects upon his or her thinking before and after--or even in the middle of--the process, pondering how to proceed and how to improve" (52). Swartz and Perkins (1989) stress the importance of "*varied* (italics in original) practice and *reflective and deliberate* (italics in original) practice" (85).

Schoenfeld (1989) says that "Broadly speaking, metacognition refers to people's understandings about their own thought process . . . [an] . . . aspect of metacognition . . . [that] . . . is referred to as 'self-regulation'" (94).

In defining metacognition as a "thinking skill," the implication is that this interaction with the internal environment of the mind can be constructed; and, anything that can be constructed can be improved upon. "While inner language, thought to be a prerequisite, begins in most children around age five, metacognition--a key attribute of formal thought--flowers at about age eleven" (Costa 1984, 57).

Developing Metacognitive Skill.

Swartz and Perkins (1989) posit that metacognition can be improved through use. They recognize three levels of metacognitive instruction: aware use, strategic use, and reflective thinking. "Aware use . . . involves . . . *classifying* (italics in original) our thinking" (179). "Strategic use . . . [involves] . . . thinking according to a plan" (180). "Reflective thinking" according to Swartz and Perkins (1989) has two critical ingredients, "monitoring . . . and directing our thinking according to some well thought-out ideas about how we *should* (italics in original) think" (182).

My stated purpose to the students who participated in my research was "to learn how people think about integers." Obviously I needed to know how they thought and what they thought and, therefore, I had to ask them to think about their thinking and to talk about their thinking. Their "aware use" of metacognition would also be true to the Constructivist viewpoint by putting them on the road to independence and autonomy.

In addition to the actual instruction cited by Swartz and Perkins (1989), Costa (1991) (while defining metacognition) gives a prescription for helping students develop and improve their metacognitive abilities:

. . . by having discussions with students about what is going on inside their heads while they are thinking; comparing different students' approaches to problem solving and decision making; identifying what is known, what needs to be known, and how to produce that knowledge; or having students think aloud while problem solving. (32)

Costa (1984) also lists twelve strategies for enhancing metacognition

"independent of grade level and subject area" (59):

1. Planning strategy
2. Generating questions
3. Choosing consciously
4. Evaluating with multiple criteria
5. Taking credit
6. Outlawing "I Can't"
7. Paraphrasing or reflecting back students' ideas
8. Labeling students' behaviors
9. Clarifying students' terminology
10. Role playing and simulations

11. Journal keeping
12. Modeling (59-61).

No attempt will be made to explicate this list of strategies. Some of these strategies are teacher behaviors, some are student behaviors, and some are both teacher and student behaviors. (Presenting this list could be viewed as a strategy to encourage the reader to "think about thinking.")

Metacognition and Constructivism.

Metacognition, as a thinking skill, has had a long existence; it is not a new phenomenon. However, the term "metacognition" is relatively new; according to Schoenfeld (1989), "the term entered the psychological literature in the late 1970's, although it has a long and distinguished heritage in both philosophy and psychology" (94). Constructivists place importance on the achievement of intellectual autonomy. Only the students who can control, monitor, and direct their own thinking can achieve autonomy. Therefore, the concept of metacognition must be inherent in their philosophy even though the term itself is not used.

Kamii (1985) defines intellectual autonomy as "being governed by oneself and making decisions for oneself" (45). Skemp (1987) did not use the term "metacognition" in his work; however, his use of the term "reflective awareness" (36) includes the concept of "introspective awareness" (36). Von Glasersfeld (1987a) uses the term "reflection" in his work. He uses this term to mean "the ability of the mind to observe its own operations" (11). Certainly this awareness and observation could be labeled metacognition.

Constructivism recognizes two environments with which each learner is involved. One is an external environment (the physical world), and the other is an internal environment (the mind itself). The internal environment is where "the life of the mind is a dynamic reality" (Piaget 1971, 146) and where transformations take place. Within these transformations, the concept of metacognition must exist.

Metacognition as an Evaluative Tool.

In using metacognition as an evaluative tool, I was taking a risk. Costa (1984) indicates that students may not wonder at all about what they are doing and may be unable to explain their strategies. The children I was working with were twelve years old; therefore, they would show some vestige of metacognitive ability.

Why use metacognition as an evaluative tool if it is risky? Because it is important that children be able to think for themselves and be in control of their own learning. I am convinced that my prime directive is to teach children how to learn. Therefore, children must know how to think if they are ever to take charge of their learning and become the autonomous learners Piaget and his followers have talked about.

My underlying assumption was that these children were capable of metacognition and were already engaging in metacognition. My instructions to the children had been to "talk aloud" to me and to tell me whatever they were thinking.

I was struck by the differences in metacognitive responses among the children. I am not certain to what to attribute the differences; there are probably several reasons. Perhaps the idea of "talking aloud one's thinking" was a new idea, and the difference was an awareness issue. Perhaps it was a question of trust; and, as the children and I built a growing level of trust, they were more willing to share their thoughts. Perhaps it was practice; if metacognition is a skill, then as practice takes place, the skill grows.

Metacognitive Performance.

It was very interesting to listen to the students. Their ability to describe their thinking differed. Some of the students articulated their thinking easily; others lapsed into an "I don't know" answer and were reluctant to verbalize their thinking.

Metacognitive monitoring. Neil gives us an example of Perkins' misplaced knowledge when he uses subtraction terminology in the integer problem. The problem he was discussing, $-2 + -5$, was one of the pre-test questions.

NEIL: I got minus three--minus three--Oh, I see what I did wrong. I added, I mean I subtracted instead of adding. It should be minus seven.

Neil was monitoring his thinking and was able to self-correct and arrive at a proper answer.

My prompt of "How would you describe a number below zero?" elicited the following response.

Neil: I mean negative seven.

Neil showed reluctance to accept the term "negative" and needed additional prompting before he assimilated the term "negative" rather than "minus."

Metacognitive visualization. Paul was working on the problem $-2 - -8$ and gave an answer of negative six--an incorrect answer. When I asked him, "What were you thinking as you did this problem?", his response was as follows:

PAUL: Negative six. I have a picture in my mind that I'd need six more to take eight away. We're borrowing zero sandwiches. Oh, so I'd have positive six.

Metacognition by personal analogy. Kathy presented a very interesting mode of thinking. She had internalized and personalized the subsets of integers. (In fact, she arrived at what would be an explanation for the number-line model for teaching integers.) When asked about her thinking as she completed the problem $+2 + -5$, this was her response.

KATHY: Well, I thought of where the zero would be on an elevator. And, if you started at two floors up, by the time you went down to the regular level, you'd have three left out of five, and so you'd subtract three from the ground level, and you'd be three below zero. And stairs, too. I have a set of stairs in my kitchen that goes down to my cellar and a set of stairs that goes up to the bedroom. And I think of it that the kitchen floor would be zero and my upstairs stairs would be above zero, and my downstairs stairs would be below zero.

Metacognitive directing. The Swartz and Perkins (1989) definition of metacognition includes both monitoring thinking and directing thinking for a specific purpose. In the following example, Carol has assumed the existence of rules in the operation of integers and is directing her thinking towards the establishment of a rule to follow when solving subtraction problems. The problem she was working on was $+2 - -3$.

CAROL: Positive five. Like, can you just almost add these two? It seems like on all these you can add these two and then like the one you have left on, it seems you just use that sign. Oh, so like--do you--do you like--well, like would the first sign always be right and then you change the second sign? Like if that was a plus and that was a minus, it would both be a plus.

As can be seen in this instance she is attempting to find the mechanics of a rule which suggests that she prefers "Learning Style I" (Davidson 1983, 8). The "Rules" model which Carol is trying to create for herself is the traditional model given in a typical algebra textbook. For example the general rule for subtraction is:

For any numbers a and b , $a - b = a + (-b)$.

This then requires the rules for addition which are as follows:

1. The sum of two positive numbers is positive.
2. The sum of two negative numbers is negative.
3. The sum of a positive number and a negative number is the difference of the two absolute values with the answer being:
 - a. positive if the positive number has the greater absolute value.
 - b. negative if the negative number has the greater absolute value.
 - c. zero if the numbers have the same absolute value.

Since manipulative models historically have been relegated to elementary schools, the "Rules" model has been the principal means used to teach integers to generations of algebra students. As Carol has shown us in her thinking, the Colored Chip Model shows the "why" and, therefore, the "rules" proceed naturally out of the work that the child is doing with the chips.

Metacognitive "I don't know's". Bill exhibited an intuitive approach to problem solving. He often arrived at an answer quickly but could not seem to find the words to describe how he had solved the problem. He frequently seemed surprised that he had the proper answer. In this example, Bill was working on the problem $-5 + +2$.

BILL: Well, I was thinking that negative five--and you took two away from it, you'd have to go up because it goes down from zero. So I just guessed, and I put negative three. I don't know how to explain it.

Bill exhibits the characteristics of a "Learning Style II" (Davidson 1983, 24) child. His greater strength is in his intuitive approach and his understanding of the logic of the structure, rather than in his ability to verbalize his insight.

Susan was another "I don't know" as she worked on the problem $+2 + -5$. She also arrived at the absolute value answer but was not convinced that this was the case.

SUSAN: If you start at zero--if you start at zero, and you have zero plus two, then you have positive two--and that makes it zero plus two--and you start at zero, then you have plus two and plus the negative side, you have two from zero, so you subtract five--I don't know--five minus two is three. I don't know.

I wondered about the "I don't know" responses and relegated them to my "Things to Think About in the Future" pile.

The third evaluation technique chosen was "the child as teacher." Again, this was an opportunity for an active rather than a passive evaluation.

The Child as Teacher

To discover how much you know about a topic, try teaching it to someone else. You certainly discover where your gaps or weaknesses are! Since this was one way to test the breadth and depth of understanding, since it was easy to do, and since it would give me another opportunity to listen to the students, I decided that "the child as teacher" would be my third evaluation measure. It was particularly apropos since my initial question to each of the children had been "How would you explain to a fourth grader that $+2 + -5 = -3$?" Again, when subtraction of integers was introduced, the question was "How would you explain that $+5 - +7$ is -2 to a fourth grader?"

Since we had been talking about these mythical fourth graders, I arranged for real fourth graders to join us during Session Five but did not mention that fact ahead of time to the sixth graders in the study. At the beginning of the fifth session, the sixth graders were told that some fourth graders would be joining them, and they were given about five minutes to prepare. They could decide between themselves who would teach addition and who would teach subtraction.

After the teaching session, Kathy and Paul shared some of their thoughts and summed up the experience pretty well.

PAUL: It was fun.

KATHY: It's fun. Like if you--like the games like war. I mean, like I bet there are a lot of other games you could use to play with them [the fourth graders]. Like when you're teaching the fourth graders--I think when you're teaching kids if you use the chips it will help them to understand.

PAUL: It's fun to learn.

KATHY: Before the fourth graders came in, I was thinking, I wonder if they're going to understand it or not, 'cause at first doing subtraction I didn't understand it. So I thought it would kind of be hard to teach them, but it wasn't hard. I like the chips.

PAUL: Me too. I feel comfortable with [the chips].

This particular evaluative technique was very successful. The sixth graders did not appear nervous nor were they apprehensive about their understanding of the addition and subtraction of integers. Their use of the Colored Chip Model was accurate, as was their dialogue. Their body language indicated comfort and control. I must admit that I was amazed by the rapidity with which the fourth graders assimilated the model and the integer concepts. These fourth graders said they had never heard of positive or negative numbers or integers before; yet, the speed with which they were able to work with integers was amazing.

But let's go back to my original question: How does the teacher know when a child's construction of knowledge has resulted in what Skemp (1987) calls relational understanding: knowing both what to do and why? My answer would be, in this particular project, when the child knows what to do and why and can demonstrate that understanding through reverse processing, metacognition, and becoming the teacher.

In the next chapter, Chapter IV, the clinical interview process will be detailed. Particular attention will be given to the scripted questions, some of which were introduced in this chapter for purposes of examining metacognition.

CHAPTER IV

RESEARCH THROUGH CLINICAL INTERVIEWS

Chapter Overview

As a neophyte Teacher as Researcher concerned with constructing knowledge about content (integers) and constructing knowledge about how children construct knowledge about integers, I discuss my selection of the Clinical interview as a vehicle for my research and focus on the script used during the interviews.

Rationale for the Clinical Interview

The clinical interview, a form of qualitative research (Ginsburg et al., 1983; Labinowicz, 1985; Borg and Gall, 1989), seeks to discover through the personal interview how a person "thinks" in the fullest possible sense of the word. The interviewer examines the knowledge, perceptions, perspectives, attitudes, emotions, misconceptions, judgment, and learning processes of the interviewee.

This seemed to be the ideal vehicle for constructing knowledge about how children construct knowledge about integers. It fit right into my preferred verbal mode of operation, and it had no constraints. I could go wherever the students led. I could ask my favorite question, "Why?" again and again. "Why did you think that?" "Why did you say that?" "Why was your answer 'x'?" "How did you know that?"

In my role of "Teacher As Researcher," it was fitting to follow the model set by Piaget. He spent his lifetime talking and listening to children in what has come to be called the clinical interview method (Labinowicz 1985). To quote Rousseau again, "Begin by studying your pupils, for assuredly you do not know them at all . . . [for] . . . the child has its own peculiar ways of seeing, of thinking, and of feeling" (Rousseau in Piaget 1971, 140). The clinical interview had the further advantage of being a method

which could be attempted by a novice researcher with some real measure of success. It did not require specific training or statistical knowledge.

My teaching experience would be a definite plus. A teacher constantly needs to be able to "think on one's feet"--to respond to the moment and use the situation. Since the students' thought processes were more important than right or wrong answers, the clinical interview method would allow me to listen as the students constructed knowledge. It would also allow me to test the construction of my own knowledge about how children construct knowledge about integers by allowing me to compare my predictions of what a child would say or do against the actual situation. It would also allow me the opportunity to ask questions at any juncture to enhance my own understanding. In short, the clinical interview seemed ideal for my purposes.

Design of the Clinical Interview

The clinical interview is an excellent choice for the Teacher as Researcher. A teacher is accustomed to planning for short-term and long-term projects. This ability to look holistically at a project requires not only the ability to articulate parameters, but also the ability to "think on one's feet" and react to the tenor and direction of the lesson--attributes necessary for a clinical interviewer.

Outlining the parameters of the clinical interviews to be held presented little difficulty, therefore. They consisted of the following:

- (1) A goal of discovering how knowledge is constructed;
- (2) A person or persons to be interviewed;
- (3) An interviewer;
- (4) A topic about which knowledge is being constructed;
- (5) A script: a series of questions designed to elicit the information desired;
- (6) Manipulatives and/or symbolic materials, if designed as part of the process;
- (7) A time and location in which to conduct the interviews;

- (8) A means of recording the information being given and any observations being made.

The Script.

One of the most critical components of the clinical interview is the script. The script is a series of questions which are designed to elicit the information you desire. In essence, these questions form the skeleton of the interview. Called an interview guide by Borg and Gall (1983), the script "lists in the desired sequence, the questions that are to be asked during the interview" (441).

I needed to have questions that were clear and concise. I didn't want these sixth graders playing a version of "what's on the teacher's mind." The questions must be general enough to encourage the child to verbalize his/her thoughts but narrow enough so that the child stays on target.

Some questions are fixed; they are asked of every subject. Other questions are impromptu--a "seize the moment" kind of question that allows you to interact with the child's thinking and explanations. The impromptu questions help to "flesh" out the skeleton, clarify something that has been said, or probe for further information.

Impromptu questions would be questions like "Can you explain that to me in another way?" or "How did you know that?." Labinowicz (1985) talks about "probing questions . . . that elicit explanations or justifications" (27). As Ginsburg et al. (1983) write, "the question may be contingent on the previous response" (11), for "the clinical interview is designed to permit a kind of naturalistic observation of unanticipated results and a flexible exploration of their meaning" (12). Spontaneous questions often take the form of "How did you know . . .?," "Why did you do or say . . .?," or "Can you tell me more about . . .?"

The scripted questions that have been developed must, of course, "be framed in language that ensures effective communication between the interviewer and the respondent" (Borg and Gall 1983, 451). Labinowicz (1985) tells us that it is "important

to establish rapport and to prepare . . . [the child] . . . for the process before beginning: and offers some suggestions for this purpose (28). He goes on to say, "When an unexpected response is given by a child, it is important to have the child elaborate on or clarify this response" (30). Labinowicz (1985) also emphasizes the importance of withholding judgment and giving encouragement when accepting a child's response. He says the interviewer then "communicates both respect for the child's thinking and a genuine curiosity to learn more" (33).

Manipulatives.

Since the purpose of the clinical interview is to explore the child's knowledge, "the adult interviewer involves the child in conversation through verbally-presented materials" (Labinowicz 1985, 26). The physical materials, or manipulatives, represent a recognition of the child as an active agent and presents the child an opportunity to interact with the environment.

Time and Location.

The time and location of the interview are not crucial to its success as long as the child is mentally alert and the location provides relative privacy and noise control. When children are completely engaged in what they are doing, they are capable of screening out irritants.

Recording.

The only attendees at a clinical interview are usually the interviewer and the interviewee; therefore, some means of recording the information collected during the interview is required. Choices range from on-the-spot note taking by the interviewer to voice recordings or to videotaping. Videotaping allows the researcher to see and hear the entire interview as frequently as wished. It also frees the interviewer from mechanical details so that complete attention can be focused on the interview process as it is happening and avoids the possibility of bias in note taking or note transcribing.

Since I was learning about the clinical interview process at the same time I was learning how children construct knowledge about integers, I videotaped the sessions. Not only would the videotape provide an audio-visual record for examination and re-examination, it would also allow me the freedom to concentrate on the children's answers to the scripted questions and to generate additional impromptu questions.

At my initial screening of the videos, I found it very difficult to get beyond myself: my performance, my interaction with the children, the way my voice sounded, my mannerisms and facial expressions, and the way I photographed.

After the initial viewings, however, I was able to concentrate on some aspect of the students: their faces as they taught the fourth graders, their hands as they worked with the Colored Chip Model, and their body language throughout. I did not realize at the time how fortuitous the decision to videotape the interviews was. Each time I view the videotapes, I am struck by some new question about or new insight into how these children constructed their knowledge about integers.

The videos also provide a benchmark for me in my role of Teacher as Researcher. I can analyze my performance in several contexts and work towards becoming more proficient as an interviewer and as a camera person. While my camera work will not win any awards, the product was sufficient for my purposes. I thought about standing behind the video camera but decided that I then would be too far away from my students to accomplish the role of interviewer. Therefore, I decided to use a stationary tripod arrangement for the video camera and to use a wide-angle focus. In this manner I hoped to capture as much of the activity as possible and yet to provide as little distraction as possible.

There were obvious drawbacks to being one's own camera person and to using a stationary arrangement. For example, a fixed camera position did not allow any "panning" to faces or hands, nor did it allow any close-up shots for emphasis. Sometimes the students moved to the right or to the left and out of the range of the camera. At other

times, the students needed to be reminded to speak loudly so that the microphone on the camera could pick up their voices. Despite the drawbacks cited, however, the decision to be my own camera person was a good one. I could concentrate on listening to the children, creating questions, and teaching the model. Also, there was no other person in the room causing a possible distraction.

Interview Content and Structure

Session One: General Introduction.

Session One was obviously vital to the success of the project. My first task was to put the students at ease with me and with the subject matter of the interviews. Only if they were at ease could I get the kind of "stream of consciousness" verbalizations I would need to "tap into" their thinking.

I again introduced myself to the students and reiterated the purpose of our sessions together, emphasizing that no grades were to be given nor would it affect their class averages in math. I asked the students to tell me something about themselves, such as age and interests.

Since I believe my first step as a teacher is to start from where my students are, I tried to get some sense of pre-knowledge the students might have. Scripted questions for that purpose were:

1. Have you ever heard of integers--positive and negative numbers?
2. Have you ever seen numbers written like this (+5 or -2) before?
3. Have you ever heard or seen numbers like these used in everyday life?

I also administered a simple four-question test, which we discussed afterwards.

This test was a written test consisting of the following four questions:

1. $+2 + +5 = \underline{\hspace{2cm}}$

2. $+2 + -5 = \underline{\hspace{2cm}}$

$$3. -2 + +5 = \underline{\hspace{2cm}}$$

$$4. -2 + -5 = \underline{\hspace{2cm}}$$

The students wrote answers without consulting other students or the interviewer. The assumption had been made that these students had never studied integers before; therefore, this was not a test of pre-knowledge. This exercise allowed an opportunity to hear a child's thinking on an unfamiliar topic.

The discussion was scripted as

4. How did you know this one?

(I would select the initial pre-test question and ask the student the scripted question.

Usually the answer was that it was just like regular addition. I then would select one of the other questions--which the child was told had been answered correctly--involving a negative number and ask the child the scripted question again for the purpose of listening to the logic which the child had constructed to answer the question.) After answers had been discussed, the script continued with the following question:

5. Have you ever been in an elevator?

The meaning of integers and real life examples were presented, as well as the concepts of relative value, absolute value, equalities, and inequalities.

My second step as a teacher is to facilitate communication by focusing on vocabulary. If our definitions differ, then we are communicating at cross purposes and misunderstandings will occur.

The next scripted question was:

6. Have you ever seen a number line before?

My assumption was that the students had been introduced to the number line, since this is a common teaching tool in the elementary grades. I had constructed the skeleton of a number line out of oak tag. Small squares of oak tag, each bearing a different integer, were also made. The students' job was to put the two together and build a number line.

This activity would present the students with a visual representation of the subsets of integers and their position on the number line.

I believe that learning takes place more quickly when the student enjoys the learning process; therefore, another activity planned was the card game of War. Most children have played the card game of War; therefore, to work from a known to an unknown, I created an oak tag deck of integer cards to evaluate the students' understanding of inequalities and equalities. The rules of War were observed: all cards were dealt, a face-down pile was made by each player, and one card from each pile was exposed. The winner had to "justify" his win verbally, using "less than or greater than" language. For example, the player whose card had the greater integer value would say "My $+6$ is greater than your 0 ; therefore, I win." Or "Your -5 is less than my $+3$; therefore, I win." (The game of War could also have been played with the smaller integer value as the win; however, I chose the greater integer value as the win for this project.) If an equality appeared, War was declared and three cards from each pile were placed face down as "spoils" or booty. The next card was exposed from each pile and to the victor went the spoils!

At the end of this session, the scripted question was:

8. I've asked you questions. Are there any questions you would like to ask me?

(I considered the first interview session a success when Paul's question was, "Can I come back next week?!")

Session Two: Colored Chip Model for Addition.

Session Two began with a review of signed numbers in two different formats. The first format was in the form of scripted questions and the second format was a game of War. The scripted questions were:

1. Can you give me an example of a positive number?
2. Can you give me an example of a negative number?

It had been a week since I had seen the students, and I wanted them to feel comfortable. Their interaction with one another would give them some feeling of comfort and would also help them to review the concepts presented in Session One. Once their body language and behavior signified that knowledge was intact, the scripted question which would set the stage for introduction of the manipulative model was asked:

3. How would you explain that $+2 + -5$ is -3 to a fourth grader?

This particular integer problem was one of the pre-test problems. After the students had attempted explanations without a great degree of success, the next scripted question was asked:

4. May I show you a model I know?

(Here was the opportunity to interact with the environment. Granted, pure constructivists would argue that this was a contrived situation; however, a teacher from time to time must employ the view of the pragmatist: if it works, use it!)

At this point I presented the Colored Chip Model for addition (see Appendix A). A variety of problems were presented: adding integers with like signs (positive and negative) and adding integers with unlike signs. Any questions asked by the students were, of course, answered immediately. Once the complete model for addition had been presented and all initial questions answered, the scripted question was:

5. Would this model make sense to a fourth grader?

The students were encouraged to talk about the model and their reactions and impressions; then, the opportunity to work with the model directly was given to the students. I had previously prepared flash cards bearing addition of integers with like signs and addition of integers with unlike signs. These cards had been arranged in the order that examples in the model had been presented.

After the students had had an opportunity to work through all the flash cards and become conversant with the model, the next scripted question was asked.

6. Could you create a story line about this problem?

One of the problems on a flash card (an addition of integers with unlike signs) was chosen as the problem. The students were encouraged to talk to one another and to create a story they could agree on. None of the students seemed to have any difficulty with this activity, although all the stories seemed to take violent turns with killer bees and aliens from outer space! Susan and Neil created the story with four spiders and seven killer bees. Spiders were positive (yellow), and killer bees were negative (blue).

NEIL: The killer bees were coming to town and looking for spiders to sting.

SUSAN: Yeah, and the four spiders couldn't find a place to hide, so

NEIL: The killer bees "got" them and stung them to death.

SUSAN: Yeah! But the spiders had fought back and killed the killer bees--so four spiders and four bees died. And--

NEIL: Three killer bees flew off to another town!

Carol and Bill preferred outer space and aliens invading the earth. When asked the initial question, this was their story for $+2 + -5 = -3$.

BILL: They had a big war.

ME: Who did, Bill?

BILL: The positives and the negatives.

CAROL: The positives live on earth, and the negatives--they're from outer space.

BILL: Yeah, and they got in a fight and two of each of them died--

CAROL: And three of them survived.

ME: Who survived?

BILL: The negatives from outer space.

In addition to the creative thinking shown in the stories, three modes of reverse processing (as discussed in Chapter II) were used during these interviews: (1) From the problem to the chips, (2) From the chips to the problem, and (3) From an answer to a problem. No matter which reverse processing mode was used, students were continually asked, "Can you do another?" to evaluate their fluency and flexibility. In this particular

session, reverse processing mode two was used: from the chips to the problem. The scripted question asked was:

7. If these are the chips, what was the problem?

(Once the question was asked, if facial expression or body language indicated that the question had not been completely understood, a further explication was: "If I had placed the chips like this (in this configuration on the table), what problem would have been written on the flash card?") The students quickly understood what they were to do and wrote down on paper what the flash-card problem would have been. This led to the ancillary question:

8. Can you do another? What if the chips looked like this? Or this?

Once it was obvious the students understood what to do and could create the "problem" without any difficulty, the next activity was presented.

A variation of War--Double War--was presented so that the students could continue to construct their knowledge of how integers operated under addition. The rules for Double War were explained. Instead of turning over just one top card, two cards would be turned over and placed in front of the player. The student must then add the integers together to arrive at an answer. Once the student had arrived at an answer, then the opponents would compare the sums with one another. The winner must verbalize the value of the sum of the integers on each player's cards and use the words "less than and/or greater than" in collecting the cards. (For example: "You have negative thirteen and I have negative four. Negative four is greater than negative thirteen; therefore, I win.") Equalities, of course, resulted in War with three additional cards being dealt face down from the top of the pack. Each player took two more cards, compared the sums, and the winner verbally declared victory.

As before, at the end of the session, the last scripted question was:

9. I've asked you questions. Are there any questions you would like to ask me?

Session Three: Addition of Integers and Reverse Processing.

By the third session the students were feeling quite comfortable with our project and arrived eager to begin work. The session began with a short game of Double War used to verify that the concepts about integers were intact. In this session, the three modes of reverse processing would be interspersed with activities focused on writing integers, solving equations from flash cards, and writing equations to solve word problems.

Although I was anxious to address the problem of "Naked Numbers" introduced in Chapter II, I wanted to do so in an appropriate manner. Because of my classroom teaching experience, I knew that the best way to counteract the many years of working with "Naked Numbers" was to "clothe" the "Naked Numbers" gradually, in stages. Perhaps because these students were not in a traditional classroom setting, they might not have the usual violent reaction to "WORD PROBLEMS"! Therefore, the body of the lesson began simply enough with the writing of integers. (Written examples had been previously prepared and were placed before the students. The examples were in the nature of "losing ten pounds," or "fifty feet above sea level," or "not winning and not losing.") The students were asked to write the integer that corresponded to the example. Once the students had completed the examples, the first scripted question was asked:

1. Why do you know this?

The students, of course, easily completed these examples and their explanations were accurate. (I always collected any written work that the students did so that I might refer to it later.) The next portion of the session was a reverse processing exercise and was introduced with a flash card.

2. If this is the problem, what would the chips look like?

As before, a series of flash cards were used (in random order). The same question was asked as each flash card was shown.

After this activity, the students were given written addition problems to solve. Although I wanted the majority of the children's experiences to be interactive, there does come a time when an individual written record of knowledge is necessary. This activity was one of those times. The students were asked to solve problems similar to those that had appeared on the flash cards. (By this time the students had correctly assigned integers to word descriptions; now they were being asked to solve problems in a familiar format as a prelude to the word problems which would come next.) Once the "Naked Number" exercises had been solved, the next reverse processing mode was introduced with the following question:

3. If these are the chips, what was the problem?

The third level of exercise was then introduced. At last the students were not dealing with "Naked Numbers." Integers were presented in real-world contexts which would enhance the knowledge that was being constructed. (An example would be: Jane joined Weight Watchers during November. During the first week she lost 3 pounds; during the second week she lost 2 pounds. Over the Thanksgiving holiday she gained back 3 pounds. This week Jane lost 4 pounds. If she weighed 133 pounds when she went to Weight Watchers, how much does she weigh now?) Problems included gains and losses on stock prices, yards gained and penalties given during football games, increases and decreases in temperatures, and other similar problems. Once these problems had been completed, the scripted question was:

4. If this is the answer, what was the problem?

The students really enjoyed the freedom that this reverse processing activity allowed and were soon busily creating quantities of problems which would generate the answer represented in chips. Patterns were recognized and used. In fact, it was the topic of "patterns" that the next scripted question addressed.

5. How would you explain this problem in writing to a fourth grader? What's my rule?

Student textbooks often use a similar approach, so "What's my rule?" was not a foreign question. (An example would be: What's my rule for these numbers: 4, 9, 19, 39? One possible rule would be to add 5, add 10, add 20, add 40, etc., doubling what is added each time. Another more general rule would be "multiply by two and add one to the preceding number. Four times two is eight; plus one equals nine.) Since one of the central ideas of mathematics is that of finding and studying patterns and the creating of generalizations based on patterns, this activity encouraged the children to engage in the work of mathematicians.

Vocabulary review of the terms "inverse" and "negative" as a preview for Session Four was done with the question:

6. What is the negative or inverse of . . . ?

A short game of Double War was played, and the session ended with the usual scripted question:

7. I've asked you questions. Are there any questions you would like to ask me?

Session Four: The Colored Chip Model for Subtraction.

Double War was the first activity in Session Four. (Since there was a week between sessions, the majority of sessions began with a review; and Session Four was no exception. Because the new concept of subtracting with integers was being introduced, the game of Double War was very short.) The first scripted question was asked as soon as the game of Double War ended:

1. How would you explain that $+5 - +7$ is -2 to a fourth grader?

Attempts were made to create an explanation, but the end result was an "I don't know."

The second scripted question was:

2. May I show you a model for subtracting with integers?

Upon receiving an affirmative answer, the Colored Chip Model for subtraction (see Appendix B) was presented. The initial subtraction problems involved positive integers

with which they were already familiar ($+5 - +2$, for example). The transition was then made to negative integers so that they could see that a problem like $-5 - -2$ worked in the same way. The students were then ready for the next question:

3. Have you ever owed anyone money?

We then talked about the concept of owing money to someone, not having the cash on hand to make repayment, and possible solutions. One of the solutions offered by the students, of course, was to get a loan from a bank. The Colored Chip Model for subtraction involving positive and negative integers was then introduced. This portion of the Colored Chip Model involves obtaining "zero sandwiches" from a bank to solve the problem (see Appendix B), and involves problems such as $+4 - -2$ or $-2 - -5$. The language for the subtraction of integers often seems confusing and contradictory to students, so time was spent on working a variety of problems. After this practice, I asked the question:

4. What do you think about subtracting integers?

This provided the students with an opportunity to voice their confusion or cite contradictory instances. Once the facial expressions and body language seemed to indicate comfort with the subtraction problems, a reverse processing activity was tried. The question was:

5. If this is what the chips looked like, what was the problem?

Can you do another?

When that went well, I asked

6. If this is the answer, what could the problem have been? Can you do another?

Once the students seemed to have no difficulty presenting a variety of problems, a change of pace was in order. A game of Double War was played for subtraction. The only change in the rules was that the top card was turned over and placed on the table; the second card was turned over and placed below the first card. The integer on the second card was then subtracted from the integer on the first card.

The last scripted question remained the same throughout all the sessions:

7. I've asked you questions. Are there any questions you would like to ask me?

Session Five: Concluding Activities

The last session began with a game of Double War to review subtraction with integers. Once it was apparent the students remembered how to subtract, a reverse processing activity was used. The question was:

1. If this is the answer, what was the subtraction problem?

To further view their understanding of integers, the next reverse processing question was:

2. (Using the same quantity of chips) If this is the answer, what was the addition problem?

The students seemed to enjoy the challenge and provided examples of addition problems. At this point, the final variation on Double War was introduced. In addition to the deck of cards, a bag of yellow and blue chips was placed between the students. Once the cards had been dealt and the piles established, then the students would take turns drawing one chip from the bag. The yellow chip signified an addition problem, and the blue chip signified a subtraction problem. Two cards were turned over from each pile, the second being placed below the first, and the operation signified by the chip would be performed.

Since one of my beliefs is that you can tell how well you know a subject by trying to teach it to someone else and since two of the important scripted questions had been "How would you explain . . .to a fourth grader?," I had arranged for two different sets of two fourth grade students to attend a portion of the last sessions for my sixth graders. My sixth grade students were a little surprised when I told them that some fourth graders would be joining them. They were even more surprised when I told them that they were to be the teachers for the fourth graders. I left the decision of "who would teach what" up to them and went to escort the fourth graders to the interview location.

After determining who would teach addition, the teaching session began. It was very interesting to listen to the sixth graders explain the Colored Chip Model, the

vocabulary of integers, and the processes of addition and subtraction. This activity certainly provided me with the opportunity to observe and evaluate the knowledge that the sixth graders had constructed about integers. It also afforded me the opportunity to hear additional questions from the fourth grade students who were also constructing knowledge. After the teaching of subtraction, the fourth graders returned to their classrooms. My question for the sixth graders was:

1. What were you thinking about before the fourth graders came in?

Bill expressed surprise that the fourth graders did so well:

BILL: Well, I thought all they really could understand since they're not really getting into math yet--I mean, I didn't think they would really understand positive and negative--like the negative number, because they're--they're into multiplication, division--and I didn't think they'd really understand.

After discussing the teaching project specifically, my next question was:

2. What did you think of this whole project?

From their answers, it was obvious that they had enjoyed working with the Colored Chip Model and learning about integers. The sixth graders were then asked:

3. Is there anything else you'd like to tell me?

The last question was the same question asked at the end of each session:

4. I've asked you questions. Are there any questions you'd like to ask me?

(Other than Paul's question in the first session about coming back next week, there were no questions at the end of the sessions.) Session Five ended with a free-writing exercise critiquing the project and my words of thanks to the students.

As with any undertaking, learning takes place. The clinical interviewing was a very positive experience for the most part; however, there are some things I would do differently another time. I now understand the importance of location from the aspects of noise and distractions. Two locations were used during the interviewing: the sixth grade students' regular classroom and the media center. The media center was very noisy with telephones, intercom announcements, bells, and people. Luckily the sixth grade students were enjoying the project and were not distracted. In fact, we must have looked as if we

were having fun because a student passing through the media center came over to the table and asked what we were doing and could he join us!

In the next chapter, Chapter V, discussion will center on what I learned from the clinical interviewing that has implications for me in my role of Teacher as Researcher, for me as a constructor of knowledge, and for me as a classroom teacher.

CHAPTER V

IMPLICATIONS OF THE TWO-TIERED MODEL FOR THE TEACHER

Chapter Overview

In an attempt to enhance my effectiveness in the classroom, learn content I had never taught before, learn how children construct knowledge about integers, and learn firsthand how the Constructivist theory of creating knowledge works, I created a two-tiered model whereby I constructed knowledge by having children construct knowledge. This chapter looks at what I have learned and the implications of the two-tiered model for me as a teacher in the following ways: (a) in my role of Teacher as Researcher, (b) in answer to the question as to how teachers can "teach a mathematics that they never learned, in ways they never experienced" (Cohen 1990, 233), and (c) in my role of classroom teacher.

Implications for My Role of Teacher as Researcher

The implications of the two-tiered model for my role of Teacher as Researcher are straightforward. The two-tiered model created an empathy between the students and the researcher because both were united in a common quest--the pursuit of knowledge. I stated my intent to the students in our very first meeting by saying that I was trying to understand how people learn about integers. The students involved were very cooperative. I learned a great deal from them and was reminded of a situation which I would like to discuss. The situation is one of communicating with the children as they are constructing knowledge. I found that sometimes language is not enough. Or, in the student's words, "If this is subtracting, why are we adding?"

Language Is Not Always Enough.

To call children's attention to the complexity of language, I often choose a generic noun, such as "sign" and tell my students to hold onto the first thought that comes into their heads when I say the generic word. They are surprised at how many different thoughts are being uncovered by just one generic noun. My purpose, I tell them, is to help them understand how much knowledge they have and how many connections or associations they have made between pieces of knowledge. If understanding is to occur and if a student's knowledge is to be "robust" (Perkins and Martin 1986, 2), then the student must know that the knowledge exists and can be called upon, that it fits or connects with the topic under discussion, and that it is complete. It is important to use a common vocabulary and to use common definitions, or to explain when a new meaning is being given to a familiar concept.

If This Is Adding, Why Am I Subtracting?.

As the students began to construct knowledge about the addition of integers, they were able to work initially in a "comfort zone." They were able to draw upon prior knowledge of what happens when two numbers are added together, as in $+5 + +2$, for example. This example proves to be no problem, and $+7$ is easily selected as the appropriate answer. And, because they understand the logic of $+5 + +2$, they are able to transfer that logic to the problem $-5 + -2$ and arrive at the answer -7 .

Such is not the case, however, when the problem looks like $-3 + +7$ or $+4 + -9$. In these instances, even though the addition sign is familiar, the student senses that the two terms cannot be joined together in the regular way. The use of the term "difference" is foreign in an addition problem and, for a short time, ambiguity may be present until the

KATHY: It's weird--cause if you're adding it--but you're not adding it, you're subtracting it.

SUSAN: How can you add more negatives than positives? When you add negatives, doesn't it take away positives?

Neil has grasped the idea of an addition that operates like a subtraction. He has answered the "how" question. Kathy agrees that this is not just plain old adding anymore, while Susan has reacted to the ambiguity created by trying to find the "why."

If This Is Subtracting, Why Am I Adding?

Ambiguity is often difficult for adults to live with, so it was not surprising that the children were really uncomfortable when they began to study the subtraction of integers. It was important that the first problems they encountered were familiar and, therefore, comfortable. Problems such as $+9 - +6$ were familiar kinds of problems even though the positive signs were used. Just as the logic of the first addition problem led to the completion of the second, so the logic of the first subtraction problem enabled the students to solve the second problem $-4 - -4$. However, when confronted with problems such as $+6 - -4$ or $-7 - +3$, the students' "take away" skills were not adequate, and language was not enough.

Paul and Kathy, in constructing knowledge about integers had encountered a situation where the problems they were solving did not follow the rules of logic which they had been using. Listen to their attempts to construct understanding.

PAUL: Negative two take away negative four. You can't do that-- in subtraction you can't do that. I keep thinking of "subtract." When I look at it, I think of subtracting. I think I have to borrow and stuff--a bigger number.

KATHY: It's strange the way a positive two and a negative three come out with positive five. So, it's like you're adding two and three without the negative and positive signs. And, when you subtract--when you add positive two and negative three, it comes out negative one. So, it gets confusing. Sometimes when you subtract positive and negative numbers,

you can get confused because sometimes you have to do something that almost seems like the opposite.

To aid in the construction of knowledge, the Teacher as Researcher needs to keep asking questions of the students and encouraging them to explain their thinking as clearly as possible to clarify where the construction process is not complete. My students understood that I was attempting to construct an understanding of how they were learning, so they were very open and cooperative.

Enhancing Classroom Effectiveness

Cohen's (1990) question, "How can teachers teach a mathematics that they never learned, in ways they never experienced?" (233), is provocative and timely. With reductions in force, a diminishing pool of teachers from which to draw, and increased emphasis on mathematics in the school curriculum, one approach has been to re-assign teachers to the mathematics area, whether that is their area of expertise or not. What does the conscientious teacher do when this happens? The conscientious teacher, of course, looks for a means of increasing effectiveness in the classroom.

Some of the different ways of achieving effectiveness in the classroom I have chosen are: taking courses at a university, participating in in-service work within the school system, finding a mentor and advisor, creating a symbiotic relationship with a peer mathematics teacher, attending workshops and seminars, and engaging in research to learn the answers to questions of content and teaching approach.

When I decided to take on the role of Teacher as Researcher, I did not fully appreciate the richness or depth of the model I had created. On one level, I was learning how children come to understand the concept of integers--what they understand easily and what they don't understand well; the kinds of real-life situations with which they can identify and those with which they may not be familiar; what questions they are likely to ask and which questions are too obscure for them to be able to ask. And, on a second level, I was creating an understanding of how a teacher can construct knowledge about

the construction of knowledge. From that knowledge should come an understanding of how best to teach the topic of integers to children in my classroom..

This two-tiered model, I am sure, could have been used just as successfully with any topic in mathematics. It affords a teacher the opportunity to observe students interacting with their physical and social environments (particularly when a manipulative model such as the Colored Chip Model is being used) and also allows the teacher access to the internal intellectual environment of the students through immediate questioning and through viewing outward performance.

This two-tiered research model can be used within or outside the regular classroom, has the advantage of being relatively easy to accomplish, and meets the needs of the teacher who wishes to enhance classroom effectiveness.

Implications for My Classroom

When I began this work, I had three questions relative to my students to which I was seeking answers. The questions were: How do children construct knowledge about integers using the Colored Chip Model? How do I assess the strength of the knowledge constructed about integers using the Colored Chip Model? What are the implications for my teaching?

How Do Children Construct Knowledge about Integers Using the Colored Chip Model?

The first question about the Colored Chip Model has to be answered as follows. Children construct knowledge successfully and easily when using the Colored Chip Model. It was evident that the students were able to answer their own questions and that they enjoyed working with the model. To those who have concerns about students being unable to perform without the model, I can say based on my research that the minute the student has constructed solid, sound knowledge, the model is put away and not used

again. In every instance, the manipulative model was used as a tool and not as a crutch. As soon as the "tool" had done its job, it was returned to the "toolbox."

One of the amazing things about the Colored Chip Model for me as a teacher was the ease with which the students were able to generalize what had been learned and, from those generalizations, create knowledge that paralleled the logic behind other models used in the teaching of integers (see Metacognition in Chapter III).

How Do I Assess the Strength of the Knowledge Constructed about Integers Using the Colored Chip Model?.

Assessing the strength of the children's knowledge was done using three techniques: reverse processing, metacognition, and "the child as teacher." These techniques are discussed in detail in Chapter III. Each technique was successfully used with the children and gave me insight into the depth and breadth of knowledge being constructed about integers.. These three evaluation techniques are valid for use within the regular classroom.

The success that I met using the technique of reverse processing (going from an answer back to reconstruction of a question) was so encouraging that I shall use the technique more frequently in my classroom. Reverse processing can be used with any mathematics topic, and the students enjoy the opportunity to show their creativity.

Students must be encouraged to share their thinking with one another and must be encouraged to gain self-awareness of their own thinking processes. Metacognition can be improved through awareness and practice, and that improvement could take place in the regular classroom.

In some classrooms, students are tutoring (re-teaching) other students who are not succeeding. This is not what I mean by the term "child as teacher." In my concept of the "child as teacher," the "student" is being introduced to a concept for the first time. This is not a re-teaching; it is an initial introduction. Although the "child as teacher" was only used briefly in this project, it is my belief that this technique can be very powerful in

terms of developing self-esteem and self-confidence. It also can clarify and solidify the knowledge and understanding of the child/teacher and underscore for the child/student the idea that knowledge is to be shared and that everyone has something to teach.

What Are the Implications for My Teaching?

The implications for my teaching fall into three general categories: the classroom environment, the teacher *per se*, and the continuing role of Teacher as Researcher.

Classroom environment. It is important that my classroom be a place for discovery, investigation, problem solving, and decision making. There must be opportunities for students to investigate how the real world works and how to communicate that information symbolically.

The classroom should be an environment in which exploration is encouraged. Answers should not be "right" or "wrong;" they should be examined in terms of their "fit" with the knowledge that the child is constructing. It is important to consider very carefully a child's "errors." Errors can give the teacher insight into the child's thinking. Did the child do the same problem as everyone else? or What question is the child answering? Labinowicz (1985) believes that the child may be giving "the right answer to a different question" (4). If the opportunity for investigation is provided, then examination of the error may provide information about a misconception the child may hold.

The classroom should be a child-centered environment. Children should be able to problem solve and talk together. As the children talk together, they test the knowledge that is being constructed and adjust that construction accordingly. Every opportunity should be given the child to interact with the physical environment supported by as many concrete materials as are applicable. The environment should exhibit, teach, and encourage good thinking skills. Critical and creative skills should be taught and reinforced, and the student's metacognitive skills should be cultivated.

The teacher *per se*. Since students must construct their own knowledge by linking existing knowledge with new knowledge, the burden of construction is on the student. However, I can assist the student in identifying which bits and pieces of existing knowledge are important. (This is "starting from where the student is.")

While the student is in the process of gaining experience and constructing "robust knowledge" (Perkins and Martin 1986, 2), questions do arise. Students may experience symptoms of "fragile knowledge" (Perkins and Martin 1986, 2). At that point, I must decide what and how much help I give the student. Will it take only a "hint" (7) for the student to continue? Will it take a "prompt" (7)? Or, does it take a "provide" (8)--almost a re-teaching. I must listen carefully and give the student only what is needed at that moment in time. To do more is to rob the students of their right to construct knowledge.

It is important for me to take the time necessary to allow the students to articulate what they understand. I must listen carefully and ask the students questions to comprehend what they are saying. I must also encourage the students to ask questions and to give examples of higher-order thinking questions such as "What if?" It is also necessary for me to be creative about the means used to view students' understandings. The old-fashioned written test is not enough.

I can also model for my students being a life-long learner. If the goal of education is to produce students who can exercise control and direction over their own lives, then through my research I can model autonomy. It is important that my students see that I am still learning, and that I am an enthusiastic student.

The teacher as researcher. A very important implication for me is that I should continue in my role of Teacher as Researcher. My classroom is rich in topics for research and every day brings to light a new misconception or misunderstanding. The research can be done on a one-to-one basis, can be done with the whole class, or with a selected group of students within the class. The topic could be one small misconception or one large question; it is the teacher's choice. The more I understand the problems and pitfalls

of learning encountered by my students, the more effective I can be in helping them to construct their own knowledge.

In the last chapter, Chapter VI, the discussion will center on another of the teacher's responsibilities--that of determining what kind of error the student is making and why this error is being made.

CHAPTER VI

IMPLICATIONS OF THE TWO-TIERED MODEL FOR THE STUDENT

Chapter Overview

When knowledge is being constructed, it is important that the knowledge be "robust" (Perkins and Martin 1986, 2). Newly constructed knowledge can sometimes be very "fragile," (Perkins and Martin 1986, 2) and fragile knowledge can lead to errors. It had not been my intention to focus on errors in my role of "Teacher as Researcher;" however, when a "significant error" occurs, it deserves attention.

"Naked Numbers" and a Misconception between "Less Than One" and "Below Zero"

What is a "significant error" you might ask? A "significant error" is an error that an individual makes that is (1) deep seated, believed with conviction, and persistent, (2) very wrong on basic issues, not just some small piece on the fringe, and (3) a challenge for even the most experienced teacher to know how to help the student. It is, in other words, a "massive misconception," such as the one held by Neil. It becomes the teacher's job to determine whether the misconception is held only by an individual or whether it is representative of a larger group of students.

Neil's Misconception.

In Session One, Neil's answer to the question "Have you ever heard of positive or negative numbers?" was surprising. His answer was

NEIL: . . . like tenths and hundredths and thousandths and--and tens, ones, hundreds, thousands, ten thousands and all that. (Susan, his partner, jumped right in at this point.)

SUSAN: A negative number is below zero, like minus 10 is a negative number; and just plain 10 is a positive number. Anything above zero is a positive number, and anything below zero is a negative number.

Since Susan had given such a clear definition of integers, I decided to continue on to see whether Neil would communicate in some way with Susan regarding her definition. We talked about how to write positive and negative numbers and practiced some examples. Both Neil and Susan read them correctly. Although only whole numbers had been used in our practice, a short while later I gave a pre-test on adding integers. Neil was working on the problem $-2 + -5$. As he wrote the answer, he put a decimal point between the sign and the number in the answer like this: $-.3$. As we talked about this pretest, Neil's dialogue went like this:

ME: What were you thinking about on this last one?

NEIL: I got minus three.

ME: Why did you choose that answer?

NEIL: Minus three is point three. Oh,---I see what I did wrong.

ME: What do you think you did wrong?

NEIL: I added--I subtracted instead of adding.

ME: If you were to correct it, what answer would you put?

NEIL: Minus seven.

ME: Why don't you correct your answer.

(Neil CORRECTS his answer by writing $-.7$ --using a sign and a decimal point.)

ME: Why did you put a point?

NEIL: Because it's after zero--a decimal. Before the zero, you don't have a decimal; but after it, you put a decimal and then put the number. And, as you go along, you put a zero and another number--like seven tenths would be point seven--whatever.

ME: Are you working with decimals in your math class right now?

NEIL: No.

ME: How long ago did you work on them?

NEIL: About three weeks ago.

In mathematics our errors can be as revealing as our successes. Neil's misconception appears to be an example of "partial knowledge" (Perkins and Martin 1986, 2) and specifically "misplaced knowledge," (14) or knowledge that is used in a situation where it is not applicable. The problem may be universal or may affect only one student.

Interpreting the Misconception.

Aha! Time to put on my detective's hat, search for clues, and find evidence that might solve the mystery of Neil's misconception. Who or what did it? Where was it done and how? When did it happen? And, more to the point, why did it happen?

The "where" question at first seemed obvious--at school. After all, most families don't sit around the dinner table discussing mathematics. But the obvious answer is not always correct. Neil's misconception could have occurred while he was home alone with his textbook--and without his teacher.

When did it happen? To answer this question, I re-examined Neil's "testimony" given during Session One of the clinical interviews. According to his own words, the misconception occurred at least "three weeks ago" since that was when his study of decimals took place.

Who was the catalyst for Neil's misconception? Could it have been the sixth-grade teacher? After cross-examining the other sixth-grade students participating in the study, I eliminated faulty teaching. Certainly if there had been faulty teaching, some of the other students would have exhibited the same misconception.

Why does Neil hold this misconception? Somewhere, somehow is it possible that an over-generalization or a faulty assumption was made about what was heard or what was said. Other possibilities exist, of course. Neil gives us a possible clue when he says,

and equated "below zero" with "after zero." In a sense, looking at a thermometer in a "top down" fashion, the negative numbers do appear "after" the zero.

Since decimals had been introduced only a short time before, Neil could be experiencing a fragile knowledge state. "Partial knowledge" (Perkins and Martin 1986, 9) could be the cause. Neil knows something about the topic but has gaps in that knowledge. In a sense, "misplaced knowledge" (Perkins and Martin 1986, 14) is occurring because Neil is trying to fit his partial knowledge about decimals in a situation where it does not belong.

Neil may have some deficit in his concept of zero. He also may not understand the difference between "less than one" and "below zero," and his place value concepts may not be intact. In the logic of place value, the place before the decimal point is "one's place." Could it be that in Neil's view, if there is a zero in "one's place," it means that there are no "one's"--there is nothing there. Zero means "nothing," so if there is a decimal point and a number such as "7" following the decimal point, as in 0.7, then the "7" is a negative number. It is a negative number because it is "less than nothing" and because negative numbers are below zero--which is nothing.

One of the difficulties a teacher faces daily is preventing misconceptions from occurring and correcting any misconceptions that can be identified. It is important that knowledge become robust. This is why teachers work diligently on their questioning skills and encourage students to ask questions. In listening carefully to students' questions, misconceptions may be identified and corrected. A teacher needs to provide many and varied examples of the subject matter since it is difficult to predict just which example will produce an "aha" experience in the child.

But what of Neil and his misconception? How can his misconception be undone? Today's students seem to understand money. To "undo" Neil's misconception then, the analogy of money was used. The analogy went something like this. Ten cents can be written as \$.10; fifty cents can be written as \$.50; seventy cents can be written as \$.70.

Seven tenths or 0.7 can be thought of as seventy cents; therefore, it can be written as \$.70. Seventy cents is less than one dollar but is more than zero. So, this decimal is between one and zero and is not less than or below zero.

Neil's misconception, however, may be caused by a much larger problem on which I would like to focus my attention. Although Neil's sixth-grade teacher was not at fault, there was both an accomplice and an accessory before, during, and after the fact. The accessory before, during, and after the fact is Neil's mathematics textbook, and the accomplice goes by the name of "Naked Numbers."

"Naked Numbers".

By "Naked Numbers," I mean numbers that are "bare" of any contextual setting-- numbers that are not "clothed" with any meaning. By "Naked Numbers," I mean the numbers of which elementary, middle, junior, and senior high school books are full-- "Naked Numbers" such as $2 + 3 = 5$, or $4x = 32$, or $3x + 2y$. A student using a textbook sees page after page of "Naked Numbers" (numbers disassociated from any meaningful context)--the only printed words being simple commands: Multiply. Simplify. Add. Evaluate. Solve. In many instances, even the examples cited as teaching examples are disassociated from any meaningful context and are confusing and frustrating for the student who seeks to understand. The HOW is shown, but a discussion of the WHY is missing.

It is easy to see why textbooks are filled with "Naked Numbers." Non-Constructivist philosophies believe that knowledge is transmitted to the student from the outside and that lots of practice is needed. Using "Naked Numbers" allows space for lots of practice material in textbooks--the old "Drill and Kill" approach to learning.

The "Naked Numbers" problem. How can one construct or integrate understanding when the only relationship the mind perceives from "Naked Numbers" is "manyness" and that it may be a perception that is incorrect? For example, $2 + 3$ always gives 5--or does it? Two dollars and three quarters do not equal five anything. Two feet

and three inches do not equal five somethings. In these and other instances of unlike units, "two plus three equals five" is a false statement.

Because the majority of textbook problems have no context associated with the numbers, a child "sees" the numbers as being of the same units. With no context accompanying a problem, the child may not apply social knowledge, as in 2 strikes + 3 balls, and realize that there is no "answer" until after the next pitch, in which case the possible answers are "1" or "0" (one base or zero bases and zero opportunities to bat again at this time). "Naked Numbers" may explain why Neil could have a misconception that confuses "less than one" and "below zero."

"Clothing" the "Naked Numbers". In the Constructivist viewpoint, knowledge is constructed from within by students interacting with their environment. It will be the student who ultimately "clothes" the "Naked Numbers" presented in the textbook. It is the teacher, however, who can guide the student in selecting the proper "wardrobe" with which to "clothe" the "Naked Numbers." The teacher can introduce students to the different "stores" of knowledge from which wardrobes can be selected. Certainly the teacher can supply the missing context and prevent the construction of faulty knowledge.

Although the teacher may make recommendations as to the wardrobe to be selected and purchased, only the student may determine the actual "fit." It is the student who must coordinate the various pieces of clothing to make a complete wardrobe, keeping those pieces of clothing that fit and tossing out those that do not. "Wrong ideas have to be modified by the child. They cannot be eliminated by the teacher" (Kamii 1985, 36).

Because I have been sensitive to the problem of "Naked Numbers," I made a concerted effort to "clothe" the "naked" integers with which the students would be working. Real-life examples of integers were discussed: thermometers, elevators, stock market, losing and gaining weight, yards gained and lost, and above, at, and below sea level, to name just a few. Students were frequently asked to "clothe" a flash-card

problem with a real world "for instance." At other times, students used their creativity to create a story to "clothe" a problem. This mixture of fact and fantasy was used in an attempt to prevent misconceptions similar to the misconception which Neil held.

In Every Ending There Is a Beginning

And so my first experience in the role of Teacher as Researcher is over. My attempt to enhance my effectiveness in the classroom by constructing knowledge through the clinical interview method about the way in which children construct knowledge about integers through the Colored Chip Model created a rich two-tiered research model. In essence, what I did for the children, they did for me.

I now view my classroom and my day-to-day experiences with my students in a new light. I am Teacher as Researcher on a daily basis as I seek answers to questions and deal with the misconceptions that are part of the daily classroom interaction between teacher and student. I am in a sense "captain of my fate." I no longer have to rely on "outside experts" to answer the questions that arise with my students in my classroom. "Researcher" is the newest hat I have added to my collection. It is a hat I would encourage any classroom teacher to try on.

In every ending, there is a beginning. I began with a question, and I have ended with a question. I wore my detective's hat to find clues that would solve the mystery of Neil's misconception that 0.7 is a negative integer, but I did not find THE answer. In the time it has taken me to complete this thesis, I have used what I have learned from my two-tiered research model in the classroom and have discovered that Neil's misconception is indeed held by other children. Again, I wonder how and why this misconception is occurring. What part do "Naked Numbers" play in this misconception? I wonder!

WORKS CITED

- Amabile, Theresa M. 1983. "The Social Psychology of Creativity: A Componential Conceptualization." Journal of Personality and Social Psychology 45 (August): 357-376.
- Belsky, Nancy Ann. 1990. "Manipulatives: Are They Worth The Time, Money, and Effort?" New England Mathematics Journal. (May): 22-27.
- Bennett, Albert B., Jr., and Gary L. Musser. 1976. "A Concrete Approach to Integer Addition and Subtraction." Arithmetic Teacher. 23 (May): 332-336.
- Borg, Walter R., and Meredith D. Gall. 1983. Educational Research: An Introduction. 4th Edition. New York: Longman.
- _____. 1989. Educational Research: An Introduction. 5th Edition. New York: Longman.
- Burton, Frederick R. 1986. "Research Currents: A Teacher's Conception of the Action Research Process." Language Arts. 63 (November): 718-723.
- Cohen, David. 1990. "Policy and Practice: An Overview." Educational Evaluation and Policy Analysis. 12(3). 233-239.
- Costa, Arthur L. 1984. "Mediating the Metacognitive." Educational Leadership. (November): 57-62.
- _____. 1991. "Teaching For, Of, and About Thinking." Developing Minds: A Resource Book for Teaching Thinking. Revised Edition. Volume 1. Alexandria, VA: Association for Supervision and Curriculum Development.
- _____, ed. 1991. Developing Minds: A Resource Book for Teaching Thinking. Revised Edition. Volume 1. Alexandria, VA: Association for Supervision and Curriculum Development.
- Costa, Arthur L. and Lawrence F. Lowery. 1989. Techniques for Teaching Thinking. The Practitioners' Guide to Teaching Thinking Series. Pacific Grove, CA: Midwest Publications.
- Davidson, Patricia S., Principal Investigator. 1983. Mathematics Learning Viewed From A Neurobiological Model for Intellectual Functioning. Final Report: Volume One. Grant Number: NIE-G-79-0089. Project Number: 9-0087. December.
- _____. 1989. Class notes presented at course "Thinking Skills in Mathematics." Critical and Creative Thinking Department. University of Massachusetts at Boston.
- Davis, Robert B. 1984. Learning Mathematics: The Cognitive Science Approach to Mathematics Education. Norwood, NJ: Ablex Publishing Corporation.
- Driscoll, Mark J. 1981. Research Within Reach: Elementary School Mathematics. CEMREL, Inc. National Institute of Education. Reston, VA: National Council of Teachers of Mathematics.

- Duckworth, Eleanor. 1986. "Teaching as Research." Harvard Educational Review. 56 (November): 481-495.
- _____. 1987. "The Having of Wonderful Ideas" And Other Essays on Teaching and Learning. New York: Teachers College Press.
- Ennis, Robert H. 1991. "Goals for a Critical Thinking Curriculum." Developing Minds: A Resource Book for Teaching Thinking, Revised Edition, Volume I. ed. A. L. Costa. Alexandria, VA: Association for Supervision and Curriculum Development.
- Fennema, Elizabeth H. 1972. "Models and Mathematics." Arithmetic Teacher. 19 (December): 635-640.
- Filby, Nikola N. 1991. An Action Research Approach to Authentic Curriculum and Instruction. Office of Education Research and Improvement. RP 91002996. San Francisco: Far West Laboratory for Education Research and Development.
- Ginsburg, Herbert P., Nancy E. Kossan, Robert Schwartz, and David Swanson. 1983. "Protocol Methods in Research on Mathematical Thinking." The Development of Mathematical Thinking. ed. Herbert P. Ginsburg. New York: Academic Press.
- Greenes, Carole. 1990. "Materials, Models, and Mathematics." New England Mathematics Journal. May : 3-11.
- Guilford, J. P. 1959. "Traits of Creativity." Creativity And Its Cultivation. ed. H. H. Anderson. New York: Harper and Row.
- Hayes, J. R. 1981. The Complete Problem Solver. Philadelphia: Franklin Institute Press.
- Kamii, Constance Kazuko. 1985. Young Children Reinvent Arithmetic. New York: Teachers College Press.
- Kennedy, Leonard M. 1986. "Manipulatives: A Rationale." Arithmetic Teacher. February: 6-7.
- Krutetskii, V. A. 1976. The Psychology of Mathematical Abilities in Schoolchildren. Translated by Joan Teller. Eds. Jeremy Kilpatrick and Izaak Wirszup. Chicago: The University of Chicago Press.
- Kutz, Eleanor. 1992. "Teacher Research: Myths and Realities." Language Arts. 69 (March): 193-197.
- Labinowicz, Ed. 1985. Learning from Children: New Beginnings for Teaching Numerical Thinking. Reading, MA: Addison-Wesley Publishing Co.
- Marzano, Robert J., Ronald S. Brandt, Carolyn Sue Hughes, Beau Fly Jones, and others, eds. 1988. Dimensions of Thinking: A Framework for Curriculum and Instruction. Alexandria, VA: Association for Supervision and Curriculum Development.

- McIntosh, Margaret E.. 1984. The Efficacy of Teacher as Researcher: A Testimonial. Opinion Paper, Meadows Excellence in Teaching Program, North Texas State University. Washington, DC: U. S. Department of Education, Office of Educational Research and Improvement.
- McTighe, James J. 1987. "Teaching For Thinking, Of Thinking, And About Thinking." Thinking Skills Instruction: Concepts and Techniques. eds. Marcia Heiman and Joshua Slomianko. Washington, DC: National Education Association of the United States.
- Moser, James M. 1986. "Curricular Issues." Arithmetic Teacher. February: 8-10.
- National Council of Teachers of Mathematics. Commission on Standards for School Mathematics. 1989. Curriculum and Evaluation Standards for School Mathematics. Reston, VA: The National Council of Teachers of Mathematics.
- Patterson, Leslie, and John C. Stansell. 1987. "Teachers and Researchers: A New Mutualism." Language Arts. 64 (November). 717-721.
- Perkins, D. N. 1986. Knowledge as Design. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.
- _____. 1991. "What Creative Thinking Is." Developing Minds: A Resource Book for Teaching Thinking. Revised Edition, Volume 1. ed., A. L. Costa. Alexandria, VA: Association for Supervision and Curriculum Development.
- Perkins, D. N. and F. Martin. 1986. "Fragile Knowledge and Neglected Strategies in Novice Programmers." In E. Soloway and S. Iyengar (eds.) Empirical Studies of Programmers. Norwood, NJ: Ablex Publishing Company.
- Piaget, Jean. 1952. The Child's Conception of Number. London: Routledge & Kegan Paul Ltd.
- _____. 1971. Psychologie et Pedagogie [Science of Education and the Psychology of the Child]. Paris: Editions Denoel, 1969. Translation © 1970 by Grosman Publishers, Inc. Reprint: New York: The Viking Press.
- _____. 1980. Adaptation and Intelligence: Organic Selection and Phenocopy. Chicago: University of Chicago Press.
- Schoenfeld, Alan H. 1989. "Teaching Mathematical Thinking and Problem Solving." In 1989 ASCD Yearbook Toward The Thinking Curriculum: Current Cognitive Research. Alexandria, VA: Association for Supervision and Curriculum Development. 83-103.
- Schön, Donald A. 1983. The Reflective Practitioner. New York: Basic Books, Inc., Publishers.
- Skemp, Richard R. 1987. The Psychology of Learning Mathematics. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.
- Swartz, Robert J., and D. N. Perkins. 1989. Teaching Thinking: Issues and Approaches. Pacific Grove, CA: Midwest Publications.

von Glasersfeld, Ernst. 1987a. "Learning as a Constructive Activity." Problems of Representation in the Teaching and Learning of Mathematics. ed., Claude Janvier. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

_____. 1987b. "Preliminaries to Any Theory of Representation." Problems of Representation in the Teaching and Learning of Mathematics. ed., Claude Janvier. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Wann, Kenneth D. 1952. "Teachers as Researchers." Educational Leadership. 9 (May): 489-495.

WORKS CONSULTED

- Adams, James L. 1986. Conceptual Blockbusting: A Guide to Better Ideas. Reading, MA: Addison-Wesley Publishing Company, Inc.
- Ashlock, Robert B., and Tommie A. West. 1967. "Physical Representations for Signed-number Operations." Arithmetic Teacher. 14 (November): 549-554.
- Baroody, Arthur J. 1989. "Manipulatives Don't Come with Guarantees." Arithmetic Teacher. 37 (October): 4-5.
- Bissex, Glenda. 1986. "On Becoming Teacher Experts: What's a Teacher-Researcher?." Language Arts. 63 (September): 482-484.
- Bruner, Jerome. 1977. The Process of Education. Cambridge, MA: Harvard University Press.
- Campbell, Sarah F., ed. 1977. Piaget Sampler: An Introduction to Jean Piaget through His Own Words. New York: Jason Aronson, Inc.
- Cohen, Louis S. 1965. "A Rationale in Working with Signed Numbers." Arithmetic Teacher. 12 (November): 563-567.
- Davis, Robert B. 1980. Discovery in Mathematics. New Rochelle, NY: Cuisenaire Company of America, Inc.
- Glesne, Corrine E. 1991. "Yet Another Role? The Teacher as Researcher." Action in Teacher Education. 13 (Spring): 7-13.
- Houser, Neil O. 1990. "Teacher-Researcher: The Synthesis of Roles for Teacher Empowerment." Action in Teacher Education. 12 (Spring): 55-60.
- Howard, Darlene V. 1983. Cognitive Psychology. New York: Macmillan Publishing Company.
- Johnson, Donovan A., and Gerald R. Rising. 1967. Guidelines for Teaching Mathematics. Belmont, CA: Wadsworth Publishing Company.
- Lehman, Barbara A. 1991. "Practicing What We Preach: A Personal Perspective on 'Knowing and Doing' in University Teacher Education Classes." Action in Teacher Education. 13 (Spring): 22-27.
- Long, Madeleine J., and Meir Ben-Hur. 1991. "Informing Learning through the Clinical Interview." Arithmetic Teacher. 38 (February): 44-46.
- McConaghy, June. 1986. "On Becoming Teacher Experts: Research as a Way of Knowing." Language Arts. 63 (November): 724-728.
- National Council of Teachers of Mathematics. Commission on Teaching Standards for School Mathematics. 1991. Professional Standards for Teaching Mathematics. Reston, VA: The National Council of Teachers of Mathematics.

Perkins, D. N. 1987. "Knowledge as Design: Teaching Thinking through Content." Teaching Thinking Skills: Theory and Practice. eds. Joan Boykoff Baron and Robert J. Sternberg. New York: W. H. Freeman and Company.

_____. 1991. "What Creative Thinking Is." Developing Minds: A Resource Book for Teaching Thinking. Revised Edition, Volume 1. ed., A. L. Costa. Alexandria, VA: Association for Supervision and Curriculum Development.

Piaget, Jean. 1973. To Understand Is to Invent. New York: Grossman Publishers.

Willoughby, Stephen S. 1990. Mathematics Education for a Changing World. Alexandria, VA: Association for Supervision and Curriculum Development.

APPENDIX A
THE COLORED CHIP MODEL
FOR ADDITION OF INTEGERS

Introduction

The set of numbers known as integers can be looked at as having three subsets: negative integers, zero, and positive integers. In the Colored Chip Model presented in this thesis, yellow chips were used to represent positive integers (based on the psychological connections of feeling positive when the weather is bright and sunny). Blue chips were used to represent negative numbers (based on the emotional reaction of feeling blue when feeling negative). These analogies helped the students keep track of which colored chips to use to represent which type of integer.

The absolute value of an integer is the number of chips in the collection representing that integer, regardless of which color they are. One way of describing the notion of absolute value is to have the students feel and count the chips representing an integer with their eyes closed. In this way, they can grasp the concept that the absolute value of $+3$ is 3, and also that the absolute value of -3 is 3.

The integer zero can be represented in two ways: either with no chips in a collection or with the same number of yellow chips and blue chips in a collection. A yellow chip ($+1$) and a blue chip (-1) cancel each other out. When a yellow chip and a blue chip are used together to represent zero, the students can stack them and call the representation of zero a "zero sandwich." When the chips are used in this manner, the set of integers can be concretely modeled.

Addition of Integers



Addition problems illustrated through the Colored Chip Model can be grouped into four types of situations: a positive integer plus a positive integer, a negative integer plus a negative integer, a positive integer plus a negative integer, and a negative integer

plus a positive integer. The Colored Chip Model very efficiently and effectively illustrates integer addition problems. In this manuscript, the key for the representation of integers with chips is as follows:

Positive integer: yellow =  Negative integer: blue = 

Positive Integer Plus Positive Integer.

Problem: $+2 + +3 =$

Step 1: 


Represent problem with colored chips.

Step 2: 





Join the two collections and form all possible "zero sandwiches" by matching blue and yellow chips. (There are none .)

Step 3: 





Remove all "zero sandwiches." (There are none to be removed.)

Step 4: $+5$

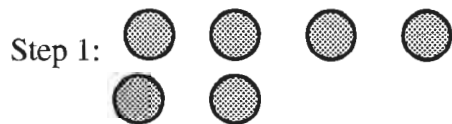
Represent answer with integer.

Step 5: $+2 + +3 = +5$

Write the completed problem.

Negative Integer Plus Negative Integer.

Problem: $-4 + -2 =$



Represent problem with colored chips.



Join the two collections and form all possible "zero sandwiches" by matching blue and yellow chips. (There are none.)



Remove all "zero sandwiches." (There are none to be removed.)

Step 4: -6

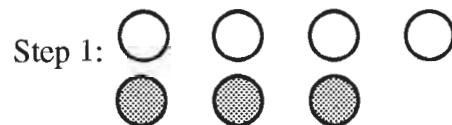
Represent answer with integer.

Step 5: $-4 + -2 = -6$

Write the completed problem.

Positive Integer Plus Negative Integer.

Problem: $+4 + -3 =$



Represent problem with colored chips.



Join the two collections and form all possible "zero sandwiches" by matching blue and yellow chips.



Remove all "zero sandwiches."

Step 4: $+1$

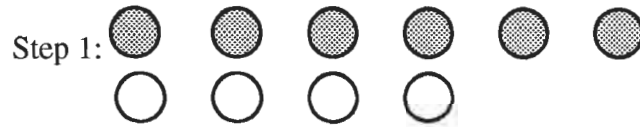
Represent answer with integer.

Step 5: $+4 + -3 = +1$

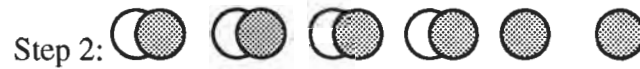
Write the completed problem.

Negative Integer Plus Positive Integer.

Problem: $-6 + +4 =$



Represent problem with colored chips.



Join the two collections and form all possible "zero sandwiches" by matching blue and yellow chips.



Remove all "zero sandwiches."

Step 4: -2

Represent answer with integer.



Step 5: $-6 + +4 = -2$

Write the completed problem.

APPENDIX B
THE COLORED CHIP MODEL
FOR SUBTRACTION OF INTEGERS

Introduction

It is traditionally the subtraction of integers that students find the most difficult. The strategy used here is to start from the known and work to the unknown. The concept of subtraction has already been constructed with whole numbers in elementary school, as "take-away," and the Colored Chip Model for integers builds on this knowledge. The key for the representation of integers with chips is as follows:

Positive integer: yellow =  Negative integer: blue = 

Subtraction of Integers with Like Signs with the First Term
a Larger Absolute Value than the Second Term

Positive Integer Minus Positive Integer.

Problem: $+6 - +4 =$

Step 1:  Represent the first term with colored chips.

Step 2:  Remove the number of colored chips represented by the second term and keep the chips that are left.

Step 3: $+2$ Represent answer with integer.

Step 4: $+6 - +4 = +2$ Write the completed problem.

Negative Integer Minus Negative Integer.

Problem: $-5 - -3 =$

Step 1: 

Represent the first term with colored chips.

Step 2: 

Remove the number of colored chips represented by the second term and keep the chips that are left.

Step 3: -2

Represent answer with integer.

Step 4: $-5 - -3 = -2$

Write the completed problem.

Subtraction of Integers with Like Signs with the Second Term a Larger Absolute Value than the First Term

It is at this point that students begin to experience difficulty with the subtraction of integers. There is no prior elementary school knowledge of subtraction that can be applied. It is at this point that other real-world knowledge can be brought to bear. The analogy of needing money but not having any can be discussed. The idea of setting up a "Bank" follows logically.

The Bank.

When the Colored Chip Model is used to teach subtraction, a "Bank" (a reserve pile of yellow and blue chips) is needed. The student may go the "Bank" to get "zero sandwiches." A "zero sandwich" has been previously defined as one yellow and one blue chip. The student may obtain as many "zero sandwiches" from the "Bank" as are needed to complete the task at hand.

Positive Integer Minus Positive Integer.

Problem: $+4 - +6 =$



Represent the first term with colored chips.



Get "zero sandwiches" to allow "taking away" the number in the second term.



Remove the chips representing the second term and keep the chips that are left.

Step 4: -2

Represent the answer with integers.

Step 5: $+4 - +6 = -2$

Write the completed problem.

Negative Integer Minus Negative Integer.

Problem: $-3 - -5 =$



Represent the first term with colored chips.



Get "zero sandwiches" to allow "taking away" the number in the second term.



Remove the chips representing the second term and keep the chips that are left.

Step 4: $+2$

Represent the answer with integers.

Step 5: $-3 - -5 = +2$

Write the completed problem.

Subtraction of Integers with Unlike Signs

Positive Integer Minus Negative Integer.

Problem: $+4 - -2 =$

Step 1: ○ ○ ○ ○

Represent the first term with colored chips.

Step 2: ○ ○ ○ ○ ○ ○

Get "zero sandwiches" to allow "taking away" the second term.

Step 3: ○ ○ ○ ○ ○ ○

Remove the chips representing the second term and keep the chips that are left. (Note that the result is $+4 + +2 = +6$.)

Step 4: $+6$ Represent the answer with integers.

Step 5: $+4 - -2 = +6$ Write the completed problem. (Note that the model shows that $+4 - -2 = +4 + +2 = +6$.)

Negative Integer Minus Positive Integer:

Problem: $-3 - +2 =$



Represent the first term with colored chips.



Get "zero sandwiches" to allow "taking away" the second term.



Remove the chips representing the second term and keep the chips that are left. (Note that the result is $-3 + -2 = -5$.)

Step 4: -5

Represent the answer with integers.

Step 5: $-3 - +2 = -5$

Write the completed problem. (Note that the model shows that $-3 - +2 = -3 + -2 = -5$.)

In general, algebra students learn the subtraction rule as "add the opposite of the integer to be subtracted." The Colored Chip Model helps students to concretize this rule.